Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.3-Tangent/101-4.3.1.2-d-sec-^m-a+b-tan-^n

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [700]. This is test number [101].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (700)	0.00 (0)
Mathematica	100.00 (700)	0.00 (0)
Fricas	81.86 (573)	18.14 (127)
Maple	81.57 (571)	18.43 (129)
Maxima	57.86 (405)	42.14 (295)
Mupad	52.71 (369)	47.29 (331)
Giac	36.86 (258)	63.14 (442)
Sympy	17.71 (124)	82.29 (576)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

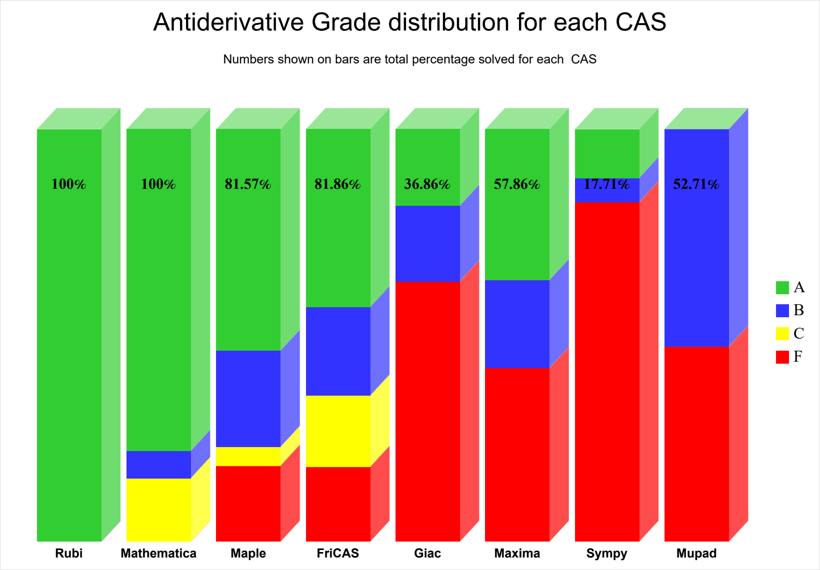
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

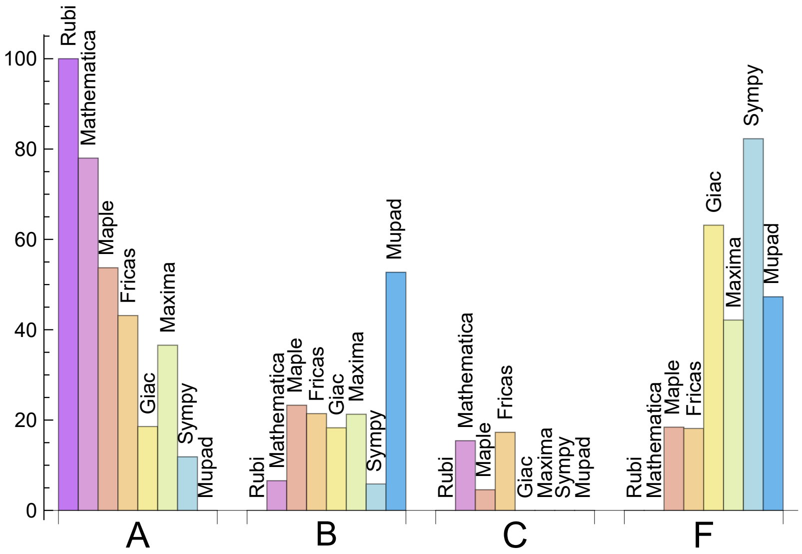
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	78.000	6.571	15.429	0.000
Maple	53.714	23.286	4.571	18.429
Fricas	43.143	21.429	17.286	18.143
Maxima	36.571	21.286	0.000	42.143
Giac	18.571	18.286	0.000	63.143
Sympy	11.857	5.857	0.000	82.286
Mupad	0.000	52.714	0.000	47.286

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	127	78.74	19.69	1.57
Maple	129	91.47	8.53	0.00
Maxima	295	62.03	5.08	32.88
Mupad	331	0.00	100.00	0.00
Giac	442	96.38	1.81	1.81
Sympy	576	73.26	26.39	0.35

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.22
Rubi	0.50
Maxima	1.06
Sympy	1.06
Giac	3.14
Mathematica	3.68
Mupad	5.49
Maple	25.28

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	139.41	1.01	114.00	1.00
Fricas	159.43	1.37	119.00	1.15
Mupad	170.09	1.72	112.00	1.36
Sympy	212.51	2.48	186.00	1.57
Mathematica	353.54	1.50	104.00	0.91
Maxima	371.27	2.40	134.00	1.22
Giac	1467.48	13.97	151.00	1.78
Maple	1500.57	4.17	137.00	1.17

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

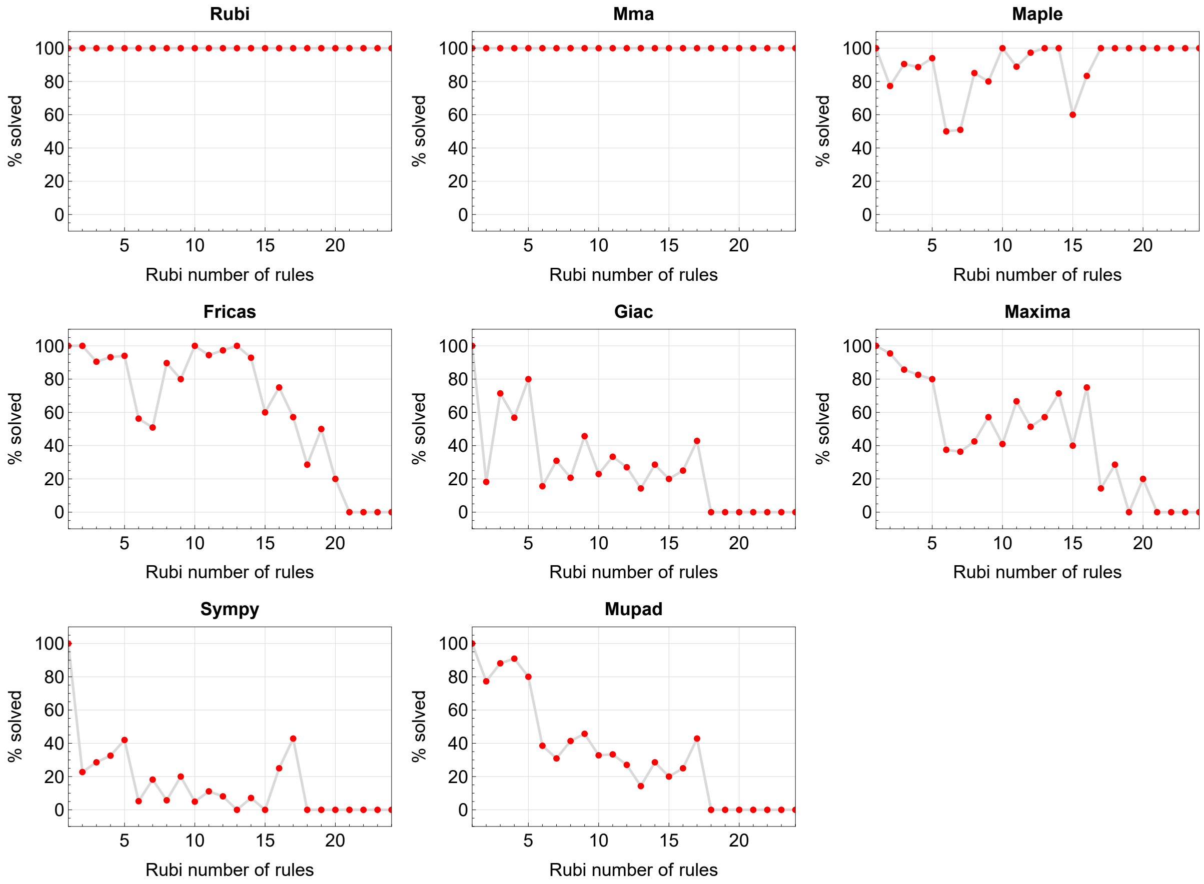

Rubi
Mma
Maple
Fricas
Giac
Maxima
Sympy
Mupad

Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

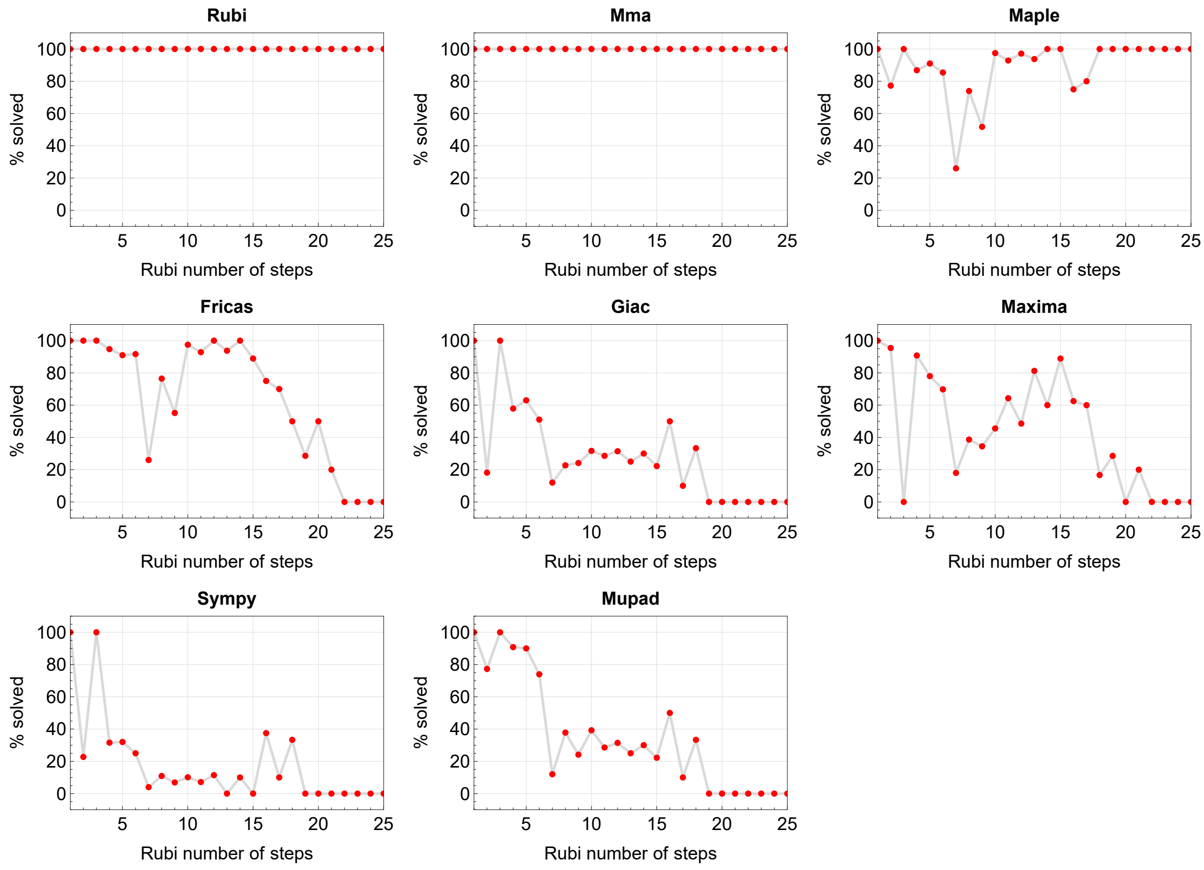


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

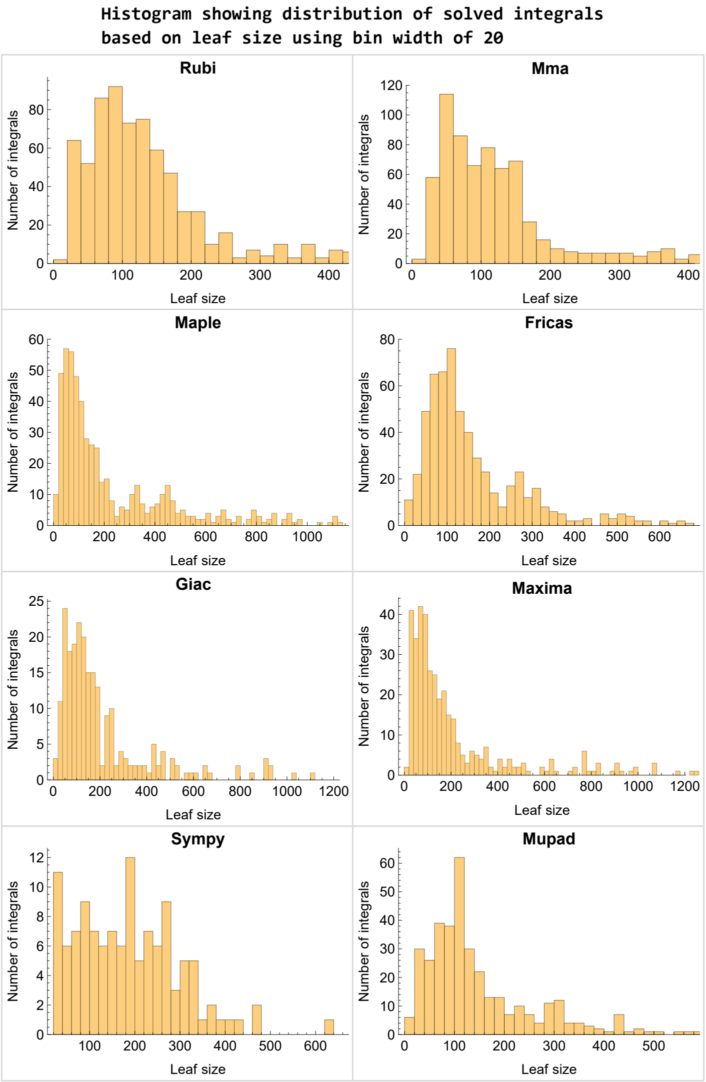
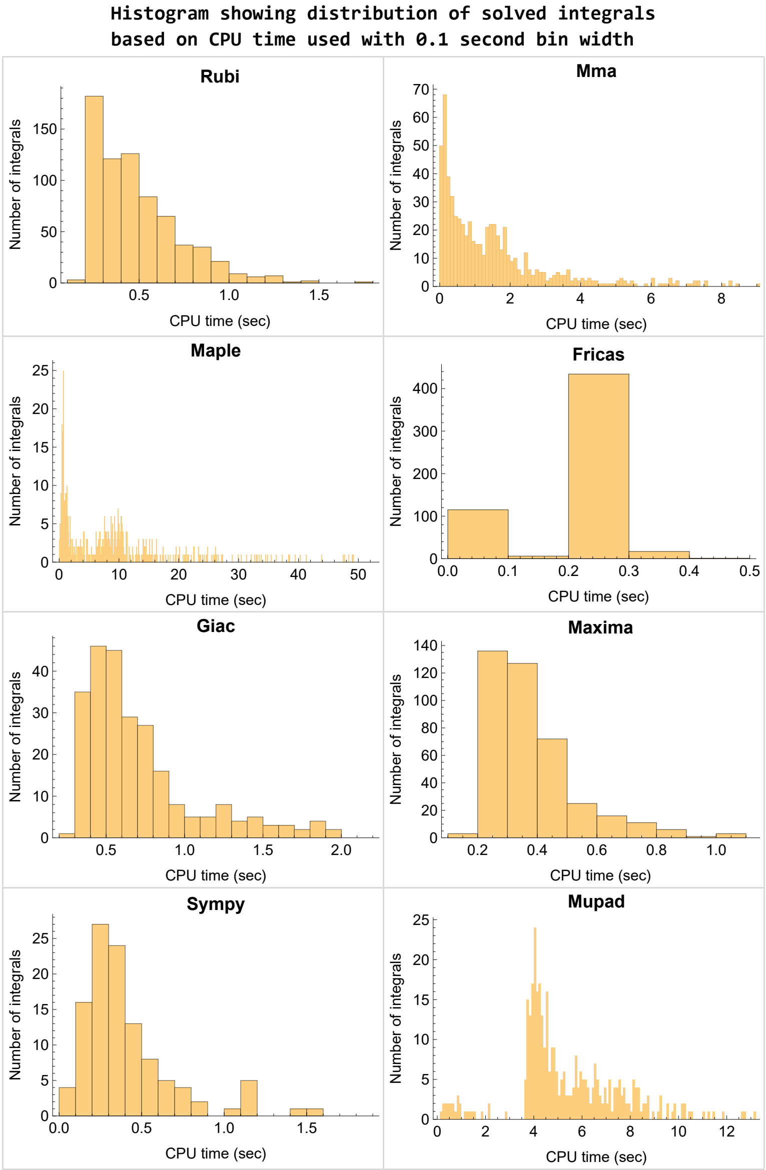


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.



Histogram showing distribution of solved integrals based on CPU time used with 0.1 second bin width

Rubi

Mma

Maple

Fricas

Giac

Maxima

Sympy

Mupad

Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

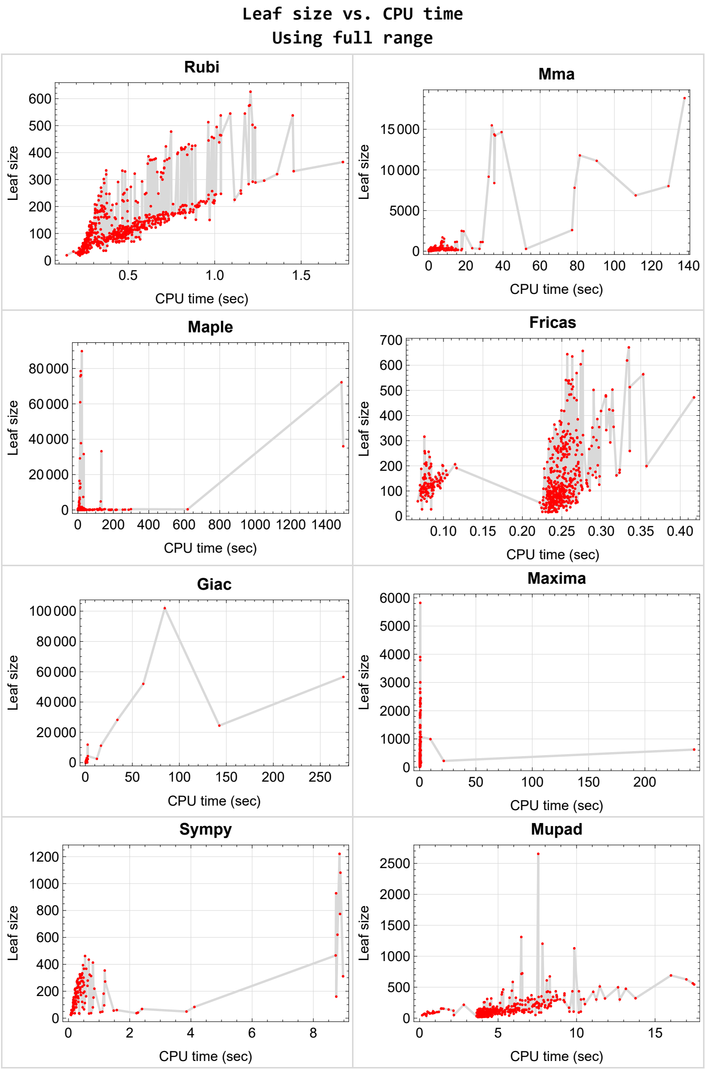


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {283, 284, 285, 297, 298, 299, 310, 311, 312, 323, 324, 325, 337, 338, 339, 351, 352, 353, 367, 368, 383, 384, 443, 444, 445, 560, 561, 562, 563, 564, 565, 572, 573, 574, 575, 576, 577, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 632, 633, 634, 635, 643, 698}

Mathematica {92, 93, 176, 177, 431, 476, 603, 605, 607, 609, 610, 612, 614, 616, 617, 619, 621, 623, 632, 633, 634, 635, 636, 637, 638, 639, 643, 644, 645, 651, 652, 653, 654, 658, 662, 664, 670, 691, 698, 699, 700}

Maple {215, 227, 373, 374, 388, 389, 403, 411, 416, 483, 484, 485, 486, 487, 504, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 686}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

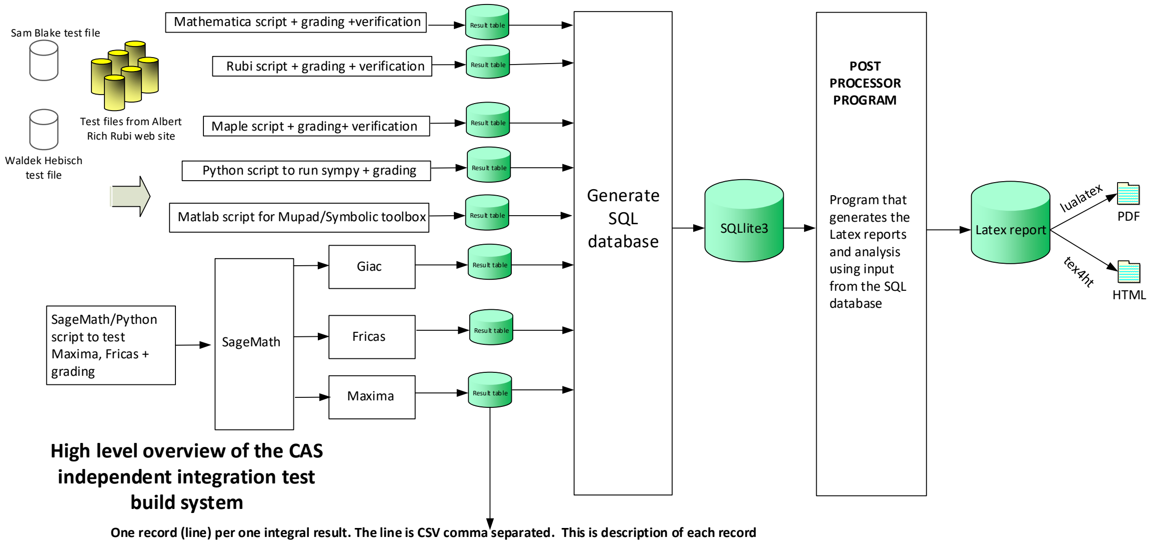
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

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2.1.8 Sympy 29

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 500, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 569, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 606, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 646, 647, 648, 649, 650, 655, 657, 661, 663, 665, 667, 669, 671, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697 }

B grade { 31, 47, 54, 55, 56, 62, 80, 88, 92, 93, 94, 95, 122, 123, 125, 150, 159, 160, 176, 177, 278, 329, 450, 497, 499, 501, 502, 503, 520, 522, 534, 535, 536, 537, 538, 539, 549, 570, 571, 573, 601, 603, 635, 637, 639, 691 }

C grade { 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 283, 284, 285, 298, 299, 311, 312, 323, 324, 325, 337, 338, 339, 351, 352, 353, 367, 368, 383, 384, 547, 560, 561, 572, 574, 604, 605, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 632, 633, 634, 636, 638, 643, 644, 645, 651, 652, 653, 654, 656, 658, 659, 660, 662, 664, 666, 668, 670, 672, 698, 699, 700 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 63, 64, 65, 66, 70, 71, 72, 73, 74, 81, 82, 83, 91, 92, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 306, 307, 308, 309, 313, 315, 319, 320, 321, 322, 327, 328, 333, 334, 335, 336, 340, 341, 342, 343, 347, 348, 349, 350, 362, 363, 364, 365, 366, 378, 379, 380, 381, 382, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 432, 433, 434, 435, 436, 467, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 575, 576, 577, 648, 655, 657, 658, 665, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687 }

B grade { 39, 58, 59, 62, 67, 68, 69, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 109, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 283, 284, 285, 290, 291, 292, 297, 298, 299, 304, 305, 310, 311, 312, 314, 316, 317, 318, 323, 324, 325, 326, 329, 330, 331, 332, 337, 338, 339, 344, 345, 346, 351, 352, 353, 358, 359, 360, 361, 367, 368, 373, 374, 375, 376, 377, 383, 384, 388, 389, 390, 391, 392, 393, 416, 424, 430, 431, 465, 466, 520, 534, 549, 560, 572, 574, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 646, 647, 656, 660, 661, 662, 663, 664, 666, 686 }

C grade { 483, 484, 485, 486, 487, 504, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 659 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 354, 355, 356, 357, 369, 370, 371, 372, 385, 386, 387 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 6, 7, 8, 9, 10, 15, 16, 17, 18, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 117, 118, 119, 120, 121, 125, 126, 127, 128, 129, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 279, 280, 281, 284, 285, 286, 287, 288, 289, 292, 293, 299, 300, 301, 302, 303, 305, 312, 313, 314, 315, 318, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 352, 353, 354, 355, 356, 361, 362, 363, 364, 365, 368, 369, 370, 376, 377, 378, 379, 380, 381, 383, 384, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 426, 427, 428, 429, 430, 431, 433, 434, 435, 436, 443, 444, 445, 446, 447, 448, 449, 483, 484, 485, 486, 491, 493, 505, 506, 507, 508, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 553, 558, 559, 564, 565, 647, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 685, 686, 687 }

B grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 67, 70, 77, 78, 79, 80, 82, 88, 107, 108, 109, 114, 115, 116, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 148, 149, 150, 166, 172, 282, 283, 290, 291, 294, 295, 296, 297, 298, 304, 306, 307, 308, 309, 310, 311, 316, 317, 319, 320, 321, 322, 323, 324, 325, 329, 337, 344, 345, 350, 351, 357, 358, 359, 360, 366, 367, 371, 372, 373, 374, 375, 382, 385, 386, 387, 388, 389, 390, 391, 396, 404, 412, 418, 425, 432, 465, 466, 467, 487, 495, 504, 512, 520, 534, 545, 546, 550, 551, 552, 554, 555, 556, 557, 560, 561, 562, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 646, 648, 684 }

C grade { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 604, 610, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 603, 606, 607, 608, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 632, 633, 634, 635, 636, 637, 638, 639 }

F(-2) exception fail { 605, 611 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62, 63, 64, 65, 68, 69, 70, 71, 72, 75, 76, 80, 81, 82, 83, 84, 89, 90, 99, 100, 101, 102, 103, 110, 114, 115, 116, 117, 118, 126, 130, 131, 132, 134, 135, 136, 143, 144, 148, 149, 151, 152, 154, 160, 161, 162, 166, 167, 168, 172, 178, 179, 180, 181, 182, 279, 280, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 306, 307, 308, 309, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 335, 336, 337, 338, 339, 347, 348, 349, 350, 351, 352, 353, 362, 363, 364, 365, 366, 367, 368, 378, 379, 380, 381, 382, 383, 384, 397, 398, 399, 405, 406, 407, 413, 414, 415, 417, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 467, 483, 484, 485, 486, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 551, 552, 554, 555, 556, 557, 558, 566, 567, 568, 569, 646, 647, 648, 673, 674, 675, 681, 682, 683, 685 }

B grade { 42, 48, 56, 61, 66, 67, 73, 74, 77, 78, 79, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 122, 123, 124, 125, 133, 140, 141, 142, 150, 153, 158, 159, 169, 170, 171, 176, 177, 288, 290, 291, 292, 300, 301, 303, 304, 313, 315, 316, 317, 327, 328, 329, 330, 333, 334, 340, 341, 342, 343, 345, 346, 354, 355, 356, 357, 358, 360, 361, 369, 370, 371, 372, 373, 374, 376, 377, 385, 386, 387, 388, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 412, 416, 418, 423, 424, 425, 430, 431, 432, 443, 444, 445, 446, 447, 448, 449, 487, 491, 493, 495, 504, 505, 549, 550, 553, 559, 560, 561, 562, 563, 564, 565, 570, 571, 572, 573, 574, 575, 576, 577, 676, 677, 678, 679, 680, 684, 686, 687 }

C grade { }

F normal fail { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 263, 264, 265, 266, 267, 268, 269, 270, 289, 302, 314, 326, 344, 359, 375, 391, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 503, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 613, 614, 615, 616, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 688, 689, 690, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 286, 287, 305, 318, 331, 332, 389, 595, 610, 612, 617, 618, 619, 620, 636 }

F(-2) exception fail { 104, 105, 106, 111, 112, 113, 119, 120, 121, 127, 128, 129, 137, 138, 139, 145, 146, 147, 155, 156, 157, 163, 164, 165, 173, 174, 175, 183, 184, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 271, 272, 273, 274, 275, 276, 277, 278, 454, 455, 456, 501, 502, 506, 609, 623, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 691, 692 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 19, 20, 21, 22, 23, 24, 26, 30, 31, 36, 37, 38, 41, 45, 46, 52, 53, 59, 60, 64, 70, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 507, 509, 511, 517, 518, 519, 531, 532, 533, 544, 545, 546, 550, 551, 552, 553, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 576 }

B grade { 11, 12, 13, 14, 15, 16, 17, 18, 25, 27, 28, 29, 32, 33, 34, 35, 39, 40, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 61, 62, 63, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 103, 104, 112, 113, 117, 129, 133, 134, 135, 136, 150, 152, 154, 166, 170, 171, 172, 179, 282, 336, 467, 508, 510, 512, 513, 514, 515, 516, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 547, 548, 549, 559, 560, 570, 571, 572, 573, 574, 575, 577 }

C grade { }

F normal fail { 185, 186, 187, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 297, 298, 299, 311, 312, 323, 324, 325 }

F(-2) exception fail { 188, 194, 204, 214, 646, 647, 648, 659 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 188, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 306, 307, 308, 309, 313, 314, 315, 316, 319, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 336, 340, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 362, 363, 364, 365, 366, 369, 370, 371, 372, 378, 379, 380, 382, 385, 386, 387, 397, 398, 399, 405, 406, 407, 412, 413, 414, 415, 418, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 446, 447, 448, 449, 465, 466, 467, 483, 484, 485, 486, 491, 493, 495, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 581, 648, 659, 673, 674, 675, 681, 682, 683 }

C grade { }

F normal fail { }

F(-1) timedout fail { 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 283, 284, 285, 290, 291, 292, 297, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 325, 330, 331, 332, 337, 338, 339, 344, 345, 346, 351, 352, 353, 358, 359, 360, 361, 367, 368, 373, 374, 375, 376, 377, 381, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 416, 417, 423, 424, 425, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 487, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 578, 579, 580, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 676, 677, 678, 679, 680, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 23, 24, 25, 26, 27, 31, 32, 40, 41, 43, 44, 47, 48, 49, 50, 54, 55, 56, 57, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 104, 105, 106, 110, 119, 120, 121, 137, 138, 139, 155, 156, 157, 163, 173, 174, 175, 183, 184, 487, 507, 509, 511, 512 }

B grade { 16, 17, 18, 33, 34, 35, 42, 51, 58, 67, 87, 88, 111, 112, 113, 118, 126, 127, 128, 129, 136, 143, 144, 145, 146, 147, 153, 154, 161, 162, 164, 165, 169, 170, 171, 172, 179, 180, 181, 182, 486 }

C grade { }

F normal fail { 11, 12, 13, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 70, 77, 78, 79, 80, 99, 100, 101, 102, 103, 107, 108, 109, 114, 115, 116, 117, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 140, 141, 142, 148, 149, 150, 151, 152, 158, 159, 160, 166, 167, 168, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 217, 218, 226, 227, 228, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 250, 251, 252, 253, 254, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 294, 295, 296, 297, 300, 301, 302, 303, 309, 314, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 380, 381, 382, 388, 389, 390, 391, 394, 395, 396, 397, 398, 402, 403, 404, 405, 417, 418, 419, 420, 421, 425, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 444, 445, 446, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 478, 479, 480, 481, 482, 483, 484, 485, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 508, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 566, 567, 568, 569, 572, 573, 574, 575, 576, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 595, 596, 597, 598, 599, 600, 604, 605, 606, 607, 608, 609, 612, 613, 614, 615, 616, 619, 620, 621, 622, 623, 624, 625, 626, 627, 629, 630, 631, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 648, 649, 650, 651, 652, 653, 658, 659, 660, 666, 667, 676, 677, 678, 682, 683, 684, 685, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 185, 199, 200, 201, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 233, 234, 235, 236, 237, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 267, 291, 292, 293, 298, 299, 304, 305, 306, 307, 308, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 346, 354, 362, 369, 370, 378, 379, 383, 384, 385, 386, 387, 392, 393, 399, 400, 401, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 422, 423, 424, 429, 430, 431, 432, 436, 437, 438, 443, 447, 448, 449, 457, 458, 470, 475, 476, 477, 558, 559, 565, 577, 578, 593, 594, 601, 602, 603, 610, 611, 617, 618, 628, 632, 636, 646, 654, 655, 656, 657, 661, 662, 663, 664, 665, 668, 669, 670, 671, 672, 673, 674, 675, 679, 680, 681, 686, 687 }

F(-2) exception fail $\{\ 570, 571\ \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	82	79	78	114	189	83	114	106
N.S.	1	0.87	0.84	0.83	1.21	2.01	0.88	1.21	1.13
time (sec)	N/A	0.280	0.312	176.202	0.280	0.236	4.114	0.432	4.023

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	68	63	67	92	153	68	92	149
N.S.	1	0.91	0.84	0.89	1.23	2.04	0.91	1.23	1.99
time (sec)	N/A	0.299	0.097	63.452	0.237	0.228	2.402	0.427	4.202

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	58	55	56	70	117	60	70	112
N.S.	1	0.94	0.89	0.90	1.13	1.89	0.97	1.13	1.81
time (sec)	N/A	0.285	0.093	18.230	0.242	0.233	1.579	0.413	3.708

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	48	81	48	48	48
N.S.	1	1.00	0.93	0.98	1.04	1.76	1.04	1.04	1.04
time (sec)	N/A	0.281	0.036	3.463	0.238	0.232	1.108	0.407	3.995

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	30	30	28	21	45	37	26	23
N.S.	1	1.11	1.11	1.04	0.78	1.67	1.37	0.96	0.85
time (sec)	N/A	0.259	0.012	0.625	0.237	0.228	0.738	0.379	3.873

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	23	17	18	24	18	17
N.S.	1	1.00	1.00	1.21	0.89	0.95	1.26	0.95	0.89
time (sec)	N/A	0.152	0.010	0.062	0.258	0.238	0.084	0.297	3.720

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	46	48	22	38	23	39	23	22
N.S.	1	1.02	1.07	0.49	0.84	0.51	0.87	0.51	0.49
time (sec)	N/A	0.255	0.036	0.624	0.332	0.240	0.090	0.359	4.170

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	46	53	61	56	136	103	64
N.S.	1	1.07	0.69	0.79	0.91	0.84	2.03	1.54	0.96
time (sec)	N/A	0.331	0.035	3.062	0.337	0.247	0.156	0.383	3.707

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	98	56	63	82	80	211	127	108
N.S.	1	1.10	0.63	0.71	0.92	0.90	2.37	1.43	1.21
time (sec)	N/A	0.405	0.045	14.070	0.340	0.235	0.210	0.415	4.036

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	124	68	73	103	104	279	151	152
N.S.	1	1.12	0.61	0.66	0.93	0.94	2.51	1.36	1.37
time (sec)	N/A	0.493	0.149	48.319	0.320	0.240	0.284	0.463	5.553

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	107	98	74	106	372	0	181	247
N.S.	1	1.09	1.00	0.76	1.08	3.80	0.00	1.85	2.52
time (sec)	N/A	0.527	0.021	36.220	0.254	0.249	0.000	0.410	8.358

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	76	64	86	276	0	139	178
N.S.	1	1.07	1.00	0.84	1.13	3.63	0.00	1.83	2.34
time (sec)	N/A	0.434	0.010	8.212	0.239	0.241	0.000	0.370	7.574

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	55	54	51	61	180	0	97	107
N.S.	1	1.02	1.00	0.94	1.13	3.33	0.00	1.80	1.98
time (sec)	N/A	0.333	0.011	1.750	0.236	0.241	0.000	0.358	5.955

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	32	82	41	52	39
N.S.	1	1.00	1.00	1.26	1.19	3.04	1.52	1.93	1.44
time (sec)	N/A	0.238	0.007	0.358	0.247	0.243	2.268	0.366	4.042

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	51	17	22	15	26	84	20
N.S.	1	1.00	1.96	0.65	0.85	0.58	1.00	3.23	0.77
time (sec)	N/A	0.238	0.034	0.349	0.256	0.233	0.074	0.361	3.947

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	42	105	196	54
N.S.	1	1.00	1.00	0.80	0.78	0.91	2.28	4.26	1.17
time (sec)	N/A	0.269	0.006	1.448	0.225	0.251	0.161	0.396	3.706

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	58	62	47	49	66	184	220	70
N.S.	1	0.94	1.00	0.76	0.79	1.06	2.97	3.55	1.13
time (sec)	N/A	0.273	0.008	6.787	0.253	0.232	0.242	0.534	5.998

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	66	76	57	58	90	253	244	93
N.S.	1	0.87	1.00	0.75	0.76	1.18	3.33	3.21	1.22
time (sec)	N/A	0.286	0.025	27.385	0.268	0.234	0.321	0.525	6.665

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	94	79	80	108	189	0	108	151
N.S.	1	0.86	0.72	0.73	0.99	1.73	0.00	0.99	1.39
time (sec)	N/A	0.291	0.544	114.000	0.227	0.228	0.000	0.555	4.316

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	63	69	95	151	0	95	132
N.S.	1	0.88	0.77	0.84	1.16	1.84	0.00	1.16	1.61
time (sec)	N/A	0.267	0.358	34.889	0.260	0.233	0.000	0.545	3.696

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	34	58	56	113	0	56	56
N.S.	1	0.91	0.62	1.05	1.02	2.05	0.00	1.02	1.02
time (sec)	N/A	0.253	0.170	8.362	0.251	0.234	0.000	0.503	3.825

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	47	21	75	0	42	35
N.S.	1	1.00	1.85	1.74	0.78	2.78	0.00	1.56	1.30
time (sec)	N/A	0.224	0.092	1.974	0.262	0.227	0.000	0.461	4.062

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	34	40	41	56	53	66	29
N.S.	1	1.00	0.89	1.05	1.08	1.47	1.39	1.74	0.76
time (sec)	N/A	0.251	0.133	0.091	0.368	0.237	0.132	0.382	3.635

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	31	19	32	17	36	17	18
N.S.	1	1.00	1.24	0.76	1.28	0.68	1.44	0.68	0.72
time (sec)	N/A	0.219	0.132	1.348	0.362	0.234	0.102	0.495	3.820

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	71	52	44	67	41	87	257	50
N.S.	1	1.13	0.83	0.70	1.06	0.65	1.38	4.08	0.79
time (sec)	N/A	0.273	0.207	6.554	0.373	0.237	0.147	0.559	3.818

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	115	99	79	92	78	185	169	88
N.S.	1	0.98	0.85	0.68	0.79	0.67	1.58	1.44	0.75
time (sec)	N/A	0.310	0.306	26.162	0.355	0.245	0.220	0.605	3.639

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	159	142	114	115	106	270	342	144
N.S.	1	0.93	0.83	0.67	0.67	0.62	1.58	2.00	0.84
time (sec)	N/A	0.336	0.362	82.579	0.377	0.247	0.275	0.677	4.951

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	121	108	133	181	364	0	237	290
N.S.	1	1.03	0.92	1.13	1.53	3.08	0.00	2.01	2.46
time (sec)	N/A	0.611	0.243	19.593	0.280	0.249	0.000	0.553	7.897

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	95	84	111	130	256	0	173	198
N.S.	1	1.01	0.89	1.18	1.38	2.72	0.00	1.84	2.11
time (sec)	N/A	0.492	0.131	3.992	0.285	0.245	0.000	0.513	7.315

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	65	56	86	83	148	0	107	104
N.S.	1	0.96	0.82	1.26	1.22	2.18	0.00	1.57	1.53
time (sec)	N/A	0.363	0.141	0.788	0.490	0.255	0.000	0.426	4.632

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	180	56	61	52	68	56	41
N.S.	1	1.00	3.91	1.22	1.33	1.13	1.48	1.22	0.89
time (sec)	N/A	0.279	0.522	0.631	0.358	0.240	0.177	0.456	3.748

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	52	38	52	34	75	531	78
N.S.	1	1.00	1.02	0.75	1.02	0.67	1.47	10.41	1.53
time (sec)	N/A	0.275	0.076	3.128	0.654	0.235	0.152	0.520	3.710

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	66	68	67	79	62	153	613	71
N.S.	1	0.96	0.99	0.97	1.14	0.90	2.22	8.88	1.03
time (sec)	N/A	0.304	0.133	14.235	0.315	0.250	0.232	0.598	5.115

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	78	85	102	98	90	238	641	256
N.S.	1	0.90	0.98	1.17	1.13	1.03	2.74	7.37	2.94
time (sec)	N/A	0.322	0.146	49.181	0.280	0.232	0.314	0.670	4.098

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	86	104	131	119	118	314	669	330
N.S.	1	0.82	0.99	1.25	1.13	1.12	2.99	6.37	3.14
time (sec)	N/A	0.323	0.315	135.736	0.271	0.237	0.387	0.721	6.135

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	94	79	91	108	215	0	108	151
N.S.	1	0.86	0.72	0.83	0.99	1.97	0.00	0.99	1.39
time (sec)	N/A	0.272	0.644	182.417	0.483	0.232	0.000	0.596	3.798

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	63	80	108	177	0	108	151
N.S.	1	0.88	0.77	0.98	1.32	2.16	0.00	1.32	1.84
time (sec)	N/A	0.264	0.367	64.761	0.392	0.239	0.000	0.539	4.170

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	34	69	82	139	0	82	114
N.S.	1	0.91	0.62	1.25	1.49	2.53	0.00	1.49	2.07
time (sec)	N/A	0.242	0.300	19.529	0.510	0.232	0.000	0.543	3.867

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	58	21	101	0	56	56
N.S.	1	1.00	1.85	2.15	0.78	3.74	0.00	2.07	2.07
time (sec)	N/A	0.216	0.153	4.088	0.267	0.233	0.000	0.489	3.847

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	67	46	51	76	97	94	118	41
N.S.	1	1.06	0.73	0.81	1.21	1.54	1.49	1.87	0.65
time (sec)	N/A	0.326	0.148	0.096	0.363	0.239	0.162	0.399	4.199

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	43	40	50	62	36	61	36	39
N.S.	1	0.88	0.82	1.02	1.27	0.73	1.24	0.73	0.80
time (sec)	N/A	0.244	0.108	2.852	0.380	0.239	0.169	0.614	3.635

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	38	57	34	80	135	36
N.S.	1	1.00	0.89	1.41	2.11	1.26	2.96	5.00	1.33
time (sec)	N/A	0.222	0.079	13.760	0.375	0.237	0.174	0.708	3.944

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	93	65	62	105	55	131	457	77
N.S.	1	1.03	0.72	0.69	1.17	0.61	1.46	5.08	0.86
time (sec)	N/A	0.284	0.245	47.766	0.461	0.234	0.202	0.814	4.152

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	137	137	97	128	92	226	514	125
N.S.	1	0.95	0.95	0.67	0.89	0.64	1.57	3.57	0.87
time (sec)	N/A	0.314	0.312	133.412	0.428	0.237	0.291	0.919	4.011

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	134	102	122	155	310	0	189	228
N.S.	1	1.06	0.80	0.96	1.22	2.44	0.00	1.49	1.80
time (sec)	N/A	0.670	1.723	8.870	0.293	0.238	0.000	0.518	7.624

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	102	93	100	109	202	0	125	136
N.S.	1	1.03	0.94	1.01	1.10	2.04	0.00	1.26	1.37
time (sec)	N/A	0.501	1.476	1.834	0.272	0.244	0.000	0.484	6.255

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	62	123	93	82	107	107	234	102
N.S.	1	1.02	2.02	1.52	1.34	1.75	1.75	3.84	1.67
time (sec)	N/A	0.370	1.564	1.460	0.291	0.253	0.195	0.604	4.196

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	19	75	17	36	901	66
N.S.	1	1.00	0.97	0.59	2.34	0.53	1.12	28.16	2.06
time (sec)	N/A	0.221	0.192	6.812	0.313	0.225	0.132	0.707	3.984

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	87	145	56	105	48	116	929	130
N.S.	1	0.99	1.65	0.64	1.19	0.55	1.32	10.56	1.48
time (sec)	N/A	0.414	0.746	26.521	0.355	0.227	0.217	0.851	4.788

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	99	180	85	123	76	190	465	134
N.S.	1	0.93	1.70	0.80	1.16	0.72	1.79	4.39	1.26
time (sec)	N/A	0.417	0.613	81.925	0.406	0.238	0.311	0.941	5.109

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	107	226	120	145	104	275	1039	330
N.S.	1	0.86	1.82	0.97	1.17	0.84	2.22	8.38	2.66
time (sec)	N/A	0.417	0.910	215.938	0.312	0.251	0.421	0.780	5.731

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	173	171	133	246	364	0	237	290
N.S.	1	1.06	1.05	0.82	1.51	2.23	0.00	1.45	1.78
time (sec)	N/A	0.834	2.055	20.175	0.278	0.241	0.000	0.598	8.388

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	139	237	111	180	256	0	173	198
N.S.	1	1.05	1.78	0.83	1.35	1.92	0.00	1.30	1.49
time (sec)	N/A	0.649	1.533	3.806	0.309	0.249	0.000	0.556	7.601

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	100	906	107	137	162	153	372	159
N.S.	1	1.03	9.34	1.10	1.41	1.67	1.58	3.84	1.64
time (sec)	N/A	0.494	6.766	3.187	0.274	0.267	0.244	0.665	6.342

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	85	246	79	121	68	109	1299	88
N.S.	1	1.09	3.15	1.01	1.55	0.87	1.40	16.65	1.13
time (sec)	N/A	0.405	0.944	13.925	0.435	0.249	0.244	0.972	4.155

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	145	38	118	34	80	915	130
N.S.	1	1.00	2.20	0.58	1.79	0.52	1.21	13.86	1.97
time (sec)	N/A	0.334	0.552	47.411	0.416	0.239	0.227	1.083	4.878

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	107	181	74	149	62	156	1327	186
N.S.	1	1.05	1.77	0.73	1.46	0.61	1.53	13.01	1.82
time (sec)	N/A	0.435	0.608	131.887	0.314	0.248	0.286	0.756	4.932

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	119	216	233	181	90	228	1409	145
N.S.	1	0.99	1.80	1.94	1.51	0.75	1.90	11.74	1.21
time (sec)	N/A	0.455	0.911	0.825	0.295	0.237	0.392	0.809	6.090

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	94	81	377	160	267	0	160	146
N.S.	1	0.86	0.74	3.46	1.47	2.45	0.00	1.47	1.34
time (sec)	N/A	0.283	0.842	1.150	0.297	0.233	0.000	0.863	4.235

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	63	102	108	229	0	108	151
N.S.	1	0.88	0.77	1.24	1.32	2.79	0.00	1.32	1.84
time (sec)	N/A	0.261	0.529	181.819	0.347	0.236	0.000	0.809	4.047

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	36	91	108	191	0	108	151
N.S.	1	0.91	0.65	1.65	1.96	3.47	0.00	1.96	2.75
time (sec)	N/A	0.246	0.382	66.858	0.335	0.237	0.000	0.777	3.893

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	72	80	21	153	0	82	114
N.S.	1	1.00	2.67	2.96	0.78	5.67	0.00	3.04	4.22
time (sec)	N/A	0.215	0.345	20.703	0.359	0.229	0.000	0.749	3.750

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	125	71	72	165	177	178	222	73
N.S.	1	1.07	0.61	0.62	1.41	1.51	1.52	1.90	0.62
time (sec)	N/A	0.547	0.179	0.151	0.459	0.248	0.249	0.399	4.399

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	78	62	85	86	125	131	146	70
N.S.	1	0.94	0.75	1.02	1.04	1.51	1.58	1.76	0.84
time (sec)	N/A	0.260	0.594	14.134	0.451	0.237	0.242	0.939	3.722

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	65	49	68	88	51	102	450	64
N.S.	1	0.89	0.67	0.93	1.21	0.70	1.40	6.16	0.88
time (sec)	N/A	0.268	0.270	48.941	0.392	0.249	0.252	0.704	4.011

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	36	38	93	34	80	187	53
N.S.	1	0.91	0.65	0.69	1.69	0.62	1.45	3.40	0.96
time (sec)	N/A	0.251	0.116	135.100	0.514	0.234	0.256	0.699	4.173

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	301	103	62	162	267	63
N.S.	1	1.00	0.89	11.15	3.81	2.30	6.00	9.89	2.33
time (sec)	N/A	0.223	0.176	1.381	0.594	0.236	0.308	0.785	3.741

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	137	123	331	164	83	209	857	122
N.S.	1	0.95	0.85	2.30	1.14	0.58	1.45	5.95	0.85
time (sec)	N/A	0.308	0.332	0.696	1.334	0.248	0.373	0.902	4.743

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	181	159	361	187	120	301	914	171
N.S.	1	0.91	0.80	1.82	0.94	0.61	1.52	4.62	0.86
time (sec)	N/A	0.347	0.639	0.788	0.430	0.260	0.492	0.883	5.556

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	176	115	122	215	310	0	189	228
N.S.	1	1.05	0.69	0.73	1.29	1.86	0.00	1.13	1.37
time (sec)	N/A	0.805	2.105	8.776	0.330	0.249	0.000	0.722	7.799

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	137	151	118	173	216	197	510	222
N.S.	1	1.05	1.16	0.91	1.33	1.66	1.52	3.92	1.71
time (sec)	N/A	0.645	1.781	7.326	0.317	0.240	0.292	0.812	8.254

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	103	130	111	154	122	148	1683	162
N.S.	1	1.05	1.33	1.13	1.57	1.24	1.51	17.17	1.65
time (sec)	N/A	0.533	1.965	26.349	0.301	0.246	0.320	1.176	6.136

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	19	152	17	36	1669	104
N.S.	1	1.00	0.97	0.59	4.75	0.53	1.12	52.16	3.25
time (sec)	N/A	0.219	0.420	81.448	0.631	0.233	0.204	0.887	4.313

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	104	122	56	187	48	121	1697	186
N.S.	1	1.03	1.21	0.55	1.85	0.48	1.20	16.80	1.84
time (sec)	N/A	0.457	0.615	212.039	0.405	0.236	0.321	0.935	4.712

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	140	146	287	217	76	192	1725	79
N.S.	1	0.99	1.04	2.04	1.54	0.54	1.36	12.23	0.56
time (sec)	N/A	0.559	0.730	0.793	0.309	0.243	0.399	0.947	4.862

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	148	170	317	246	104	265	1807	139
N.S.	1	0.93	1.07	1.99	1.55	0.65	1.67	11.36	0.87
time (sec)	N/A	0.566	1.043	0.668	0.349	0.250	0.492	1.030	5.811

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	94	56	611	186	345	0	186	153
N.S.	1	0.86	0.51	5.61	1.71	3.17	0.00	1.71	1.40
time (sec)	N/A	0.288	1.929	0.593	0.282	0.242	0.000	1.200	5.595

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	44	475	173	307	0	173	190
N.S.	1	0.88	0.54	5.79	2.11	3.74	0.00	2.11	2.32
time (sec)	N/A	0.270	1.012	0.582	0.339	0.234	0.000	1.113	5.868

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	34	124	134	269	0	134	107
N.S.	1	0.91	0.62	2.25	2.44	4.89	0.00	2.44	1.95
time (sec)	N/A	0.246	0.721	286.484	0.352	0.241	0.000	1.130	4.728

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	102	113	21	231	0	120	83
N.S.	1	1.00	3.78	4.19	0.78	8.56	0.00	4.44	3.07
time (sec)	N/A	0.228	0.667	111.631	0.302	0.238	0.000	1.064	4.383

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	212	100	103	121	297	301	378	113
N.S.	1	1.06	0.50	0.52	0.60	1.48	1.50	1.89	0.56
time (sec)	N/A	0.966	0.509	0.338	0.475	0.235	0.379	0.536	4.289

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	121	96	118	124	245	257	302	102
N.S.	1	0.91	0.72	0.89	0.93	1.84	1.93	2.27	0.77
time (sec)	N/A	0.297	0.890	84.154	0.355	0.258	0.366	0.907	3.795

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	113	86	114	135	179	216	785	111
N.S.	1	0.91	0.69	0.92	1.09	1.44	1.74	6.33	0.90
time (sec)	N/A	0.297	1.513	219.138	0.366	0.244	0.381	1.044	4.114

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	101	77	319	146	113	172	799	103
N.S.	1	0.89	0.68	2.80	1.28	0.99	1.51	7.01	0.90
time (sec)	N/A	0.290	0.916	0.839	0.341	0.243	0.406	1.094	4.221

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	42	31	451	136	17	36	381	66
N.S.	1	0.98	0.72	10.49	3.16	0.40	0.84	8.86	1.53
time (sec)	N/A	0.229	1.610	1.405	0.333	0.244	0.346	1.123	3.842

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	70	44	588	152	48	121	409	82
N.S.	1	0.88	0.55	7.35	1.90	0.60	1.51	5.11	1.02
time (sec)	N/A	0.265	0.240	1.177	0.354	0.243	0.447	1.229	4.493

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	34	639	162	76	197	437	82
N.S.	1	0.91	0.62	11.62	2.95	1.38	3.58	7.95	1.49
time (sec)	N/A	0.249	0.413	2.809	0.334	0.261	0.522	1.270	3.843

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	116	689	171	104	279	465	105
N.S.	1	1.00	4.30	25.52	6.33	3.85	10.33	17.22	3.89
time (sec)	N/A	0.220	2.932	1.402	0.343	0.276	0.630	1.310	3.898

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	203	152	739	246	125	323	1457	195
N.S.	1	0.90	0.68	3.28	1.09	0.56	1.44	6.48	0.87
time (sec)	N/A	0.359	0.976	2.592	0.375	0.297	0.693	1.529	5.673

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	247	186	789	269	162	413	1514	231
N.S.	1	0.89	0.67	2.83	0.96	0.58	1.48	5.43	0.83
time (sec)	N/A	0.406	1.170	2.284	0.393	0.319	0.799	1.629	6.040

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	248	205	151	396	378	320	924	399
N.S.	1	1.06	0.87	0.64	1.69	1.61	1.36	3.93	1.70
time (sec)	N/A	1.204	2.765	90.461	0.274	0.247	0.434	1.437	9.230

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	205	215	1540	147	352	284	277	2835	343
N.S.	1	1.05	7.51	0.72	1.72	1.39	1.35	13.83	1.67
time (sec)	N/A	1.012	8.297	253.242	0.288	0.255	0.458	1.346	8.747

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	182	1162	322	326	190	235	2849	281
N.S.	1	1.05	6.72	1.86	1.88	1.10	1.36	16.47	1.62
time (sec)	N/A	0.845	8.064	2.376	0.277	0.252	0.454	1.421	8.578

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	163	305	385	309	96	187	2863	207
N.S.	1	1.07	2.01	2.53	2.03	0.63	1.23	18.84	1.36
time (sec)	N/A	0.730	3.173	1.562	0.268	0.254	0.484	1.544	8.247

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	146	447	302	34	80	2451	37
N.S.	1	1.00	2.21	6.77	4.58	0.52	1.21	37.14	0.56
time (sec)	N/A	0.338	0.807	1.588	0.265	0.252	0.461	1.615	4.140

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	142	151	567	355	62	162	2863	65
N.S.	1	1.04	1.11	4.17	2.61	0.46	1.19	21.05	0.48
time (sec)	N/A	0.617	0.959	1.875	0.435	0.260	0.575	1.734	4.812

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	218	168	617	405	90	240	2891	93
N.S.	1	1.03	0.80	2.92	1.92	0.43	1.14	13.70	0.44
time (sec)	N/A	0.936	1.784	1.807	0.323	0.272	0.621	1.842	4.837

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	209	173	667	453	118	313	2919	222
N.S.	1	0.99	0.82	3.15	2.14	0.56	1.48	13.77	1.05
time (sec)	N/A	0.784	2.106	1.804	0.500	0.283	0.738	1.889	6.412

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	92	56	58	87	146	0	87	92
N.S.	1	0.86	0.52	0.54	0.81	1.36	0.00	0.81	0.86
time (sec)	N/A	0.275	0.269	0.451	0.283	0.234	0.000	0.405	4.347

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	70	46	47	67	109	0	67	114
N.S.	1	0.88	0.58	0.59	0.84	1.36	0.00	0.84	1.42
time (sec)	N/A	0.270	0.168	0.369	0.255	0.227	0.000	0.392	3.944

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	50	36	47	72	0	47	77
N.S.	1	0.91	0.91	0.65	0.85	1.31	0.00	0.85	1.40
time (sec)	N/A	0.256	0.098	0.348	0.299	0.230	0.000	0.379	4.516

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	27	34	23	27	33	0	27	25
N.S.	1	0.79	1.00	0.68	0.79	0.97	0.00	0.79	0.74
time (sec)	N/A	0.226	0.054	1.345	0.355	0.224	0.000	0.415	3.901

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	24	23	23	20	26	0	57	19
N.S.	1	1.04	1.00	1.00	0.87	1.13	0.00	2.48	0.83
time (sec)	N/A	0.222	0.026	0.536	0.369	0.243	0.000	0.362	4.314

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	35	26	0	32	60	58	29
N.S.	1	1.00	1.06	0.79	0.00	0.97	1.82	1.76	0.88
time (sec)	N/A	0.189	0.081	0.178	0.000	0.230	0.097	0.359	4.242

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	93	87	61	0	54	151	95	60
N.S.	1	1.13	1.06	0.74	0.00	0.66	1.84	1.16	0.73
time (sec)	N/A	0.292	0.181	0.938	0.000	0.239	0.170	0.369	4.052

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	137	131	96	0	76	219	116	123
N.S.	1	1.02	0.98	0.72	0.00	0.57	1.63	0.87	0.92
time (sec)	N/A	0.318	0.209	1.589	0.000	0.237	0.218	0.428	4.993

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	85	60	122	288	266	0	138	193
N.S.	1	1.01	0.71	1.45	3.43	3.17	0.00	1.64	2.30
time (sec)	N/A	0.460	0.819	0.714	0.273	0.251	0.000	0.377	8.363

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	59	50	100	186	174	0	99	116
N.S.	1	0.98	0.83	1.67	3.10	2.90	0.00	1.65	1.93
time (sec)	N/A	0.363	0.540	0.648	0.357	0.236	0.000	0.427	5.914

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	70	83	80	0	58	43
N.S.	1	1.00	1.10	2.26	2.68	2.58	0.00	1.87	1.39
time (sec)	N/A	0.274	0.444	1.189	0.369	0.239	0.000	0.391	4.112

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	19	29	17	34	21	25
N.S.	1	1.00	0.89	0.68	1.04	0.61	1.21	0.75	0.89
time (sec)	N/A	0.205	0.191	0.644	0.351	0.232	0.355	0.335	4.048

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	50	49	0	41	126	67	78
N.S.	1	1.00	1.06	1.04	0.00	0.87	2.68	1.43	1.66
time (sec)	N/A	0.278	0.307	0.750	0.000	0.235	0.170	0.412	3.966

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	64	72	84	0	63	196	119	134
N.S.	1	0.96	1.07	1.25	0.00	0.94	2.93	1.78	2.00
time (sec)	N/A	0.307	0.432	1.267	0.000	0.239	0.238	0.403	5.730

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	76	94	119	0	85	264	171	188
N.S.	1	0.89	1.11	1.40	0.00	1.00	3.11	2.01	2.21
time (sec)	N/A	0.319	0.606	0.585	0.000	0.246	0.338	0.428	8.209

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	44	47	77	138	0	77	93
N.S.	1	0.88	0.54	0.57	0.94	1.68	0.00	0.94	1.13
time (sec)	N/A	0.273	0.217	0.307	0.373	0.241	0.000	0.484	3.906

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	50	36	47	97	0	47	77
N.S.	1	0.91	0.91	0.65	0.85	1.76	0.00	0.85	1.40
time (sec)	N/A	0.251	0.133	0.346	0.731	0.232	0.000	0.502	3.803

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	20	35	54	0	35	33
N.S.	1	1.00	1.85	0.74	1.30	2.00	0.00	1.30	1.22
time (sec)	N/A	0.218	0.054	0.334	0.224	0.222	0.000	0.486	4.260

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	39	38	30	32	70	0	100	28
N.S.	1	1.03	1.00	0.79	0.84	1.84	0.00	2.63	0.74
time (sec)	N/A	0.240	0.048	0.663	0.233	0.244	0.000	0.468	3.746

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	25	19	19	21	17	65	30	22
N.S.	1	0.96	0.73	0.73	0.81	0.65	2.50	1.15	0.85
time (sec)	N/A	0.228	0.117	1.259	0.229	0.229	0.509	0.461	3.888

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	65	52	44	0	43	117	68	39
N.S.	1	1.07	0.85	0.72	0.00	0.70	1.92	1.11	0.64
time (sec)	N/A	0.274	0.122	0.253	0.000	0.238	0.150	0.337	4.110

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	115	97	79	0	65	189	103	71
N.S.	1	1.01	0.85	0.69	0.00	0.57	1.66	0.90	0.62
time (sec)	N/A	0.300	0.291	1.746	0.000	0.236	0.231	0.532	3.944

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	159	142	114	0	87	258	123	149
N.S.	1	0.96	0.86	0.69	0.00	0.53	1.56	0.75	0.90
time (sec)	N/A	0.332	0.306	0.747	0.000	0.240	0.299	0.508	5.252

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	130	294	133	421	326	0	203	191
N.S.	1	1.05	2.37	1.07	3.40	2.63	0.00	1.64	1.54
time (sec)	N/A	0.628	2.421	0.771	0.257	0.244	0.000	0.492	7.191

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	215	111	295	230	0	151	136
N.S.	1	1.04	2.15	1.11	2.95	2.30	0.00	1.51	1.36
time (sec)	N/A	0.498	1.456	0.668	0.247	0.245	0.000	0.492	6.668

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	146	89	167	134	0	95	104
N.S.	1	1.00	1.97	1.20	2.26	1.81	0.00	1.28	1.41
time (sec)	N/A	0.385	0.765	0.694	0.272	0.240	0.000	0.473	4.915

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	184	54	117	64	0	57	44
N.S.	1	1.00	3.83	1.12	2.44	1.33	0.00	1.19	0.92
time (sec)	N/A	0.295	0.437	0.727	0.563	0.240	0.000	0.419	4.056

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	64	38	38	45	30	112	47	79
N.S.	1	0.98	0.58	0.58	0.69	0.46	1.72	0.72	1.22
time (sec)	N/A	0.312	0.250	1.067	0.406	0.228	0.535	0.405	3.912

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	68	68	67	0	52	163	93	90
N.S.	1	0.96	0.96	0.94	0.00	0.73	2.30	1.31	1.27
time (sec)	N/A	0.306	0.495	1.127	0.000	0.235	0.237	0.446	4.847

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	80	95	102	0	74	231	145	161
N.S.	1	0.90	1.07	1.15	0.00	0.83	2.60	1.63	1.81
time (sec)	N/A	0.315	0.557	0.710	0.000	0.234	0.319	0.518	7.588

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	88	117	137	0	96	299	197	216
N.S.	1	0.82	1.09	1.28	0.00	0.90	2.79	1.84	2.02
time (sec)	N/A	0.320	0.846	0.705	0.000	0.232	0.400	0.494	6.544

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	94	56	58	87	194	0	87	119
N.S.	1	0.86	0.51	0.53	0.80	1.78	0.00	0.80	1.09
time (sec)	N/A	0.281	0.281	0.479	0.219	0.255	0.000	0.680	4.539

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	46	47	87	153	0	87	103
N.S.	1	0.88	0.56	0.57	1.06	1.87	0.00	1.06	1.26
time (sec)	N/A	0.270	0.201	0.458	0.223	0.244	0.000	0.766	4.256

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	34	36	67	112	0	67	114
N.S.	1	0.91	0.62	0.65	1.22	2.04	0.00	1.22	2.07
time (sec)	N/A	0.254	0.124	0.439	0.227	0.239	0.000	0.636	3.794

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	21	47	69	0	47	77
N.S.	1	1.00	1.85	0.78	1.74	2.56	0.00	1.74	2.85
time (sec)	N/A	0.223	0.097	0.424	0.208	0.235	0.000	0.596	4.449

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	48	68	45	113	0	128	41
N.S.	1	1.00	0.83	1.17	0.78	1.95	0.00	2.21	0.71
time (sec)	N/A	0.253	0.097	0.501	0.219	0.244	0.000	0.587	3.667

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	45	44	56	66	55	0	100	42
N.S.	1	0.90	0.88	1.12	1.32	1.10	0.00	2.00	0.84
time (sec)	N/A	0.246	0.084	0.438	0.220	0.252	0.000	0.654	3.927

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	24	21	30	153	57	20
N.S.	1	1.00	0.89	0.89	0.78	1.11	5.67	2.11	0.74
time (sec)	N/A	0.219	0.130	0.433	0.204	0.235	0.817	0.546	4.372

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	97	65	62	0	54	155	78	50
N.S.	1	1.10	0.74	0.70	0.00	0.61	1.76	0.89	0.57
time (sec)	N/A	0.354	0.161	0.296	0.000	0.236	0.172	0.386	3.999

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	137	137	97	0	76	224	115	124
N.S.	1	0.97	0.97	0.69	0.00	0.54	1.59	0.82	0.88
time (sec)	N/A	0.304	0.244	0.731	0.000	0.237	0.266	0.663	5.221

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	181	159	129	0	98	292	136	173
N.S.	1	0.93	0.82	0.66	0.00	0.50	1.50	0.70	0.89
time (sec)	N/A	0.346	0.365	0.758	0.000	0.234	0.312	0.679	5.634

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	128	113	122	341	278	0	164	150
N.S.	1	1.08	0.95	1.03	2.87	2.34	0.00	1.38	1.26
time (sec)	N/A	0.661	1.000	0.928	0.234	0.246	0.000	0.728	6.976

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	98	63	100	215	182	0	112	135
N.S.	1	1.05	0.68	1.08	2.31	1.96	0.00	1.20	1.45
time (sec)	N/A	0.522	0.828	0.853	0.317	0.236	0.000	0.616	6.066

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	70	108	86	319	112	0	110	105
N.S.	1	1.08	1.66	1.32	4.91	1.72	0.00	1.69	1.62
time (sec)	N/A	0.413	0.715	0.757	0.408	0.251	0.000	0.598	4.421

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	19	29	17	80	36	68
N.S.	1	1.00	1.00	0.59	0.91	0.53	2.50	1.12	2.12
time (sec)	N/A	0.215	0.360	0.794	0.305	0.237	0.797	0.560	3.925

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	54	56	69	41	219	73	133
N.S.	1	1.04	0.55	0.57	0.70	0.42	2.23	0.74	1.36
time (sec)	N/A	0.442	0.345	0.595	0.268	0.246	0.846	0.547	4.737

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	106	76	85	0	63	197	119	134
N.S.	1	1.05	0.75	0.84	0.00	0.62	1.95	1.18	1.33
time (sec)	N/A	0.454	0.416	0.663	0.000	0.226	0.304	0.637	6.588

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	120	98	120	0	85	265	171	188
N.S.	1	0.99	0.81	0.99	0.00	0.70	2.19	1.41	1.55
time (sec)	N/A	0.471	0.570	0.709	0.000	0.245	0.363	0.764	7.193

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	128	120	155	0	107	333	223	136
N.S.	1	0.92	0.86	1.12	0.00	0.77	2.40	1.60	0.98
time (sec)	N/A	0.467	1.070	0.715	0.000	0.230	0.450	0.699	6.768

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	72	44	47	97	168	0	97	120
N.S.	1	0.88	0.54	0.57	1.18	2.05	0.00	1.18	1.46
time (sec)	N/A	0.277	0.253	0.469	0.223	0.267	0.000	0.869	4.060

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	34	36	67	127	0	67	113
N.S.	1	0.91	0.62	0.65	1.22	2.31	0.00	1.22	2.05
time (sec)	N/A	0.255	0.166	0.456	0.250	0.243	0.000	0.799	4.040

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	58	20	55	84	0	55	93
N.S.	1	1.00	2.15	0.74	2.04	3.11	0.00	2.04	3.44
time (sec)	N/A	0.226	0.130	0.434	0.350	0.234	0.000	0.750	4.135

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	79	56	84	53	156	0	154	60
N.S.	1	0.88	0.62	0.93	0.59	1.73	0.00	1.71	0.67
time (sec)	N/A	0.271	0.113	0.529	0.284	0.251	0.000	0.727	3.761

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	59	52	69	95	102	0	146	55
N.S.	1	0.94	0.83	1.10	1.51	1.62	0.00	2.32	0.87
time (sec)	N/A	0.257	0.261	0.454	0.227	0.256	0.000	0.720	4.286

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	28	36	19	66	17	95	44	25
N.S.	1	0.97	1.24	0.66	2.28	0.59	3.28	1.52	0.86
time (sec)	N/A	0.222	0.054	0.449	0.216	0.235	1.166	0.616	3.915

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	21	41	272	85	19
N.S.	1	1.00	0.81	0.89	0.78	1.52	10.07	3.15	0.70
time (sec)	N/A	0.227	0.184	0.477	0.189	0.230	1.197	0.627	3.745

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	129	113	80	0	65	189	88	60
N.S.	1	1.11	0.97	0.69	0.00	0.56	1.63	0.76	0.52
time (sec)	N/A	0.470	0.222	0.338	0.000	0.236	0.243	0.407	4.521

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	159	142	115	0	87	258	123	90
N.S.	1	0.94	0.84	0.68	0.00	0.51	1.53	0.73	0.53
time (sec)	N/A	0.315	0.289	0.736	0.000	0.232	0.297	0.802	4.598

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	203	164	143	0	109	326	143	197
N.S.	1	0.91	0.73	0.64	0.00	0.49	1.46	0.64	0.88
time (sec)	N/A	0.352	0.502	0.651	0.000	0.240	0.370	0.808	6.070

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	143	237	111	295	230	0	151	197
N.S.	1	1.08	1.78	0.83	2.22	1.73	0.00	1.14	1.48
time (sec)	N/A	0.692	1.537	0.899	0.230	0.250	0.000	0.794	7.751

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	113	988	107	457	160	0	113	162
N.S.	1	1.06	9.23	1.00	4.27	1.50	0.00	1.06	1.51
time (sec)	N/A	0.544	6.644	0.881	0.319	0.247	0.000	0.770	6.256

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	89	247	71	141	76	0	71	88
N.S.	1	1.09	3.01	0.87	1.72	0.93	0.00	0.87	1.07
time (sec)	N/A	0.448	0.614	0.867	0.298	0.248	0.000	0.647	4.388

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	38	53	30	182	73	133
N.S.	1	1.00	0.59	0.56	0.78	0.44	2.68	1.07	1.96
time (sec)	N/A	0.336	0.384	0.816	0.214	0.235	1.148	0.644	4.411

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	140	73	74	91	52	354	99	64
N.S.	1	1.06	0.55	0.56	0.69	0.39	2.68	0.75	0.48
time (sec)	N/A	0.577	0.396	0.547	0.230	0.231	1.184	0.622	4.221

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	144	95	103	0	74	231	145	161
N.S.	1	1.07	0.71	0.77	0.00	0.55	1.72	1.08	1.20
time (sec)	N/A	0.599	0.425	0.707	0.000	0.235	0.324	0.807	8.141

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	160	117	138	0	96	299	197	216
N.S.	1	1.03	0.75	0.88	0.00	0.62	1.92	1.26	1.38
time (sec)	N/A	0.635	0.787	0.677	0.000	0.236	0.421	0.852	6.742

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	168	139	173	0	118	367	249	262
N.S.	1	0.97	0.80	0.99	0.00	0.68	2.11	1.43	1.51
time (sec)	N/A	0.628	1.235	0.752	0.000	0.240	0.504	0.777	7.931

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	121	98	124	229	273	0	250	105
N.S.	1	0.90	0.73	0.93	1.71	2.04	0.00	1.87	0.78
time (sec)	N/A	0.292	0.845	0.561	0.209	0.297	0.000	1.925	4.128

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	113	88	120	212	199	0	224	114
N.S.	1	0.90	0.70	0.95	1.68	1.58	0.00	1.78	0.90
time (sec)	N/A	0.287	0.836	0.515	0.222	0.268	0.000	1.982	3.813

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	101	79	108	191	124	0	199	104
N.S.	1	0.87	0.68	0.93	1.65	1.07	0.00	1.72	0.90
time (sec)	N/A	0.282	0.796	0.514	0.222	0.252	0.000	1.789	4.528

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	42	36	19	158	17	160	70	73
N.S.	1	0.98	0.84	0.44	3.67	0.40	3.72	1.63	1.70
time (sec)	N/A	0.228	0.053	0.497	0.229	0.242	8.749	1.574	3.969

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	69	44	50	141	41	466	137	85
N.S.	1	0.85	0.54	0.62	1.74	0.51	5.75	1.69	1.05
time (sec)	N/A	0.256	0.153	0.529	0.204	0.236	8.730	1.496	4.234

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	34	36	121	63	774	163	85
N.S.	1	0.91	0.62	0.65	2.20	1.15	14.07	2.96	1.55
time (sec)	N/A	0.247	0.125	0.457	0.204	0.236	8.873	1.309	4.345

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	21	85	1081	189	19
N.S.	1	1.00	0.81	0.89	0.78	3.15	40.04	7.00	0.70
time (sec)	N/A	0.222	0.206	0.415	0.206	0.232	8.885	1.206	4.007

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	257	152	152	0	109	325	128	198
N.S.	1	1.12	0.66	0.66	0.00	0.48	1.42	0.56	0.86
time (sec)	N/A	1.011	0.482	0.679	0.000	0.230	0.397	0.681	6.005

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	247	189	169	0	131	394	163	235
N.S.	1	0.89	0.68	0.61	0.00	0.47	1.42	0.59	0.85
time (sec)	N/A	0.381	0.813	0.643	0.000	0.234	0.475	1.280	6.250

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	291	208	197	0	153	462	184	294
N.S.	1	0.87	0.62	0.59	0.00	0.46	1.39	0.55	0.88
time (sec)	N/A	0.433	1.160	0.743	0.000	0.233	0.541	1.248	6.410

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	205	225	1704	147	786	267	0	195	344
N.S.	1	1.10	8.31	0.72	3.83	1.30	0.00	0.95	1.68
time (sec)	N/A	1.159	7.392	1.092	0.386	0.285	0.000	1.881	8.501

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	183	199	1244	143	531	182	0	165	284
N.S.	1	1.09	6.80	0.78	2.90	0.99	0.00	0.90	1.55
time (sec)	N/A	0.932	7.084	0.958	0.343	0.268	0.000	1.812	8.674

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	171	304	115	185	98	0	123	207
N.S.	1	1.10	1.95	0.74	1.19	0.63	0.00	0.79	1.33
time (sec)	N/A	0.771	1.492	0.947	0.323	0.265	0.000	1.644	8.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	38	53	30	311	125	37
N.S.	1	1.00	0.59	0.56	0.78	0.44	4.57	1.84	0.54
time (sec)	N/A	0.331	0.513	0.903	0.226	0.241	8.971	1.488	4.216

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	148	73	74	97	52	620	151	64
N.S.	1	1.07	0.53	0.54	0.70	0.38	4.49	1.09	0.46
time (sec)	N/A	0.633	0.857	0.974	0.221	0.237	8.789	1.422	4.381

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	228	95	110	141	74	928	177	159
N.S.	1	1.07	0.45	0.52	0.66	0.35	4.36	0.83	0.75
time (sec)	N/A	1.022	0.751	0.779	0.287	0.234	8.743	1.272	5.170

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	292	117	146	179	96	1221	203	224
N.S.	1	1.09	0.43	0.54	0.67	0.36	4.54	0.75	0.83
time (sec)	N/A	1.258	0.862	0.494	0.276	0.232	8.856	1.167	6.148

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	296	139	175	0	118	367	249	262
N.S.	1	1.09	0.51	0.65	0.00	0.44	1.35	0.92	0.97
time (sec)	N/A	1.345	1.314	0.732	0.000	0.233	0.579	1.256	7.471

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	320	161	210	0	140	435	301	308
N.S.	1	1.06	0.53	0.70	0.00	0.47	1.45	1.00	1.02
time (sec)	N/A	1.398	1.631	0.770	0.000	0.236	0.664	1.302	10.461

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	125	156	434	0	208	0	0	0
N.S.	1	1.02	1.27	3.53	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.587	2.223	27.132	0.000	0.084	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	95	57	163	0	154	0	0	0
N.S.	1	1.01	0.61	1.73	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.462	0.839	24.867	0.000	0.076	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	91	102	411	0	116	0	0	0
N.S.	1	1.01	1.13	4.57	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.456	1.013	3.329	0.000	0.073	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	96	0	59	0	0	40
N.S.	1	1.00	0.73	1.60	0.00	0.98	0.00	0.00	0.67
time (sec)	N/A	0.333	0.588	7.391	0.000	0.067	0.000	0.000	4.516

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	73	301	0	27	0	0	0
N.S.	1	1.00	1.22	5.02	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.330	0.699	6.644	0.000	0.073	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	62	152	0	76	0	0	0
N.S.	1	1.01	0.65	1.58	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.466	0.790	6.753	0.000	0.074	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	97	99	320	0	109	0	0	0
N.S.	1	1.01	1.03	3.33	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.464	1.129	6.186	0.000	0.075	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	133	121	179	0	118	0	0	0
N.S.	1	1.06	0.97	1.43	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.587	1.150	5.825	0.000	0.074	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	135	267	430	0	166	0	0	0
N.S.	1	0.98	1.93	3.12	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.656	3.301	14.594	0.000	0.076	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	104	67	144	0	108	0	0	0
N.S.	1	0.98	0.63	1.36	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.506	1.605	10.731	0.000	0.073	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	111	132	807	0	69	0	0	0
N.S.	1	1.04	1.23	7.54	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.508	2.182	10.343	0.000	0.076	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	156	0	82	0	0	0
N.S.	1	1.00	1.34	1.84	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.388	1.532	9.085	0.000	0.077	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	324	0	94	0	0	0
N.S.	1	1.00	1.34	3.81	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.395	1.986	13.223	0.000	0.074	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	121	133	172	0	96	0	0	0
N.S.	1	1.04	1.15	1.48	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.505	1.836	11.330	0.000	0.077	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	121	133	337	0	131	0	0	0
N.S.	1	1.04	1.15	2.91	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.499	2.563	19.859	0.000	0.078	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	157	155	190	0	142	0	0	0
N.S.	1	1.07	1.05	1.29	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.636	2.258	20.160	0.000	0.085	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	212	161	488	0	316	0	0	0
N.S.	1	1.05	0.80	2.42	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	1.021	4.837	11.294	0.000	0.076	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	182	89	184	0	258	0	0	0
N.S.	1	1.04	0.51	1.05	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.882	2.745	7.500	0.000	0.076	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	178	129	452	0	208	0	0	0
N.S.	1	1.02	0.74	2.58	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.838	3.367	16.207	0.000	0.076	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	147	79	157	0	148	0	0	0
N.S.	1	1.06	0.57	1.13	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.657	2.368	14.257	0.000	0.075	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	154	101	1114	0	123	0	0	0
N.S.	1	1.24	0.81	8.98	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.700	2.671	12.234	0.000	0.084	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	108	123	165	0	82	0	0	0
N.S.	1	0.97	1.11	1.49	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.513	1.971	12.282	0.000	0.071	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	108	108	324	0	94	0	0	0
N.S.	1	0.97	0.97	2.92	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.522	2.403	14.919	0.000	0.075	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	130	133	181	0	96	0	0	0
N.S.	1	1.05	1.07	1.46	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.602	1.867	13.451	0.000	0.074	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	130	118	339	0	108	0	0	0
N.S.	1	1.05	0.95	2.73	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.605	2.949	14.156	0.000	0.079	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	166	148	201	0	110	0	0	0
N.S.	1	1.07	0.95	1.30	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.789	2.106	20.099	0.000	0.076	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	166	145	352	0	145	0	0	0
N.S.	1	1.07	0.94	2.27	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.756	4.309	25.767	0.000	0.086	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	202	170	217	0	156	0	0	0
N.S.	1	1.09	0.91	1.17	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.897	2.588	24.652	0.000	0.090	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	221	327	480	0	250	0	0	0
N.S.	1	1.03	1.52	2.23	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	1.037	7.208	24.515	0.000	0.077	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	190	101	172	0	188	0	0	0
N.S.	1	1.04	0.55	0.94	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.877	2.935	19.377	0.000	0.072	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	178	179	123	1378	0	164	0	0	0
N.S.	1	1.01	0.69	7.74	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.822	5.107	19.089	0.000	0.082	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	150	130	179	0	129	0	0	0
N.S.	1	1.03	0.89	1.23	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.679	2.973	17.233	0.000	0.076	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	159	110	1721	0	94	0	0	0
N.S.	1	1.02	0.71	11.03	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.705	4.006	22.493	0.000	0.078	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	133	133	183	0	96	0	0	0
N.S.	1	1.06	1.06	1.46	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.569	2.711	14.523	0.000	0.079	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	133	108	339	0	108	0	0	0
N.S.	1	1.06	0.86	2.71	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.557	4.183	25.448	0.000	0.077	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	169	148	201	0	110	0	0	0
N.S.	1	1.08	0.95	1.29	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.762	2.497	22.542	0.000	0.076	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	169	121	352	0	122	0	0	0
N.S.	1	1.08	0.78	2.26	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.707	8.208	26.291	0.000	0.087	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	205	155	217	0	124	0	0	0
N.S.	1	1.10	0.83	1.16	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.916	3.189	33.738	0.000	0.084	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	135	128	443	0	205	0	0	0
N.S.	1	0.99	0.94	3.26	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.630	2.233	8.742	0.000	0.078	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	62	158	0	152	0	0	0
N.S.	1	1.00	0.59	1.50	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.503	1.579	7.632	0.000	0.074	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	410	0	123	0	0	0
N.S.	1	1.00	1.01	4.06	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.488	1.748	6.543	0.000	0.070	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	49	137	0	63	0	0	0
N.S.	1	1.00	0.70	1.96	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.384	1.358	6.628	0.000	0.072	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	70	70	74	780	0	96	0	0	0
N.S.	1	1.00	1.06	11.14	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.390	1.369	4.506	0.000	0.072	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	83	160	0	90	0	0	0
N.S.	1	1.00	1.04	2.00	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.394	1.090	6.354	0.000	0.071	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	109	440	0	107	0	0	0
N.S.	1	1.00	1.36	5.50	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.391	1.568	7.539	0.000	0.075	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	125	174	0	115	0	0	0
N.S.	1	1.02	1.10	1.53	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.521	1.611	8.862	0.000	0.083	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	134	477	0	129	0	0	0
N.S.	1	1.02	1.18	4.18	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.512	2.043	10.559	0.000	0.084	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	152	142	190	0	137	0	0	0
N.S.	1	1.05	0.98	1.31	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.650	1.842	8.916	0.000	0.089	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	188	302	473	0	256	0	0	0
N.S.	1	1.03	1.65	2.58	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.830	2.881	9.205	0.000	0.081	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	158	85	174	0	201	0	0	0
N.S.	1	1.04	0.56	1.14	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.654	1.560	8.128	0.000	0.078	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	154	123	432	0	170	0	0	0
N.S.	1	1.01	0.81	2.84	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.656	1.899	7.878	0.000	0.081	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	120	67	149	0	115	0	0	0
N.S.	1	1.01	0.56	1.25	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.521	1.384	6.863	0.000	0.073	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	116	80	431	0	100	0	0	0
N.S.	1	1.01	0.70	3.75	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.537	1.490	8.129	0.000	0.078	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	101	164	0	97	0	0	0
N.S.	1	1.00	1.12	1.82	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.402	1.407	7.510	0.000	0.073	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	102	451	0	108	0	0	0
N.S.	1	1.00	1.13	5.01	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.405	1.821	7.070	0.000	0.072	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	126	112	177	0	101	0	0	0
N.S.	1	1.09	0.97	1.53	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.528	1.480	5.155	0.000	0.070	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	126	123	474	0	118	0	0	0
N.S.	1	1.09	1.06	4.09	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.540	2.395	8.478	0.000	0.081	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	162	134	190	0	126	0	0	0
N.S.	1	1.08	0.89	1.27	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.672	1.543	8.663	0.000	0.080	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	162	149	511	0	140	0	0	0
N.S.	1	1.08	0.99	3.41	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.658	2.823	11.155	0.000	0.095	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	198	151	206	0	148	0	0	0
N.S.	1	1.09	0.83	1.14	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.825	1.904	9.955	0.000	0.091	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	188	128	454	0	213	0	0	0
N.S.	1	1.06	0.72	2.55	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.900	2.328	9.276	0.000	0.078	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	154	74	160	0	158	0	0	0
N.S.	1	1.09	0.52	1.13	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.725	1.639	8.774	0.000	0.076	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	150	93	444	0	150	0	0	0
N.S.	1	1.06	0.66	3.15	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.730	1.850	10.353	0.000	0.078	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	123	125	117	0	97	0	0	0
N.S.	1	1.06	1.08	1.01	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.583	1.448	9.267	0.000	0.074	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	123	117	473	0	114	0	0	0
N.S.	1	1.06	1.01	4.08	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.564	1.636	8.082	0.000	0.086	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	133	104	191	0	111	0	0	0
N.S.	1	1.01	0.79	1.45	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.596	1.570	6.607	0.000	0.073	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	133	140	507	0	120	0	0	0
N.S.	1	1.01	1.06	3.84	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.574	1.742	7.500	0.000	0.080	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	170	129	142	0	112	0	0	0
N.S.	1	1.12	0.85	0.93	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.719	1.378	6.304	0.000	0.075	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	170	145	528	0	129	0	0	0
N.S.	1	1.12	0.95	3.47	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.710	2.182	8.695	0.000	0.085	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	206	151	217	0	137	0	0	0
N.S.	1	1.11	0.81	1.17	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.931	1.670	9.313	0.000	0.092	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	203	124	466	0	193	0	0	0
N.S.	1	1.06	0.65	2.43	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.884	2.169	11.665	0.000	0.083	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	171	134	172	0	147	0	0	0
N.S.	1	1.09	0.85	1.10	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.733	1.559	10.102	0.000	0.075	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	169	106	482	0	114	0	0	0
N.S.	1	1.04	0.65	2.96	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.725	1.594	9.680	0.000	0.077	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	143	137	191	0	111	0	0	0
N.S.	1	1.08	1.04	1.45	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.631	1.495	7.632	0.000	0.076	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	143	149	509	0	128	0	0	0
N.S.	1	1.08	1.13	3.86	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.590	1.719	8.038	0.000	0.081	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	179	144	143	0	125	0	0	0
N.S.	1	1.10	0.88	0.88	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.711	1.614	6.852	0.000	0.086	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	179	142	541	0	132	0	0	0
N.S.	1	1.10	0.87	3.32	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.732	2.183	8.175	0.000	0.077	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	214	137	152	0	123	0	0	0
N.S.	1	1.12	0.72	0.80	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.899	1.328	6.550	0.000	0.076	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	71	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	0.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	61	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	61	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	0.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	63	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	0.237	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	111	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	0.472	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	102	0	0	0	0	0	0
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.375	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	80	103	0	0	0	0	0	0
N.S.	1	0.96	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	0.355	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	82	105	0	0	0	0	0	0
N.S.	1	0.96	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	0.514	0.000	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	84	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	1.498	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	103	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	1.345	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	112	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	1.735	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	119	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.495	1.895	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	86	128	0	0	0	0	0	0
N.S.	1	0.99	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	1.620	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	86	121	0	0	0	0	0	0
N.S.	1	0.99	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	1.410	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	141	0	0	0	0	0	0
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	2.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	143	0	0	0	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	1.822	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	70	82	76	154	0	0	474
N.S.	1	0.87	0.60	0.70	0.65	1.32	0.00	0.00	4.05
time (sec)	N/A	0.282	0.326	2.157	0.293	0.251	0.000	0.000	13.139

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	78	58	63	58	119	0	0	352
N.S.	1	0.89	0.66	0.72	0.66	1.35	0.00	0.00	4.00
time (sec)	N/A	0.275	0.203	1.201	0.311	0.249	0.000	0.000	7.763

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	48	44	40	84	0	0	230
N.S.	1	0.92	0.81	0.75	0.68	1.42	0.00	0.00	3.90
time (sec)	N/A	0.256	0.136	1.040	0.273	0.253	0.000	0.000	7.365

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	46	0	55	82
N.S.	1	1.00	1.00	0.83	0.72	1.59	0.00	1.90	2.83
time (sec)	N/A	0.239	0.070	0.661	0.222	0.240	0.000	0.522	0.641

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	112	51	384	122	253	0	0	0
N.S.	1	0.93	0.42	3.20	1.02	2.11	0.00	0.00	0.00
time (sec)	N/A	0.286	0.105	38.564	0.307	0.245	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	191	53	411	176	275	0	0	0
N.S.	1	0.99	0.27	2.13	0.91	1.42	0.00	0.00	0.00
time (sec)	N/A	0.323	0.127	109.911	0.341	0.252	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	270	53	438	230	297	0	0	0
N.S.	1	1.02	0.20	1.65	0.86	1.12	0.00	0.00	0.00
time (sec)	N/A	0.353	0.235	108.030	0.303	0.249	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	155	95	116	0	132	0	0	289
N.S.	1	1.05	0.65	0.79	0.00	0.90	0.00	0.00	1.97
time (sec)	N/A	0.707	0.976	8.633	0.000	0.281	0.000	0.000	9.182

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	114	77	89	0	97	0	0	102
N.S.	1	1.04	0.70	0.81	0.00	0.88	0.00	0.00	0.93
time (sec)	N/A	0.532	0.674	7.643	0.000	0.253	0.000	0.000	6.945

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	62	222	62	0	0	88
N.S.	1	1.00	0.86	0.85	3.04	0.85	0.00	0.00	1.21
time (sec)	N/A	0.375	0.526	7.947	21.457	0.236	0.000	0.000	6.646

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	37	0	25	0	0	61
N.S.	1	1.00	1.26	1.19	0.00	0.81	0.00	0.00	1.97
time (sec)	N/A	0.207	0.289	5.633	0.000	0.234	0.000	0.000	0.379

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	87	364	774	184	0	0	0
N.S.	1	1.00	1.05	4.39	9.33	2.22	0.00	0.00	0.00
time (sec)	N/A	0.358	0.684	17.334	0.413	0.254	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	163	126	394	935	245	0	0	0
N.S.	1	1.06	0.82	2.56	6.07	1.59	0.00	0.00	0.00
time (sec)	N/A	0.672	0.718	23.321	0.477	0.275	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	245	152	421	2220	267	0	0	0
N.S.	1	1.10	0.68	1.89	9.96	1.20	0.00	0.00	0.00
time (sec)	N/A	1.035	0.981	22.484	0.616	0.260	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	170	0	0	544
N.S.	1	0.87	0.62	0.70	0.65	1.45	0.00	0.00	4.65
time (sec)	N/A	0.274	0.429	1.248	0.243	0.283	0.000	0.000	17.489

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	78	61	63	58	134	0	0	420
N.S.	1	0.89	0.69	0.72	0.66	1.52	0.00	0.00	4.77
time (sec)	N/A	0.263	0.215	1.222	0.249	0.262	0.000	0.000	8.311

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	51	44	40	98	0	0	296
N.S.	1	0.92	0.86	0.75	0.68	1.66	0.00	0.00	5.02
time (sec)	N/A	0.267	0.160	1.230	0.237	0.249	0.000	0.000	7.041

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	59	0	0	153
N.S.	1	1.00	1.00	0.83	0.72	2.03	0.00	0.00	5.28
time (sec)	N/A	0.242	0.092	0.998	0.212	0.244	0.000	0.000	1.524

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	84	92	576	98	244	0	0	0
N.S.	1	0.90	0.99	6.19	1.05	2.62	0.00	0.00	0.00
time (sec)	N/A	0.276	0.170	15.406	0.311	0.239	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	159	53	659	158	287	0	0	0
N.S.	1	0.96	0.32	3.97	0.95	1.73	0.00	0.00	0.00
time (sec)	N/A	0.304	0.114	14.370	0.314	0.257	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	238	53	686	212	311	0	0	0
N.S.	1	1.00	0.22	2.87	0.89	1.30	0.00	0.00	0.00
time (sec)	N/A	0.348	0.153	14.053	0.324	0.269	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	155	109	169	994	125	0	0	293
N.S.	1	1.05	0.74	1.15	6.76	0.85	0.00	0.00	1.99
time (sec)	N/A	0.706	1.260	7.158	9.667	0.267	0.000	0.000	8.402

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	114	91	123	580	89	0	0	103
N.S.	1	1.04	0.83	1.12	5.27	0.81	0.00	0.00	0.94
time (sec)	N/A	0.502	0.782	7.283	0.610	0.252	0.000	0.000	6.534

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	80	0	53	0	0	98
N.S.	1	1.00	0.83	1.16	0.00	0.77	0.00	0.00	1.42
time (sec)	N/A	0.327	0.431	6.757	0.000	0.258	0.000	0.000	5.445

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	47	201	40	0	0	60
N.S.	1	1.00	1.00	1.52	6.48	1.29	0.00	0.00	1.94
time (sec)	N/A	0.221	0.336	15.156	0.396	0.256	0.000	0.000	0.250

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	123	101	648	884	222	0	0	0
N.S.	1	1.01	0.83	5.31	7.25	1.82	0.00	0.00	0.00
time (sec)	N/A	0.496	0.997	11.931	0.825	0.249	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	203	160	676	0	274	0	0	0
N.S.	1	1.06	0.83	3.52	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.856	1.608	15.235	0.000	0.260	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	190	0	0	626
N.S.	1	0.87	0.62	0.70	0.65	1.62	0.00	0.00	5.35
time (sec)	N/A	0.271	0.519	1.924	0.216	0.286	0.000	0.000	16.993

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	78	61	63	58	152	0	0	498
N.S.	1	0.89	0.69	0.72	0.66	1.73	0.00	0.00	5.66
time (sec)	N/A	0.260	0.314	0.160	0.213	0.252	0.000	0.000	12.640

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	51	44	40	114	0	0	370
N.S.	1	0.92	0.86	0.75	0.68	1.93	0.00	0.00	6.27
time (sec)	N/A	0.259	0.260	185.671	0.239	0.251	0.000	0.000	7.129

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	73	0	0	242
N.S.	1	1.00	1.00	0.83	0.72	2.52	0.00	0.00	8.34
time (sec)	N/A	0.236	0.133	4.082	0.224	0.254	0.000	0.000	7.426

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	76	83	603	98	236	0	0	0
N.S.	1	0.85	0.93	6.78	1.10	2.65	0.00	0.00	0.00
time (sec)	N/A	0.263	0.460	18.585	0.321	0.253	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	131	53	846	140	263	0	0	0
N.S.	1	0.96	0.39	6.18	1.02	1.92	0.00	0.00	0.00
time (sec)	N/A	0.299	0.087	123.884	0.318	0.248	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	206	53	933	194	309	0	0	0
N.S.	1	0.98	0.25	4.44	0.92	1.47	0.00	0.00	0.00
time (sec)	N/A	0.326	0.122	5.970	0.318	0.264	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	155	103	224	624	121	0	0	301
N.S.	1	1.05	0.70	1.52	4.24	0.82	0.00	0.00	2.05
time (sec)	N/A	0.692	1.209	41.252	243.694	0.261	0.000	0.000	8.781

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	108	93	182	0	83	0	0	105
N.S.	1	1.04	0.89	1.75	0.00	0.80	0.00	0.00	1.01
time (sec)	N/A	0.454	0.591	7.418	0.000	0.246	0.000	0.000	7.168

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	88	331	45	0	0	64
N.S.	1	1.00	0.71	1.35	5.09	0.69	0.00	0.00	0.98
time (sec)	N/A	0.345	0.534	24.898	0.410	0.231	0.000	0.000	0.445

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	69	66	328	59	0	0	89
N.S.	1	1.00	1.97	1.89	9.37	1.69	0.00	0.00	2.54
time (sec)	N/A	0.243	0.779	35.408	0.460	0.243	0.000	0.000	0.968

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	163	118	916	1076	244	0	0	0
N.S.	1	1.03	0.74	5.76	6.77	1.53	0.00	0.00	0.00
time (sec)	N/A	0.671	1.414	3.305	0.467	0.268	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	243	155	945	0	300	0	0	0
N.S.	1	1.05	0.67	4.09	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	1.027	1.849	3.719	0.000	0.266	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	202	0	0	690
N.S.	1	0.87	0.62	0.70	0.65	1.73	0.00	0.00	5.90
time (sec)	N/A	0.268	0.470	0.609	0.218	0.296	0.000	0.000	16.013

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	78	63	63	58	164	0	0	562
N.S.	1	0.89	0.72	0.72	0.66	1.86	0.00	0.00	6.39
time (sec)	N/A	0.262	0.431	0.161	0.214	0.272	0.000	0.000	17.405

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	51	44	40	126	0	0	434
N.S.	1	0.92	0.86	0.75	0.68	2.14	0.00	0.00	7.36
time (sec)	N/A	0.253	0.209	187.494	0.207	0.259	0.000	0.000	8.677

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	85	0	0	306
N.S.	1	1.00	1.00	0.83	0.72	2.93	0.00	0.00	10.55
time (sec)	N/A	0.247	0.176	4.134	0.364	0.253	0.000	0.000	6.924

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	100	51	641	117	235	0	0	0
N.S.	1	0.86	0.44	5.53	1.01	2.03	0.00	0.00	0.00
time (sec)	N/A	0.273	0.119	38.355	0.449	0.256	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	125	53	869	138	261	0	0	0
N.S.	1	0.91	0.39	6.34	1.01	1.91	0.00	0.00	0.00
time (sec)	N/A	0.290	0.114	123.981	0.472	0.243	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	178	53	1136	176	277	0	0	0
N.S.	1	0.98	0.29	6.28	0.97	1.53	0.00	0.00	0.00
time (sec)	N/A	0.322	0.117	4.652	0.298	0.254	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	147	109	235	0	109	0	0	286
N.S.	1	1.06	0.78	1.69	0.00	0.78	0.00	0.00	2.06
time (sec)	N/A	0.621	1.069	7.877	0.000	0.267	0.000	0.000	6.743

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	59	122	418	71	0	0	102
N.S.	1	1.00	0.57	1.17	4.02	0.68	0.00	0.00	0.98
time (sec)	N/A	0.505	0.809	29.973	0.401	0.258	0.000	0.000	5.741

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	88	504	59	0	0	85
N.S.	1	1.00	1.21	1.24	7.10	0.83	0.00	0.00	1.20
time (sec)	N/A	0.368	0.965	34.537	0.397	0.264	0.000	0.000	0.935

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	73	63	454	73	0	0	112
N.S.	1	1.00	2.09	1.80	12.97	2.09	0.00	0.00	3.20
time (sec)	N/A	0.240	1.309	4.059	0.694	0.258	0.000	0.000	5.756

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	203	131	1185	1253	258	0	0	0
N.S.	1	1.04	0.67	6.05	6.39	1.32	0.00	0.00	0.00
time (sec)	N/A	0.853	2.410	3.878	0.704	0.256	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	283	188	1211	0	314	0	0	0
N.S.	1	1.06	0.70	4.52	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	1.222	3.649	4.768	0.000	0.290	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	365	194	1238	0	342	0	0	0
N.S.	1	1.07	0.57	3.62	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	1.750	6.019	6.019	0.000	0.306	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	297	150	0	0	434
N.S.	1	0.87	0.62	0.70	2.54	1.28	0.00	0.00	3.71
time (sec)	N/A	0.274	0.260	1.384	0.245	0.261	0.000	0.000	9.736

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	78	63	63	169	113	0	0	306
N.S.	1	0.89	0.72	0.72	1.92	1.28	0.00	0.00	3.48
time (sec)	N/A	0.265	0.177	1.346	0.238	0.264	0.000	0.000	7.157

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	54	49	44	79	76	0	0	155
N.S.	1	0.92	0.83	0.75	1.34	1.29	0.00	0.00	2.63
time (sec)	N/A	0.260	0.118	1.118	0.229	0.239	0.000	0.000	1.432

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	37	0	55	47
N.S.	1	1.00	1.00	0.89	0.78	1.37	0.00	2.04	1.74
time (sec)	N/A	0.233	0.074	0.770	0.227	0.235	0.000	0.598	0.168

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	144	51	406	138	271	0	0	0
N.S.	1	0.99	0.35	2.78	0.95	1.86	0.00	0.00	0.00
time (sec)	N/A	0.294	0.135	10.673	0.318	0.253	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	223	53	540	192	293	0	0	0
N.S.	1	1.02	0.24	2.47	0.88	1.34	0.00	0.00	0.00
time (sec)	N/A	0.340	0.198	11.100	0.321	0.255	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	302	53	674	246	315	0	0	0
N.S.	1	1.03	0.18	2.31	0.84	1.08	0.00	0.00	0.00
time (sec)	N/A	0.374	0.361	9.443	0.326	0.253	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	155	95	126	608	153	0	0	301
N.S.	1	1.05	0.65	0.86	4.14	1.04	0.00	0.00	2.05
time (sec)	N/A	0.724	1.200	9.328	0.426	0.293	0.000	0.000	10.282

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	114	77	99	474	116	0	0	105
N.S.	1	1.04	0.70	0.90	4.31	1.05	0.00	0.00	0.95
time (sec)	N/A	0.530	0.860	6.937	0.383	0.256	0.000	0.000	6.625

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	72	340	79	0	0	91
N.S.	1	1.00	0.89	0.99	4.66	1.08	0.00	0.00	1.25
time (sec)	N/A	0.377	0.613	7.516	0.352	0.250	0.000	0.000	10.144

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	40	44	206	40	0	0	98
N.S.	1	1.00	1.14	1.26	5.89	1.14	0.00	0.00	2.80
time (sec)	N/A	0.238	0.439	7.635	0.326	0.248	0.000	0.000	1.111

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	70	122	0	149	0	0	0
N.S.	1	1.00	1.35	2.35	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.235	0.449	7.038	0.000	0.266	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	123	96	350	837	245	0	0	0
N.S.	1	1.01	0.79	2.87	6.86	2.01	0.00	0.00	0.00
time (sec)	N/A	0.495	0.602	9.742	0.439	0.251	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	205	117	484	1938	267	0	0	0
N.S.	1	1.06	0.61	2.51	10.04	1.38	0.00	0.00	0.00
time (sec)	N/A	0.855	0.897	11.700	0.517	0.252	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	149	0	0	370
N.S.	1	0.87	0.62	0.70	0.65	1.27	0.00	0.00	3.16
time (sec)	N/A	0.276	0.405	1.369	0.220	0.253	0.000	0.000	8.730

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	78	59	63	58	108	0	0	242
N.S.	1	0.89	0.67	0.72	0.66	1.23	0.00	0.00	2.75
time (sec)	N/A	0.276	0.180	1.422	0.222	0.249	0.000	0.000	7.568

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	52	37	44	38	67	0	0	85
N.S.	1	0.91	0.65	0.77	0.67	1.18	0.00	0.00	1.49
time (sec)	N/A	0.265	0.113	1.003	0.226	0.238	0.000	0.000	4.707

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	49	0	0	67
N.S.	1	1.00	1.00	0.89	0.78	1.81	0.00	0.00	2.48
time (sec)	N/A	0.237	0.055	0.992	0.231	0.234	0.000	0.000	0.286

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	176	51	595	153	294	0	0	0
N.S.	1	1.01	0.29	3.40	0.87	1.68	0.00	0.00	0.00
time (sec)	N/A	0.308	0.190	10.010	0.315	0.265	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	255	53	729	207	316	0	0	0
N.S.	1	1.03	0.21	2.94	0.83	1.27	0.00	0.00	0.00
time (sec)	N/A	0.359	0.344	8.622	0.311	0.251	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	334	53	811	261	338	0	0	0
N.S.	1	1.04	0.17	2.53	0.81	1.05	0.00	0.00	0.00
time (sec)	N/A	0.389	0.527	9.310	0.320	0.257	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	155	108	0	764	184	0	0	301
N.S.	1	1.05	0.73	0.00	5.20	1.25	0.00	0.00	2.05
time (sec)	N/A	0.719	1.899	0.000	0.479	0.323	0.000	0.000	11.256

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	114	92	0	626	143	0	0	105
N.S.	1	1.04	0.84	0.00	5.69	1.30	0.00	0.00	0.95
time (sec)	N/A	0.536	1.501	0.000	0.426	0.281	0.000	0.000	9.548

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	80	0	488	102	0	0	91
N.S.	1	1.00	1.10	0.00	6.68	1.40	0.00	0.00	1.25
time (sec)	N/A	0.384	1.101	0.000	0.373	0.285	0.000	0.000	7.741

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	59	0	350	59	0	0	139
N.S.	1	1.00	1.69	0.00	10.00	1.69	0.00	0.00	3.97
time (sec)	N/A	0.240	0.842	0.000	0.344	0.251	0.000	0.000	1.851

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	101	223	813	196	0	0	0
N.S.	1	1.00	1.17	2.59	9.45	2.28	0.00	0.00	0.00
time (sec)	N/A	0.376	1.062	8.938	0.447	0.244	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	416	0	246	0	0	0
N.S.	1	1.00	1.09	4.78	0.00	2.83	0.00	0.00	0.00
time (sec)	N/A	0.339	0.784	8.674	0.000	0.259	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	163	120	539	1821	270	0	0	0
N.S.	1	1.04	0.76	3.43	11.60	1.72	0.00	0.00	0.00
time (sec)	N/A	0.633	1.115	9.948	0.549	0.245	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	247	145	673	2632	292	0	0	0
N.S.	1	1.06	0.62	2.89	11.30	1.25	0.00	0.00	0.00
time (sec)	N/A	1.029	1.842	10.019	0.525	0.274	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	126	85	101	94	175	0	0	434
N.S.	1	0.86	0.58	0.69	0.64	1.20	0.00	0.00	2.97
time (sec)	N/A	0.287	0.396	1.286	0.235	0.272	0.000	0.000	10.150

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	102	73	82	76	134	0	0	306
N.S.	1	0.87	0.62	0.70	0.65	1.15	0.00	0.00	2.62
time (sec)	N/A	0.285	0.321	1.121	0.239	0.270	0.000	0.000	7.327

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	76	51	63	58	93	0	0	155
N.S.	1	0.88	0.59	0.73	0.67	1.08	0.00	0.00	1.80
time (sec)	N/A	0.270	0.164	1.041	0.239	0.252	0.000	0.000	1.370

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	36	42	44	50	0	0	72
N.S.	1	0.91	0.65	0.76	0.80	0.91	0.00	0.00	1.31
time (sec)	N/A	0.259	0.110	1.033	0.226	0.249	0.000	0.000	0.328

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	61	0	0	23
N.S.	1	1.00	1.00	0.83	0.72	2.10	0.00	0.00	0.79
time (sec)	N/A	0.238	0.112	1.216	0.216	0.243	0.000	0.000	4.592

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	208	51	780	175	305	0	0	0
N.S.	1	1.02	0.25	3.82	0.86	1.50	0.00	0.00	0.00
time (sec)	N/A	0.317	0.344	10.474	0.445	0.249	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	287	53	914	229	327	0	0	0
N.S.	1	1.04	0.19	3.30	0.83	1.18	0.00	0.00	0.00
time (sec)	N/A	0.363	0.489	9.155	0.440	0.256	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	155	112	0	902	199	0	0	301
N.S.	1	1.05	0.76	0.00	6.14	1.35	0.00	0.00	2.05
time (sec)	N/A	0.740	1.995	0.000	1.051	0.357	0.000	0.000	12.751

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	114	94	0	764	158	0	0	105
N.S.	1	1.04	0.85	0.00	6.95	1.44	0.00	0.00	0.95
time (sec)	N/A	0.544	1.739	0.000	0.479	0.298	0.000	0.000	9.813

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	80	0	626	117	0	0	91
N.S.	1	1.00	1.10	0.00	8.58	1.60	0.00	0.00	1.25
time (sec)	N/A	0.377	1.615	0.000	0.425	0.275	0.000	0.000	7.064

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	57	0	488	74	0	0	50
N.S.	1	1.00	1.63	0.00	13.94	2.11	0.00	0.00	1.43
time (sec)	N/A	0.233	1.407	0.000	0.378	0.252	0.000	0.000	2.188

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	129	82	268	1074	270	0	0	0
N.S.	1	1.05	0.67	2.18	8.73	2.20	0.00	0.00	0.00
time (sec)	N/A	0.547	1.478	9.840	0.482	0.249	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	86	126	149	396	827	245	0	0	0
N.S.	1	1.47	1.73	4.60	9.62	2.85	0.00	0.00	0.00
time (sec)	N/A	0.511	1.512	9.184	0.704	0.254	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	127	121	606	0	267	0	0	0
N.S.	1	1.04	0.99	4.97	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.466	1.088	9.230	0.000	0.255	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	203	143	730	2297	289	0	0	0
N.S.	1	1.06	0.74	3.80	11.96	1.51	0.00	0.00	0.00
time (sec)	N/A	0.813	1.405	10.355	0.512	0.251	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	289	165	864	3789	311	0	0	0
N.S.	1	1.07	0.61	3.20	14.03	1.15	0.00	0.00	0.00
time (sec)	N/A	1.272	1.927	9.111	0.598	0.261	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	126	81	101	94	160	0	0	370
N.S.	1	0.86	0.55	0.69	0.64	1.10	0.00	0.00	2.53
time (sec)	N/A	0.286	0.489	1.612	0.246	0.272	0.000	0.000	9.227

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	98	61	82	76	119	0	0	242
N.S.	1	0.87	0.54	0.73	0.67	1.05	0.00	0.00	2.14
time (sec)	N/A	0.285	0.235	1.317	0.225	0.262	0.000	0.000	7.488

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	74	49	63	62	77	0	0	110
N.S.	1	0.88	0.58	0.75	0.74	0.92	0.00	0.00	1.31
time (sec)	N/A	0.268	0.136	1.204	0.380	0.244	0.000	0.000	0.863

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	52	52	44	32	61	0	0	0
N.S.	1	0.91	0.91	0.77	0.56	1.07	0.00	0.00	0.00
time (sec)	N/A	0.281	0.180	1.304	0.408	0.252	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	72	0	0	23
N.S.	1	1.00	1.00	0.83	0.72	2.48	0.00	0.00	0.79
time (sec)	N/A	0.252	0.201	1.317	0.487	0.257	0.000	0.000	4.092

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	240	51	977	195	316	0	0	0
N.S.	1	1.03	0.22	4.19	0.84	1.36	0.00	0.00	0.00
time (sec)	N/A	0.343	0.445	10.355	0.312	0.251	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	319	53	1111	249	338	0	0	0
N.S.	1	1.04	0.17	3.63	0.81	1.10	0.00	0.00	0.00
time (sec)	N/A	0.386	0.725	9.966	0.315	0.253	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	114	92	0	902	173	0	0	105
N.S.	1	1.04	0.84	0.00	8.20	1.57	0.00	0.00	0.95
time (sec)	N/A	0.565	2.003	0.000	0.569	0.323	0.000	0.000	10.273

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	82	0	764	132	0	0	91
N.S.	1	1.00	1.12	0.00	10.47	1.81	0.00	0.00	1.25
time (sec)	N/A	0.387	1.662	0.000	0.469	0.281	0.000	0.000	7.618

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	59	0	626	89	0	0	50
N.S.	1	1.00	1.69	0.00	17.89	2.54	0.00	0.00	1.43
time (sec)	N/A	0.235	1.473	0.000	0.721	0.262	0.000	0.000	7.382

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	172	130	313	1164	326	0	0	0
N.S.	1	1.08	0.81	1.96	7.28	2.04	0.00	0.00	0.00
time (sec)	N/A	0.727	2.049	10.794	0.717	0.261	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	121	167	126	425	0	245	0	0	0
N.S.	1	1.38	1.04	3.51	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.686	1.780	10.215	0.000	0.254	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	170	120	795	977	267	0	0	0
N.S.	1	1.36	0.96	6.36	7.82	2.14	0.00	0.00	0.00
time (sec)	N/A	0.648	1.862	10.651	0.457	0.256	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	167	119	798	0	278	0	0	0
N.S.	1	1.06	0.76	5.08	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	0.606	1.878	9.874	0.000	0.252	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	243	141	923	2779	300	0	0	0
N.S.	1	1.07	0.62	4.07	12.24	1.32	0.00	0.00	0.00
time (sec)	N/A	1.007	2.318	12.797	0.485	0.260	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	331	175	1057	5821	322	0	0	0
N.S.	1	1.08	0.57	3.44	18.96	1.05	0.00	0.00	0.00
time (sec)	N/A	1.492	2.974	11.076	0.621	0.277	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	413	373	323	1870	418	0	0	0
N.S.	1	0.79	0.71	0.62	3.57	0.80	0.00	0.00	0.00
time (sec)	N/A	0.890	2.123	11.390	0.484	0.253	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	333	277	157	1400	323	0	0	0
N.S.	1	1.03	0.86	0.49	4.33	1.00	0.00	0.00	0.00
time (sec)	N/A	0.482	1.896	11.343	0.482	0.255	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	64	0	0	0
N.S.	1	1.00	1.00	0.89	2.11	1.78	0.00	0.00	0.00
time (sec)	N/A	0.229	0.690	8.489	0.317	0.239	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	48	52	54	75	0	0	86
N.S.	1	1.00	0.59	0.64	0.67	0.93	0.00	0.00	1.06
time (sec)	N/A	0.384	0.791	10.644	0.377	0.245	0.000	0.000	5.273

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	127	63	62	130	86	0	0	101
N.S.	1	1.04	0.52	0.51	1.07	0.70	0.00	0.00	0.83
time (sec)	N/A	0.547	0.970	10.524	0.384	0.248	0.000	0.000	5.796

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	174	80	80	178	97	0	0	109
N.S.	1	1.06	0.49	0.49	1.09	0.59	0.00	0.00	0.66
time (sec)	N/A	0.754	1.262	10.089	0.392	0.240	0.000	0.000	5.785

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	465	376	524	3005	644	0	0	0
N.S.	1	1.03	0.83	1.16	6.63	1.42	0.00	0.00	0.00
time (sec)	N/A	1.049	4.189	9.605	0.617	0.257	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	458	375	422	2367	538	0	0	0
N.S.	1	0.80	0.66	0.74	4.15	0.94	0.00	0.00	0.00
time (sec)	N/A	1.004	3.657	10.541	0.529	0.258	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	373	338	359	1871	420	0	0	0
N.S.	1	1.02	0.93	0.99	5.14	1.15	0.00	0.00	0.00
time (sec)	N/A	0.642	3.272	10.476	0.488	0.249	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	520	411	401	502	1462	460	0	0	0
N.S.	1	0.79	0.77	0.97	2.81	0.88	0.00	0.00	0.00
time (sec)	N/A	0.823	2.829	10.454	0.502	0.257	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	76	76	0	0	0
N.S.	1	1.00	1.00	1.34	2.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.242	1.070	9.750	0.348	0.241	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	84	68	59	79	0	0	102
N.S.	1	1.00	1.04	0.84	0.73	0.98	0.00	0.00	1.26
time (sec)	N/A	0.383	1.342	9.542	0.416	0.253	0.000	0.000	5.665

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	128	98	77	84	91	0	0	110
N.S.	1	1.02	0.78	0.62	0.67	0.73	0.00	0.00	0.88
time (sec)	N/A	0.575	1.452	9.705	0.412	0.246	0.000	0.000	5.852

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	174	113	95	160	103	0	0	125
N.S.	1	1.04	0.68	0.57	0.96	0.62	0.00	0.00	0.75
time (sec)	N/A	0.787	1.578	9.886	0.416	0.248	0.000	0.000	6.535

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	612	503	386	556	3005	635	0	0	0
N.S.	1	0.82	0.63	0.91	4.91	1.04	0.00	0.00	0.00
time (sec)	N/A	1.251	3.201	9.464	0.650	0.263	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	418	387	449	2431	525	0	0	0
N.S.	1	1.02	0.94	1.09	5.91	1.28	0.00	0.00	0.00
time (sec)	N/A	0.828	4.090	10.241	0.872	0.261	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	454	359	661	2013	484	0	0	0
N.S.	1	0.81	0.64	1.17	3.58	0.86	0.00	0.00	0.00
time (sec)	N/A	1.014	3.413	10.548	1.024	0.262	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	362	372	343	471	1492	505	0	0	0
N.S.	1	1.03	0.95	1.30	4.12	1.40	0.00	0.00	0.00
time (sec)	N/A	0.651	3.806	10.553	0.504	0.263	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	57	76	80	0	0	104
N.S.	1	1.00	1.00	1.50	2.00	2.11	0.00	0.00	2.74
time (sec)	N/A	0.248	1.161	9.623	0.343	0.240	0.000	0.000	5.026

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	76	94	94	0	0	112
N.S.	1	1.00	1.14	0.94	1.16	1.16	0.00	0.00	1.38
time (sec)	N/A	0.433	1.397	9.868	0.434	0.243	0.000	0.000	5.988

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	128	104	87	96	99	0	0	127
N.S.	1	1.02	0.83	0.70	0.77	0.79	0.00	0.00	1.02
time (sec)	N/A	0.567	1.457	9.877	0.394	0.239	0.000	0.000	6.674

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	175	121	97	124	99	0	0	133
N.S.	1	1.04	0.72	0.57	0.73	0.59	0.00	0.00	0.79
time (sec)	N/A	0.790	1.605	9.815	0.843	0.241	0.000	0.000	6.803

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	369	375	350	765	2258	461	0	0	0
N.S.	1	1.02	0.95	2.07	6.12	1.25	0.00	0.00	0.00
time (sec)	N/A	0.653	3.779	15.948	0.778	0.269	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	483	373	302	157	726	385	0	0	0
N.S.	1	0.77	0.63	0.33	1.50	0.80	0.00	0.00	0.00
time (sec)	N/A	0.631	1.846	14.688	0.468	0.270	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	64	0	0	40
N.S.	1	1.00	1.00	0.89	2.11	1.78	0.00	0.00	1.11
time (sec)	N/A	0.233	0.620	11.600	0.345	0.238	0.000	0.000	5.704

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	48	42	80	78	0	0	78
N.S.	1	1.00	0.60	0.52	1.00	0.98	0.00	0.00	0.98
time (sec)	N/A	0.370	0.664	10.194	0.422	0.234	0.000	0.000	0.845

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	127	68	61	130	89	0	0	86
N.S.	1	1.05	0.56	0.50	1.07	0.74	0.00	0.00	0.71
time (sec)	N/A	0.593	0.863	10.354	0.413	0.236	0.000	0.000	4.560

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	173	79	70	178	100	0	0	101
N.S.	1	1.05	0.48	0.42	1.08	0.61	0.00	0.00	0.61
time (sec)	N/A	0.742	1.131	10.228	0.593	0.235	0.000	0.000	5.345

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	220	87	88	226	111	0	0	109
N.S.	1	1.07	0.42	0.43	1.10	0.54	0.00	0.00	0.53
time (sec)	N/A	0.944	1.399	10.602	0.798	0.240	0.000	0.000	4.938

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	464	337	712	1817	483	0	0	0
N.S.	1	0.88	0.64	1.35	3.43	0.91	0.00	0.00	0.00
time (sec)	N/A	1.064	3.655	16.846	0.507	0.259	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	378	338	1090	778	539	0	0	0
N.S.	1	1.04	0.93	2.99	2.13	1.48	0.00	0.00	0.00
time (sec)	N/A	0.672	4.028	16.180	0.467	0.259	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	55	76	67	0	0	0
N.S.	1	1.00	1.00	1.45	2.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.242	1.183	11.311	0.376	0.239	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	59	80	75	0	0	84
N.S.	1	1.00	0.79	0.74	1.00	0.94	0.00	0.00	1.05
time (sec)	N/A	0.383	0.945	14.185	0.429	0.239	0.000	0.000	4.839

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	126	83	69	130	89	0	0	104
N.S.	1	1.04	0.69	0.57	1.07	0.74	0.00	0.00	0.86
time (sec)	N/A	0.558	1.101	9.737	0.402	0.243	0.000	0.000	4.875

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	173	100	91	178	100	0	0	112
N.S.	1	1.05	0.61	0.55	1.08	0.61	0.00	0.00	0.68
time (sec)	N/A	0.800	1.414	9.505	0.407	0.242	0.000	0.000	5.095

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	219	100	97	226	111	0	0	127
N.S.	1	1.05	0.48	0.46	1.08	0.53	0.00	0.00	0.61
time (sec)	N/A	0.953	1.513	9.826	0.764	0.242	0.000	0.000	5.433

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	426	370	1612	2449	541	0	0	0
N.S.	1	1.04	0.90	3.92	5.96	1.32	0.00	0.00	0.00
time (sec)	N/A	0.919	5.504	15.103	0.920	0.255	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	527	420	357	920	1466	543	0	0	0
N.S.	1	0.80	0.68	1.75	2.78	1.03	0.00	0.00	0.00
time (sec)	N/A	0.856	4.273	15.139	0.665	0.263	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	59	76	71	0	0	0
N.S.	1	1.00	1.00	1.55	2.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.244	1.187	11.109	0.343	0.249	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	73	86	79	0	0	102
N.S.	1	1.00	0.79	0.91	1.08	0.99	0.00	0.00	1.28
time (sec)	N/A	0.393	1.224	14.333	0.408	0.233	0.000	0.000	4.768

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	126	85	69	130	86	0	0	109
N.S.	1	1.04	0.70	0.57	1.07	0.71	0.00	0.00	0.90
time (sec)	N/A	0.553	1.347	13.263	0.390	0.234	0.000	0.000	5.343

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	172	102	85	178	89	0	0	118
N.S.	1	1.06	0.63	0.52	1.10	0.55	0.00	0.00	0.73
time (sec)	N/A	0.740	1.448	10.019	0.771	0.247	0.000	0.000	5.121

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	219	107	102	226	111	0	0	135
N.S.	1	1.06	0.52	0.50	1.10	0.54	0.00	0.00	0.66
time (sec)	N/A	0.994	1.567	8.522	0.643	0.250	0.000	0.000	5.954

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	118	0	0	0	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.484	1.689	0.000	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	116	0	0	0	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	1.268	0.000	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	116	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	1.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	1.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	1.346	0.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	112	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.479	1.376	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	437	248	240	0	3902	528	0	0	0
N.S.	1	0.57	0.55	0.00	8.93	1.21	0.00	0.00	0.00
time (sec)	N/A	0.702	2.443	0.000	0.547	0.259	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	378	201	220	0	1906	515	0	0	0
N.S.	1	0.53	0.58	0.00	5.04	1.36	0.00	0.00	0.00
time (sec)	N/A	0.652	1.945	0.000	0.441	0.264	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	340	154	161	0	1753	367	0	0	0
N.S.	1	0.45	0.47	0.00	5.16	1.08	0.00	0.00	0.00
time (sec)	N/A	0.446	1.160	0.000	0.467	0.259	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	47	0	107	55	0	0	81
N.S.	1	1.00	1.27	0.00	2.89	1.49	0.00	0.00	2.19
time (sec)	N/A	0.273	0.984	0.000	0.625	0.246	0.000	0.000	5.786

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	316	58	0	0	90
N.S.	1	1.00	0.86	0.00	3.90	0.72	0.00	0.00	1.11
time (sec)	N/A	0.446	1.093	0.000	0.578	0.244	0.000	0.000	5.075

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	126	100	0	402	107	0	0	122
N.S.	1	1.03	0.82	0.00	3.30	0.88	0.00	0.00	1.00
time (sec)	N/A	0.598	1.499	0.000	0.892	0.244	0.000	0.000	6.360

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	171	116	0	983	133	0	0	303
N.S.	1	1.05	0.71	0.00	6.03	0.82	0.00	0.00	1.86
time (sec)	N/A	0.780	2.039	0.000	0.402	0.236	0.000	0.000	8.950

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	88	210	0	0	0	0	0	0
N.S.	1	1.02	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	6.415	0.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	88	147	0	0	0	0	0	0
N.S.	1	1.02	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	1.546	0.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	114	114	0	0	0	0	0	0
N.S.	1	1.33	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	0.513	0.000	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	114	67	0	0	0	0	0	0
N.S.	1	1.39	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	88	152	0	0	0	0	0	0
N.S.	1	1.02	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	2.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	88	156	0	0	0	0	0	0
N.S.	1	1.02	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	1.966	0.000	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	88	153	0	0	0	0	0	0
N.S.	1	1.02	1.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	2.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	121	186	0	0	0	0	0	0
N.S.	1	1.11	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	3.291	0.000	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	121	186	0	0	0	0	0	0
N.S.	1	1.11	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	5.210	0.000	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	114	186	0	0	0	0	0	0
N.S.	1	1.07	1.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	2.739	0.000	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	114	174	0	0	0	0	0	0
N.S.	1	1.07	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	1.754	0.000	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	118	174	0	0	0	0	0	0
N.S.	1	1.11	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	2.492	0.000	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	121	178	0	0	0	0	0	0
N.S.	1	1.11	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.496	3.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	121	178	0	0	0	0	0	0
N.S.	1	1.11	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.496	5.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	165	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	10.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	86	80	272	0	247	0	0	168
N.S.	1	0.89	0.82	2.80	0.00	2.55	0.00	0.00	1.73
time (sec)	N/A	0.278	0.881	1.832	0.000	0.252	0.000	0.000	9.520

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	60	60	150	0	142	0	0	216
N.S.	1	0.92	0.92	2.31	0.00	2.18	0.00	0.00	3.32
time (sec)	N/A	0.272	0.278	1.014	0.000	0.263	0.000	0.000	2.817

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	28	60	0	62	104
N.S.	1	1.00	1.00	0.97	0.88	1.88	0.00	1.94	3.25
time (sec)	N/A	0.240	0.187	0.446	0.224	0.241	0.000	0.730	0.506

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	60	54	0	0	0	0	0	0
N.S.	1	1.07	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	64	58	0	0	0	0	0	0
N.S.	1	1.07	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.250	0.292	0.000	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	64	58	0	0	0	0	0	0
N.S.	1	1.07	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.542	0.000	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.492	14.720	0.000	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	149	0	0	0	0	0	0
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.461	13.910	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	146	0	0	0	0	0	0
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	9.963	0.000	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	146	0	0	0	0	0	0
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	13.431	0.000	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	14.216	0.000	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.478	15.681	0.000	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	181	0	0	0	0	0	0
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	10.898	0.000	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	170	0	0	0	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.478	11.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	170	0	0	0	0	0	0
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	10.434	0.000	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	143	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	11.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	143	0	0	0	0	0	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	12.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	157	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	12.477	0.000	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	269	259	165	4331	435	335	0	0	511
N.S.	1	0.96	0.61	16.10	1.62	1.25	0.00	0.00	1.90
time (sec)	N/A	1.129	1.889	6.426	0.817	0.252	0.000	0.000	11.480

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	205	209	119	4982	346	265	0	0	425
N.S.	1	1.02	0.58	24.30	1.69	1.29	0.00	0.00	2.07
time (sec)	N/A	0.891	1.891	6.876	0.401	0.244	0.000	0.000	11.061

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	147	82	2581	175	178	0	0	227
N.S.	1	0.99	0.55	17.44	1.18	1.20	0.00	0.00	1.53
time (sec)	N/A	0.637	1.378	6.424	0.397	0.247	0.000	0.000	10.534

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	94	97	58	2484	113	129	246	0	121
N.S.	1	1.03	0.62	26.43	1.20	1.37	2.62	0.00	1.29
time (sec)	N/A	0.418	1.289	7.101	0.377	0.249	0.507	0.000	2.159

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	37	37	37	842	86	84	49	0	0
N.S.	1	1.00	1.00	22.76	2.32	2.27	1.32	0.00	0.00
time (sec)	N/A	0.235	0.325	5.522	0.332	0.243	3.856	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	156	87	0	0	0	0	0	0
N.S.	1	1.32	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.549	5.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	146	112	0	0	0	0	0	0
N.S.	1	1.29	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	14.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	156	116	0	0	0	0	0	0
N.S.	1	1.29	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.573	17.560	0.000	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	159	122	0	1067	166	0	0	318
N.S.	1	1.02	0.78	0.00	6.84	1.06	0.00	0.00	2.04
time (sec)	N/A	0.653	3.579	0.000	1.366	0.252	0.000	0.000	11.811

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	142	166	0	0	0	0	0	0
N.S.	1	1.46	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.551	15.718	0.000	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	102	91	0	595	134	0	0	174
N.S.	1	1.04	0.93	0.00	6.07	1.37	0.00	0.00	1.78
time (sec)	N/A	0.445	2.560	0.000	1.084	0.245	0.000	0.000	7.467

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	140	166	0	0	0	0	0	0
N.S.	1	1.44	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.551	14.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	59	0	217	109	0	0	106
N.S.	1	1.00	1.28	0.00	4.72	2.37	0.00	0.00	2.30
time (sec)	N/A	0.247	1.966	0.000	0.364	0.251	0.000	0.000	5.628

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	138	154	0	0	0	0	0	0
N.S.	1	1.45	1.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.529	10.340	0.000	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	69	146	0	0	0	0	0	0
N.S.	1	1.06	2.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	2.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	137	157	0	0	0	0	0	0
N.S.	1	1.44	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.535	14.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	78	151	0	0	0	0	0	0
N.S.	1	1.05	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	14.688	0.000	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	142	166	0	0	0	0	0	0
N.S.	1	1.46	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.575	14.985	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	70	165	0	0	0	0	0	0
N.S.	1	1.06	2.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.546	13.412	0.000	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	70	165	0	0	0	0	0	0
N.S.	1	1.06	2.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.529	12.187	0.000	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	67	150	0	0	0	0	0	0
N.S.	1	1.06	2.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	2.494	0.000	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	40	40	40	1261	137	115	0	0	62
N.S.	1	1.00	1.00	31.52	3.42	2.88	0.00	0.00	1.55
time (sec)	N/A	0.253	1.680	8.248	0.337	0.246	0.000	0.000	5.442

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	61	0	304	139	0	0	260
N.S.	1	1.00	0.66	0.00	3.30	1.51	0.00	0.00	2.83
time (sec)	N/A	0.413	2.436	0.000	0.443	0.256	0.000	0.000	9.241

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	145	129	0	0	172	0	0	321
N.S.	1	0.98	0.87	0.00	0.00	1.16	0.00	0.00	2.17
time (sec)	N/A	0.611	3.298	0.000	0.000	0.254	0.000	0.000	13.755

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	56	53	69	70	57	56	70	68
N.S.	1	0.93	0.88	1.15	1.17	0.95	0.93	1.17	1.13
time (sec)	N/A	0.292	0.198	14.753	0.204	0.244	1.477	0.394	4.097

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	79	74	63	86	88	0	141	175
N.S.	1	1.07	1.00	0.85	1.16	1.19	0.00	1.91	2.36
time (sec)	N/A	0.430	0.015	8.658	0.350	0.265	0.000	0.406	7.984

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	47	48	45	44	48	46
N.S.	1	1.00	0.93	1.07	1.09	1.02	1.00	1.09	1.05
time (sec)	N/A	0.285	0.100	3.540	0.394	0.243	1.040	0.388	4.043

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	53	52	50	61	74	0	99	105
N.S.	1	1.02	1.00	0.96	1.17	1.42	0.00	1.90	2.02
time (sec)	N/A	0.335	0.015	2.253	0.281	0.248	0.000	0.385	6.547

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	20	30	34	25	23
N.S.	1	1.00	1.00	0.89	0.71	1.07	1.21	0.89	0.82
time (sec)	N/A	0.262	0.017	0.855	0.200	0.246	0.673	0.353	4.035

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	31	54	37	54	38
N.S.	1	1.00	1.00	1.33	1.29	2.25	1.54	2.25	1.58
time (sec)	N/A	0.239	0.013	0.572	0.204	0.262	2.224	0.367	4.531

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	23	23	23	0	129	38
N.S.	1	1.00	1.92	0.96	0.96	0.96	0.00	5.38	1.58
time (sec)	N/A	0.232	0.019	0.720	0.209	0.256	0.000	0.358	4.211

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	44	46	36	38	35	0	146	31
N.S.	1	1.02	1.07	0.84	0.88	0.81	0.00	3.40	0.72
time (sec)	N/A	0.253	0.061	1.136	0.304	0.261	0.000	0.369	4.026

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	38	0	11886	47
N.S.	1	1.00	1.00	0.82	0.80	0.86	0.00	270.14	1.07
time (sec)	N/A	0.267	0.009	2.894	0.312	0.253	0.000	2.422	4.653

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	70	62	52	61	51	0	426	41
N.S.	1	1.08	0.95	0.80	0.94	0.78	0.00	6.55	0.63
time (sec)	N/A	0.322	0.101	4.856	0.519	0.262	0.000	0.519	4.060

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	129	133	138	133	122	0	156	132
N.S.	1	1.08	1.12	1.16	1.12	1.03	0.00	1.31	1.11
time (sec)	N/A	0.309	0.613	113.345	0.659	0.274	0.000	0.532	4.540

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	107	104	110	104	100	0	118	102
N.S.	1	1.10	1.07	1.13	1.07	1.03	0.00	1.22	1.05
time (sec)	N/A	0.295	0.763	28.988	0.216	0.251	0.000	0.531	4.092

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	64	54	82	71	79	0	80	71
N.S.	1	0.85	0.72	1.09	0.95	1.05	0.00	1.07	0.95
time (sec)	N/A	0.280	0.222	7.983	0.209	0.264	0.000	0.512	4.212

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	46	48	20	55	0	41	39
N.S.	1	1.00	2.09	2.18	0.91	2.50	0.00	1.86	1.77
time (sec)	N/A	0.219	0.044	2.439	0.249	0.270	0.000	0.455	4.360

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	78	52	64	55	52	0	245	50
N.S.	1	1.59	1.06	1.31	1.12	1.06	0.00	5.00	1.02
time (sec)	N/A	0.253	0.448	1.998	0.290	0.268	0.000	0.502	3.991

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	139	197	97	85	75	0	2286	83
N.S.	1	1.58	2.24	1.10	0.97	0.85	0.00	25.98	0.94
time (sec)	N/A	0.296	3.325	8.991	0.623	0.268	0.000	2.597	4.788

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	159	216	177	220	163	0	437	432
N.S.	1	0.98	1.33	1.09	1.35	1.00	0.00	2.68	2.65
time (sec)	N/A	0.435	0.083	67.511	0.315	0.289	0.000	0.587	8.173

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	130	168	149	180	142	0	343	328
N.S.	1	0.99	1.28	1.14	1.37	1.08	0.00	2.62	2.50
time (sec)	N/A	0.412	0.069	15.695	0.580	0.274	0.000	0.540	6.891

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	101	120	118	129	120	0	249	216
N.S.	1	1.02	1.21	1.19	1.30	1.21	0.00	2.52	2.18
time (sec)	N/A	0.402	0.060	4.717	0.212	0.254	0.000	0.509	7.249

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	68	67	83	82	96	0	122	106
N.S.	1	1.05	1.03	1.28	1.26	1.48	0.00	1.88	1.63
time (sec)	N/A	0.351	0.045	1.068	0.200	0.249	0.000	0.512	4.654

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	46	84	53	60	62	0	1100	66
N.S.	1	0.98	1.79	1.13	1.28	1.32	0.00	23.40	1.40
time (sec)	N/A	0.346	0.579	1.181	0.209	0.250	0.000	0.702	4.971

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	54	67	52	52	53	0	11162	77
N.S.	1	0.60	0.74	0.58	0.58	0.59	0.00	124.02	0.86
time (sec)	N/A	0.403	0.061	4.723	0.209	0.242	0.000	16.591	4.240

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	77	69	88	77	74	0	28204	115
N.S.	1	0.68	0.61	0.77	0.68	0.65	0.00	247.40	1.01
time (sec)	N/A	0.413	0.178	21.460	0.334	0.242	0.000	34.076	4.573

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	100	92	108	98	94	0	52002	176
N.S.	1	0.72	0.67	0.78	0.71	0.68	0.00	376.83	1.28
time (sec)	N/A	0.431	0.319	68.937	0.358	0.260	0.000	61.655	4.663

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	192	177	219	176	150	0	220	175
N.S.	1	0.99	0.91	1.13	0.91	0.77	0.00	1.13	0.90
time (sec)	N/A	0.370	2.254	191.796	0.312	0.261	0.000	0.821	4.111

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	115	115	173	142	128	0	166	139
N.S.	1	0.83	0.83	1.25	1.03	0.93	0.00	1.20	1.01
time (sec)	N/A	0.331	0.670	69.606	0.216	0.282	0.000	0.787	4.856

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	64	54	127	98	105	0	112	97
N.S.	1	0.85	0.72	1.69	1.31	1.40	0.00	1.49	1.29
time (sec)	N/A	0.276	0.405	16.388	0.206	0.240	0.000	0.754	4.001

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	57	72	20	78	0	57	55
N.S.	1	1.00	2.59	3.27	0.91	3.55	0.00	2.59	2.50
time (sec)	N/A	0.216	0.181	4.500	0.237	0.240	0.000	0.739	4.407

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	119	401	98	81	79	0	561	141
N.S.	1	1.38	4.66	1.14	0.94	0.92	0.00	6.52	1.64
time (sec)	N/A	0.314	0.837	4.698	0.294	0.257	0.000	0.886	4.418

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	130	488	114	110	100	0	2496	109
N.S.	1	1.55	5.81	1.36	1.31	1.19	0.00	29.71	1.30
time (sec)	N/A	0.282	1.224	20.988	0.483	0.247	0.000	12.250	4.114

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	637	248	208	170	0	465	423
N.S.	1	1.00	4.01	1.56	1.31	1.07	0.00	2.92	2.66
time (sec)	N/A	0.575	3.324	31.313	0.298	0.258	0.000	0.841	8.106

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	130	464	198	157	147	0	333	293
N.S.	1	1.03	3.68	1.57	1.25	1.17	0.00	2.64	2.33
time (sec)	N/A	0.545	2.463	9.239	0.391	0.265	0.000	0.754	8.175

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	97	293	146	111	123	0	171	160
N.S.	1	1.07	3.22	1.60	1.22	1.35	0.00	1.88	1.76
time (sec)	N/A	0.481	2.440	2.562	0.215	0.260	0.000	0.705	6.262

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	70	131	96	84	109	0	4309	116
N.S.	1	0.83	1.56	1.14	1.00	1.30	0.00	51.30	1.38
time (sec)	N/A	0.447	1.818	2.797	0.221	0.276	0.000	2.617	4.693

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	82	81	75	77	77	0	24430	104
N.S.	1	1.17	1.16	1.07	1.10	1.10	0.00	349.00	1.49
time (sec)	N/A	0.514	0.781	10.368	0.276	0.255	0.000	142.573	4.221

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	113	121	125	107	102	0	56572	147
N.S.	1	1.08	1.15	1.19	1.02	0.97	0.00	538.78	1.40
time (sec)	N/A	0.558	1.326	40.215	0.214	0.260	0.000	274.601	4.527

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	130	141	145	126	123	0	101962	214
N.S.	1	0.92	0.99	1.02	0.89	0.87	0.00	718.04	1.51
time (sec)	N/A	0.592	3.084	119.541	0.374	0.260	0.000	84.494	4.491

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	103	99	106	108	183	0	120	119
N.S.	1	0.89	0.85	0.91	0.93	1.58	0.00	1.03	1.03
time (sec)	N/A	0.302	1.318	32.677	0.348	0.271	0.000	0.384	4.368

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	52	52	53	53	117	0	54	57
N.S.	1	0.88	0.88	0.90	0.90	1.98	0.00	0.92	0.97
time (sec)	N/A	0.266	0.152	7.497	0.623	0.257	0.000	0.381	4.048

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	59	0	19	18
N.S.	1	1.00	1.00	1.06	1.00	3.28	0.00	1.06	1.00
time (sec)	N/A	0.219	0.018	1.451	0.207	0.256	0.000	0.367	3.938

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	158	143	120	141	119	0	182	156
N.S.	1	1.70	1.54	1.29	1.52	1.28	0.00	1.96	1.68
time (sec)	N/A	0.381	0.605	2.289	0.310	0.263	0.000	0.363	4.834

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	245	225	197	271	208	0	322	318
N.S.	1	1.61	1.48	1.30	1.78	1.37	0.00	2.12	2.09
time (sec)	N/A	0.483	1.329	8.190	0.333	0.271	0.000	0.400	4.565

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	150	321	269	361	259	0	278	724
N.S.	1	1.07	2.29	1.92	2.58	1.85	0.00	1.99	5.17
time (sec)	N/A	1.003	2.594	16.287	0.304	0.336	0.000	0.408	6.543

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	80	109	129	163	191	0	136	310
N.S.	1	1.01	1.38	1.63	2.06	2.42	0.00	1.72	3.92
time (sec)	N/A	0.498	0.526	4.092	0.474	0.272	0.000	0.405	4.308

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	43	80	131	0	74	39
N.S.	1	1.00	0.98	0.93	1.74	2.85	0.00	1.61	0.85
time (sec)	N/A	0.222	0.139	1.343	0.514	0.250	0.000	0.378	4.571

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	83	79	90	142	187	0	118	110
N.S.	1	0.92	0.88	1.00	1.58	2.08	0.00	1.31	1.22
time (sec)	N/A	0.492	0.565	1.595	0.455	0.265	0.000	0.393	4.449

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	151	137	221	379	262	0	286	342
N.S.	1	0.92	0.83	1.34	2.30	1.59	0.00	1.73	2.07
time (sec)	N/A	0.909	1.715	4.153	0.317	0.269	0.000	0.404	7.586

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	156	229	201	186	386	0	253	258
N.S.	1	0.88	1.29	1.13	1.04	2.17	0.00	1.42	1.45
time (sec)	N/A	0.367	2.623	265.495	0.212	0.295	0.000	0.496	4.140

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	100	122	114	115	281	0	149	130
N.S.	1	0.86	1.05	0.98	0.99	2.42	0.00	1.28	1.12
time (sec)	N/A	0.314	5.389	58.769	0.225	0.270	0.000	0.445	4.287

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	51	51	57	60	178	0	71	67
N.S.	1	0.84	0.84	0.93	0.98	2.92	0.00	1.16	1.10
time (sec)	N/A	0.274	0.170	15.500	0.221	0.262	0.000	0.417	4.563

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	32	21	20	57	0	20	20
N.S.	1	1.00	1.60	1.05	1.00	2.85	0.00	1.00	1.00
time (sec)	N/A	0.227	0.164	3.420	0.340	0.253	0.000	0.429	4.005

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	210	304	154	282	279	0	250	246
N.S.	1	1.38	2.00	1.01	1.86	1.84	0.00	1.64	1.62
time (sec)	N/A	0.431	4.411	4.168	0.417	0.288	0.000	0.441	5.129

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	322	416	248	502	424	0	464	463
N.S.	1	1.37	1.77	1.06	2.14	1.80	0.00	1.97	1.97
time (sec)	N/A	0.554	3.599	16.901	0.479	0.310	0.000	0.472	5.330

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	222	1152	454	827	472	0	530	2654
N.S.	1	0.94	4.90	1.93	3.52	2.01	0.00	2.26	11.29
time (sec)	N/A	0.511	6.762	152.135	0.316	0.417	0.000	0.530	7.560

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	161	709	259	471	355	0	280	585
N.S.	1	0.91	4.03	1.47	2.68	2.02	0.00	1.59	3.32
time (sec)	N/A	0.394	6.577	32.082	0.331	0.316	0.000	0.497	5.935

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	104	120	135	212	293	0	166	383
N.S.	1	1.14	1.32	1.48	2.33	3.22	0.00	1.82	4.21
time (sec)	N/A	0.334	1.345	7.865	0.296	0.311	0.000	0.478	5.232

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	103	78	118	182	215	0	138	136
N.S.	1	1.26	0.95	1.44	2.22	2.62	0.00	1.68	1.66
time (sec)	N/A	0.299	0.631	1.872	0.358	0.271	0.000	0.435	4.646

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	173	153	172	348	302	0	286	286
N.S.	1	1.10	0.97	1.10	2.22	1.92	0.00	1.82	1.82
time (sec)	N/A	0.383	0.938	2.315	0.422	0.294	0.000	0.507	6.540

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	273	249	320	772	418	0	438	674
N.S.	1	1.13	1.03	1.33	3.20	1.73	0.00	1.82	2.80
time (sec)	N/A	0.533	1.803	8.249	0.463	0.300	0.000	0.511	8.298

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	160	272	195	200	476	0	243	234
N.S.	1	0.86	1.47	1.05	1.08	2.57	0.00	1.31	1.26
time (sec)	N/A	0.370	1.509	1.076	0.236	0.306	0.000	0.621	4.492

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	102	140	115	128	354	0	140	143
N.S.	1	0.84	1.16	0.95	1.06	2.93	0.00	1.16	1.18
time (sec)	N/A	0.307	4.392	174.192	0.213	0.297	0.000	0.581	4.732

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	57	57	63	78	284	0	62	80
N.S.	1	0.83	0.83	0.91	1.13	4.12	0.00	0.90	1.16
time (sec)	N/A	0.270	0.225	37.257	0.246	0.270	0.000	0.563	4.288

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	142	0	20	39
N.S.	1	1.00	1.00	0.95	0.91	6.45	0.00	0.91	1.77
time (sec)	N/A	0.219	0.219	7.224	0.232	0.260	0.000	0.516	4.522

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	263	458	219	458	503	0	439	419
N.S.	1	1.30	2.27	1.08	2.27	2.49	0.00	2.17	2.07
time (sec)	N/A	0.480	6.365	10.265	0.307	0.314	0.000	0.635	5.260

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	386	596	312	738	671	0	587	715
N.S.	1	1.31	2.02	1.06	2.50	2.27	0.00	1.99	2.42
time (sec)	N/A	0.600	6.295	43.898	0.533	0.335	0.000	0.613	6.490

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	223	688	444	902	564	0	510	1203
N.S.	1	0.93	2.88	1.86	3.77	2.36	0.00	2.13	5.03
time (sec)	N/A	0.477	3.614	299.230	0.609	0.353	0.000	0.646	7.829

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	164	396	269	518	513	0	314	1311
N.S.	1	1.11	2.68	1.82	3.50	3.47	0.00	2.12	8.86
time (sec)	N/A	0.378	3.682	94.516	0.339	0.336	0.000	0.607	6.480

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	117	132	191	326	294	0	221	260
N.S.	1	1.23	1.39	2.01	3.43	3.09	0.00	2.33	2.74
time (sec)	N/A	0.313	0.487	17.398	0.306	0.273	0.000	0.612	6.927

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	170	110	280	412	352	0	293	443
N.S.	1	1.10	0.71	1.81	2.66	2.27	0.00	1.89	2.86
time (sec)	N/A	0.369	1.663	3.616	0.304	0.289	0.000	0.540	5.879

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	250	183	283	658	480	0	399	610
N.S.	1	1.13	0.83	1.28	2.98	2.17	0.00	1.81	2.76
time (sec)	N/A	0.476	2.713	4.994	0.319	0.306	0.000	0.626	8.102

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	359	371	457	1229	619	0	640	1128
N.S.	1	1.16	1.20	1.47	3.96	2.00	0.00	2.06	3.64
time (sec)	N/A	0.614	3.039	23.375	0.360	0.333	0.000	0.635	9.856

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	123	69	436	0	145	0	0	0
N.S.	1	1.02	0.57	3.60	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.607	1.382	26.102	0.000	0.096	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	58	162	0	123	0	0	0
N.S.	1	1.01	0.63	1.76	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.459	0.896	24.297	0.000	0.093	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	89	58	412	0	120	0	0	0
N.S.	1	1.01	0.66	4.68	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.450	0.958	3.589	0.000	0.093	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	42	95	0	74	0	0	39
N.S.	1	1.00	0.72	1.64	0.00	1.28	0.00	0.00	0.67
time (sec)	N/A	0.334	0.813	11.108	0.000	0.084	0.000	0.000	0.431

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	306	0	89	0	0	0
N.S.	1	1.00	0.93	5.28	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.331	0.815	10.435	0.000	0.087	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	95	69	155	0	103	0	0	0
N.S.	1	1.01	0.73	1.65	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.440	0.697	10.849	0.000	0.087	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	95	74	438	0	111	0	0	0
N.S.	1	1.01	0.79	4.66	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.442	1.419	6.150	0.000	0.096	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	131	94	178	0	118	0	0	0
N.S.	1	1.07	0.76	1.45	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.558	1.112	6.184	0.000	0.095	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	139	127	317	0	172	0	0	0
N.S.	1	0.97	0.89	2.22	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.692	2.437	146.292	0.000	0.094	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	135	126	833	0	171	0	0	0
N.S.	1	0.94	0.88	5.83	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.685	1.851	21.000	0.000	0.094	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	104	87	239	0	133	0	0	0
N.S.	1	1.01	0.84	2.32	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.528	1.650	15.539	0.000	0.087	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	64	805	0	122	0	0	0
N.S.	1	1.00	0.67	8.47	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.539	2.654	16.287	0.000	0.093	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	132	101	295	0	131	0	0	0
N.S.	1	0.95	0.73	2.12	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.690	2.433	15.536	0.000	0.095	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	141	92	863	0	139	0	0	0
N.S.	1	0.97	0.63	5.95	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.704	2.831	23.411	0.000	0.099	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	177	127	336	0	154	0	0	0
N.S.	1	0.96	0.69	1.83	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.879	4.655	21.609	0.000	0.097	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	177	126	931	0	164	0	0	0
N.S.	1	0.96	0.68	5.06	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.848	6.007	33.432	0.000	0.105	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	193	157	349	0	202	0	0	0
N.S.	1	0.97	0.79	1.76	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.361	4.211	619.103	0.000	0.099	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	186	155	872	0	200	0	0	0
N.S.	1	1.06	0.88	4.95	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.359	3.858	22.069	0.000	0.099	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	152	132	298	0	164	0	0	0
N.S.	1	1.18	1.02	2.31	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.339	3.913	21.260	0.000	0.095	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	184	130	1114	0	169	0	0	0
N.S.	1	1.03	0.73	6.26	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.342	4.903	21.299	0.000	0.099	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	161	117	323	0	148	0	0	0
N.S.	1	1.10	0.80	2.21	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.356	3.805	23.956	0.000	0.097	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	199	150	1289	0	164	0	0	0
N.S.	1	0.98	0.74	6.32	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.372	3.943	25.996	0.000	0.104	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	183	150	366	0	181	0	0	0
N.S.	1	1.08	0.88	2.15	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.359	5.489	23.648	0.000	0.103	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	181	372	976	0	191	0	0	0
N.S.	1	1.03	2.11	5.55	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.371	9.408	32.975	0.000	0.117	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	213	296	407	0	206	0	0	0
N.S.	1	0.98	1.36	1.87	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.388	10.232	39.402	0.000	0.115	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	456	324	11117	31613	0	0	0	0	0
N.S.	1	0.71	24.38	69.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.705	90.481	33.062	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	396	300	290	7347	0	0	0	0	0
N.S.	1	0.76	0.73	18.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.729	27.353	30.430	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	334	246	276	3634	0	0	0	0	0
N.S.	1	0.74	0.83	10.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.614	3.428	9.428	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	324	250	218	3038	0	0	0	0	0
N.S.	1	0.77	0.67	9.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.613	2.866	8.017	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	451	344	285	6968	0	0	0	0	0
N.S.	1	0.76	0.63	15.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	4.475	9.987	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	422	327	418	6052	0	0	0	0	0
N.S.	1	0.77	0.99	14.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.725	7.171	10.466	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	568	425	2596	12841	0	0	0	0	0
N.S.	1	0.75	4.57	22.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.922	77.215	14.007	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	480	328	1129	33211	0	0	0	0	0
N.S.	1	0.68	2.35	69.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.690	29.031	132.901	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	440	309	378	4812	0	0	0	0	0
N.S.	1	0.70	0.86	10.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.717	12.021	128.711	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	477	342	1125	16454	0	0	0	0	0
N.S.	1	0.72	2.36	34.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.738	27.971	8.872	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	430	324	422	12206	0	0	0	0	0
N.S.	1	0.75	0.98	28.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.715	10.760	11.210	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	555	431	8379	29178	0	0	0	0	0
N.S.	1	0.78	15.10	52.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.898	35.310	11.809	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	520	413	528	14917	0	0	0	0	0
N.S.	1	0.79	1.02	28.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.885	12.230	13.293	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	700	538	9161	37753	0	0	0	0	0
N.S.	1	0.77	13.09	53.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.066	32.275	17.401	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	583	395	14225	72185	0	0	0	0	0
N.S.	1	0.68	24.40	123.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.821	35.712	1486.381	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	532	374	352	36014	0	0	0	0	0
N.S.	1	0.70	0.66	67.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.770	10.263	1496.337	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	566	413	14364	60919	0	0	0	0	0
N.S.	1	0.73	25.38	107.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.839	35.268	13.111	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	515	392	369	75607	0	0	0	0	0
N.S.	1	0.76	0.72	146.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.833	9.699	14.532	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	664	513	14652	78532	0	0	0	0	0
N.S.	1	0.77	22.07	118.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.029	39.230	15.802	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	620	495	409	76277	0	0	0	0	0
N.S.	1	0.80	0.66	123.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.007	12.852	17.743	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	814	626	15481	89815	0	0	0	0	0
N.S.	1	0.77	19.02	110.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.204	33.968	22.569	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	69	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.519	0.000	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	59	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.362	0.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	59	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	61	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	0.320	0.000	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	124	109	0	0	0	0	0	0
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	0.759	0.000	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	122	106	0	0	0	0	0	0
N.S.	1	1.03	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.568	0.901	0.000	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	122	107	0	0	0	0	0	0
N.S.	1	1.03	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.583	1.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	124	118	0	0	0	0	0	0
N.S.	1	1.04	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	1.384	0.000	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	552	379	276	0	0	0	0	0	0
N.S.	1	0.69	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.769	17.906	0.000	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	552	369	280	0	0	0	0	0	0
N.S.	1	0.67	0.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.712	14.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	579	423	285	0	0	0	0	0	0
N.S.	1	0.73	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	52.388	0.000	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	581	415	6862	0	0	0	0	0	0
N.S.	1	0.71	11.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.800	111.464	0.000	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	687	545	8003	0	0	0	0	0	0
N.S.	1	0.79	11.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.194	129.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	687	545	7801	0	0	0	0	0	0
N.S.	1	0.79	11.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.160	78.547	0.000	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	715	574	18832	0	0	0	0	0	0
N.S.	1	0.80	26.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.219	137.803	0.000	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	717	576	11783	0	0	0	0	0	0
N.S.	1	0.80	16.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.234	81.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	206	159	0	0	0	0	0	0
N.S.	1	1.19	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.422	2.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	119	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.654	0.824	0.000	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	65	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	141	160	1158	0	0	0	0	0	0
N.S.	1	1.13	8.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	15.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	227	219	2453	0	0	0	0	0	0
N.S.	1	0.96	10.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.467	18.792	0.000	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	181	186	699	0	0	0	0	0	0
N.S.	1	1.03	3.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.416	10.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	138	161	462	286	420	0	0	0
N.S.	1	0.86	1.00	2.87	1.78	2.61	0.00	0.00	0.00
time (sec)	N/A	0.342	3.408	0.115	0.415	0.315	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	77	71	204	116	176	0	0	0
N.S.	1	0.88	0.81	2.32	1.32	2.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.205	109.936	0.555	0.293	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	64	0	0	51
N.S.	1	1.00	1.00	1.04	1.00	2.46	0.00	0.00	1.96
time (sec)	N/A	0.227	0.093	8.988	0.190	0.268	0.000	0.000	5.245

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	293	225	0	0	0	0	0	0
N.S.	1	1.08	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	1.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	434	478	360	0	0	0	0	0	0
N.S.	1	1.10	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.744	4.712	0.000	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	159	185	306	0	0	0	0	0	0
N.S.	1	1.16	1.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.378	14.538	0.000	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	159	185	340	0	0	0	0	0	0
N.S.	1	1.16	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	5.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	161	185	341	0	0	0	0	0	0
N.S.	1	1.15	2.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	6.574	0.000	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	161	185	341	0	0	0	0	0	0
N.S.	1	1.15	2.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	8.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	158	133	229	0	110	0	0	0
N.S.	1	1.27	1.07	1.85	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.861	1.243	9.940	0.000	0.086	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	122	250	205	0	80	0	0	0
N.S.	1	1.36	2.78	2.28	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.672	4.265	7.905	0.000	0.081	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	122	100	168	0	58	0	0	0
N.S.	1	1.36	1.11	1.87	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.645	0.666	5.496	0.000	0.079	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	60	85	192	108	0	27	0	0	0
N.S.	1	1.42	3.20	1.80	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.487	3.681	4.256	0.000	0.084	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	85	143	73	0	89	0	0	74
N.S.	1	1.42	2.38	1.22	0.00	1.48	0.00	0.00	1.23
time (sec)	N/A	0.472	2.237	3.056	0.000	0.079	0.000	0.000	0.661

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	116	190	214	0	151	0	0	0
N.S.	1	1.30	2.13	2.40	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.638	5.023	5.444	0.000	0.083	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	120	57	260	0	190	0	0	0
N.S.	1	1.25	0.59	2.71	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.633	1.605	6.702	0.000	0.082	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	130	150	666	385	0	235	0	0	0
N.S.	1	1.15	5.12	2.96	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.774	7.515	11.274	0.000	0.083	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	223	156	387	0	151	0	0	0
N.S.	1	1.17	0.82	2.04	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	1.044	2.628	9.441	0.000	0.102	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	187	464	351	0	140	0	0	0
N.S.	1	1.21	3.01	2.28	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.844	6.001	8.423	0.000	0.094	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	187	131	315	0	115	0	0	0
N.S.	1	1.21	0.85	2.05	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.884	1.946	7.257	0.000	0.088	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	151	268	277	0	102	0	0	0
N.S.	1	1.26	2.23	2.31	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.721	7.247	6.174	0.000	0.085	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	151	158	239	0	91	0	0	0
N.S.	1	1.26	1.32	1.99	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.683	1.898	5.089	0.000	0.081	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	115	114	206	0	94	0	0	0
N.S.	1	1.25	1.24	2.24	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.564	3.546	4.182	0.000	0.079	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	115	116	171	0	79	0	0	0
N.S.	1	1.25	1.26	1.86	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.567	1.557	3.523	0.000	0.086	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	122	141	1106	135	0	113	0	0	0
N.S.	1	1.16	9.07	1.11	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.696	7.390	3.715	0.000	0.081	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	145	67	208	0	152	0	0	0
N.S.	1	1.15	0.53	1.65	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.760	1.575	4.561	0.000	0.083	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	179	414	321	0	198	0	0	0
N.S.	1	1.09	2.52	1.96	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.920	5.212	7.546	0.000	0.084	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	199	80	78	202	100	0	0	96
N.S.	1	1.11	0.45	0.44	1.13	0.56	0.00	0.00	0.54
time (sec)	N/A	0.944	2.094	8.878	0.431	0.254	0.000	0.000	6.146

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	152	63	62	148	86	0	0	84
N.S.	1	1.15	0.48	0.47	1.12	0.65	0.00	0.00	0.64
time (sec)	N/A	0.700	1.709	8.991	0.393	0.265	0.000	0.000	0.836

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	106	56	50	59	68	0	0	88
N.S.	1	1.25	0.66	0.59	0.69	0.80	0.00	0.00	1.04
time (sec)	N/A	0.535	1.202	8.578	0.731	0.249	0.000	0.000	0.583

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	52	0	0	0
N.S.	1	1.00	1.00	0.89	2.11	1.44	0.00	0.00	0.00
time (sec)	N/A	0.357	1.104	7.312	0.495	0.261	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	330	125	155	1400	313	0	0	0
N.S.	1	0.99	0.37	0.46	4.18	0.93	0.00	0.00	0.00
time (sec)	N/A	0.499	1.794	9.278	0.496	0.269	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	438	274	319	1836	470	0	0	0
N.S.	1	0.84	0.52	0.61	3.50	0.90	0.00	0.00	0.00
time (sec)	N/A	1.034	5.149	10.437	0.468	0.266	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	445	227	359	2249	569	0	0	0
N.S.	1	0.87	0.44	0.70	4.39	1.11	0.00	0.00	0.00
time (sec)	N/A	1.017	3.708	10.117	0.571	0.269	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	719	538	305	459	2661	657	0	0	0
N.S.	1	0.75	0.42	0.64	3.70	0.91	0.00	0.00	0.00
time (sec)	N/A	1.467	4.506	8.771	0.594	0.277	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	198	80	70	202	103	0	0	110
N.S.	1	1.13	0.46	0.40	1.15	0.59	0.00	0.00	0.63
time (sec)	N/A	0.958	2.061	7.941	0.799	0.260	0.000	0.000	6.142

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	152	63	59	136	83	0	0	100
N.S.	1	1.21	0.50	0.47	1.08	0.66	0.00	0.00	0.79
time (sec)	N/A	0.695	1.597	8.204	0.686	0.242	0.000	0.000	1.225

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	105	48	42	80	68	0	0	82
N.S.	1	1.31	0.60	0.52	1.00	0.85	0.00	0.00	1.02
time (sec)	N/A	0.520	1.032	7.824	0.391	0.247	0.000	0.000	0.797

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	58	0	0	0
N.S.	1	1.00	1.00	0.89	2.11	1.61	0.00	0.00	0.00
time (sec)	N/A	0.368	1.038	10.041	0.325	0.269	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	373	209	161	714	341	0	0	0
N.S.	1	0.75	0.42	0.33	1.44	0.69	0.00	0.00	0.00
time (sec)	N/A	0.646	2.588	12.923	0.489	0.268	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	470	400	250	765	2147	502	0	0	0
N.S.	1	0.85	0.53	1.63	4.57	1.07	0.00	0.00	0.00
time (sec)	N/A	0.831	3.366	12.387	0.489	0.290	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	682	493	245	502	2254	604	0	0	0
N.S.	1	0.72	0.36	0.74	3.30	0.89	0.00	0.00	0.00
time (sec)	N/A	1.241	3.420	12.958	0.873	0.274	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	202	0	0	0	0	0	0
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	14.502	0.000	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.564	9.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	78	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.344	0.000	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	433	0	0	0	0	0	0
N.S.	1	1.00	5.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.577	7.522	0.000	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	100	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.570	3.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	117	122	0	0	0	0	0	0
N.S.	1	1.11	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.614	2.502	0.000	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	116	143	0	0	0	0	0	0
N.S.	1	1.12	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	2.716	0.000	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	220	212	0	0	0	0	0	0
N.S.	1	1.20	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.556	6.571	0.000	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	175	140	0	0	0	0	0	0
N.S.	1	1.13	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.844	5.484	0.000	0.000	0.000	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	113	75	0	0	0	0	0	0
N.S.	1	1.26	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.508	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	140	157	1132	0	0	0	0	0	0
N.S.	1	1.12	8.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	14.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	227	215	2502	0	0	0	0	0	0
N.S.	1	0.95	11.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.577	17.714	0.000	0.000	0.000	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	187	182	365	0	0	0	0	0	0
N.S.	1	0.97	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	23.409	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [173] had the largest ratio of [1.13332999999999995]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	0.87	22	0.227
2	A	6	5	0.91	22	0.227
3	A	6	5	0.94	22	0.227
4	A	6	5	1.00	22	0.227
5	A	6	5	1.11	22	0.227
6	A	1	1	1.00	13	0.077
7	A	5	5	1.02	22	0.227
8	A	7	7	1.07	22	0.318
9	A	9	9	1.10	22	0.409
10	A	11	11	1.12	22	0.500
11	A	10	10	1.09	22	0.455
12	A	8	8	1.07	22	0.364
13	A	6	6	1.02	22	0.273
14	A	4	4	1.00	20	0.200
15	A	4	4	1.00	20	0.200
16	A	6	5	1.00	22	0.227
17	A	6	5	0.94	22	0.227
18	A	6	5	0.87	22	0.227
19	A	5	4	0.86	24	0.167
20	A	5	4	0.88	24	0.167
21	A	5	4	0.91	24	0.167
22	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	4	1.00	15	0.267
24	A	4	3	1.00	24	0.125
25	A	5	4	1.13	24	0.167
26	A	5	4	0.98	24	0.167
27	A	5	4	0.93	24	0.167
28	A	10	10	1.03	24	0.417
29	A	8	8	1.01	24	0.333
30	A	6	6	0.96	22	0.273
31	A	4	4	1.00	22	0.182
32	A	4	4	1.00	24	0.167
33	A	6	5	0.96	24	0.208
34	A	6	5	0.90	24	0.208
35	A	6	5	0.82	24	0.208
36	A	5	4	0.86	24	0.167
37	A	5	4	0.88	24	0.167
38	A	5	4	0.91	24	0.167
39	A	4	3	1.00	24	0.125
40	A	6	6	1.06	15	0.400
41	A	5	4	0.88	24	0.167
42	A	4	3	1.00	24	0.125
43	A	5	4	1.03	24	0.167
44	A	5	4	0.95	24	0.167
45	A	10	10	1.06	24	0.417
46	A	8	8	1.03	22	0.364
47	A	6	6	1.02	22	0.273
48	A	2	2	1.00	24	0.083
49	A	8	7	0.99	24	0.292
50	A	8	7	0.93	24	0.292
51	A	8	7	0.86	24	0.292
52	A	12	12	1.06	24	0.500
53	A	10	10	1.05	22	0.455
54	A	8	8	1.03	22	0.364
55	A	6	6	1.09	24	0.250
56	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	8	7	1.05	24	0.292
58	A	8	7	0.99	24	0.292
59	A	5	4	0.86	24	0.167
60	A	5	4	0.88	24	0.167
61	A	5	4	0.91	24	0.167
62	A	4	3	1.00	24	0.125
63	A	10	10	1.07	15	0.667
64	A	5	4	0.94	24	0.167
65	A	5	4	0.89	24	0.167
66	A	5	4	0.91	24	0.167
67	A	4	3	1.00	24	0.125
68	A	5	4	0.95	24	0.167
69	A	5	4	0.91	24	0.167
70	A	12	12	1.05	22	0.545
71	A	10	10	1.05	22	0.455
72	A	8	8	1.05	24	0.333
73	A	2	2	1.00	24	0.083
74	A	6	6	1.03	24	0.250
75	A	10	9	0.99	24	0.375
76	A	10	9	0.93	24	0.375
77	A	5	4	0.86	24	0.167
78	A	5	4	0.88	24	0.167
79	A	5	4	0.91	24	0.167
80	A	4	3	1.00	24	0.125
81	A	16	16	1.06	15	1.067
82	A	5	4	0.91	24	0.167
83	A	5	4	0.91	24	0.167
84	A	5	4	0.89	24	0.167
85	A	4	3	0.98	24	0.125
86	A	5	4	0.88	24	0.167
87	A	5	4	0.91	24	0.167
88	A	4	3	1.00	24	0.125
89	A	5	4	0.90	24	0.167
90	A	5	4	0.89	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	16	16	1.06	22	0.727
92	A	14	14	1.05	24	0.583
93	A	12	12	1.05	24	0.500
94	A	10	10	1.07	24	0.417
95	A	4	4	1.00	24	0.167
96	A	8	8	1.04	24	0.333
97	A	12	12	1.03	24	0.500
98	A	12	11	0.99	24	0.458
99	A	5	4	0.86	24	0.167
100	A	5	4	0.88	24	0.167
101	A	5	4	0.91	24	0.167
102	A	4	3	0.79	24	0.125
103	A	4	3	1.04	24	0.125
104	A	3	3	1.00	15	0.200
105	A	5	4	1.13	24	0.167
106	A	5	4	1.02	24	0.167
107	A	8	8	1.01	24	0.333
108	A	6	6	0.98	24	0.250
109	A	4	4	1.00	24	0.167
110	A	2	2	1.00	22	0.091
111	A	4	4	1.00	22	0.182
112	A	6	5	0.96	24	0.208
113	A	6	5	0.89	24	0.208
114	A	5	4	0.88	24	0.167
115	A	5	4	0.91	24	0.167
116	A	4	3	1.00	24	0.125
117	A	5	4	1.03	24	0.167
118	A	4	3	0.96	24	0.125
119	A	5	5	1.07	15	0.333
120	A	5	4	1.01	24	0.167
121	A	5	4	0.96	24	0.167
122	A	10	10	1.05	24	0.417
123	A	8	8	1.04	24	0.333
124	A	6	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	4	4	1.00	24	0.167
126	A	4	4	0.98	22	0.182
127	A	6	5	0.96	22	0.227
128	A	6	5	0.90	24	0.208
129	A	6	5	0.82	24	0.208
130	A	5	4	0.86	24	0.167
131	A	5	4	0.88	24	0.167
132	A	5	4	0.91	24	0.167
133	A	4	3	1.00	24	0.125
134	A	5	4	1.00	24	0.167
135	A	5	4	0.90	24	0.167
136	A	4	3	1.00	24	0.125
137	A	7	7	1.10	15	0.467
138	A	5	4	0.97	24	0.167
139	A	5	4	0.93	24	0.167
140	A	10	10	1.08	24	0.417
141	A	8	8	1.05	24	0.333
142	A	6	6	1.08	24	0.250
143	A	2	2	1.00	24	0.083
144	A	6	6	1.04	22	0.273
145	A	8	7	1.05	22	0.318
146	A	8	7	0.99	24	0.292
147	A	8	7	0.92	24	0.292
148	A	5	4	0.88	24	0.167
149	A	5	4	0.91	24	0.167
150	A	4	3	1.00	24	0.125
151	A	5	4	0.88	24	0.167
152	A	5	4	0.94	24	0.167
153	A	4	3	0.97	24	0.125
154	A	4	3	1.00	24	0.125
155	A	9	9	1.11	15	0.600
156	A	5	4	0.94	24	0.167
157	A	5	4	0.91	24	0.167
158	A	10	10	1.08	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	8	8	1.06	24	0.333
160	A	6	6	1.09	24	0.250
161	A	4	4	1.00	24	0.167
162	A	8	8	1.06	22	0.364
163	A	10	9	1.07	22	0.409
164	A	10	9	1.03	24	0.375
165	A	10	9	0.97	24	0.375
166	A	5	4	0.90	24	0.167
167	A	5	4	0.90	24	0.167
168	A	5	4	0.87	24	0.167
169	A	4	3	0.98	24	0.125
170	A	5	4	0.85	24	0.167
171	A	5	4	0.91	24	0.167
172	A	4	3	1.00	24	0.125
173	A	17	17	1.12	15	1.133
174	A	5	4	0.89	24	0.167
175	A	5	4	0.87	24	0.167
176	A	14	14	1.10	24	0.583
177	A	12	12	1.09	24	0.500
178	A	10	10	1.10	24	0.417
179	A	4	4	1.00	24	0.167
180	A	8	8	1.07	24	0.333
181	A	12	12	1.07	24	0.500
182	A	16	16	1.09	22	0.727
183	A	18	17	1.09	22	0.773
184	A	18	17	1.06	24	0.708
185	A	10	10	1.02	26	0.385
186	A	8	8	1.01	26	0.308
187	A	8	8	1.01	26	0.308
188	A	6	6	1.00	26	0.231
189	A	6	6	1.00	26	0.231
190	A	8	8	1.01	26	0.308
191	A	8	8	1.01	26	0.308
192	A	10	10	1.06	26	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	10	10	0.98	28	0.357
194	A	8	8	0.98	28	0.286
195	A	8	8	1.04	28	0.286
196	A	6	6	1.00	28	0.214
197	A	6	6	1.00	28	0.214
198	A	8	8	1.04	28	0.286
199	A	8	8	1.04	28	0.286
200	A	10	10	1.07	28	0.357
201	A	14	14	1.05	28	0.500
202	A	12	12	1.04	28	0.429
203	A	12	12	1.02	28	0.429
204	A	10	10	1.06	28	0.357
205	A	10	10	1.24	28	0.357
206	A	8	8	0.97	28	0.286
207	A	8	8	0.97	28	0.286
208	A	8	8	1.05	28	0.286
209	A	8	8	1.05	28	0.286
210	A	10	10	1.07	28	0.357
211	A	10	10	1.07	28	0.357
212	A	12	12	1.09	28	0.429
213	A	14	14	1.03	28	0.500
214	A	12	12	1.04	28	0.429
215	A	12	12	1.01	28	0.429
216	A	10	10	1.03	28	0.357
217	A	10	10	1.02	28	0.357
218	A	8	8	1.06	28	0.286
219	A	8	8	1.06	28	0.286
220	A	10	10	1.08	28	0.357
221	A	10	10	1.08	28	0.357
222	A	12	12	1.10	28	0.429
223	A	10	10	0.99	28	0.357
224	A	8	8	1.00	28	0.286
225	A	8	8	1.00	28	0.286
226	A	6	6	1.00	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	6	6	1.00	28	0.214
228	A	6	6	1.00	28	0.214
229	A	6	6	1.00	28	0.214
230	A	8	8	1.02	28	0.286
231	A	8	8	1.02	28	0.286
232	A	10	10	1.05	28	0.357
233	A	12	12	1.03	28	0.429
234	A	10	10	1.04	28	0.357
235	A	10	10	1.01	28	0.357
236	A	8	8	1.01	28	0.286
237	A	8	8	1.01	28	0.286
238	A	6	6	1.00	28	0.214
239	A	6	6	1.00	28	0.214
240	A	8	8	1.09	28	0.286
241	A	8	8	1.09	28	0.286
242	A	10	10	1.08	28	0.357
243	A	10	10	1.08	28	0.357
244	A	12	12	1.09	28	0.429
245	A	12	12	1.06	28	0.429
246	A	10	10	1.09	28	0.357
247	A	10	10	1.06	28	0.357
248	A	8	8	1.06	28	0.286
249	A	8	8	1.06	28	0.286
250	A	8	8	1.01	28	0.286
251	A	8	8	1.01	28	0.286
252	A	10	10	1.12	28	0.357
253	A	10	10	1.12	28	0.357
254	A	12	12	1.11	28	0.429
255	A	12	12	1.06	28	0.429
256	A	10	10	1.09	28	0.357
257	A	10	10	1.04	28	0.357
258	A	8	8	1.08	28	0.286
259	A	8	8	1.08	28	0.286
260	A	10	10	1.10	28	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	10	10	1.10	28	0.357
262	A	12	12	1.12	28	0.429
263	A	8	7	1.00	26	0.269
264	A	8	7	1.00	26	0.269
265	A	8	7	1.00	26	0.269
266	A	8	7	1.00	26	0.269
267	A	8	7	1.00	28	0.250
268	A	8	7	1.00	28	0.250
269	A	8	7	0.96	28	0.250
270	A	8	7	0.96	28	0.250
271	A	8	7	1.00	28	0.250
272	A	8	7	1.00	28	0.250
273	A	8	7	1.00	28	0.250
274	A	8	7	1.00	28	0.250
275	A	8	7	0.99	28	0.250
276	A	8	7	0.99	28	0.250
277	A	8	7	1.00	28	0.250
278	A	8	7	1.00	28	0.250
279	A	5	4	0.87	26	0.154
280	A	5	4	0.89	26	0.154
281	A	5	4	0.92	26	0.154
282	A	4	3	1.00	26	0.115
283	A	7	6	0.93	26	0.231
284	A	9	8	0.99	26	0.308
285	A	11	10	1.02	26	0.385
286	A	8	8	1.05	26	0.308
287	A	6	6	1.04	26	0.231
288	A	4	4	1.00	26	0.154
289	A	2	2	1.00	24	0.083
290	A	6	5	1.00	24	0.208
291	A	10	9	1.06	26	0.346
292	A	14	13	1.10	26	0.500
293	A	5	4	0.87	26	0.154
294	A	5	4	0.89	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	5	4	0.92	26	0.154
296	A	4	3	1.00	26	0.115
297	A	6	5	0.90	26	0.192
298	A	8	7	0.96	26	0.269
299	A	10	9	1.00	26	0.346
300	A	8	8	1.05	26	0.308
301	A	6	6	1.04	26	0.231
302	A	4	4	1.00	24	0.167
303	A	2	2	1.00	24	0.083
304	A	8	7	1.01	26	0.269
305	A	12	11	1.06	26	0.423
306	A	5	4	0.87	26	0.154
307	A	5	4	0.89	26	0.154
308	A	5	4	0.92	26	0.154
309	A	4	3	1.00	26	0.115
310	A	6	5	0.85	26	0.192
311	A	7	6	0.96	26	0.231
312	A	9	8	0.98	26	0.308
313	A	8	8	1.05	26	0.308
314	A	6	6	1.04	24	0.250
315	A	4	4	1.00	24	0.167
316	A	2	2	1.00	26	0.077
317	A	10	9	1.03	26	0.346
318	A	14	13	1.05	26	0.500
319	A	5	4	0.87	26	0.154
320	A	5	4	0.89	26	0.154
321	A	5	4	0.92	26	0.154
322	A	4	3	1.00	26	0.115
323	A	7	6	0.86	26	0.231
324	A	7	6	0.91	26	0.231
325	A	8	7	0.98	26	0.269
326	A	8	8	1.06	24	0.333
327	A	6	6	1.00	24	0.250
328	A	4	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	2	2	1.00	26	0.077
330	A	12	11	1.04	26	0.423
331	A	16	15	1.06	26	0.577
332	A	20	19	1.07	26	0.731
333	A	5	4	0.87	26	0.154
334	A	5	4	0.89	26	0.154
335	A	5	4	0.92	26	0.154
336	A	4	3	1.00	26	0.115
337	A	8	7	0.99	26	0.269
338	A	10	9	1.02	26	0.346
339	A	12	11	1.03	26	0.423
340	A	8	8	1.05	26	0.308
341	A	6	6	1.04	26	0.231
342	A	4	4	1.00	26	0.154
343	A	2	2	1.00	26	0.077
344	A	4	3	1.00	24	0.125
345	A	8	7	1.01	24	0.292
346	A	12	11	1.06	26	0.423
347	A	5	4	0.87	26	0.154
348	A	5	4	0.89	26	0.154
349	A	5	4	0.91	26	0.154
350	A	4	3	1.00	26	0.115
351	A	9	8	1.01	26	0.308
352	A	11	10	1.03	26	0.385
353	A	13	12	1.04	26	0.462
354	A	8	8	1.05	26	0.308
355	A	6	6	1.04	26	0.231
356	A	4	4	1.00	26	0.154
357	A	2	2	1.00	26	0.077
358	A	6	5	1.00	26	0.192
359	A	6	5	1.00	24	0.208
360	A	10	9	1.04	24	0.375
361	A	14	13	1.06	26	0.500
362	A	5	4	0.86	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	5	4	0.87	26	0.154
364	A	5	4	0.88	26	0.154
365	A	5	4	0.91	26	0.154
366	A	4	3	1.00	26	0.115
367	A	10	9	1.02	26	0.346
368	A	12	11	1.04	26	0.423
369	A	8	8	1.05	26	0.308
370	A	6	6	1.04	26	0.231
371	A	4	4	1.00	26	0.154
372	A	2	2	1.00	26	0.077
373	A	8	7	1.05	26	0.269
374	A	8	7	1.47	26	0.269
375	A	8	7	1.04	24	0.292
376	A	12	11	1.06	24	0.458
377	A	16	15	1.07	26	0.577
378	A	5	4	0.86	26	0.154
379	A	5	4	0.87	26	0.154
380	A	5	4	0.88	26	0.154
381	A	5	4	0.91	26	0.154
382	A	4	3	1.00	26	0.115
383	A	11	10	1.03	26	0.385
384	A	13	12	1.04	26	0.462
385	A	6	6	1.04	26	0.231
386	A	4	4	1.00	26	0.154
387	A	2	2	1.00	26	0.077
388	A	10	9	1.08	26	0.346
389	A	10	9	1.38	26	0.346
390	A	10	9	1.36	26	0.346
391	A	10	9	1.06	24	0.375
392	A	14	13	1.07	24	0.542
393	A	18	17	1.08	26	0.654
394	A	15	14	0.79	30	0.467
395	A	11	10	1.03	30	0.333
396	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	4	4	1.00	30	0.133
398	A	6	6	1.04	30	0.200
399	A	8	8	1.06	30	0.267
400	A	17	16	1.03	30	0.533
401	A	17	16	0.80	30	0.533
402	A	13	12	1.02	30	0.400
403	A	15	14	0.79	30	0.467
404	A	2	2	1.00	30	0.067
405	A	4	4	1.00	30	0.133
406	A	6	6	1.02	30	0.200
407	A	8	8	1.04	30	0.267
408	A	19	18	0.82	30	0.600
409	A	15	14	1.02	30	0.467
410	A	17	16	0.81	30	0.533
411	A	13	12	1.03	30	0.400
412	A	2	2	1.00	30	0.067
413	A	4	4	1.00	30	0.133
414	A	6	6	1.02	30	0.200
415	A	8	8	1.04	30	0.267
416	A	13	12	1.02	30	0.400
417	A	13	12	0.77	30	0.400
418	A	2	2	1.00	30	0.067
419	A	4	4	1.00	30	0.133
420	A	6	6	1.05	30	0.200
421	A	8	8	1.05	30	0.267
422	A	10	10	1.07	30	0.333
423	A	17	16	0.88	30	0.533
424	A	13	12	1.04	30	0.400
425	A	2	2	1.00	30	0.067
426	A	4	4	1.00	30	0.133
427	A	6	6	1.04	30	0.200
428	A	8	8	1.05	30	0.267
429	A	10	10	1.05	30	0.333
430	A	15	14	1.04	30	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	15	14	0.80	30	0.467
432	A	2	2	1.00	30	0.067
433	A	4	4	1.00	30	0.133
434	A	6	6	1.04	30	0.200
435	A	8	8	1.06	30	0.267
436	A	10	10	1.06	30	0.333
437	A	8	7	1.00	30	0.233
438	A	8	7	1.00	30	0.233
439	A	8	7	1.00	30	0.233
440	A	8	7	1.00	30	0.233
441	A	8	7	1.00	30	0.233
442	A	8	7	1.00	30	0.233
443	A	13	12	0.57	30	0.400
444	A	12	11	0.53	30	0.367
445	A	9	8	0.45	30	0.267
446	A	2	2	1.00	30	0.067
447	A	4	4	1.00	30	0.133
448	A	6	6	1.03	30	0.200
449	A	8	8	1.05	30	0.267
450	A	7	6	1.02	26	0.231
451	A	7	6	1.02	26	0.231
452	A	7	6	1.33	26	0.231
453	A	7	6	1.39	24	0.250
454	A	7	6	1.02	26	0.231
455	A	7	6	1.02	26	0.231
456	A	7	6	1.02	26	0.231
457	A	7	6	1.11	28	0.214
458	A	7	6	1.11	28	0.214
459	A	7	6	1.07	28	0.214
460	A	7	6	1.07	28	0.214
461	A	7	6	1.11	28	0.214
462	A	7	6	1.11	28	0.214
463	A	7	6	1.11	28	0.214
464	A	7	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
465	A	5	4	0.89	24	0.167
466	A	5	4	0.92	24	0.167
467	A	4	3	1.00	24	0.125
468	A	4	3	1.07	24	0.125
469	A	4	3	1.07	24	0.125
470	A	4	3	1.07	24	0.125
471	A	7	6	1.00	24	0.250
472	A	7	6	1.00	24	0.250
473	A	7	6	1.00	22	0.273
474	A	7	6	1.00	22	0.273
475	A	7	6	1.00	24	0.250
476	A	7	6	1.00	24	0.250
477	A	7	6	1.00	28	0.214
478	A	7	6	1.00	28	0.214
479	A	7	6	1.00	28	0.214
480	A	7	6	1.00	28	0.214
481	A	7	6	1.00	28	0.214
482	A	7	6	1.00	28	0.214
483	A	10	10	0.96	30	0.333
484	A	8	8	1.02	30	0.267
485	A	6	6	0.99	30	0.200
486	A	4	4	1.03	30	0.133
487	A	2	2	1.00	28	0.071
488	A	7	6	1.32	30	0.200
489	A	7	6	1.29	30	0.200
490	A	7	6	1.29	30	0.200
491	A	6	6	1.02	30	0.200
492	A	7	6	1.46	30	0.200
493	A	4	4	1.04	30	0.133
494	A	7	6	1.44	30	0.200
495	A	2	2	1.00	30	0.067
496	A	7	6	1.45	30	0.200
497	A	6	5	1.06	28	0.179
498	A	7	6	1.44	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	8	7	1.05	30	0.233
500	A	7	6	1.46	30	0.200
501	A	8	7	1.06	32	0.219
502	A	8	7	1.06	32	0.219
503	A	6	5	1.06	30	0.167
504	A	2	2	1.00	32	0.062
505	A	4	4	1.00	32	0.125
506	A	6	6	0.98	32	0.188
507	A	6	5	0.93	19	0.263
508	A	8	8	1.07	19	0.421
509	A	6	5	1.00	19	0.263
510	A	6	6	1.02	19	0.316
511	A	6	5	1.00	19	0.263
512	A	4	4	1.00	17	0.235
513	A	4	4	1.00	17	0.235
514	A	5	5	1.02	19	0.263
515	A	6	5	1.00	19	0.263
516	A	7	7	1.08	19	0.368
517	A	6	5	1.08	21	0.238
518	A	6	5	1.10	21	0.238
519	A	6	5	0.85	21	0.238
520	A	4	3	1.00	21	0.143
521	A	6	5	1.59	21	0.238
522	A	7	6	1.58	21	0.286
523	A	13	12	0.98	21	0.571
524	A	12	11	0.99	21	0.524
525	A	11	10	1.02	21	0.476
526	A	10	9	1.05	19	0.474
527	A	9	8	0.98	19	0.421
528	A	9	8	0.60	21	0.381
529	A	10	9	0.68	21	0.429
530	A	10	9	0.72	21	0.429
531	A	6	5	0.99	21	0.238
532	A	6	5	0.83	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	6	5	0.85	21	0.238
534	A	4	3	1.00	21	0.143
535	A	7	6	1.38	21	0.286
536	A	7	6	1.55	21	0.286
537	A	14	13	1.00	21	0.619
538	A	13	12	1.03	21	0.571
539	A	12	11	1.07	19	0.579
540	A	12	11	0.83	19	0.579
541	A	8	7	1.17	21	0.333
542	A	10	9	1.08	21	0.429
543	A	10	9	0.92	21	0.429
544	A	6	5	0.89	21	0.238
545	A	6	5	0.88	21	0.238
546	A	4	3	1.00	21	0.143
547	A	8	7	1.70	21	0.333
548	A	10	9	1.61	21	0.429
549	A	15	14	1.07	21	0.667
550	A	9	8	1.01	21	0.381
551	A	4	3	1.00	19	0.158
552	A	9	8	0.92	19	0.421
553	A	15	14	0.92	21	0.667
554	A	6	5	0.88	21	0.238
555	A	6	5	0.86	21	0.238
556	A	6	5	0.84	21	0.238
557	A	4	3	1.00	21	0.143
558	A	8	7	1.38	21	0.333
559	A	10	9	1.37	21	0.429
560	A	12	11	0.94	21	0.524
561	A	10	9	0.91	21	0.429
562	A	8	7	1.14	21	0.333
563	A	6	5	1.26	19	0.263
564	A	9	8	1.10	19	0.421
565	A	13	12	1.13	21	0.571
566	A	6	5	0.86	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
567	A	6	5	0.84	21	0.238
568	A	6	5	0.83	21	0.238
569	A	4	3	1.00	21	0.143
570	A	8	7	1.30	21	0.333
571	A	10	9	1.31	21	0.429
572	A	13	12	0.93	21	0.571
573	A	11	10	1.11	21	0.476
574	A	6	5	1.23	21	0.238
575	A	8	7	1.10	19	0.368
576	A	11	10	1.13	19	0.526
577	A	16	15	1.16	21	0.714
578	A	10	10	1.02	23	0.435
579	A	8	8	1.01	23	0.348
580	A	8	8	1.01	23	0.348
581	A	6	6	1.00	23	0.261
582	A	6	6	1.00	23	0.261
583	A	8	8	1.01	23	0.348
584	A	8	8	1.01	23	0.348
585	A	10	10	1.07	23	0.435
586	A	11	11	0.97	25	0.440
587	A	11	11	0.94	25	0.440
588	A	9	9	1.01	25	0.360
589	A	9	9	1.00	25	0.360
590	A	11	11	0.95	25	0.440
591	A	11	11	0.97	25	0.440
592	A	13	13	0.96	25	0.520
593	A	13	13	0.96	25	0.520
594	A	9	8	0.97	25	0.320
595	A	9	8	1.06	25	0.320
596	A	8	7	1.18	25	0.280
597	A	9	8	1.03	25	0.320
598	A	7	6	1.10	25	0.240
599	A	8	7	0.98	25	0.280
600	A	7	6	1.08	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	A	7	6	1.03	25	0.240
602	A	8	7	0.98	25	0.280
603	A	18	17	0.71	25	0.680
604	A	19	18	0.76	25	0.720
605	A	13	12	0.74	25	0.480
606	A	15	14	0.77	25	0.560
607	A	19	18	0.76	25	0.720
608	A	19	18	0.77	25	0.720
609	A	21	20	0.75	25	0.800
610	A	17	16	0.68	25	0.640
611	A	18	17	0.70	25	0.680
612	A	18	17	0.72	25	0.680
613	A	19	18	0.75	25	0.720
614	A	21	20	0.78	25	0.800
615	A	21	20	0.79	25	0.800
616	A	22	21	0.77	25	0.840
617	A	19	18	0.68	25	0.720
618	A	20	19	0.70	25	0.760
619	A	21	20	0.73	25	0.800
620	A	22	21	0.76	25	0.840
621	A	23	22	0.77	25	0.880
622	A	24	23	0.80	25	0.920
623	A	25	24	0.77	25	0.960
624	A	6	6	1.00	23	0.261
625	A	6	6	1.00	23	0.261
626	A	6	6	1.00	23	0.261
627	A	6	6	1.00	23	0.261
628	A	9	9	1.04	25	0.360
629	A	9	9	1.03	25	0.360
630	A	9	9	1.03	25	0.360
631	A	9	9	1.04	25	0.360
632	A	16	15	0.69	25	0.600
633	A	16	15	0.67	25	0.600
634	A	17	16	0.73	25	0.640

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	17	16	0.71	25	0.640
636	A	5	4	0.79	25	0.160
637	A	5	4	0.79	25	0.160
638	A	5	4	0.80	25	0.160
639	A	5	4	0.80	25	0.160
640	A	8	7	1.19	23	0.304
641	A	9	9	1.00	23	0.391
642	A	6	6	1.00	21	0.286
643	A	7	6	1.13	23	0.261
644	A	5	4	0.96	23	0.174
645	A	4	3	1.03	23	0.130
646	A	6	5	0.86	21	0.238
647	A	6	5	0.88	21	0.238
648	A	4	3	1.00	21	0.143
649	A	8	7	1.08	21	0.333
650	A	10	9	1.10	21	0.429
651	A	5	4	1.16	21	0.190
652	A	5	4	1.16	19	0.211
653	A	5	4	1.15	19	0.211
654	A	5	4	1.15	21	0.190
655	A	12	12	1.27	26	0.462
656	A	10	10	1.36	26	0.385
657	A	10	10	1.36	26	0.385
658	A	8	8	1.42	26	0.308
659	A	8	8	1.42	26	0.308
660	A	10	10	1.30	26	0.385
661	A	10	10	1.25	26	0.385
662	A	12	12	1.15	26	0.462
663	A	14	14	1.17	28	0.500
664	A	12	12	1.21	28	0.429
665	A	12	12	1.21	28	0.429
666	A	10	10	1.26	28	0.357
667	A	10	10	1.26	28	0.357
668	A	8	8	1.25	28	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
669	A	8	8	1.25	28	0.286
670	A	10	10	1.16	28	0.357
671	A	10	10	1.15	28	0.357
672	A	12	12	1.09	28	0.429
673	A	10	10	1.11	30	0.333
674	A	8	8	1.15	30	0.267
675	A	6	6	1.25	30	0.200
676	A	4	4	1.00	30	0.133
677	A	11	10	0.99	30	0.333
678	A	17	16	0.84	30	0.533
679	A	17	16	0.87	30	0.533
680	A	21	20	0.75	30	0.667
681	A	10	10	1.13	30	0.333
682	A	8	8	1.21	30	0.267
683	A	6	6	1.31	30	0.200
684	A	4	4	1.00	30	0.133
685	A	13	12	0.75	30	0.400
686	A	15	14	0.85	30	0.467
687	A	19	18	0.72	30	0.600
688	A	9	8	1.00	26	0.308
689	A	9	8	1.00	26	0.308
690	A	9	8	1.00	24	0.333
691	A	9	8	1.00	26	0.308
692	A	9	8	1.00	26	0.308
693	A	9	8	1.11	28	0.286
694	A	9	8	1.12	28	0.286
695	A	10	9	1.20	23	0.391
696	A	11	11	1.13	23	0.478
697	A	8	8	1.26	21	0.381
698	A	9	8	1.12	23	0.348
699	A	7	6	0.95	23	0.261
700	A	6	5	0.97	23	0.217

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 249
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3.5 $\int \sec^{2}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 273
3.6 $\int (a+ia\tan(c+dx))\,dx$ . . . 278
3.7 $\int \cos^{2}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 282
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3.10 $\int \cos^{8}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 299
3.11 $\int \sec^{7}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 306
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3.13 $\int \sec^{3}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 319
3.14 $\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 325
3.15 $\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 330
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3.17 $\int \cos^{5}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 341
3.18 $\int \cos^{7}(c+dx)(a+ia\tan(c+dx))\,dx$ . . . 347
3.19 $\int \sec^{8}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 353
3.20 $\int \sec^{6}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 359
3.21 $\int \sec^{4}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 365
3.22 $\int \sec^{2}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 370
3.23 $\int (a+ia\tan(c+dx))^2\,dx$ . . . 375
3.24 $\int \cos^{2}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 380
3.25 $\int \cos^{4}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 385
3.26 $\int \cos^{6}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 390
3.27 $\int \cos^{8}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 396
3.28 $\int \sec^{5}(c+dx)(a+ia\tan(c+dx))^2\,dx$ . . . 402

3.29 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$ 410
3.30 $\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$ 417
3.31 $\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$ 423
3.32 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$ 428
3.33 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$ 433
3.34 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$ 439
3.35 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$ 446
3.36 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$ 454
3.37 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$ 460
3.38 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$ 466
3.39 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$ 471
3.40 $\int (a+ia\tan(c+dx))^3\,dx$ 476
3.41 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$ 482
3.42 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$ 487
3.43 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$ 492
3.44 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$ 498
3.45 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$ 505
3.46 $\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$ 512
3.47 $\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$ 518
3.48 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$ 524
3.49 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$ 529
3.50 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$ 536
3.51 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$ 543
3.52 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$ 552
3.53 $\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$ 560
3.54 $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$ 567
3.55 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$ 576
3.56 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$ 583
3.57 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$ 589
3.58 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$ 596
3.59 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$ 603
3.60 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$ 609
3.61 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$ 615
3.62 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$ 621
3.63 $\int (a+ia\tan(c+dx))^5\,dx$ 626
3.64 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$ 633
3.65 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$ 639
3.66 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$ 645
3.67 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$ 651
3.68 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$ 657

3.69 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 664
3.70 $\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 671
3.71 $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 679
3.72 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 687
3.73 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 694
3.74 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 699
3.75 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 706
3.76 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$ . . . 713
3.77 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 721
3.78 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 728
3.79 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 735
3.80 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 742
3.81 $\int (a+ia\tan(c+dx))^8\,dx$ . . . 748
3.82 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 756
3.83 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 762
3.84 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 769
3.85 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 775
3.86 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 780
3.87 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 786
3.88 $\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 793
3.89 $\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 799
3.90 $\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 806
3.91 $\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 813
3.92 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 823
3.93 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 833
3.94 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 843
3.95 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 851
3.96 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 857
3.97 $\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 865
3.98 $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$ . . . 873
3.99 $\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 882
3.100 $\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 887
3.101 $\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 892
3.102 $\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 897
3.103 $\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 902
3.104 $\int \frac{1}{a+ia\tan(c+dx)}\,dx$ . . . 907
3.105 $\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 912
3.106 $\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 917

3.107 $\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 923

3.108 $\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 930

3.109 $\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 936

3.110 $\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 941

3.111 $\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 945

3.112 $\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 950

3.113 $\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$ . . . 956

3.114 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 962

3.115 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 967

3.116 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 972

3.117 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 977

3.118 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 982

3.119 $\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx$ . . . 987

3.120 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 992

3.121 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 997

3.122 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1003

3.123 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1010

3.124 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1017

3.125 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1023

3.126 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1028

3.127 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1034

3.128 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1040

3.129 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$ . . . 1046

3.130 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1052

3.131 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1058

3.132 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1064

3.133 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1069

3.134 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1074

3.135 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1079

3.136 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1084

3.137 $\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1089

3.138 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1094

3.139 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$ . . . 1100

3.140 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1106
3.141 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1114
3.142 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1120
3.143 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1126
3.144 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1131
3.145 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1137
3.146 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1143
3.147 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$ . . . 1149
3.148 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1156
3.149 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1162
3.150 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1167
3.151 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1172
3.152 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1177
3.153 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1183
3.154 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1188
3.155 $\int \frac{1}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1193
3.156 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1199
3.157 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1205
3.158 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1211
3.159 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1218
3.160 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1226
3.161 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1232
3.162 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1238
3.163 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1244
3.164 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1251
3.165 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$ . . . 1258
3.166 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$ . . . 1265
3.167 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$ . . . 1271
3.168 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$ . . . 1277
3.169 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$ . . . 1283
3.170 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$ . . . 1288
3.171 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$ . . . 1294
3.172 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$ . . . 1300

3.173 $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$ 1305
3.174 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1313
3.175 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1320
3.176 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1327
3.177 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1337
3.178 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1346
3.179 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1353
3.180 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1359
3.181 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1366
3.182 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1375
3.183 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1390
3.184 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$ 1409
3.185 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$ 1428
3.186 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$ 1435
3.187 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$ 1441
3.188 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx$ 1447
3.189 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx$ 1452
3.190 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$ 1457
3.191 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$ 1463
3.192 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$ 1469
3.193 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx$ 1476
3.194 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$ 1483
3.195 $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$ 1489
3.196 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx$ 1496
3.197 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx$ 1502
3.198 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx$ 1508
3.199 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx$ 1514
3.200 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$ 1520
3.201 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$ 1527
3.202 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx$ 1535
3.203 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx$ 1542
3.204 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$ 1550
3.205 $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$ 1557
3.206 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx$ 1564
3.207 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx$ 1570

3.208 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx$ 1576

3.209 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx$ 1582

3.210 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx$ 1588

3.211 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$ 1595

3.212 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$ 1602

3.213 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$ 1610

3.214 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$ 1618

3.215 $\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$ 1625

3.216 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx$ 1633

3.217 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$ 1640

3.218 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx$ 1647

3.219 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx$ 1653

3.220 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx$ 1659

3.221 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$ 1666

3.222 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$ 1673

3.223 $\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx$ 1681

3.224 $\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx$ 1688

3.225 $\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx$ 1694

3.226 $\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx$ 1700

3.227 $\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx$ 1705

3.228 $\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx$ 1711

3.229 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx$ 1716

3.230 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx$ 1721

3.231 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx$ 1727

3.232 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx$ 1733

3.233 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$ 1740

3.234 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx$ 1747

3.235 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx$ 1753

3.236 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx$ 1760

3.237 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$ 1766

3.238 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$ 1772

3.239 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$ 1777

3.240 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$ 1782

3.241 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$ 1788
3.242 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$ 1794
3.243 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$ 1801
3.244 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$ 1808
3.245 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$ 1816
3.246 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx$ 1824
3.247 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$ 1831
3.248 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx$ 1838
3.249 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx$ 1844
3.250 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$ 1850
3.251 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx$ 1856
3.252 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$ 1863
3.253 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$ 1870
3.254 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$ 1878
3.255 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx$ 1886
3.256 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$ 1894
3.257 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$ 1901
3.258 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx$ 1908
3.259 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx$ 1914
3.260 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx$ 1920
3.261 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx$ 1927
3.262 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$ 1934
3.263 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx$ 1942
3.264 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx$ 1947
3.265 $\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$ 1952
3.266 $\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$ 1958
3.267 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx$ 1963
3.268 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$ 1968
3.269 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$ 1973
3.270 $\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$ 1979
3.271 $\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx$ 1984
3.272 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx$ 1989

3.273 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx$ 1994
3.274 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx$ 2000
3.275 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$ 2005
3.276 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$ 2011
3.277 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx$ 2017
3.278 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx$ 2023
3.279 $\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2029
3.280 $\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2035
3.281 $\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2041
3.282 $\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2046
3.283 $\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2051
3.284 $\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2057
3.285 $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2065
3.286 $\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2076
3.287 $\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2082
3.288 $\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2087
3.289 $\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2092
3.290 $\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2096
3.291 $\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2102
3.292 $\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$ 2110
3.293 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2120
3.294 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2126
3.295 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2132
3.296 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2137
3.297 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2141
3.298 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2147
3.299 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2154
3.300 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2164
3.301 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2170
3.302 $\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2176
3.303 $\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2181
3.304 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2186
3.305 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$ 2193
3.306 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ 2201
3.307 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ 2207
3.308 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ 2213
3.309 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ 2218
3.310 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ 2223

3.311 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2229
3.312 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2235
3.313 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2243
3.314 $\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2250
3.315 $\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2255
3.316 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2260
3.317 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2265
3.318 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$ . . . 2273
3.319 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2281
3.320 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2287
3.321 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2293
3.322 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2298
3.323 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2303
3.324 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2309
3.325 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2315
3.326 $\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2322
3.327 $\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2328
3.328 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2334
3.329 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2339
3.330 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2344
3.331 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2352
3.332 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$ . . . 2361
3.333 $\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2371
3.334 $\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2377
3.335 $\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2383
3.336 $\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2388
3.337 $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2393
3.338 $\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2400
3.339 $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2409
3.340 $\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2424
3.341 $\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2431
3.342 $\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2437
3.343 $\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2442
3.344 $\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2447
3.345 $\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2452
3.346 $\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$ . . . 2459

3.347 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2468
3.348 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2474
3.349 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2479
3.350 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2484
3.351 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2489
3.352 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2496
3.353 $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2506
3.354 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2523
3.355 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2529
3.356 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2535
3.357 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2540
3.358 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2545
3.359 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2551
3.360 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2557
3.361 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$ . . . 2565
3.362 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2574
3.363 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2580
3.364 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2585
3.365 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2590
3.366 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2595
3.367 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2600
3.368 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2608
3.369 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2620
3.370 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2627
3.371 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2633
3.372 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2639
3.373 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2644
3.374 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2651
3.375 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2658
3.376 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2664
3.377 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$ . . . 2673
3.378 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ . . . 2685

3.379 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2691
3.380 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2696
3.381 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2701
3.382 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2706
3.383 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2711
3.384 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2720
3.385 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2734
3.386 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2740
3.387 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2746
3.388 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2751
3.389 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2759
3.390 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2766
3.391 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2774
3.392 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2781
3.393 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ 2792
3.394 $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$ 2806
3.395 $\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$ 2817
3.396 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx$ 2827
3.397 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$ 2832
3.398 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$ 2837
3.399 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$ 2843
3.400 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$ 2849
3.401 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$ 2861
3.402 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$ 2873
3.403 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$ 2883
3.404 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx$ 2894
3.405 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx$ 2899
3.406 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx$ 2904
3.407 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx$ 2909
3.408 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$ 2915
3.409 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$ 2928
3.410 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$ 2940
3.411 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$ 2953

3.412 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx$ 2963

3.413 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx$ 2968

3.414 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx$ 2973

3.415 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx$ 2978

3.416 $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 2984

3.417 $\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 2994

3.418 $\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 3003

3.419 $\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$ 3008

3.420 $\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$ 3013

3.421 $\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$ 3019

3.422 $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$ 3025

3.423 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$ 3032

3.424 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$ 3045

3.425 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$ 3055

3.426 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$ 3060

3.427 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$ 3065

3.428 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx$ 3070

3.429 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$ 3076

3.430 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$ 3083

3.431 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$ 3095

3.432 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$ 3106

3.433 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$ 3111

3.434 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx$ 3116

3.435 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$ 3121

3.436 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$ 3127

3.437 $\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 3135

3.438 $\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 3141

3.439 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 3147

3.440 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 3153

3.441 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$ 3159

3.442 $\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx$ 3165

3.443 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$ 3171

3.444 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$ 3181

3.445 $\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$ 3190

3.446 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx$ 3198

3.447 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx$ 3203

3.448 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$ 3208

3.449 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx$ 3214

3.450 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx$ 3221

3.451 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx$ 3227

3.452 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx$ 3232

3.453 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$ 3237

3.454 $\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx$ 3242

3.455 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$ 3247

3.456 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$ 3252

3.457 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx$ 3257

3.458 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx$ 3262

3.459 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx$ 3267

3.460 $\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$ 3272

3.461 $\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 3277

3.462 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$ 3282

3.463 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx$ 3287

3.464 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx$ 3292

3.465 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3297

3.466 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3302

3.467 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3307

3.468 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3312

3.469 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3316

3.470 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3320

3.471 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3324

3.472 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3329

3.473 $\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3334

3.474 $\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3339

3.475 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3344

3.476 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$ 3349

3.477 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$ 3354

3.478 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$ 3359

3.479 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx$ 3364

3.480 $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx$ 3369

3.481 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx$ 3374

3.482 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx$ 3379

3.483 $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$ 3384

3.484 $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$ 3392

3.485 $\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$ 3400

3.486 $\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$ 3406

3.487 $\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx$ 3412

3.488 $\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$ 3417

3.489 $\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$ 3423

3.490 $\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$ 3429

3.491 $\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$ 3435

3.492 $\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx$ 3442

3.493 $\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$ 3447

3.494 $\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$ 3453

3.495 $\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$ 3458

3.496 $\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$ 3463

3.497 $\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$ 3468

3.498 $\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$ 3473

3.499 $\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$ 3478

3.500 $\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$ 3484

3.501 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$ 3489

3.502 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$ 3494

3.503 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx$ 3499

3.504 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$ 3504

3.505 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$ 3510

3.506 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$ 3515

3.507 $\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$ 3521

3.508 $\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$ 3526

3.509 $\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$ 3532

3.510 $\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$ 3537

3.511 $\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$ 3542

3.512 $\int \sec(c+dx)(a+b\tan(c+dx))\,dx$ 3547

3.513 $\int \cos(c+dx)(a+b\tan(c+dx))\,dx$ 3552

3.514 $\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$ 3557

3.515 $\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$ 3562

3.516 $\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$ 3568

3.517 $\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$ 3573

3.518 $\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$ 3579

3.519 $\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$ 3585

3.520 $\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$ 3590

3.521 $\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$ 3595
3.522 $\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$ 3600
3.523 $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$ 3607
3.524 $\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$ 3616
3.525 $\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$ 3624
3.526 $\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$ 3631
3.527 $\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$ 3637
3.528 $\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$ 3643
3.529 $\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$ 3649
3.530 $\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$ 3656
3.531 $\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$ 3664
3.532 $\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$ 3670
3.533 $\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$ 3676
3.534 $\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$ 3682
3.535 $\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$ 3687
3.536 $\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$ 3694
3.537 $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$ 3701
3.538 $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$ 3710
3.539 $\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$ 3718
3.540 $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$ 3725
3.541 $\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$ 3732
3.542 $\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$ 3738
3.543 $\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$ 3745
3.544 $\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$ 3753
3.545 $\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$ 3759
3.546 $\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx$ 3764
3.547 $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$ 3769
3.548 $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$ 3776
3.549 $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$ 3784
3.550 $\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$ 3793
3.551 $\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$ 3799
3.552 $\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$ 3804
3.553 $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$ 3810
3.554 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ 3818
3.555 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ 3825
3.556 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ 3831
3.557 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ 3836

3.558 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3841
3.559 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3848
3.560 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3856
3.561 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3868
3.562 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3878
3.563 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3885
3.564 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3891
3.565 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$ . . . 3899
3.566 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3910
3.567 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3917
3.568 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3923
3.569 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3928
3.570 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3933
3.571 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3941
3.572 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3951
3.573 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3963
3.574 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3973
3.575 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3980
3.576 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3988
3.577 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$ . . . 3998
3.578 $\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$ . . . 4010
3.579 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$ . . . 4017
3.580 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$ . . . 4023
3.581 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$ . . . 4029
3.582 $\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 4034
3.583 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 4039
3.584 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 4045
3.585 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$ . . . 4051
3.586 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$ . . . 4057
3.587 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$ . . . 4064
3.588 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$ . . . 4071
3.589 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$ . . . 4077
3.590 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$ . . . 4083
3.591 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$ . . . 4090

3.592 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$ 4097
3.593 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx$ 4104
3.594 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$ 4112
3.595 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$ 4119
3.596 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$ 4127
3.597 $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$ 4134
3.598 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx$ 4142
3.599 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$ 4148
3.600 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx$ 4155
3.601 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx$ 4161
3.602 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$ 4168
3.603 $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$ 4175
3.604 $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$ 4186
3.605 $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$ 4200
3.606 $\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$ 4209
3.607 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$ 4219
3.608 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$ 4231
3.609 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$ 4245
3.610 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$ 4260
3.611 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$ 4272
3.612 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$ 4287
3.613 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$ 4299
3.614 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$ 4313
3.615 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$ 4326
3.616 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$ 4341
3.617 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$ 4359
3.618 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$ 4374
3.619 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$ 4389
3.620 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$ 4402
3.621 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$ 4417
3.622 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$ 4434
3.623 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$ 4455
3.624 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$ 4479
3.625 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$ 4484

3.626 $\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$ 4489
3.627 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$ 4494
3.628 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$ 4499
3.629 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$ 4505
3.630 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$ 4511
3.631 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$ 4517
3.632 $\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$ 4523
3.633 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$ 4533
3.634 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$ 4543
3.635 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$ 4555
3.636 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$ 4567
3.637 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$ 4573
3.638 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$ 4579
3.639 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$ 4586
3.640 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$ 4592
3.641 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$ 4598
3.642 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx$ 4604
3.643 $\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$ 4609
3.644 $\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$ 4615
3.645 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$ 4621
3.646 $\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$ 4626
3.647 $\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$ 4632
3.648 $\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$ 4637
3.649 $\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$ 4641
3.650 $\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$ 4647
3.651 $\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$ 4654
3.652 $\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$ 4659
3.653 $\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$ 4664
3.654 $\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$ 4669
3.655 $\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$ 4674
3.656 $\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$ 4681
3.657 $\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$ 4688
3.658 $\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$ 4694
3.659 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$ 4700
3.660 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$ 4706

3.661 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$ 4712
3.662 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$ 4718
3.663 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$ 4726
3.664 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$ 4735
3.665 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$ 4743
3.666 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$ 4751
3.667 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$ 4758
3.668 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$ 4765
3.669 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$ 4771
3.670 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$ 4777
3.671 $\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx$ 4785
3.672 $\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx$ 4791
3.673 $\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx$ 4798
3.674 $\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx$ 4805
3.675 $\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$ 4811
3.676 $\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$ 4817
3.677 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$ 4822
3.678 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$ 4831
3.679 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$ 4842
3.680 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$ 4854
3.681 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 4870
3.682 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 4877
3.683 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 4883
3.684 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$ 4889
3.685 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$ 4894
3.686 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$ 4903
3.687 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$ 4913
3.688 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx$ 4925
3.689 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx$ 4931
3.690 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx$ 4936
3.691 $\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx$ 4941
3.692 $\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$ 4947
3.693 $\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$ 4952
3.694 $\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$ 4958

3.695 $\int (d\cos(e+fx))^m (a+b\tan(e+fx))^3\,dx$ 4964
3.696 $\int (d\cos(e+fx))^m (a+b\tan(e+fx))^2\,dx$ 4970
3.697 $\int (d\cos(e+fx))^m (a+b\tan(e+fx))\,dx$ 4976
3.698 $\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$ 4981
3.699 $\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$ 4987
3.700 $\int (d\cos(e+fx))^m (a+b\tan(e+fx))^n\,dx$ 4993

3.1 $\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$

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3.1.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\tan(c+dx)}{d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{a\tan^9(c+dx)}{9d}$$

output

```
1/10*I*a*sec(d*x+c)^10/d+a*tan(d*x+c)/d+4/3*a*tan(d*x+c)^3/d+6/5*a*tan(d*x
+c)^5/d+4/7*a*tan(d*x+c)^7/d+1/9*a*tan(d*x+c)^9/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\left(\tan(c+dx)+\frac{4}{3}\tan^3(c+dx)+\frac{6}{5}\tan^5(c+dx)+\frac{4}{7}\tan^7(c+dx)+\frac{1}{9}\tan^9(c+dx)\right)}{d}$$

input

```
Integrate[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/10)*a*Sec[c + d*x]^10)/d + (a*(Tan[c + d*x] + (4*Tan[c + d*x]^3)/3 + (
6*Tan[c + d*x]^5)/5 + (4*Tan[c + d*x]^7)/7 + Tan[c + d*x]^9/9))/d
```

3.1.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^{10}(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^{10}(c+dx)dx+\frac{ia\sec^{10}(c+dx)}{10d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^{10}dx+\frac{ia\sec^{10}(c+dx)}{10d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int\left(\tan^8(c+dx)+4\tan^6(c+dx)+6\tan^4(c+dx)+4\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{9}\tan^9(c+dx)-\frac{4}{7}\tan^7(c+dx)-\frac{6}{5}\tan^5(c+dx)-\frac{4}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

input
```
Int[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

output

```
((I/10)*a*Sec[c + d*x]^10)/d - (a*(-Tan[c + d*x] - (4*Tan[c + d*x]^3)/3 -
(6*Tan[c + d*x]^5)/5 - (4*Tan[c + d*x]^7)/7 - Tan[c + d*x]^9/9))/d
```

3.1.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.1.4 Maple [A] (verified)

Time = 176.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

method	result
risch	$\frac{256ia\left(252\,\mathrm{e}^{10i(dx+c)}+210\,\mathrm{e}^{8i(dx+c)}+120\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{2i(dx+c)}+1\right)}{315d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{10}}$
derivativedivides	$\frac{a\bigg(\tan(dx+c)+\frac{i\left(\tan^{10}(dx+c)\right)}{10}+\frac{\left(\tan^{9}(dx+c)\right)}{9}+\frac{i\left(\tan^{8}(dx+c)\right)}{2}+\frac{4\left(\tan^{7}(dx+c)\right)}{7}+i\left(\tan^{6}(dx+c)\right)+\frac{6\left(\tan^{5}(dx+c)\right)}{5}+i(\tan^{4}}{d}$
default	$\frac{a\bigg(\tan(dx+c)+\frac{i\left(\tan^{10}(dx+c)\right)}{10}+\frac{\left(\tan^{9}(dx+c)\right)}{9}+\frac{i\left(\tan^{8}(dx+c)\right)}{2}+\frac{4\left(\tan^{7}(dx+c)\right)}{7}+i\left(\tan^{6}(dx+c)\right)+\frac{6\left(\tan^{5}(dx+c)\right)}{5}+i(\tan^{4}}{d}$

input

```
int(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
256/315*I*a*(252*exp(10*I*(d*x+c))+210*exp(8*I*(d*x+c))+120*exp(6*I*(d*x+c
))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^10
```

3.1.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(82) = 164$.

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx =$$

$$-\frac{256\left(-252i\,ae^{(10i\,dx+10i\,c)}-210i\,ae^{(8i\,dx+8i\,c)}-120i\,ae^{(6i\,dx+6i}\right.}{315\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de\right.}$$

input
```
integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-256/315*(-252*I*a*e^(10*I*d*x + 10*I*c) - 210*I*a*e^(8*I*d*x + 8*I*c) - 1
20*I*a*e^(6*I*d*x + 6*I*c) - 45*I*a*e^(4*I*d*x + 4*I*c) - 10*I*a*e^(2*I*d*
x + 2*I*c) - I*a)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) +
45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d
*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 1
20*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*
I*c) + d)
```

3.1.6 Sympy [A] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\begin{cases}\frac{a\left(\frac{\tan^{9}(c+dx)}{9}+\frac{4\tan^{7}(c+dx)}{7}+\frac{6\tan^{5}(c+dx)}{5}+\frac{4\tan^{3}(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^{10}(c+dx)}{10}}{d} & \text{for } d\neq 0\\ x(ia\tan(c)+a)\sec^{10}(c) & \text{otherwise}\end{cases}$$

input
```
integrate(sec(d*x+c)**10*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**9/9 + 4*tan(c + d*x)**7/7 + 6*tan(c + d*x)**5
/5 + 4*tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**10/10)/d, Ne(
d, 0)), (x*(I*a*tan(c) + a)*sec(c)**10, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{63i\,a\tan\left(dx+c\right)^{10}+70\,a\tan\left(dx+c\right)^{9}+315i\,a\tan\left(dx+c\right)^{8}+360\,a\tan\left(dx+c\right)^{7}+630i\,a\tan\left(dx+c\right)}{}$$

input
```
integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/630*(63*I*a*tan(d*x + c)^10 + 70*a*tan(d*x + c)^9 + 315*I*a*tan(d*x + c)
^8 + 360*a*tan(d*x + c)^7 + 630*I*a*tan(d*x + c)^6 + 756*a*tan(d*x + c)^5
+ 630*I*a*tan(d*x + c)^4 + 840*a*tan(d*x + c)^3 + 315*I*a*tan(d*x + c)^2 +
 630*a*tan(d*x + c))/d
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-63i\,a\tan\left(dx+c\right)^{10}-70\,a\tan\left(dx+c\right)^{9}-315i\,a\tan\left(dx+c\right)^{8}-360\,a\tan\left(dx+c\right)^{7}-630i\,a\tan\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/630*(-63*I*a*tan(d*x + c)^10 - 70*a*tan(d*x + c)^9 - 315*I*a*tan(d*x +
c)^8 - 360*a*tan(d*x + c)^7 - 630*I*a*tan(d*x + c)^6 - 756*a*tan(d*x + c)^
5 - 630*I*a*tan(d*x + c)^4 - 840*a*tan(d*x + c)^3 - 315*I*a*tan(d*x + c)^2
 - 630*a*tan(d*x + c))/d
```

3.1.9 Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$

$$= \frac{a\left(-\cos\left(c+d\,x\right)^{10}\,63\mathrm{i}+256\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^9+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^7+96\,\sin\left(c+d\right.\right.}{630\,d\cos\left(c+d\,x\right)^{10}}$$

input

```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^10,x)
```

output

```
(a*(70*cos(c + d*x)*sin(c + d*x) + 80*cos(c + d*x)^3*sin(c + d*x) + 96*cos
(c + d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + 256*cos(c + d
*x)^9*sin(c + d*x) - cos(c + d*x)^10*63i + 63i))/(630*d*cos(c + d*x)^10)
```

3.2 $\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$

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3.2.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^8(c+dx)}{8d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^7(c+dx)}{7d}$$

output
```
1/8*I*a*sec(d*x+c)^8/d+a*tan(d*x+c)/d+a*tan(d*x+c)^3/d+3/5*a*tan(d*x+c)^5/
d+1/7*a*tan(d*x+c)^7/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^8(c+dx)}{8d}+\frac{a\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/8)*a*Sec[c + d*x]^8)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c
+ d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

3.2.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^8(c+dx)dx+\frac{ia\sec^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^8dx+\frac{ia\sec^8(c+dx)}{8d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int\left(\tan^6(c+dx)+3\tan^4(c+dx)+3\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a(-\frac{1}{7}\tan^7(c+dx)-\frac{3}{5}\tan^5(c+dx)-\tan^3(c+dx)-\tan(c+dx))}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/8)*a*Sec[c + d*x]^8)/d - (a*(-Tan[c + d*x] - Tan[c + d*x]^3 - (3*Tan[c
 + d*x]^5)/5 - Tan[c + d*x]^7/7))/d
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.2.4 Maple [A] (verified)

Time = 63.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

method	result
risch	$\frac{32 i a \left(70 \,{\mathrm e}^{8 i \left(d x +c \right)}+56 \,{\mathrm e}^{6 i \left(d x +c \right)}+28 \,{\mathrm e}^{4 i \left(d x +c \right)}+8 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{35 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{8}}$
derivativedivides	$\frac{a \Bigl(\tan \left(d x +c \right)+\frac{i \left(\tan^{8}\left(d x +c \right)\right)}{8}+\frac{\left(\tan^{7}\left(d x +c \right)\right)}{7}+\frac{i \left(\tan^{6}\left(d x +c \right)\right)}{2}+\frac{3 \left(\tan^{5}\left(d x +c \right)\right)}{5}+\frac{3 i \left(\tan^{4}\left(d x +c \right)\right)}{4}+\tan^{3}\left(d x +c \right)+\frac{i \bigl(\tan^{2}(d x}{2}}{d}$
default	$\frac{a \Bigl(\tan \left(d x +c \right)+\frac{i \left(\tan^{8}\left(d x +c \right)\right)}{8}+\frac{\left(\tan^{7}\left(d x +c \right)\right)}{7}+\frac{i \left(\tan^{6}\left(d x +c \right)\right)}{2}+\frac{3 \left(\tan^{5}\left(d x +c \right)\right)}{5}+\frac{3 i \left(\tan^{4}\left(d x +c \right)\right)}{4}+\tan^{3}\left(d x +c \right)+\frac{i \bigl(\tan^{2}(d x}{2}}{d}$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
32/35*I*a*(70*exp(8*I*(d*x+c))+56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*e
xp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^8
```

3.2.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(67) = 134.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.04

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx =$$

$$-\frac{32\left(-70i\,ae^{(8i\,dx+8i\,c)}-56i\,ae^{(6i\,dx+6i\,c)}-28i\,ae^{(4i\,dx+4i\,c)}-8i\,ae^{(2i\,dx+2i\,c}\right.}{35\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i}\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-32/35*(-70*I*a*e^(8*I*d*x + 8*I*c) - 56*I*a*e^(6*I*d*x + 6*I*c) - 28*I*a*
e^(4*I*d*x + 4*I*c) - 8*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(16*I*d*x + 16
*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(1
0*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) +
28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

3.2.6 Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^7(c+dx)}{7}+\frac{3\tan^5(c+dx)}{5}+\tan^3(c+dx)+\tan(c+dx)\right)+\frac{ia\sec^8(c+dx)}{8}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^8(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**7/7 + 3*tan(c + d*x)**5/5 + tan(c + d*x)**3 +
 tan(c + d*x)) + I*a*sec(c + d*x)**8/8)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*
sec(c)**8, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{35i\,a\tan\left(dx+c\right)^8+40\,a\tan\left(dx+c\right)^7+140i\,a\tan\left(dx+c\right)^6+168\,a\tan\left(dx+c\right)^5+210i\,a\tan\left(dx+c\right)^4}{280\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/280*(35*I*a*tan(d*x + c)^8 + 40*a*tan(d*x + c)^7 + 140*I*a*tan(d*x + c)^
6 + 168*a*tan(d*x + c)^5 + 210*I*a*tan(d*x + c)^4 + 280*a*tan(d*x + c)^3 +
 140*I*a*tan(d*x + c)^2 + 280*a*tan(d*x + c))/d
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-35i\,a\tan\left(dx+c\right)^8-40\,a\tan\left(dx+c\right)^7-140i\,a\tan\left(dx+c\right)^6-168\,a\tan\left(dx+c\right)^5-210i\,a\tan\left(dx\right.}{280\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/280*(-35*I*a*tan(d*x + c)^8 - 40*a*tan(d*x + c)^7 - 140*I*a*tan(d*x + c
)^6 - 168*a*tan(d*x + c)^5 - 210*I*a*tan(d*x + c)^4 - 280*a*tan(d*x + c)^3
 - 140*I*a*tan(d*x + c)^2 - 280*a*tan(d*x + c))/d
```

3.2.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\,\sin\left(c+d\,x\right)\,\left(280\cos\left(c+d\,x\right)^7+\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)\,140\mathrm{i}+280\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)^2+\mathrm{c}\right.}{}$$

input

```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^8,x)
```

output

```
(a*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c + d
*x)*140i + 280*cos(c + d*x)^7 + sin(c + d*x)^7*35i + cos(c + d*x)^2*sin(c
+ d*x)^5*140i + 168*cos(c + d*x)^3*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c +
 d*x)^3*210i + 280*cos(c + d*x)^5*sin(c + d*x)^2))/(280*d*cos(c + d*x)^8)
```

3.2. $\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$

3.3 $\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$

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3.3.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^6(c+dx)}{6d} + \frac{a\tan(c+dx)}{d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d}$$

output
```
1/6*I*a*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c
)^5/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^6(c+dx)}{6d} + \frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/6)*a*Sec[c + d*x]^6)/d + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan
[c + d*x]^5/5))/d
```

3.3.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^6(c+dx)dx+\frac{ia\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{ia\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int \left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

input

```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output

```
((I/6)*a*Sec[c + d*x]^6)/d - (a*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Ta
n[c + d*x]^5/5))/d
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.3.4 Maple [A] (verified)

Time = 18.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{16ia\left(20\,e^{6i(dx+c)}+15\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}+1\right)}{15d\left(e^{2i(dx+c)}+1\right)^{6}}$	56
derivativedivides	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^{6}(dx+c)\right)}{6}+\frac{\left(\tan^{5}(dx+c)\right)}{5}+\frac{i\left(\tan^{4}(dx+c)\right)}{2}+\frac{2\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}\right)}{d}$	66
default	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^{6}(dx+c)\right)}{6}+\frac{\left(\tan^{5}(dx+c)\right)}{5}+\frac{i\left(\tan^{4}(dx+c)\right)}{2}+\frac{2\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}\right)}{d}$	66

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
16/15*I*a*(20*exp(6*I*(d*x+c))+15*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d
/(exp(2*I*(d*x+c))+1)^6
```

3.3.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(54) = 108$.

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.89

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{16\left(-20i\,ae^{(6i\,dx+6i\,c)} - 15i\,ae^{(4i\,dx+4i\,c)} - 6i\,ae^{(2i\,dx+2i\,c)} - i\,a\right)}{15\left(de^{(12i\,dx+12i\,c)} + 6\,de^{(10i\,dx+10i\,c)} + 15\,de^{(8i\,dx+8i\,c)} + 20\,de^{(6i\,dx+6i\,c)} + 15\,de^{(4i\,dx+4i\,c)} + 6\,de^{(2i\,dx+2i\,c)} + \right.}$$

input

```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output

```
-16/15*(-20*I*a*e^(6*I*d*x + 6*I*c) - 15*I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e
^(2*I*d*x + 2*I*c) - I*a)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*
I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d
*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

3.3.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^5{(c+dx)}}{5} + \frac{2\tan^3{(c+dx)}}{3} + \tan{(c+dx)}\right) + \frac{ia\sec^6{(c+dx)}}{6}}{d} & \text{for } d \neq 0 \\ x(ia\tan{(c)} + a)\sec^6{(c)} & \text{otherwise} \end{cases}$$

input

```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

output

```
Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + I
*a*sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**6, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{5i\,a\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15i\,a\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15i\,a\tan\left(dx+c\right)^2+30}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(5*I*a*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*I*a*tan(d*x + c)^4 +
20*a*tan(d*x + c)^3 + 15*I*a*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-5i\,a\tan\left(dx+c\right)^6-6\,a\tan\left(dx+c\right)^5-15i\,a\tan\left(dx+c\right)^4-20\,a\tan\left(dx+c\right)^3-15i\,a\tan\left(dx+c\right)^2-}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/30*(-5*I*a*tan(d*x + c)^6 - 6*a*tan(d*x + c)^5 - 15*I*a*tan(d*x + c)^4
- 20*a*tan(d*x + c)^3 - 15*I*a*tan(d*x + c)^2 - 30*a*tan(d*x + c))/d
```

3.3.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a\,\sin\left(c+d\,x\right)\,\left(30\cos\left(c+d\,x\right)^5+\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,15\mathrm{i}+20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2+\cos\left(c\right.\right.}{30\,d\cos\left(c+d\,x\right)^6}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^6,x)
```

output
```
(a*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*
x)*15i + 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*
x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

3.3. $\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$

3.4 $\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$

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3.4.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

output

```
1/4*I*a*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^4(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input

```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output

```
((I/4)*a*Sec[c + d*x]^4)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.4.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^4(c+dx)dx+\frac{ia\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{ia\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx))}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

input

```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output

```
((I/4)*a*Sec[c + d*x]^4)/d - (a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

3.4.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.4.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^{4}(dx+c)\right)}{4}+\frac{\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}\right)}{d}$	45
default	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^{4}(dx+c)\right)}{4}+\frac{\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}\right)}{d}$	45
risch	$\frac{4ia\left(6\,e^{4i(dx+c)}+4\,e^{2i(dx+c)}+1\right)}{3d\left(e^{2i(dx+c)}+1\right)^{4}}$	45

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
a/d*(tan(d*x+c)+1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3+1/2*I*tan(d*x+c)^2)
```

3.4.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(40) = 80$.

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{4\left(-6i\,ae^{(4i\,dx+4i\,c)}-4i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-4/3*(-6*I*a*e^(4*I*d*x + 4*I*c) - 4*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(
8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d
*e^(2*I*d*x + 2*I*c) + d)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^4(c+dx)}{4}}{d} & \text{for } d \neq 0 \\ x(ia\tan(c)+a)\sec^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**4/4)/
d, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**4, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3i\,a\tan{(dx+c)}^4+4\,a\tan{(dx+c)}^3+6i\,a\tan{(dx+c)}^2+12\,a\tan{(dx+c)}}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*I*a*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*I*a*tan(d*x + c)^2 + 1
2*a*tan(d*x + c))/d
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{-3i\,a\tan{(dx+c)}^4-4\,a\tan{(dx+c)}^3-6i\,a\tan{(dx+c)}^2-12\,a\tan{(dx+c)}}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(-3*I*a*tan(d*x + c)^4 - 4*a*tan(d*x + c)^3 - 6*I*a*tan(d*x + c)^2 -
 12*a*tan(d*x + c))/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\frac{1{}\mathrm{i}\,a\,\tan(c+d\,x)^4}{4}+\frac{a\,\tan(c+d\,x)^3}{3}+\frac{1{}\mathrm{i}\,a\,\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

input

```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^4,x)
```

output

```
(a*tan(c + d*x) + (a*tan(c + d*x)^2*1i)/2 + (a*tan(c + d*x)^3)/3 + (a*tan(
c + d*x)^4*1i)/4)/d
```

3.4. $\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$

3.5 $\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx$

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3.5.1 Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{i(a+ia\tan(c+dx))^2}{2ad}$$

output
```
-1/2*I*(a+I*a*tan(d*x+c))^2/a/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^2(c+dx)dx+\frac{ia\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{ia\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{a\int 1d(-\tan(c+dx))}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{a\tan(c+dx)}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

input

```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output

```
((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

3.5.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{ia\left(-\frac{\left(\tan^{2}(dx+c)\right)}{2}+i\tan(dx+c)\right)}{d}$	28
default	$-\frac{ia\left(-\frac{\left(\tan^{2}(dx+c)\right)}{2}+i\tan(dx+c)\right)}{d}$	28
risch	$\frac{2ia\left(2\,\mathrm{e}^{2i(dx+c)}+1\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}$	34

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I*a/d*(-1/2*tan(d*x+c)^2+I*tan(d*x+c))
```

3.5.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\left(-2i\,ae^{(2i\,dx+2i\,c)} - i\,a\right)}{de^{(4i\,dx+4i\,c)} + 2\,de^{(2i\,dx+2i\,c)} + d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(-2*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*
d*x + 2*I*c) + d)
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{\frac{ia\tan^2(c+dx)}{2} + a\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(ia\tan(c)+a)\sec^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((I*a*tan(c + d*x)**2/2 + a*tan(c + d*x))/d, Ne(d, 0)), (x*(I*a*
tan(c) + a)*sec(c)**2, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\,(i\,a\tan(dx+c)+a)^2}{2\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*I*(I*a*tan(d*x + c) + a)^2/(a*d)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{-i\,a\tan\left(dx+c\right)^2-2\,a\tan\left(dx+c\right)}{2\,d}$$

input

```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output

```
-1/2*(-I*a*tan(d*x + c)^2 - 2*a*tan(d*x + c))/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\tan(c+d\,x)\ (2+\tan(c+d\,x)\ 1\mathrm{i})}{2\,d}$$

input

```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^2,x)
```

output

```
(a*tan(c + d*x)*(tan(c + d*x)*1i + 2))/(2*d)
```

3.6 $\int (a+ia\tan(c+dx))\,dx$

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3.6.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int (a+ia\tan(c+dx))\,dx=ax-\frac{ia\log(\cos(c+dx))}{d}$$

output
```
a*x-I*a*ln(cos(d*x+c))/d
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+ia\tan(c+dx))\,dx=ax-\frac{ia\log(\cos(c+dx))}{d}$$

input
```
Integrate[a + I*a*Tan[c + d*x],x]
```

output
```
a*x - (I*a*Log[Cos[c + d*x]])/d
```

3.6.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax-\frac{ia\log(\cos(c+dx))}{d}$$

input
```
Int[a + I*a*Tan[c + d*x],x]
```

output
```
a*x - (I*a*Log[Cos[c + d*x]])/d
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
default	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
norman	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
parallelrisch	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
parts	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
derivativedivides	$\frac{a\left(\frac{i\ln(1+\tan^2(dx+c))}{2}+\arctan(\tan(dx+c))\right)}{d}$	28
risch	$-\frac{ia\ln(e^{2i(dx+c)}+1)}{d}-\frac{2ac}{d}$	28

input
```
int(a+I*a*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*I*a/d*ln(1+tan(d*x+c)^2)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int (a+ia\tan(c+dx))\,dx = -\frac{i\,a\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x, algorithm="fricas")
```

output
```
-I*a*log(e^(2*I*d*x + 2*I*c) + 1)/d
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int (a+ia\tan(c+dx))\,dx = -\frac{ia\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x)
```

output
```
-I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(c+dx))\,dx = ax+\frac{i\,a\log\left(\sec\left(dx+c\right)\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x, algorithm="maxima")
```

output
```
a*x + I*a*log(sec(d*x + c))/d
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int (a+ia\tan(c+dx))\,dx=ax-\frac{i\,a\log\left(\left|\cos\left(dx+c\right)\right|\right)}{d}$$

input
```
integrate(a+I*a*tan(d*x+c),x, algorithm="giac")
```

output
```
a*x - I*a*log(abs(cos(d*x + c)))/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(c+dx))\,dx=\frac{a\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d}$$

input
```
int(a + a*tan(c + d*x)*1i,x)
```

output
```
(a*log(tan(c + d*x) + 1i)*1i)/d
```

3.7 $\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$

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3.7.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ax}{2} - \frac{ia\cos^2(c+dx)}{2d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x-1/2*I*a*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(c+dx)}{2d} - \frac{ia\cos^2(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Sin[2*(c + d*x)])/(4
*d)
```

3.7.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3967}$$

$$a\int \cos^2(c+dx)dx-\frac{ia\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{ia\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{ia\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 24}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{ia\cos^2(c+dx)}{2d}$$

input

```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

output

```
((-1/2*I)*a*Cos[c + d*x]^2)/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)
)
```

3.7.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.7.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{ax}{2}-\frac{ia\,e^{2i(dx+c)}}{4d}$	22
derivativedivides	$\frac{-\frac{ia\left(\cos^{2}(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42
default	$\frac{-\frac{ia\left(\cos^{2}(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x-1/4*I/d*a*exp(2*I*(d*x+c))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,adx - i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ax}{2} + \begin{cases} -\frac{iae^{2ic}e^{2idx}}{4d} & \text{for } d \neq 0 \\ \frac{axe^{2ic}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

output
```
a*x/2 + Piecewise((-I*a*exp(2*I*c)*exp(2*I*d*x)/(4*d), Ne(d, 0)), (a*x*exp
(2*I*c)/2, True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{(dx+c)a + \frac{a\tan(dx+c)-i\,a}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)*a + (a*tan(d*x + c) - I*a)/(tan(d*x + c)^2 + 1))/d
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,adx - i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\,x}{2} + \frac{a}{2\,d\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i),x)
```

output
```
(a*x)/2 + a/(2*d*(tan(c + d*x) + 1i))
```

3.8 $\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$

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3.8.8 Giac [A] (verification not implemented) 291
3.8.9 Mupad [B] (verification not implemented) 291

3.8.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3ax}{8}-\frac{ia\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
3/8*a*x-1/4*I*a*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x
+c)^3*sin(d*x+c)/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(12c+12dx-8i\cos^4(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(12*c + 12*d*x - (8*I)*Cos[c + d*x]^4 + 8*Sin[2*(c + d*x)] + Sin[4*(c +
 d*x)]))/(32*d)
```

3.8.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^4(c+dx)dx-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{ia\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{ia\cos^4(c+dx)}{4d}$$

input

```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/4*I)*a*Cos[c + d*x]^4)/d + a*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (
3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.8.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.8.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^{4}(dx+c)\right)}{4}+a\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
default	$\frac{-\frac{ia\left(\cos^{4}(dx+c)\right)}{4}+a\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
risch	$\frac{3ax}{8}-\frac{ia\,e^{4i(dx+c)}}{32d}-\frac{ia\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	53

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*I*a*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(12\,adxe^{(2i\,dx+2i\,c)} - i\,ae^{(6i\,dx+6i\,c)} - 6i\,ae^{(4i\,dx+4i\,c)} + 2i\,a\right)e^{(-2i\,dx-2i\,c)}}{32\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) - I*a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*
I*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*I*c)/d
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3ax}{8} + \begin{cases} \frac{\left(-256iad^2e^{6ic}e^{4idx} - 1536iad^2e^{4ic}e^{2idx} + 512iad^2e^{-2idx}\right)e^{-2ic}}{8192d^3} & \text{for } d^3e^{2ic} \neq 0 \\ x\left(-\frac{3a}{8} + \frac{\left(ae^{6ic} + 3ae^{4ic} + 3ae^{2ic} + a\right)e^{-2ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

output
```
3*a*x/8 + Piecewise(((-256*I*a*d**2*exp(6*I*c)*exp(4*I*d*x) - 1536*I*a*d**
2*exp(4*I*c)*exp(2*I*d*x) + 512*I*a*d**2*exp(-2*I*d*x))*exp(-2*I*c)/(8192*
d**3), Ne(d**3*exp(2*I*c), 0)), (x*(-3*a/8 + (a*exp(6*I*c) + 3*a*exp(4*I*c
) + 3*a*exp(2*I*c) + a)*exp(-2*I*c)/8), True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,(dx+c)a + \frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2i\,a}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*I*a)/(tan(
d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{\big(12\,adxe^{(2i\,dx+2i\,c)} + i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) - i\,ae^{(6i\,dx+6i\,c)}}{32\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) + I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) - I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) -
I*a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*I*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x -
 2*I*c)/d
```

3.8.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{3\,a\,x}{8} + \frac{\frac{3\,a\,\tan(c+d\,x)^2}{8} + \frac{3i\,a\,\tan(c+d\,x)}{8} + \frac{a}{4}}{d\,\left(\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,1i+\tan\left(c+d\,x\right)+1i\right)}$$

input

```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i),x)
```

output

```
(3*a*x)/8 + (a/4 + (a*tan(c + d*x)*3i)/8 + (3*a*tan(c + d*x)^2)/8)/(d*(tan
(c + d*x) + tan(c + d*x)^2*1i + tan(c + d*x)^3 + 1i))
```

3.8. $\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$

3.9 $\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$

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3.9.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5ax}{16}-\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

output
```
5/16*a*x-1/6*I*a*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(
d*x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(60c+60dx-32i\cos^6(c+dx)+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx)))}{192d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(60*c + 60*d*x - (32*I)*Cos[c + d*x]^6 + 45*Sin[2*(c + d*x)] + 9*Sin[4*
(c + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^6(c+dx)dx-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)-\frac{ia\cos^6(c+dx)}{6d}$$

$$\Big\downarrow 24$$

$$a\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)\right)-\frac{ia\cos^6(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/6*I)*a*Cos[c + d*x]^6)/d + a*((Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + (
5*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d
*x])/(2*d)))/4))/6)
```

3.9.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.9.4 Maple [A] (verified)

Time = 14.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^{6}(dx+c)\right)}{6}+a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
default	$\frac{-\frac{ia\left(\cos^{6}(dx+c)\right)}{6}+a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
risch	$\frac{5ax}{16}-\frac{ia\,\mathrm{e}^{6i(dx+c)}}{192d}-\frac{ia\cos(4dx+4c)}{32d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{5ia\cos(2dx+2c)}{64d}+\frac{15a\sin(2dx+2c)}{64d}$	84

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/6*I*a*cos(d*x+c)^6+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(
d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \cos^{6}(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{\left(120\,adxe^{(4i\,dx+4i\,c)}-2i\,ae^{(10i\,dx+10i\,c)}-15i\,ae^{(8i\,dx+8i\,c)}-60i\,ae^{(6i\,dx+6i\,c)}+30i\,ae^{(2i\,dx+2i\,c)}+3i\,a\right)e^{(-4i\,dx-}}{384\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/384*(120*a*d*x*e^(4*I*d*x + 4*I*c) - 2*I*a*e^(10*I*d*x + 10*I*c) - 15*I*
a*e^(8*I*d*x + 8*I*c) - 60*I*a*e^(6*I*d*x + 6*I*c) + 30*I*a*e^(2*I*d*x + 2
*I*c) + 3*I*a)*e^(-4*I*d*x - 4*I*c)/d
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.37

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5ax}{16} + \begin{cases} \frac{(-33554432iad^4e^{12ic}e^{6idx}-251658240iad^4e^{10ic}e^{4idx}-1006632960iad^4e^{8ic}e^{2idx}+503316480iad^4e^{4ic}e^{-2idx}+50331648iad^4e^{2ic}e^{-4idx})e^{-}}{6442450944d^5} \\ x\left(-\frac{5a}{16}+\frac{(ae^{10ic}+5ae^{8ic}+10ae^{6ic}+10ae^{4ic}+5ae^{2ic}+a)e^{-4ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

output
```
5*a*x/16 + Piecewise(((-33554432*I*a*d**4*exp(12*I*c)*exp(6*I*d*x) - 25165
8240*I*a*d**4*exp(10*I*c)*exp(4*I*d*x) - 1006632960*I*a*d**4*exp(8*I*c)*ex
p(2*I*d*x) + 503316480*I*a*d**4*exp(4*I*c)*exp(-2*I*d*x) + 50331648*I*a*d*
*4*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(6442450944*d**5), Ne(d**5*exp(6*
I*c), 0)), (x*(-5*a/16 + (a*exp(10*I*c) + 5*a*exp(8*I*c) + 10*a*exp(6*I*c)
 + 10*a*exp(4*I*c) + 5*a*exp(2*I*c) + a)*exp(-4*I*c)/32), True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{15\,(dx+c)a+\frac{15\,a\tan(dx+c)^5+40\,a\tan(dx+c)^3+33\,a\tan(dx+c)-8i\,a}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(15*(d*x + c)*a + (15*a*tan(d*x + c)^5 + 40*a*tan(d*x + c)^3 + 33*a*t
an(d*x + c) - 8*I*a)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2
 + 1))/d
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.43

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{\left(120\,adxe^{(4i\,dx+2i\,c)} + 12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) - 2i\,ae^{(10}}{384\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/384*(120*a*d*x*e^(4*I*d*x + 2*I*c) + 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) - 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*
I*c)) - 2*I*a*e^(10*I*d*x + 8*I*c) - 15*I*a*e^(8*I*d*x + 6*I*c) - 60*I*a*e
^(6*I*d*x + 4*I*c) + 30*I*a*e^(2*I*d*x) + 3*I*a*e^(-2*I*c))*e^(-4*I*d*x -
2*I*c)/d
```

3.9.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5\,a\,x}{16}$$
$$+ \frac{\frac{5\,a\tan(c+d\,x)^4}{16} + \frac{5\mathrm{i}\,a\tan(c+d\,x)^3}{16} + \frac{25\,a\tan(c+d\,x)^2}{48} + \frac{25\mathrm{i}\,a\tan(c+d\,x)}{48} + \frac{a}{6}}{d\,\left(\tan\left(c+d\,x\right)^5 + \tan\left(c+d\,x\right)^4\,1\mathrm{i} + 2\tan\left(c+d\,x\right)^3 + \tan\left(c+d\,x\right)^2\,2\mathrm{i} + \tan\left(c+d\,x\right) + 1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i),x)
```

output
```
(5*a*x)/16 + (a/6 + (a*tan(c + d*x)*25i)/48 + (25*a*tan(c + d*x)^2)/48 + (
a*tan(c + d*x)^3*5i)/16 + (5*a*tan(c + d*x)^4)/16)/(d*(tan(c + d*x) + tan(
c + d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x)^4*1i + tan(c + d*x)^5 + 1i
))
```

3.10 $\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$

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3.10.1 Optimal result

Integrand size = 22, antiderivative size = 111

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = {} & \frac{35ax}{128}-\frac{ia\cos^8(c+dx)}{8d}\\ & +\frac{35a\cos(c+dx)\sin(c+dx)}{128d}\\ & +\frac{35a\cos^3(c+dx)\sin(c+dx)}{192d}\\ & +\frac{7a\cos^5(c+dx)\sin(c+dx)}{48d}\\ & +\frac{a\cos^7(c+dx)\sin(c+dx)}{8d}\end{aligned}$$

output
```
35/128*a*x-1/8*I*a*cos(d*x+c)^8/d+35/128*a*cos(d*x+c)*sin(d*x+c)/d+35/192*
a*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a*cos(d*x
+c)^7*sin(d*x+c)/d
```

3.10.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a(840c+840dx-384i\cos^8(c+dx)+672\sin(2(c+dx))+168\sin(4(c+dx))+32\sin(6(c+dx))+3\sin}{3072d}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(840*c + 840*d*x - (384*I)*Cos[c + d*x]^8 + 672*Sin[2*(c + d*x)] + 168*
Sin[4*(c + d*x)] + 32*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)]))/(3072*d)
```

3.10.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^8}dx$$
$$\Big\downarrow \ 3967$$
$$a\int \cos^8(c+dx)dx-\frac{ia\cos^8(c+dx)}{8d}$$
$$\Big\downarrow \ 3042$$
$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^8dx-\frac{ia\cos^8(c+dx)}{8d}$$
$$\Big\downarrow \ 3115$$
$$a\left(\frac{7}{8}\int \cos^6(c+dx)dx+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{8}\int \sin\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(c+dx)dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+dx)}{8d}\right)-\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7(c+d}{8d}$$

$$\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\frac{\sin(c+dx)\cos^7}{8d}$$

$$\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)+\frac{\sin(c+dx)\cos^5(c+dx)}{6d}\right)+\sin(c$$

$$\frac{ia\cos^8(c+dx)}{8d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7}{8}\left(\frac{\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5}{6}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c-}{2d}\right.\right.\right.\right.$$

$$\frac{ia\cos^8(c+dx)}{8d}$$

3.10. $\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/8*I)*a*Cos[c + d*x]^8)/d + a*((Cos[c + d*x]^7*Sin[c + d*x])/(8*d) + (
7*((Cos[c + d*x]^5*Sin[c + d*x])/(6*d) + (5*((Cos[c + d*x]^3*Sin[c + d*x])
/(4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6))/8)
```

3.10.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.10.4 Maple [A] (verified)

Time = 48.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{ia\left(\cos^{8}(dx+c)\right)}{8}+a\left(\frac{\left(\cos^{7}(dx+c)+\frac{7\left(\cos^{5}(dx+c)\right)}{6}+\frac{35\left(\cos^{3}(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
default	$\frac{-\frac{ia\left(\cos^{8}(dx+c)\right)}{8}+a\left(\frac{\left(\cos^{7}(dx+c)+\frac{7\left(\cos^{5}(dx+c)\right)}{6}+\frac{35\left(\cos^{3}(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
risch	$\frac{35ax}{128}-\frac{ia\,\mathrm{e}^{8i(dx+c)}}{1024d}-\frac{ia\cos(6dx+6c)}{128d}+\frac{a\sin(6dx+6c)}{96d}-\frac{7ia\cos(4dx+4c)}{256d}+\frac{7a\sin(4dx+4c)}{128d}-\frac{7ia\cos(2dx+}{128d}$

input

```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/8*I*a*cos(d*x+c)^8+a*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos
(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \cos^{8}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\big(840\,adxe^{(6i\,dx+6i\,c)}-3i\,ae^{(14i\,dx+14i\,c)}-28i\,ae^{(12i\,dx+12i\,c)}-126i\,ae^{(10i\,dx+10i\,c)}-420i\,ae^{(8i\,dx+8i\,c)}+252i\,ae}{3072\,d}$$

input

```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output

```
1/3072*(840*a*d*x*e^(6*I*d*x + 6*I*c) - 3*I*a*e^(14*I*d*x + 14*I*c) - 28*I
*a*e^(12*I*d*x + 12*I*c) - 126*I*a*e^(10*I*d*x + 10*I*c) - 420*I*a*e^(8*I*
d*x + 8*I*c) + 252*I*a*e^(4*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x + 2*I*c) +
4*I*a)*e^(-6*I*d*x - 6*I*c)/d
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.51

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{35ax}{128} + \begin{cases} \frac{(-10133099161583616iad^6e^{20ic}e^{8idx}-94575592174780416iad^6e^{18ic}e^{6idx}-425590164786511872iad^6e^{16ic}e^{4idx}-1418633882621706240iad}{1037629354146} \\ x\left(-\frac{35a}{128}+\frac{(ae^{14ic}+7ae^{12ic}+21ae^{10ic}+35ae^{8ic}+35ae^{6ic}+21ae^{4ic}+7ae^{2ic}+a)e^{-6ic}}{128}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

output
```
35*a*x/128 + Piecewise(((-10133099161583616*I*a*d**6*exp(20*I*c)*exp(8*I*d
*x) - 94575592174780416*I*a*d**6*exp(18*I*c)*exp(6*I*d*x) - 42559016478651
1872*I*a*d**6*exp(16*I*c)*exp(4*I*d*x) - 1418633882621706240*I*a*d**6*exp(
14*I*c)*exp(2*I*d*x) + 851180329573023744*I*a*d**6*exp(10*I*c)*exp(-2*I*d*
x) + 141863388262170624*I*a*d**6*exp(8*I*c)*exp(-4*I*d*x) + 13510798882111
488*I*a*d**6*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(10376293541461622784*
d**7), Ne(d**7*exp(12*I*c), 0)), (x*(-35*a/128 + (a*exp(14*I*c) + 7*a*exp(
12*I*c) + 21*a*exp(10*I*c) + 35*a*exp(8*I*c) + 35*a*exp(6*I*c) + 21*a*exp(
4*I*c) + 7*a*exp(2*I*c) + a)*exp(-6*I*c)/128), True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{105\,(dx+c)a+\frac{105\,a\tan(dx+c)^7+385\,a\tan(dx+c)^5+511\,a\tan(dx+c)^3+279\,a\tan(dx+c)-48i\,a}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{384\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/384*(105*(d*x + c)*a + (105*a*tan(d*x + c)^7 + 385*a*tan(d*x + c)^5 + 51
1*a*tan(d*x + c)^3 + 279*a*tan(d*x + c) - 48*I*a)/(tan(d*x + c)^8 + 4*tan(
d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.36

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$

$$= \frac{\left(840\,adxe^{(6i\,dx+2i\,c)} + 84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) - 3i\,ae^{(14}}{}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/3072*(840*a*d*x*e^(6*I*d*x + 2*I*c) + 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(
2*I*d*x + 2*I*c) + 1) - 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2
*I*c)) - 3*I*a*e^(14*I*d*x + 10*I*c) - 28*I*a*e^(12*I*d*x + 8*I*c) - 126*I
*a*e^(10*I*d*x + 6*I*c) - 420*I*a*e^(8*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x
- 2*I*c) + 252*I*a*e^(4*I*d*x) + 4*I*a*e^(-4*I*c))*e^(-6*I*d*x - 2*I*c)/d
```

3.10.9 Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{35\,a\,x}{128}$$

$$+ \frac{\frac{35\,a\tan(c+d\,x)^6}{128} + \frac{35\mathrm{i}\,a\tan(c+d\,x)^5}{128} + \frac{35\,a\tan(c+d\,x)^4}{48} + \frac{35\mathrm{i}\,a\tan(c+d\,x)^3}{48} + \frac{77\,a\tan(c+d\,x)^2}{128} + \frac{77\mathrm{i}\,a\tan(}{128}}{d\,\left(\tan\left(c+d\,x\right)^7 + \tan\left(c+d\,x\right)^6\,1\mathrm{i} + 3\tan\left(c+d\,x\right)^5 + \tan\left(c+d\,x\right)^4\,3\mathrm{i} + 3\tan\left(c+d\,x\right)^3 + \tan\left(c+d\right.\right.}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i),x)
```

output
```
(35*a*x)/128 + (a/8 + (a*tan(c + d*x)*77i)/128 + (77*a*tan(c + d*x)^2)/128
 + (a*tan(c + d*x)^3*35i)/48 + (35*a*tan(c + d*x)^4)/48 + (a*tan(c + d*x)^
5*35i)/128 + (35*a*tan(c + d*x)^6)/128)/(d*(tan(c + d*x) + tan(c + d*x)^2*
3i + 3*tan(c + d*x)^3 + tan(c + d*x)^4*3i + 3*tan(c + d*x)^5 + tan(c + d*x
)^6*1i + tan(c + d*x)^7 + 1i))
```

3.11 $\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$

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3.11.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d}$$

output
```
5/16*a*arctanh(sin(d*x+c))/d+1/7*I*a*sec(d*x+c)^7/d+5/16*a*sec(d*x+c)*tan(
d*x+c)/d+5/24*a*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a*sec(d*x+c)^5*tan(d*x+c)/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d}$$

input
```
Integrate[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
(5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((I/7)*a*Sec[c + d*x]^7)/d + (5*a*Sec
[c + d*x]*Tan[c + d*x])/(16*d) + (5*a*Sec[c + d*x]^3*Tan[c + d*x])/(24*d)
+ (a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

3.11.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^7(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^7(c+dx)dx+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^7dx+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{5}{6}\int \sec^5(c+dx)dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5}{6}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \ 3042$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)+\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \ 4255$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx}{6d}\right.$$
$$\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \ 3042$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(}{6d}\right.$$
$$\frac{ia\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \ 4257$$

$$a\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+}{6d}\right.$$
$$\frac{ia\sec^7(c+dx)}{7d}$$

input
```
Int[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/7)*a*Sec[c + d*x]^7)/d + a*((Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (5*(
(Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (S
ec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/6)
```

3.11. $\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$

3.11.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.11.4 Maple [A] (verified)

Time = 36.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{ia}{7\cos(dx+c)^{7}}+a\left(-\left(-\frac{\left(\sec^{5}(dx+c)\right)}{6}-\frac{5\left(\sec^{3}(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
default	$\frac{\frac{ia}{7\cos(dx+c)^{7}}+a\left(-\left(-\frac{\left(\sec^{5}(dx+c)\right)}{6}-\frac{5\left(\sec^{3}(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
risch	$-\frac{ia\left(105\,\mathrm{e}^{13i(dx+c)}+700\,\mathrm{e}^{11i(dx+c)}+1981\,\mathrm{e}^{9i(dx+c)}-3072\,\mathrm{e}^{7i(dx+c)}-1981\,\mathrm{e}^{5i(dx+c)}-700\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{168d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{7}}+$

input
```
int(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/7*I*a/cos(d*x+c)^7+a*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*se
c(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

3.11.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(86) = 172$.

Time = 0.25 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.80

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{-210i\,ae^{(13i\,dx+13i\,c)} - 1400i\,ae^{(11i\,dx+11i\,c)} - 3962i\,ae^{(9i\,dx+9i\,c)} + 6144i\,ae^{(7i\,dx+7i\,c)} + 3962i\,ae^{(5i\,dx+5i\,c)} + 1}{}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/336*(-210*I*a*e^(13*I*d*x + 13*I*c) - 1400*I*a*e^(11*I*d*x + 11*I*c) - 3
962*I*a*e^(9*I*d*x + 9*I*c) + 6144*I*a*e^(7*I*d*x + 7*I*c) + 3962*I*a*e^(5
*I*d*x + 5*I*c) + 1400*I*a*e^(3*I*d*x + 3*I*c) + 210*I*a*e^(I*d*x + I*c) +
 105*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d
*x + 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*
e^(4*I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I
) - 105*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*
I*d*x + 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21
*a*e^(4*I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c)
- I))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*
d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d
*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

3.11.6 Sympy [F]

$$\begin{aligned}\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = ia\bigg(&\int \left(-i\sec^7\left(c+dx\right)\right)\,dx\\ &+ \int \tan\left(c+dx\right)\sec^7\left(c+dx\right)dx\bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sec(c + d*x)**7, x) + Integral(tan(c + d*x)*sec(c + d*x)*
*7, x))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx =$$

$$-\frac{7\,a\left(\frac{2\left(15\,\sin(dx+c)^5-40\,\sin(dx+c)^3+33\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-}{672\,d}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/672*(7*a*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(
sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x
 + c) + 1) + 15*log(sin(d*x + c) - 1)) - 96*I*a/cos(d*x + c)^7)/d
```

3.11.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(86) = 172.

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.85

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$

$$= \frac{105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(231\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-336i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{1}\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/336*(105*a*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a*log(tan(1/2*d*x + 1/2*c
) - 1) + 2*(231*a*tan(1/2*d*x + 1/2*c)^13 - 336*I*a*tan(1/2*d*x + 1/2*c)^1
2 - 196*a*tan(1/2*d*x + 1/2*c)^11 + 595*a*tan(1/2*d*x + 1/2*c)^9 - 1680*I*
a*tan(1/2*d*x + 1/2*c)^8 - 595*a*tan(1/2*d*x + 1/2*c)^5 - 1008*I*a*tan(1/2
*d*x + 1/2*c)^4 + 196*a*tan(1/2*d*x + 1/2*c)^3 - 231*a*tan(1/2*d*x + 1/2*c
) - 48*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.52

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

$$-\frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}-\frac{85\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+10\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{85\,a\tan(}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan(\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^7,x)
```

output
```
(5*a*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a*2i)/7 + (11*a*tan(c/2 + (d*x)/
2))/8 - (7*a*tan(c/2 + (d*x)/2)^3)/6 + a*tan(c/2 + (d*x)/2)^4*6i + (85*a*t
an(c/2 + (d*x)/2)^5)/24 + a*tan(c/2 + (d*x)/2)^8*10i - (85*a*tan(c/2 + (d*
x)/2)^9)/24 + (7*a*tan(c/2 + (d*x)/2)^11)/6 + a*tan(c/2 + (d*x)/2)^12*2i -
 (11*a*tan(c/2 + (d*x)/2)^13)/8)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 +
 (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c
/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.12 $\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$

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3.12.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+1/5*I*a*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*
x+c)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output
```
(3*a*ArcTanh[Sin[c + d*x]])/(8*d) + ((I/5)*a*Sec[c + d*x]^5)/d + (3*a*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^5(c+dx)dx+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/5)*a*Sec[c + d*x]^5)/d + a*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(
ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

3.12.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.12.4 Maple [A] (verified)

Time = 8.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{ia\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}-128\,\mathrm{e}^{5i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{20d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}+\frac{3a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}-\frac{3a\ln(\mathrm{e}^{i(dx+c)}-i)}{8d}$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*I*a/cos(d*x+c)^5+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c
)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.12.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(66) = 132.

Time = 0.24 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.63

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{-30i\,ae^{(9i\,dx+9i\,c)} - 140i\,ae^{(7i\,dx+7i\,c)} + 256i\,ae^{(5i\,dx+5i\,c)} + 140i\,ae^{(3i\,dx+3i\,c)} + 30i\,ae^{(i\,dx+i\,c)} + 15\,(ae^{(10i\,dx+1}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/40*(-30*I*a*e^(9*I*d*x + 9*I*c) - 140*I*a*e^(7*I*d*x + 7*I*c) + 256*I*a*
e^(5*I*d*x + 5*I*c) + 140*I*a*e^(3*I*d*x + 3*I*c) + 30*I*a*e^(I*d*x + I*c)
 + 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10*a*e^(6*I*d*x
 + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c) + a)*log(e^
(I*d*x + I*c) + I) - 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c)
 + 10*a*e^(6*I*d*x + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x +
2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*
I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d
*e^(2*I*d*x + 2*I*c) + d)
```

3.12.6 Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sec^5{(c+dx)}\right)\,dx + \int \tan{(c+dx)}\sec^5{(c+dx)}\,dx\right)$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sec(c + d*x)**5, x) + Integral(tan(c + d*x)*sec(c + d*x)*
*5, x))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{16i\,a}{\cos(dx+c)^5}}{80\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*
I*a/cos(d*x + c)^5)/d
```

3.12.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(66) = 132.

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.83

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{15\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right) - 15\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right) + \frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 40i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 10\,a}{}}{40\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*a*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a*log(tan(1/2*d*x + 1/2*c) -
 1) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*I*a*tan(1/2*d*x + 1/2*c)^8 - 10*
a*tan(1/2*d*x + 1/2*c)^7 - 80*I*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*
x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*I*a)/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.34

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^5,x)
```

output
```
(3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*2i)/5 + (5*a*tan(c/2 + (d*x)/2
))/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + a*tan(c/2 + (d*x)/2)^4*4i + (a*tan(c/2
 + (d*x)/2)^7)/2 + a*tan(c/2 + (d*x)/2)^8*2i - (5*a*tan(c/2 + (d*x)/2)^9)/
4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*
x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.13 $\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$

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3.13.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\,\text{arctanh}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+1/3*I*a*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*
x+c)/d
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\,\text{arctanh}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c +
d*x]*Tan[c + d*x])/(2*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4257}$$

$$a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/3)*a*Sec[c + d*x]^3)/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*d))
```

3.13.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.13.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
default	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
risch	$-\frac{ia\left(3\,\mathrm{e}^{5i(dx+c)}-8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$	94

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*I*a/cos(d*x+c)^3+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+t
an(d*x+c))))
```

3.13.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(46) = 92$.

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.33

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{-6i\,ae^{(5i\,dx+5i\,c)} + 16i\,ae^{(3i\,dx+3i\,c)} + 6i\,ae^{(i\,dx+i\,c)} + 3\left(ae^{(6i\,dx+6i\,c)} + 3\,ae^{(4i\,dx+4i\,c)} + 3\,ae^{(2i\,dx+2i\,c)} + a\right)\log}{6\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,d\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(-6*I*a*e^(5*I*d*x + 5*I*c) + 16*I*a*e^(3*I*d*x + 3*I*c) + 6*I*a*e^(I*
d*x + I*c) + 3*(a*e^(6*I*d*x + 6*I*c) + 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2
*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 3*(a*e^(6*I*d*x + 6*I*c) +
 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c
) - I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x
+ 2*I*c) + d)
```

3.13.6 Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sec^3\left(c+dx\right)\right)\,dx + \int \tan\left(c+dx\right)\sec^3\left(c+dx\right)dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sec(c + d*x)**3, x) + Integral(tan(c + d*x)*sec(c + d*x)*
*3, x))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=-\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{4i\,a}{\cos(dx+c)^3}}{12\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 4*I*a/cos(d*x + c)^3)/d
```

3.13.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(46) = 92.

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\,(3\,a\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-6i\,a\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4-3\,a\tan(\frac{1}{2}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a*log(tan(1/2*d*x + 1/2*c) - 1)
 + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*I*a*tan(1/2*d*x + 1/2*c)^4 - 3*a*tan(
1/2*d*x + 1/2*c) - 2*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.98

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + a\tan\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{a\,2\mathrm{i}}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input

```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^3,x)
```

output

```
(a*atanh(tan(c/2 + (d*x)/2)))/d - ((a*2i)/3 + a*tan(c/2 + (d*x)/2) + a*tan
(c/2 + (d*x)/2)^4*2i - a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2
- 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.14 $\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$

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3.14.6 Sympy [A] (verification not implemented) 328
3.14.7 Maxima [A] (verification not implemented) 328
3.14.8 Giac [B] (verification not implemented) 328
3.14.9 Mupad [B] (verification not implemented) 329

3.14.1 Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+I*a*sec(d*x+c)/d
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

3.14.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

3.14.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.14.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
default	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
risch	$\frac{2i\mathrm{e}^{i(dx+c)}a}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$	68

input

```
int(sec(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d*(I*a/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

3.14.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{2i\,ae^{(i\,dx+i\,c)}+\left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{de^{(2i\,dx+2i\,c)}+d}$$

input

```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output

```
(2*I*a*e^(I*d*x + I*c) + (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) +
 I) - (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x
+ 2*I*c) + d)
```

3.14.6 Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+ia\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + I*a*sec(c + d*x))/d, Ne(d
, 0)), (x*(I*a*tan(c) + a)*sec(c), True))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\log(\sec(dx+c)+\tan(dx+c))+\frac{ia}{\cos(dx+c)}}{d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + I*a/cos(d*x + c))/d
```

3.14.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-a\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)-\frac{2ia}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(tan(1/2*d*x + 1/2*c) + 1) - a*log(tan(1/2*d*x + 1/2*c) - 1) - 2*I*a
/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.14.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\big(\tan\big(\frac{c}{2}+\frac{d\,x}{2}\big)\big)}{d}-\frac{a\,2\mathrm{i}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a*2i)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.15 $\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$

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3.15.8 Giac [B] (verification not implemented) 333
3.15.9 Mupad [B] (verification not implemented) 334

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
-I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

3.15.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))\,dx={}&-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}\\&+\frac{a\cos(c)\sin(dx)}{d}+\frac{ia\sin(c)\sin(dx)}{d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
 + (I*a*Sin[c]*Sin[d*x])/d
```

3.15.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3967, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)}dx$$

$$\Big\downarrow \; 3967$$

$$a\int \cos(c+dx)dx-\frac{ia\cos(c+dx)}{d}$$

$$\Big\downarrow \; 3042$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-\frac{ia\cos(c+dx)}{d}$$

$$\Big\downarrow \; 3117$$

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

3.15.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.15.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{ia\,\mathrm{e}^{i(dx+c)}}{d}$	17
derivativedivides	$\frac{-ia\cos(dx+c)+a\sin(dx+c)}{d}$	24
default	$\frac{-ia\cos(dx+c)+a\sin(dx+c)}{d}$	24

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/d*a*exp(I*(d*x+c))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-I*a*e^(I*d*x + I*c)/d
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for } d\neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \frac{-i\,a\cos(dx+c)+a\sin(dx+c)}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(-I*a*cos(d*x + c) + a*sin(d*x + c))/d
```

3.15.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(24) = 48.

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx =$$

$$-\frac{4i\,ae^{(i\,dx+i\,c)}+a\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+a\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-a\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-a\log\left(-i\,e^{(i\,dx+}\right.}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*I*a*e^(I*d*x + I*c) + a*log(I*e^(I*d*x + I*c) + 1) + a*log(I*e^(I*
d*x + I*c) - 1) - a*log(-I*e^(I*d*x + I*c) + 1) - a*log(-I*e^(I*d*x + I*c)
 - 1))/d
```

3.15.9 Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx=\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input

```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i),x)
```

output

```
(2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))
```

3.16 $\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$

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3.16.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

output

```
-1/3*I*a*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.16.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

input

```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output

```
((-1/3*I)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3
*d)
```

3.16.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3967}$$

$$a\int \cos^3(c+dx)dx-\frac{ia\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{ia\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/3*I)*a*Cos[c + d*x]^3)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d
```

3.16.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.16.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^{3}(dx+c)\right)}{3}+\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}}{d}$	37
default	$\frac{-\frac{ia\left(\cos^{3}(dx+c)\right)}{3}+\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}}{d}$	37
risch	$-\frac{ia\,\mathrm{e}^{3i(dx+c)}}{12d}-\frac{ia\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}$	43

input

```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d*(-1/3*I*a*cos(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(-i\,ae^{(4i\,dx+4i\,c)} - 6i\,ae^{(2i\,dx+2i\,c)} + 3i\,a\right)e^{(-i\,dx-i\,c)}}{12\,d}$$

input

```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output

```
1/12*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) + 3*I*a)*e^(-I*
d*x - I*c)/d
```

3.16.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.28

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \begin{cases} \frac{\left(-8iad^2e^{4ic}e^{3idx} - 48iad^2e^{2ic}e^{idx} + 24iad^2e^{-idx}\right)e^{-ic}}{96d^3} & \text{for } d^3e^{ic} \neq 0 \\ \frac{x\left(ae^{4ic} + 2ae^{2ic} + a\right)e^{-ic}}{4} & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

output

```
Piecewise(((-8*I*a*d**2*exp(4*I*c)*exp(3*I*d*x) - 48*I*a*d**2*exp(2*I*c)*e
xp(I*d*x) + 24*I*a*d**2*exp(-I*d*x))*exp(-I*c)/(96*d**3), Ne(d**3*exp(I*c)
, 0)), (x*(a*exp(4*I*c) + 2*a*exp(2*I*c) + a)*exp(-I*c)/4, True))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\,a\cos\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(I*a*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

3.16.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(40) = 80.

Time = 0.40 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.26

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx =$$

$$-\frac{\left(9\,ae^{\left(i\,dx+i\,c\right)}\log\left(i\,e^{\left(i\,dx+i\,c\right)}+1\right)+6\,ae^{\left(i\,dx+i\,c\right)}\log\left(i\,e^{\left(i\,dx+i\,c\right)}-1\right)-9\,ae^{\left(i\,dx+i\,c\right)}\log\left(-i\,e^{\left(i\,dx+i\,c\right)}+1\right)-}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/48*(9*a*e^(I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 6*a*e^(I*d*x + I*c
)*log(I*e^(I*d*x + I*c) - 1) - 9*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 6*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3*a*e^(I*d*x + I*
c)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 4*I*a*e^(4*I*d*x + 4*I*c) + 24*I*a*e^(2*I*d*x + 2*I*c) - 12*I*a)
*e^(-I*d*x - I*c)/d
```

3.16.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{2\,a\left(-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}}{4}-\frac{\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2\,1\mathrm{i}}{4}+\frac{9\,\sin(c+d\,x)}{8}+\frac{\sin(3\,c+3\,d\,x)}{8}\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i),x)
```

output
```
(2*a*((9*sin(c + d*x))/8 + sin(3*c + 3*d*x)/8 - (cos(c/2 + (d*x)/2)^2*3i)/
4 - (cos((3*c)/2 + (3*d*x)/2)^2*1i)/4))/(3*d)
```

3.17 $\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$

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3.17.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

output
```
-1/5*I*a*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+
c)^5/d
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/5*I)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/
(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

3.17.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^5(c+dx)dx-\frac{ia\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{ia\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx))}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

input

```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

output

```
((-1/5*I)*a*Cos[c + d*x]^5)/d - (a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 -
 Sin[c + d*x]^5/5))/d
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.17.4 Maple [A] (verified)

Time = 6.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^5(dx+c)\right)}{5}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	47
default	$\frac{-\frac{ia\left(\cos^5(dx+c)\right)}{5}+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	47
risch	$-\frac{ia\,e^{5i(dx+c)}}{80d}-\frac{ia\cos(dx+c)}{8d}+\frac{5a\sin(dx+c)}{8d}-\frac{ia\cos(3dx+3c)}{16d}+\frac{5a\sin(3dx+3c)}{48d}$	74

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5*I*a*cos(d*x+c)^5+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d
*x+c))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(-3i\,ae^{(8i\,dx+8i\,c)}-20i\,ae^{(6i\,dx+6i\,c)}-90i\,ae^{(4i\,dx+4i\,c)}+60i\,ae^{(2i\,dx+2i\,c)}+5i\,a\right)e^{(-3i\,dx-3i\,c)}}{240\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/240*(-3*I*a*e^(8*I*d*x + 8*I*c) - 20*I*a*e^(6*I*d*x + 6*I*c) - 90*I*a*e^
(4*I*d*x + 4*I*c) + 60*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*e^(-3*I*d*x - 3*I*
c)/d
```

3.17.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{\left(-18432iad^4e^{9ic}e^{5idx}-122880iad^4e^{7ic}e^{3idx}-552960iad^4e^{5ic}e^{idx}+368640iad^4e^{3ic}e^{-idx}+30720iad^4e^{ic}e^{-3idx}\right)e^{-4ic}}{1474560d^5} & \text{for } d^5e^{4ic}\neq 0 \\ \frac{x\left(ae^{8ic}+4ae^{6ic}+6ae^{4ic}+4ae^{2ic}+a\right)e^{-3ic}}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-18432*I*a*d**4*exp(9*I*c)*exp(5*I*d*x) - 122880*I*a*d**4*exp(
7*I*c)*exp(3*I*d*x) - 552960*I*a*d**4*exp(5*I*c)*exp(I*d*x) + 368640*I*a*d
**4*exp(3*I*c)*exp(-I*d*x) + 30720*I*a*d**4*exp(I*c)*exp(-3*I*d*x))*exp(-4
*I*c)/(1474560*d**5), Ne(d**5*exp(4*I*c), 0)), (x*(a*exp(8*I*c) + 4*a*exp(
6*I*c) + 6*a*exp(4*I*c) + 4*a*exp(2*I*c) + a)*exp(-3*I*c)/16, True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{3i\,a\cos\left(dx+c\right)^5 - \left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/15*(3*I*a*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*a)/d
```

3.17.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(54) = 108.

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.55

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{(135\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 90\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right) - 135\,ae^{(3i\,dx+i\,c)}\log(-i\,e^{(i\,dx+i\,c}}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/960*(135*a*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a*e^(3*I*d
*x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 135*a*e^(3*I*d*x + I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 90*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 45
*a*e^(3*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 45*a*e^(3*I*d*x + I*c)*
log(-I*e^(I*d*x) + e^(-I*c)) + 12*I*a*e^(8*I*d*x + 6*I*c) + 80*I*a*e^(6*I*
d*x + 4*I*c) + 360*I*a*e^(4*I*d*x + 2*I*c) - 240*I*a*e^(2*I*d*x) - 20*I*a*
e^(-2*I*c))*e^(-3*I*d*x - I*c)/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\,a\left(-\frac{75\,\sin(c+d\,x)}{16}-\frac{25\,\sin(3\,c+3\,d\,x)}{32}-\frac{3\,\sin(5\,c+5\,d\,x)}{32}+\frac{\cos(c+d\,x)\,15\mathrm{i}}{16}+\frac{\cos(3\,c+3\,d\,x)\,15\mathrm{i}}{32}+\frac{\cos(5\,c+5\,d\,x)\,3\mathrm{i}}{32}\right)}{15\,d}$$

input

```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i),x)
```

output

```
-(2*a*((cos(c + d*x)*15i)/16 - (75*sin(c + d*x))/16 + (cos(3*c + 3*d*x)*15
i)/32 + (cos(5*c + 5*d*x)*3i)/32 - (25*sin(3*c + 3*d*x))/32 - (3*sin(5*c +
 5*d*x))/32))/(15*d)
```

3.18 $\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$

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3.18.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}$$

output
```
-1/7*I*a*cos(d*x+c)^7/d+a*sin(d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5
/d-1/7*a*sin(d*x+c)^7/d
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/7*I)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d
+ (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{ 3967}$$

$$a\int \cos^7(c+dx)dx-\frac{ia\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{ia\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{a\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/7*I)*a*Cos[c + d*x]^7)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Si
n[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/d
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.18.4 Maple [A] (verified)

Time = 27.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{i a \left(\cos^{7}\left(d x +c \right)\right)}{7}+\frac{a \left(\frac{16}{5}+\cos^{6}\left(d x +c \right)+\frac{6 \left(\cos^{4}\left(d x +c \right)\right)}{5}+\frac{8 \left(\cos^{2}\left(d x +c \right)\right)}{5}\right) \sin \left(d x +c \right)}{7}}{d}$
default	$\frac{-\frac{i a \left(\cos^{7}\left(d x +c \right)\right)}{7}+\frac{a \left(\frac{16}{5}+\cos^{6}\left(d x +c \right)+\frac{6 \left(\cos^{4}\left(d x +c \right)\right)}{5}+\frac{8 \left(\cos^{2}\left(d x +c \right)\right)}{5}\right) \sin \left(d x +c \right)}{7}}{d}$
risch	$-\frac{i a \,{\mathrm e}^{7 i \left(d x +c \right)}}{448 d}-\frac{5 i a \cos \left(d x +c \right)}{64 d}+\frac{35 a \sin \left(d x +c \right)}{64 d}-\frac{i a \cos \left(5 d x +5 c \right)}{64 d}+\frac{7 a \sin \left(5 d x +5 c \right)}{320 d}-\frac{3 i a \cos \left(3 d x +3 c \right)}{64 d}+$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/7*I*a*cos(d*x+c)^7+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*c
os(d*x+c)^2)*sin(d*x+c))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{\left(-5i\,ae^{(12i\,dx+12i\,c)} - 42i\,ae^{(10i\,dx+10i\,c)} - 175i\,ae^{(8i\,dx+8i\,c)} - 700i\,ae^{(6i\,dx+6i\,c)} + 525i\,ae^{(4i\,dx+4i\,c)} + 70i\,ae^{(2i}\right.}{2240\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2240*(-5*I*a*e^(12*I*d*x + 12*I*c) - 42*I*a*e^(10*I*d*x + 10*I*c) - 175*
I*a*e^(8*I*d*x + 8*I*c) - 700*I*a*e^(6*I*d*x + 6*I*c) + 525*I*a*e^(4*I*d*x
 + 4*I*c) + 70*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*e^(-5*I*d*x - 5*I*c)/d
```

3.18.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(65) = 130.

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.33

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \begin{cases} \frac{(-107374182400iad^6e^{16ic}e^{7idx} - 901943132160iad^6e^{14ic}e^{5idx} - 3758096384000iad^6e^{12ic}e^{3idx} - 15032385536000iad^6e^{10ic}e^{idx} + 11274289152}{48103633715200d^7} \\ \frac{x(ae^{12ic}+6ae^{10ic}+15ae^{8ic}+20ae^{6ic}+15ae^{4ic}+6ae^{2ic}+a)e^{-5ic}}{64} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-107374182400*I*a*d**6*exp(16*I*c)*exp(7*I*d*x) - 901943132160
*I*a*d**6*exp(14*I*c)*exp(5*I*d*x) - 3758096384000*I*a*d**6*exp(12*I*c)*ex
p(3*I*d*x) - 15032385536000*I*a*d**6*exp(10*I*c)*exp(I*d*x) + 112742891520
00*I*a*d**6*exp(8*I*c)*exp(-I*d*x) + 1503238553600*I*a*d**6*exp(6*I*c)*exp
(-3*I*d*x) + 150323855360*I*a*d**6*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(
48103633715200*d**7), Ne(d**7*exp(9*I*c), 0)), (x*(a*exp(12*I*c) + 6*a*exp
(10*I*c) + 15*a*exp(8*I*c) + 20*a*exp(6*I*c) + 15*a*exp(4*I*c) + 6*a*exp(2
*I*c) + a)*exp(-5*I*c)/64, True))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{5i\,a\cos(dx+c)^7+\left(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c)\right)a}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/35*(5*I*a*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*s
in(d*x + c)^3 - 35*sin(d*x + c))*a)/d
```

3.18.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.

Time = 0.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.21

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{\left(1015\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+700\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-1015\,ae^{(5i\,dx+i\,c)}\log\left(-i\,e^{(i\,d}\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/8960*(1015*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 700*a*e^(5*
I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 1015*a*e^(5*I*d*x + I*c)*log(-I*
e^(I*d*x + I*c) + 1) - 700*a*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1)
 - 315*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 315*a*e^(5*I*d*x
+ I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 20*I*a*e^(12*I*d*x + 8*I*c) + 168*I*
a*e^(10*I*d*x + 6*I*c) + 700*I*a*e^(8*I*d*x + 4*I*c) + 2800*I*a*e^(6*I*d*x
 + 2*I*c) - 280*I*a*e^(2*I*d*x - 2*I*c) - 2100*I*a*e^(4*I*d*x) - 28*I*a*e^
(-4*I*c))*e^(-5*I*d*x - I*c)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx =$$

$$-\frac{2\,a\,\left(-\frac{1225\,\sin(c+d\,x)}{128}-\frac{245\,\sin(3\,c+3\,d\,x)}{128}-\frac{49\,\sin(5\,c+5\,d\,x)}{128}-\frac{5\,\sin(7\,c+7\,d\,x)}{128}+\frac{\cos(c+d\,x)\,175\mathrm{i}}{128}+\frac{\cos(3\,c+3\,d\,x)\,105\mathrm{i}}{128}+\cos(\right.}{35\,d}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i),x)
```

output
```
-(2*a*((cos(c + d*x)*175i)/128 - (1225*sin(c + d*x))/128 + (cos(3*c + 3*d*
x)*105i)/128 + (cos(5*c + 5*d*x)*35i)/128 + (cos(7*c + 7*d*x)*5i)/128 - (2
45*sin(3*c + 3*d*x))/128 - (49*sin(5*c + 5*d*x))/128 - (5*sin(7*c + 7*d*x)
)/128))/(35*d)
```

3.19 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.19.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{4i(a+ia\tan(c+dx))^6}{3a^4d} + \frac{12i(a+ia\tan(c+dx))^7}{7a^5d} - \frac{3i(a+ia\tan(c+dx))^8}{4a^6d} + \frac{i(a+ia\tan(c+dx))^9}{9a^7d}$$

output
```
-4/3*I*(a+I*a*tan(d*x+c))^6/a^4/d+12/7*I*(a+I*a*tan(d*x+c))^7/a^5/d-3/4*I*
(a+I*a*tan(d*x+c))^8/a^6/d+1/9*I*(a+I*a*tan(d*x+c))^9/a^7/d
```

3.19.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\sec^8(c+dx)(\cos(6(c+dx))+i\sin(6(c+dx)))(-40i+170i\cos(2(c+dx))+83\sec(c+dx)\sin(3(c}{504d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/504*(a^2*Sec[c + d*x]^8*(Cos[6*(c + d*x)] + I*Sin[6*(c + d*x)])*(-40*I
+ (170*I)*Cos[2*(c + d*x)] + 83*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c +
 d*x]))/d
```

3.19.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^5d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int\left(-(i\tan(c+dx)a+a)^8+6a(i\tan(c+dx)a+a)^7-12a^2(i\tan(c+dx)a+a)^6+8a^3(i\tan(c+dx)a+a)^5\right)}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{3}a^3(a+ia\tan(c+dx))^6-\frac{12}{7}a^2(a+ia\tan(c+dx))^7-\frac{1}{9}(a+ia\tan(c+dx))^9+\frac{3}{4}a(a+ia\tan(c+dx))^8\right)}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((4*a^3*(a + I*a*Tan[c + d*x])^6)/3 - (12*a^2*(a + I*a*Tan[c + d*x])
^7)/7 + (3*a*(a + I*a*Tan[c + d*x])^8)/4 - (a + I*a*Tan[c + d*x])^9/9))/(a
^7*d)
```

3.19.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.19.4 Maple [A] (verified)

Time = 114.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result
risch	$\frac{64 i a^{2} \left(126 \,{\mathrm e}^{10 i \left(d x +c \right)}+126 \,{\mathrm e}^{8 i \left(d x +c \right)}+84 \,{\mathrm e}^{6 i \left(d x +c \right)}+36 \,{\mathrm e}^{4 i \left(d x +c \right)}+9 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{63 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{9}}$
derivativedivides	$\frac{-a^{2} \left(\frac{\sin^{3}\left(d x +c \right)}{9 \cos\left(d x +c \right)^{9}}+\frac{2 \left(\sin^{3}\left(d x +c \right)\right)}{21 \cos\left(d x +c \right)^{7}}+\frac{8 \left(\sin^{3}\left(d x +c \right)\right)}{105 \cos\left(d x +c \right)^{5}}+\frac{16 \left(\sin^{3}\left(d x +c \right)\right)}{315 \cos\left(d x +c \right)^{3}}\right)+\frac{i a^{2}}{4 \cos\left(d x +c \right)^{8}}-a^{2} \left(-\frac{16}{35}-\frac{\left(\sec^{6}\left(d x +c \right)\right)}{7}-\frac{6 \left(\sec^{4}\left(d\right.\right.}{35}\right.}{d}$
default	$\frac{-a^{2} \left(\frac{\sin^{3}\left(d x +c \right)}{9 \cos\left(d x +c \right)^{9}}+\frac{2 \left(\sin^{3}\left(d x +c \right)\right)}{21 \cos\left(d x +c \right)^{7}}+\frac{8 \left(\sin^{3}\left(d x +c \right)\right)}{105 \cos\left(d x +c \right)^{5}}+\frac{16 \left(\sin^{3}\left(d x +c \right)\right)}{315 \cos\left(d x +c \right)^{3}}\right)+\frac{i a^{2}}{4 \cos\left(d x +c \right)^{8}}-a^{2} \left(-\frac{16}{35}-\frac{\left(\sec^{6}\left(d x +c \right)\right)}{7}-\frac{6 \left(\sec^{4}\left(d\right.\right.}{35}\right.}{d}$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
64/63*I*a^2*(126*exp(10*I*(d*x+c))+126*exp(8*I*(d*x+c))+84*exp(6*I*(d*x+c)
)+36*exp(4*I*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^9
```

3.19.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(85) = 170$.

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.73

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{64\,(-126i\,a^2e^{(10i\,dx+10i\,c)} - 126i\,a^2e^{(8i\,dx+8i\,c)} - 84i\,a^2e^{(6i\,dx+6i\,c)} - 36i\,a^2e^{(4i}}{63\,(de^{(18i\,dx+18i\,c)} + 9\,de^{(16i\,dx+16i\,c)} + 36\,de^{(14i\,dx+14i\,c)} + 84\,de^{(12i\,dx+12i\,c)} + 126\,de^{(10i\,dx+10i\,c)} + 126\,de^{(8i\,d}}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-64/63*(-126*I*a^2*e^(10*I*d*x + 10*I*c) - 126*I*a^2*e^(8*I*d*x + 8*I*c) -
 84*I*a^2*e^(6*I*d*x + 6*I*c) - 36*I*a^2*e^(4*I*d*x + 4*I*c) - 9*I*a^2*e^(
2*I*d*x + 2*I*c) - I*a^2)/(d*e^(18*I*d*x + 18*I*c) + 9*d*e^(16*I*d*x + 16*
I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*I*c) + 126*d*e^(
10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6*I*d*x + 6*I*c)
+ 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

3.19.6 Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^8(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^8(c+dx)\right)dx + \int \left(-\sec^8(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**8, x) + Integral(-sec(c + d*x)**8, x))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{28\,a^2\tan\left(dx+c\right)^9-63i\,a^2\tan\left(dx+c\right)^8+72\,a^2\tan\left(dx+c\right)^7-252i\,a^2\tan\left(dx+c\right)^6-378i\,a^2\tan\left(dx\right.}{252\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
 c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(
d*x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{28\,a^2\tan\left(dx+c\right)^9-63i\,a^2\tan\left(dx+c\right)^8+72\,a^2\tan\left(dx+c\right)^7-252i\,a^2\tan\left(dx+c\right)^6-378i\,a^2\tan\left(dx\right.}{252\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
 c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(
d*x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

3.19.9 Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{a^2\,\sin\left(c+d\,x\right)\,\left(252\cos\left(c+d\,x\right)^8+\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)\,252\mathrm{i}+168\cos\left(c+d\,x\right)^6\sin\left(c+d\,x\right)^2+c\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^8,x)
```

output
```
(a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^7*63i + cos(c + d*x)^7*sin(c
+ d*x)*252i + 252*cos(c + d*x)^8 - 28*sin(c + d*x)^8 - 72*cos(c + d*x)^2*s
in(c + d*x)^6 + cos(c + d*x)^3*sin(c + d*x)^5*252i + cos(c + d*x)^5*sin(c
+ d*x)^3*378i + 168*cos(c + d*x)^6*sin(c + d*x)^2))/(252*d*cos(c + d*x)^9)
```

3.19. $\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.20 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.20.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{i(a+ia\tan(c+dx))^7}{7a^5d}$$

output

```
-4/5*I*(a+I*a*tan(d*x+c))^5/a^3/d+2/3*I*(a+I*a*tan(d*x+c))^6/a^4/d-1/7*I*(
a+I*a*tan(d*x+c))^7/a^5/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx =\frac{a^2\sec^7(c+dx)(7+22\cos(2(c+dx))-20i\sin(2(c+dx)))(-i\cos(5(c+dx))+\sin(5(c+dx)))}{105d}$$

input

```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output

```
(a^2*Sec[c + d*x]^7*(7 + 22*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((
-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)]))/(105*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^4d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^6-4a(i\tan(c+dx)a+a)^5+4a^2(i\tan(c+dx)a+a)^4\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i(\frac{4}{5}a^2(a+ia\tan(c+dx))^5+\frac{1}{7}(a+ia\tan(c+dx))^7-\frac{2}{3}a(a+ia\tan(c+dx))^6)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((4*a^2*(a + I*a*Tan[c + d*x])^5)/5 - (2*a*(a + I*a*Tan[c + d*x])^6)
/3 + (a + I*a*Tan[c + d*x])^7/7))/(a^5*d)
```

3.20.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.20.4 Maple [A] (verified)

Time = 34.89 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
risch	$\frac{128ia^2\left(35\,\mathrm{e}^{8i(dx+c)}+35\,\mathrm{e}^{6i(dx+c)}+21\,\mathrm{e}^{4i(dx+c)}+7\,\mathrm{e}^{2i(dx+c)}+1\right)}{105d\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+}{d}$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*a^2*(35*exp(8*I*(d*x+c))+35*exp(6*I*(d*x+c))+21*exp(4*I*(d*x+c))
+7*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^7
```

3.20.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{128\,(-35i\,a^2e^{(8i\,dx+8i\,c)} - 35i\,a^2e^{(6i\,dx+6i\,c)} - 21i\,a^2e^{(4i\,dx+4i\,c)} - 7i\,a^2e^{(2i\,dx+2i\,c)} - i\,a}{105\,(de^{(14i\,dx+14i\,c)} + 7\,de^{(12i\,dx+12i\,c)} + 21\,de^{(10i\,dx+10i\,c)} + 35\,de^{(8i\,dx+8i\,c)} + 35\,de^{(6i\,dx+6i\,c)} + 21\,de^{(4i\,dx+4i}}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-128/105*(-35*I*a^2*e^(8*I*d*x + 8*I*c) - 35*I*a^2*e^(6*I*d*x + 6*I*c) - 2
1*I*a^2*e^(4*I*d*x + 4*I*c) - 7*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(1
4*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c)
 + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x +
 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

3.20.6 Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^6(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^6(c+dx)\right)dx + \int \left(-\sec^6(c+dx)\right)dx\right)$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**6, x) + Integral(-sec(c + d*x)**6, x))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{15\,a^2\tan\left(dx+c\right)^7-35i\,a^2\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5-105i\,a^2\tan\left(dx+c\right)^4-35\,a^2\tan\left(dx+\right.}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
 c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d
*x + c)^2 - 105*a^2*tan(d*x + c))/d
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{15\,a^2\tan\left(dx+c\right)^7-35i\,a^2\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5-105i\,a^2\tan\left(dx+c\right)^4-35\,a^2\tan\left(dx+\right.}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
 c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d
*x + c)^2 - 105*a^2*tan(d*x + c))/d
```

3.20.9 Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.61

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$=\frac{a^2\,\sin\left(c+d\,x\right)\,\left(105\cos\left(c+d\,x\right)^6+\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)\,105\mathrm{i}+35\cos\left(c+d\,x\right)^4\sin\left(c+d\,x\right)^2+\mathrm{co}\right.}{105}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^6,x)
```

output
```
(a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*35i + cos(c + d*x)^5*sin(c
+ d*x)*105i + 105*cos(c + d*x)^6 - 15*sin(c + d*x)^6 - 21*cos(c + d*x)^2*s
in(c + d*x)^4 + cos(c + d*x)^3*sin(c + d*x)^3*105i + 35*cos(c + d*x)^4*sin
(c + d*x)^2))/(105*d*cos(c + d*x)^7)
```

3.20. $\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.21 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.21.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{i(a+ia\tan(c+dx))^4}{2a^2d}+\frac{i(a+ia\tan(c+dx))^5}{5a^3d}$$

output
```
-1/2*I*(a+I*a*tan(d*x+c))^4/a^2/d+1/5*I*(a+I*a*tan(d*x+c))^5/a^3/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{a^2(-i+\tan(c+dx))^4(3i+2\tan(c+dx))}{10d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/10*(a^2*(-I + Tan[c + d*x])^4*(3*I + 2*Tan[c + d*x]))/d
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^3-(i\tan(c+dx)a+a)^4\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{1}{2}a(a+ia\tan(c+dx))^4-\frac{1}{5}(a+ia\tan(c+dx))^5\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((a*(a + I*a*Tan[c + d*x])^4)/2 - (a + I*a*Tan[c + d*x])^5/5))/(a^3*
d)
```

3.21.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.21.4 Maple [A] (verified)

Time = 8.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{8 i a^{2} \left(10 \,{\mathrm e}^{6 i \left(d x +c \right)}+10 \,{\mathrm e}^{4 i \left(d x +c \right)}+5 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{5 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{5}}$	58
derivativedivides	$\frac{-a^{2} \left(\frac{\sin^{3}\left(d x +c \right)}{5 \cos\left(d x +c \right)^{5}}+\frac{2 \left(\sin^{3}\left(d x +c \right)\right)}{15 \cos\left(d x +c \right)^{3}}\right)+\frac{i a^{2}}{2 \cos\left(d x +c \right)^{4}}-a^{2} \left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(d x +c \right)\right)}{3}\right) \tan\left(d x +c \right)}{d}$	85
default	$\frac{-a^{2} \left(\frac{\sin^{3}\left(d x +c \right)}{5 \cos\left(d x +c \right)^{5}}+\frac{2 \left(\sin^{3}\left(d x +c \right)\right)}{15 \cos\left(d x +c \right)^{3}}\right)+\frac{i a^{2}}{2 \cos\left(d x +c \right)^{4}}-a^{2} \left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(d x +c \right)\right)}{3}\right) \tan\left(d x +c \right)}{d}$	85

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
8/5*I*a^2*(10*exp(6*I*(d*x+c))+10*exp(4*I*(d*x+c))+5*exp(2*I*(d*x+c))+1)/d
/(exp(2*I*(d*x+c))+1)^5
```

3.21.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(43) = 86.

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{8\left(-10i\,a^2e^{(6i\,dx+6i\,c)}-10i\,a^2e^{(4i\,dx+4i\,c)}-5i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{5\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-8/5*(-10*I*a^2*e^(6*I*d*x + 6*I*c) - 10*I*a^2*e^(4*I*d*x + 4*I*c) - 5*I*a
^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x
+ 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*
I*d*x + 2*I*c) + d)
```

3.21.6 Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^4(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^4(c+dx)\right)\,dx + \int \left(-\sec^4(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**4, x) + Integral(-sec(c + d*x)**4, x))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\tan(dx+c)^5 - 5i\,a^2\tan(dx+c)^4 - 10i\,a^2\tan(dx+c)^2 - 10\,a^2\tan(dx+c)}{10\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x +
c)^2 - 10*a^2*tan(d*x + c))/d
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= -\frac{2\,a^2\tan\left(dx+c\right)^5 - 5i\,a^2\tan\left(dx+c\right)^4 - 10i\,a^2\tan\left(dx+c\right)^2 - 10\,a^2\tan\left(dx+c\right)}{10\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x +
c)^2 - 10*a^2*tan(d*x + c))/d
```

3.21.9 Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{-\frac{a^2\tan(c+d\,x)^5}{5} + \frac{a^2\tan(c+d\,x)^4\,1\mathrm{i}}{2} + a^2\tan(c+d\,x)^2\,1\mathrm{i} + a^2\tan(c+d\,x)}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^4,x)
```

output
```
(a^2*tan(c + d*x) + a^2*tan(c + d*x)^2*1i + (a^2*tan(c + d*x)^4*1i)/2 - (a
^2*tan(c + d*x)^5)/5)/d
```

3.22 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.22.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

output
```
-1/3*I*(a+I*a*tan(d*x+c))^3/a/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(c+dx)}{d} + \frac{ia^2\tan^2(c+dx)}{d} - \frac{a^2\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*Tan[c + d*x])/d + (I*a^2*Tan[c + d*x]^2)/d - (a^2*Tan[c + d*x]^3)/(3*
d)
```

3.22.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-1/3*I)*(a + I*a*Tan[c + d*x])^3)/(a*d)
```

3.22.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.22.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

method	result	size
risch	$\frac{8ia^2\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	47
derivativedivides	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51
default	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
8/3*I*a^2*(3*exp(4*I*(d*x+c))+3*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)
^3
```

3.22.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{8\left(-3i\,a^2e^{(4i\,dx+4i\,c)}-3i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-8/3*(-3*I*a^2*e^(4*I*d*x + 4*I*c) - 3*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/
(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

3.22.6 Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^2(c+dx)\,dx + \int (-2i\tan(c+dx)\sec^2(c+dx))\,dx + \int (-\sec^2(c+dx))\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**2, x) + Integral(-sec(c + d*x)**2, x))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i\,(i\,a\tan(dx+c)+a)^3}{3\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*I*(I*a*tan(d*x + c) + a)^3/(a*d)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\tan(dx+c)^3 - 3i\,a^2\tan(dx+c)^2 - 3\,a^2\tan(dx+c)}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(a^2*tan(d*x + c)^3 - 3*I*a^2*tan(d*x + c)^2 - 3*a^2*tan(d*x + c))/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x)\,\left(-\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}+3\right)}{3\,d}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^2,x)
```

output
```
(a^2*tan(c + d*x)*(tan(c + d*x)*3i - tan(c + d*x)^2 + 3))/(3*d)
```

3.23 $\int (a+ia\tan(c+dx))^2\,dx$

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3.23.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int (a+ia\tan(c+dx))^2\,dx = 2a^2x - \frac{2ia^2\log(\cos(c+dx))}{d} - \frac{a^2\tan(c+dx)}{d}$$

output
```
2*a^2*x-2*I*a^2*ln(cos(d*x+c))/d-a^2*tan(d*x+c)/d
```

3.23.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(c+dx))^2\,dx = -\frac{ia(-2a\log(i+\tan(c+dx)) - ia\tan(c+dx))}{d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a*(-2*a*Log[I + Tan[c + d*x]] - I*a*Tan[c + d*x]))/d
```

3.23.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{ 3958}$$

$$2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{ 3042}$$

$$2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

input
```
Int[(a + I*a*Tan[c + d*x])^2,x]
```

output
```
2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d
```

3.23.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

3.23.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{a^2(-\tan(dx+c)+i\ln(1+\tan^2(dx+c))+2\arctan(\tan(dx+c)))}{d}$	40
default	$\frac{a^2(-\tan(dx+c)+i\ln(1+\tan^2(dx+c))+2\arctan(\tan(dx+c)))}{d}$	40
parallelrisch	$\frac{ia^2\ln(1+\tan^2(dx+c))+2a^2dx-a^2\tan(dx+c)}{d}$	41
norman	$2a^2x-\frac{a^2\tan(dx+c)}{d}+\frac{ia^2\ln(1+\tan^2(dx+c))}{d}$	42
parts	$a^2x+\frac{ia^2\ln(1+\tan^2(dx+c))}{d}-\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	51
risch	$-\frac{4a^2c}{d}-\frac{2ia^2}{d(\mathrm{e}^{2i(dx+c)}+1)}-\frac{2ia^2\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}$	54

input
```
int((a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^2*(-tan(d*x+c)+I*ln(1+tan(d*x+c)^2)+2*arctan(tan(d*x+c)))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int (a+ia\tan(c+dx))^2\,dx=-\frac{2\left(i\,a^2+\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2*(I*a^2 + (I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*log(e^(2*I*d*x + 2*I*c) +
1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int (a+ia\tan(c+dx))^2\,dx=-\frac{2ia^2}{de^{2ic}e^{2idx}+d}-\frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))**2,x)
```

output
```
-2*I*a**2/(d*exp(2*I*c)*exp(2*I*d*x) + d) - 2*I*a**2*log(exp(2*I*d*x) + ex
p(-2*I*c))/d
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int (a+ia\tan(c+dx))^2\,dx=a^2x+\frac{(dx+c-\tan(dx+c))a^2}{d}+\frac{2i\,a^2\log(\sec(dx+c))}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + (d*x + c - tan(d*x + c))*a^2/d + 2*I*a^2*log(sec(d*x + c))/d
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int (a+ia\tan(c+dx))^2\,dx$$
$$=-\frac{2\left(i\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\right)}{de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*(I*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + I*a^2*log(e^(
2*I*d*x + 2*I*c) + 1) + I*a^2)/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int (a+ia\tan(c+dx))^2\,dx=\frac{a^2\left(-\tan(c+d\,x)+\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}\right)}{d}$$

input

```
int((a + a*tan(c + d*x)*1i)^2,x)
```

output

```
(a^2*(log(tan(c + d*x) + 1i)*2i - tan(c + d*x)))/d
```

3.24 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.24.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{ia^3}{d(a-ia\tan(c+dx))}$$

output
```
-I*a^3/d/(a-I*a*tan(d*x+c))
```

3.24.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{ia^2(\cos(c+dx)+i\sin(c+dx))^2}{2d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-1/2*I)*a^2*(Cos[c + d*x] + I*Sin[c + d*x])^2)/d
```

3.24.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{ia^3}{d(a-ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^3)/(d*(a - I*a*Tan[c + d*x]))
```

3.24.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.24.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{i a^{2} {\mathrm e}^{2 i (d x +c)}}{2 d}$	19
derivativedivides	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^{2} \left(\cos^{2}\left(d x +c \right)\right)+a^{2} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73
default	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^{2} \left(\cos^{2}\left(d x +c \right)\right)+a^{2} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d*a^2*exp(2*I*(d*x+c))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } d \neq 0 \\ a^2xe^{2ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(d, 0)), (a**2*x*exp(2
*I*c), True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan\left(dx+c\right) - i\,a^2}{\left(\tan\left(dx+c\right)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(a^2*tan(d*x + c) - I*a^2)/((tan(d*x + c)^2 + 1)*d)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{a^2}{d\ (\tan\left(c+d\,x\right)+1\mathrm{i})}$$

input

```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2,x)
```

output

```
a^2/(d*(tan(c + d*x) + 1i))
```

3.25 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.25.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2} - \frac{ia^3}{4d(a-ia\tan(c+dx))}$$

output
```
1/4*a^2*x-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2-1/4*I*a^3/d/(a-I*a*tan(d*x+c))
```

3.25.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2(2i+\tan(c+dx)+\arctan(\tan(c+dx))(i+\tan(c+dx))^2)}{4d(i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(2*I + Tan[c + d*x] + ArcTan[Tan[c + d*x]]*(I + Tan[c + d*x])^2))/(4*
d*(I + Tan[c + d*x])^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 54$$

$$-\frac{ia^5\int \left(\frac{1}{2(a-ia\tan(c+dx))^3a}+\frac{1}{4(\tan^2(c+dx)a^2+a^2)a^2}+\frac{1}{4(a-ia\tan(c+dx))^2a^2}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{ia^5\left(\frac{i\arctan(\tan(c+dx))}{4a^3}+\frac{1}{4a^2(a-ia\tan(c+dx))}+\frac{1}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^5*(((I/4)*ArcTan[Tan[c + d*x]])/a^3 + 1/(4*a*(a - I*a*Tan[c + d*x]
)^2) + 1/(4*a^2*(a - I*a*Tan[c + d*x]))))/d
```

3.25.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.25.4 Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

method	result
risch	$\frac{a^{2}x}{4}-\frac{i a^{2}{\mathrm e}^{4 i (d x +c)}}{16 d}-\frac{i a^{2}{\mathrm e}^{2 i (d x +c)}}{4 d}$
derivativedivides	$\frac{-a^{2}\left(-\frac{\left(\cos^{3}\left(d x +c \right)\right)\sin\left(d x +c \right)}{4}+\frac{\sin\left(d x +c \right)\cos\left(d x +c \right)}{8}+\frac{d x}{8}+\frac{c}{8}\right)-\frac{i a^{2}\left(\cos^{4}\left(d x +c \right)\right)}{2}+a^{2}\left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3\cos\left(d x +c \right)}{2}\right)\sin(d x +}{4}\right.}{d}$
default	$\frac{-a^{2}\left(-\frac{\left(\cos^{3}\left(d x +c \right)\right)\sin\left(d x +c \right)}{4}+\frac{\sin\left(d x +c \right)\cos\left(d x +c \right)}{8}+\frac{d x}{8}+\frac{c}{8}\right)-\frac{i a^{2}\left(\cos^{4}\left(d x +c \right)\right)}{2}+a^{2}\left(\frac{\left(\cos^{3}\left(d x +c \right)+\frac{3\cos\left(d x +c \right)}{2}\right)\sin(d x +}{4}\right.}{d}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x-1/16*I/d*a^2*exp(4*I*(d*x+c))-1/4*I/d*a^2*exp(2*I*(d*x+c))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{4\,a^2dx - i\,a^2e^{(4i\,dx+4i\,c)} - 4i\,a^2e^{(2i\,dx+2i\,c)}}{16\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*a^2*d*x - I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c))
/d
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} + \begin{cases} \frac{-4ia^2de^{4ic}e^{4idx}-16ia^2de^{2ic}e^{2idx}}{64d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{4ic}}{4}+\frac{a^2e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

output
```
a**2*x/4 + Piecewise(((-4*I*a**2*d*exp(4*I*c)*exp(4*I*d*x) - 16*I*a**2*d*e
xp(2*I*c)*exp(2*I*d*x))/(64*d**2), Ne(d**2, 0)), (x*(a**2*exp(4*I*c)/4 + a
**2*exp(2*I*c)/2), True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{(dx+c)a^2 + \frac{a^2\tan(dx+c)^3+3\,a^2\tan(dx+c)-2i\,a^2}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{4\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*((d*x + c)*a^2 + (a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 2*I*a^2)/(
tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

3.25.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(49) = 98$.

Time = 0.56 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.08

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= \frac{8\,a^2dxe^{(4i\,dx+2i\,c)}+16\,a^2dxe^{(2i\,dx)}+8\,a^2dxe^{(-2i\,c)}-i\,a^2e^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-2i\,a^2e^{(2i\,dx)}\log\left(e^{(}\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/32*(8*a^2*d*x*e^(4*I*d*x + 2*I*c) + 16*a^2*d*x*e^(2*I*d*x) + 8*a^2*d*x*e
^(-2*I*c) - I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2*I*a
^2*e^(2*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - I*a^2*e^(-2*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) + I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)
) + 2*I*a^2*e^(2*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) + I*a^2*e^(-2*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) - 2*I*a^2*e^(8*I*d*x + 6*I*c) - 12*I*a^2*e^(6
*I*d*x + 4*I*c) - 18*I*a^2*e^(4*I*d*x + 2*I*c) - 8*I*a^2*e^(2*I*d*x))/(d*e
^(4*I*d*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\,x}{4}+\frac{\frac{a^2\tan(c+d\,x)}{4}+\frac{a^2\,1\mathrm{i}}{2}}{d\,\left(\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(a^2*x)/4 + ((a^2*tan(c + d*x))/4 + (a^2*1i)/2)/(d*(tan(c + d*x)*2i + tan(
c + d*x)^2 - 1))
```

3.26 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.26.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\begin{aligned}\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = {} & \frac{a^2x}{4}-\frac{ia^5}{12d(a-ia\tan(c+dx))^3}\\ & -\frac{ia^4}{8d(a-ia\tan(c+dx))^2}\\ & -\frac{3ia^3}{16d(a-ia\tan(c+dx))}\\ & +\frac{ia^3}{16d(a+ia\tan(c+dx))}\end{aligned}$$

output
```
1/4*a^2*x-1/12*I*a^5/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^4/d/(a-I*a*tan(d*x+c))
^2-3/16*I*a^3/d/(a-I*a*tan(d*x+c))+1/16*I*a^3/d/(a+I*a*tan(d*x+c))
```

3.26.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= \frac{a^2(4i-\tan(c+dx)+6i\tan^2(c+dx)+3\tan^3(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))(i+\mathrm{ta}}{12d(-i+\tan(c+dx))(i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(4*I - Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 + 3*Tan[c + d*x]^3 + 3*Arc
Tan[Tan[c + d*x]]*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^3))/(12*d*(-I + T
an[c + d*x])*(I + Tan[c + d*x])^3)
```

3.26.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^7\int \Bigg(\frac{3}{16a^4(a-ia\tan(c+dx))^2}+\frac{1}{16a^4(i\tan(c+dx)a+a)^2}+\frac{1}{4a^3(a-ia\tan(c+dx))^3}+\frac{1}{4a^2(a-ia\tan(c+dx))^4}+\frac{1}{4a^4(\tan^2(c+dx)a^2+a^2)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^7\left(\frac{i\arctan(\tan(c+dx))}{4a^5}+\frac{3}{16a^4(a-ia\tan(c+dx))}-\frac{1}{16a^4(a+ia\tan(c+dx))}+\frac{1}{8a^3(a-ia\tan(c+dx))^2}+\frac{1}{12a^2(a-ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^7*(((I/4)*ArcTan[Tan[c + d*x]])/a^5 + 1/(12*a^2*(a - I*a*Tan[c + d
*x])^3) + 1/(8*a^3*(a - I*a*Tan[c + d*x])^2) + 3/(16*a^4*(a - I*a*Tan[c +
d*x])) - 1/(16*a^4*(a + I*a*Tan[c + d*x]))))/d
```

3.26.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.26.4 Maple [A] (verified)

Time = 26.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

method	result
risch	$\frac{a^2 x}{4}-\frac{i a^2 \mathrm{e}^{6 i (d x+c)}}{96 d}-\frac{i a^2 \mathrm{e}^{4 i (d x+c)}}{16 d}-\frac{5 i a^2 \cos(2 d x+2 c)}{32 d}+\frac{7 a^2 \sin(2 d x+2 c)}{32 d}$
derivativedivides	$\frac{-a^2\left(-\frac{\left(\cos^5(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\left(\cos^6(dx+c)\right)}{3}+a^2}{d}$
default	$\frac{-a^2\left(-\frac{\left(\cos^5(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\left(\cos^6(dx+c)\right)}{3}+a^2}{d}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x-1/96*I/d*a^2*exp(6*I*(d*x+c))-1/16*I/d*a^2*exp(4*I*(d*x+c))-5/32
*I/d*a^2*cos(2*d*x+2*c)+7/32/d*a^2*sin(2*d*x+2*c)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{\left(24\,a^2dxe^{(2i\,dx+2i\,c)} - i\,a^2e^{(8i\,dx+8i\,c)} - 6i\,a^2e^{(6i\,dx+6i\,c)} - 18i\,a^2e^{(4i\,dx+4i\,c)} + 3i\,a^2\right)e^{(-2i\,dx-2i\,c)}}{96\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/96*(24*a^2*d*x*e^(2*I*d*x + 2*I*c) - I*a^2*e^(8*I*d*x + 8*I*c) - 6*I*a^2
*e^(6*I*d*x + 6*I*c) - 18*I*a^2*e^(4*I*d*x + 4*I*c) + 3*I*a^2)*e^(-2*I*d*x
 - 2*I*c)/d
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2x}{4} + \begin{cases} \frac{(-8192ia^2d^3e^{8ic}e^{6idx}-49152ia^2d^3e^{6ic}e^{4idx}-147456ia^2d^3e^{4ic}e^{2idx}+24576ia^2d^3e^{-2idx})e^{-2ic}}{786432d^4} & \text{for } d^4e^{2ic} \neq 0 \\ x\left(-\frac{a^2}{4} + \frac{(a^2e^{8ic}+4a^2e^{6ic}+6a^2e^{4ic}+4a^2e^{2ic}+a^2)e^{-2ic}}{16}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

output
```
a**2*x/4 + Piecewise(((-8192*I*a**2*d**3*exp(8*I*c)*exp(6*I*d*x) - 49152*I
*a**2*d**3*exp(6*I*c)*exp(4*I*d*x) - 147456*I*a**2*d**3*exp(4*I*c)*exp(2*I
*d*x) + 24576*I*a**2*d**3*exp(-2*I*d*x))*exp(-2*I*c)/(786432*d**4), Ne(d**
4*exp(2*I*c), 0)), (x*(-a**2/4 + (a**2*exp(8*I*c) + 4*a**2*exp(6*I*c) + 6*
a**2*exp(4*I*c) + 4*a**2*exp(2*I*c) + a**2)*exp(-2*I*c)/16), True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{3\,(dx+c)a^2 + \frac{3\,a^2\tan(dx+c)^5+8\,a^2\tan(dx+c)^3+9\,a^2\tan(dx+c)-4i\,a^2}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{12\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/12*(3*(d*x + c)*a^2 + (3*a^2*tan(d*x + c)^5 + 8*a^2*tan(d*x + c)^3 + 9*a
^2*tan(d*x + c) - 4*I*a^2)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x
+ c)^2 + 1))/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.44

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{24\,a^2dxe^{(6i\,dx+4i\,c)} + 48\,a^2dxe^{(4i\,dx+2i\,c)} + 24\,a^2dxe^{(2i\,dx)} - i\,a^2e^{(12i\,dx+10i\,c)} - 8i\,a^2e^{(10i\,dx+8i\,c)} - 31i\,a^2e^{(8i\,dx-}}{96\,(de^{(6i\,dx+4i\,c)} + 2\,de^{(4i\,dx+2i\,c)} + de^{(2i\,d}}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/96*(24*a^2*d*x*e^(6*I*d*x + 4*I*c) + 48*a^2*d*x*e^(4*I*d*x + 2*I*c) + 24
*a^2*d*x*e^(2*I*d*x) - I*a^2*e^(12*I*d*x + 10*I*c) - 8*I*a^2*e^(10*I*d*x +
 8*I*c) - 31*I*a^2*e^(8*I*d*x + 6*I*c) - 42*I*a^2*e^(6*I*d*x + 4*I*c) - 15
*I*a^2*e^(4*I*d*x + 2*I*c) + 6*I*a^2*e^(2*I*d*x) + 3*I*a^2*e^(-2*I*c))/(d*
e^(6*I*d*x + 4*I*c) + 2*d*e^(4*I*d*x + 2*I*c) + d*e^(2*I*d*x))
```

3.26.9 Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{a^2\,x}{4}+\frac{\frac{a^2\tan(c+d\,x)^3}{4}+\frac{a^2\tan(c+d\,x)^2\,1\mathrm{i}}{2}-\frac{a^2\tan(c+d\,x)}{12}+\frac{a^2\,1\mathrm{i}}{3}}{d\,\left(\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,2\mathrm{i}+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(a^2*x)/4 + ((a^2*1i)/3 - (a^2*tan(c + d*x))/12 + (a^2*tan(c + d*x)^2*1i)/
2 + (a^2*tan(c + d*x)^3)/4)/(d*(tan(c + d*x)*2i + tan(c + d*x)^3*2i + tan(
c + d*x)^4 - 1))
```

3.27 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.27.1 Optimal result

Integrand size = 24, antiderivative size = 171

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx=&\frac{15a^2x}{64}-\frac{ia^6}{32d(a-ia\tan(c+dx))^4}\\&-\frac{ia^5}{16d(a-ia\tan(c+dx))^3}\\&-\frac{3ia^4}{32d(a-ia\tan(c+dx))^2}\\&-\frac{5ia^3}{32d(a-ia\tan(c+dx))}\\&+\frac{ia^4}{64d(a+ia\tan(c+dx))^2}\\&+\frac{5ia^3}{64d(a+ia\tan(c+dx))}\end{aligned}$$

output
```
15/64*a^2*x-1/32*I*a^6/d/(a-I*a*tan(d*x+c))^4-1/16*I*a^5/d/(a-I*a*tan(d*x+
c))^3-3/32*I*a^4/d/(a-I*a*tan(d*x+c))^2-5/32*I*a^3/d/(a-I*a*tan(d*x+c))+1/
64*I*a^4/d/(a+I*a*tan(d*x+c))^2+5/64*I*a^3/d/(a+I*a*tan(d*x+c))
```

3.27.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\sec^6(c+dx)(-80i-65i\cos(2(c+dx))+16i\cos(4(c+dx))+i\cos(6(c+dx))+120\arctan(\tan(c-}{512d(-i+\tan(c+dx)}$$

input

```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output

```
-1/512*(a^2*Sec[c + d*x]^6*(-80*I - (65*I)*Cos[2*(c + d*x)] + (16*I)*Cos[4
*(c + d*x)] + I*Cos[6*(c + d*x)] + 120*ArcTan[Tan[c + d*x]]*(Cos[2*(c + d*
x)] - I*Sin[2*(c + d*x)]) - 5*Sin[2*(c + d*x)] + 32*Sin[4*(c + d*x)] + 3*S
in[6*(c + d*x)]))/(d*(-I + Tan[c + d*x])^2*(I + Tan[c + d*x])^4)
```

3.27.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^9\int \frac{1}{(a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^9\int \left(\frac{5}{32a^6(a-ia\tan(c+dx))^2}+\frac{5}{64a^6(i\tan(c+dx)a+a)^2}+\frac{3}{16a^5(a-ia\tan(c+dx))^3}+\frac{1}{32a^5(i\tan(c+dx)a+a)^3}+\frac{3}{16a^4(a-ia\tan(c+dx)}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^9\left(\frac{15i\arctan(\tan(c+dx))}{64a^7}+\frac{5}{32a^6(a-ia\tan(c+dx))}-\frac{5}{64a^6(a+ia\tan(c+dx))}+\frac{3}{32a^5(a-ia\tan(c+dx))^2}-\frac{1}{64a^5(a+ia\tan(c+dx))^2}+\right.}{d}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^9*((((15*I)/64)*ArcTan[Tan[c + d*x]])/a^7 + 1/(32*a^3*(a - I*a*Tan
[c + d*x])^4) + 1/(16*a^4*(a - I*a*Tan[c + d*x])^3) + 3/(32*a^5*(a - I*a*T
an[c + d*x])^2) + 5/(32*a^6*(a - I*a*Tan[c + d*x])) - 1/(64*a^5*(a + I*a*T
an[c + d*x])^2) - 5/(64*a^6*(a + I*a*Tan[c + d*x]))))/d
```

3.27.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.27.4 Maple [A] (verified)

Time = 82.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

method	result
risch	$\frac{15a^{2}x}{64}-\frac{ia^{2}\mathrm{e}^{8i(dx+c)}}{512d}-\frac{ia^{2}\mathrm{e}^{6i(dx+c)}}{64d}-\frac{7ia^{2}\cos(4dx+4c)}{128d}+\frac{a^{2}\sin(4dx+4c)}{16d}-\frac{7ia^{2}\cos(2dx+2c)}{64d}+\frac{13a^{2}\sin(2}{64}$
derivativedivides	$\frac{-a^{2}\left(-\frac{\sin(dx+c)\left(\cos^{7}(dx+c)\right)}{8}+\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^{2}\left(\cos^{8}(dx+c)\right)}{4}+a}{d}$
default	$\frac{-a^{2}\left(-\frac{\sin(dx+c)\left(\cos^{7}(dx+c)\right)}{8}+\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^{2}\left(\cos^{8}(dx+c)\right)}{4}+a}{d}$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
15/64*a^2*x-1/512*I/d*a^2*exp(8*I*(d*x+c))-1/64*I/d*a^2*exp(6*I*(d*x+c))-7
/128*I/d*a^2*cos(4*d*x+4*c)+1/16/d*a^2*sin(4*d*x+4*c)-7/64*I/d*a^2*cos(2*d
*x+2*c)+13/64/d*a^2*sin(2*d*x+2*c)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int\cos^{8}(c+dx)(a+ia\tan(c+dx))^{2}\,dx$$
$$=\frac{\left(120\,a^{2}dxe^{(4i\,dx+4i\,c)}-i\,a^{2}e^{(12i\,dx+12i\,c)}-8i\,a^{2}e^{(10i\,dx+10i\,c)}-30i\,a^{2}e^{(8i\,dx+8i\,c)}-80i\,a^{2}e^{(6i\,dx+6i\,c)}+24i\,a^{2}e^{(2}}{512\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/512*(120*a^2*d*x*e^(4*I*d*x + 4*I*c) - I*a^2*e^(12*I*d*x + 12*I*c) - 8*I
*a^2*e^(10*I*d*x + 10*I*c) - 30*I*a^2*e^(8*I*d*x + 8*I*c) - 80*I*a^2*e^(6*
I*d*x + 6*I*c) + 24*I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*e^(-4*I*d*x - 4*I
*c)/d
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.58

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15a^2x}{64} + \begin{cases} \frac{(-8589934592ia^2d^5e^{14ic}e^{8idx}-68719476736ia^2d^5e^{12ic}e^{6idx}-257698037760ia^2d^5e^{10ic}e^{4idx}-687194767360ia^2d^5e^{8ic}e^{2idx}+206158430208}{4398046511104d^6} \\ x\left(-\frac{15a^2}{64}+\frac{(a^2e^{12ic}+6a^2e^{10ic}+15a^2e^{8ic}+20a^2e^{6ic}+15a^2e^{4ic}+6a^2e^{2ic}+a^2)e^{-4ic}}{64}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

output
```
15*a**2*x/64 + Piecewise(((-8589934592*I*a**2*d**5*exp(14*I*c)*exp(8*I*d*x
) - 68719476736*I*a**2*d**5*exp(12*I*c)*exp(6*I*d*x) - 257698037760*I*a**2
*d**5*exp(10*I*c)*exp(4*I*d*x) - 687194767360*I*a**2*d**5*exp(8*I*c)*exp(2
*I*d*x) + 206158430208*I*a**2*d**5*exp(4*I*c)*exp(-2*I*d*x) + 17179869184*
I*a**2*d**5*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(4398046511104*d**6), Ne
(d**6*exp(6*I*c), 0)), (x*(-15*a**2/64 + (a**2*exp(12*I*c) + 6*a**2*exp(10
*I*c) + 15*a**2*exp(8*I*c) + 20*a**2*exp(6*I*c) + 15*a**2*exp(4*I*c) + 6*a
**2*exp(2*I*c) + a**2)*exp(-4*I*c)/64), True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15\,(dx+c)a^2+\frac{15\,a^2\tan(dx+c)^7+55\,a^2\tan(dx+c)^5+73\,a^2\tan(dx+c)^3+49\,a^2\tan(dx+c)-16i\,a^2}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{64\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/64*(15*(d*x + c)*a^2 + (15*a^2*tan(d*x + c)^7 + 55*a^2*tan(d*x + c)^5 +
73*a^2*tan(d*x + c)^3 + 49*a^2*tan(d*x + c) - 16*I*a^2)/(tan(d*x + c)^8 +
4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

3.27.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(133) = 266.

Time = 0.68 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.00

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{120\,a^2dxe^{(8i\,dx+4i\,c)} + 240\,a^2dxe^{(6i\,dx+2i\,c)} + 120\,a^2dxe^{(4i\,dx)} + 8i\,a^2e^{(8i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + 16i\,a^2e}{}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/512*(120*a^2*d*x*e^(8*I*d*x + 4*I*c) + 240*a^2*d*x*e^(6*I*d*x + 2*I*c) +
 120*a^2*d*x*e^(4*I*d*x) + 8*I*a^2*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) + 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 8*
I*a^2*e^(4*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^2*e^(8*I*d*x + 4*I*
c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I
*d*x) + e^(-2*I*c)) - 8*I*a^2*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) -
I*a^2*e^(16*I*d*x + 12*I*c) - 10*I*a^2*e^(14*I*d*x + 10*I*c) - 47*I*a^2*e^
(12*I*d*x + 8*I*c) - 148*I*a^2*e^(10*I*d*x + 6*I*c) - 190*I*a^2*e^(8*I*d*x
 + 4*I*c) - 56*I*a^2*e^(6*I*d*x + 2*I*c) + 28*I*a^2*e^(2*I*d*x - 2*I*c) +
50*I*a^2*e^(4*I*d*x) + 2*I*a^2*e^(-4*I*c))/(d*e^(8*I*d*x + 4*I*c) + 2*d*e^
(6*I*d*x + 2*I*c) + d*e^(4*I*d*x))
```

3.27.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15\,a^2\,x}{64}$$
$$+ \frac{\frac{15\,a^2\tan(c+d\,x)^5}{64} + \frac{a^2\tan(c+d\,x)^4\,15\mathrm{i}}{32} + \frac{5\,a^2\tan(c+d\,x)^3}{32} + \frac{a^2\tan(c+d\,x)^2\,25\mathrm{i}}{32} - \frac{17\,a^2\tan(c+d\,x)}{64} + \frac{a^2\,1\mathrm{i}}{4}}{d\,\left(\tan\left(c+d\,x\right)^6 + \tan\left(c+d\,x\right)^5\,2\mathrm{i} + \tan\left(c+d\,x\right)^4 + \tan\left(c+d\,x\right)^3\,4\mathrm{i} - \tan\left(c+d\,x\right)^2 + \tan\left(c+d\,x\right)\right.}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(15*a^2*x)/64 + ((a^2*1i)/4 - (17*a^2*tan(c + d*x))/64 + (a^2*tan(c + d*x)
^2*25i)/32 + (5*a^2*tan(c + d*x)^3)/32 + (a^2*tan(c + d*x)^4*15i)/32 + (15
*a^2*tan(c + d*x)^5)/64)/(d*(tan(c + d*x)*2i - tan(c + d*x)^2 + tan(c + d*
x)^3*4i + tan(c + d*x)^4 + tan(c + d*x)^5*2i + tan(c + d*x)^6 - 1))
```

3.28 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.28.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\begin{aligned}\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx &= \frac{7a^2\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{7ia^2\sec^5(c+dx)}{30d}\\&\quad+\frac{7a^2\sec(c+dx)\tan(c+dx)}{16d}\\&\quad+\frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d}\\&\quad+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}\end{aligned}$$

output
```
7/16*a^2*arctanh(sin(d*x+c))/d+7/30*I*a^2*sec(d*x+c)^5/d+7/16*a^2*sec(d*x+
c)*tan(d*x+c)/d+7/24*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*I*sec(d*x+c)^5*(a^2
+I*a^2*tan(d*x+c))/d
```

3.28.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx &= \frac{7a^2\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{2ia^2\sec^5(c+dx)}{5d}\\&\quad+\frac{7a^2\sec(c+dx)\tan(c+dx)}{16d}\\&\quad+\frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d}\\&\quad-\frac{a^2\sec^5(c+dx)\tan(c+dx)}{6d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(7*a^2*ArcTanh[Sin[c + d*x]])/(16*d) + (((2*I)/5)*a^2*Sec[c + d*x]^5)/d +
(7*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^2*Sec[c + d*x]^3*Tan[c + d
*x])/(24*d) - (a^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

3.28.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3979, 3042, 3967, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{6}a\int \sec^5(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}a\int \sec(c+dx)^5(i\tan(c+dx)a+a)dx+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{7}{6}a\left(a\int \sec^5(c+dx)dx+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \ 3042$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \ 4255$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \ 3042$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)$$
$$\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

$$\Big\downarrow \ 4257$$

$$\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{7}{6}a\left(a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{ia\sec^5(c+dx)}{5d}\right)$$

input

```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output

```
((I/6)*Sec[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d + (7*a*(((I/5)*a*Sec[c
 + d*x]^5)/d + a*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c
+ d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)))/6
```

3.28. $\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.28.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.28.4 Maple [A] (verified)

Time = 19.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{i a^2 (105\,e^{11 i (dx+c)}+595\,e^{9 i (dx+c)}-1686\,e^{7 i (dx+c)}-1386\,e^{5 i (dx+c)}-595\,e^{3 i (dx+c)}-105\,e^{i (dx+c)})}{120 d (e^{2 i (dx+c)}+1)^6}-\frac{7a^2 \ln(e^{i(dx+c)}}{16d}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ia^2}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{(\sec^3(d}{4}\right.\right.}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ia^2}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{(\sec^3(d}{4}\right.\right.}{d}$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/120*I*a^2/d/(exp(2*I*(d*x+c))+1)^6*(105*exp(11*I*(d*x+c))+595*exp(9*I*(
d*x+c))-1686*exp(7*I*(d*x+c))-1386*exp(5*I*(d*x+c))-595*exp(3*I*(d*x+c))-1
05*exp(I*(d*x+c)))-7/16/d*a^2*ln(exp(I*(d*x+c))-I)+7/16/d*a^2*ln(exp(I*(d*
x+c))+I)
```

3.28.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(102) = 204$.

Time = 0.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.08

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{-210i\,a^2e^{(11i\,dx+11i\,c)} - 1190i\,a^2e^{(9i\,dx+9i\,c)} + 3372i\,a^2e^{(7i\,dx+7i\,c)} + 2772i\,a^2e^{(5i\,dx+5i\,c)} + 1190i\,a^2e^{(3i\,dx+3i\,c)}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/240*(-210*I*a^2*e^(11*I*d*x + 11*I*c) - 1190*I*a^2*e^(9*I*d*x + 9*I*c) +
 3372*I*a^2*e^(7*I*d*x + 7*I*c) + 2772*I*a^2*e^(5*I*d*x + 5*I*c) + 1190*I*
a^2*e^(3*I*d*x + 3*I*c) + 210*I*a^2*e^(I*d*x + I*c) + 105*(a^2*e^(12*I*d*x
 + 12*I*c) + 6*a^2*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20
*a^2*e^(6*I*d*x + 6*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x +
 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) - 105*(a^2*e^(12*I*d*x + 12*I*c) +
 6*a^2*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2*e^(6*I*
d*x + 6*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*c) + a^
2)*log(e^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x +
10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*
I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

3.28.6 Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^5(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^5(c+dx)\right)\,dx + \int \left(-\sec^5(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**5, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**5, x) + Integral(-sec(c + d*x)**5, x))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.53

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{5\,a^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+30\,a^2\left(\right.}{480\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/480*(5*a^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) + 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 192*I*a^2/cos(d*x + c)^5)/d
```

3.28.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(102) = 204$.

Time = 0.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.01

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= \frac{105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(135\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480i\,a^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(105*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^2*log(tan(1/2*d*x + 1
/2*c) - 1) + 2*(135*a^2*tan(1/2*d*x + 1/2*c)^11 - 480*I*a^2*tan(1/2*d*x +
1/2*c)^10 - 445*a^2*tan(1/2*d*x + 1/2*c)^9 + 480*I*a^2*tan(1/2*d*x + 1/2*c
)^8 - 330*a^2*tan(1/2*d*x + 1/2*c)^7 - 960*I*a^2*tan(1/2*d*x + 1/2*c)^6 -
330*a^2*tan(1/2*d*x + 1/2*c)^5 + 960*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 445*a^
2*tan(1/2*d*x + 1/2*c)^3 - 96*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 135*a^2*tan(1
/2*d*x + 1/2*c) + 96*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

$$-\frac{-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}4\mathrm{i}+\frac{89\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}4\mathrm{i}+\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+a^2\,\mathrm{ta}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^5,x)
```

output

```
(7*a^2*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/5
 + (89*a^2*tan(c/2 + (d*x)/2)^3)/24 - a^2*tan(c/2 + (d*x)/2)^4*8i + (11*a^
2*tan(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*8i + (11*a^2*tan(c/2
+ (d*x)/2)^7)/4 - a^2*tan(c/2 + (d*x)/2)^8*4i + (89*a^2*tan(c/2 + (d*x)/2)
^9)/24 + a^2*tan(c/2 + (d*x)/2)^10*4i - (9*a^2*tan(c/2 + (d*x)/2)^11)/8 -
(a^2*4i)/5 - (9*a^2*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6
*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8
- 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.28. $\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.29 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.29.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{5a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

output
```
5/8*a^2*arctanh(sin(d*x+c))/d+5/12*I*a^2*sec(d*x+c)^3/d+5/8*a^2*sec(d*x+c)
*tan(d*x+c)/d+1/4*I*sec(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

3.29.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{5a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{2ia^2\sec^3(c+dx)}{3d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output

```
(5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((2*I)/3)*a^2*Sec[c + d*x]^3)/d + (
5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d)
```

3.29.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3979, 3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow 3042$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow 3979$$

$$\frac{5}{4}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow 3042$$

$$\frac{5}{4}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow 3967$$

$$\frac{5}{4}a\left(a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow 3042$$

$$\frac{5}{4}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow 4255$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow 3042$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow 4257$$

$$\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5}{4}a\left(a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/4)*Sec[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]))/d + (5*a*(((I/3)*a*Sec[c
 + d*x]^3)/d + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x]
)/(2*d))))/4
```

3.29.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.29.4 Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{i a^{2} \left(15 \,{\mathrm e}^{7 i \left(d x +c \right)}-73 \,{\mathrm e}^{5 i \left(d x +c \right)}-55 \,{\mathrm e}^{3 i \left(d x +c \right)}-15 \,{\mathrm e}^{i \left(d x +c \right)}\right)}{12 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{4}}+\frac{5 a^{2} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{8 d}-\frac{5 a^{2} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{8 d}$
derivativedivides	$\frac{-a^{2} \left(\frac{\sin^{3}\left(d x +c \right)}{4 \cos \left(d x +c \right)^{4}}+\frac{\sin^{3}\left(d x +c \right)}{8 \cos \left(d x +c \right)^{2}}+\frac{\sin \left(d x +c \right)}{8}-\frac{\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{8}\right)+\frac{2 i a^{2}}{3 \cos \left(d x +c \right)^{3}}+a^{2} \left(\frac{\sec \left(d x +c \right) \tan \left(d x +c \right)}{2}+\ln(\sec(d\right.}{d}$
default	$\frac{-a^{2} \left(\frac{\sin^{3}\left(d x +c \right)}{4 \cos \left(d x +c \right)^{4}}+\frac{\sin^{3}\left(d x +c \right)}{8 \cos \left(d x +c \right)^{2}}+\frac{\sin \left(d x +c \right)}{8}-\frac{\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{8}\right)+\frac{2 i a^{2}}{3 \cos \left(d x +c \right)^{3}}+a^{2} \left(\frac{\sec \left(d x +c \right) \tan \left(d x +c \right)}{2}+\ln(\sec(d\right.}{d}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I*a^2/d/(exp(2*I*(d*x+c))+1)^4*(15*exp(7*I*(d*x+c))-73*exp(5*I*(d*x+
c))-55*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+5/8/d*a^2*ln(exp(I*(d*x+c))+I)-
5/8/d*a^2*ln(exp(I*(d*x+c))-I)
```

3.29.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(80) = 160.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.72

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{-30i\,a^2e^{(7i\,dx+7i\,c)}+146i\,a^2e^{(5i\,dx+5i\,c)}+110i\,a^2e^{(3i\,dx+3i\,c)}+30i\,a^2e^{(i\,dx+i\,c)}+15\left(a^2e^{(8i\,dx+8i\,c)}+4\,a^2e^{(6i\,dx}\right.}{24\left(de^{(8i}\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(-30*I*a^2*e^(7*I*d*x + 7*I*c) + 146*I*a^2*e^(5*I*d*x + 5*I*c) + 110*
I*a^2*e^(3*I*d*x + 3*I*c) + 30*I*a^2*e^(I*d*x + I*c) + 15*(a^2*e^(8*I*d*x
+ 8*I*c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e
^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) - 15*(a^2*e^(8*I*d*x +
8*I*c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(
2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) +
 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*
c) + d)
```

3.29.6 Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^3(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^3(c+dx)\right)\,dx + \int \left(-\sec^3(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(-2*I*tan(c
+ d*x)*sec(c + d*x)**3, x) + Integral(-sec(c + d*x)**3, x))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{3\,a^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log}{48\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/48*(3*a^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) - 32*I*a^2/cos(d*x + c)^3)/d
```

3.29.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(80) = 160.

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= \frac{15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-}{}}{24\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(15*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^2*log(tan(1/2*d*x + 1/2*
c) - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*I*a^2*tan(1/2*d*x + 1/2*c)^
6 - 33*a^2*tan(1/2*d*x + 1/2*c)^5 + 48*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 33*a
^2*tan(1/2*d*x + 1/2*c)^3 - 16*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*tan(1/
2*d*x + 1/2*c) + 16*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.29.9 Mupad [B] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.11

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

$$-\frac{-\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,4{}\mathrm{i}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4{}\mathrm{i}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^3,x)
```

output

```
(5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/3
 + (11*a^2*tan(c/2 + (d*x)/2)^3)/4 - a^2*tan(c/2 + (d*x)/2)^4*4i + (11*a^2
*tan(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*4i - (3*a^2*tan(c/2 +
(d*x)/2)^7)/4 - (a^2*4i)/3 - (3*a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 +
 (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 +
(d*x)/2)^8 + 1))
```

3.29. $\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.30 $\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.30.1 Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{3a^2\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

output
```
3/2*a^2*arctanh(sin(d*x+c))/d+3/2*I*a^2*sec(d*x+c)/d+1/2*I*sec(d*x+c)*(a^2
+I*a^2*tan(d*x+c))/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{3a^2\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{2ia^2\sec(c+dx)}{d}-\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*I)*a^2*Sec[c + d*x])/d - (a^2*Se
c[c + d*x]*Tan[c + d*x])/(2*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2dx$$

$$\Big\downarrow \ 3979$$

$$\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \ 3967$$

$$\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \ 4257$$

$$\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(3*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[
c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

3.30.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.30.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-a^{2}\left(\frac{\sin^{3}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^{2}}{\cos(dx+c)}+a^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
default	$\frac{-a^{2}\left(\frac{\sin^{3}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^{2}}{\cos(dx+c)}+a^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
risch	$\frac{ia^{2}\left(5\,\mathrm{e}^{3i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{3a^{2}\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{3a^{2}\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$	89

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+
tan(d*x+c)))+2*I*a^2/cos(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.30.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{10i\,a^2e^{(3i\,dx+3i\,c)} + 6i\,a^2e^{(i\,dx+i\,c)} + 3\,(a^2e^{(4i\,dx+4i\,c)} + 2\,a^2e^{(2i\,dx+2i\,c)} + a^2)\log\,(e^{(i\,dx+i\,c)} + i) - 3\,(a^2e^{(4i\,dx+4}}{2\,(de^{(4i\,dx+4i\,c)} + 2\,de^{(2i\,dx+2i\,c)} + d)}$$

input

```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output

```
1/2*(10*I*a^2*e^(3*I*d*x + 3*I*c) + 6*I*a^2*e^(I*d*x + I*c) + 3*(a^2*e^(4*
I*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I)
 - 3*(a^2*e^(4*I*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*
d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.30.6 Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec(c+dx)\,dx + \int (-2i\tan(c+dx)\sec(c+dx))\,dx + \int (-\sec(c+dx))\,dx\right)$$

input

```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

output

```
-a**2*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(-2*I*tan(c + d
*x)*sec(c + d*x), x) + Integral(-sec(c + d*x), x))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)\right) + 4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right) +}{4\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*I*a^2/co
s(d*x + c))/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.57

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{3\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right) - 3\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right) - \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a^2*log(tan(1/2*d*x + 1/2*c)
- 1) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 + 4*I*a^2*tan(1/2*d*x + 1/2*c)^2 + a^
2*tan(1/2*d*x + 1/2*c) - 4*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^2\,4\mathrm{i}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x),x)
```

output
```
(3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2*4i + a^2*t
an(c/2 + (d*x)/2)^3 - a^2*4i + a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)
/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.31 $\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.31.1 Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

output
```
-a^2*arctanh(sin(d*x+c))/d-2*I*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))/d
```

3.31.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 180 vs. 2(46) = 92.

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.91

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(Cos[(c + d*x)/2]*(-2*I + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 - I*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[(c + d
*x)/2])*(Cos[(c + 5*d*x)/2] + I*Sin[(c + 5*d*x)/2]))/(d*(Cos[d*x] + I*Sin[
d*x])^2)
```

3.31.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3977, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3977}$$

$$a^2\left(-\int \sec(c+dx)dx\right)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$a^2\left(-\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-((a^2*ArcTanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c
+ d*x]))/d
```

3.31.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.31.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
default	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
risch	$-\frac{2ia^2 e^{i(dx+c)}}{d}+\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}$	61

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*I*a^2*cos(d*x+c)+a^2*s
in(d*x+c))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-2i\,a^2e^{(i\,dx+i\,c)} - a^2\log\left(e^{(i\,dx+i\,c)}+i\right) + a^2\log\left(e^{(i\,dx+i\,c)}-i\right)}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
(-2*I*a^2*e^(I*d*x + I*c) - a^2*log(e^(I*d*x + I*c) + I) + a^2*log(e^(I*d*
x + I*c) - I))/d
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} -\frac{2ia^2e^{ic}e^{idx}}{d} & \text{for } d\neq 0 \\ 2a^2xe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

output
```
a**2*(log(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise((-2*I*a**2*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (2*a**2*x*exp(I*c),
True))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) - 2\sin(dx+c)) + 4i\,a^2\cos(dx+c) - 2\,a^2\sin(dx+}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
 + 4*I*a^2*cos(d*x + c) - 2*a^2*sin(d*x + c))/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+a^2\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
(-2*I*a^2*e^(I*d*x + I*c) - a^2*log(I*e^(I*d*x + I*c) - 1) + a^2*log(-I*e^
(I*d*x + I*c) - 1))/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(4*a^2)/(d*(tan(c/2 + (d*x)/2) + 1i)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)))/
d
```

3.32 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.32.1 Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

output
```
1/3*a^2*sin(d*x+c)/d-2/3*I*cos(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

3.32.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{2ia^2\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/3)*a^2*Cos[c + d*x]^3)/d + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c +
d*x]^3)/(3*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3977, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{3}a^2\int \cos(c+dx)dx-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*Sin[c + d*x])/(3*d) - (((2*I)/3)*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c +
d*x]))/d
```

3.32.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.32.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{i a^{2} {\mathrm e}^{3 i \left(d x +c \right)}}{6 d}-\frac{i a^{2} {\mathrm e}^{i \left(d x +c \right)}}{2 d}$	38
derivativedivides	$\frac{-\frac{a^{2} \left(\sin^{3}\left(d x +c \right)\right)}{3}-\frac{2 i a^{2} \left(\cos^{3}\left(d x +c \right)\right)}{3}+\frac{a^{2} \left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)}{3}}{d}$	54
default	$\frac{-\frac{a^{2} \left(\sin^{3}\left(d x +c \right)\right)}{3}-\frac{2 i a^{2} \left(\cos^{3}\left(d x +c \right)\right)}{3}+\frac{a^{2} \left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)}{3}}{d}$	54

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*I/d*a^2*exp(3*I*(d*x+c))-1/2*I/d*a^2*exp(I*(d*x+c))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-i\,a^2e^{(3i\,dx+3i\,c)}-3i\,a^2e^{(i\,dx+i\,c)}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(-I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*a^2*e^(I*d*x + I*c))/d
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} \frac{-2ia^2de^{3ic}e^{3idx}-6ia^2de^{ic}e^{idx}}{12d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{3ic}}{2}+\frac{a^2e^{ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-2*I*a**2*d*exp(3*I*c)*exp(3*I*d*x) - 6*I*a**2*d*exp(I*c)*exp(
I*d*x))/(12*d**2), Ne(d**2, 0)), (x*(a**2*exp(3*I*c)/2 + a**2*exp(I*c)/2),
 True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= -\frac{2i\,a^2\cos\left(dx+c\right)^3+a^2\sin\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(2*I*a^2*cos(d*x + c)^3 + a^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*si
n(d*x + c))*a^2)/d
```

3.32.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(43) = 86.

Time = 0.52 (sec) , antiderivative size = 531, normalized size of antiderivative = 10.41

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{24\,a^2e^{(4i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+48\,a^2e^{(2i\,dx)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+24\,a^2e^{(-2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/96*(24*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 48*a^2*e^(2
*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 24*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 27*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 54*a^2*e
^(2*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 27*a^2*e^(-2*I*c)*log(I*e^(I*d*x +
 I*c) - 1) - 24*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 48*a
^2*e^(2*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 24*a^2*e^(-2*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 27*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 54*a^2*e^(2*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 27*a^2*e^(-2*I*c)*log(-
I*e^(I*d*x + I*c) - 1) + 3*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I
*c)) + 6*a^2*e^(2*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^2*e^(-2*I*c)*lo
g(I*e^(I*d*x) + e^(-I*c)) - 3*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) - 6*a^2*e^(2*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 3*a^2*e^(-2*I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) + 16*I*a^2*e^(7*I*d*x + 5*I*c) + 80*I*a^2*
e^(5*I*d*x + 3*I*c) + 112*I*a^2*e^(3*I*d*x + I*c) + 48*I*a^2*e^(I*d*x - I*
c))/(d*e^(4*I*d*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

3.32.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}-2\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
-(2*a^2*(tan(c/2 + (d*x)/2)*3i + 3*tan(c/2 + (d*x)/2)^2 - 2))/(3*d*(3*tan(
c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.33 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.33.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{3a^2\sin(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

output
```
3/5*a^2*sin(d*x+c)/d-1/5*a^2*sin(d*x+c)^3/d-2/5*I*cos(d*x+c)^5*(a^2+I*a^2*
tan(d*x+c))/d
```

3.33.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{2ia^2\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin^5(c+dx)}{5d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/5)*a^2*Cos[c + d*x]^5)/d + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*
x]^3)/d + (2*a^2*Sin[c + d*x]^5)/(5*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \textcolor{blue}{3977}$$

$$\frac{3}{5}a^2\int \cos^3(c+dx)dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{3}{5}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \textcolor{blue}{3113}$$

$$-\frac{3a^2\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{3a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-3*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*d) - (((2*I)/5)*Cos[c + d*x
]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.33.4 Maple [A] (verified)

Time = 14.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i a^{2} {\mathrm e}^{5 i (d x +c)}}{40 d}-\frac{i a^{2} {\mathrm e}^{3 i (d x +c)}}{8 d}-\frac{i a^{2} \cos (d x +c)}{4 d}+\frac{a^{2} \sin (d x +c)}{2 d}$
derivativedivides	$\frac{-a^{2} \left(-\frac{\sin (d x +c) \left(\cos^{4}(d x +c)\right)}{5}+\frac{\left(2+\cos^{2}(d x +c)\right) \sin (d x +c)}{15}\right)-\frac{2 i a^{2} \left(\cos^{5}(d x +c)\right)}{5}+\frac{a^{2} \left(\frac{8}{3}+\cos^{4}(d x +c)+\frac{4 \left(\cos^{2}(d x +c)\right)}{3}\right) \sin (}{5}}{d}$
default	$\frac{-a^{2} \left(-\frac{\sin (d x +c) \left(\cos^{4}(d x +c)\right)}{5}+\frac{\left(2+\cos^{2}(d x +c)\right) \sin (d x +c)}{15}\right)-\frac{2 i a^{2} \left(\cos^{5}(d x +c)\right)}{5}+\frac{a^{2} \left(\frac{8}{3}+\cos^{4}(d x +c)+\frac{4 \left(\cos^{2}(d x +c)\right)}{3}\right) \sin (}{5}}{d}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/40*I/d*a^2*exp(5*I*(d*x+c))-1/8*I/d*a^2*exp(3*I*(d*x+c))-1/4*I/d*a^2*co
s(d*x+c)+1/2*a^2*sin(d*x+c)/d
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\left(-i\,a^2e^{(6i\,dx+6i\,c)}-5i\,a^2e^{(4i\,dx+4i\,c)}-15i\,a^2e^{(2i\,dx+2i\,c)}+5i\,a^2\right)e^{(-i\,dx-i\,c)}}{40\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/40*(-I*a^2*e^(6*I*d*x + 6*I*c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) - 15*I*a^2*
e^(2*I*d*x + 2*I*c) + 5*I*a^2)*e^(-I*d*x - I*c)/d
```

3.33.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(60) = 120.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.22

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} \frac{\left(-512ia^2d^3e^{6ic}e^{5idx}-2560ia^2d^3e^{4ic}e^{3idx}-7680ia^2d^3e^{2ic}e^{idx}+2560ia^2d^3e^{-idx}\right)e^{-ic}}{20480d^4} & \text{for } d^4e^{ic}\neq 0 \\ \frac{x\left(a^2e^{6ic}+3a^2e^{4ic}+3a^2e^{2ic}+a^2\right)e^{-ic}}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-512*I*a**2*d**3*exp(6*I*c)*exp(5*I*d*x) - 2560*I*a**2*d**3*ex
p(4*I*c)*exp(3*I*d*x) - 7680*I*a**2*d**3*exp(2*I*c)*exp(I*d*x) + 2560*I*a*
*2*d**3*exp(-I*d*x))*exp(-I*c)/(20480*d**4), Ne(d**4*exp(I*c), 0)), (x*(a*
*2*exp(6*I*c) + 3*a**2*exp(4*I*c) + 3*a**2*exp(2*I*c) + a**2)*exp(-I*c)/8,
 True))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{6i\,a^2\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)a^2-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\mathrm{si}}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*(6*I*a^2*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2
- (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2)/d
```

3.33.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(59) = 118.

Time = 0.60 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{45\,a^2e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+90\,a^2e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+45\,a^2e^{(i\,dx-i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/160*(45*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a^2*e^(
3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 45*a^2*e^(I*d*x - I*c)*log(I*e
^(I*d*x + I*c) + 1) + 40*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1
) + 80*a^2*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 40*a^2*e^(I*d*x
- I*c)*log(I*e^(I*d*x + I*c) - 1) - 45*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 90*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) -
45*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 40*a^2*e^(5*I*d*x + 3
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 80*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d
*x + I*c) - 1) - 40*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5*a^
2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 10*a^2*e^(3*I*d*x + I*
c)*log(I*e^(I*d*x) + e^(-I*c)) - 5*a^2*e^(I*d*x - I*c)*log(I*e^(I*d*x) + e
^(-I*c)) + 5*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 10*a^2
*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5*a^2*e^(I*d*x - I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) + 4*I*a^2*e^(10*I*d*x + 8*I*c) + 28*I*a^2*e^(8*
I*d*x + 6*I*c) + 104*I*a^2*e^(6*I*d*x + 4*I*c) + 120*I*a^2*e^(4*I*d*x + 2*
I*c) + 20*I*a^2*e^(2*I*d*x) - 20*I*a^2*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c)
+ 2*d*e^(3*I*d*x + I*c) + d*e^(I*d*x - I*c))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{2\,a^2\,\left(\frac{5\,\sin(3\,c+3\,d\,x)}{16}-\frac{\cos(5\,c+5\,d\,x)\,1\mathrm{i}}{16}-\frac{\cos(3\,c+3\,d\,x)\,5\mathrm{i}}{16}+\frac{\sin(5\,c+5\,d\,x)}{16}+\frac{5\,\sqrt{3}\,\sin\left(c+d\,x-\frac{\ln(3)\,1\mathrm{i}}{2}\right)}{8}\right)}{5\,d}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(2*a^2*((5*sin(3*c + 3*d*x))/16 - (cos(5*c + 5*d*x)*1i)/16 - (cos(3*c + 3*
d*x)*5i)/16 + sin(5*c + 5*d*x)/16 + (5*3^(1/2)*sin(c - (log(3)*1i)/2 + d*x
))/8))/(5*d)
```

3.34 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.34.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5a^2\sin(c+dx)}{7d} - \frac{10a^2\sin^3(c+dx)}{21d} + \frac{a^2\sin^5(c+dx)}{7d} - \frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

output

```
5/7*a^2*sin(d*x+c)/d-10/21*a^2*sin(d*x+c)^3/d+1/7*a^2*sin(d*x+c)^5/d-2/7*I
*cos(d*x+c)^7*(a^2+I*a^2*tan(d*x+c))/d
```

3.34.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2\cos^7(c+dx)}{7d} + \frac{a^2\sin(c+dx)}{d} - \frac{4a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin^5(c+dx)}{d} - \frac{2a^2\sin^7(c+dx)}{7d}$$

input

```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/7)*a^2*Cos[c + d*x]^7)/d + (a^2*Sin[c + d*x])/d - (4*a^2*Sin[c +
d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/d - (2*a^2*Sin[c + d*x]^7)/(7*d)
```

3.34.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{5}{7}a^2\int \cos^5(c+dx)dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{5a^2\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5a^2\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-5*a^2*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*d) -
 (((2*I)/7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.34.4 Maple [A] (verified)

Time = 49.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{i a^{2} {\mathrm e}^{7 i \left(d x +c \right)}}{224 d}-\frac{i a^{2} {\mathrm e}^{5 i \left(d x +c \right)}}{32 d}-\frac{5 i a^{2} \cos \left(d x +c \right)}{32 d}+\frac{15 a^{2} \sin \left(d x +c \right)}{32 d}-\frac{3 i a^{2} \cos \left(3 d x +3 c \right)}{32 d}+\frac{11 a^{2} \sin \left(3 d x +3 c \right)}{96 d}$
derivativedivides	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \left(\cos^{6}\left(d x +c \right)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}\left(d x +c \right)+\frac{4 \left(\cos^{2}\left(d x +c \right)\right)}{3}\right) \sin \left(d x +c \right)}{35}\right)-\frac{2 i a^{2} \left(\cos^{7}\left(d x +c \right)\right)}{7}+ a^{2} \left(\frac{16}{5}+\cos^{6}\left(d x +c \right)+\right.}{d}$
default	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \left(\cos^{6}\left(d x +c \right)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}\left(d x +c \right)+\frac{4 \left(\cos^{2}\left(d x +c \right)\right)}{3}\right) \sin \left(d x +c \right)}{35}\right)-\frac{2 i a^{2} \left(\cos^{7}\left(d x +c \right)\right)}{7}+ a^{2} \left(\frac{16}{5}+\cos^{6}\left(d x +c \right)+\right.}{d}$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/224*I/d*a^2*exp(7*I*(d*x+c))-1/32*I/d*a^2*exp(5*I*(d*x+c))-5/32*I/d*a^2
*cos(d*x+c)+15/32*a^2*sin(d*x+c)/d-3/32*I/d*a^2*cos(3*d*x+3*c)+11/96/d*a^2
*sin(3*d*x+3*c)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\left(-3i\,a^2e^{(10i\,dx+10i\,c)} - 21i\,a^2e^{(8i\,dx+8i\,c)} - 70i\,a^2e^{(6i\,dx+6i\,c)} - 210i\,a^2e^{(4i\,dx+4i\,c)} + 105i\,a^2e^{(2i\,dx+2i\,c)} + 7i\,a^2\right)}{672\,d}$$

input

```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output

```
1/672*(-3*I*a^2*e^(10*I*d*x + 10*I*c) - 21*I*a^2*e^(8*I*d*x + 8*I*c) - 70*
I*a^2*e^(6*I*d*x + 6*I*c) - 210*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2
*I*d*x + 2*I*c) + 7*I*a^2)*e^(-3*I*d*x - 3*I*c)/d
```

3.34.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(76) = 152.

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.74

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} \frac{(-75497472ia^2d^5e^{11ic}e^{7idx}-528482304ia^2d^5e^{9ic}e^{5idx}-1761607680ia^2d^5e^{7ic}e^{3idx}-5284823040ia^2d^5e^{5ic}e^{idx}+2642411520ia^2d^5e^{3ic}e^{-idx}+}{16911433728d^6} \\ \frac{x(a^2e^{10ic}+5a^2e^{8ic}+10a^2e^{6ic}+10a^2e^{4ic}+5a^2e^{2ic}+a^2)e^{-3ic}}{32} \end{cases}$$

input

```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-75497472*I*a**2*d**5*exp(11*I*c)*exp(7*I*d*x) - 528482304*I*a
**2*d**5*exp(9*I*c)*exp(5*I*d*x) - 1761607680*I*a**2*d**5*exp(7*I*c)*exp(3
*I*d*x) - 5284823040*I*a**2*d**5*exp(5*I*c)*exp(I*d*x) + 2642411520*I*a**2
*d**5*exp(3*I*c)*exp(-I*d*x) + 176160768*I*a**2*d**5*exp(I*c)*exp(-3*I*d*x
))*exp(-4*I*c)/(16911433728*d**6), Ne(d**6*exp(4*I*c), 0)), (x*(a**2*exp(1
0*I*c) + 5*a**2*exp(8*I*c) + 10*a**2*exp(6*I*c) + 10*a**2*exp(4*I*c) + 5*a
**2*exp(2*I*c) + a**2)*exp(-3*I*c)/32, True))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{30i\,a^2\cos(dx+c)^7 + \left(15\sin(dx+c)^7 - 42\sin(dx+c)^5 + 35\sin(dx+c)^3\right)a^2 + 3\left(5\sin(dx+c)^7 -\right.}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(30*I*a^2*cos(d*x + c)^7 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 +
 35*sin(d*x + c)^3)*a^2 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin
(d*x + c)^3 - 35*sin(d*x + c))*a^2)/d
```

3.34.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(75) = 150.

Time = 0.67 (sec) , antiderivative size = 641, normalized size of antiderivative = 7.37

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{2583\,a^2e^{(7i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 5166\,a^2e^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 2583\,a^2e^{(3i\,dx-i\,c)}\log\left(i\,e\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output

```
-1/10752*(2583*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5166*a
^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 2583*a^2*e^(3*I*d*x - I*
c)*log(I*e^(I*d*x + I*c) + 1) + 2121*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 4242*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 21
21*a^2*e^(3*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 2583*a^2*e^(7*I*d*x
+ 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5166*a^2*e^(5*I*d*x + I*c)*log(-I*e
^(I*d*x + I*c) + 1) - 2583*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) +
1) - 2121*a^2*e^(7*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4242*a^2*e
^(5*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2121*a^2*e^(3*I*d*x - I*c)*
log(-I*e^(I*d*x + I*c) - 1) - 462*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 924*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 462*
a^2*e^(3*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 462*a^2*e^(7*I*d*x + 3
*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 924*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) + 462*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c))
 + 48*I*a^2*e^(14*I*d*x + 10*I*c) + 432*I*a^2*e^(12*I*d*x + 8*I*c) + 1840*
I*a^2*e^(10*I*d*x + 6*I*c) + 5936*I*a^2*e^(8*I*d*x + 4*I*c) + 6160*I*a^2*e
^(6*I*d*x + 2*I*c) - 1904*I*a^2*e^(2*I*d*x - 2*I*c) - 112*I*a^2*e^(4*I*d*x
) - 112*I*a^2*e^(-4*I*c))/(d*e^(7*I*d*x + 3*I*c) + 2*d*e^(5*I*d*x + I*c) +
 d*e^(3*I*d*x - I*c))
```

3.34. $\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.34.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.94

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{2\,a^2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} + \frac{256\,a^2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{7\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7} - \frac{8\,a^2\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\mathrm{i}\right)}{3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2} - \frac{128\,a^2\left(6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\mathrm{i}\right)}{7\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6} + \frac{16\,a^2\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15\mathrm{i}\right)}{3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3} - \frac{32\,a^2\left(22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-35\mathrm{i}\right)}{7\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4} + \frac{32\,a^2\left(31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-42\mathrm{i}\right)}{7\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}$$

input

```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2,x)
```

output

```
(2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (256*a^
2*(tan(c/2 + (d*x)/2) - 1i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (8*a^2*(
4*tan(c/2 + (d*x)/2) - 9i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (128*a^2*
(6*tan(c/2 + (d*x)/2) - 7i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (16*a^2*
(8*tan(c/2 + (d*x)/2) - 15i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (32*a^2
*(22*tan(c/2 + (d*x)/2) - 35i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (32*a
^2*(31*tan(c/2 + (d*x)/2) - 42i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.35 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$

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3.35.1 Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{7a^2\sin(c+dx)}{9d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}-\frac{a^2\sin^7(c+dx)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

output
```
7/9*a^2*sin(d*x+c)/d-7/9*a^2*sin(d*x+c)^3/d+7/15*a^2*sin(d*x+c)^5/d-1/9*a^
2*sin(d*x+c)^7/d-2/9*I*cos(d*x+c)^9*(a^2+I*a^2*tan(d*x+c))/d
```

3.35.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{2ia^2\cos^9(c+dx)}{9d}+\frac{a^2\sin(c+dx)}{d}-\frac{5a^2\sin^3(c+dx)}{3d}+\frac{9a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^7(c+dx)}{d}+\frac{2a^2\sin^9(c+dx)}{9d}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/9)*a^2*Cos[c + d*x]^9)/d + (a^2*Sin[c + d*x])/d - (5*a^2*Sin[c +
d*x]^3)/(3*d) + (9*a^2*Sin[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/d + (2
*a^2*Sin[c + d*x]^9)/(9*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{7}{9}a^2\int \cos^7(c+dx)dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{9}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{7a^2\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7a^2\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-7*a^2*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d
*x]^7/7))/(9*d) - (((2*I)/9)*Cos[c + d*x]^9*(a^2 + I*a^2*Tan[c + d*x]))/d
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.35.4 Maple [A] (verified)

Time = 135.74 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-a^{2}\left(-\frac{\left(\cos^{8}(dx+c)\right)\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^{2}\left(\cos^{9}(dx+c)\right)}{9}+ a^{2}(}{d}$
default	$\frac{-a^{2}\left(-\frac{\left(\cos^{8}(dx+c)\right)\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^{2}\left(\cos^{9}(dx+c)\right)}{9}+ a^{2}(}{d}$
risch	$-\frac{ia^{2}e^{9i(dx+c)}}{1152d}-\frac{ia^{2}e^{7i(dx+c)}}{128d}-\frac{7ia^{2}\cos(dx+c)}{64d}+\frac{7a^{2}\sin(dx+c)}{16d}-\frac{ia^{2}\cos(5dx+5c)}{32d}+\frac{11a^{2}\sin(5dx+5c)}{320d}-$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-2/9*I*a^2*cos(d*x+c)^9+1/9*a^2*(128/3
5+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin
(d*x+c))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{\left(-5i\,a^2e^{(14i\,dx+14i\,c)}-45i\,a^2e^{(12i\,dx+12i\,c)}-189i\,a^2e^{(10i\,dx+10i\,c)}-525i\,a^2e^{(8i\,dx+8i\,c)}-1575i\,a^2e^{(6i\,dx+6i\,c)}+\right.}{5760\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/5760*(-5*I*a^2*e^(14*I*d*x + 14*I*c) - 45*I*a^2*e^(12*I*d*x + 12*I*c) -
189*I*a^2*e^(10*I*d*x + 10*I*c) - 525*I*a^2*e^(8*I*d*x + 8*I*c) - 1575*I*a
^2*e^(6*I*d*x + 6*I*c) + 945*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I*
d*x + 2*I*c) + 9*I*a^2)*e^(-5*I*d*x - 5*I*c)/d
```

3.35.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(94) = 188.

Time = 0.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.99

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\begin{cases}\frac{\left(-126663739519795200ia^2d^7e^{18ic}e^{9idx}-1139973655678156800ia^2d^7e^{16ic}e^{7idx}-4787889353848258560ia^2d^7e^{14ic}e^{5idx}-1329969264957849\right.}{} \\ \frac{x\left(a^2e^{14ic}+7a^2e^{12ic}+21a^2e^{10ic}+35a^2e^{8ic}+35a^2e^{6ic}+21a^2e^{4ic}+7a^2e^{2ic}+a^2\right)e^{-5ic}}{128}\end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-126663739519795200*I*a**2*d**7*exp(18*I*c)*exp(9*I*d*x) - 113
9973655678156800*I*a**2*d**7*exp(16*I*c)*exp(7*I*d*x) - 478788935384825856
0*I*a**2*d**7*exp(14*I*c)*exp(5*I*d*x) - 13299692649578496000*I*a**2*d**7*
exp(12*I*c)*exp(3*I*d*x) - 39899077948735488000*I*a**2*d**7*exp(10*I*c)*ex
p(I*d*x) + 23939446769241292800*I*a**2*d**7*exp(8*I*c)*exp(-I*d*x) + 26599
38529915699200*I*a**2*d**7*exp(6*I*c)*exp(-3*I*d*x) + 227994731135631360*I
*a**2*d**7*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(145916627926804070400*d*
*8), Ne(d**8*exp(9*I*c), 0)), (x*(a**2*exp(14*I*c) + 7*a**2*exp(12*I*c) +
21*a**2*exp(10*I*c) + 35*a**2*exp(8*I*c) + 35*a**2*exp(6*I*c) + 21*a**2*ex
p(4*I*c) + 7*a**2*exp(2*I*c) + a**2)*exp(-5*I*c)/128, True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{70i\,a^2\cos(dx+c)^9 - \left(35\sin(dx+c)^9 - 135\sin(dx+c)^7 + 189\sin(dx+c)^5 - 105\sin(dx+c)^3\right)a^2}{315}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/315*(70*I*a^2*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 135*sin(d*x + c)^7
+ 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^2 - (35*sin(d*x + c)^9 - 180*
sin(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c
))*a^2)/d
```

3.35.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(91) = 182.

Time = 0.72 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.37

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{18585\,a^2e^{(9i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 37170\,a^2e^{(7i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 18585\,a^2e^{(5i\,dx-i\,c)}\log}{}$$

input

```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output

```
-1/92160*(18585*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 37170
*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 18585*a^2*e^(5*I*d*x -
 I*c)*log(I*e^(I*d*x + I*c) + 1) + 14625*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(
I*d*x + I*c) - 1) + 29250*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 14625*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 18585*a^2*e^(9
*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 37170*a^2*e^(7*I*d*x + I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 18585*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 14625*a^2*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
29250*a^2*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 14625*a^2*e^(5*I
*d*x - I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3960*a^2*e^(9*I*d*x + 3*I*c)*log
(I*e^(I*d*x) + e^(-I*c)) - 7920*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^
(-I*c)) - 3960*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3960*a^
2*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 7920*a^2*e^(7*I*d*x +
 I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(5*I*d*x - I*c)*log(-I*e^(
I*d*x) + e^(-I*c)) + 80*I*a^2*e^(18*I*d*x + 12*I*c) + 880*I*a^2*e^(16*I*d*
x + 10*I*c) + 4544*I*a^2*e^(14*I*d*x + 8*I*c) + 15168*I*a^2*e^(12*I*d*x +
6*I*c) + 45024*I*a^2*e^(10*I*d*x + 4*I*c) + 43680*I*a^2*e^(8*I*d*x + 2*I*c
) - 18624*I*a^2*e^(4*I*d*x - 2*I*c) - 1968*I*a^2*e^(2*I*d*x - 4*I*c) - 672
0*I*a^2*e^(6*I*d*x) - 144*I*a^2*e^(-6*I*c))/(d*e^(9*I*d*x + 3*I*c) + 2*d*e
^(7*I*d*x + I*c) + d*e^(5*I*d*x - I*c))
```

3.35. $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.35.9 Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.14

$$
\begin{aligned}
\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx &= \frac{2\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)} \\
&\quad + \frac{1024\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^9} \\
&\quad - \frac{8\,a^2\,\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2} \\
&\quad - \frac{512\,a^2\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^8} \\
&\quad + \frac{128\,a^2\,\left(19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-24\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7} \\
&\quad - \frac{64\,a^2\,\left(19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-35\mathrm{i}\right)}{5\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4} \\
&\quad + \frac{56\,a^2\,\left(19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-40\mathrm{i}\right)}{15\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3} \\
&\quad - \frac{128\,a^2\,\left(59\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-84\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6} \\
&\quad + \frac{32\,a^2\,\left(781\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1260\mathrm{i}\right)}{45\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}
\end{aligned}
$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (1024*a
^2*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (8*a^2*
(5*tan(c/2 + (d*x)/2) - 12i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (512*a^
2*(8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) + (128*a
^2*(19*tan(c/2 + (d*x)/2) - 24i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (64
*a^2*(19*tan(c/2 + (d*x)/2) - 35i))/(5*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (
56*a^2*(19*tan(c/2 + (d*x)/2) - 40i))/(15*d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
- (128*a^2*(59*tan(c/2 + (d*x)/2) - 84i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^
6) + (32*a^2*(781*tan(c/2 + (d*x)/2) - 1260i))/(45*d*(tan(c/2 + (d*x)/2)^2
 + 1)^5)
```

3.35. $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$

3.36 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.36.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}$$

output

```
-8/7*I*(a+I*a*tan(d*x+c))^7/a^4/d+3/2*I*(a+I*a*tan(d*x+c))^8/a^5/d-2/3*I*(
a+I*a*tan(d*x+c))^9/a^6/d+1/10*I*(a+I*a*tan(d*x+c))^10/a^7/d
```

3.36.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\sec^9(c+dx)(\cos(7(c+dx))+i\sin(7(c+dx)))(-66i+242i\cos(2(c+dx))+119\sec(c+dx)\sin(3(}{840d}$$

input

```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output

```
-1/840*(a^3*Sec[c + d*x]^9*(Cos[7*(c + d*x)] + I*Sin[7*(c + d*x)])*(-66*I
+ (242*I)*Cos[2*(c + d*x)] + 119*Sec[c + d*x]*Sin[3*(c + d*x)] + 35*Tan[c
+ d*x]))/d
```

3.36.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^6d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^9+6a(i\tan(c+dx)a+a)^8-12a^2(i\tan(c+dx)a+a)^7+8a^3(i\tan(c+dx)a+a)^6\right)}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{7}a^3(a+ia\tan(c+dx))^7-\frac{3}{2}a^2(a+ia\tan(c+dx))^8-\frac{1}{10}(a+ia\tan(c+dx))^{10}+\frac{2}{3}a(a+ia\tan(c+dx))^9\right)}{a^7d}$$

input

```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((-I)*((8*a^3*(a + I*a*Tan[c + d*x])^7)/7 - (3*a^2*(a + I*a*Tan[c + d*x])^
8)/2 + (2*a*(a + I*a*Tan[c + d*x])^9)/3 - (a + I*a*Tan[c + d*x])^10/10))/(
a^7*d)
```

3.36.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.36.4 Maple [A] (verified)

Time = 182.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
risch	$\frac{128ia^{3}\left(210\,\mathrm{e}^{12i(dx+c)}+252\,\mathrm{e}^{10i(dx+c)}+210\,\mathrm{e}^{8i(dx+c)}+120\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{2i(dx+c)}+1\right)}{105d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{10}}$
derivativedivides	$\frac{-ia^{3}\left(\frac{\sin^{4}(dx+c)}{10\cos(dx+c)^{10}}+\frac{3\left(\sin^{4}(dx+c)\right)}{40\cos(dx+c)^{8}}+\frac{\sin^{4}(dx+c)}{20\cos(dx+c)^{6}}+\frac{\sin^{4}(dx+c)}{40\cos(dx+c)^{4}}\right)-3a^{3}\left(\frac{\sin^{3}(dx+c)}{9\cos(dx+c)^{9}}+\frac{2\left(\sin^{3}(dx+c)\right)}{21\cos(dx+c)^{7}}+\frac{8\left(\sin^{3}(dx+c)\right.}{105\cos(dx+c)}\right.}{d}$
default	$\frac{-ia^{3}\left(\frac{\sin^{4}(dx+c)}{10\cos(dx+c)^{10}}+\frac{3\left(\sin^{4}(dx+c)\right)}{40\cos(dx+c)^{8}}+\frac{\sin^{4}(dx+c)}{20\cos(dx+c)^{6}}+\frac{\sin^{4}(dx+c)}{40\cos(dx+c)^{4}}\right)-3a^{3}\left(\frac{\sin^{3}(dx+c)}{9\cos(dx+c)^{9}}+\frac{2\left(\sin^{3}(dx+c)\right)}{21\cos(dx+c)^{7}}+\frac{8\left(\sin^{3}(dx+c)\right.}{105\cos(dx+c)}\right.}{d}$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*a^3*(210*exp(12*I*(d*x+c))+252*exp(10*I*(d*x+c))+210*exp(8*I*(d*
x+c))+120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)/d/(e
xp(2*I*(d*x+c))+1)^10
```

3.36.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(85) = 170$.

Time = 0.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.97

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{128\left(-210i\,a^3e^{(12i\,dx+12i\,c)} - 252i\,a^3e^{(10i\,dx+10i\,c)} - 210i\,a^3e^{(8i\,dx+8i\,c)} - 120i\right.}{105\left(de^{(20i\,dx+20i\,c)} + 10\,de^{(18i\,dx+18i\,c)} + 45\,de^{(16i\,dx+16i\,c)} + 120\,de^{(14i\,dx+14i\,c)} + 210\,de^{(12i\,dx+12i\,c)} + 252\,de\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-128/105*(-210*I*a^3*e^(12*I*d*x + 12*I*c) - 252*I*a^3*e^(10*I*d*x + 10*I*
c) - 210*I*a^3*e^(8*I*d*x + 8*I*c) - 120*I*a^3*e^(6*I*d*x + 6*I*c) - 45*I*
a^3*e^(4*I*d*x + 4*I*c) - 10*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(20*I
*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) +
 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I
*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 4
5*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

3.36.6 Sympy [F]

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\Bigg(&\int i\sec^8{(c+dx)}\,dx\\ &+\int\left(-3\tan{(c+dx)}\sec^8{(c+dx)}\right)dx\\ &+\int\tan^3{(c+dx)}\sec^8{(c+dx)}\,dx\\ &+\int\left(-3i\tan^2{(c+dx)}\sec^8{(c+dx)}\right)dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**8, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**8, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**8, x))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{21i\,a^3\tan(dx+c)^{10}+70\,a^3\tan(dx+c)^9+240\,a^3\tan(dx+c)^7-210i\,a^3\tan(dx+c)^6+252\,a^3\tan(}{210\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
 + c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*ta
n(d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{21i\,a^3\tan(dx+c)^{10}+70\,a^3\tan(dx+c)^9+240\,a^3\tan(dx+c)^7-210i\,a^3\tan(dx+c)^6+252\,a^3\tan(}{210\,d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
 + c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*ta
n(d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{a^3\,\sin(c+d\,x)\,\left(-210\cos(c+d\,x)^9-\cos(c+d\,x)^8\,\sin(c+d\,x)\,315\mathrm{i}-\cos(c+d\,x)^6\,\sin(c+d\,x)^3\,42}{}$$

input

```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^8,x)
```

output

```
-(a^3*sin(c + d*x)*(70*cos(c + d*x)*sin(c + d*x)^8 - cos(c + d*x)^8*sin(c
+ d*x)*315i - 210*cos(c + d*x)^9 + sin(c + d*x)^9*21i + 240*cos(c + d*x)^3
*sin(c + d*x)^6 - cos(c + d*x)^4*sin(c + d*x)^5*210i + 252*cos(c + d*x)^5*
sin(c + d*x)^4 - cos(c + d*x)^6*sin(c + d*x)^3*420i))/(210*d*cos(c + d*x)^
10)
```

3.36. $\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.37 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.37.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2i(a+ia\tan(c+dx))^6}{3a^3d} + \frac{4i(a+ia\tan(c+dx))^7}{7a^4d} - \frac{i(a+ia\tan(c+dx))^8}{8a^5d}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^6/a^3/d+4/7*I*(a+I*a*tan(d*x+c))^7/a^4/d-1/8*I*(
a+I*a*tan(d*x+c))^8/a^5/d
```

3.37.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^8(c+dx)(8+29\cos(2(c+dx))-27i\sin(2(c+dx)))(-i\cos(6(c+dx))+\sin(6(c+dx)))}{168d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Sec[c + d*x]^8*(8 + 29*Cos[2*(c + d*x)] - (27*I)*Sin[2*(c + d*x)])*((
-I)*Cos[6*(c + d*x)] + Sin[6*(c + d*x)]))/(168*d)
```

3.37.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^5d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^7-4a(i\tan(c+dx)a+a)^6+4a^2(i\tan(c+dx)a+a)^5\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{2}{3}a^2(a+ia\tan(c+dx))^6+\frac{1}{8}(a+ia\tan(c+dx))^8-\frac{4}{7}a(a+ia\tan(c+dx))^7\right)}{a^5d}$$

input

```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((-I)*((2*a^2*(a + I*a*Tan[c + d*x])^6)/3 - (4*a*(a + I*a*Tan[c + d*x])^7)
/7 + (a + I*a*Tan[c + d*x])^8/8))/(a^5*d)
```

3.37.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.37.4 Maple [A] (verified)

Time = 64.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
risch	$\frac{32 i a^{3} \left(56 \,{\mathrm e}^{10 i \left(d x +c \right)}+70 \,{\mathrm e}^{8 i \left(d x +c \right)}+56 \,{\mathrm e}^{6 i \left(d x +c \right)}+28 \,{\mathrm e}^{4 i \left(d x +c \right)}+8 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{21 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{8}}$
derivativedivides	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{8 \cos\left(d x +c \right)^{8}}+\frac{\sin^{4}\left(d x +c \right)}{12 \cos\left(d x +c \right)^{6}}+\frac{\sin^{4}\left(d x +c \right)}{24 \cos\left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin^{3}\left(d x +c \right)}{7 \cos\left(d x +c \right)^{7}}+\frac{4 \left(\sin^{3}\left(d x +c \right)\right)}{35 \cos\left(d x +c \right)^{5}}+\frac{8 \left(\sin^{3}\left(d x +c \right)\right)}{105 \cos\left(d x +c \right)^{3}}\right)+\frac{i a^{3}}{2 \cos\left(d x +c \right)^{6}}}{d}$
default	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{8 \cos\left(d x +c \right)^{8}}+\frac{\sin^{4}\left(d x +c \right)}{12 \cos\left(d x +c \right)^{6}}+\frac{\sin^{4}\left(d x +c \right)}{24 \cos\left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin^{3}\left(d x +c \right)}{7 \cos\left(d x +c \right)^{7}}+\frac{4 \left(\sin^{3}\left(d x +c \right)\right)}{35 \cos\left(d x +c \right)^{5}}+\frac{8 \left(\sin^{3}\left(d x +c \right)\right)}{105 \cos\left(d x +c \right)^{3}}\right)+\frac{i a^{3}}{2 \cos\left(d x +c \right)^{6}}}{d}$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
32/21*I*a^3*(56*exp(10*I*(d*x+c))+70*exp(8*I*(d*x+c))+56*exp(6*I*(d*x+c))+
28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^8
```

3.37.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(64) = 128$.

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{32\,(-56i\,a^3e^{(10i\,dx+10i\,c)} - 70i\,a^3e^{(8i\,dx+8i\,c)} - 56i\,a^3e^{(6i\,dx+6i\,c)} - 28i\,a^3e^{(4i\,dx+4i\,c)} - 8i}{21\,(de^{(16i\,dx+16i\,c)} + 8\,de^{(14i\,dx+14i\,c)} + 28\,de^{(12i\,dx+12i\,c)} + 56\,de^{(10i\,dx+10i\,c)} + 70\,de^{(8i\,dx+8i\,c)} + 56\,de^{(6i\,dx+6i}}$$

input

```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
-32/21*(-56*I*a^3*e^(10*I*d*x + 10*I*c) - 70*I*a^3*e^(8*I*d*x + 8*I*c) - 5
6*I*a^3*e^(6*I*d*x + 6*I*c) - 28*I*a^3*e^(4*I*d*x + 4*I*c) - 8*I*a^3*e^(2*
I*d*x + 2*I*c) - I*a^3)/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*
c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I
*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*
e^(2*I*d*x + 2*I*c) + d)
```

3.37.6 Sympy [F]

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\Bigg(\int i\sec^6(c+dx)\,dx \\ + \int \left(-3\tan(c+dx)\sec^6(c+dx)\right)\,dx \\ + \int \tan^3(c+dx)\sec^6(c+dx)\,dx \\ + \int \left(-3i\tan^2(c+dx)\sec^6(c+dx)\right)\,dx\Bigg)\end{aligned}$$

input

```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

output

```
-I*a**3*(Integral(I*sec(c + d*x)**6, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**6, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**6, x))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{21i\,a^3\tan(dx+c)^8+72\,a^3\tan(dx+c)^7-28i\,a^3\tan(dx+c)^6+168\,a^3\tan(dx+c)^5-210i\,a^3\tan(dx}{168\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
 + c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d
*x + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{21i\,a^3\tan(dx+c)^8+72\,a^3\tan(dx+c)^7-28i\,a^3\tan(dx+c)^6+168\,a^3\tan(dx+c)^5-210i\,a^3\tan(dx}{168\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
 + c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d
*x + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{a^3\,\sin(c+d\,x)\,\left(-168\cos(c+d\,x)^7-\cos(c+d\,x)^6\,\sin(c+d\,x)\,252\mathrm{i}+56\cos(c+d\,x)^5\,\sin(c+d\,x)^2}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^6,x)
```

output
```
-(a^3*sin(c + d*x)*(72*cos(c + d*x)*sin(c + d*x)^6 - cos(c + d*x)^6*sin(c
+ d*x)*252i - 168*cos(c + d*x)^7 + sin(c + d*x)^7*21i - cos(c + d*x)^2*sin
(c + d*x)^5*28i + 168*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c
 + d*x)^3*210i + 56*cos(c + d*x)^5*sin(c + d*x)^2))/(168*d*cos(c + d*x)^8)
```

3.37. $\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.38 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.38.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}+\frac{i(a+ia\tan(c+dx))^6}{6a^3d}$$

output
```
-2/5*I*(a+I*a*tan(d*x+c))^5/a^2/d+1/6*I*(a+I*a*tan(d*x+c))^6/a^3/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{a^3(7-5i\tan(c+dx))(-i+\tan(c+dx))^5}{30d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(7 - (5*I)*Tan[c + d*x])*(-I + Tan[c + d*x])^5)/(30*d)
```

3.38.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^4d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^4-(i\tan(c+dx)a+a)^5\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{5}a(a+ia\tan(c+dx))^5-\frac{1}{6}(a+ia\tan(c+dx))^6\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^5)/5 - (a + I*a*Tan[c + d*x])^6/6))/(a^
3*d)
```

3.38.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.38.4 Maple [A] (verified)

Time = 19.53 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

method	result
risch	$\frac{32ia^3\left(15\,\mathrm{e}^{8i(dx+c)}+20\,\mathrm{e}^{6i(dx+c)}+15\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3ia^3}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(d}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3ia^3}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(d}{d}$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
32/15*I*a^3*(15*exp(8*I*(d*x+c))+20*exp(6*I*(d*x+c))+15*exp(4*I*(d*x+c))+6
*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^6
```

3.38.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(43) = 86.

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.53

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{32\left(-15i\,a^3e^{(8i\,dx+8i\,c)}-20i\,a^3e^{(6i\,dx+6i\,c)}-15i\,a^3e^{(4i\,dx+4i\,c)}-6i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-32/15*(-15*I*a^3*e^(8*I*d*x + 8*I*c) - 20*I*a^3*e^(6*I*d*x + 6*I*c) - 15*
I*a^3*e^(4*I*d*x + 4*I*c) - 6*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(12*
I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 2
0*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*
c) + d)
```

3.38.6 Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\Bigg(\int i\sec^4{(c+dx)}\,dx + \int \left(-3\tan{(c+dx)}\sec^4{(c+dx)}\right)\,dx + \int \tan^3{(c+dx)}\sec^4{(c+dx)}\,dx + \int \left(-3i\tan^2{(c+dx)}\sec^4{(c+dx)}\right)\,dx\Bigg)$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**4, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**4, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**4, x))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{5i\,a^3\tan{(dx+c)}^6 + 18\,a^3\tan{(dx+c)}^5 - 15i\,a^3\tan{(dx+c)}^4 + 20\,a^3\tan{(dx+c)}^3 - 45i\,a^3\tan{(dx+}}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
 c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x +
 c))/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{5i\,a^3\tan(dx+c)^6+18\,a^3\tan(dx+c)^5-15i\,a^3\tan(dx+c)^4+20\,a^3\tan(dx+c)^3-45i\,a^3\tan(dx+}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
 c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x +
 c))/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{a^3\sin(c+d\,x)\,\left(-30\cos(c+d\,x)^5-\cos(c+d\,x)^4\sin(c+d\,x)\,45\mathrm{i}+20\cos(c+d\,x)^3\sin(c+d\,x)^2-}{30\,d\cos(c+d\,x)^6}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^4,x)
```

output
```
-(a^3*sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c
+ d*x)*45i - 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c
+ d*x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

3.39 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.39.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

output
```
-1/4*I*(a+I*a*tan(d*x+c))^4/a/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx \\ =\frac{a^3\tan(c+dx)\left(4+6i\tan(c+dx)-4\tan^2(c+dx)-i\tan^3(c+dx)\right)}{4d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Tan[c + d*x]*(4 + (6*I)*Tan[c + d*x] - 4*Tan[c + d*x]^2 - I*Tan[c + d
*x]^3))/(4*d)
```

3.39.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

input

```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((-1/4*I)*(a + I*a*Tan[c + d*x])^4)/(a*d)
```

3.39.3.1 Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.39.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(23) = 46$.

Time = 4.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

method	result	size
risch	$\frac{4 i a^{3} \left(4 \,{\mathrm e}^{6 i \left(d x +c \right)}+6 \,{\mathrm e}^{4 i \left(d x +c \right)}+4 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{4}}$	58
derivativedivides	$\frac{-\frac{i a^{3} \left(\sin^{4}\left(d x +c \right)\right)}{4 \cos \left(d x +c \right)^{4}}-\frac{a^{3} \left(\sin^{3}\left(d x +c \right)\right)}{\cos \left(d x +c \right)^{3}}+\frac{3 i a^{3}}{2 \cos \left(d x +c \right)^{2}}+a^{3} \tan \left(d x +c \right)}{d}$	73
default	$\frac{-\frac{i a^{3} \left(\sin^{4}\left(d x +c \right)\right)}{4 \cos \left(d x +c \right)^{4}}-\frac{a^{3} \left(\sin^{3}\left(d x +c \right)\right)}{\cos \left(d x +c \right)^{3}}+\frac{3 i a^{3}}{2 \cos \left(d x +c \right)^{2}}+a^{3} \tan \left(d x +c \right)}{d}$	73

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
4*I*a^3*(4*exp(6*I*(d*x+c))+6*exp(4*I*(d*x+c))+4*exp(2*I*(d*x+c))+1)/d/(ex
p(2*I*(d*x+c))+1)^4
```

3.39.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{4\left(-4i\,a^3e^{(6i\,dx+6i\,c)}-6i\,a^3e^{(4i\,dx+4i\,c)}-4i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)}{de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-4*(-4*I*a^3*e^(6*I*d*x + 6*I*c) - 6*I*a^3*e^(4*I*d*x + 4*I*c) - 4*I*a^3*e
^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*
c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.39.6 Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^2(c+dx)\,dx + \int \left(-3\tan(c+dx)\sec^2(c+dx)\right)\,dx + \int \tan^3(c+dx)\sec^2(c+dx)\,dx + \int \left(-3i\tan^2(c+dx)\sec^2(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**2, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**2, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**2, x))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,(i\,a\tan(dx+c)+a)^4}{4\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*I*(I*a*tan(d*x + c) + a)^4/(a*d)
```

3.39.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(21) = 42.

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,a^3\tan(dx+c)^4 + 4\,a^3\tan(dx+c)^3 - 6i\,a^3\tan(dx+c)^2 - 4\,a^3\tan(dx+c)}{4\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/4*(I*a^3*tan(d*x + c)^4 + 4*a^3*tan(d*x + c)^3 - 6*I*a^3*tan(d*x + c)^2
 - 4*a^3*tan(d*x + c))/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-\frac{a^3\tan(c+d\,x)^4\,1\mathrm{i}}{4}-a^3\tan(c+d\,x)^3+\frac{a^3\tan(c+d\,x)^2\,3\mathrm{i}}{2}+a^3\tan(c+d\,x)}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^2,x)
```

output
```
(a^3*tan(c + d*x) + (a^3*tan(c + d*x)^2*3i)/2 - a^3*tan(c + d*x)^3 - (a^3*
tan(c + d*x)^4*1i)/4)/d
```

3.40 $\int (a+ia\tan(c+dx))^3\,dx$

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3.40.1 Optimal result

Integrand size = 15, antiderivative size = 63

$$\int (a+ia\tan(c+dx))^3\,dx = 4a^3x - \frac{4ia^3\log(\cos(c+dx))}{d} - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d}$$

output
```
4*a^3*x-4*I*a^3*ln(cos(d*x+c))/d-2*a^3*tan(d*x+c)/d+1/2*I*a*(a+I*a*tan(d*x
+c))^2/d
```

3.40.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int (a+ia\tan(c+dx))^3\,dx = \frac{ia^3(8\log(i+\tan(c+dx))+6i\tan(c+dx)-\tan^2(c+dx))}{2d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/2)*a^3*(8*Log[I + Tan[c + d*x]] + (6*I)*Tan[c + d*x] - Tan[c + d*x]^2)
)/d
```

3.40.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3959}$$

$$2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3958}$$

$$2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)*a^2*Log[Cos[c
 + d*x]])/d - (a^2*Tan[c + d*x])/d)
```

3.40.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

3.40.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a^{3}\left(-3\tan\left(dx+c\right)-\frac{i\left(\tan^{2}\left(dx+c\right)\right)}{2}+2i\ln\left(1+\tan^{2}\left(dx+c\right)\right)+4\arctan\left(\tan\left(dx+c\right)\right)\right)}{d}$
default	$\frac{a^{3}\left(-3\tan\left(dx+c\right)-\frac{i\left(\tan^{2}\left(dx+c\right)\right)}{2}+2i\ln\left(1+\tan^{2}\left(dx+c\right)\right)+4\arctan\left(\tan\left(dx+c\right)\right)\right)}{d}$
parallelrisch	$\frac{-ia^{3}\left(\tan^{2}\left(dx+c\right)\right)+4ia^{3}\ln\left(1+\tan^{2}\left(dx+c\right)\right)+8a^{3}dx-6a^{3}\tan\left(dx+c\right)}{2d}$
norman	$4a^{3}x-\frac{3a^{3}\tan\left(dx+c\right)}{d}-\frac{ia^{3}\left(\tan^{2}\left(dx+c\right)\right)}{2d}+\frac{2ia^{3}\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{d}$
risch	$-\frac{8a^{3}c}{d}-\frac{2ia^{3}\left(4\,e^{2i\left(dx+c\right)}+3\right)}{d\left(e^{2i\left(dx+c\right)}+1\right)^{2}}-\frac{4ia^{3}\ln\left(e^{2i\left(dx+c\right)}+1\right)}{d}$
parts	$a^{3}x-\frac{ia^{3}\left(\frac{\left(\tan^{2}\left(dx+c\right)\right)}{2}-\frac{\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2}\right)}{d}+\frac{3ia^{3}\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}-\frac{3a^{3}\left(\tan\left(dx+c\right)-\arctan\left(\tan\left(dx+c\right)\right)\right)}{d}$

input
```
int((a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^3*(-3*tan(d*x+c)-1/2*I*tan(d*x+c)^2+2*I*ln(1+tan(d*x+c)^2)+4*arctan(
tan(d*x+c)))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{2\left(4i\,a^3e^{(2i\,dx+2i\,c)}+3i\,a^3+2\left(i\,a^3e^{(4i\,dx+4i\,c)}+2i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2*(4*I*a^3*e^(2*I*d*x + 2*I*c) + 3*I*a^3 + 2*(I*a^3*e^(4*I*d*x + 4*I*c) +
 2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(
4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{4ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-8ia^3e^{2ic}e^{2idx}-6ia^3}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))**3,x)
```

output
```
-4*I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-8*I*a**3*exp(2*I*c)*exp(2*
I*d*x) - 6*I*a**3)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x
) + d)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int (a + ia\tan(c + dx))^3\,dx = a^3x + \frac{3\,(dx + c - \tan{(dx + c)})a^3}{d} + \frac{i\,a^3\left(\frac{1}{\sin{(dx+c)}^2-1} - \log{\left(\sin{(dx + c)}^2 - 1\right)}\right)}{2\,d} + \frac{3i\,a^3\log{(\sec{(dx + c)})}}{d}$$

input

```
integrate((a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
a^3*x + 3*(d*x + c - tan(d*x + c))*a^3/d + 1/2*I*a^3*(1/(sin(d*x + c)^2 -
1) - log(sin(d*x + c)^2 - 1))/d + 3*I*a^3*log(sec(d*x + c))/d
```

3.40.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(55) = 110.

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int (a + ia\tan(c + dx))^3\,dx = -\frac{2\left(2i\,a^3e^{(4i\,dx+4i\,c)}\log{\left(e^{(2i\,dx+2i\,c)} + 1\right)} + 4i\,a^3e^{(2i\,dx+2i\,c)}\log{\left(e^{(2i\,dx+2i\,c)} + 1\right)} + 4i\,a^3e^{(2i\,dx+2i\,c)} + 2i\,a^3\log}{de^{(4i\,dx+4i\,c)} + 2\,de^{(2i\,dx+2i\,c)} + d}$$

input

```
integrate((a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-2*(2*I*a^3*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(
2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(2*I*d*x + 2*I*c
) + 2*I*a^3*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^3)/(d*e^(4*I*d*x + 4*I*c)
 + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int (a+ia\tan(c+dx))^3\,dx=-\frac{a^3\,\left(6\tan(c+d\,x)-\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,8\mathrm{i}+{\tan(c+d\,x)}^2\,1\mathrm{i}\right)}{2\,d}$$

input

```
int((a + a*tan(c + d*x)*1i)^3,x)
```

output

```
-(a^3*(6*tan(c + d*x) - log(tan(c + d*x) + 1i)*8i + tan(c + d*x)^2*1i))/(2
*d)
```

3.41 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.41.1 Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-a^3x+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{2ia^4}{d(a-ia\tan(c+dx))}$$

output

```
-a^3*x+I*a^3*ln(cos(d*x+c))/d-2*I*a^4/d/(a-I*a*tan(d*x+c))
```

3.41.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{ia^3\left(\log(i+\tan(c+dx))+\frac{2a}{a-ia\tan(c+dx)}\right)}{d}$$

input

```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((-I)*a^3*(Log[I + Tan[c + d*x]] + (2*a)/(a - I*a*Tan[c + d*x])))/d
```

3.41.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{i\tan(c+dx)a+a}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^3\int \left(\frac{2a}{(a-ia\tan(c+dx))^2}+\frac{1}{ia\tan(c+dx)-a}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{2a}{a-ia\tan(c+dx)}+\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*(Log[a - I*a*Tan[c + d*x]] + (2*a)/(a - I*a*Tan[c + d*x])))/d
```

3.41.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.41.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{i a^{3} {\mathrm e}^{2 i \left(d x +c \right)}}{d}+\frac{2 a^{3} c}{d}+\frac{i a^{3} \ln \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{d}$
derivativedivides	$\frac{-i a^{3} \left(-\frac{\left(\sin^{2}\left(d x +c \right)\right)}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)-3 a^{3} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-\frac{3 i a^{3} \left(\cos^{2}\left(d x +c \right)\right)}{2}+a^{3} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c\right.}{2}\right.}{d}$
default	$\frac{-i a^{3} \left(-\frac{\left(\sin^{2}\left(d x +c \right)\right)}{2}-\ln \left(\cos \left(d x +c \right)\right)\right)-3 a^{3} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-\frac{3 i a^{3} \left(\cos^{2}\left(d x +c \right)\right)}{2}+a^{3} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c\right.}{2}\right.}{d}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-I/d*a^3*exp(2*I*(d*x+c))+2/d*a^3*c+I/d*a^3*ln(exp(2*I*(d*x+c))+1)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{-i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
(-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \begin{cases} -\frac{ia^3e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 2a^3xe^{2ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise((-I*a**3*exp(2*I*c)*e
xp(2*I*d*x)/d, Ne(d, 0)), (2*a**3*x*exp(2*I*c), True))
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= -\frac{2\,(dx+c)a^3 + i\,a^3\log\left(\tan\left(dx+c\right)^2+1\right) - \frac{4\,\left(a^3\tan(dx+c)-i\,a^3\right)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(d*x + c)*a^3 + I*a^3*log(tan(d*x + c)^2 + 1) - 4*(a^3*tan(d*x + c
) - I*a^3)/(tan(d*x + c)^2 + 1))/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-i\,a^3e^{(2i\,dx+2i\,c)} + i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
(-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{2\,a^3}{d\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)} - \frac{a^3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d}$$

input

```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3,x)
```

output

```
(2*a^3)/(d*(tan(c + d*x) + 1i)) - (a^3*log(tan(c + d*x) + 1i)*1i)/d
```

3.42 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.42.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

output
```
-1/2*I*a^5/d/(a-I*a*tan(d*x+c))^2
```

3.42.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia^3}{2d(i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/2)*a^3)/(d*(I + Tan[c + d*x])^2)
```

3.42.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-1/2*I)*a^5)/(d*(a - I*a*Tan[c + d*x])^2)
```

3.42.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.42.4 Maple [A] (verified)

Time = 13.76 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{i a^3 \mathrm{e}^{4i(dx+c)}}{8d}-\frac{i a^3 \mathrm{e}^{2i(dx+c)}}{4d}$
derivativedivides	$\frac{-\frac{i a^3\left(\sin^4(dx+c)\right)}{4}-3a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3i a^3\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$
default	$\frac{-\frac{i a^3\left(\sin^4(dx+c)\right)}{4}-3a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3i a^3\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+\right.}{}\right.}{d}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I/d*a^3*exp(4*I*(d*x+c))-1/4*I/d*a^3*exp(2*I*(d*x+c))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{-i\,a^3e^{(4i\,dx+4i\,c)}-2i\,a^3e^{(2i\,dx+2i\,c)}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(-I*a^3*e^(4*I*d*x + 4*I*c) - 2*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

3.42.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(22) = 44$.

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} \frac{-4ia^3de^{4ic}e^{4idx}-8ia^3de^{2ic}e^{2idx}}{32d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^3e^{4ic}}{2}+\frac{a^3e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-4*I*a**3*d*exp(4*I*c)*exp(4*I*d*x) - 8*I*a**3*d*exp(2*I*c)*ex
p(2*I*d*x))/(32*d**2), Ne(d**2, 0)), (x*(a**3*exp(4*I*c)/2 + a**3*exp(2*I*
c)/2), True))
```

3.42.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{-i\,a^3\tan\left(dx+c\right)^2 - 2\,a^3\tan\left(dx+c\right) + i\,a^3}{2\left(\tan\left(dx+c\right)^4 + 2\,\tan\left(dx+c\right)^2 + 1\right)d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(-I*a^3*tan(d*x + c)^2 - 2*a^3*tan(d*x + c) + I*a^3)/((tan(d*x + c)^4
 + 2*tan(d*x + c)^2 + 1)*d)
```

3.42.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(21) = 42$.

Time = 0.71 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.00

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,a^3e^{(12i\,dx+8i\,c)}+6i\,a^3e^{(10i\,dx+6i\,c)}+14i\,a^3e^{(8i\,dx+4i\,c)}+16i\,a^3e^{(6i\,dx+2i\,c)}+2i\,a^3e^{(2i\,dx-2i\,c)}+9i\,a^3e^{(4i\,dx)}}{8\left(de^{(8i\,dx+4i\,c)}+4\,de^{(6i\,dx+2i\,c)}+4\,de^{(2i\,dx-2i\,c)}+6\,de^{(4i\,dx)}+de^{(-4i\,c)}\right)}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/8*(I*a^3*e^(12*I*d*x + 8*I*c) + 6*I*a^3*e^(10*I*d*x + 6*I*c) + 14*I*a^3
*e^(8*I*d*x + 4*I*c) + 16*I*a^3*e^(6*I*d*x + 2*I*c) + 2*I*a^3*e^(2*I*d*x -
 2*I*c) + 9*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x + 2
*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

3.42.9 Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\left(\frac{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}}{4}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-(a^3*(exp(c*2i + d*x*2i)/2 + exp(c*4i + d*x*4i)/4)*1i)/(2*d)
```

3.43 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.43.1 Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3x}{8} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3} - \frac{ia^5}{8d(a-ia\tan(c+dx))^2} - \frac{ia^4}{8d(a-ia\tan(c+dx))}$$

output
```
1/8*a^3*x-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^5/d/(a-I*a*tan(d*x+c))^
2-1/8*I*a^4/d/(a-I*a*tan(d*x+c))
```

3.43.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3(-10+9i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(i+\tan(c+dx))^3)}{24d(i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(-10 + (9*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]
*(I + Tan[c + d*x])^3))/(24*d*(I + Tan[c + d*x])^3)
```

3.43.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^7\int \left(\frac{1}{2(a-ia\tan(c+dx))^4a}+\frac{1}{4(a-ia\tan(c+dx))^3a^2}+\frac{1}{8(\tan^2(c+dx)a^2+a^2)a^3}+\frac{1}{8(a-ia\tan(c+dx))^2a^3}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^7\left(\frac{i\arctan(\tan(c+dx))}{8a^4}+\frac{1}{8a^3(a-ia\tan(c+dx))}+\frac{1}{8a^2(a-ia\tan(c+dx))^2}+\frac{1}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^7*(((I/8)*ArcTan[Tan[c + d*x]])/a^4 + 1/(6*a*(a - I*a*Tan[c + d*x]
)^3) + 1/(8*a^2*(a - I*a*Tan[c + d*x])^2) + 1/(8*a^3*(a - I*a*Tan[c + d*x]
))))/d
```

3.43.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.43.4 Maple [A] (verified)

Time = 47.77 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.69

method	result
risch	$\frac{a^3 x}{8}-\frac{i a^3 \mathrm{e}^{6 i (d x+c)}}{48 d}-\frac{3 i a^3 \mathrm{e}^{4 i (d x+c)}}{32 d}-\frac{3 i a^3 \mathrm{e}^{2 i (d x+c)}}{16 d}$
derivativedivides	$\frac{-i a^3 \Bigg(-\frac{\big(\cos^4(d x+c)\big)\big(\sin^2(d x+c)\big)}{6}-\frac{\big(\cos^4(d x+c)\big)}{12}\Bigg)-3 a^3 \Bigg(-\frac{\big(\cos^5(d x+c)\big)\sin(d x+c)}{6}+\frac{\big(\cos^3(d x+c)+\frac{3\cos(d x+c)}{2}\big)\sin(d x+c}{24}}{d}$
default	$\frac{-i a^3 \Bigg(-\frac{\big(\cos^4(d x+c)\big)\big(\sin^2(d x+c)\big)}{6}-\frac{\big(\cos^4(d x+c)\big)}{12}\Bigg)-3 a^3 \Bigg(-\frac{\big(\cos^5(d x+c)\big)\sin(d x+c)}{6}+\frac{\big(\cos^3(d x+c)+\frac{3\cos(d x+c)}{2}\big)\sin(d x+c}{24}}{d}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^3*x-1/48*I/d*a^3*exp(6*I*(d*x+c))-3/32*I/d*a^3*exp(4*I*(d*x+c))-3/16
*I/d*a^3*exp(2*I*(d*x+c))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{12\,a^3dx - 2i\,a^3e^{(6i\,dx+6i\,c)} - 9i\,a^3e^{(4i\,dx+4i\,c)} - 18i\,a^3e^{(2i\,dx+2i\,c)}}{96\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/96*(12*a^3*d*x - 2*I*a^3*e^(6*I*d*x + 6*I*c) - 9*I*a^3*e^(4*I*d*x + 4*I*
c) - 18*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3x}{8} + \begin{cases} \frac{-512ia^3d^2e^{6ic}e^{6idx}-2304ia^3d^2e^{4ic}e^{4idx}-4608ia^3d^2e^{2ic}e^{2idx}}{24576d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^3e^{6ic}}{8}+\frac{3a^3e^{4ic}}{8}+\frac{3a^3e^{2ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

output
```
a**3*x/8 + Piecewise(((-512*I*a**3*d**2*exp(6*I*c)*exp(6*I*d*x) - 2304*I*a
**3*d**2*exp(4*I*c)*exp(4*I*d*x) - 4608*I*a**3*d**2*exp(2*I*c)*exp(2*I*d*x
))/(24576*d**3), Ne(d**3, 0)), (x*(a**3*exp(6*I*c)/8 + 3*a**3*exp(4*I*c)/8
 + 3*a**3*exp(2*I*c)/8), True))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{3\,(dx+c)a^3 + \frac{3\,a^3\tan(dx+c)^5+8\,a^3\tan(dx+c)^3+6i\,a^3\tan(dx+c)^2+21\,a^3\tan(dx+c)-10i\,a^3}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{24\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/24*(3*(d*x + c)*a^3 + (3*a^3*tan(d*x + c)^5 + 8*a^3*tan(d*x + c)^3 + 6*I
*a^3*tan(d*x + c)^2 + 21*a^3*tan(d*x + c) - 10*I*a^3)/(tan(d*x + c)^6 + 3*
tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

3.43.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(70) = 140$.

Time = 0.81 (sec) , antiderivative size = 457, normalized size of antiderivative = 5.08

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{12\,a^3dxe^{(8i\,dx+4i\,c)} + 48\,a^3dxe^{(6i\,dx+2i\,c)} + 48\,a^3dxe^{(2i\,dx-2i\,c)} + 72\,a^3dxe^{(4i\,dx)} + 12\,a^3dxe^{(-4i\,c)} - 3i\,a^3e^{(8i\,dx+}}{}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/96*(12*a^3*d*x*e^(8*I*d*x + 4*I*c) + 48*a^3*d*x*e^(6*I*d*x + 2*I*c) + 48
*a^3*d*x*e^(2*I*d*x - 2*I*c) + 72*a^3*d*x*e^(4*I*d*x) + 12*a^3*d*x*e^(-4*I
*c) - 3*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^3*
e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^3*e^(2*I*d*x - 2
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 18*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x +
 2*I*c) + 1) - 3*I*a^3*e^(-4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^3*e
^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 12*I*a^3*e^(6*I*d*x + 2
*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 12*I*a^3*e^(2*I*d*x - 2*I*c)*log(e^(
2*I*d*x) + e^(-2*I*c)) + 18*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)
) + 3*I*a^3*e^(-4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 2*I*a^3*e^(14*I*d*x
 + 10*I*c) - 17*I*a^3*e^(12*I*d*x + 8*I*c) - 66*I*a^3*e^(10*I*d*x + 6*I*c)
 - 134*I*a^3*e^(8*I*d*x + 4*I*c) - 146*I*a^3*e^(6*I*d*x + 2*I*c) - 18*I*a^
3*e^(2*I*d*x - 2*I*c) - 81*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d
*e^(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4
*I*c))
```

3.43.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3\,x}{8} - \frac{\frac{a^3\tan(c+d\,x)^2}{8} + \frac{a^3\tan(c+d\,x)\,3\mathrm{i}}{8} - \frac{5\,a^3}{12}}{d\,\left(-\tan\left(c+d\,x\right)^3 - \tan\left(c+d\,x\right)^2\,3\mathrm{i} + 3\tan\left(c+d\,x\right) + 1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(a^3*x)/8 - ((a^3*tan(c + d*x)*3i)/8 - (5*a^3)/12 + (a^3*tan(c + d*x)^2)/8
)/(d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan(c + d*x)^3 + 1i))
```

3.44 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.44.7 Maxima [A] (verification not implemented) 502
3.44.8 Giac [B] (verification not implemented) 503
3.44.9 Mupad [B] (verification not implemented) 503

3.44.1 Optimal result

Integrand size = 24, antiderivative size = 144

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = {} & \frac{5a^3x}{32}-\frac{ia^7}{16d(a-ia\tan(c+dx))^4} \\ & -\frac{ia^6}{12d(a-ia\tan(c+dx))^3} \\ & -\frac{3ia^5}{32d(a-ia\tan(c+dx))^2} \\ & -\frac{ia^4}{8d(a-ia\tan(c+dx))} \\ & +\frac{ia^4}{32d(a+ia\tan(c+dx))}\end{aligned}$$

output
```
5/32*a^3*x-1/16*I*a^7/d/(a-I*a*tan(d*x+c))^4-1/12*I*a^6/d/(a-I*a*tan(d*x+c
))^3-3/32*I*a^5/d/(a-I*a*tan(d*x+c))^2-1/8*I*a^4/d/(a-I*a*tan(d*x+c))+1/32
*I*a^4/d/(a+I*a*tan(d*x+c))
```

3.44.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{ia^9\Bigg(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{16a^2(a-ia\tan(c+dx))^4}+\frac{1}{12a^3(a-ia\tan(c+dx))^3}+\frac{3}{32a^4(a-ia\tan(c+dx))^2}+\frac{1}{8a^5(a-ia\tan(c+dx))}}{d}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^9*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(16*a^2*(a - I*a*Tan[
c + d*x])^4) + 1/(12*a^3*(a - I*a*Tan[c + d*x])^3) + 3/(32*a^4*(a - I*a*Ta
n[c + d*x])^2) + 1/(8*a^5*(a - I*a*Tan[c + d*x])) - 1/(32*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

3.44.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^9\int \frac{1}{(a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^9\int \Bigg(\frac{1}{8a^5(a-ia\tan(c+dx))^2}+\frac{1}{32a^5(i\tan(c+dx)a+a)^2}+\frac{3}{16a^4(a-ia\tan(c+dx))^3}+\frac{1}{4a^3(a-ia\tan(c+dx))^4}+\frac{1}{4a^2(a-ia\tan(c+dx))^5}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ia^9\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{8a^5(a-ia\tan(c+dx))}-\frac{1}{32a^5(a+ia\tan(c+dx))}+\frac{3}{32a^4(a-ia\tan(c+dx))^2}+\frac{1}{12a^3(a-ia\tan(c+dx))^3}+\frac{}{16}\right.}{d}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^9*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(16*a^2*(a - I*a*Tan[
c + d*x])^4) + 1/(12*a^3*(a - I*a*Tan[c + d*x])^3) + 3/(32*a^4*(a - I*a*Ta
n[c + d*x])^2) + 1/(8*a^5*(a - I*a*Tan[c + d*x])) - 1/(32*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

3.44.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.44.4 Maple [A] (verified)

Time = 133.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

method	result
risch	$\frac{5a^3x}{32}-\frac{ia^3\mathrm{e}^{8i(dx+c)}}{256d}-\frac{5ia^3\mathrm{e}^{6i(dx+c)}}{192d}-\frac{5ia^3\mathrm{e}^{4i(dx+c)}}{64d}-\frac{9ia^3\cos(2dx+2c)}{64d}+\frac{11a^3\sin(2dx+2c)}{64d}$
derivativedivides	$-ia^3\left(-\frac{\left(\cos^6(dx+c)\right)\left(\sin^2(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos$
default	$-ia^3\left(-\frac{\left(\cos^6(dx+c)\right)\left(\sin^2(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
5/32*a^3*x-1/256*I/d*a^3*exp(8*I*(d*x+c))-5/192*I/d*a^3*exp(6*I*(d*x+c))-5
/64*I/d*a^3*exp(4*I*(d*x+c))-9/64*I/d*a^3*cos(2*d*x+2*c)+11/64/d*a^3*sin(2
*d*x+2*c)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\left(120\,a^3dxe^{(2i\,dx+2i\,c)}-3i\,a^3e^{(10i\,dx+10i\,c)}-20i\,a^3e^{(8i\,dx+8i\,c)}-60i\,a^3e^{(6i\,dx+6i\,c)}-120i\,a^3e^{(4i\,dx+4i\,c)}+12i\,a^3\right)}{768\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/768*(120*a^3*d*x*e^(2*I*d*x + 2*I*c) - 3*I*a^3*e^(10*I*d*x + 10*I*c) - 2
0*I*a^3*e^(8*I*d*x + 8*I*c) - 60*I*a^3*e^(6*I*d*x + 6*I*c) - 120*I*a^3*e^(
4*I*d*x + 4*I*c) + 12*I*a^3)*e^(-2*I*d*x - 2*I*c)/d
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.57

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3x}{32} + \begin{cases} \frac{(-25165824ia^3d^4e^{10ic}e^{8idx}-167772160ia^3d^4e^{8ic}e^{6idx}-503316480ia^3d^4e^{6ic}e^{4idx}-1006632960ia^3d^4e^{4ic}e^{2idx}+100663296ia^3d^4e^{-2idx})e^{-}}{6442450944d^5} \\ x\left(-\frac{5a^3}{32}+\frac{(a^3e^{10ic}+5a^3e^{8ic}+10a^3e^{6ic}+10a^3e^{4ic}+5a^3e^{2ic}+a^3)e^{-2ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

output
```
5*a**3*x/32 + Piecewise(((-25165824*I*a**3*d**4*exp(10*I*c)*exp(8*I*d*x) -
 167772160*I*a**3*d**4*exp(8*I*c)*exp(6*I*d*x) - 503316480*I*a**3*d**4*exp
(6*I*c)*exp(4*I*d*x) - 1006632960*I*a**3*d**4*exp(4*I*c)*exp(2*I*d*x) + 10
0663296*I*a**3*d**4*exp(-2*I*d*x))*exp(-2*I*c)/(6442450944*d**5), Ne(d**5*
exp(2*I*c), 0)), (x*(-5*a**3/32 + (a**3*exp(10*I*c) + 5*a**3*exp(8*I*c) +
10*a**3*exp(6*I*c) + 10*a**3*exp(4*I*c) + 5*a**3*exp(2*I*c) + a**3)*exp(-2
*I*c)/32), True))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{15\,(dx+c)a^3 + \frac{15\,a^3\tan(dx+c)^7+55\,a^3\tan(dx+c)^5+73\,a^3\tan(dx+c)^3+16i\,a^3\tan(dx+c)^2+81\,a^3\tan(dx+c)-32i\,a^3}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{96\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/96*(15*(d*x + c)*a^3 + (15*a^3*tan(d*x + c)^7 + 55*a^3*tan(d*x + c)^5 +
73*a^3*tan(d*x + c)^3 + 16*I*a^3*tan(d*x + c)^2 + 81*a^3*tan(d*x + c) - 32
*I*a^3)/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x
+ c)^2 + 1))/d
```

3.44.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(112) = 224$.

Time = 0.92 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.57

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{240\,a^3dxe^{(10i\,dx+6i\,c)} + 960\,a^3dxe^{(8i\,dx+4i\,c)} + 1440\,a^3dxe^{(6i\,dx+2i\,c)} + 240\,a^3dxe^{(2i\,dx-2i\,c)} + 960\,a^3dxe^{(4i\,dx)} -}{}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/1536*(240*a^3*d*x*e^(10*I*d*x + 6*I*c) + 960*a^3*d*x*e^(8*I*d*x + 4*I*c)
 + 1440*a^3*d*x*e^(6*I*d*x + 2*I*c) + 240*a^3*d*x*e^(2*I*d*x - 2*I*c) + 96
0*a^3*d*x*e^(4*I*d*x) - 33*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) - 132*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 19
8*I*a^3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 33*I*a^3*e^(2*I
*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 132*I*a^3*e^(4*I*d*x)*log(e^(
2*I*d*x + 2*I*c) + 1) + 33*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^
(-2*I*c)) + 132*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) +
198*I*a^3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 33*I*a^3*e^(
2*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 132*I*a^3*e^(4*I*d*x)*log
(e^(2*I*d*x) + e^(-2*I*c)) - 6*I*a^3*e^(18*I*d*x + 14*I*c) - 64*I*a^3*e^(1
6*I*d*x + 12*I*c) - 316*I*a^3*e^(14*I*d*x + 10*I*c) - 984*I*a^3*e^(12*I*d*
x + 8*I*c) - 1846*I*a^3*e^(10*I*d*x + 6*I*c) - 1936*I*a^3*e^(8*I*d*x + 4*I
*c) - 984*I*a^3*e^(6*I*d*x + 2*I*c) + 96*I*a^3*e^(2*I*d*x - 2*I*c) - 96*I*
a^3*e^(4*I*d*x) + 24*I*a^3*e^(-4*I*c))/(d*e^(10*I*d*x + 6*I*c) + 4*d*e^(8*
I*d*x + 4*I*c) + 6*d*e^(6*I*d*x + 2*I*c) + d*e^(2*I*d*x - 2*I*c) + 4*d*e^(
4*I*d*x))
```

3.44.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5\,a^3\,x}{32}$$
$$-\frac{\frac{5\,a^3\,\tan(c+d\,x)^4}{32}+\frac{a^3\,\tan(c+d\,x)^3\,15\mathrm{i}}{32}-\frac{35\,a^3\,\tan(c+d\,x)^2}{96}+\frac{a^3\,\tan(c+d\,x)\,5\mathrm{i}}{32}-\frac{a^3}{3}}{d\,\left(-{\tan\left(c+d\,x\right)}^5-{\tan\left(c+d\,x\right)}^4\,3\mathrm{i}+2\,{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,2\mathrm{i}+3\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(5*a^3*x)/32 - ((a^3*tan(c + d*x)*5i)/32 - a^3/3 - (35*a^3*tan(c + d*x)^2)
/96 + (a^3*tan(c + d*x)^3*15i)/32 + (5*a^3*tan(c + d*x)^4)/32)/(d*(3*tan(c
 + d*x) - tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 - tan(c + d*x)^4*3i - tan(c
 + d*x)^5 + 1i))
```

3.44. $\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.45 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.45.1 Optimal result

Integrand size = 24, antiderivative size = 127

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx &= \frac{7a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{7ia^3\sec^3(c+dx)}{12d}\\ &\quad+\frac{7a^3\sec(c+dx)\tan(c+dx)}{8d}\\ &\quad+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}\\ &\quad+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}\end{aligned}$$

output
```
7/8*a^3*arctanh(sin(d*x+c))/d+7/12*I*a^3*sec(d*x+c)^3/d+7/8*a^3*sec(d*x+c)
*tan(d*x+c)/d+1/5*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2/d+7/20*I*sec(d*x+c
)^3*(a^3+I*a^3*tan(d*x+c))/d
```

3.45.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{a^3(\cos(3dx)+i\sin(3dx))\left(1680\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(448i+640i\cos(2(c+d}{960d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(1680*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2
]] + Sec[c + d*x]^5*(448*I + (640*I)*Cos[2*(c + d*x)] - 150*Sin[2*(c + d*x
)] + 105*Sin[4*(c + d*x)])))/(960*d*(Cos[d*x] + I*Sin[d*x])^3)
```

3.45.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{5}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+\right.}{4d}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2t\right.}{4d}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7}{5}a\left(\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5}{4}a\left(a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c}{3d}\right.\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/5)*a*Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2)/d + (7*a*(((I/4)*Sec[c +
 d*x]^3*(a^2 + I*a^2*Tan[c + d*x]))/d + (5*a*(((I/3)*a*Sec[c + d*x]^3)/d +
 a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/4))
/5
```

3.45. $\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.45.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.45.4 Maple [A] (verified)

Time = 8.87 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{i a^3 \left(105\,\mathrm{e}^{9 i (d x+c)}-790\,\mathrm{e}^{7 i (d x+c)}-896\,\mathrm{e}^{5 i (d x+c)}-490\,\mathrm{e}^{3 i (d x+c)}-105\,\mathrm{e}^{i (d x+c)}\right)}{60 d \left(\mathrm{e}^{2 i (d x+c)}+1\right)^{5}}+\frac{7 a^3 \ln\left(\mathrm{e}^{i (d x+c)}+i\right)}{8 d}-$
derivativedivides	$-i a^3 \left(\frac{\sin^4(d x+c)}{5 \cos(d x+c)^5}+\frac{\sin^4(d x+c)}{15 \cos(d x+c)^3}-\frac{\sin^4(d x+c)}{15 \cos(d x+c)}-\frac{\left(2+\sin^2(d x+c)\right) \cos(d x+c)}{15}\right)-3 a^3 \left(\frac{\sin^3(d x+c)}{4 \cos(d x+c)^4}+\frac{\sin^3(d x+c)}{8 \cos(d x+c)^2}+\right.$
default	$-i a^3 \left(\frac{\sin^4(d x+c)}{5 \cos(d x+c)^5}+\frac{\sin^4(d x+c)}{15 \cos(d x+c)^3}-\frac{\sin^4(d x+c)}{15 \cos(d x+c)}-\frac{\left(2+\sin^2(d x+c)\right) \cos(d x+c)}{15}\right)-3 a^3 \left(\frac{\sin^3(d x+c)}{4 \cos(d x+c)^4}+\frac{\sin^3(d x+c)}{8 \cos(d x+c)^2}+\right.$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I*a^3/d/(exp(2*I*(d*x+c))+1)^5*(105*exp(9*I*(d*x+c))-790*exp(7*I*(d*
x+c))-896*exp(5*I*(d*x+c))-490*exp(3*I*(d*x+c))-105*exp(I*(d*x+c)))+7/8/d*
a^3*ln(exp(I*(d*x+c))+I)-7/8/d*a^3*ln(exp(I*(d*x+c))-I)
```

3.45.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(107) = 214$.

Time = 0.24 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.44

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{-210i\,a^3e^{(9i\,dx+9i\,c)} + 1580i\,a^3e^{(7i\,dx+7i\,c)} + 1792i\,a^3e^{(5i\,dx+5i\,c)} + 980i\,a^3e^{(3i\,dx+3i\,c)} + 210i\,a^3e^{(i\,dx+i\,c)} + 105}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(-210*I*a^3*e^(9*I*d*x + 9*I*c) + 1580*I*a^3*e^(7*I*d*x + 7*I*c) + 1
792*I*a^3*e^(5*I*d*x + 5*I*c) + 980*I*a^3*e^(3*I*d*x + 3*I*c) + 210*I*a^3*
e^(I*d*x + I*c) + 105*(a^3*e^(10*I*d*x + 10*I*c) + 5*a^3*e^(8*I*d*x + 8*I*
c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d*x + 4*I*c) + 5*a^3*e^(2*
I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 105*(a^3*e^(10*I*d*x + 10
*I*c) + 5*a^3*e^(8*I*d*x + 8*I*c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^
(4*I*d*x + 4*I*c) + 5*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) -
 I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x
+ 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

3.45.6 Sympy [F]

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\Bigg(\int i\sec^3{(c+dx)}\,dx \\ + \int\left(-3\tan{(c+dx)}\sec^3{(c+dx)}\right)\,dx \\ + \int \tan^3{(c+dx)}\sec^3{(c+dx)}\,dx \\ + \int\left(-3i\tan^2{(c+dx)}\sec^3{(c+dx)}\right)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x)**3, x) + Integral(-3*tan(c + d*x)*sec(c +
 d*x)**3, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(-3*
I*tan(c + d*x)**2*sec(c + d*x)**3, x))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{45\,a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\mathrm{lo}\right.}{240\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/240*(45*a^3*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^3*
(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) - 240*I*a^3/cos(d*x + c)^3 - 16*I*(5*cos(d*x + c)^2 - 3)*a^3/c
os(d*x + c)^5)/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.49

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
1/120*(105*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^3*log(tan(1/2*d*x + 1
/2*c) - 1) + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^9 - 360*I*a^3*tan(1/2*d*x + 1/
2*c)^8 - 390*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*I*a^3*tan(1/2*d*x + 1/2*c)^6
 - 400*I*a^3*tan(1/2*d*x + 1/2*c)^4 + 390*a^3*tan(1/2*d*x + 1/2*c)^3 + 320
*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 136*I*a^3)/(
tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{-\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,6\mathrm{i}+\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,16\mathrm{i}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,20\mathrm{i}}{3}-13\,a^3\,\mathrm{ta}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

input

```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^3,x)
```

output

```
(7*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^3*tan(c/2 + (d*x)/2)^4*20i)/
3 - (13*a^3*tan(c/2 + (d*x)/2)^3)/2 - (a^3*tan(c/2 + (d*x)/2)^2*16i)/3 - a
^3*tan(c/2 + (d*x)/2)^6*16i + (13*a^3*tan(c/2 + (d*x)/2)^7)/2 + a^3*tan(c/
2 + (d*x)/2)^8*6i - (a^3*tan(c/2 + (d*x)/2)^9)/4 + (a^3*34i)/15 + (a^3*tan
(c/2 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 +
 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
- 1))
```

3.46 $\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.46.1 Optimal result

Integrand size = 22, antiderivative size = 99

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx &= \frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{5ia^3\sec(c+dx)}{2d}\\ &\quad+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\\ &\quad+\frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}\end{aligned}$$

output
```
5/2*a^3*arctanh(sin(d*x+c))/d+5/2*I*a^3*sec(d*x+c)/d+1/3*I*a*sec(d*x+c)*(a
+I*a*tan(d*x+c))^2/d+5/6*I*sec(d*x+c)*(a^3+I*a^3*tan(d*x+c))/d
```

3.46.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{a^3(\cos(3dx)+i\sin(3dx))\,(60\operatorname{arctanh}(\sin(c)+\cos(c)\tan(\tfrac{dx}{2}))+i\sec^3(c+dx)(20+24\cos(2(c+dx))+}{12d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]
 + I*Sec[c + d*x]^3*(20 + 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/
(12*d*(Cos[d*x] + I*Sin[d*x])^3)
```

3.46.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

input

```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((I/3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh
[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 +
I*a^2*Tan[c + d*x]))/d))/3
```

3.46.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.46.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
risch	$\frac{i a^{3} \left(33 \,{\mathrm e}^{5 i \left(d x +c \right)}+40 \,{\mathrm e}^{3 i \left(d x +c \right)}+15 \,{\mathrm e}^{i \left(d x +c \right)}\right)}{3 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{3}}+\frac{5 a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{2 d}-\frac{5 a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{2 d}$
derivativedivides	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{3 \cos\left(d x +c \right)^{3}}-\frac{\sin^{4}\left(d x +c \right)}{3 \cos\left(d x +c \right)}-\frac{\left(2+\sin^{2}\left(d x +c \right)\right) \cos\left(d x +c \right)}{3}\right)-3 a^{3} \left(\frac{\sin^{3}\left(d x +c \right)}{2 \cos\left(d x +c \right)^{2}}+\frac{\sin\left(d x +c \right)}{2}-\frac{\ln\left(\sec\left(d x +c \right)+\tan\left(d x +c \right)\right)}{2}\right)}{d}$
default	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{3 \cos\left(d x +c \right)^{3}}-\frac{\sin^{4}\left(d x +c \right)}{3 \cos\left(d x +c \right)}-\frac{\left(2+\sin^{2}\left(d x +c \right)\right) \cos\left(d x +c \right)}{3}\right)-3 a^{3} \left(\frac{\sin^{3}\left(d x +c \right)}{2 \cos\left(d x +c \right)^{2}}+\frac{\sin\left(d x +c \right)}{2}-\frac{\ln\left(\sec\left(d x +c \right)+\tan\left(d x +c \right)\right)}{2}\right)}{d}$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*I*a^3/d/(exp(2*I*(d*x+c))+1)^3*(33*exp(5*I*(d*x+c))+40*exp(3*I*(d*x+c)
)+15*exp(I*(d*x+c)))+5/2/d*a^3*ln(exp(I*(d*x+c))+I)-5/2/d*a^3*ln(exp(I*(d*
x+c))-I)
```

3.46.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(81) = 162.

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.04

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{66i\,a^3e^{(5i\,dx+5i\,c)}+80i\,a^3e^{(3i\,dx+3i\,c)}+30i\,a^3e^{(i\,dx+i\,c)}+15\left(a^3e^{(6i\,dx+6i\,c)}+3\,a^3e^{(4i\,dx+4i\,c)}+3\,a^3e^{(2i\,dx+2i\,c)}+\right.}{6\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\right.}}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(66*I*a^3*e^(5*I*d*x + 5*I*c) + 80*I*a^3*e^(3*I*d*x + 3*I*c) + 30*I*a^
3*e^(I*d*x + I*c) + 15*(a^3*e^(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c
) + 3*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 15*(a^3*e^
(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c) + 3*a^3*e^(2*I*d*x + 2*I*c)
+ a^3)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x +
 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.46.6 Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec(c+dx)\,dx + \int(-3\tan(c+dx)\sec(c+dx))\,dx + \int\tan^3(c+dx)\sec(c+dx)\,dx + \int\left(-3i\tan^2(c+dx)\sec(c+dx)\right)\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sec(c + d*x), x) + Integral(-3*tan(c + d*x)*sec(c + d*
x), x) + Integral(tan(c + d*x)**3*sec(c + d*x), x) + Integral(-3*I*tan(c +
 d*x)**2*sec(c + d*x), x))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{9\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right) + 12\,a^3\log(\sec(dx+c)+\tan(dx+c)}{12\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(9*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 12*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*I*a
^3/cos(d*x + c) + 4*I*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-4\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{6\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^3*log(tan(1/2*d*x + 1/2*c
) - 1) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 + 18*I*a^3*tan(1/2*d*x + 1/2*c)^4
 - 48*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) + 22*I*a^3
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6\mathrm{i}-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3\,22\mathrm{i}}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x),x)
```

output
```
(5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^4*6i - a^3*t
an(c/2 + (d*x)/2)^2*16i + 3*a^3*tan(c/2 + (d*x)/2)^5 + (a^3*22i)/3 - 3*a^3
*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^6 - 1))
```

3.47 $\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.47.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{3a^3\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3ia^3\sec(c+dx)}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

output
```
-3*a^3*arctanh(sin(d*x+c))/d-3*I*a^3*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(a+I*a*
tan(d*x+c))^2/d
```

3.47.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 123 vs. 2(61) = 122.

Time = 1.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3\cos^2(c+dx)\bigl(6\operatorname{arctanh}\bigl(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\bigr)\cos(c+dx)(i\cos(3c)+\sin(3c))+(-\cos(2c-dx)+}{d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Cos[c + d*x]^2*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*
(I*Cos[3*c] + Sin[3*c]) + (-Cos[2*c - d*x] + I*Sin[2*c - d*x])*(5*Cos[c +
d*x] - I*Sin[c + d*x]))*(-I + Tan[c + d*x])^3)/(d*(Cos[d*x] + I*Sin[d*x])^
3)
```

3.47.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3977, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3977}$$

$$-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3967}$$

$$-3a^2\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$-3a^2\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{4257}$$

$$-3a^2\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-3*a^2*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d) - ((2*I)*a*Cos
[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d
```

3.47.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.47.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{4 i a^{3} {\mathrm e}^{i \left(d x +c \right)}}{d}-\frac{2 i {\mathrm e}^{i \left(d x +c \right)} a^{3}}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}-\frac{3 a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{d}+\frac{3 a^{3} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{d}$
derivativedivides	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(2+\sin^{2}\left(d x +c \right)\right) \cos \left(d x +c \right)\right)-3 a^{3} \left(-\sin \left(d x +c \right)+\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)\right)-3 i a^{3} \cos \left(d x +c \right)+a^{3} \mathrm{si}}{d}$
default	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(2+\sin^{2}\left(d x +c \right)\right) \cos \left(d x +c \right)\right)-3 a^{3} \left(-\sin \left(d x +c \right)+\ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)\right)-3 i a^{3} \cos \left(d x +c \right)+a^{3} \mathrm{si}}{d}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4*I/d*a^3*exp(I*(d*x+c))-2*I*exp(I*(d*x+c))*a^3/d/(exp(2*I*(d*x+c))+1)-3/
d*a^3*ln(exp(I*(d*x+c))+I)+3/d*a^3*ln(exp(I*(d*x+c))-I)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{-4i\,a^3e^{(3i\,dx+3i\,c)} - 6i\,a^3e^{(i\,dx+i\,c)} - 3\,(a^3e^{(2i\,dx+2i\,c)}+a^3)\log(e^{(i\,dx+i\,c)}+i) + 3\,(a^3e^{(2i\,dx+2i\,c)}+a^3)\log(e^{(i}}{de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
(-4*I*a^3*e^(3*I*d*x + 3*I*c) - 6*I*a^3*e^(I*d*x + I*c) - 3*(a^3*e^(2*I*d*
x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) + 3*(a^3*e^(2*I*d*x + 2*I*c) +
a^3)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia^3e^{ic}e^{idx}}{de^{2ic}e^{2idx}+d} + \frac{3a^3\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} -\frac{4ia^3e^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ 4a^3xe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-2*I*a**3*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 3*a**3*(lo
g(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise
((-4*I*a**3*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (4*a**3*x*exp(I*c), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2i\,a^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+3\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))+}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*I*a^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a^3*(log(sin(d*x + c) +
1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 6*I*a^3*cos(d*x + c) - 2*a^
3*sin(d*x + c))/d
```

3.47.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(55) = 110.

Time = 0.60 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.84

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{63\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-33\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-63\,a^3e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/32*(63*a^3*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 33*a^3*e^(2*
I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*e^(2*I*d*x + 2*I*c)*log
(-I*e^(I*d*x + I*c) + 1) + 33*a^3*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 128*I*a^3*e^(3*I*d*x + 3*I*c) - 192*I*a^3*e^(I*d*x + I*c) + 63*a
^3*log(I*e^(I*d*x + I*c) + 1) - 33*a^3*log(I*e^(I*d*x + I*c) - 1) - 63*a^3
*log(-I*e^(I*d*x + I*c) + 1) + 33*a^3*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(2
*I*d*x + 2*I*c) + d)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= -\frac{6\,a^3\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,2\mathrm{i}-10\,a^3}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
- (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (8*a^3*tan(c/2 + (d*x)/2)^2 - 10*a
^3 + a^3*tan(c/2 + (d*x)/2)*2i)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2
)^2*1i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.48 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.48.1 Optimal result 524
3.48.2 Mathematica [A] (verified) 524
3.48.3 Rubi [A] (verified) 525
3.48.4 Maple [A] (verified) 526
3.48.5 Fricas [A] (verification not implemented) 526
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3.48.7 Maxima [B] (verification not implemented) 527
3.48.8 Giac [B] (verification not implemented) 527
3.48.9 Mupad [B] (verification not implemented) 528

3.48.1 Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

output
```
-1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{ia^3(\cos(c+dx)+i\sin(c+dx))^3}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-1/3*I)*a^3*(Cos[c + d*x] + I*Sin[c + d*x])^3)/d
```

3.48.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3969}$$

$$-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-1/3*I)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d
```

3.48.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.48.4 Maple [A] (verified)

Time = 6.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result	size
risch	$-\frac{i a^3 \mathrm{e}^{3 i (dx+c)}}{3d}$	19
derivativedivides	$\frac{\frac{i a^3 \left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-a^3\left(\sin^3(dx+c)\right)-i a^3\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	76
default	$\frac{\frac{i a^3 \left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-a^3\left(\sin^3(dx+c)\right)-i a^3\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	76

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d*a^3*exp(3*I*(d*x+c))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,a^3e^{(3i\,dx+3i\,c)}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} -\frac{ia^3e^{3ic}e^{3idx}}{3d} & \text{for } d \neq 0 \\ a^3xe^{3ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(d, 0)), (a**3*x*exp(3
*I*c), True))
```

3.48.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{3i\,a^3\cos(dx+c)^3+3\,a^3\sin(dx+c)^3+i\,\big(\cos(dx+c)^3-3\,\cos(dx+c)\big)a^3+\big(\sin(dx+c)^3-3\,\sin(d}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/3*(3*I*a^3*cos(d*x + c)^3 + 3*a^3*sin(d*x + c)^3 + I*(cos(d*x + c)^3 -
3*cos(d*x + c))*a^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a^3)/d
```

3.48.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(26) = 52$.

Time = 0.71 (sec) , antiderivative size = 901, normalized size of antiderivative = 28.16

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-1/384*(108*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e
^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^(2*I*d*x - 2*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 648*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) +
 1) + 108*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 111*a^3*e^(8*I*d*x +
 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 444*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(
I*d*x + I*c) - 1) + 444*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 666*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 111*a^3*e^(-4*I*c)*log
(I*e^(I*d*x + I*c) - 1) - 108*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*
c) + 1) - 432*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 432*a^
3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 648*a^3*e^(4*I*d*x)*lo
g(-I*e^(I*d*x + I*c) + 1) - 108*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
 - 111*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(6*
I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(2*I*d*x - 2*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 666*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) -
1) - 111*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 3*a^3*e^(8*I*d*x + 4
*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 12*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*
d*x) + e^(-I*c)) + 12*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c))
+ 18*a^3*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^3*e^(-4*I*c)*log(I*
e^(I*d*x) + e^(-I*c)) - 3*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) - 12*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3...
```

3.48.9 Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=-\frac{2\,a^3\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3{}\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

input

```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3,x)
```

output

```
-(2*a^3*(3*tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*tan(c/2 + (d*x)/2) - tan(c/2
 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.49 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.49.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^3\cos^3(c+dx)}{15d} + \frac{a^3\sin(c+dx)}{5d} - \frac{a^3\sin^3(c+dx)}{15d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

output
```
-1/15*I*a^3*cos(d*x+c)^3/d+1/5*a^3*sin(d*x+c)/d-1/15*a^3*sin(d*x+c)^3/d-2/
5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.65

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3(-i\cos(2(c+dx))+\sin(2(c+dx)))\Bigl(\cos(c+dx)\Bigl(4+25\sqrt{\cos^2(c+dx)}\Bigr)+\Bigl(4+3\sqrt{\cos^2(c+dx)}\Bigr)\,\mathrm{co}}{60d\sqrt{\cos^2(c+}}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(Cos[c + d*x]*(4 + 25*Sqrt
[Cos[c + d*x]^2]) + (4 + 3*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] - I*((4
+ 5*Sqrt[Cos[c + d*x]^2])*Sin[c + d*x] + (4 - 3*Sqrt[Cos[c + d*x]^2])*Sin[
3*(c + d*x)])))/(60*d*Sqrt[Cos[c + d*x]^2])
```

3.49.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{5}a^2\int \cos^3(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a^2\int \frac{i\tan(c+dx)a+a}{\sec(c+dx)^3}dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{5}a^2\left(a\int \cos^3(c+dx)dx-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a^2\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{5}a^2\left(-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^2\left(-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^3(c+dx)}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^2*(((-1/3*I)*a*Cos[c + d*x]^3)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3/3
))/d))/5 - (((2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2)/d
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.49.4 Maple [A] (verified)

Time = 26.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{i a^{3} \mathrm{e}^{5 i (d x+c)}}{20 d}-\frac{i a^{3} \mathrm{e}^{3 i (d x+c)}}{6 d}-\frac{i a^{3} \mathrm{e}^{i (d x+c)}}{4 d}$
derivativedivides	$\frac{-i a^{3} \left(-\frac{\left(\cos^{3}(d x+c)\right) \left(\sin^{2}(d x+c)\right)}{5}-\frac{2 \left(\cos^{3}(d x+c)\right)}{15}\right)-3 a^{3} \left(-\frac{\sin(d x+c) \left(\cos^{4}(d x+c)\right)}{5}+\frac{\left(2+\cos^{2}(d x+c)\right) \sin(d x+c)}{15}\right)-\frac{3 i a^{3} \biggl(}{}}{d}$
default	$\frac{-i a^{3} \left(-\frac{\left(\cos^{3}(d x+c)\right) \left(\sin^{2}(d x+c)\right)}{5}-\frac{2 \left(\cos^{3}(d x+c)\right)}{15}\right)-3 a^{3} \left(-\frac{\sin(d x+c) \left(\cos^{4}(d x+c)\right)}{5}+\frac{\left(2+\cos^{2}(d x+c)\right) \sin(d x+c)}{15}\right)-\frac{3 i a^{3} \biggl(}{}}{d}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/20*I/d*a^3*exp(5*I*(d*x+c))-1/6*I/d*a^3*exp(3*I*(d*x+c))-1/4*I/d*a^3*ex
p(I*(d*x+c))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-3i\,a^3e^{(5i\,dx+5i\,c)} - 10i\,a^3e^{(3i\,dx+3i\,c)} - 15i\,a^3e^{(i\,dx+i\,c)}}{60\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(-3*I*a^3*e^(5*I*d*x + 5*I*c) - 10*I*a^3*e^(3*I*d*x + 3*I*c) - 15*I*a
^3*e^(I*d*x + I*c))/d
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} \frac{-24ia^3d^2e^{5ic}e^{5idx}-80ia^3d^2e^{3ic}e^{3idx}-120ia^3d^2e^{ic}e^{idx}}{480d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^3e^{5ic}}{4}+\frac{a^3e^{3ic}}{2}+\frac{a^3e^{ic}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-24*I*a**3*d**2*exp(5*I*c)*exp(5*I*d*x) - 80*I*a**3*d**2*exp(3
*I*c)*exp(3*I*d*x) - 120*I*a**3*d**2*exp(I*c)*exp(I*d*x))/(480*d**3), Ne(d
**3, 0)), (x*(a**3*exp(5*I*c)/4 + a**3*exp(3*I*c)/2 + a**3*exp(I*c)/4), Tr
ue))
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{9i\,a^3\cos(dx+c)^5+i\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^3-3\left(3\sin(dx+c)^5-5\sin(dx+c)^3\right)a^3-}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/15*(9*I*a^3*cos(d*x + c)^5 + I*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
3 - 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^3 - (3*sin(d*x + c)^5 - 10*s
in(d*x + c)^3 + 15*sin(d*x + c))*a^3)/d
```

3.49.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(74) = 148$.

Time = 0.85 (sec) , antiderivative size = 929, normalized size of antiderivative = 10.56

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
1/7680*(1785*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3
*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*e^(2*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 10710*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I
*c) + 1) + 1785*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1530*a^3*e^(8*
I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6120*a^3*e^(6*I*d*x + 2*I*c)*l
og(I*e^(I*d*x + I*c) - 1) + 6120*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 9180*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1530*a^3*e^(
-4*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1785*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e
^(I*d*x + I*c) + 1) - 7140*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 7140*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 10710*a^
3*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1785*a^3*e^(-4*I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 1530*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 6120*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^3*e
^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9180*a^3*e^(4*I*d*x)*log(
-I*e^(I*d*x + I*c) - 1) - 1530*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 255*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(6*
I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(2*I*d*x - 2*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 1530*a^3*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I
*c)) - 255*a^3*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(8*I*d*x
 + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1020*a^3*e^(6*I*d*x + 2*I*c)*l...
```

3.49.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{2\,a^3\,\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,30\mathrm{i}-40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,20\mathrm{i}+7\right)}{15\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,5\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,10\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input

```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3,x)
```

output

```
(2*a^3*(tan(c/2 + (d*x)/2)^3*30i - 40*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)*20i + 15*tan(c/2 + (d*x)/2)^4 + 7))/(15*d*(5*tan(c/2 + (d*x)/2) - ta
n(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i
 + tan(c/2 + (d*x)/2)^5 + 1i))
```

3.49. $\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.50 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.50.9 Mupad [B] (verification not implemented) 541

3.50.1 Optimal result

Integrand size = 24, antiderivative size = 106

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx =& -\frac{3ia^3\cos^5(c+dx)}{35d}+\frac{3a^3\sin(c+dx)}{7d}\\ &-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{35d}\\ &-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\end{aligned}$$

output
```
-3/35*I*a^3*cos(d*x+c)^5/d+3/7*a^3*sin(d*x+c)/d-2/7*a^3*sin(d*x+c)^3/d+3/3
5*a^3*sin(d*x+c)^5/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d
```

3.50.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.70

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{a^3(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(35\sqrt{\cos^2(c+dx)}+\left(8+84\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))+\Big(}{}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35*Sqrt[Cos[c + d*x]^2] +
 (8 + 84*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (8 - 15*Sqrt[Cos[c + d*x
]^2])*Cos[4*(c + d*x)] - (8*I)*Sin[2*(c + d*x)] - (56*I)*Sqrt[Cos[c + d*x]
^2]*Sin[2*(c + d*x)] - (8*I)*Sin[4*(c + d*x)] + (20*I)*Sqrt[Cos[c + d*x]^2
]*Sin[4*(c + d*x)]))/(280*d*Sqrt[Cos[c + d*x]^2])
```

3.50.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{3}{7}a^2\int \cos^5(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}a^2\int \frac{i\tan(c+dx)a+a}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{3}{7}a^2\left(a\int \cos^5(c+dx)dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{7}a^2\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{3}{7}a^2\left(-\frac{a\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{7}a^2\left(-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(3*a^2*(((-1/5*I)*a*Cos[c + d*x]^5)/d - (a*(-Sin[c + d*x] + (2*Sin[c + d*x
]^3)/3 - Sin[c + d*x]^5/5))/d))/7 - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*T
an[c + d*x])^2)/d
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.50.4 Maple [A] (verified)

Time = 81.92 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{ia^3 e^{7i(dx+c)}}{112d}-\frac{ia^3 e^{5i(dx+c)}}{20d}-\frac{ia^3 e^{3i(dx+c)}}{8d}-\frac{3ia^3\cos(dx+c)}{16d}+\frac{5a^3\sin(dx+c)}{16d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)-3a^3\Bigg(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)}{35}}{d}$
default	$\frac{-ia^3\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)-3a^3\Bigg(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)}{35}}{d}$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/112*I/d*a^3*exp(7*I*(d*x+c))-1/20*I/d*a^3*exp(5*I*(d*x+c))-1/8*I/d*a^3*
exp(3*I*(d*x+c))-3/16*I/d*a^3*cos(d*x+c)+5/16*a^3*sin(d*x+c)/d
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int\cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\left(-5i\,a^3e^{(8i\,dx+8i\,c)}-28i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-140i\,a^3e^{(2i\,dx+2i\,c)}+35i\,a^3\right)e^{(-i\,dx-i\,c)}}{560\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/560*(-5*I*a^3*e^(8*I*d*x + 8*I*c) - 28*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*
a^3*e^(4*I*d*x + 4*I*c) - 140*I*a^3*e^(2*I*d*x + 2*I*c) + 35*I*a^3)*e^(-I*
d*x - I*c)/d
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.79

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\begin{cases} \frac{(-10240ia^3d^4e^{8ic}e^{7idx}-57344ia^3d^4e^{6ic}e^{5idx}-143360ia^3d^4e^{4ic}e^{3idx}-286720ia^3d^4e^{2ic}e^{idx}+71680ia^3d^4e^{-idx})e^{-ic}}{1146880d^5} & \text{for } d^5e^{ic}\neq 0 \\ \frac{x(a^3e^{8ic}+4a^3e^{6ic}+6a^3e^{4ic}+4a^3e^{2ic}+a^3)e^{-ic}}{16} & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**3,x)
```

output

```
Piecewise(((-10240*I*a**3*d**4*exp(8*I*c)*exp(7*I*d*x) - 57344*I*a**3*d**4
*exp(6*I*c)*exp(5*I*d*x) - 143360*I*a**3*d**4*exp(4*I*c)*exp(3*I*d*x) - 28
6720*I*a**3*d**4*exp(2*I*c)*exp(I*d*x) + 71680*I*a**3*d**4*exp(-I*d*x))*ex
p(-I*c)/(1146880*d**5), Ne(d**5*exp(I*c), 0)), (x*(a**3*exp(8*I*c) + 4*a**
3*exp(6*I*c) + 6*a**3*exp(4*I*c) + 4*a**3*exp(2*I*c) + a**3)*exp(-I*c)/16,
 True))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{15i\,a^3\cos(dx+c)^7+i\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^3+\left(15\sin(dx+c)^7-42\sin(dx+c)^5+3\right.}{35\,d}$$

input

```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
-1/35*(15*I*a^3*cos(d*x + c)^7 + I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a
^3 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^3 + (5*
sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*
a^3)/d
```

3.50.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(90) = 180$.

Time = 0.94 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.39

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{19635\,a^3e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+39270\,a^3e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+19635\,a^3e^{(i\,dx-i\,c)}\log\left(i\,e^{(}\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/71680*(19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 39270*
a^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(I*d*x - I*
c)*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 39270*a^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) +
19635*a^3*e^(I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(5*I*d*
x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 39270*a^3*e^(3*I*d*x + I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 19635*a^3*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 39270*a
^3*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(I*d*x - I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 640*I*a^3*e^(12*I*d*x + 10*I*c) - 4864*I*
a^3*e^(10*I*d*x + 8*I*c) - 16768*I*a^3*e^(8*I*d*x + 6*I*c) - 39424*I*a^3*e
^(6*I*d*x + 4*I*c) - 40320*I*a^3*e^(4*I*d*x + 2*I*c) - 8960*I*a^3*e^(2*I*d
*x) + 4480*I*a^3*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x + I*c
) + d*e^(I*d*x - I*c))
```

3.50.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{17\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{17\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}+\frac{31\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{2}-\frac{5\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{2}+\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,35\mathrm{i}}{8}-\cos\left(\right.\right.}{35\,d\,\left(\cos\left(3\,c+3\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-(2*a^3*cos(c/2 + (d*x)/2)*((cos(c/2 + (d*x)/2)*35i)/8 - (cos((3*c)/2 + (3
*d*x)/2)*35i)/8 + (cos((5*c)/2 + (5*d*x)/2)*119i)/8 - (cos((7*c)/2 + (7*d*
x)/2)*15i)/8 + (17*sin(c/2 + (d*x)/2))/2 - (17*sin((3*c)/2 + (3*d*x)/2))/2
 + (31*sin((5*c)/2 + (5*d*x)/2))/2 - (5*sin((7*c)/2 + (7*d*x)/2))/2))/(35*
d*(cos(3*c + 3*d*x) - sin(3*c + 3*d*x)*1i))
```

3.50. $\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.51 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$

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3.51.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = &-\frac{5ia^3\cos^7(c+dx)}{63d}\\ &+\frac{5a^3\sin(c+dx)}{9d}-\frac{5a^3\sin^3(c+dx)}{9d}\\ &+\frac{a^3\sin^5(c+dx)}{3d}-\frac{5a^3\sin^7(c+dx)}{63d}\\ &-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\end{aligned}$$

output

```
-5/63*I*a^3*cos(d*x+c)^7/d+5/9*a^3*sin(d*x+c)/d-5/9*a^3*sin(d*x+c)^3/d+1/3
*a^3*sin(d*x+c)^5/d-5/63*a^3*sin(d*x+c)^7/d-2/9*I*a*cos(d*x+c)^9*(a+I*a*ta
n(d*x+c))^2/d
```

3.51.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.82

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{a^3(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(210\sqrt{\cos^2(c+dx)}+\left(32+567\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(210*Sqrt[Cos[c + d*x]^2]
+ (32 + 567*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (32 - 162*Sqrt[Cos[c
+ d*x]^2])*Cos[4*(c + d*x)] - 7*Sqrt[Cos[c + d*x]^2]*Cos[6*(c + d*x)] - (3
2*I)*Sin[2*(c + d*x)] - (378*I)*Sqrt[Cos[c + d*x]^2]*Sin[2*(c + d*x)] - (3
2*I)*Sin[4*(c + d*x)] + (216*I)*Sqrt[Cos[c + d*x]^2]*Sin[4*(c + d*x)] + (1
4*I)*Sqrt[Cos[c + d*x]^2]*Sin[6*(c + d*x)]))/(2016*d*Sqrt[Cos[c + d*x]^2])
```

3.51.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{5}{9}a^2\int \cos^7(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{9}a^2\int \frac{i\tan(c+dx)a+a}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{5}{9}a^2\left(a\int \cos^7(c+dx)dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{9}a^2\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{5}{9}a^2\left(-\frac{a\int\left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{9}a^2\left(-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(5*a^2*(((-1/7*I)*a*Cos[c + d*x]^7)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3
 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/d))/9 - (((2*I)/9)*a*Cos[c +
d*x]^9*(a + I*a*Tan[c + d*x])^2)/d
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.51.4 Maple [A] (verified)

Time = 215.94 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i a^{3} {\mathrm e}^{9 i \left(d x +c \right)}}{576 d}-\frac{3 i a^{3} {\mathrm e}^{7 i \left(d x +c \right)}}{224 d}-\frac{3 i a^{3} {\mathrm e}^{5 i \left(d x +c \right)}}{64 d}-\frac{9 i a^{3} \cos \left(d x +c \right)}{64 d}+\frac{21 a^{3} \sin \left(d x +c \right)}{64 d}-\frac{19 i a^{3} \cos \left(3 d x +3 c \right)}{192 d}+$
derivativedivides	$\frac{-i a^{3} \left(-\frac{\left(\cos^{7}\left(d x +c \right)\right) \left(\sin^{2}\left(d x +c \right)\right)}{9}-\frac{2 \left(\cos^{7}\left(d x +c \right)\right)}{63}\right)-3 a^{3} \left(-\frac{\left(\cos^{8}\left(d x +c \right)\right) \sin \left(d x +c \right)}{9}+\frac{\left(\frac{16}{5}+\cos^{6}\left(d x +c \right)+\frac{6 \left(\cos^{4}\left(d x +c \right)\right)}{5}+\right.}{63}\right.}{}$
default	$\frac{-i a^{3} \left(-\frac{\left(\cos^{7}\left(d x +c \right)\right) \left(\sin^{2}\left(d x +c \right)\right)}{9}-\frac{2 \left(\cos^{7}\left(d x +c \right)\right)}{63}\right)-3 a^{3} \left(-\frac{\left(\cos^{8}\left(d x +c \right)\right) \sin \left(d x +c \right)}{9}+\frac{\left(\frac{16}{5}+\cos^{6}\left(d x +c \right)+\frac{6 \left(\cos^{4}\left(d x +c \right)\right)}{5}+\right.}{63}\right.}{}$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/576*I/d*a^3*exp(9*I*(d*x+c))-3/224*I/d*a^3*exp(7*I*(d*x+c))-3/64*I/d*a^
3*exp(5*I*(d*x+c))-9/64*I/d*a^3*cos(d*x+c)+21/64*a^3*sin(d*x+c)/d-19/192*I
/d*a^3*cos(3*d*x+3*c)+7/64/d*a^3*sin(3*d*x+3*c)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\left(-7i\,a^3e^{(12i\,dx+12i\,c)}-54i\,a^3e^{(10i\,dx+10i\,c)}-189i\,a^3e^{(8i\,dx+8i\,c)}-420i\,a^3e^{(6i\,dx+6i\,c)}-945i\,a^3e^{(4i\,dx+4i\,c)}+378\right.}{4032\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4032*(-7*I*a^3*e^(12*I*d*x + 12*I*c) - 54*I*a^3*e^(10*I*d*x + 10*I*c) -
189*I*a^3*e^(8*I*d*x + 8*I*c) - 420*I*a^3*e^(6*I*d*x + 6*I*c) - 945*I*a^3*
e^(4*I*d*x + 4*I*c) + 378*I*a^3*e^(2*I*d*x + 2*I*c) + 21*I*a^3)*e^(-3*I*d*
x - 3*I*c)/d
```

3.51.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(112) = 224$.

Time = 0.42 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.22

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \begin{cases} \frac{(-270582939648ia^3d^6e^{13ic}e^{9idx}-2087354105856ia^3d^6e^{11ic}e^{7idx}-7305739370496ia^3d^6e^{9ic}e^{5idx}-16234976378880ia^3d^6e^{7ic}e^{3idx}-36528696}{155855773237248d^7} \\ \frac{x(a^3e^{12ic}+6a^3e^{10ic}+15a^3e^{8ic}+20a^3e^{6ic}+15a^3e^{4ic}+6a^3e^{2ic}+a^3)e^{-3ic}}{64} \end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-270582939648*I*a**3*d**6*exp(13*I*c)*exp(9*I*d*x) - 208735410
5856*I*a**3*d**6*exp(11*I*c)*exp(7*I*d*x) - 7305739370496*I*a**3*d**6*exp(
9*I*c)*exp(5*I*d*x) - 16234976378880*I*a**3*d**6*exp(7*I*c)*exp(3*I*d*x) -
 36528696852480*I*a**3*d**6*exp(5*I*c)*exp(I*d*x) + 14611478740992*I*a**3*
d**6*exp(3*I*c)*exp(-I*d*x) + 811748818944*I*a**3*d**6*exp(I*c)*exp(-3*I*d
*x))*exp(-4*I*c)/(155855773237248*d**7), Ne(d**7*exp(4*I*c), 0)), (x*(a**3
*exp(12*I*c) + 6*a**3*exp(10*I*c) + 15*a**3*exp(8*I*c) + 20*a**3*exp(6*I*c
) + 15*a**3*exp(4*I*c) + 6*a**3*exp(2*I*c) + a**3)*exp(-3*I*c)/64, True))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{105i\,a^3\cos\left(dx+c\right)^9+5i\left(7\cos\left(dx+c\right)^9-9\cos\left(dx+c\right)^7\right)a^3-3\left(35\sin\left(dx+c\right)^9-135\sin\left(dx+c\right)}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/315*(105*I*a^3*cos(d*x + c)^9 + 5*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^3 - 3*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 -
105*sin(d*x + c)^3)*a^3 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*si
n(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^3)/d
```

3.51.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1039 vs. 2(106) = 212.

Time = 0.78 (sec) , antiderivative size = 1039, normalized size of antiderivative = 8.38

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
1/516096*(119511*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 478
044*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 717066*a^3*e^(7*I
*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 478044*a^3*e^(5*I*d*x - I*c)*log(
I*e^(I*d*x + I*c) + 1) + 119511*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 128898*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 51
5592*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 773388*a^3*e^(7*
I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*a^3*e^(5*I*d*x - I*c)*log
(I*e^(I*d*x + I*c) - 1) + 128898*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x +
I*c) - 1) - 119511*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
478044*a^3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 717066*a^3*e^
(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^3*e^(5*I*d*x - I*c)
*log(-I*e^(I*d*x + I*c) + 1) - 119511*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 128898*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) -
 1) - 515592*a^3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 773388*
a^3*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*a^3*e^(5*I*d*x
- I*c)*log(-I*e^(I*d*x + I*c) - 1) - 128898*a^3*e^(3*I*d*x - 3*I*c)*log(-I
*e^(I*d*x + I*c) - 1) + 9387*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x) + e^
(-I*c)) + 37548*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 5632
2*a^3*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a^3*e^(5*I*d*x
 - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 9387*a^3*e^(3*I*d*x - 3*I*c)*log(...
```

3.51. $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.51.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.66

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{2a^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\mathrm{i}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} + \frac{2048a^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)}{9d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^9} - \frac{1024a^3\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-9\mathrm{i}\right)}{9d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8} - \frac{4a^3\left(14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-39\mathrm{i}\right)}{3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2} + \frac{8a^3\left(43\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-97\mathrm{i}\right)}{3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^3} - \frac{16a^3\left(188\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-357\mathrm{i}\right)}{7d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4} + \frac{128a^3\left(263\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-333\mathrm{i}\right)}{21d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^7} - \frac{64a^3\left(1598\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2289\mathrm{i}\right)}{63d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6} + \frac{32a^3\left(2041\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3339\mathrm{i}\right)}{63d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^5}$$

input

```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3,x)
```

output

```
(2*a^3*(tan(c/2 + (d*x)/2) - 3i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2048*a
^3*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (1024*a
^3*(8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) - (4*a^
3*(14*tan(c/2 + (d*x)/2) - 39i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) + (8*a
^3*(43*tan(c/2 + (d*x)/2) - 97i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (16
*a^3*(188*tan(c/2 + (d*x)/2) - 357i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) +
 (128*a^3*(263*tan(c/2 + (d*x)/2) - 333i))/(21*d*(tan(c/2 + (d*x)/2)^2 + 1
)^7) - (64*a^3*(1598*tan(c/2 + (d*x)/2) - 2289i))/(63*d*(tan(c/2 + (d*x)/2
)^2 + 1)^6) + (32*a^3*(2041*tan(c/2 + (d*x)/2) - 3339i))/(63*d*(tan(c/2 +
(d*x)/2)^2 + 1)^5)
```

3.51. $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$

3.52 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

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3.52.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx &= \frac{21a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{7ia^4\sec^3(c+dx)}{8d}\\ &+\frac{21a^4\sec(c+dx)\tan(c+dx)}{16d}\\ &+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}\\ &+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d}\\ &+\frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d}\end{aligned}$$

output
```
21/16*a^4*arctanh(sin(d*x+c))/d+7/8*I*a^4*sec(d*x+c)^3/d+21/16*a^4*sec(d*x
+c)*tan(d*x+c)/d+1/6*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+3/10*I*sec(d*
x+c)^3*(a^2+I*a^2*tan(d*x+c))^2/d+21/40*I*sec(d*x+c)^3*(a^4+I*a^4*tan(d*x+
c))/d
```

3.52.2 Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{a^4\sec^2(c+dx)(\cos(4c)-i\sin(4c))\big(-4608i\cos(c+dx)+5040\cos^6(c+dx)\big(\log\big(\cos\big(\tfrac{1}{2}(c+dx)\big)-\mathrm{si}}{}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/3840*(a^4*Sec[c + d*x]^2*(Cos[4*c] - I*Sin[4*c])*((-4608*I)*Cos[c + d*x
] + 5040*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-512*I)*Cos[3*(c + d*x)] + 90*Si
n[c + d*x] + 155*Sin[3*(c + d*x)] - 63*Sin[5*(c + d*x)]))*(-I + Tan[c + d*
x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4)
```

3.52.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^4dx$$

$$\Big\downarrow \ 3979$$

$$\frac{3}{2}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{2}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^3dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \ 3979$$

$$\frac{3}{2}a\left(\frac{7}{5}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}\right)+$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{2}a\left(\frac{7}{5}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^2dx+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}\right)+$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 3979$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a-}{5}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)dx+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(a-}{5}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 3967$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \sec^3(c+dx)dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c+dx)(}{}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\right)+\frac{ia\sec^3(c}{}\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 4255$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{ia\sec^3(c+dx)}{3d}\right)+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan\right.}{4d}\right.\right.$$
$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

3.52. $\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\Bigg(\frac{7}{5}a\Bigg(\frac{5}{4}a\Bigg(a\Bigg(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\Bigg)+\frac{ia\sec^3(c+dx)}{3d}\Bigg)+\frac{i\sec^3(c+dx)\left(a^2+\right.}{4d}$$

$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3}{2}a\Bigg(\frac{7}{5}a\Bigg(\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5}{4}a\Bigg(a\Bigg(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\Bigg)+ia\,\mathrm{s}$$

$$\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/6)*a*Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d + (3*a*(((I/5)*a*Sec[c
 + d*x]^3*(a + I*a*Tan[c + d*x])^2)/d + (7*a*(((I/4)*Sec[c + d*x]^3*(a^2 +
 I*a^2*Tan[c + d*x]))/d + (5*a*(((I/3)*a*Sec[c + d*x]^3)/d + a*(ArcTanh[Si
n[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))))/4))/5))/2
```

3.52.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.52.4 Maple [A] (verified)

Time = 20.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{i a^{4} \left(315 \,\mathrm{e}^{11 i (d x +c)}-3335 \,\mathrm{e}^{9 i (d x +c)}-5058 \,\mathrm{e}^{7 i (d x +c)}-4158 \,\mathrm{e}^{5 i (d x +c)}-1785 \,\mathrm{e}^{3 i (d x +c)}-315 \,\mathrm{e}^{i (d x +c)}\right)}{120 d \left(\mathrm{e}^{2 i (d x +c)}+1\right)^{6}}+\frac{21 a^{4} \ln (\mathrm{e}^{i(dx}}{16 d}$
derivativedivides	$\frac{a^{4} \left(\frac{\sin^{5}(d x +c)}{6 \cos(d x +c)^{6}}+\frac{\sin^{5}(d x +c)}{24 \cos(d x +c)^{4}}-\frac{\sin^{5}(d x +c)}{48 \cos(d x +c)^{2}}-\frac{\left(\sin^{3}(d x +c)\right)}{48}-\frac{\sin(d x +c)}{16}+\frac{\ln(\sec(d x +c)+\tan(d x +c))}{16}\right)-4 i a^{4} \left(\frac{\sin^{4}(d x +c}{5 \cos(d x +c)}\right.}{}$
default	$\frac{a^{4} \left(\frac{\sin^{5}(d x +c)}{6 \cos(d x +c)^{6}}+\frac{\sin^{5}(d x +c)}{24 \cos(d x +c)^{4}}-\frac{\sin^{5}(d x +c)}{48 \cos(d x +c)^{2}}-\frac{\left(\sin^{3}(d x +c)\right)}{48}-\frac{\sin(d x +c)}{16}+\frac{\ln(\sec(d x +c)+\tan(d x +c))}{16}\right)-4 i a^{4} \left(\frac{\sin^{4}(d x +c}{5 \cos(d x +c)}\right.}{}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/120*I*a^4/d/(exp(2*I*(d*x+c))+1)^6*(315*exp(11*I*(d*x+c))-3335*exp(9*I*
(d*x+c))-5058*exp(7*I*(d*x+c))-4158*exp(5*I*(d*x+c))-1785*exp(3*I*(d*x+c))
-315*exp(I*(d*x+c)))+21/16*a^4/d*ln(exp(I*(d*x+c))+I)-21/16*a^4/d*ln(exp(I
*(d*x+c))-I)
```

3.52.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(137) = 274.

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.23

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$= \frac{-630i\,a^4e^{(11i\,dx+11i\,c)} + 6670i\,a^4e^{(9i\,dx+9i\,c)} + 10116i\,a^4e^{(7i\,dx+7i\,c)} + 8316i\,a^4e^{(5i\,dx+5i\,c)} + 3570i\,a^4e^{(3i\,dx+3i\,c)}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/240*(-630*I*a^4*e^(11*I*d*x + 11*I*c) + 6670*I*a^4*e^(9*I*d*x + 9*I*c) +
 10116*I*a^4*e^(7*I*d*x + 7*I*c) + 8316*I*a^4*e^(5*I*d*x + 5*I*c) + 3570*I
*a^4*e^(3*I*d*x + 3*I*c) + 630*I*a^4*e^(I*d*x + I*c) + 315*(a^4*e^(12*I*d*
x + 12*I*c) + 6*a^4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 2
0*a^4*e^(6*I*d*x + 6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x
+ 2*I*c) + a^4)*log(e^(I*d*x + I*c) + I) - 315*(a^4*e^(12*I*d*x + 12*I*c)
+ 6*a^4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^4*e^(6*I
*d*x + 6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I*c) + a
^4)*log(e^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x +
 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4
*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

3.52.6 Sympy [F]

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = a^4\Bigg(&\int \left(-6\tan^2(c+dx)\sec^3(c+dx)\right)\,dx\\ &+\int \tan^4(c+dx)\sec^3(c+dx)\,dx\\ &+\int 4i\tan(c+dx)\sec^3(c+dx)\,dx\\ &+\int \left(-4i\tan^3(c+dx)\sec^3(c+dx)\right)\,dx\\ &+\int \sec^3(c+dx)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(tan(c + d
*x)**4*sec(c + d*x)**3, x) + Integral(4*I*tan(c + d*x)*sec(c + d*x)**3, x)
 + Integral(-4*I*tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(sec(c + d*
x)**3, x))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.51

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{5\,a^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+180\,a^4}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/480*(5*a^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) + 180*a^4*(2*(sin(d*x + c)^3 + sin(d*x
 + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log
(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*I*a^4/cos(d*x + c)^3 - 1
28*I*(5*cos(d*x + c)^2 - 3)*a^4/cos(d*x + c)^5)/d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$=\frac{315\,a^4\log\left(\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)+1\right)-315\,a^4\log\left(\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)-1\right)-\frac{2\left(75\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+960i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(315*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^4*log(tan(1/2*d*x + 1
/2*c) - 1) - 2*(75*a^4*tan(1/2*d*x + 1/2*c)^11 + 960*I*a^4*tan(1/2*d*x + 1
/2*c)^10 + 1175*a^4*tan(1/2*d*x + 1/2*c)^9 - 4800*I*a^4*tan(1/2*d*x + 1/2*
c)^8 - 1890*a^4*tan(1/2*d*x + 1/2*c)^7 + 4480*I*a^4*tan(1/2*d*x + 1/2*c)^6
 - 1890*a^4*tan(1/2*d*x + 1/2*c)^5 - 1920*I*a^4*tan(1/2*d*x + 1/2*c)^4 + 1
175*a^4*tan(1/2*d*x + 1/2*c)^3 + 1728*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 75*a^
4*tan(1/2*d*x + 1/2*c) - 448*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.78

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{21\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

$$-\frac{\frac{5\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}8\mathrm{i}+\frac{235\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}40\mathrm{i}-\frac{63\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{a^4\tan}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x)^3,x)
```

output
```
(21*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^4*tan(c/2 + (d*x)/2)^2*72i)
/5 + (235*a^4*tan(c/2 + (d*x)/2)^3)/24 - a^4*tan(c/2 + (d*x)/2)^4*16i - (6
3*a^4*tan(c/2 + (d*x)/2)^5)/4 + (a^4*tan(c/2 + (d*x)/2)^6*112i)/3 - (63*a^
4*tan(c/2 + (d*x)/2)^7)/4 - a^4*tan(c/2 + (d*x)/2)^8*40i + (235*a^4*tan(c/
2 + (d*x)/2)^9)/24 + a^4*tan(c/2 + (d*x)/2)^10*8i + (5*a^4*tan(c/2 + (d*x)
/2)^11)/8 - (a^4*56i)/15 + (5*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 +
(d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2
 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.53 $\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$

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3.53.1 Optimal result

Integrand size = 22, antiderivative size = 133

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx &= \frac{35a^4\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{35ia^4\sec(c+dx)}{8d}\\&\quad+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\\&\quad+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}\\&\quad+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d}\end{aligned}$$

output
```
35/8*a^4*arctanh(sin(d*x+c))/d+35/8*I*a^4*sec(d*x+c)/d+1/4*I*a*sec(d*x+c)*
(a+I*a*tan(d*x+c))^3/d+7/12*I*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+35/24*
I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

3.53.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{a^4\sec^4(c+dx)\,(-896i\cos(c+dx)+3(-128i\cos(3(c+dx))+105\log(\cos(\tfrac{1}{2}(c+dx))-\sin(\tfrac{1}{2}(c+d}{}$$

input

```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

output

```
-1/192*(a^4*Sec[c + d*x]^4*((-896*I)*Cos[c + d*x] + 3*((-128*I)*Cos[3*(c +
 d*x)] + 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)
]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) - 105*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c +
d*x)])))/d
```

3.53.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia}{3d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+ia}{3d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+}{3d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+d}{}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\,\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c+dx)(}{}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

3.53. $\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$

output

```
((I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + (7*a*(((I/3)*a*Sec[c +
 d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]])/
d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*
x]))/d))/3))/4
```

3.53.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.53.4 Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
risch	$\frac{i a^4 (279 \mathrm{e}^{7 i (d x+c)}+511 \mathrm{e}^{5 i (d x+c)}+385 \mathrm{e}^{3 i (d x+c)}+105 \mathrm{e}^{i (d x+c)})}{12 d (\mathrm{e}^{2 i (d x+c)}+1)^4}-\frac{35 a^4 \ln(\mathrm{e}^{i (d x+c)}-i)}{8 d}+\frac{35 a^4 \ln(\mathrm{e}^{i (d x+c)}+i)}{8 d}$
derivativedivides	$a^4 (\frac{\sin^5(d x+c)}{4 \cos(d x+c)^4}-\frac{\sin^5(d x+c)}{8 \cos(d x+c)^2}-\frac{(\sin^3(d x+c))}{8}-\frac{3 \sin(d x+c)}{8}+\frac{3 \ln(\sec(d x+c)+\tan(d x+c))}{8})-4 i a^4 (\frac{\sin^4(d x+c)}{3 \cos(d x+c)^3}-$
default	$a^4 (\frac{\sin^5(d x+c)}{4 \cos(d x+c)^4}-\frac{\sin^5(d x+c)}{8 \cos(d x+c)^2}-\frac{(\sin^3(d x+c))}{8}-\frac{3 \sin(d x+c)}{8}+\frac{3 \ln(\sec(d x+c)+\tan(d x+c))}{8})-4 i a^4 (\frac{\sin^4(d x+c)}{3 \cos(d x+c)^3}-$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*I*a^4/d/(exp(2*I*(d*x+c))+1)^4*(279*exp(7*I*(d*x+c))+511*exp(5*I*(d*x
+c))+385*exp(3*I*(d*x+c))+105*exp(I*(d*x+c)))-35/8*a^4/d*ln(exp(I*(d*x+c))
-I)+35/8*a^4/d*ln(exp(I*(d*x+c))+I)
```

3.53.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(109) = 218$.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.92

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{558i\,a^4e^{(7i\,dx+7i\,c)} + 1022i\,a^4e^{(5i\,dx+5i\,c)} + 770i\,a^4e^{(3i\,dx+3i\,c)} + 210i\,a^4e^{(i\,dx+i\,c)} + 105\,\left(a^4e^{(8i\,dx+8i\,c)} + 4\,a^4e^{(6}\right.}{24\,(de^{(}}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(558*I*a^4*e^(7*I*d*x + 7*I*c) + 1022*I*a^4*e^(5*I*d*x + 5*I*c) + 770
*I*a^4*e^(3*I*d*x + 3*I*c) + 210*I*a^4*e^(I*d*x + I*c) + 105*(a^4*e^(8*I*d
*x + 8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^
4*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) + I) - 105*(a^4*e^(8*I*d*
x + 8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4
*e^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*
c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x +
2*I*c) + d)
```

3.53.6 Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = a^4\left(\int \left(-6\tan^2{(c+dx)}\sec{(c+dx)}\right)\,dx + \int \tan^4{(c+dx)}\sec{(c+dx)}\,dx + \int 4i\tan{(c+dx)}\sec{(c+dx)}\,dx + \int \left(-4i\tan^3{(c+dx)}\sec{(c+dx)}\right)\,dx + \int \sec{(c+dx)}\,dx\right)$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)
**4*sec(c + d*x), x) + Integral(4*I*tan(c + d*x)*sec(c + d*x), x) + Integr
al(-4*I*tan(c + d*x)**3*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.35

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{3\,a^4\left(\frac{2\,\left(5\,\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+72\,a^4\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}+\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/48*(3*a^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin
(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 72
*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1)) + 48*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*I*a^4/cos
(d*x + c) + 64*I*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.30

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$= \frac{105\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\,(81\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7+96i\,a^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c}{}}{24\,d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^4*log(tan(1/2*d*x + 1/
2*c) - 1) - 2*(81*a^4*tan(1/2*d*x + 1/2*c)^7 + 96*I*a^4*tan(1/2*d*x + 1/2*
c)^6 - 105*a^4*tan(1/2*d*x + 1/2*c)^5 - 480*I*a^4*tan(1/2*d*x + 1/2*c)^4 -
 105*a^4*tan(1/2*d*x + 1/2*c)^3 + 544*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 81*a^
4*tan(1/2*d*x + 1/2*c) - 160*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.49

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

$$-\frac{\frac{27\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,8\mathrm{i}-\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,40\mathrm{i}-\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a^4\tan(}{}}{d\,(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x),x)
```

output
```
(35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^4*tan(c/2 + (d*x)/2)^2*136i
)/3 - (35*a^4*tan(c/2 + (d*x)/2)^3)/4 - a^4*tan(c/2 + (d*x)/2)^4*40i - (35
*a^4*tan(c/2 + (d*x)/2)^5)/4 + a^4*tan(c/2 + (d*x)/2)^6*8i + (27*a^4*tan(c
/2 + (d*x)/2)^7)/4 - (a^4*40i)/3 + (27*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*ta
n(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan
(c/2 + (d*x)/2)^8 + 1))
```

3.54 $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$

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3.54.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = &-\frac{15a^4\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{15ia^4\sec(c+dx)}{2d}\\ &-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}\\ &-\frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}\end{aligned}$$

output

```
-15/2*a^4*arctanh(sin(d*x+c))/d-15/2*I*a^4*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(
a+I*a*tan(d*x+c))^3/d-5/2*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

3.54.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 906 vs. $2(97) = 194$.

Time = 6.77 (sec) , antiderivative size = 906, normalized size of antiderivative = 9.34

$$\begin{aligned}
&\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx \\
&= \frac{15\cos(4c)\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4} \\
&\quad - \frac{15\cos(4c)\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4} \\
&\quad + \frac{\cos(dx)\cos^4(c+dx)(-8i\cos(3c)-8\sin(3c))(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4} \\
&\quad + \frac{\cos^4(c+dx)\sec(c)(-4i\cos(4c)-4\sin(4c))(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4} \\
&\quad - \frac{15i\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(4c)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4} \\
&\quad + \frac{15i\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(4c)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4} \\
&\quad + \frac{\cos^4(c+dx)(8\cos(3c)-8i\sin(3c))\sin(dx)(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4} \\
&\quad + \frac{\cos^4(c+dx)\left(\frac{1}{4}\cos(4c)-\frac{1}{4}i\sin(4c)\right)(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad - \frac{i\cos^4(c+dx)(4\cos(4c)-4i\sin(4c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^4}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&\quad + \frac{\cos^4(c+dx)\left(-\frac{1}{4}\cos(4c)+\frac{1}{4}i\sin(4c)\right)(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad + \frac{i\cos^4(c+dx)(4\cos(4c)-4i\sin(4c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^4}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(15*Cos[4*c]*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(
a + I*a*Tan[c + d*x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) - (15*Cos[4*c]*Co
s[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c +
 d*x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[d*x]*Cos[c + d*x]^4*((-8*
I)*Cos[3*c] - 8*Sin[3*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d
*x])^4) + (Cos[c + d*x]^4*Sec[c]*((-4*I)*Cos[4*c] - 4*Sin[4*c])*(a + I*a*T
an[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) - (((15*I)/2)*Cos[c + d*x]^4
*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*
x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) + (((15*I)/2)*Cos[c + d*x]^4*Log[Cos[
c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(d
*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(8*Cos[3*c] - (8*I)*Sin[3*c]
)*Sin[d*x]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[
c + d*x]^4*(Cos[4*c]/4 - (I/4)*Sin[4*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos
[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) - (I*Co
s[c + d*x]^4*(4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d
*x])^4)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x
)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^4*(-1/4*Cos[4*c] + (I/4)*Sin[4
*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2])^2) + (I*Cos[c + d*x]^4*(4*Cos[4*c] - (4*I)*Sin
[4*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[c/2] + Sin[c/2])*...
```

3.54.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3977, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3977}$$

$$-5a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$-5a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3979}$$

$$-5a^2\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$-5a^2\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3967}$$

$$-5a^2\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$-5a^2\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{4257}$$

$$-5a^2\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\,\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - 5*a^2*((3*a*((a*ArcTa
nh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2
+ I*a^2*Tan[c + d*x]))/d)
```

3.54. $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$

3.54.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.54.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{8ia^4\mathrm{e}^{i(dx+c)}}{d}-\frac{ia^4\left(9\,\mathrm{e}^{3i(dx+c)}+7\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{15a^4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}+\frac{15a^4\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$
derivativedivides	$\frac{a^4\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c}{d}$
default	$\frac{a^4\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c}{d}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-8*I*a^4/d*exp(I*(d*x+c))-I*a^4/d/(exp(2*I*(d*x+c))+1)^2*(9*exp(3*I*(d*x+c
))+7*exp(I*(d*x+c)))-15/2*a^4/d*ln(exp(I*(d*x+c))+I)+15/2*a^4/d*ln(exp(I*(
d*x+c))-I)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{-16i\,a^4e^{(5i\,dx+5i\,c)}-50i\,a^4e^{(3i\,dx+3i\,c)}-30i\,a^4e^{(i\,dx+i\,c)}-15\left(a^4e^{(4i\,dx+4i\,c)}+2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx+}\right.}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2*(-16*I*a^4*e^(5*I*d*x + 5*I*c) - 50*I*a^4*e^(3*I*d*x + 3*I*c) - 30*I*a
^4*e^(I*d*x + I*c) - 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2*I*d*x + 2*I*
c) + a^4)*log(e^(I*d*x + I*c) + I) + 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e
^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c)
 + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{15a^4\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d} + \frac{-9ia^4e^{3ic}e^{3idx}-7ia^4e^{ic}e^{idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} + \begin{cases} -\frac{8ia^4e^{ic}e^{idx}}{d} & \text{for } d\neq 0 \\ 8a^4xe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
15*a**4*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2
)/d + (-9*I*a**4*exp(3*I*c)*exp(3*I*d*x) - 7*I*a**4*exp(I*c)*exp(I*d*x))/(
d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise((
-8*I*a**4*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (8*a**4*x*exp(I*c), True))
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log\left(\sin\left(dx+c\right)+1\right)-3\log\left(\sin\left(dx+c\right)-1\right)-4\sin\left(dx+c\right)\right)+16i\,a^4\left(\frac{1}{\cos(dx+c)}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/4*(a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 16*I*a^4*(1/cos(d*x + c) + co
s(d*x + c)) + 12*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*si
n(d*x + c)) + 16*I*a^4*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

3.54.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(81) = 162.

Time = 0.67 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.84

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{235\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+470\,a^4e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-5\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/32*(235*a^4*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 470*a^4*e^(
2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*e^(4*I*d*x + 4*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) - 10*a^4*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c
) - 1) - 235*a^4*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 470*a^4
*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*e^(4*I*d*x + 4*I*
c)*log(-I*e^(I*d*x + I*c) - 1) + 10*a^4*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 256*I*a^4*e^(5*I*d*x + 5*I*c) - 800*I*a^4*e^(3*I*d*x + 3*I
*c) - 480*I*a^4*e^(I*d*x + I*c) + 235*a^4*log(I*e^(I*d*x + I*c) + 1) - 5*a
^4*log(I*e^(I*d*x + I*c) - 1) - 235*a^4*log(-I*e^(I*d*x + I*c) + 1) + 5*a^
4*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2
*I*c) + d)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.64

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{15\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$
$$+\frac{17\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 9\mathrm{i}-39\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,7\mathrm{i}+24\,a^4}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 1\mathrm{i}-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 2\mathrm{i}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4,x)
```

output

```
(a^4*tan(c/2 + (d*x)/2)^3*9i - 39*a^4*tan(c/2 + (d*x)/2)^2 + 17*a^4*tan(c/
2 + (d*x)/2)^4 + 24*a^4 - a^4*tan(c/2 + (d*x)/2)*7i)/(d*(tan(c/2 + (d*x)/2
) - tan(c/2 + (d*x)/2)^2*2i - 2*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^
4*1i + tan(c/2 + (d*x)/2)^5 + 1i)) - (15*a^4*atanh(tan(c/2 + (d*x)/2)))/d
```

3.54. $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$

3.55 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

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3.55.1 Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx=\frac{a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}+\frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}$$

output
```
a^4*arctanh(sin(d*x+c))/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+2*I*
cos(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

3.55.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 246 vs. 2(78) = 156.

Time = 0.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.15

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{a^4\left(-3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input

```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output

```
(a^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*
Sin[3*c] + (3*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*c] - (3*I)
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*c] + Cos[3*c]*((6*I)*Cos[d
*x] - 6*Sin[d*x]) + (6*I)*Sin[3*c]*Sin[d*x] - (2*I)*Sin[c]*Sin[3*d*x] + 2*
Cos[c]*((-I)*Cos[3*d*x] + Sin[3*d*x]))*(Cos[c + d*x] + I*Sin[c + d*x])^4)/
(3*d*(Cos[d*x] + I*Sin[d*x])^4)
```

3.55.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3977, 3042, 3977, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3977}$$

$$-a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 3977}$$

$$-a^2\left(a^2(-\int \sec(c+dx)dx)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-a^2\left(a^2\left(-\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4257}$$

$$-a^2\left(-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - a^2*(-((a^2*Arc
Tanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
)
```

3.55.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.55.4 Maple [A] (verified)

Time = 13.92 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{2ia^4 \mathrm{e}^{3i(dx+c)}}{3d}+\frac{2ia^4 \mathrm{e}^{i(dx+c)}}{d}+\frac{a^4 \ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a^4 \ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^4\left(\sin^3(dx+c)\right)-\frac{4ia^4(c}{}}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^4\left(\sin^3(dx+c)\right)-\frac{4ia^4(c}{}}{d}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*a^4/d*exp(3*I*(d*x+c))+2*I*a^4/d*exp(I*(d*x+c))+a^4/d*ln(exp(I*(d*x
+c))+I)-a^4/d*ln(exp(I*(d*x+c))-I)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-2i\,a^4e^{(3i\,dx+3i\,c)}+6i\,a^4e^{(i\,dx+i\,c)}+3\,a^4\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,a^4\log\left(e^{(i\,dx+i\,c)}-i\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(-2*I*a^4*e^(3*I*d*x + 3*I*c) + 6*I*a^4*e^(I*d*x + I*c) + 3*a^4*log(e^
(I*d*x + I*c) + I) - 3*a^4*log(e^(I*d*x + I*c) - I))/d
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} \frac{-2ia^4de^{3ic}e^{3idx}+6ia^4de^{ic}e^{idx}}{3d^2} & \text{for } d^2 \neq 0 \\ x\left(2a^4e^{3ic}-2a^4e^{ic}\right) & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

output

```
a**4*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d +
Piecewise(((-2*I*a**4*d*exp(3*I*c)*exp(3*I*d*x) + 6*I*a**4*d*exp(I*c)*exp(
I*d*x))/(3*d**2), Ne(d**2, 0)), (x*(2*a**4*exp(3*I*c) - 2*a**4*exp(I*c)),
True))
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.55

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{8i\,a^4\cos\left(dx+c\right)^3+12\,a^4\sin\left(dx+c\right)^3+8i\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^4+\left(2\,\sin\left(dx+c\right)^3-3\,\mathrm{l}\right.}{6}$$

input

```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output

```
-1/6*(8*I*a^4*cos(d*x + c)^3 + 12*a^4*sin(d*x + c)^3 + 8*I*(cos(d*x + c)^3
 - 3*cos(d*x + c))*a^4 + (2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*l
og(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^4 + 2*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*a^4)/d
```

3.55.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(68) = 136.

Time = 0.97 (sec) , antiderivative size = 1299, normalized size of antiderivative = 16.65

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/768*(1110*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6660*a^4
*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(8*I*d*x +
2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 6660*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) +
1) + 22200*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1110*a^4*e^(-6*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 1875*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 11250*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1)
+ 28125*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125*a^4*e^(
4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11250*a^4*e^(2*I*d*x - 4*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 37500*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c)
 - 1) + 1875*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1110*a^4*e^(12*I*
d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(10*I*d*x + 4*I*c)*l
og(-I*e^(I*d*x + I*c) + 1) - 16650*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 16650*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
 6660*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22200*a^4*e^(6
*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1110*a^4*e^(-6*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 1875*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
11250*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(
8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(4*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 11250*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e...
```

3.55.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$
\begin{aligned}
&\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx \\
&= \frac{2\,a^4\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} \\
&\quad - \frac{\frac{8\,a^4}{3}-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,8\mathrm{i}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}
\end{aligned}
$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(2*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((8*a^4)/3 - a^4*tan(c/2 + (d*x)/2)*
8i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2
)^3 + 1i))
```

3.56 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$

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3.56.1 Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

output

```
-1/15*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-1/5*I*cos(d*x+c)^5*(a+I*a*ta
n(d*x+c))^4/d
```

3.56.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 145 vs. 2(66) = 132.

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.20

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4(-i\cos(2(c+dx))+\sin(2(c+dx)))\Big(\cos(c+dx)\left(8+5\sqrt{\cos^2(c+dx)}\right)+\left(8+3\sqrt{\cos^2(c+dx)}\right)\cos}{30d\sqrt{\cos^2(c}}$$

input

```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(Cos[c + d*x]*(8 + 5*Sqrt[
Cos[c + d*x]^2]) + (8 + 3*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] + I*((-8
+ 5*Sqrt[Cos[c + d*x]^2])*Sin[c + d*x] + (-8 + 3*Sqrt[Cos[c + d*x]^2])*Sin
[3*(c + d*x)])))/(30*d*Sqrt[Cos[c + d*x]^2])
```

3.56.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{1}{5}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}a\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3969}$$

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/15*I)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d
*x]^5*(a + I*a*Tan[c + d*x])^4)/d
```

3.56.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.56.4 Maple [A] (verified)

Time = 47.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{i a^{4} {\mathrm e}^{5 i (d x +c)}}{10 d}-\frac{i a^{4} {\mathrm e}^{3 i (d x +c)}}{6 d}$
derivativedivides	$\frac{\frac{a^{4} \left(\sin^{5}(d x +c)\right)}{5}-4 i a^{4} \left(-\frac{\left(\cos^{3}(d x +c)\right) \left(\sin^{2}(d x +c)\right)}{5}-\frac{2 \left(\cos^{3}(d x +c)\right)}{15}\right)-6 a^{4} \left(-\frac{\sin (d x +c) \left(\cos^{4}(d x +c)\right)}{5}+\frac{\left(2+\cos^{2}(d x +c)\right) \mathrm{si}}{15}\right.}{d}$
default	$\frac{\frac{a^{4} \left(\sin^{5}(d x +c)\right)}{5}-4 i a^{4} \left(-\frac{\left(\cos^{3}(d x +c)\right) \left(\sin^{2}(d x +c)\right)}{5}-\frac{2 \left(\cos^{3}(d x +c)\right)}{15}\right)-6 a^{4} \left(-\frac{\sin (d x +c) \left(\cos^{4}(d x +c)\right)}{5}+\frac{\left(2+\cos^{2}(d x +c)\right) \mathrm{si}}{15}\right.}{d}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/10*I*a^4/d*exp(5*I*(d*x+c))-1/6*I*a^4/d*exp(3*I*(d*x+c))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-3i\,a^4e^{(5i\,dx+5i\,c)}-5i\,a^4e^{(3i\,dx+3i\,c)}}{30\,d}$$

input

```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output

```
1/30*(-3*I*a^4*e^(5*I*d*x + 5*I*c) - 5*I*a^4*e^(3*I*d*x + 3*I*c))/d
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \begin{cases} \frac{-6ia^4de^{5ic}e^{5idx}-10ia^4de^{3ic}e^{3idx}}{60d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^4e^{5ic}}{2}+\frac{a^4e^{3ic}}{2}\right) & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**4,x)
```

output

```
Piecewise(((-6*I*a**4*d*exp(5*I*c)*exp(5*I*d*x) - 10*I*a**4*d*exp(3*I*c)*e
xp(3*I*d*x))/(60*d**2), Ne(d**2, 0)), (x*(a**4*exp(5*I*c)/2 + a**4*exp(3*I
*c)/2), True))
```

3.56.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(54) = 108.

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.79

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{12i\,a^4\cos(dx+c)^5-3\,a^4\sin(dx+c)^5+4i\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^4-6\left(3\,\sin(dx+c)^5-\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/15*(12*I*a^4*cos(d*x + c)^5 - 3*a^4*sin(d*x + c)^5 + 4*I*(3*cos(d*x + c
)^5 - 5*cos(d*x + c)^3)*a^4 - 6*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^4
- (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4)/d
```

3.56.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(54) = 108$.

Time = 1.08 (sec) , antiderivative size = 915, normalized size of antiderivative = 13.86

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/7680*(9075*a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^
4*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4*e^(2*I*d*x -
2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 54450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x +
 I*c) + 1) + 9075*a^4*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9000*a^4*e^(
8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(6*I*d*x + 2*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 54000*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 9000*a^
4*e^(-4*I*c)*log(I*e^(I*d*x + I*c) - 1) - 9075*a^4*e^(8*I*d*x + 4*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 36300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 36300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5
4450*a^4*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 9075*a^4*e^(-4*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 9000*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 36000*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 36
000*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54000*a^4*e^(4*I
*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 9000*a^4*e^(-4*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 75*a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a
^4*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(2*I*d*x -
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x) +
 e^(-I*c)) - 75*a^4*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(8*I
*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 300*a^4*e^(6*I*d*x + 2*I*c...
```

3.56.9 Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{2\,a^4\,\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,15\mathrm{i}-25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5\mathrm{i}+4\right)}{15\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,5\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,10\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(2*a^4*(tan(c/2 + (d*x)/2)^3*15i - 25*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)*5i + 15*tan(c/2 + (d*x)/2)^4 + 4))/(15*d*(5*tan(c/2 + (d*x)/2) - tan
(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i
+ tan(c/2 + (d*x)/2)^5 + 1i))
```

3.57 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$

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3.57.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{3a^4\sin(c+dx)}{35d}-\frac{a^4\sin^3(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

output
```
3/35*a^4*sin(d*x+c)/d-1/35*a^4*sin(d*x+c)^3/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*
tan(d*x+c))^3/d-2/35*I*cos(d*x+c)^5*(a^4+I*a^4*tan(d*x+c))/d
```

3.57.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$= \frac{a^4(-i\cos(3(c+dx))+\sin(3(c+dx)))\Big(35\sqrt{\cos^2(c+dx)}+8\Big(4+7\sqrt{\cos^2(c+dx)}\Big)\cos(2(c+dx))+\Big(3}{}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

output

```
(a^4*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35*Sqrt[Cos[c + d*x]^2] +
 8*(4 + 7*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (32 + 5*Sqrt[Cos[c + d*
x]^2])*Cos[4*(c + d*x)] - (32*I)*Sin[2*(c + d*x)] - (14*I)*Sqrt[Cos[c + d*
x]^2]*Sin[2*(c + d*x)] - (32*I)*Sin[4*(c + d*x)] + (5*I)*Sqrt[Cos[c + d*x]
^2]*Sin[4*(c + d*x)]))/(280*d*Sqrt[Cos[c + d*x]^2])
```

3.57.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3977, 3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{1}{7}a^2\int \cos^5(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{1}{7}a^2\left(\frac{3}{5}a^2\int \cos^3(c+dx)dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}a^2\left(\frac{3}{5}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{1}{7}a^2\left(-\frac{3a^2\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}a^2\left(-\frac{3a^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3)/d + (a^2*((-3*a^2*(
-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*d) - (((2*I)/5)*Cos[c + d*x]^5*(a^2
+ I*a^2*Tan[c + d*x]))/d))/7
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.57.4 Maple [A] (verified)

Time = 131.89 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{i a^{4} {\mathrm e}^{7 i \left(d x +c \right)}}{56 d}-\frac{3 i a^{4} {\mathrm e}^{5 i \left(d x +c \right)}}{40 d}-\frac{i a^{4} {\mathrm e}^{3 i \left(d x +c \right)}}{8 d}-\frac{i a^{4} {\mathrm e}^{i \left(d x +c \right)}}{8 d}$
derivativedivides	$a^{4} \left(-\frac{\left(\sin^{3}\left(d x +c \right)\right) \left(\cos^{4}\left(d x +c \right)\right)}{7}-\frac{3 \sin \left(d x +c \right) \left(\cos^{4}\left(d x +c \right)\right)}{35}+\frac{\left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)}{35}\right)-4 i a^{4} \left(-\frac{\left(\cos^{5}\left(d x +c \right)\right) \left(\sin^{2}\left(d x +c\right.\right.}{7}\right.$
default	$a^{4} \left(-\frac{\left(\sin^{3}\left(d x +c \right)\right) \left(\cos^{4}\left(d x +c \right)\right)}{7}-\frac{3 \sin \left(d x +c \right) \left(\cos^{4}\left(d x +c \right)\right)}{35}+\frac{\left(2+\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)}{35}\right)-4 i a^{4} \left(-\frac{\left(\cos^{5}\left(d x +c \right)\right) \left(\sin^{2}\left(d x +c\right.\right.}{7}\right.$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*I*a^4/d*exp(7*I*(d*x+c))-3/40*I*a^4/d*exp(5*I*(d*x+c))-1/8*I*a^4/d*e
xp(3*I*(d*x+c))-1/8*I*a^4/d*exp(I*(d*x+c))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-5i\,a^4e^{(7i\,dx+7i\,c)}-21i\,a^4e^{(5i\,dx+5i\,c)}-35i\,a^4e^{(3i\,dx+3i\,c)}-35i\,a^4e^{(i\,dx+i\,c)}}{280\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/280*(-5*I*a^4*e^(7*I*d*x + 7*I*c) - 21*I*a^4*e^(5*I*d*x + 5*I*c) - 35*I*
a^4*e^(3*I*d*x + 3*I*c) - 35*I*a^4*e^(I*d*x + I*c))/d
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \begin{cases} \frac{-2560ia^4d^3e^{7ic}e^{7idx}-10752ia^4d^3e^{5ic}e^{5idx}-17920ia^4d^3e^{3ic}e^{3idx}-17920ia^4d^3e^{ic}e^{idx}}{143360d^4} & \text{for } d^4\neq 0 \\ x\left(\frac{a^4e^{7ic}}{8}+\frac{3a^4e^{5ic}}{8}+\frac{3a^4e^{3ic}}{8}+\frac{a^4e^{ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-2560*I*a**4*d**3*exp(7*I*c)*exp(7*I*d*x) - 10752*I*a**4*d**3*
exp(5*I*c)*exp(5*I*d*x) - 17920*I*a**4*d**3*exp(3*I*c)*exp(3*I*d*x) - 1792
0*I*a**4*d**3*exp(I*c)*exp(I*d*x))/(143360*d**4), Ne(d**4, 0)), (x*(a**4*e
xp(7*I*c)/8 + 3*a**4*exp(5*I*c)/8 + 3*a**4*exp(3*I*c)/8 + a**4*exp(I*c)/8)
, True))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.46

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{20i\,a^4\cos(dx+c)^7+4i\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^4+2\left(15\sin(dx+c)^7-42\sin(dx+c)^5\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/35*(20*I*a^4*cos(d*x + c)^7 + 4*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a^4 + 2*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^4 +
 (5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^4 + (5*sin(d*x + c)^7 - 21*sin(d*
x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4)/d
```

3.57.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(86) = 172$.

Time = 0.76 (sec) , antiderivative size = 1327, normalized size of antiderivative = 13.01

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/143360*(89950*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5397
00*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(8*
I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(4*I*d*x - 2*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 539700*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d
*x + I*c) + 1) + 1799000*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 8995
0*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 86065*a^4*e^(12*I*d*x + 6*I*
c)*log(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I
*d*x + I*c) - 1) + 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 1290975*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516390*
a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1721300*a^4*e^(6*I*d*
x)*log(I*e^(I*d*x + I*c) - 1) + 86065*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c)
 - 1) - 89950*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 53970
0*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(8*
I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(4*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 539700*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 1799000*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) -
89950*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 86065*a^4*e^(12*I*d*x +
 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516390*a^4*e^(10*I*d*x + 4*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 1290975*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - ...
```

3.57.9 Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.82

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{2\,a^4\,\Big(35\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^6+\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^5\,105\mathrm{i}-210\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^4-\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^3\,210\mathrm{i}+147\tan\big(}{35\,d\,\Big(-\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^7-\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^6\,7\mathrm{i}+21\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^5+\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^4\,35\mathrm{i}-35\tan\big(\frac{c}{2}+\frac{dx}{2}\big)^3-}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
-(2*a^4*(tan(c/2 + (d*x)/2)*49i + 147*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)^3*210i - 210*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*105i + 35*t
an(c/2 + (d*x)/2)^6 - 12))/(35*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2
)^2*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2
+ (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.57. $\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$

3.58 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$

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3.58.9 Mupad [B] (verification not implemented) 602

3.58.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{5a^4\sin(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{a^4\sin^5(c+dx)}{21d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}$$

output
```
5/21*a^4*sin(d*x+c)/d-10/63*a^4*sin(d*x+c)^3/d+1/21*a^4*sin(d*x+c)^5/d-2/9
*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3/d-2/21*I*cos(d*x+c)^7*(a^4+I*a^4*ta
n(d*x+c))/d
```

3.58.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.80

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4(-i\cos(4(c+dx))+\sin(4(c+dx)))\left(168\cos(c+dx)\sqrt{\cos^2(c+dx)}+4\left(16+45\sqrt{\cos^2(c+dx)}\right)\cos(}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(168*Cos[c + d*x]*Sqrt[Cos
[c + d*x]^2] + 4*(16 + 45*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] + 64*Cos[
5*(c + d*x)] - 28*Sqrt[Cos[c + d*x]^2]*Cos[5*(c + d*x)] - (42*I)*Sqrt[Cos[
c + d*x]^2]*Sin[c + d*x] - (64*I)*Sin[3*(c + d*x)] - (135*I)*Sqrt[Cos[c +
d*x]^2]*Sin[3*(c + d*x)] - (64*I)*Sin[5*(c + d*x)] + (35*I)*Sqrt[Cos[c + d
*x]^2]*Sin[5*(c + d*x)]))/(1008*d*Sqrt[Cos[c + d*x]^2])
```

3.58.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3977, 3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{3}a^2\int \cos^7(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{3}a^2\left(\frac{5}{7}a^2\int \cos^5(c+dx)dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}a^2\left(\frac{5}{7}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \textcolor{blue}{3113}$$

$$\frac{1}{3}a^2\left(-\frac{5a^2\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{1}{3}a^2\left(-\frac{5a^2\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3)/d + (a^2*((-5*a^2*(
-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*d) - (((2*I)/
7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d))/3
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.58.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(106) = 212.

Time = 0.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.94

$$a^4 \left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9} - \frac{\sin(dx+c)(\cos^6(dx+c))}{21} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{105} \right) - 4ia^4\left(-\frac{(\cos^7(dx+c))(\mathrm{s}}{9}\right.$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x)
```

output
```
1/d*(a^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/10
5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-4*I*a^4*(-1/9*cos(d*x+c)
^7*sin(d*x+c)^2-2/63*cos(d*x+c)^7)-6*a^4*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/6
3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-4/9*I*
a^4*cos(d*x+c)^9+1/9*a^4*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d
*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.75

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{\left(-7i\,a^4e^{(10i\,dx+10i\,c)} - 45i\,a^4e^{(8i\,dx+8i\,c)} - 126i\,a^4e^{(6i\,dx+6i\,c)} - 210i\,a^4e^{(4i\,dx+4i\,c)} - 315i\,a^4e^{(2i\,dx+2i\,c)} + 63i\,a^4\right.}{2016\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2016*(-7*I*a^4*e^(10*I*d*x + 10*I*c) - 45*I*a^4*e^(8*I*d*x + 8*I*c) - 12
6*I*a^4*e^(6*I*d*x + 6*I*c) - 210*I*a^4*e^(4*I*d*x + 4*I*c) - 315*I*a^4*e^
(2*I*d*x + 2*I*c) + 63*I*a^4)*e^(-I*d*x - I*c)/d
```

3.58.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(107) = 214.

Time = 0.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$= \begin{cases} \frac{(-176160768ia^4d^5e^{10ic}e^{9idx}-1132462080ia^4d^5e^{8ic}e^{7idx}-3170893824ia^4d^5e^{6ic}e^{5idx}-5284823040ia^4d^5e^{4ic}e^{3idx}-7927234560ia^4d^5e^{2ic}e^{idx}}{50734301184d^6} \\ \frac{x(a^4e^{10ic}+5a^4e^{8ic}+10a^4e^{6ic}+10a^4e^{4ic}+5a^4e^{2ic}+a^4)e^{-ic}}{32} \end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-176160768*I*a**4*d**5*exp(10*I*c)*exp(9*I*d*x) - 1132462080*I
*a**4*d**5*exp(8*I*c)*exp(7*I*d*x) - 3170893824*I*a**4*d**5*exp(6*I*c)*exp
(5*I*d*x) - 5284823040*I*a**4*d**5*exp(4*I*c)*exp(3*I*d*x) - 7927234560*I*
a**4*d**5*exp(2*I*c)*exp(I*d*x) + 1585446912*I*a**4*d**5*exp(-I*d*x))*exp(
-I*c)/(50734301184*d**6), Ne(d**6*exp(I*c), 0)), (x*(a**4*exp(10*I*c) + 5*
a**4*exp(8*I*c) + 10*a**4*exp(6*I*c) + 10*a**4*exp(4*I*c) + 5*a**4*exp(2*I
*c) + a**4)*exp(-I*c)/32, True))
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{140i\,a^4\cos(dx+c)^9+20i\,\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^4-\left(35\sin(dx+c)^9-90\sin(dx+c)^7\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/315*(140*I*a^4*cos(d*x + c)^9 + 20*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)
^7)*a^4 - (35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^4
- 6*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin
(d*x + c)^3)*a^4 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x +
 c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4)/d
```

3.58.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(102) = 204$.

Time = 0.81 (sec) , antiderivative size = 1409, normalized size of antiderivative = 11.74

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/516096*(435267*a^4*e^(13*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 261
1602*a^4*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(
9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 8705340*a^4*e^(7*I*d*x + I*c
)*log(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*
x + I*c) + 1) + 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 435267*a^4*e^(I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 427896*a^4*e^
(13*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(11*I*d*x +
5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6418440*a^4*e^(9*I*d*x + 3*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 8557920*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c)
 - 1) + 6418440*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376
*a^4*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 427896*a^4*e^(I*d*x
- 5*I*c)*log(I*e^(I*d*x + I*c) - 1) - 435267*a^4*e^(13*I*d*x + 7*I*c)*log(
-I*e^(I*d*x + I*c) + 1) - 2611602*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 6529005*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1)
 - 8705340*a^4*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6529005*a^4
*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2611602*a^4*e^(3*I*d*x -
3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 435267*a^4*e^(I*d*x - 5*I*c)*log(-I*e
^(I*d*x + I*c) + 1) - 427896*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 2567376*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6
418440*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 8557920*a^...
```

3.58.9 Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{2a^4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{89\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}-\frac{55\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{4}+\frac{55\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{4}-\frac{355\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{16}+\frac{35\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{16}-\cos\right.}{63\,d\,\left(\cos\left(4\,c+4\,d\,x\right)-\sin\left(4\,c+4\,d\,x\right)\,1\mathrm{i}\right.}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(2*a^4*cos(c/2 + (d*x)/2)*((cos((5*c)/2 + (5*d*x)/2)*21i)/2 - (cos((3*c)/2
 + (3*d*x)/2)*21i)/2 - (cos((7*c)/2 + (7*d*x)/2)*87i)/4 + (cos((9*c)/2 + (
9*d*x)/2)*7i)/4 + (89*sin(c/2 + (d*x)/2))/8 - (55*sin((3*c)/2 + (3*d*x)/2)
)/4 + (55*sin((5*c)/2 + (5*d*x)/2))/4 - (355*sin((7*c)/2 + (7*d*x)/2))/16
+ (35*sin((9*c)/2 + (9*d*x)/2))/16))/(63*d*(cos(4*c + 4*d*x) - sin(4*c + 4
*d*x)*1i))
```

3.59 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.59.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{8i(a+ia\tan(c+dx))^9}{9a^4d} + \frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d} - \frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d} + \frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}$$

output
```
-8/9*I*(a+I*a*tan(d*x+c))^9/a^4/d+6/5*I*(a+I*a*tan(d*x+c))^10/a^5/d-6/11*I
*(a+I*a*tan(d*x+c))^11/a^6/d+1/12*I*(a+I*a*tan(d*x+c))^12/a^7/d
```

3.59.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\sec^{12}(c+dx)(78\cos(c+dx)+221\cos(3(c+dx))-3i(18\sin(c+dx)+73\sin(3(c+dx))))(-i\cos(9(c}{1980d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]^12*(78*Cos[c + d*x] + 221*Cos[3*(c + d*x)] - (3*I)*(18*S
in[c + d*x] + 73*Sin[3*(c + d*x)]))*((-I)*Cos[9*(c + d*x)] + Sin[9*(c + d*
x)]))/(1980*d)
```

3.59.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^8d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{11}+6a(i\tan(c+dx)a+a)^{10}-12a^2(i\tan(c+dx)a+a)^9+8a^3(i\tan(c+dx)a+a)^8\right.}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{8}{9}a^3(a+ia\tan(c+dx))^9-\frac{6}{5}a^2(a+ia\tan(c+dx))^{10}-\frac{1}{12}(a+ia\tan(c+dx))^{12}+\frac{6}{11}a(a+ia\tan(c+dx))^{11}}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*((8*a^3*(a + I*a*Tan[c + d*x])^9)/9 - (6*a^2*(a + I*a*Tan[c + d*x])^
10)/5 + (6*a*(a + I*a*Tan[c + d*x])^11)/11 - (a + I*a*Tan[c + d*x])^12/12)
)/(a^7*d)
```

3.59.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.59.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(93) = 186.

Time = 1.15 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.46

$$ia^5\left(\frac{\sin^6(dx+c)}{12\cos(dx+c)^{12}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{40\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{120\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{11\cos(dx+c)^{11}}+\frac{2(\sin^5(dx+c))}{33\cos(dx+c)^9}+\frac{8(\sin^5(dx+c))}{231\cos(dx+c)}\right.$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^1
0+1/40*sin(d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)+5*a^5*(1
/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*
x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^5*(1/10*sin(
d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/co
s(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-10*a^5*(1/9*sin(d*x+c)^3/cos(d*
x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/3
15*sin(d*x+c)^3/cos(d*x+c)^3)+5/8*I*a^5/cos(d*x+c)^8-a^5*(-16/35-1/7*sec(d
*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

3.59.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.45

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{1024\left(-495i\,a^5e^{(16i\,dx+16i\,c)} - 792i\,a^5e^{(14i\,dx+14i\,c)} - 924i\,a^5e^{(12i\,dx+12i\,c)} - 792\right.}{495\left(de^{(24i\,dx+24i\,c)} + 12\,de^{(22i\,dx+22i\,c)} + 66\,de^{(20i\,dx+20i\,c)} + 220\,de^{(18i\,dx+18i\,c)} + 495\,de^{(16i\,dx+16i\,c)} + 792\,de\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1024/495*(-495*I*a^5*e^(16*I*d*x + 16*I*c) - 792*I*a^5*e^(14*I*d*x + 14*I
*c) - 924*I*a^5*e^(12*I*d*x + 12*I*c) - 792*I*a^5*e^(10*I*d*x + 10*I*c) -
495*I*a^5*e^(8*I*d*x + 8*I*c) - 220*I*a^5*e^(6*I*d*x + 6*I*c) - 66*I*a^5*e
^(4*I*d*x + 4*I*c) - 12*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(24*I*d*x
+ 24*I*c) + 12*d*e^(22*I*d*x + 22*I*c) + 66*d*e^(20*I*d*x + 20*I*c) + 220*
d*e^(18*I*d*x + 18*I*c) + 495*d*e^(16*I*d*x + 16*I*c) + 792*d*e^(14*I*d*x
+ 14*I*c) + 924*d*e^(12*I*d*x + 12*I*c) + 792*d*e^(10*I*d*x + 10*I*c) + 49
5*d*e^(8*I*d*x + 8*I*c) + 220*d*e^(6*I*d*x + 6*I*c) + 66*d*e^(4*I*d*x + 4*
I*c) + 12*d*e^(2*I*d*x + 2*I*c) + d)
```

3.59.6 Sympy [F]

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\Bigg(&\int \left(-i\sec^8(c+dx)\right)\,dx\\ &+\int 5\tan(c+dx)\sec^8(c+dx)\,dx\\ &+\int \left(-10\tan^3(c+dx)\sec^8(c+dx)\right)\,dx\\ &+\int \tan^5(c+dx)\sec^8(c+dx)\,dx\\ &+\int 10i\tan^2(c+dx)\sec^8(c+dx)\,dx\\ &+\int \left(-5i\tan^4(c+dx)\sec^8(c+dx)\right)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x)**8, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**8, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**8, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**8, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**8, x))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{-165i\,a^5\tan(dx+c)^{12} - 900\,a^5\tan(dx+c)^{11} + 1386i\,a^5\tan(dx+c)^{10} - 1100\,a^5\tan(dx+c)^9 + 544}{}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5
*tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 3
960*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)
^5 - 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(
d*x + c)^2 - 1980*a^5*tan(d*x + c))/d
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{-165i\,a^5\tan(dx+c)^{12} - 900\,a^5\tan(dx+c)^{11} + 1386i\,a^5\tan(dx+c)^{10} - 1100\,a^5\tan(dx+c)^9 + 544}{}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5
*tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 3
960*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)
^5 - 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(
d*x + c)^2 - 1980*a^5*tan(d*x + c))/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.34

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\,(-\cos(c+d\,x)^{12}\,1749\mathrm{i}+2048\,\sin(c+d\,x)\,\cos(c+d\,x)^{11}+1024\,\sin(c+d\,x)\,\cos(c+d\,x)^9+768\,\sin}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^8,x)
```

output
```
(a^5*(900*cos(c + d*x)*sin(c + d*x) - 3400*cos(c + d*x)^3*sin(c + d*x) + 6
40*cos(c + d*x)^5*sin(c + d*x) + 768*cos(c + d*x)^7*sin(c + d*x) + 1024*co
s(c + d*x)^9*sin(c + d*x) + 2048*cos(c + d*x)^11*sin(c + d*x) - cos(c + d*
x)^2*2376i + cos(c + d*x)^4*3960i - cos(c + d*x)^12*1749i + 165i))/(1980*d
*cos(c + d*x)^12)
```

3.60 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.60.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^8}{2a^3d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}$$

output
```
-1/2*I*(a+I*a*tan(d*x+c))^8/a^3/d+4/9*I*(a+I*a*tan(d*x+c))^9/a^4/d-1/10*I*
(a+I*a*tan(d*x+c))^10/a^5/d
```

3.60.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sec^{10}(c+dx)(5+23\cos(2(c+dx))-22i\sin(2(c+dx)))(-i\cos(8(c+dx))+\sin(8(c+dx)))}{180d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]^10*(5 + 23*Cos[2*(c + d*x)] - (22*I)*Sin[2*(c + d*x)])*(
(-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]))/(180*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^7d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^9-4a(i\tan(c+dx)a+a)^8+4a^2(i\tan(c+dx)a+a)^7\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{1}{2}a^2(a+ia\tan(c+dx))^8+\frac{1}{10}(a+ia\tan(c+dx))^{10}-\frac{4}{9}a(a+ia\tan(c+dx))^9)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*((a^2*(a + I*a*Tan[c + d*x])^8)/2 - (4*a*(a + I*a*Tan[c + d*x])^9)/9
 + (a + I*a*Tan[c + d*x])^10/10))/(a^5*d)
```

3.60.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.60.4 Maple [A] (verified)

Time = 181.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$\frac{128 i a^{5} \left(120 \,{\mathrm e}^{14 i \left(d x +c \right)}+210 \,{\mathrm e}^{12 i \left(d x +c \right)}+252 \,{\mathrm e}^{10 i \left(d x +c \right)}+210 \,{\mathrm e}^{8 i \left(d x +c \right)}+120 \,{\mathrm e}^{6 i \left(d x +c \right)}+45 \,{\mathrm e}^{4 i \left(d x +c \right)}+10 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{45 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{10}}$
derivativedivides	$i a^{5} \left(\frac{\sin^{6}\left(d x +c \right)}{10 \cos\left(d x +c \right)^{10}}+\frac{\sin^{6}\left(d x +c \right)}{20 \cos\left(d x +c \right)^{8}}+\frac{\sin^{6}\left(d x +c \right)}{60 \cos\left(d x +c \right)^{6}}\right)+5 a^{5} \left(\frac{\sin^{5}\left(d x +c \right)}{9 \cos\left(d x +c \right)^{9}}+\frac{4 \left(\sin^{5}\left(d x +c \right)\right)}{63 \cos\left(d x +c \right)^{7}}+\frac{8 \left(\sin^{5}\left(d x +c \right)\right)}{315 \cos\left(d x +c \right)^{5}}\right)-10 i a^{5} ($
default	$i a^{5} \left(\frac{\sin^{6}\left(d x +c \right)}{10 \cos\left(d x +c \right)^{10}}+\frac{\sin^{6}\left(d x +c \right)}{20 \cos\left(d x +c \right)^{8}}+\frac{\sin^{6}\left(d x +c \right)}{60 \cos\left(d x +c \right)^{6}}\right)+5 a^{5} \left(\frac{\sin^{5}\left(d x +c \right)}{9 \cos\left(d x +c \right)^{9}}+\frac{4 \left(\sin^{5}\left(d x +c \right)\right)}{63 \cos\left(d x +c \right)^{7}}+\frac{8 \left(\sin^{5}\left(d x +c \right)\right)}{315 \cos\left(d x +c \right)^{5}}\right)-10 i a^{5} ($

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
128/45*I*a^5*(120*exp(14*I*(d*x+c))+210*exp(12*I*(d*x+c))+252*exp(10*I*(d*
x+c))+210*exp(8*I*(d*x+c))+120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp
(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^10
```

3.60.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(64) = 128$.

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.79

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{128\left(-120i\,a^5e^{(14i\,dx+14i\,c)}-210i\,a^5e^{(12i\,dx+12i\,c)}-252i\,a^5e^{(10i\,dx+10i\,c)}-210i\,a^5e^{(8i\,dx+}\right.}{45\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-128/45*(-120*I*a^5*e^(14*I*d*x + 14*I*c) - 210*I*a^5*e^(12*I*d*x + 12*I*c
) - 252*I*a^5*e^(10*I*d*x + 10*I*c) - 210*I*a^5*e^(8*I*d*x + 8*I*c) - 120*
I*a^5*e^(6*I*d*x + 6*I*c) - 45*I*a^5*e^(4*I*d*x + 4*I*c) - 10*I*a^5*e^(2*I
*d*x + 2*I*c) - I*a^5)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*
c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(1
2*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c
) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x
 + 2*I*c) + d)
```

3.60.6 Sympy [F]

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\Bigg(\int\left(-i\sec^6{(c+dx)}\right)\,dx\\ +\int 5\tan{(c+dx)}\sec^6{(c+dx)}\,dx\\ +\int\left(-10\tan^3{(c+dx)}\sec^6{(c+dx)}\right)\,dx\\ +\int\tan^5{(c+dx)}\sec^6{(c+dx)}\,dx\\ +\int 10i\tan^2{(c+dx)}\sec^6{(c+dx)}\,dx\\ +\int\left(-5i\tan^4{(c+dx)}\sec^6{(c+dx)}\right)\,dx\Bigg)\end{aligned}$$

input

```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

output

```
I*a**5*(Integral(-I*sec(c + d*x)**6, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**6, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**6, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**6, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**6, x))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{-9i\,a^5\tan\left(dx+c\right)^{10}-50\,a^5\tan\left(dx+c\right)^{9}+90i\,a^5\tan\left(dx+c\right)^{8}+210i\,a^5\tan\left(dx+c\right)^{6}+252\,a^5\tan (}{90\,d}$$

input

```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output

```
-1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
 + c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(
d*x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{-9i\,a^5\tan\left(dx+c\right)^{10}-50\,a^5\tan\left(dx+c\right)^{9}+90i\,a^5\tan\left(dx+c\right)^{8}+210i\,a^5\tan\left(dx+c\right)^{6}+252\,a^5\tan (}{90\,d}$$

input

```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output

```
-1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
 + c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(
d*x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$=\frac{a^5\,\sin\left(c+d\,x\right)\,\left(90\cos\left(c+d\,x\right)^9+\cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)\,225\mathrm{i}-240\cos\left(c+d\,x\right)^7\sin\left(c+d\,x\right)^2-2\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^6,x)
```

output
```
(a^5*sin(c + d*x)*(50*cos(c + d*x)*sin(c + d*x)^8 + cos(c + d*x)^8*sin(c +
 d*x)*225i + 90*cos(c + d*x)^9 + sin(c + d*x)^9*9i - cos(c + d*x)^2*sin(c
+ d*x)^7*90i - cos(c + d*x)^4*sin(c + d*x)^5*210i - 252*cos(c + d*x)^5*sin
(c + d*x)^4 - 240*cos(c + d*x)^7*sin(c + d*x)^2))/(90*d*cos(c + d*x)^10)
```

3.61 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.61.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx=-\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}+\frac{i(a+ia\tan(c+dx))^8}{8a^3d}$$

output
```
-2/7*I*(a+I*a*tan(d*x+c))^7/a^2/d+1/8*I*(a+I*a*tan(d*x+c))^8/a^3/d
```

3.61.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{ia^5(-i+\tan(c+dx))^7(9i+7\tan(c+dx))}{56d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/56)*a^5*(-I + Tan[c + d*x])^7*(9*I + 7*Tan[c + d*x]))/d
```

3.61.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^6d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^6-(i\tan(c+dx)a+a)^7\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{7}a(a+ia\tan(c+dx))^7-\frac{1}{8}(a+ia\tan(c+dx))^8\right)}{a^3d}$$

input

```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output

```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^7)/7 - (a + I*a*Tan[c + d*x])^8/8))/(a^
3*d)
```

3.61.3.1 Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.61.4 Maple [A] (verified)

Time = 66.86 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

method	result
risch	$\frac{32ia^5\left(28\,\mathrm{e}^{12i(dx+c)}+56\,\mathrm{e}^{10i(dx+c)}+70\,\mathrm{e}^{8i(dx+c)}+56\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{7d\left(\mathrm{e}^{2i(dx+c)}+1\right)^8}$
derivativedivides	$ia^5\left(\frac{\sin^6(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{24\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-10a^5\left(\right.$ d
default	$ia^5\left(\frac{\sin^6(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{24\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-10a^5\left(\right.$ d

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
32/7*I*a^5*(28*exp(12*I*(d*x+c))+56*exp(10*I*(d*x+c))+70*exp(8*I*(d*x+c))+
56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(
d*x+c))+1)^8
```

3.61.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.47

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{32\left(-28i\,a^5e^{(12i\,dx+12i\,c)}-56i\,a^5e^{(10i\,dx+10i\,c)}-70i\,a^5e^{(8i\,dx+8i\,c)}-56i\,a^5e^{(6i\,dx+6i\,c)}-28i\,a^5e^{(4i\,dx}\right.}{7\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c}\right.}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-32/7*(-28*I*a^5*e^(12*I*d*x + 12*I*c) - 56*I*a^5*e^(10*I*d*x + 10*I*c) -
70*I*a^5*e^(8*I*d*x + 8*I*c) - 56*I*a^5*e^(6*I*d*x + 6*I*c) - 28*I*a^5*e^(
4*I*d*x + 4*I*c) - 8*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(16*I*d*x + 1
6*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(
10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) +
 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

3.61.6 Sympy [F]

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\Bigg(&\int \left(-i\sec^4\left(c+dx\right)\right)\,dx\\
&+\int 5\tan\left(c+dx\right)\sec^4\left(c+dx\right)dx\\
&+\int \left(-10\tan^3\left(c+dx\right)\sec^4\left(c+dx\right)\right)\,dx\\
&+\int \tan^5\left(c+dx\right)\sec^4\left(c+dx\right)dx\\
&+\int 10i\tan^2\left(c+dx\right)\sec^4\left(c+dx\right)dx\\
&+\int \left(-5i\tan^4\left(c+dx\right)\sec^4\left(c+dx\right)\right)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x)**4, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**4, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**4, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**4, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**4, x))
```

3.61.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{-7i\,a^5\tan{(dx+c)}^8 - 40\,a^5\tan{(dx+c)}^7 + 84i\,a^5\tan{(dx+c)}^6 + 56\,a^5\tan{(dx+c)}^5 + 70i\,a^5\tan{(dx -}}{56\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x
+ c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x
 + c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

3.61.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(43) = 86$.

Time = 0.78 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{-7i\,a^5\tan{(dx+c)}^8 - 40\,a^5\tan{(dx+c)}^7 + 84i\,a^5\tan{(dx+c)}^6 + 56\,a^5\tan{(dx+c)}^5 + 70i\,a^5\tan{(dx -}}{56\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x
+ c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x
 + c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\,\sin\left(c+d\,x\right)\,\left(56\cos\left(c+d\,x\right)^7+\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)\,140\mathrm{i}-168\cos\left(c+d\,x\right)^5\sin\left(c+d\,x\right)^2-}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^4,x)
```

output
```
(a^5*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c +
 d*x)*140i + 56*cos(c + d*x)^7 + sin(c + d*x)^7*7i - cos(c + d*x)^2*sin(c
+ d*x)^5*84i - 56*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d
*x)^3*70i - 168*cos(c + d*x)^5*sin(c + d*x)^2))/(56*d*cos(c + d*x)^8)
```

3.62 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.62.1 Optimal result 621
3.62.2 Mathematica [B] (verified) 621
3.62.3 Rubi [A] (verified) 622
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3.62.8 Giac [B] (verification not implemented) 625
3.62.9 Mupad [B] (verification not implemented) 625

3.62.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

output
```
-1/6*I*(a+I*a*tan(d*x+c))^6/a/d
```

3.62.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 72 vs. 2(27) = 54.

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\tan(c+dx)\left(6+15i\tan(c+dx)-20\tan^2(c+dx)-15i\tan^3(c+dx)+6\tan^4(c+dx)+i\tan^5(c+d\right.}{6d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Tan[c + d*x]*(6 + (15*I)*Tan[c + d*x] - 20*Tan[c + d*x]^2 - (15*I)*Ta
n[c + d*x]^3 + 6*Tan[c + d*x]^4 + I*Tan[c + d*x]^5))/(6*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (i\tan(c+dx)a+a)^5d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/6*I)*(a + I*a*Tan[c + d*x])^6)/(a*d)
```

3.62.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.62.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(23) = 46$.

Time = 20.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

method	result	size
risch	$\frac{32ia^5\left(6\,\mathrm{e}^{10i(dx+c)}+15\,\mathrm{e}^{8i(dx+c)}+20\,\mathrm{e}^{6i(dx+c)}+15\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$	80
derivativedivides	$\frac{\frac{ia^5\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+\frac{a^5\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}-\frac{5ia^5\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}-\frac{10a^5\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{5ia^5}{2\cos(dx+c)^2}+a^5\tan(dx+c)}{d}$	115
default	$\frac{\frac{ia^5\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+\frac{a^5\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}-\frac{5ia^5\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}-\frac{10a^5\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{5ia^5}{2\cos(dx+c)^2}+a^5\tan(dx+c)}{d}$	115

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
32/3*I*a^5*(6*exp(10*I*(d*x+c))+15*exp(8*I*(d*x+c))+20*exp(6*I*(d*x+c))+15
*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^6
```

3.62.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{32\left(-6i\,a^5e^{(10i\,dx+10i\,c)}-15i\,a^5e^{(8i\,dx+8i\,c)}-20i\,a^5e^{(6i\,dx+6i\,c)}-15i\,a^5e^{(4i\,dx+4i\,c)}-6i\,a^5e^{(2i\,dx+2i\,c)}-i\,a^5\right)}{3\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right.}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-32/3*(-6*I*a^5*e^(10*I*d*x + 10*I*c) - 15*I*a^5*e^(8*I*d*x + 8*I*c) - 20*
I*a^5*e^(6*I*d*x + 6*I*c) - 15*I*a^5*e^(4*I*d*x + 4*I*c) - 6*I*a^5*e^(2*I*
d*x + 2*I*c) - I*a^5)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c)
 + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x +
 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

3.62.6 Sympy [F]

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\biggl(\int \left(-i\sec^2{(c+dx)}\right)\,dx \\ +\int 5\tan{(c+dx)}\sec^2{(c+dx)}\,dx \\ +\int \left(-10\tan^3{(c+dx)}\sec^2{(c+dx)}\right)\,dx \\ +\int \tan^5{(c+dx)}\sec^2{(c+dx)}\,dx \\ +\int 10i\tan^2{(c+dx)}\sec^2{(c+dx)}\,dx \\ +\int \left(-5i\tan^4{(c+dx)}\sec^2{(c+dx)}\right)\,dx\biggr)\end{aligned}$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x)**2, x) + Integral(5*tan(c + d*x)*sec(c +
d*x)**2, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(
tan(c + d*x)**5*sec(c + d*x)**2, x) + Integral(10*I*tan(c + d*x)**2*sec(c
+ d*x)**2, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**2, x))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,(i\,a\tan{(dx+c)}+a)^6}{6\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/6*I*(I*a*tan(d*x + c) + a)^6/(a*d)
```

3.62.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(21) = 42$.

Time = 0.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{-i\,a^5\tan(dx+c)^6 - 6\,a^5\tan(dx+c)^5 + 15i\,a^5\tan(dx+c)^4 + 20\,a^5\tan(dx+c)^3 - 15i\,a^5\tan(dx+c}{6\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/6*(-I*a^5*tan(d*x + c)^6 - 6*a^5*tan(d*x + c)^5 + 15*I*a^5*tan(d*x + c)
^4 + 20*a^5*tan(d*x + c)^3 - 15*I*a^5*tan(d*x + c)^2 - 6*a^5*tan(d*x + c))
/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.22

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{a^5\,\sin(c+d\,x)\,\big(6\cos(c+d\,x)^5 + \cos(c+d\,x)^4\,\sin(c+d\,x)\,15\mathrm{i} - 20\cos(c+d\,x)^3\sin(c+d\,x)^2 - \cos(c}{6\,d\cos(c+d\,x)^6}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^2,x)
```

output
```
(a^5*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c +
d*x)*15i + 6*cos(c + d*x)^5 + sin(c + d*x)^5*1i - cos(c + d*x)^2*sin(c + d
*x)^3*15i - 20*cos(c + d*x)^3*sin(c + d*x)^2))/(6*d*cos(c + d*x)^6)
```

3.63 $\int (a+ia\tan(c+dx))^5\,dx$

3.63.1 Optimal result 626
3.63.2 Mathematica [A] (verified) 626
3.63.3 Rubi [A] (verified) 627
3.63.4 Maple [A] (verified) 629
3.63.5 Fricas [A] (verification not implemented) 630
3.63.6 Sympy [A] (verification not implemented) 630
3.63.7 Maxima [A] (verification not implemented) 631
3.63.8 Giac [B] (verification not implemented) 631
3.63.9 Mupad [B] (verification not implemented) 632

3.63.1 Optimal result

Integrand size = 15, antiderivative size = 117

$$\begin{aligned}\int (a+ia\tan(c+dx))^5\,dx = 16a^5x &- \frac{16ia^5\log(\cos(c+dx))}{d} \\ &- \frac{8a^5\tan(c+dx)}{d} + \frac{2ia^2(a+ia\tan(c+dx))^3}{3d} \\ &+ \frac{ia(a+ia\tan(c+dx))^4}{4d} + \frac{2ia(a^2+ia^2\tan(c+dx))^2}{d}\end{aligned}$$

output
```
16*a^5*x-16*I*a^5*ln(cos(d*x+c))/d-8*a^5*tan(d*x+c)/d+2/3*I*a^2*(a+I*a*tan
(d*x+c))^3/d+1/4*I*a*(a+I*a*tan(d*x+c))^4/d+2*I*a*(a^2+I*a^2*tan(d*x+c))^2
/d
```

3.63.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int (a+ia\tan(c+dx))^5\,dx \\ = \frac{a^5(35i+192i\log(i+\tan(c+dx))-180\tan(c+dx)-66i\tan^2(c+dx)+20\tan^3(c+dx)+3i\tan^4(c+}{12d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*(35*I + (192*I)*Log[I + Tan[c + d*x]] - 180*Tan[c + d*x] - (66*I)*Tan
[c + d*x]^2 + 20*Tan[c + d*x]^3 + (3*I)*Tan[c + d*x]^4))/(12*d)
```

3.63.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3959, 3042, 3959, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3959}$$

$$2a\int (i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\int (i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3959}$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3959}$$

$$2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\left(2a\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3958}$$

$$2a\Bigg(2a\Bigg(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\Bigg)+\frac{ia(a+ia\tan(c+dx)}{3d}\quad \frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$2a\Bigg(2a\Bigg(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\Bigg)+\frac{ia(a+ia\tan(c+dx)}{3d}\quad \frac{ia(a+ia\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3956}$$

$$2a\Bigg(2a\Bigg(2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\Bigg)+\frac{ia(a+ia\tan(c+d}{3d}\quad \frac{ia(a+ia\tan(c+dx))^4}{4d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/4)*a*(a + I*a*Tan[c + d*x])^4)/d + 2*a*(((I/3)*a*(a + I*a*Tan[c + d*x]
)^3)/d + 2*a*(((I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)
*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d)))
```

3.63.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

3.63.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{a^5\left(-15\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{5\left(\tan^3(dx+c)\right)}{3}-\frac{11i\left(\tan^2(dx+c)\right)}{2}+8i\ln(1+\tan^2(dx+c))+16\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^5\left(-15\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{5\left(\tan^3(dx+c)\right)}{3}-\frac{11i\left(\tan^2(dx+c)\right)}{2}+8i\ln(1+\tan^2(dx+c))+16\arctan(\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{3ia^5(\tan^4(dx+c))-66ia^5(\tan^2(dx+c))+20(\tan^3(dx+c))a^5+96ia^5\ln(1+\tan^2(dx+c))+192a^5xd-180a^5\tan(dx+c)}{12d}$
risch	$-\frac{32a^5c}{d}-\frac{4ia^5\left(48\,\mathrm{e}^{6i(dx+c)}+108\,\mathrm{e}^{4i(dx+c)}+88\,\mathrm{e}^{2i(dx+c)}+25\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{16ia^5\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$
norman	$16a^5x-\frac{15a^5\tan(dx+c)}{d}+\frac{5a^5(\tan^3(dx+c))}{3d}-\frac{11ia^5(\tan^2(dx+c))}{2d}+\frac{ia^5(\tan^4(dx+c))}{4d}+\frac{8ia^5\ln(1+\tan^2(dx}{d}$
parts	$a^5x-\frac{10ia^5\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{ia^5\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{5ia}{}$

input
```
int((a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^5*(-15*tan(d*x+c)+1/4*I*tan(d*x+c)^4+5/3*tan(d*x+c)^3-11/2*I*tan(d*x
+c)^2+8*I*ln(1+tan(d*x+c)^2)+16*arctan(tan(d*x+c)))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int (a+ia\tan(c+dx))^5\,dx = -\frac{4\,(48i\,a^5e^{(6i\,dx+6i\,c)} + 108i\,a^5e^{(4i\,dx+4i\,c)} + 88i\,a^5e^{(2i\,dx+2i\,c)} + 25i\,a^5 + 12\,(i\,a^5e^{(8i\,dx+8i\,c)} + 4i\,a^5e^{(6i\,dx+6i\,c)}}{3\,(de^{(8i\,dx+8i\,c)} + 4\,de^{(6i\,dx+6i\,c)} + 6\,de^{(4i\,dx+4i\,c)} + 4\,de^{(}}$$

input
```
integrate((a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-4/3*(48*I*a^5*e^(6*I*d*x + 6*I*c) + 108*I*a^5*e^(4*I*d*x + 4*I*c) + 88*I*
a^5*e^(2*I*d*x + 2*I*c) + 25*I*a^5 + 12*(I*a^5*e^(8*I*d*x + 8*I*c) + 4*I*a
^5*e^(6*I*d*x + 6*I*c) + 6*I*a^5*e^(4*I*d*x + 4*I*c) + 4*I*a^5*e^(2*I*d*x
+ 2*I*c) + I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int (a+ia\tan(c+dx))^5\,dx = -\frac{16ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-192ia^5e^{6ic}e^{6idx} - 432ia^5e^{4ic}e^{4idx} - 352ia^5e^{2ic}e^{2idx} - 100ia^5}{3de^{8ic}e^{8idx} + 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} + 12de^{2ic}e^{2idx} + 3d}$$

input
```
integrate((a+I*a*tan(d*x+c))**5,x)
```

output
```
-16*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-192*I*a**5*exp(6*I*c)*exp
(6*I*d*x) - 432*I*a**5*exp(4*I*c)*exp(4*I*d*x) - 352*I*a**5*exp(2*I*c)*exp
(2*I*d*x) - 100*I*a**5)/(3*d*exp(8*I*c)*exp(8*I*d*x) + 12*d*exp(6*I*c)*exp
(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) + 12*d*exp(2*I*c)*exp(2*I*d*x) +
3*d)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

$$\int (a+ia\tan(c+dx))^5\,dx = a^5x + \frac{5\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^5}{3\,d} + \frac{10\,(dx+c-\tan(dx+c))a^5}{d} + \frac{i\,a^5\left(\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 2\,\log\left(\sin(dx+c)^2-1\right)\right)}{4\,d} + \frac{5i\,a^5\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin(dx+c)^2-1\right)\right)}{d} + \frac{5i\,a^5\log(\sec(dx+c))}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
a^5*x + 5/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^5/d + 10*(d*
x + c - tan(d*x + c))*a^5/d + 1/4*I*a^5*((4*sin(d*x + c)^2 - 3)/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*(1/
(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*log(sec(d*x +
c))/d
```

3.63.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(99) = 198.

Time = 0.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.90

$$\int (a+ia\tan(c+dx))^5\,dx = -\frac{4\,(12i\,a^5e^{(8i\,dx+8i\,c)}\log(e^{(2i\,dx+2i\,c)}+1) + 48i\,a^5e^{(6i\,dx+6i\,c)}\log(e^{(2i\,dx+2i\,c)}+1) + 72i\,a^5e^{(4i\,dx+4i\,c)}\log(e^{}}{3\,(de^{(8i\,dx+}}$$

input
```
integrate((a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-4/3*(12*I*a^5*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 48*I*a^5
*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 72*I*a^5*e^(4*I*d*x +
4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 48*I*a^5*e^(2*I*d*x + 2*I*c)*log(e^(
2*I*d*x + 2*I*c) + 1) + 48*I*a^5*e^(6*I*d*x + 6*I*c) + 108*I*a^5*e^(4*I*d*
x + 4*I*c) + 88*I*a^5*e^(2*I*d*x + 2*I*c) + 12*I*a^5*log(e^(2*I*d*x + 2*I*
c) + 1) + 25*I*a^5)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d
*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int (a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,16\mathrm{i}-15\,a^5\,\tan(c+d\,x)-\frac{a^5\,\tan(c+d\,x)^2\,11\mathrm{i}}{2}+\frac{5\,a^5\,\tan(c+d\,x)^3}{3}+\frac{a^5\,\tan(c+d\,x)^4\,1\mathrm{i}}{4}}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*log(tan(c + d*x) + 1i)*16i - 15*a^5*tan(c + d*x) - (a^5*tan(c + d*x)^
2*11i)/2 + (5*a^5*tan(c + d*x)^3)/3 + (a^5*tan(c + d*x)^4*1i)/4)/d
```

3.64 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.64.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -12a^5x+\frac{12ia^5\log(\cos(c+dx))}{d}+\frac{5a^5\tan(c+dx)}{d}+\frac{ia^5\tan^2(c+dx)}{2d}-\frac{8ia^6}{d(a-ia\tan(c+dx))}$$

output
```
-12*a^5*x+12*I*a^5*ln(cos(d*x+c))/d+5*a^5*tan(d*x+c)/d+1/2*I*a^5*tan(d*x+c
)^2/d-8*I*a^6/d/(a-I*a*tan(d*x+c))
```

3.64.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\left(24\log(i+\tan(c+dx))+10i\tan(c+dx)-\tan^2(c+dx)+\frac{16i}{i+\tan(c+dx)}\right)}{2d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/2*I)*a^5*(24*Log[I + Tan[c + d*x]] + (10*I)*Tan[c + d*x] - Tan[c + d*
x]^2 + (16*I)/(I + Tan[c + d*x])))/d
```

3.64.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \color{blue}{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \color{blue}{3968}$$

$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^3}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \color{blue}{49}$$

$$-\frac{ia^3\int \left(\frac{8a^3}{(a-ia\tan(c+dx))^2}-\frac{12a^2}{a-ia\tan(c+dx)}+i\tan(c+dx)a+5a\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \color{blue}{2009}$$

$$-\frac{ia^3\left(\frac{8a^3}{a-ia\tan(c+dx)}-\frac{1}{2}a^2\tan^2(c+dx)+5ia^2\tan(c+dx)+12a^2\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^3*(12*a^2*Log[a - I*a*Tan[c + d*x]] + (5*I)*a^2*Tan[c + d*x] - (a^
2*Tan[c + d*x]^2)/2 + (8*a^3)/(a - I*a*Tan[c + d*x])))/d
```

3.64.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.64.4 Maple [A] (verified)

Time = 14.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{4 i a^{5} {\mathrm e}^{2 i \left(d x +c \right)}}{d}+\frac{24 a^{5} c}{d}+\frac{2 i a^{5} \left(6 \,{\mathrm e}^{2 i \left(d x +c \right)}+5\right)}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{2}}+\frac{12 i a^{5} \ln \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{d}$
derivativedivides	$i a^{5} \left(\frac{\sin^{6}\left(d x +c \right)}{2 \cos \left(d x +c \right)^{2}}+\frac{\left(\sin^{4}\left(d x +c \right)\right)}{2}+\sin^{2}\left(d x +c \right)+2 \ln \left(\cos \left(d x +c \right)\right)\right)+5 a^{5} \Bigg(\frac{\sin^{5}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(\sin^{3}\left(d x +c \right)+\frac{3 \sin \left(d x +c \right)}{2}\right) \cos(dx+$
default	$i a^{5} \left(\frac{\sin^{6}\left(d x +c \right)}{2 \cos \left(d x +c \right)^{2}}+\frac{\left(\sin^{4}\left(d x +c \right)\right)}{2}+\sin^{2}\left(d x +c \right)+2 \ln \left(\cos \left(d x +c \right)\right)\right)+5 a^{5} \Bigg(\frac{\sin^{5}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(\sin^{3}\left(d x +c \right)+\frac{3 \sin \left(d x +c \right)}{2}\right) \cos(dx+$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-4*I/d*a^5*exp(2*I*(d*x+c))+24/d*a^5*c+2*I*a^5*(6*exp(2*I*(d*x+c))+5)/d/(e
xp(2*I*(d*x+c))+1)^2+12*I/d*a^5*ln(exp(2*I*(d*x+c))+1)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2\,(2i\,a^5e^{(6i\,dx+6i\,c)}+4i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}-5i\,a^5+6\,(-i\,a^5e^{(4i\,dx+4i\,c)}-2i\,a^5e^{(2i\,dx+2i\,c)}-i\,a}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-2*(2*I*a^5*e^(6*I*d*x + 6*I*c) + 4*I*a^5*e^(4*I*d*x + 4*I*c) - 4*I*a^5*e^
(2*I*d*x + 2*I*c) - 5*I*a^5 + 6*(-I*a^5*e^(4*I*d*x + 4*I*c) - 2*I*a^5*e^(2
*I*d*x + 2*I*c) - I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I
*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = {} & \frac{12ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} \\ & + \frac{12ia^5e^{2ic}e^{2idx}+10ia^5}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} \\ & + \begin{cases} -\frac{4ia^5e^{2ic}e^{2idx}}{d} & \text{for } d \neq 0 \\ 8a^5xe^{2ic} & \text{otherwise} \end{cases}\end{aligned}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

output
```
12*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (12*I*a**5*exp(2*I*c)*exp(2*
I*d*x) + 10*I*a**5)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*
x) + d) + Piecewise((-4*I*a**5*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (8*a*
*5*x*exp(2*I*c), True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-i\,a^5\tan\left(dx+c\right)^2 + 24\left(dx+c\right)a^5 + 12i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right) - 10\,a^5\tan\left(dx+c\right) - \frac{16\,(a^5\tan(dx+c}{\tan(dx+c)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/2*(-I*a^5*tan(d*x + c)^2 + 24*(d*x + c)*a^5 + 12*I*a^5*log(tan(d*x + c)
^2 + 1) - 10*a^5*tan(d*x + c) - 16*(a^5*tan(d*x + c) - I*a^5)/(tan(d*x + c
)^2 + 1))/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.76

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2\left(-6i\,a^5e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 12i\,a^5e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + 2i\,a^5e^{(6i\,dx+6i\,c)} + 4i\,a^5}{de^{(4i\,dx+4i\,c)} + 2\,de^{(2i\,dx+2i\,c)} + d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-2*(-6*I*a^5*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^5*e
^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a^5*e^(6*I*d*x + 6*I
*c) + 4*I*a^5*e^(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c) - 6*I*a^5*
log(e^(2*I*d*x + 2*I*c) + 1) - 5*I*a^5)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*
I*d*x + 2*I*c) + d)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx &= \frac{8\,a^5}{d\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}\\ &\quad -\frac{a^5\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,12\mathrm{i}}{d}\\ &\quad +\frac{5\,a^5\tan(c+d\,x)}{d}+\frac{a^5\tan(c+d\,x)^2\,1\mathrm{i}}{2\,d}\end{aligned}$$

input

```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^5,x)
```

output

```
(8*a^5)/(d*(tan(c + d*x) + 1i)) - (a^5*log(tan(c + d*x) + 1i)*12i)/d + (5*
a^5*tan(c + d*x))/d + (a^5*tan(c + d*x)^2*1i)/(2*d)
```

3.65 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.65.1 Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = a^5x - \frac{ia^5\log(\cos(c+dx))}{d} - \frac{2ia^7}{d(a-ia\tan(c+dx))^2} + \frac{4ia^6}{d(a-ia\tan(c+dx))}$$

output

```
a^5*x-I*a^5*ln(cos(d*x+c))/d-2*I*a^7/d/(a-I*a*tan(d*x+c))^2+4*I*a^6/d/(a-I
*a*tan(d*x+c))
```

3.65.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\left(-\log(i+\tan(c+dx))+\frac{2-4i\tan(c+dx)}{(i+\tan(c+dx))^2}\right)}{d}$$

input

```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output

```
((-I)*a^5*(-Log[I + Tan[c + d*x]] + (2 - (4*I)*Tan[c + d*x])/(I + Tan[c +
d*x])^2))/d
```

3.65.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{(i\tan(c+dx)a+a)^2}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^5\int \left(\frac{4a^2}{(a-ia\tan(c+dx))^3}-\frac{4a}{(a-ia\tan(c+dx))^2}+\frac{1}{a-ia\tan(c+dx)}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{2a^2}{(a-ia\tan(c+dx))^2}-\frac{4a}{a-ia\tan(c+dx)}-\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^5*(-Log[a - I*a*Tan[c + d*x]] + (2*a^2)/(a - I*a*Tan[c + d*x])^2 -
 (4*a)/(a - I*a*Tan[c + d*x])))/d
```

3.65.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.65.4 Maple [A] (verified)

Time = 48.94 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{i a^5 \mathrm{e}^{4 i (d x+c)}}{2 d}+\frac{i a^5 \mathrm{e}^{2 i (d x+c)}}{d}-\frac{2 a^5 c}{d}-\frac{i a^5 \ln\left(\mathrm{e}^{2 i (d x+c)}+1\right)}{d}$
derivativedivides	$i a^5\left(-\frac{\left(\sin^4(d x+c)\right)}{4}-\frac{\left(\sin^2(d x+c)\right)}{2}-\ln(\cos(d x+c))\right)+5 a^5\left(-\frac{\left(\sin^3(d x+c)+\frac{3 \sin(d x+c)}{2}\right) \cos(d x+c)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)-5 i a^5 (\sin$
default	$i a^5\left(-\frac{\left(\sin^4(d x+c)\right)}{4}-\frac{\left(\sin^2(d x+c)\right)}{2}-\ln(\cos(d x+c))\right)+5 a^5\left(-\frac{\left(\sin^3(d x+c)+\frac{3 \sin(d x+c)}{2}\right) \cos(d x+c)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)-5 i a^5 (\sin$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d*a^5*exp(4*I*(d*x+c))+I/d*a^5*exp(2*I*(d*x+c))-2/d*a^5*c-I/d*a^5*l
n(exp(2*I*(d*x+c))+1)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-i\,a^5e^{(4i\,dx+4i\,c)}+2i\,a^5e^{(2i\,dx+2i\,c)}-2i\,a^5\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{2\,d}$$

input

```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output

```
1/2*(-I*a^5*e^(4*I*d*x + 4*I*c) + 2*I*a^5*e^(2*I*d*x + 2*I*c) - 2*I*a^5*lo
g(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \begin{cases} \frac{-ia^5de^{4ic}e^{4idx}+2ia^5de^{2ic}e^{2idx}}{2d^2} & \text{for } d^2\neq 0 \\ x\left(2a^5e^{4ic}-2a^5e^{2ic}\right) & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

output

```
-I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise(((-I*a**5*d*exp(4*I*
c)*exp(4*I*d*x) + 2*I*a**5*d*exp(2*I*c)*exp(2*I*d*x))/(2*d**2), Ne(d**2, 0
)), (x*(2*a**5*exp(4*I*c) - 2*a**5*exp(2*I*c)), True))
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{2\,(dx+c)a^5+i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{4\left(2\,a^5\tan(dx+c)^3-3i\,a^5\tan(dx+c)^2-i\,a^5\right)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/2*(2*(d*x + c)*a^5 + I*a^5*log(tan(d*x + c)^2 + 1) - 4*(2*a^5*tan(d*x +
c)^3 - 3*I*a^5*tan(d*x + c)^2 - I*a^5)/(tan(d*x + c)^4 + 2*tan(d*x + c)^2
+ 1))/d
```

3.65.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(63) = 126$.

Time = 0.70 (sec) , antiderivative size = 450, normalized size of antiderivative = 6.16

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{2i\,a^5e^{(16i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+16i\,a^5e^{(14i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+56i\,a^5e^{(12i\,dx+4i\,c)}\log\left(e^{(2i}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/2*(2*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 16*I*a^5
*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 56*I*a^5*e^(12*I*d*x
+ 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 112*I*a^5*e^(10*I*d*x + 2*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) + 112*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x +
 2*I*c) + 1) + 56*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) +
 16*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 140*I*a^5*e^(
8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a^5*e^(-8*I*c)*log(e^(2*I*d*x
+ 2*I*c) + 1) + I*a^5*e^(20*I*d*x + 12*I*c) + 6*I*a^5*e^(18*I*d*x + 10*I*c
) + 12*I*a^5*e^(16*I*d*x + 8*I*c) - 42*I*a^5*e^(12*I*d*x + 4*I*c) - 84*I*a
^5*e^(10*I*d*x + 2*I*c) - 48*I*a^5*e^(6*I*d*x - 2*I*c) - 15*I*a^5*e^(4*I*d
*x - 4*I*c) - 2*I*a^5*e^(2*I*d*x - 6*I*c) - 84*I*a^5*e^(8*I*d*x))/(d*e^(16
*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 5
6*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*
I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

3.65.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d} - \frac{4\,a^5\tan(c+d\,x)+a^5\,2\mathrm{i}}{d\,\left(\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*log(tan(c + d*x) + 1i)*1i)/d - (4*a^5*tan(c + d*x) + a^5*2i)/(d*(tan(
c + d*x)*2i + tan(c + d*x)^2 - 1))
```

3.66 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.66.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx=-\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}+\frac{ia^7}{2d(a-ia\tan(c+dx))^2}$$

output
```
-2/3*I*a^8/d/(a-I*a*tan(d*x+c))^3+1/2*I*a^7/d/(a-I*a*tan(d*x+c))^2
```

3.66.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx=-\frac{ia^5(-i+3\tan(c+dx))}{6d(i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/6*I)*a^5*(-I + 3*Tan[c + d*x]))/(d*(I + Tan[c + d*x])^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{i\tan(c+dx)a+a}{(a-ia\tan(c+dx))^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{ia^7\int \left(\frac{2a}{(a-ia\tan(c+dx))^4}-\frac{1}{(a-ia\tan(c+dx))^3}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^7\left(\frac{2a}{3(a-ia\tan(c+dx))^3}-\frac{1}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

input

```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

output

```
((-I)*a^7*((2*a)/(3*(a - I*a*Tan[c + d*x])^3) - 1/(2*(a - I*a*Tan[c + d*x]
)^2)))/d
```

3.66.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.66.4 Maple [A] (verified)

Time = 135.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{i a^{5} {\mathrm e}^{6 i \left(d x +c \right)}}{12 d}-\frac{i a^{5} {\mathrm e}^{4 i \left(d x +c \right)}}{8 d}$
derivativedivides	$\frac{i a^{5} \left(\sin^{6}\left(d x +c \right)\right)}{6}+5 a^{5} \left(-\frac{\left(\sin^{3}\left(d x +c \right)\right) \left(\cos^{3}\left(d x +c \right)\right)}{6}-\frac{\left(\cos^{3}\left(d x +c \right)\right) \sin \left(d x +c \right)}{8}+\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{16}+\frac{d x}{16}+\frac{c}{16}\right)-10 i a^{5} \Bigg(-$
default	$\frac{i a^{5} \left(\sin^{6}\left(d x +c \right)\right)}{6}+5 a^{5} \left(-\frac{\left(\sin^{3}\left(d x +c \right)\right) \left(\cos^{3}\left(d x +c \right)\right)}{6}-\frac{\left(\cos^{3}\left(d x +c \right)\right) \sin \left(d x +c \right)}{8}+\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{16}+\frac{d x}{16}+\frac{c}{16}\right)-10 i a^{5} \Bigg(-$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d*a^5*exp(6*I*(d*x+c))-1/8*I/d*a^5*exp(4*I*(d*x+c))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-2i\,a^5e^{(6i\,dx+6i\,c)} - 3i\,a^5e^{(4i\,dx+4i\,c)}}{24\,d}$$

input

```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output

```
1/24*(-2*I*a^5*e^(6*I*d*x + 6*I*c) - 3*I*a^5*e^(4*I*d*x + 4*I*c))/d
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} \frac{-8ia^5de^{6ic}e^{6idx}-12ia^5de^{4ic}e^{4idx}}{96d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^5e^{6ic}}{2}+\frac{a^5e^{4ic}}{2}\right) & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

output

```
Piecewise(((-8*I*a**5*d*exp(6*I*c)*exp(6*I*d*x) - 12*I*a**5*d*exp(4*I*c)*e
xp(4*I*d*x))/(96*d**2), Ne(d**2, 0)), (x*(a**5*exp(6*I*c)/2 + a**5*exp(4*I
*c)/2), True))
```

3.66.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(43) = 86.

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{3i\,a^5\tan\left(dx+c\right)^4 + 10\,a^5\tan\left(dx+c\right)^3 - 12i\,a^5\tan\left(dx+c\right)^2 - 6\,a^5\tan\left(dx+c\right) + i\,a^5}{6\left(\tan\left(dx+c\right)^6 + 3\,\tan\left(dx+c\right)^4 + 3\,\tan\left(dx+c\right)^2 + 1\right)d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/6*(3*I*a^5*tan(d*x + c)^4 + 10*a^5*tan(d*x + c)^3 - 12*I*a^5*tan(d*x +
c)^2 - 6*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3
*tan(d*x + c)^2 + 1)*d)
```

3.66.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(43) = 86$.

Time = 0.70 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.40

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{2i\,a^5e^{(18i\,dx+12i\,c)}+15i\,a^5e^{(16i\,dx+10i\,c)}+48i\,a^5e^{(14i\,dx+8i\,c)}+85i\,a^5e^{(12i\,dx+6i\,c)}+90i\,a^5e^{(10i\,dx+4i\,c)}+57i\,a}{24\,(de^{(12i\,dx+6i\,c)}+6\,de^{(10i\,dx+4i\,c)}+15\,de^{(8i\,dx+2i\,c)}+15\,de^{(4i\,dx-2i\,c)}+6\,de^{(2i\,dx-4i\,c)}+}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/24*(2*I*a^5*e^(18*I*d*x + 12*I*c) + 15*I*a^5*e^(16*I*d*x + 10*I*c) + 48
*I*a^5*e^(14*I*d*x + 8*I*c) + 85*I*a^5*e^(12*I*d*x + 6*I*c) + 90*I*a^5*e^(
10*I*d*x + 4*I*c) + 57*I*a^5*e^(8*I*d*x + 2*I*c) + 3*I*a^5*e^(4*I*d*x - 2*
I*c) + 20*I*a^5*e^(6*I*d*x))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x + 4
*I*c) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*d
*x - 4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))
```

3.66.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\,\left(1+\tan(c+d\,x)\ 3\mathrm{i}\right)}{6\,d\,\left(-{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,3\mathrm{i}+3\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(a^5*(tan(c + d*x)*3i + 1))/(6*d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan
(c + d*x)^3 + 1i))
```

3.66. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.67 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.67.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

output
```
-1/4*I*a^9/d/(a-I*a*tan(d*x+c))^4
```

3.67.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5}{4d(i+\tan(c+dx))^4}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/4*I)*a^5)/(d*(I + Tan[c + d*x])^4)
```

3.67.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^9\int \frac{1}{(a-ia\tan(c+dx))^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/4*I)*a^9)/(d*(a - I*a*Tan[c + d*x])^4)
```

3.67.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.67.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(23) = 46$.

Time = 1.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 11.15

$$ia^5\left(-\frac{(\sin^4(dx+c))(\cos^4(dx+c))}{8}-\frac{(\cos^4(dx+c))(\sin^2(dx+c))}{12}-\frac{(\cos^4(dx+c))}{24}\right)+5a^5\left(-\frac{(\sin^3(dx+c))(\cos^5(dx+c))}{8}-\frac{(\cos^5(dx+c))}{16}\right.$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*cos(d*x+c)^4*sin(d*x+c)^2-
1/24*cos(d*x+c)^4)+5*a^5*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*cos(d*x+c)^5
*sin(d*x+c)+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*
c)-10*I*a^5*(-1/8*cos(d*x+c)^6*sin(d*x+c)^2-1/24*cos(d*x+c)^6)-10*a^5*(-1/
8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x
+c))*sin(d*x+c)+5/128*d*x+5/128*c)-5/8*I*a^5*cos(d*x+c)^8+a^5*(1/8*(cos(d*
x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35
/128*d*x+35/128*c))
```

3.67.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-i\,a^5e^{(8i\,dx+8i\,c)}-4i\,a^5e^{(6i\,dx+6i\,c)}-6i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}}{64\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/64*(-I*a^5*e^(8*I*d*x + 8*I*c) - 4*I*a^5*e^(6*I*d*x + 6*I*c) - 6*I*a^5*e
^(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

3.67.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 6.00

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} \frac{-8192ia^5d^3e^{8ic}e^{8idx}-32768ia^5d^3e^{6ic}e^{6idx}-49152ia^5d^3e^{4ic}e^{4idx}-32768ia^5d^3e^{2ic}e^{2idx}}{524288d^4} & \text{for } d^4\neq 0 \\ x\left(\frac{a^5e^{8ic}}{8}+\frac{3a^5e^{6ic}}{8}+\frac{3a^5e^{4ic}}{8}+\frac{a^5e^{2ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-8192*I*a**5*d**3*exp(8*I*c)*exp(8*I*d*x) - 32768*I*a**5*d**3*
exp(6*I*c)*exp(6*I*d*x) - 49152*I*a**5*d**3*exp(4*I*c)*exp(4*I*d*x) - 3276
8*I*a**5*d**3*exp(2*I*c)*exp(2*I*d*x))/(524288*d**4), Ne(d**4, 0)), (x*(a*
*5*exp(8*I*c)/8 + 3*a**5*exp(6*I*c)/8 + 3*a**5*exp(4*I*c)/8 + a**5*exp(2*I
*c)/8), True))
```

3.67.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(21) = 42$.

Time = 0.59 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.81

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,a^5\tan(dx+c)^4+4\,a^5\tan(dx+c)^3-6i\,a^5\tan(dx+c)^2-4\,a^5\tan(dx+c)+i\,a^5}{4\left(\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1\right)d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/4*(I*a^5*tan(d*x + c)^4 + 4*a^5*tan(d*x + c)^3 - 6*I*a^5*tan(d*x + c)^2
 - 4*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan
(d*x + c)^4 + 4*tan(d*x + c)^2 + 1)*d)
```

3.67.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(21) = 42$.

Time = 0.78 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.89

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{i\,a^5e^{(24i\,dx+16i\,c)} + 12i\,a^5e^{(22i\,dx+14i\,c)} + 66i\,a^5e^{(20i\,dx+12i\,c)} + 220i\,a^5e^{(18i\,dx+10i\,c)} + 494i\,a^5e^{(16i\,dx+8i\,c)} + 78}{64\,(de^{(16i\,dx+8i\,c)} + 8\,de^{(14i\,dx+6i\,c)} + 28\,de^{(12i\,dx+4i\,c)} + 56\,de}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/64*(I*a^5*e^(24*I*d*x + 16*I*c) + 12*I*a^5*e^(22*I*d*x + 14*I*c) + 66*I
*a^5*e^(20*I*d*x + 12*I*c) + 220*I*a^5*e^(18*I*d*x + 10*I*c) + 494*I*a^5*e
^(16*I*d*x + 8*I*c) + 784*I*a^5*e^(14*I*d*x + 6*I*c) + 896*I*a^5*e^(12*I*d
*x + 4*I*c) + 736*I*a^5*e^(10*I*d*x + 2*I*c) + 164*I*a^5*e^(6*I*d*x - 2*I*
c) + 38*I*a^5*e^(4*I*d*x - 4*I*c) + 4*I*a^5*e^(2*I*d*x - 6*I*c) + 425*I*a^
5*e^(8*I*d*x))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e
^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c)
 + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) +
 d*e^(-8*I*c))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{\frac{a^5\cos(c+d\,x)^4\,1\mathrm{i}}{4} + a^5\cos\left(c+d\,x\right)^6\left(\tan(c+d\,x)-2\mathrm{i}\right) - 2\,a^5\cos\left(c+d\,x\right)^8\left(\tan(c+d\,x)-\mathrm{i}\right)}{d}$$

input

```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^5,x)
```

output

```
-((a^5*cos(c + d*x)^4*1i)/4 + a^5*cos(c + d*x)^6*(tan(c + d*x) - 2i) - 2*a
^5*cos(c + d*x)^8*(tan(c + d*x) - 1i))/d
```

3.67. $\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.68 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.68.1 Optimal result

Integrand size = 24, antiderivative size = 144

$$\begin{aligned}\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx &= \frac{a^5x}{32}-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}\\ &\quad-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}\\ &\quad-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}\\ &\quad-\frac{ia^7}{32d(a-ia\tan(c+dx))^2}\\ &\quad-\frac{ia^6}{32d(a-ia\tan(c+dx))}\end{aligned}$$

output

```
1/32*a^5*x-1/10*I*a^10/d/(a-I*a*tan(d*x+c))^5-1/16*I*a^9/d/(a-I*a*tan(d*x+
c))^4-1/24*I*a^8/d/(a-I*a*tan(d*x+c))^3-1/32*I*a^7/d/(a-I*a*tan(d*x+c))^2-
1/32*I*a^6/d/(a-I*a*tan(d*x+c))
```

3.68.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\sec^5(c+dx)(500\cos(c+dx)+375\cos(3(c+dx))+149\cos(5(c+dx))-100i\sin(c+dx)-225i\sin(3}{3840d(i+\tan(c+}$$

input
```
Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]^5*(500*Cos[c + d*x] + 375*Cos[3*(c + d*x)] + 149*Cos[5*(
c + d*x)] - (100*I)*Sin[c + d*x] - (225*I)*Sin[3*(c + d*x)] - (125*I)*Sin[
5*(c + d*x)] + 120*ArcTan[Tan[c + d*x]]*(I*Cos[5*(c + d*x)] + Sin[5*(c + d
*x)])))/(3840*d*(I + Tan[c + d*x])^5)
```

3.68.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^{10}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{11}\int \frac{1}{(a-ia\tan(c+dx))^6(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^{11}\int \left(\frac{1}{2(a-ia\tan(c+dx))^6a}+\frac{1}{4(a-ia\tan(c+dx))^5a^2}+\frac{1}{8(a-ia\tan(c+dx))^4a^3}+\frac{1}{16(a-ia\tan(c+dx))^3a^4}+\frac{1}{32(\tan^2(c+dx)a^2+a^2)a^5}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{11}\Bigg(\frac{i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{32a^5(a-ia\tan(c+dx))}+\frac{1}{32a^4(a-ia\tan(c+dx))^2}+\frac{1}{24a^3(a-ia\tan(c+dx))^3}+\frac{1}{16a^2(a-ia\tan(c+dx))^4}+}{d}$$

input
```
Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^11*(((I/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(10*a*(a - I*a*Tan[c + d
*x])^5) + 1/(16*a^2*(a - I*a*Tan[c + d*x])^4) + 1/(24*a^3*(a - I*a*Tan[c +
 d*x])^3) + 1/(32*a^4*(a - I*a*Tan[c + d*x])^2) + 1/(32*a^5*(a - I*a*Tan[c
 + d*x]))))/d
```

3.68.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.68.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(122) = 244.

Time = 0.70 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.30

$$ia^5\Big(-\frac{(\sin^4(dx+c))(\cos^6(dx+c))}{10}-\frac{(\cos^6(dx+c))(\sin^2(dx+c))}{20}-\frac{(\cos^6(dx+c))}{60}\Big)+5a^5\Bigg(-\frac{(\sin^3(dx+c))(\cos^7(dx+c))}{10}-\frac{3\sin(dx+c)(}{8}$$

input
```
int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*cos(d*x+c)^6*sin(d*x+c)^2
-1/60*cos(d*x+c)^6)+5*a^5*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)
*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*
x+c)+3/256*d*x+3/256*c)-10*I*a^5*(-1/10*cos(d*x+c)^8*sin(d*x+c)^2-1/40*cos
(d*x+c)^8)-10*a^5*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*co
s(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256
*c)-1/2*I*a^5*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16
*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*
x+63/256*c))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{120\,a^5dx-12i\,a^5e^{(10i\,dx+10i\,c)}-75i\,a^5e^{(8i\,dx+8i\,c)}-200i\,a^5e^{(6i\,dx+6i\,c)}-300i\,a^5e^{(4i\,dx+4i\,c)}-300i\,a^5e^{(2i\,dx+2i}}{3840\,d}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/3840*(120*a^5*d*x - 12*I*a^5*e^(10*I*d*x + 10*I*c) - 75*I*a^5*e^(8*I*d*x
 + 8*I*c) - 200*I*a^5*e^(6*I*d*x + 6*I*c) - 300*I*a^5*e^(4*I*d*x + 4*I*c)
- 300*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5x}{32} + \begin{cases} \frac{-100663296ia^5d^4e^{10ic}e^{10idx}-629145600ia^5d^4e^{8ic}e^{8idx}-1677721600ia^5d^4e^{6ic}e^{6idx}-2516582400ia^5d^4e^{4ic}e^{4idx}-2516582400ia^5d^4e^{2ic}e^{2i}}{32212254720d^5} \\ x\left(\frac{a^5e^{10ic}}{32}+\frac{5a^5e^{8ic}}{32}+\frac{5a^5e^{6ic}}{16}+\frac{5a^5e^{4ic}}{16}+\frac{5a^5e^{2ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**5,x)
```

output
```
a**5*x/32 + Piecewise(((-100663296*I*a**5*d**4*exp(10*I*c)*exp(10*I*d*x) -
 629145600*I*a**5*d**4*exp(8*I*c)*exp(8*I*d*x) - 1677721600*I*a**5*d**4*ex
p(6*I*c)*exp(6*I*d*x) - 2516582400*I*a**5*d**4*exp(4*I*c)*exp(4*I*d*x) - 2
516582400*I*a**5*d**4*exp(2*I*c)*exp(2*I*d*x))/(32212254720*d**5), Ne(d**5
, 0)), (x*(a**5*exp(10*I*c)/32 + 5*a**5*exp(8*I*c)/32 + 5*a**5*exp(6*I*c)/
16 + 5*a**5*exp(4*I*c)/16 + 5*a**5*exp(2*I*c)/32), True))
```

3.68.7 Maxima [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.14

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{15\,(dx+c)a^5 + \frac{15\,a^5\tan(dx+c)^9+70\,a^5\tan(dx+c)^7+128\,a^5\tan(dx+c)^5-80i\,a^5\tan(dx+c)^4-230\,a^5\tan(dx+c)^3+560i\,a^5\tan(dx+c)^2+46}{\tan(dx+c)^{10}+5\,\tan(dx+c)^8+10\,\tan(dx+c)^6+10\,\tan(dx+c)^4+5\,\tan(dx+c)^2+1}}{480\,d}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/480*(15*(d*x + c)*a^5 + (15*a^5*tan(d*x + c)^9 + 70*a^5*tan(d*x + c)^7 +
 128*a^5*tan(d*x + c)^5 - 80*I*a^5*tan(d*x + c)^4 - 230*a^5*tan(d*x + c)^3
 + 560*I*a^5*tan(d*x + c)^2 + 465*a^5*tan(d*x + c) - 128*I*a^5)/(tan(d*x +
 c)^10 + 5*tan(d*x + c)^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(
d*x + c)^2 + 1))/d
```

3.68.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(112) = 224$.

Time = 0.90 (sec) , antiderivative size = 857, normalized size of antiderivative = 5.95

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output

```
1/15360*(480*a^5*d*x*e^(16*I*d*x + 8*I*c) + 3840*a^5*d*x*e^(14*I*d*x + 6*I
*c) + 13440*a^5*d*x*e^(12*I*d*x + 4*I*c) + 26880*a^5*d*x*e^(10*I*d*x + 2*I
*c) + 26880*a^5*d*x*e^(6*I*d*x - 2*I*c) + 13440*a^5*d*x*e^(4*I*d*x - 4*I*c
) + 3840*a^5*d*x*e^(2*I*d*x - 6*I*c) + 33600*a^5*d*x*e^(8*I*d*x) + 480*a^5
*d*x*e^(-8*I*c) - 195*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) +
 1) - 1560*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 5460*
I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5*e^(1
0*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5*e^(6*I*d*x - 2
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 5460*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^
(2*I*d*x + 2*I*c) + 1) - 1560*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) - 13650*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 195*I*
a^5*e^(-8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 195*I*a^5*e^(16*I*d*x + 8*I*
c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1560*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(
2*I*d*x) + e^(-2*I*c)) + 5460*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x) +
 e^(-2*I*c)) + 10920*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*
c)) + 10920*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 5460
*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1560*I*a^5*e^(2
*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 13650*I*a^5*e^(8*I*d*x)*lo
g(e^(2*I*d*x) + e^(-2*I*c)) + 195*I*a^5*e^(-8*I*c)*log(e^(2*I*d*x) + e^(-2
*I*c)) - 48*I*a^5*e^(26*I*d*x + 18*I*c) - 684*I*a^5*e^(24*I*d*x + 16*I*...
```

3.68.9 Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\,x}{32} + \frac{\frac{a^5\,{\tan(c+d\,x)}^4}{32}+\frac{a^5\,{\tan(c+d\,x)}^3\,5{}\mathrm{i}}{32}-\frac{31\,a^5\,{\tan(c+d\,x)}^2}{96}-\frac{a^5\,\tan(c+d\,x)\,35{}\mathrm{i}}{96}+\frac{4\,a^5}{15}}{d\,\left({\tan\left(c+d\,x\right)}^5+{\tan\left(c+d\,x\right)}^4\,5{}\mathrm{i}-10\,{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,10{}\mathrm{i}+5\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

input

```
int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^5,x)
```

output

```
(a^5*x)/32 + ((4*a^5)/15 - (a^5*tan(c + d*x)*35i)/96 - (31*a^5*tan(c + d*x
)^2)/96 + (a^5*tan(c + d*x)^3*5i)/32 + (a^5*tan(c + d*x)^4)/32)/(d*(5*tan(
c + d*x) - tan(c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + ta
n(c + d*x)^5 + 1i))
```

3.68. $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.69 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.69.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\begin{aligned}\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = {} & \frac{7a^5x}{128} - \frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} \\ & - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} \\ & - \frac{3ia^9}{64d(a-ia\tan(c+dx))^4} \\ & - \frac{ia^8}{24d(a-ia\tan(c+dx))^3} \\ & - \frac{5ia^7}{128d(a-ia\tan(c+dx))^2} \\ & - \frac{3ia^6}{64d(a-ia\tan(c+dx))} \\ & + \frac{ia^6}{128d(a+ia\tan(c+dx))}\end{aligned}$$

output
```
7/128*a^5*x-1/24*I*a^11/d/(a-I*a*tan(d*x+c))^6-1/20*I*a^10/d/(a-I*a*tan(d*
x+c))^5-3/64*I*a^9/d/(a-I*a*tan(d*x+c))^4-1/24*I*a^8/d/(a-I*a*tan(d*x+c))^
3-5/128*I*a^7/d/(a-I*a*tan(d*x+c))^2-3/64*I*a^6/d/(a-I*a*tan(d*x+c))+1/128
*I*a^6/d/(a+I*a*tan(d*x+c))
```

3.69.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.80

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{a^5\sec^7(c+dx)(-1750\cos(c+dx)-1575\cos(3(c+dx))-693\cos(5(c+dx))+50\cos(7(c+dx))+3}{15}$$

input
```
Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
-1/15360*(a^5*Sec[c + d*x]^7*(-1750*Cos[c + d*x] - 1575*Cos[3*(c + d*x)] -
 693*Cos[5*(c + d*x)] + 50*Cos[7*(c + d*x)] + (350*I)*Sin[c + d*x] + (945*
I)*Sin[3*(c + d*x)] - (840*I)*ArcTan[Tan[c + d*x]]*(Cos[5*(c + d*x)] - I*S
in[5*(c + d*x)]) + (525*I)*Sin[5*(c + d*x)] - (70*I)*Sin[7*(c + d*x)]))/(d
*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^6)
```

3.69.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^{12}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{13}\int \frac{1}{(a-ia\tan(c+dx))^7(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^{13}\int \left(\frac{3}{64a^7(a-ia\tan(c+dx))^2}+\frac{1}{128a^7(i\tan(c+dx)a+a)^2}+\frac{5}{64a^6(a-ia\tan(c+dx))^3}+\frac{1}{8a^5(a-ia\tan(c+dx))^4}+\frac{3}{16a^4(a-ia\tan(c+dx}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{13}\Big(\frac{7i\arctan(\tan(c+dx))}{128a^8}+\frac{3}{64a^7(a-ia\tan(c+dx))}-\frac{1}{128a^7(a+ia\tan(c+dx))}+\frac{5}{128a^6(a-ia\tan(c+dx))^2}+\frac{1}{24a^5(a-ia\tan(c+dx))^3}-}{d}$$

input
```
Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-I)*a^13*((((7*I)/128)*ArcTan[Tan[c + d*x]])/a^8 + 1/(24*a^2*(a - I*a*Ta
n[c + d*x])^6) + 1/(20*a^3*(a - I*a*Tan[c + d*x])^5) + 3/(64*a^4*(a - I*a*
Tan[c + d*x])^4) + 1/(24*a^5*(a - I*a*Tan[c + d*x])^3) + 5/(128*a^6*(a - I
*a*Tan[c + d*x])^2) + 3/(64*a^7*(a - I*a*Tan[c + d*x])) - 1/(128*a^7*(a +
I*a*Tan[c + d*x]))))/d
```

3.69.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.69.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(168) = 336$.

Time = 0.79 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.82

$$ia^5\left(-\frac{(\cos^8(dx+c))(\sin^4(dx+c))}{12}-\frac{(\cos^8(dx+c))(\sin^2(dx+c))}{30}-\frac{(\cos^8(dx+c))}{120}\right)+5a^5\left(-\frac{(\sin^3(dx+c))(\cos^9(dx+c))}{12}-\frac{\sin(dx+c)(\mathrm{co}}{40}\right.$$

input
```
int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/12*cos(d*x+c)^8*sin(d*x+c)^4-1/30*cos(d*x+c)^8*sin(d*x+c)^2
-1/120*cos(d*x+c)^8)+5*a^5*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*sin(d*x+c
)*cos(d*x+c)^9+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/
16*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-10*I*a^5*(-1/12*cos(d*x+c)^
10*sin(d*x+c)^2-1/60*cos(d*x+c)^10)-10*a^5*(-1/12*sin(d*x+c)*cos(d*x+c)^11
+1/120*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)
^3+315/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-5/12*I*a^5*cos(d*
x+c)^12+a^5*(1/12*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231
/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/
1024*d*x+231/1024*c))
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.61

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{\left(840\,a^5dxe^{(2i\,dx+2i\,c)}-10i\,a^5e^{(14i\,dx+14i\,c)}-84i\,a^5e^{(12i\,dx+12i\,c)}-315i\,a^5e^{(10i\,dx+10i\,c)}-700i\,a^5e^{(8i\,dx+8i\,c)}-1\right.}{15360\,d}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/15360*(840*a^5*d*x*e^(2*I*d*x + 2*I*c) - 10*I*a^5*e^(14*I*d*x + 14*I*c)
- 84*I*a^5*e^(12*I*d*x + 12*I*c) - 315*I*a^5*e^(10*I*d*x + 10*I*c) - 700*I
*a^5*e^(8*I*d*x + 8*I*c) - 1050*I*a^5*e^(6*I*d*x + 6*I*c) - 1260*I*a^5*e^(
4*I*d*x + 4*I*c) + 60*I*a^5)*e^(-2*I*d*x - 2*I*c)/d
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.52

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{7a^5x}{128} + \begin{cases} \frac{(-33776997205278720ia^5d^6e^{14ic}e^{12idx}-283726776524341248ia^5d^6e^{12ic}e^{10idx}-1063975411966279680ia^5d^6e^{10ic}e^{8idx}-236438980436951}{51881467707} \\ x\left(-\frac{7a^5}{128}+\frac{(a^5e^{14ic}+7a^5e^{12ic}+21a^5e^{10ic}+35a^5e^{8ic}+35a^5e^{6ic}+21a^5e^{4ic}+7a^5e^{2ic}+a^5)e^{-2ic}}{128}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**5,x)
```

output
```
7*a**5*x/128 + Piecewise(((-33776997205278720*I*a**5*d**6*exp(14*I*c)*exp(
12*I*d*x) - 283726776524341248*I*a**5*d**6*exp(12*I*c)*exp(10*I*d*x) - 106
3975411966279680*I*a**5*d**6*exp(10*I*c)*exp(8*I*d*x) - 236438980436951040
0*I*a**5*d**6*exp(8*I*c)*exp(6*I*d*x) - 3546584706554265600*I*a**5*d**6*ex
p(6*I*c)*exp(4*I*d*x) - 4255901647865118720*I*a**5*d**6*exp(4*I*c)*exp(2*I
*d*x) + 202661983231672320*I*a**5*d**6*exp(-2*I*d*x))*exp(-2*I*c)/(5188146
7707308113920*d**7), Ne(d**7*exp(2*I*c), 0)), (x*(-7*a**5/128 + (a**5*exp(
14*I*c) + 7*a**5*exp(12*I*c) + 21*a**5*exp(10*I*c) + 35*a**5*exp(8*I*c) +
35*a**5*exp(6*I*c) + 21*a**5*exp(4*I*c) + 7*a**5*exp(2*I*c) + a**5)*exp(-2
*I*c)/128), True))
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{105\,(dx+c)a^5+\frac{105\,a^5\tan(dx+c)^{11}+595\,a^5\tan(dx+c)^9+1386\,a^5\tan(dx+c)^7+1686\,a^5\tan(dx+c)^5-240i\,a^5\tan(dx+c)^4+45\,a^5\tan(dx+}{\tan(dx+c)^{12}+6\,\tan(dx+c)^{10}+15\,\tan(dx+c)^8+20\,\tan(dx+c)^6+15\,\tan(dx+c)^4+6}}{1920\,d}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/1920*(105*(d*x + c)*a^5 + (105*a^5*tan(d*x + c)^11 + 595*a^5*tan(d*x + c
)^9 + 1386*a^5*tan(d*x + c)^7 + 1686*a^5*tan(d*x + c)^5 - 240*I*a^5*tan(d*
x + c)^4 + 45*a^5*tan(d*x + c)^3 + 1824*I*a^5*tan(d*x + c)^2 + 1815*a^5*ta
n(d*x + c) - 496*I*a^5)/(tan(d*x + c)^12 + 6*tan(d*x + c)^10 + 15*tan(d*x
+ c)^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 1))/d
```

3.69.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(154) = 308$.

Time = 0.88 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.62

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
1/122880*(6720*a^5*d*x*e^(18*I*d*x + 10*I*c) + 53760*a^5*d*x*e^(16*I*d*x +
 8*I*c) + 188160*a^5*d*x*e^(14*I*d*x + 6*I*c) + 376320*a^5*d*x*e^(12*I*d*x
 + 4*I*c) + 470400*a^5*d*x*e^(10*I*d*x + 2*I*c) + 188160*a^5*d*x*e^(6*I*d*
x - 2*I*c) + 53760*a^5*d*x*e^(4*I*d*x - 4*I*c) + 6720*a^5*d*x*e^(2*I*d*x -
 6*I*c) + 376320*a^5*d*x*e^(8*I*d*x) - 2355*I*a^5*e^(18*I*d*x + 10*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 18840*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d
*x + 2*I*c) + 1) - 65940*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) - 131880*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) -
164850*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 65940*I*a
^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 18840*I*a^5*e^(4*I*d
*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2355*I*a^5*e^(2*I*d*x - 6*I*c)*
log(e^(2*I*d*x + 2*I*c) + 1) - 131880*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2
*I*c) + 1) + 2355*I*a^5*e^(18*I*d*x + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)
) + 18840*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 65940
*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 131880*I*a^5*e
^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 164850*I*a^5*e^(10*I*d
*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 65940*I*a^5*e^(6*I*d*x - 2*I*c
)*log(e^(2*I*d*x) + e^(-2*I*c)) + 18840*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2
*I*d*x) + e^(-2*I*c)) + 2355*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e
^(-2*I*c)) + 131880*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - 8...
```

3.69.9 Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{7\,a^5\,x}{128}$$

$$-\frac{-\frac{7\,a^5\,\tan(c+d\,x)^6}{128}-\frac{a^5\,\tan(c+d\,x)^5\,35\mathrm{i}}{128}+\frac{49\,a^5\,\tan(c+d\,x)^4}{96}+\frac{a^5\,\tan(c+d\,x)^3\,35\mathrm{i}}{96}+\frac{63\,a^5\,\tan(c+d\,x)^2}{640}+}{d\,\left({\tan\left(c+d\,x\right)}^7+{\tan\left(c+d\,x\right)}^6\,5\mathrm{i}-9\,{\tan\left(c+d\,x\right)}^5-{\tan\left(c+d\,x\right)}^4\,5\mathrm{i}-5\,{\tan\left(c+d\,x\right)}^3-\tan(c+d\right.}$$

input
```
int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(7*a^5*x)/128 - ((a^5*tan(c + d*x)*133i)/384 - (31*a^5)/120 + (63*a^5*tan(
c + d*x)^2)/640 + (a^5*tan(c + d*x)^3*35i)/96 + (49*a^5*tan(c + d*x)^4)/96
 - (a^5*tan(c + d*x)^5*35i)/128 - (7*a^5*tan(c + d*x)^6)/128)/(d*(5*tan(c
+ d*x) - tan(c + d*x)^2*9i - 5*tan(c + d*x)^3 - tan(c + d*x)^4*5i - 9*tan(
c + d*x)^5 + tan(c + d*x)^6*5i + tan(c + d*x)^7 + 1i))
```

3.69. $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.70 $\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.70.1 Optimal result

Integrand size = 22, antiderivative size = 167

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx &= \frac{63a^5\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{63ia^5\sec(c+dx)}{8d}\\ &\quad+\frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d}\\ &\quad+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}\\ &\quad+\frac{21ia\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{20d}\\ &\quad+\frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d}\end{aligned}$$

output
```
63/8*a^5*arctanh(sin(d*x+c))/d+63/8*I*a^5*sec(d*x+c)/d+9/20*I*a^2*sec(d*x+
c)*(a+I*a*tan(d*x+c))^3/d+1/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^4/d+21/20*
I*a*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+21/8*I*sec(d*x+c)*(a^5+I*a^5*tan
(d*x+c))/d
```

3.70.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5(\cos(5dx)+i\sin(5dx))\,\big(5040\text{arctanh}\big(\sin(c)+\cos(c)\tan\big(\frac{dx}{2}\big)\big)+i\sec^5(c+dx)(1344+1920\cos(2(c+}{320d(\cos(dx)+i\sin(dx))^5}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*(Cos[5*d*x] + I*Sin[5*d*x])*(5040*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2
]] + I*Sec[c + d*x]^5*(1344 + 1920*Cos[2*(c + d*x)] + 640*Cos[4*(c + d*x)]
 + (450*I)*Sin[2*(c + d*x)] + (325*I)*Sin[4*(c + d*x)])))/(320*d*(Cos[d*x]
 + I*Sin[d*x])^5)
```

3.70.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)(a+ia\tan(c+dx))^5dx$$
$$\Big\downarrow \text{3979}$$
$$\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$
$$\Big\downarrow \text{3042}$$
$$\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$
$$\Big\downarrow \text{3979}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+i}{4d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+dx)(a+i}{4d}\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(}{}\right.\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(}{}\right.\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)}{}\right.\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

3.70. $\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$

$$\Big\downarrow \color{blue}{3042}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c}{}\right.\right.\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

$$\Big\downarrow \color{blue}{4257}$$

$$\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c+}{}\right.\right.$$
$$\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/5)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4)/d + (9*a*((((I/4)*a*Sec[c +
 d*x]*(a + I*a*Tan[c + d*x])^3)/d + (7*a*((((I/3)*a*Sec[c + d*x]*(a + I*a*T
an[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c +
 d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d))/3))/4))
/5
```

3.70.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.70.4 Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

method	result
risch	$\frac{i a^5 \left(965\,\mathrm{e}^{9 i (d x+c)}+2370\,\mathrm{e}^{7 i (d x+c)}+2688\,\mathrm{e}^{5 i (d x+c)}+1470\,\mathrm{e}^{3 i (d x+c)}+315\,\mathrm{e}^{i (d x+c)}\right)}{20 d \left(\mathrm{e}^{2 i (d x+c)}+1\right)^5}+\frac{63 a^5 \ln\left(\mathrm{e}^{i (d x+c)}+i\right)}{8 d}-$
derivativedivides	$i a^5 \left(\frac{\sin^6(d x+c)}{5 \cos(d x+c)^5}-\frac{\sin^6(d x+c)}{15 \cos(d x+c)^3}+\frac{\sin^6(d x+c)}{5 \cos(d x+c)}+\frac{\left(\frac{8}{3}+\sin^4(d x+c)+\frac{4\left(\sin^2(d x+c)\right)}{3}\right)\cos(d x+c)}{5}\right)+5 a^5 \left(\frac{\sin^5(d x+c)}{4 \cos(d x+c)^4}-\right.$
default	$i a^5 \left(\frac{\sin^6(d x+c)}{5 \cos(d x+c)^5}-\frac{\sin^6(d x+c)}{15 \cos(d x+c)^3}+\frac{\sin^6(d x+c)}{5 \cos(d x+c)}+\frac{\left(\frac{8}{3}+\sin^4(d x+c)+\frac{4\left(\sin^2(d x+c)\right)}{3}\right)\cos(d x+c)}{5}\right)+5 a^5 \left(\frac{\sin^5(d x+c)}{4 \cos(d x+c)^4}-\right.$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/20*I*a^5/d/(exp(2*I*(d*x+c))+1)^5*(965*exp(9*I*(d*x+c))+2370*exp(7*I*(d*
x+c))+2688*exp(5*I*(d*x+c))+1470*exp(3*I*(d*x+c))+315*exp(I*(d*x+c)))+63/8
/d*a^5*ln(exp(I*(d*x+c))+I)-63/8/d*a^5*ln(exp(I*(d*x+c))-I)
```

3.70.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(137) = 274.

Time = 0.25 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.86

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{1930i\,a^5e^{(9i\,dx+9i\,c)}+4740i\,a^5e^{(7i\,dx+7i\,c)}+5376i\,a^5e^{(5i\,dx+5i\,c)}+2940i\,a^5e^{(3i\,dx+3i\,c)}+630i\,a^5e^{(i\,dx+i\,c)}+315}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/40*(1930*I*a^5*e^(9*I*d*x + 9*I*c) + 4740*I*a^5*e^(7*I*d*x + 7*I*c) + 53
76*I*a^5*e^(5*I*d*x + 5*I*c) + 2940*I*a^5*e^(3*I*d*x + 3*I*c) + 630*I*a^5*
e^(I*d*x + I*c) + 315*(a^5*e^(10*I*d*x + 10*I*c) + 5*a^5*e^(8*I*d*x + 8*I*
c) + 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d*x + 4*I*c) + 5*a^5*e^(2*
I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) + I) - 315*(a^5*e^(10*I*d*x + 10
*I*c) + 5*a^5*e^(8*I*d*x + 8*I*c) + 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^
(4*I*d*x + 4*I*c) + 5*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) -
 I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x
+ 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

3.70.6 Sympy [F]

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\Bigg(&\int(-i\sec{(c+dx)})\,dx\\
&+\int 5\tan{(c+dx)}\sec{(c+dx)}\,dx\\
&+\int\left(-10\tan^3{(c+dx)}\sec{(c+dx)}\right)\,dx\\
&+\int\tan^5{(c+dx)}\sec{(c+dx)}\,dx\\
&+\int 10i\tan^2{(c+dx)}\sec{(c+dx)}\,dx\\
&+\int\left(-5i\tan^4{(c+dx)}\sec{(c+dx)}\right)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*sec(c + d*x), x) + Integral(5*tan(c + d*x)*sec(c + d*x
), x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x), x) + Integral(tan(c + d
*x)**5*sec(c + d*x), x) + Integral(10*I*tan(c + d*x)**2*sec(c + d*x), x) +
 Integral(-5*I*tan(c + d*x)**4*sec(c + d*x), x))
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.29

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{75\,a^5\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+600\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}\right.}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/240*(75*a^5*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*s
in(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) +
600*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 240*a^5*log(sec(d*x + c) + tan(d*x + c)) + 1200*I*a^
5/cos(d*x + c) + 800*I*(3*cos(d*x + c)^2 - 1)*a^5/cos(d*x + c)^3 + 16*I*(1
5*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*a^5/cos(d*x + c)^5)/d
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(275\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+200i\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
1/40*(315*a^5*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^5*log(tan(1/2*d*x + 1/
2*c) - 1) - 2*(275*a^5*tan(1/2*d*x + 1/2*c)^9 + 200*I*a^5*tan(1/2*d*x + 1/
2*c)^8 - 750*a^5*tan(1/2*d*x + 1/2*c)^7 - 1600*I*a^5*tan(1/2*d*x + 1/2*c)^
6 + 3280*I*a^5*tan(1/2*d*x + 1/2*c)^4 + 750*a^5*tan(1/2*d*x + 1/2*c)^3 - 2
240*I*a^5*tan(1/2*d*x + 1/2*c)^2 - 275*a^5*tan(1/2*d*x + 1/2*c) + 488*I*a^
5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{63\,a^5\,\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{55\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4} + a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,10\mathrm{i} - \frac{75\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2} - a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,80\mathrm{i} + a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,164\mathrm{i} +}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 10\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x),x)
```

output
```
(63*a^5*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((75*a^5*tan(c/2 + (d*x)/2)^3)/
2 - a^5*tan(c/2 + (d*x)/2)^2*112i + a^5*tan(c/2 + (d*x)/2)^4*164i - a^5*ta
n(c/2 + (d*x)/2)^6*80i - (75*a^5*tan(c/2 + (d*x)/2)^7)/2 + a^5*tan(c/2 + (
d*x)/2)^8*10i + (55*a^5*tan(c/2 + (d*x)/2)^9)/4 + (a^5*122i)/5 - (55*a^5*t
an(c/2 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4
 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^1
0 - 1))
```

3.71 $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.71.1 Optimal result

Integrand size = 22, antiderivative size = 130

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = & -\frac{35a^5\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{35ia^5\sec(c+dx)}{2d}\\ & -\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\\ & -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}\\ & -\frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d}\end{aligned}$$

output
```
-35/2*a^5*arctanh(sin(d*x+c))/d-35/2*I*a^5*sec(d*x+c)/d-7/3*I*a^3*sec(d*x+
c)*(a+I*a*tan(d*x+c))^2/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^4/d-35/6*I*s
ec(d*x+c)*(a^5+I*a^5*tan(d*x+c))/d
```

3.71.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{a^5\cos^2(c+dx)\left(-840i\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^3(c+dx)(\cos(5c)-i\sin(5c))+(\cos(4c-dx}{24d(c}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Cos[c + d*x]^2*((-840*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c
+ d*x]^3*(Cos[5*c] - I*Sin[5*c]) + (Cos[4*c - d*x] - I*Sin[4*c - d*x])*(51
1*Cos[c + d*x] + 153*Cos[3*(c + d*x)] - I*(49*Sin[c + d*x] + 57*Sin[3*(c +
 d*x)])))*(-I + Tan[c + d*x])^5)/(24*d*(Cos[d*x] + I*Sin[d*x])^5)
```

3.71.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3977, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3977}$$

$$-7a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3042}$$

$$-7a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3979}$$

$$-7a^2\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3042}$$

$$-7a^2\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3979}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+}{3}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3042}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a+}{3}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3967}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+dx)(a}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{3042}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\sec(c+}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

$$\Big\downarrow \text{4257}$$

$$-7a^2\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)+\frac{ia\sec(c+d}{}\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4)/d - 7*a^2*(((I/3)*a*Sec[c
 + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]]
)/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c +
d*x]))/d))/3)
```

3.71. $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.71.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.71.4 Maple [A] (verified)

Time = 7.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{16ia^5\mathrm{e}^{i(dx+c)}}{d}-\frac{ia^5\left(87\,\mathrm{e}^{5i(dx+c)}+136\,\mathrm{e}^{3i(dx+c)}+57\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{35a^5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}-\frac{35a^5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+3\sin\right.}{}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+3\sin\right.}{}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-16*I/d*a^5*exp(I*(d*x+c))-1/3*I*a^5/d/(exp(2*I*(d*x+c))+1)^3*(87*exp(5*I*
(d*x+c))+136*exp(3*I*(d*x+c))+57*exp(I*(d*x+c)))+35/2/d*a^5*ln(exp(I*(d*x+
c))-I)-35/2/d*a^5*ln(exp(I*(d*x+c))+I)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.66

$$\int\cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{-96i\,a^5e^{(7i\,dx+7i\,c)}-462i\,a^5e^{(5i\,dx+5i\,c)}-560i\,a^5e^{(3i\,dx+3i\,c)}-210i\,a^5e^{(i\,dx+i\,c)}-105\left(a^5e^{(6i\,dx+6i\,c)}+3\,a^5e^{(4i}\right.}{6\left(de^{(6i\,dx+6i\,c)}+\right.}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/6*(-96*I*a^5*e^(7*I*d*x + 7*I*c) - 462*I*a^5*e^(5*I*d*x + 5*I*c) - 560*I
*a^5*e^(3*I*d*x + 3*I*c) - 210*I*a^5*e^(I*d*x + I*c) - 105*(a^5*e^(6*I*d*x
 + 6*I*c) + 3*a^5*e^(4*I*d*x + 4*I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*l
og(e^(I*d*x + I*c) + I) + 105*(a^5*e^(6*I*d*x + 6*I*c) + 3*a^5*e^(4*I*d*x
+ 4*I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e
^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d
)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.52

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{35a^5\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d} + \frac{-87ia^5e^{5ic}e^{5idx}-136ia^5e^{3ic}e^{3idx}-57ia^5e^{ic}e^{idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d} + \begin{cases} -\frac{16ia^5e^{ic}e^{idx}}{d} & \text{for } d\neq 0 \\ 16a^5xe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

output
```
35*a**5*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2
)/d + (-87*I*a**5*exp(5*I*c)*exp(5*I*d*x) - 136*I*a**5*exp(3*I*c)*exp(3*I*
d*x) - 57*I*a**5*exp(I*c)*exp(I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) + 9*d*e
xp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d) + Piecewise((-
16*I*a**5*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (16*a**5*x*exp(I*c), True))
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{15\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)+120i\,a^5\left(\frac{1}{\cos(d}\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/12*(15*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) +
1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 120*I*a^5*(1/cos(d*x + c)
 + cos(d*x + c)) + 4*I*a^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(
d*x + c)) + 60*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(
d*x + c)) + 60*I*a^5*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

3.71.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(108) = 216$.

Time = 0.81 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.92

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{8295\,a^5e^{(6i\,dx+6i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+24885\,a^5e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+24885\,a^5e^{(2i\,dx+2i\,c)}\log\left(i\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
1/1536*(8295*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^
5*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5*e^(2*I*d*x +
2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 18585*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^
(I*d*x + I*c) - 1) - 55755*a^5*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) -
 1) - 55755*a^5*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*
e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(4*I*d*x + 4
*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(2*I*d*x + 2*I*c)*log(-I*e
^(I*d*x + I*c) + 1) + 18585*a^5*e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) + 55755*a^5*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 55755*
a^5*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 24576*I*a^5*e^(7*I*d
*x + 7*I*c) - 118272*I*a^5*e^(5*I*d*x + 5*I*c) - 143360*I*a^5*e^(3*I*d*x +
 3*I*c) - 53760*I*a^5*e^(I*d*x + I*c) + 8295*a^5*log(I*e^(I*d*x + I*c) + 1
) - 18585*a^5*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*log(-I*e^(I*d*x + I*c)
 + 1) + 18585*a^5*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(6*I*d*x + 6*I*c) + 3*
d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.71

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{35\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{37\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 27\mathrm{i}-118\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 48\mathrm{i}+139\,a^5\tan\left(\frac{c}{2}\right.}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 1\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 3\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}\right.\right.}$$

input

```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^5,x)
```

output

```
- (35*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (139*a^5*tan(c/2 + (d*x)/2)^2 - a
^5*tan(c/2 + (d*x)/2)^3*48i - 118*a^5*tan(c/2 + (d*x)/2)^4 + a^5*tan(c/2 +
 (d*x)/2)^5*27i + 37*a^5*tan(c/2 + (d*x)/2)^6 - (166*a^5)/3 + (a^5*tan(c/2
 + (d*x)/2)*55i)/3)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - 3*t
an(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + 3*tan(c/2 + (d*x)/2)^5 - t
an(c/2 + (d*x)/2)^6*1i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.71. $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.72 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.72.1 Optimal result

Integrand size = 24, antiderivative size = 98

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx &= \frac{5a^5\text{arctanh}(\sin(c+dx))}{d}+\frac{5ia^5\sec(c+dx)}{d}\\ &\quad+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}\\ &\quad-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}\end{aligned}$$

output
```
5*a^5*arctanh(sin(d*x+c))/d+5*I*a^5*sec(d*x+c)/d+10/3*I*a^3*cos(d*x+c)*(a+
I*a*tan(d*x+c))^2/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4/d
```

3.72.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$=\frac{a^5\cos^4(c+dx)\big(30\text{arctanh}\big(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\big)\cos(c+dx)(i\cos(5c)+\sin(5c))-(\cos(3c-2dx)-}{3d(\cos(dx)+i\sin(dx))}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Cos[c + d*x]^4*(30*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]
*(I*Cos[5*c] + Sin[5*c]) - (Cos[3*c - 2*d*x] - I*Sin[3*c - 2*d*x])*(10 + 1
3*Cos[2*(c + d*x)] - (17*I)*Sin[2*(c + d*x)]))*(-I + Tan[c + d*x])^5)/(3*d
*(Cos[d*x] + I*Sin[d*x])^5)
```

3.72.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3977, 3042, 3977, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3977}$$

$$-\frac{5}{3}a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{5}{3}a^2\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{ 3977}$$

$$-\frac{5}{3}a^2\left(-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{5}{3}a^2\left(-3a^2\int \sec(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{ 3967}$$

$$-\frac{5}{3}a^2\left(-3a^2\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{3}a^2\left(-3a^2\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{5}{3}a^2\left(-3a^2\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

input

```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

output

```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4)/d - (5*a^2*(-3*a^2*
((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d) - ((2*I)*a*Cos[c + d*
x]*(a + I*a*Tan[c + d*x])^2)/d))/3
```

3.72.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.72.4 Maple [A] (verified)

Time = 26.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{4 i a^{5} {\mathrm e}^{3 i \left(d x +c \right)}}{3 d}+\frac{8 i a^{5} {\mathrm e}^{i \left(d x +c \right)}}{d}+\frac{2 i a^{5} {\mathrm e}^{i \left(d x +c \right)}}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}+\frac{5 a^{5} \ln \left({\mathrm e}^{i \left(d x +c \right)}+i\right)}{d}-\frac{5 a^{5} \ln \left({\mathrm e}^{i \left(d x +c \right)}-i\right)}{d}$
derivativedivides	$\frac{i a^{5} \left(\frac{\sin^{6}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(\frac{8}{3}+\sin^{4}\left(d x +c \right)+\frac{4 \left(\sin^{2}\left(d x +c \right)\right)}{3}\right) \cos \left(d x +c \right)\right)+5 a^{5} (-\frac{\left(\sin^{3}\left(d x +c \right)\right)}{3}-\sin \left(d x +c \right)+\ln (\sec \left(d x +c \right)+\tan}{d}$
default	$\frac{i a^{5} \left(\frac{\sin^{6}\left(d x +c \right)}{\cos \left(d x +c \right)}+\left(\frac{8}{3}+\sin^{4}\left(d x +c \right)+\frac{4 \left(\sin^{2}\left(d x +c \right)\right)}{3}\right) \cos \left(d x +c \right)\right)+5 a^{5} (-\frac{\left(\sin^{3}\left(d x +c \right)\right)}{3}-\sin \left(d x +c \right)+\ln (\sec \left(d x +c \right)+\tan}{d}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-4/3*I/d*a^5*exp(3*I*(d*x+c))+8*I/d*a^5*exp(I*(d*x+c))+2*I*a^5*exp(I*(d*x+
c))/d/(exp(2*I*(d*x+c))+1)+5/d*a^5*ln(exp(I*(d*x+c))+I)-5/d*a^5*ln(exp(I*(
d*x+c))-I)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{-4i\,a^5e^{(5i\,dx+5i\,c)}+20i\,a^5e^{(3i\,dx+3i\,c)}+30i\,a^5e^{(i\,dx+i\,c)}+15\left(a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\,(a^5e}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/3*(-4*I*a^5*e^(5*I*d*x + 5*I*c) + 20*I*a^5*e^(3*I*d*x + 3*I*c) + 30*I*a^
5*e^(I*d*x + I*c) + 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c)
 + I) - 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^
(2*I*d*x + 2*I*c) + d)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{2ia^5e^{ic}e^{idx}}{de^{2ic}e^{2idx}+d}+\frac{5a^5\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}\frac{-4ia^5de^{3ic}e^{3idx}+24ia^5de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0\\ x(4a^5e^{3ic}-8a^5e^{ic}) & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**5,x)
```

output
```
2*I*a**5*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 5*a**5*(-lo
g(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise
(((-4*I*a**5*d*exp(3*I*c)*exp(3*I*d*x) + 24*I*a**5*d*exp(I*c)*exp(I*d*x))/
(3*d**2), Ne(d**2, 0)), (x*(4*a**5*exp(3*I*c) - 8*a**5*exp(I*c)), True))
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.57

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{10i\,a^5\cos{(dx+c)}^3+20\,a^5\sin{(dx+c)}^3+2i\left(\cos{(dx+c)}^3-\frac{3}{\cos(dx+c)}-6\,\cos{(dx+c)}\right)a^5+20i\,(\cos}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/6*(10*I*a^5*cos(d*x + c)^3 + 20*a^5*sin(d*x + c)^3 + 2*I*(cos(d*x + c)^
3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^5 + 20*I*(cos(d*x + c)^3 - 3*cos(d*
x + c))*a^5 + 5*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1) + 6*sin(d*x + c))*a^5 + 2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^
5)/d
```

3.72.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1683 vs. $2(84) = 168$.

Time = 1.18 (sec) , antiderivative size = 1683, normalized size of antiderivative = 17.17

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/6144*(39225*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 31380
0*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1098300*a^5*e^(12*
I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(10*I*d*x + 2*I*
c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I
*d*x + I*c) + 1) + 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 313800*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2745750*
a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 39225*a^5*e^(-8*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 8520*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 68160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 238560*a
^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(10*I*d*
x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(6*I*d*x - 2*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 238560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 68160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 596
400*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 8520*a^5*e^(-8*I*c)*log(I
*e^(I*d*x + I*c) - 1) - 39225*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I
*c) + 1) - 313800*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1
098300*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*
e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(6*I*d*x
- 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1098300*a^5*e^(4*I*d*x - 4*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 313800*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d...
```

3.72.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.65

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{10\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{8\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,34\mathrm{i}-\frac{82\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,38\mathrm{i}+\frac{46\,a^5}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,3\mathrm{i}-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,4\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(10*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (a^5*tan(c/2 + (d*x)/2)^3*34i - (82
*a^5*tan(c/2 + (d*x)/2)^2)/3 + 8*a^5*tan(c/2 + (d*x)/2)^4 + (46*a^5)/3 - a
^5*tan(c/2 + (d*x)/2)*38i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2
*4i - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + tan(c/2 + (d*x)/2
)^5 + 1i))
```

3.73 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.73.1 Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx=-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

output
```
-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d
```

3.73.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx=-\frac{ia^5(\cos(c+dx)+i\sin(c+dx))^5}{5d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/5*I)*a^5*(Cos[c + d*x] + I*Sin[c + d*x])^5)/d
```

3.73.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \ 3969$$

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d
```

3.73.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.73.4 Maple [A] (verified)

Time = 81.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{i a^5 \mathrm{e}^{5 i (dx+c)}}{5d}$
derivativedivides	$-\frac{i a^5 \left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a^5\left(\sin^5(dx+c)\right)-10 i a^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)$
default	$-\frac{i a^5 \left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a^5\left(\sin^5(dx+c)\right)-10 i a^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/5*I/d*a^5*exp(5*I*(d*x+c))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,a^5 e^{(5i\,dx+5i\,c)}}{5\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/5*I*a^5*e^(5*I*d*x + 5*I*c)/d
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} -\frac{ia^5e^{5ic}e^{5idx}}{5d} & \text{for } d \neq 0 \\ a^5xe^{5ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise((-I*a**5*exp(5*I*c)*exp(5*I*d*x)/(5*d), Ne(d, 0)), (a**5*x*exp(5
*I*c), True))
```

3.73.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(26) = 52$.

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.75

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{15i\,a^5\cos\left(dx+c\right)^5 - 15\,a^5\sin\left(dx+c\right)^5 + 10i\left(3\,\cos\left(dx+c\right)^5 - 5\,\cos\left(dx+c\right)^3\right)a^5 + i\left(3\,\cos\left(dx+c\right)\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/15*(15*I*a^5*cos(d*x + c)^5 - 15*a^5*sin(d*x + c)^5 + 10*I*(3*cos(d*x +
 c)^5 - 5*cos(d*x + c)^3)*a^5 + I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 +
15*cos(d*x + c))*a^5 - 10*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^5 - (3*s
in(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5)/d
```

3.73.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(26) = 52$.

Time = 0.89 (sec) , antiderivative size = 1669, normalized size of antiderivative = 52.16

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/40960*(11375*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9100
0*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 318500*a^5*e^(12*I
*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(10*I*d*x + 2*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 318500*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 91000*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 796250*a^5*e^
(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 11375*a^5*e^(-8*I*c)*log(I*e^(I*d*x
 + I*c) + 1) + 11590*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 92720*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 324520*a^5*e^
(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(10*I*d*x + 2
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 324520*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c)
- 1) + 92720*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 811300*a
^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 11590*a^5*e^(-8*I*c)*log(I*e^(
I*d*x + I*c) - 1) - 11375*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 91000*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 318500
*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(10*I
*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(6*I*d*x - 2*I*c)
*log(-I*e^(I*d*x + I*c) + 1) - 318500*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 91000*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) ...
```

3.73.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{2\,a^5\,\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{5\,d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,5{}\mathrm{i}-10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,10{}\mathrm{i}+5\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
(2*a^5*(5*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 1))/(5*d*(5*tan
(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan
(c/2 + (d*x)/2)^4*5i + tan(c/2 + (d*x)/2)^5 + 1i))
```

3.74 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.74.1 Optimal result 699
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3.74.1 Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

output
```
-2/105*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-2/35*I*a*cos(d*x+c)^5*(a+
I*a*tan(d*x+c))^4/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d
```

3.74.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sec(c+dx)(-i\cos(4(c+dx))+\sin(4(c+dx)))\Big(77+92\cos(2(c+dx))+\Big(15+416\sqrt{\cos^2(c+dx)}\Big)}{840d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

output

```
(a^5*Sec[c + d*x]*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(77 + 92*Cos[
2*(c + d*x)] + (15 + 416*Sqrt[Cos[c + d*x]^2])*Cos[4*(c + d*x)] + (22*I)*S
in[2*(c + d*x)] + (15*I)*Sin[4*(c + d*x)] - (416*I)*Sqrt[Cos[c + d*x]^2]*S
in[4*(c + d*x)]))/(840*d)
```

3.74.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{2}{7}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{2}{7}a\left(\frac{1}{5}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\left(\frac{1}{5}a\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

$$\Big\downarrow \text{3969}$$

$$\frac{2}{7}a\left(-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5)/d + (2*a*(((-1/15*I)*a*
Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d*x]^5*(a + I*
a*Tan[c + d*x])^4)/d))/7
```

3.74.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.74.4 Maple [A] (verified)

Time = 212.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{ia^5 \mathrm{e}^{7i(dx+c)}}{28d}-\frac{ia^5 \mathrm{e}^{5i(dx+c)}}{10d}-\frac{ia^5 \mathrm{e}^{3i(dx+c)}}{12d}$
derivativedivides	$ia^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^4(dx+c)\right)}{7}-\frac{4\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{35}-\frac{8\left(\cos^3(dx+c)\right)}{105}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-3\sin$
default	$ia^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^4(dx+c)\right)}{7}-\frac{4\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{35}-\frac{8\left(\cos^3(dx+c)\right)}{105}\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-3\sin$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/28*I/d*a^5*exp(7*I*(d*x+c))-1/10*I/d*a^5*exp(5*I*(d*x+c))-1/12*I/d*a^5*
exp(3*I*(d*x+c))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-15i\,a^5e^{(7i\,dx+7i\,c)}-42i\,a^5e^{(5i\,dx+5i\,c)}-35i\,a^5e^{(3i\,dx+3i\,c)}}{420\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/420*(-15*I*a^5*e^(7*I*d*x + 7*I*c) - 42*I*a^5*e^(5*I*d*x + 5*I*c) - 35*I
*a^5*e^(3*I*d*x + 3*I*c))/d
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} \frac{-120ia^5d^2e^{7ic}e^{7idx}-336ia^5d^2e^{5ic}e^{5idx}-280ia^5d^2e^{3ic}e^{3idx}}{3360d^3} & \text{for } d^3 \neq 0 \\ x\left(\frac{a^5e^{7ic}}{4}+\frac{a^5e^{5ic}}{2}+\frac{a^5e^{3ic}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-120*I*a**5*d**2*exp(7*I*c)*exp(7*I*d*x) - 336*I*a**5*d**2*exp
(5*I*c)*exp(5*I*d*x) - 280*I*a**5*d**2*exp(3*I*c)*exp(3*I*d*x))/(3360*d**3
), Ne(d**3, 0)), (x*(a**5*exp(7*I*c)/4 + a**5*exp(5*I*c)/2 + a**5*exp(3*I*
c)/4), True))
```

3.74.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(83) = 166.

Time = 0.41 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.85

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{75i\,a^5\cos(dx+c)^7 + i\left(15\cos(dx+c)^7 - 42\cos(dx+c)^5 + 35\cos(dx+c)^3\right)a^5 + 30i\left(5\cos(dx+c)\right.}{}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/105*(75*I*a^5*cos(d*x + c)^7 + I*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5
 + 35*cos(d*x + c)^3)*a^5 + 30*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^5
 + 10*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5 + 15
*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^5 + 3*(5*sin(d*x + c)^7 - 21*sin(
d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5)/d
```

3.74.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1697 vs. $2(83) = 166$.

Time = 0.94 (sec) , antiderivative size = 1697, normalized size of antiderivative = 16.80

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/3440640*(7357770*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
58862160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 206017560*a
^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(10*I
*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(6*I*d*x - 2*I*
c)*log(I*e^(I*d*x + I*c) + 1) + 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 58862160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c
) + 1) + 515043900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7357770*a^
5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7390425*a^5*e^(16*I*d*x + 8*I*c)
*log(I*e^(I*d*x + I*c) - 1) + 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I
*d*x + I*c) - 1) + 206931900*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c
) - 1) + 413863800*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4
13863800*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 206931900*a^
5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 59123400*a^5*e^(2*I*d*x
 - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 517329750*a^5*e^(8*I*d*x)*log(I*e^(
I*d*x + I*c) - 1) + 7390425*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 73
57770*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 58862160*a^5*
e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 206017560*a^5*e^(12*I*d
*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 412035120*a^5*e^(10*I*d*x + 2*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 412035120*a^5*e^(6*I*d*x - 2*I*c)*log(-I*
e^(I*d*x + I*c) + 1) - 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*...
```

3.74.9 Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.84

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{2\,a^5\,\left(105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 210\mathrm{i}-455\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 350\mathrm{i}+273\tan}{105\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 7\mathrm{i}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right.}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
-(2*a^5*(tan(c/2 + (d*x)/2)*56i + 273*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)^3*350i - 455*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*210i + 105*
tan(c/2 + (d*x)/2)^6 - 23))/(105*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)
/2)^2*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/
2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.74. $\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.75 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.75.1 Optimal result

Integrand size = 24, antiderivative size = 141

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = & -\frac{ia^5\cos^5(c+dx)}{105d}+\frac{a^5\sin(c+dx)}{21d} \\ & -\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin^5(c+dx)}{105d} \\ & -\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d} \\ & -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}\end{aligned}$$

output
```
-1/105*I*a^5*cos(d*x+c)^5/d+1/21*a^5*sin(d*x+c)/d-2/63*a^5*sin(d*x+c)^3/d+
1/105*a^5*sin(d*x+c)^5/d-2/63*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d-2/
9*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4/d
```

3.75.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{a^5\sec(c+dx)(-i\cos(5(c+dx))+\sin(5(c+dx)))\big(678\cos(c+dx)+475\cos(3(c+dx))+175\cos(5(c+}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*Sec[c + d*x]*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)])*(678*Cos[c +
d*x] + 475*Cos[3*(c + d*x)] + 175*Cos[5*(c + d*x)] + 1472*Sqrt[Cos[c + d*x
]^2]*Cos[5*(c + d*x)] - (120*I)*Sin[c + d*x] - (260*I)*Sin[3*(c + d*x)] -
(140*I)*Sin[5*(c + d*x)] - (1472*I)*Sqrt[Cos[c + d*x]^2]*Sin[5*(c + d*x)])
)/(5040*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3977, 3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \ 3977$$

$$\frac{1}{9}a^2\int \cos^7(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{9}a^2\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \ 3977$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\int \cos^5(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\int\frac{i\tan(c+dx)a+a}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow 3967$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(a\int\cos^5(c+dx)dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(a\int\sin\left(c+dx+\frac{\pi}{2}\right)^5dx-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow 3113$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(-\frac{a\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+}{7d}\right.$$
$$\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

$$\Big\downarrow 2009$$

$$\frac{1}{9}a^2\left(\frac{3}{7}a^2\left(-\frac{a\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^5(c+dx)}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\,\mathrm{ta}}{7d}\right.$$
$$\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(((-2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4)/d + (a^2*((3*a^2*((
(-1/5*I)*a*Cos[c + d*x]^5)/d - (a*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 -
Sin[c + d*x]^5/5))/d))/7 - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*
x])^2)/d))/9
```

3.75. $\int\cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.75.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(124) = 248$.

Time = 0.79 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.04

$$ia^5\left(-\frac{(\sin^4(dx+c))(\cos^5(dx+c))}{9}-\frac{4(\cos^5(dx+c))(\sin^2(dx+c))}{63}-\frac{8(\cos^5(dx+c))}{315}\right)+5a^5\Bigg(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9}-\frac{\sin(dx+c)}{}$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*cos(d*x+c)^5*sin(d*x+c)^2-
8/315*cos(d*x+c)^5)+5*a^5*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*
cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-10*I*a^
5*(-1/9*cos(d*x+c)^7*sin(d*x+c)^2-2/63*cos(d*x+c)^7)-10*a^5*(-1/9*cos(d*x+
c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)
*sin(d*x+c))-5/9*I*a^5*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d
*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{-35i\,a^5e^{(9i\,dx+9i\,c)}-180i\,a^5e^{(7i\,dx+7i\,c)}-378i\,a^5e^{(5i\,dx+5i\,c)}-420i\,a^5e^{(3i\,dx+3i\,c)}-315i\,a^5e^{(i\,dx+i\,c)}}{5040\,d}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/5040*(-35*I*a^5*e^(9*I*d*x + 9*I*c) - 180*I*a^5*e^(7*I*d*x + 7*I*c) - 37
8*I*a^5*e^(5*I*d*x + 5*I*c) - 420*I*a^5*e^(3*I*d*x + 3*I*c) - 315*I*a^5*e^
(I*d*x + I*c))/d
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\begin{cases}\frac{-215040ia^5d^4e^{9ic}e^{9idx}-1105920ia^5d^4e^{7ic}e^{7idx}-2322432ia^5d^4e^{5ic}e^{5idx}-2580480ia^5d^4e^{3ic}e^{3idx}-1935360ia^5d^4e^{ic}e^{idx}}{30965760d^5} & \text{for } d^5\neq 0\\ x\left(\frac{a^5e^{9ic}}{16}+\frac{a^5e^{7ic}}{4}+\frac{3a^5e^{5ic}}{8}+\frac{a^5e^{3ic}}{4}+\frac{a^5e^{ic}}{16}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-215040*I*a**5*d**4*exp(9*I*c)*exp(9*I*d*x) - 1105920*I*a**5*d
**4*exp(7*I*c)*exp(7*I*d*x) - 2322432*I*a**5*d**4*exp(5*I*c)*exp(5*I*d*x)
- 2580480*I*a**5*d**4*exp(3*I*c)*exp(3*I*d*x) - 1935360*I*a**5*d**4*exp(I*
c)*exp(I*d*x))/(30965760*d**5), Ne(d**5, 0)), (x*(a**5*exp(9*I*c)/16 + a**
5*exp(7*I*c)/4 + 3*a**5*exp(5*I*c)/8 + a**5*exp(3*I*c)/4 + a**5*exp(I*c)/1
6), True))
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.54

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{175i\,a^5\cos\left(dx+c\right)^9+i\left(35\cos\left(dx+c\right)^9-90\cos\left(dx+c\right)^7+63\cos\left(dx+c\right)^5\right)a^5+50i\left(7\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/315*(175*I*a^5*cos(d*x + c)^9 + I*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^
7 + 63*cos(d*x + c)^5)*a^5 + 50*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^
5 - 5*(35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^5 - 10
*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*
x + c)^3)*a^5 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)
^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5)/d
```

3.75.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1725 vs. 2(119) = 238.

Time = 0.95 (sec) , antiderivative size = 1725, normalized size of antiderivative = 12.23

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/41287680*(69853455*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 558827640*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 19558967
40*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^
(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(6*I*d*x
- 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740*a^5*e^(4*I*d*x - 4*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d
*x + I*c) + 1) + 4889741850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 6
9853455*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 70703325*a^5*e^(16*I*d
*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(14*I*d*x + 6*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x +
 I*c) - 1) + 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 1979693100*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5656266
00*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4949232750*a^5*e^(
8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 70703325*a^5*e^(-8*I*c)*log(I*e^(I*d
*x + I*c) - 1) - 69853455*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 558827640*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 19
55896740*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480
*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(
6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(4*I*d*...
```

3.75.9 Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{a^5\left(\frac{e^{c\,1i+d\,x\,1i}\,1i}{16}+\frac{e^{c\,3i+d\,x\,3i}\,1i}{12}+\frac{e^{c\,5i+d\,x\,5i}\,3i}{40}+\frac{e^{c\,7i+d\,x\,7i}\,1i}{28}+\frac{e^{c\,9i+d\,x\,9i}\,1i}{144}\right)}{d}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
-(a^5*((exp(c*1i + d*x*1i)*1i)/16 + (exp(c*3i + d*x*3i)*1i)/12 + (exp(c*5i
 + d*x*5i)*3i)/40 + (exp(c*7i + d*x*7i)*1i)/28 + (exp(c*9i + d*x*9i)*1i)/1
44))/d
```

3.76 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$

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3.76.9 Mupad [B] (verification not implemented) 719

3.76.1 Optimal result

Integrand size = 24, antiderivative size = 159

$$\begin{aligned}\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = &-\frac{5ia^5\cos^7(c+dx)}{231d}\\ &+\frac{5a^5\sin(c+dx)}{33d}-\frac{5a^5\sin^3(c+dx)}{33d}\\ &+\frac{a^5\sin^5(c+dx)}{11d}-\frac{5a^5\sin^7(c+dx)}{231d}\\ &-\frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d}\\ &-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}\end{aligned}$$

output

```
-5/231*I*a^5*cos(d*x+c)^7/d+5/33*a^5*sin(d*x+c)/d-5/33*a^5*sin(d*x+c)^3/d+
1/11*a^5*sin(d*x+c)^5/d-5/231*a^5*sin(d*x+c)^7/d-2/33*I*a^3*cos(d*x+c)^9*(
a+I*a*tan(d*x+c))^2/d-2/11*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^4/d
```

3.76.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{ia^5\sec(c+dx)(\cos(5(c+dx))+i\sin(5(c+dx)))\,\big(-1749\cos(c+dx)-1595\cos(3(c+dx))-665\cos(5(}{}$$

input
```
Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
((I/14784)*a^5*Sec[c + d*x]*(Cos[5*(c + d*x)] + I*Sin[5*(c + d*x)])*(-1749
*Cos[c + d*x] - 1595*Cos[3*(c + d*x)] - 665*Cos[5*(c + d*x)] - 2816*Sqrt[C
os[c + d*x]^2]*Cos[5*(c + d*x)] + 105*Cos[7*(c + d*x)] + (330*I)*Sin[c + d
*x] + (946*I)*Sin[3*(c + d*x)] + (490*I)*Sin[5*(c + d*x)] + (2816*I)*Sqrt[
Cos[c + d*x]^2]*Sin[5*(c + d*x)] - (126*I)*Sin[7*(c + d*x)]))/d
```

3.76.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3977, 3042, 3977, 3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^5}{\sec(c+dx)^{11}}dx$$

$$\Big\downarrow \; 3977$$

$$\frac{3}{11}a^2\int \cos^9(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{11}a^2\int \frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^9}dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\int \cos^7(c+dx)(i\tan(c+dx)a+a)dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\int \frac{i\tan(c+dx)a+a}{\sec(c+dx)^7}dx-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(a\int \cos^7(c+dx)dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{ia\cos^7(c+dx)}{7d}\right)-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(-\frac{a\int\left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-2\right.$$
$$\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{11}a^2\left(\frac{5}{9}a^2\left(-\frac{a\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{ia\cos^7(c+dx)}{7d}\right)-2ia\cos^9(c+\right.$$
$$\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

input
```
Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

3.76. $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$

output
```
(((-2*I)/11)*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^4)/d + (3*a^2*((5*a^
2*(((-1/7*I)*a*Cos[c + d*x]^7)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3
*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/d))/9 - (((2*I)/9)*a*Cos[c + d*x]^
9*(a + I*a*Tan[c + d*x])^2)/d))/11
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.76.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(140) = 280.

Time = 0.67 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.99

$$\frac{ia^5\left(-\frac{(\cos^7(dx+c))(\sin^4(dx+c))}{11}-\frac{4(\cos^7(dx+c))(\sin^2(dx+c))}{99}-\frac{8(\cos^7(dx+c))}{693}\right)+5a^5\left(-\frac{(\sin^3(dx+c))(\cos^8(dx+c))}{11}-\frac{(\cos^8(dx+}{}}{}$$

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x)
```

output
```
1/d*(I*a^5*(-1/11*cos(d*x+c)^7*sin(d*x+c)^4-4/99*cos(d*x+c)^7*sin(d*x+c)^2
-8/693*cos(d*x+c)^7)+5*a^5*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*cos(d*x+c
)^8*sin(d*x+c)+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)
*sin(d*x+c))-10*I*a^5*(-1/11*cos(d*x+c)^9*sin(d*x+c)^2-2/99*cos(d*x+c)^9)-
10*a^5*(-1/11*sin(d*x+c)*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d
*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-5/11*I*a^5*cos(
d*x+c)^11+1/11*a^5*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c
)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{\left(-21i\,a^5e^{(12i\,dx+12i\,c)}-154i\,a^5e^{(10i\,dx+10i\,c)}-495i\,a^5e^{(8i\,dx+8i\,c)}-924i\,a^5e^{(6i\,dx+6i\,c)}-1155i\,a^5e^{(4i\,dx+4i\,c)}-\right.}{14784\,d}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/14784*(-21*I*a^5*e^(12*I*d*x + 12*I*c) - 154*I*a^5*e^(10*I*d*x + 10*I*c)
 - 495*I*a^5*e^(8*I*d*x + 8*I*c) - 924*I*a^5*e^(6*I*d*x + 6*I*c) - 1155*I*
a^5*e^(4*I*d*x + 4*I*c) - 1386*I*a^5*e^(2*I*d*x + 2*I*c) + 231*I*a^5)*e^(-
I*d*x - I*c)/d
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.67

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \begin{cases} \frac{(-90194313216ia^5d^6e^{12ic}e^{11idx}-661424963584ia^5d^6e^{10ic}e^{9idx}-2126008811520ia^5d^6e^{8ic}e^{7idx}-3968549781504ia^5d^6e^{6ic}e^{5idx}-4960687226}{63496796504064d^7} \\ \frac{x(a^5e^{12ic}+6a^5e^{10ic}+15a^5e^{8ic}+20a^5e^{6ic}+15a^5e^{4ic}+6a^5e^{2ic}+a^5)e^{-ic}}{64} \end{cases}$$

input
```
integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**5,x)
```

output
```
Piecewise(((-90194313216*I*a**5*d**6*exp(12*I*c)*exp(11*I*d*x) - 661424963
584*I*a**5*d**6*exp(10*I*c)*exp(9*I*d*x) - 2126008811520*I*a**5*d**6*exp(8
*I*c)*exp(7*I*d*x) - 3968549781504*I*a**5*d**6*exp(6*I*c)*exp(5*I*d*x) - 4
960687226880*I*a**5*d**6*exp(4*I*c)*exp(3*I*d*x) - 5952824672256*I*a**5*d*
*6*exp(2*I*c)*exp(I*d*x) + 992137445376*I*a**5*d**6*exp(-I*d*x))*exp(-I*c)
/(63496796504064*d**7), Ne(d**7*exp(I*c), 0)), (x*(a**5*exp(12*I*c) + 6*a*
*5*exp(10*I*c) + 15*a**5*exp(8*I*c) + 20*a**5*exp(6*I*c) + 15*a**5*exp(4*I
*c) + 6*a**5*exp(2*I*c) + a**5)*exp(-I*c)/64, True))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.55

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{315i\,a^5\cos{(dx+c)}^{11}+i\left(63\cos{(dx+c)}^{11}-154\cos{(dx+c)}^{9}+99\cos{(dx+c)}^{7}\right)a^5+70i\left(9\cos{(dx}\right.}{}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/693*(315*I*a^5*cos(d*x + c)^11 + I*(63*cos(d*x + c)^11 - 154*cos(d*x +
c)^9 + 99*cos(d*x + c)^7)*a^5 + 70*I*(9*cos(d*x + c)^11 - 11*cos(d*x + c)^
9)*a^5 + 2*(315*sin(d*x + c)^11 - 1540*sin(d*x + c)^9 + 2970*sin(d*x + c)^
7 - 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a^5 + 3*(105*sin(d*x + c)^1
1 - 385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 231*sin(d*x + c)^5)*a^5 + (6
3*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(d*x + c)^7 - 1386*sin(d*x
 + c)^5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*a^5)/d
```

3.76.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1807 vs. $2(135) = 270$.

Time = 1.03 (sec) , antiderivative size = 1807, normalized size of antiderivative = 11.36

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/121110528*(168111405*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 47071
19340*a^5*e^(13*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5
*e^(11*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11767798350*a^5*e^(9*I*
d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^(7*I*d*x - I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 4707119340*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*
d*x + I*c) + 1) + 1344891240*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 168111405*a^5*e^(I*d*x - 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 17025
1620*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*
e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767045360*a^5*e^(13*I*d
*x + 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 9534090720*a^5*e^(11*I*d*x + 3*I*
c)*log(I*e^(I*d*x + I*c) - 1) + 11917613400*a^5*e^(9*I*d*x + I*c)*log(I*e^
(I*d*x + I*c) - 1) + 9534090720*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x + I*c
) - 1) + 4767045360*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1
362012960*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 170251620*a
^5*e^(I*d*x - 7*I*c)*log(I*e^(I*d*x + I*c) - 1) - 168111405*a^5*e^(17*I*d*
x + 9*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(15*I*d*x + 7*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 4707119340*a^5*e^(13*I*d*x + 5*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 9414238680*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 11767798350*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x + I*...
```

3.76.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(\frac{5\,\sin(3\,c+3\,d\,x)}{64}-\frac{\cos(5\,c+5\,d\,x)\,1\mathrm{i}}{16}-\frac{\cos(7\,c+7\,d\,x)\,15\mathrm{i}}{448}-\frac{\cos(9\,c+9\,d\,x)\,1\mathrm{i}}{96}-\frac{\cos(11\,c+11\,d\,x)\,1\mathrm{i}}{704}-\frac{\cos(3\,c+3\,d\,x)\,5\mathrm{i}}{64}+\frac{\sin(5\,c+5\,d}{16}\right.}{d}$$

input

```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^5,x)
```

output

```
(a^5*((5*sin(3*c + 3*d*x))/64 - (cos(5*c + 5*d*x)*1i)/16 - (cos(7*c + 7*d*
x)*15i)/448 - (cos(9*c + 9*d*x)*1i)/96 - (cos(11*c + 11*d*x)*1i)/704 - (co
s(3*c + 3*d*x)*5i)/64 + sin(5*c + 5*d*x)/16 + (15*sin(7*c + 7*d*x))/448 +
sin(9*c + 9*d*x)/96 + sin(11*c + 11*d*x)/704 + (24^(1/2)*cos(c - atanh(7/5
)*1i + d*x))/64))/d
```

3.76. $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$

3.77 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.77.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = &-\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}\\ &+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}\\ &-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}\\ &+\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}\end{aligned}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^12/a^4/d+12/13*I*(a+I*a*tan(d*x+c))^13/a^5/d-3/7
*I*(a+I*a*tan(d*x+c))^14/a^6/d+1/15*I*(a+I*a*tan(d*x+c))^15/a^7/d
```

3.77.2 Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\begin{aligned}&\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx\\ &=\frac{a^8(-i+\tan(c+dx))^{12}\left(-144i-363\tan(c+dx)+312i\tan^2(c+dx)+91\tan^3(c+dx)\right)}{1365d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*(-I + Tan[c + d*x])^12*(-144*I - 363*Tan[c + d*x] + (312*I)*Tan[c + d
*x]^2 + 91*Tan[c + d*x]^3))/(1365*d)
```

3.77.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^8dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int\left(-(i\tan(c+dx)a+a)^{14}+6a(i\tan(c+dx)a+a)^{13}-12a^2(i\tan(c+dx)a+a)^{12}+8a^3(i\tan(c+dx)a+a)\right.}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{3}a^3(a+ia\tan(c+dx))^{12}-\frac{12}{13}a^2(a+ia\tan(c+dx))^{13}-\frac{1}{15}(a+ia\tan(c+dx))^{15}+\frac{3}{7}a(a+ia\tan(c+dx))^{14}\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*((2*a^3*(a + I*a*Tan[c + d*x])^12)/3 - (12*a^2*(a + I*a*Tan[c + d*x]
)^13)/13 + (3*a*(a + I*a*Tan[c + d*x])^14)/7 - (a + I*a*Tan[c + d*x])^15/1
5))/(a^7*d)
```

3.77.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.77.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(93) = 186.

Time = 0.59 (sec) , antiderivative size = 611, normalized size of antiderivative = 5.61

$$a^{8}\left(\frac{\sin^{9}(dx+c)}{15\cos(dx+c)^{15}}+\frac{2\left(\sin^{9}(dx+c)\right)}{65\cos(dx+c)^{13}}+\frac{8\left(\sin^{9}(dx+c)\right)}{715\cos(dx+c)^{11}}+\frac{16\left(\sin^{9}(dx+c)\right)}{6435\cos(dx+c)^{9}}\right)+56ia^{8}\left(\frac{\sin^{6}(dx+c)}{12\cos(dx+c)^{12}}+\frac{\sin^{6}(dx+c)}{20\cos(dx+c)^{10}}+\frac{\sin^{6}(dx}{40\cos(dx}\right.$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(1/15*sin(d*x+c)^9/cos(d*x+c)^15+2/65*sin(d*x+c)^9/cos(d*x+c)^13+
8/715*sin(d*x+c)^9/cos(d*x+c)^11+16/6435*sin(d*x+c)^9/cos(d*x+c)^9)+56*I*a
^8*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10+1/40*s
in(d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)-28*a^8*(1/13*sin
(d*x+c)^7/cos(d*x+c)^13+6/143*sin(d*x+c)^7/cos(d*x+c)^11+8/429*sin(d*x+c)^
7/cos(d*x+c)^9+16/3003*sin(d*x+c)^7/cos(d*x+c)^7)-8*I*a^8*(1/14*sin(d*x+c)
^8/cos(d*x+c)^14+1/28*sin(d*x+c)^8/cos(d*x+c)^12+1/70*sin(d*x+c)^8/cos(d*x
+c)^10+1/280*sin(d*x+c)^8/cos(d*x+c)^8)+70*a^8*(1/11*sin(d*x+c)^5/cos(d*x+
c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c)^5/cos(d*x+c)^7+16/11
55*sin(d*x+c)^5/cos(d*x+c)^5)+I*a^8/cos(d*x+c)^8-28*a^8*(1/9*sin(d*x+c)^3/
cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^
5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)-56*I*a^8*(1/10*sin(d*x+c)^4/cos(d*x+c)
^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin
(d*x+c)^4/cos(d*x+c)^4)-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/3
5*sec(d*x+c)^2)*tan(d*x+c))
```

3.77.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(85) = 170$.

Time = 0.24 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.17

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{8192\,(-1365i\,a^8e^{(22i\,dx+22i\,c)} - 3003i\,a^8e^{(20i\,dx+20i\,c)} - 5005i\,a^8e^{(18i\,dx+18i\,c)} - 64}{1365\,(de^{(30i\,dx+30i\,c)} + 15\,de^{(28i\,dx+28i\,c)} + 105\,de^{(26i\,dx+26i\,c)} + 455\,de^{(24i\,dx+24i\,c)} + 1365\,de^{(22i\,dx+22i\,c)} + 30}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-8192/1365*(-1365*I*a^8*e^(22*I*d*x + 22*I*c) - 3003*I*a^8*e^(20*I*d*x + 2
0*I*c) - 5005*I*a^8*e^(18*I*d*x + 18*I*c) - 6435*I*a^8*e^(16*I*d*x + 16*I*
c) - 6435*I*a^8*e^(14*I*d*x + 14*I*c) - 5005*I*a^8*e^(12*I*d*x + 12*I*c) -
 3003*I*a^8*e^(10*I*d*x + 10*I*c) - 1365*I*a^8*e^(8*I*d*x + 8*I*c) - 455*I
*a^8*e^(6*I*d*x + 6*I*c) - 105*I*a^8*e^(4*I*d*x + 4*I*c) - 15*I*a^8*e^(2*I
*d*x + 2*I*c) - I*a^8)/(d*e^(30*I*d*x + 30*I*c) + 15*d*e^(28*I*d*x + 28*I*
c) + 105*d*e^(26*I*d*x + 26*I*c) + 455*d*e^(24*I*d*x + 24*I*c) + 1365*d*e^
(22*I*d*x + 22*I*c) + 3003*d*e^(20*I*d*x + 20*I*c) + 5005*d*e^(18*I*d*x +
18*I*c) + 6435*d*e^(16*I*d*x + 16*I*c) + 6435*d*e^(14*I*d*x + 14*I*c) + 50
05*d*e^(12*I*d*x + 12*I*c) + 3003*d*e^(10*I*d*x + 10*I*c) + 1365*d*e^(8*I*
d*x + 8*I*c) + 455*d*e^(6*I*d*x + 6*I*c) + 105*d*e^(4*I*d*x + 4*I*c) + 15*
d*e^(2*I*d*x + 2*I*c) + d)
```

3.77.6 Sympy [F]

$$
\begin{aligned}
\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\bigg(\int \left(-28\tan^2{(c+dx)}\sec^8{(c+dx)}\right)\,dx \\
+ \int 70\tan^4{(c+dx)}\sec^8{(c+dx)}\,dx \\
+ \int \left(-28\tan^6{(c+dx)}\sec^8{(c+dx)}\right)\,dx \\
+ \int \tan^8{(c+dx)}\sec^8{(c+dx)}\,dx \\
+ \int 8i\tan{(c+dx)}\sec^8{(c+dx)}\,dx \\
+ \int \left(-56i\tan^3{(c+dx)}\sec^8{(c+dx)}\right)\,dx \\
+ \int 56i\tan^5{(c+dx)}\sec^8{(c+dx)}\,dx \\
+ \int \left(-8i\tan^7{(c+dx)}\sec^8{(c+dx)}\right)\,dx \\
+ \int \sec^8{(c+dx)}\,dx\bigg)
\end{aligned}
$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**8, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**8, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**8, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**8, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**8, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**8, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**8, x) + Integral(sec(c + d*x)**8, x))
```

3.77.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(85) = 170$.

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{91\,a^8\tan{(dx+c)}^{15} - 780i\,a^8\tan{(dx+c)}^{14} - 2625\,a^8\tan{(dx+c)}^{13} + 3640i\,a^8\tan{(dx+c)}^{12} - 1365\,a^8\,\mathrm{t}}{}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(
d*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 1201
2*I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c
)^7 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*t
an(d*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 136
5*a^8*tan(d*x + c))/d
```

3.77.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(85) = 170$.

Time = 1.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{91\,a^8\tan{(dx+c)}^{15} - 780i\,a^8\tan{(dx+c)}^{14} - 2625\,a^8\tan{(dx+c)}^{13} + 3640i\,a^8\tan{(dx+c)}^{12} - 1365\,a^8\,\mathrm{t}}{}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(
d*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 1201
2*I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c
)^7 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*t
an(d*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 136
5*a^8*tan(d*x + c))/d
```

3.77.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\left(\frac{\sin(9\,c+9\,d\,x)}{12}+\frac{\sin(11\,c+11\,d\,x)}{52}+\frac{\sin(13\,c+13\,d\,x)}{364}+\frac{\sin(15\,c+15\,d\,x)}{5460}+\frac{\cos(c+d\,x)\,297\mathrm{i}}{7168}+\frac{\cos(3\,c+3\,d\,x)\,33\mathrm{i}}{1024}+\frac{\cos(5\,c+5\,d\,x)\,99}{5120}\right.}{d\cos\left(c+d\,x\right)^{15}}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^8,x)
```

output
```
(a^8*((cos(c + d*x)*297i)/7168 + (cos(3*c + 3*d*x)*33i)/1024 + (cos(5*c +
5*d*x)*99i)/5120 + (cos(7*c + 7*d*x)*9i)/1024 - (cos(9*c + 9*d*x)*247i)/30
72 - (cos(11*c + 11*d*x)*19i)/1024 - (cos(13*c + 13*d*x)*19i)/7168 - (cos(
15*c + 15*d*x)*19i)/107520 + sin(9*c + 9*d*x)/12 + sin(11*c + 11*d*x)/52 +
 sin(13*c + 13*d*x)/364 + sin(15*c + 15*d*x)/5460))/(d*cos(c + d*x)^15)
```

3.78 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.78.1 Optimal result 728
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3.78.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4d}-\frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}$$

output
```
-4/11*I*(a+I*a*tan(d*x+c))^11/a^3/d+1/3*I*(a+I*a*tan(d*x+c))^12/a^4/d-1/13
*I*(a+I*a*tan(d*x+c))^13/a^5/d
```

3.78.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx=\frac{a^8(-i+\tan(c+dx))^{11}\left(-46+77i\tan(c+dx)+33\tan^2(c+dx)\right)}{429d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*(-I + Tan[c + d*x])^11*(-46 + (77*I)*Tan[c + d*x] + 33*Tan[c + d*x]^2
))/(429*d)
```

3.78.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^8dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{10}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{12}-4a(i\tan(c+dx)a+a)^{11}+4a^2(i\tan(c+dx)a+a)^{10}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{4}{11}a^2(a+ia\tan(c+dx))^{11}+\frac{1}{13}(a+ia\tan(c+dx))^{13}-\frac{1}{3}a(a+ia\tan(c+dx))^{12})}{a^5d}$$

input

```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-I)*((4*a^2*(a + I*a*Tan[c + d*x])^11)/11 - (a*(a + I*a*Tan[c + d*x])^12
)/3 + (a + I*a*Tan[c + d*x])^13/13))/(a^5*d)
```

3.78.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.78.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(70) = 140.

Time = 0.58 (sec) , antiderivative size = 475, normalized size of antiderivative = 5.79

$$a^8\left(\frac{\sin^9(dx+c)}{13\cos(dx+c)^{13}}+\frac{4(\sin^9(dx+c))}{143\cos(dx+c)^{11}}+\frac{8(\sin^9(dx+c))}{1287\cos(dx+c)^9}\right)+56ia^8\left(\frac{\sin^6(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{60\cos(dx+c)^6}\right)-28a^8\Big($$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(1/13*sin(d*x+c)^9/cos(d*x+c)^13+4/143*sin(d*x+c)^9/cos(d*x+c)^11
+8/1287*sin(d*x+c)^9/cos(d*x+c)^9)+56*I*a^8*(1/10*sin(d*x+c)^6/cos(d*x+c)^
10+1/20*sin(d*x+c)^6/cos(d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6)-28*a^8*(
1/11*sin(d*x+c)^7/cos(d*x+c)^11+4/99*sin(d*x+c)^7/cos(d*x+c)^9+8/693*sin(d
*x+c)^7/cos(d*x+c)^7)-8*I*a^8*(1/12*sin(d*x+c)^8/cos(d*x+c)^12+1/30*sin(d*
x+c)^8/cos(d*x+c)^10+1/120*sin(d*x+c)^8/cos(d*x+c)^8)+70*a^8*(1/9*sin(d*x+
c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*
x+c)^5)-56*I*a^8*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+
c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)-28*a^8*(1/7*sin(d*x+c)^3/cos(d*x+c)^7
+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+4/3*I*a^8
/cos(d*x+c)^6-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

3.78.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(64) = 128.

Time = 0.23 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.74

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{4096\,(-286i\,a^8e^{(20i\,dx+20i\,c)} - 715i\,a^8e^{(18i\,dx+18i\,c)} - 1287i\,a^8e^{(16i\,dx+16i\,c)} - 1716i\,a^8e^{(14i\,dx}}{429\,(de^{(26i\,dx+26i\,c)} + 13\,de^{(24i\,dx+24i\,c)} + 78\,de^{(22i\,dx+22i\,c)} + 286\,de^{(20i\,dx+20i\,c)} + 715\,de^{(18i\,dx+18i\,c)} + 1287\,}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-4096/429*(-286*I*a^8*e^(20*I*d*x + 20*I*c) - 715*I*a^8*e^(18*I*d*x + 18*I
*c) - 1287*I*a^8*e^(16*I*d*x + 16*I*c) - 1716*I*a^8*e^(14*I*d*x + 14*I*c)
- 1716*I*a^8*e^(12*I*d*x + 12*I*c) - 1287*I*a^8*e^(10*I*d*x + 10*I*c) - 71
5*I*a^8*e^(8*I*d*x + 8*I*c) - 286*I*a^8*e^(6*I*d*x + 6*I*c) - 78*I*a^8*e^(
4*I*d*x + 4*I*c) - 13*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(26*I*d*x +
26*I*c) + 13*d*e^(24*I*d*x + 24*I*c) + 78*d*e^(22*I*d*x + 22*I*c) + 286*d*
e^(20*I*d*x + 20*I*c) + 715*d*e^(18*I*d*x + 18*I*c) + 1287*d*e^(16*I*d*x +
 16*I*c) + 1716*d*e^(14*I*d*x + 14*I*c) + 1716*d*e^(12*I*d*x + 12*I*c) + 1
287*d*e^(10*I*d*x + 10*I*c) + 715*d*e^(8*I*d*x + 8*I*c) + 286*d*e^(6*I*d*x
 + 6*I*c) + 78*d*e^(4*I*d*x + 4*I*c) + 13*d*e^(2*I*d*x + 2*I*c) + d)
```

3.78.6 Sympy [F]

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\Bigg(\int \left(-28\tan^2(c+dx)\sec^6(c+dx)\right)dx \\ + \int 70\tan^4(c+dx)\sec^6(c+dx)\,dx \\ + \int \left(-28\tan^6(c+dx)\sec^6(c+dx)\right)dx \\ + \int \tan^8(c+dx)\sec^6(c+dx)\,dx \\ + \int 8i\tan(c+dx)\sec^6(c+dx)\,dx \\ + \int \left(-56i\tan^3(c+dx)\sec^6(c+dx)\right)dx \\ + \int 56i\tan^5(c+dx)\sec^6(c+dx)\,dx \\ + \int \left(-8i\tan^7(c+dx)\sec^6(c+dx)\right)dx \\ + \int \sec^6(c+dx)\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**6, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**6, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**6, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**6, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**6, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**6, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x))
```

3.78.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{33\,a^8\tan(dx+c)^{13} - 286i\,a^8\tan(dx+c)^{12} - 1014\,a^8\tan(dx+c)^{11} + 1716i\,a^8\tan(dx+c)^{10} + 715\,a^8\,\mathrm{ta}}{}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d
*x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*
a^8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 +
 1287*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x +
c)^3 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

3.78.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 1.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{33\,a^8\tan\left(dx+c\right)^{13} - 286i\,a^8\tan\left(dx+c\right)^{12} - 1014\,a^8\tan\left(dx+c\right)^{11} + 1716i\,a^8\tan\left(dx+c\right)^{10} + 715\,a^8\,\mathrm{ta}}{}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d
*x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*
a^8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 +
 1287*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x +
c)^3 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

3.78.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\,\sin\left(c+d\,x\right)\,\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(-184\sin\left(c+d\,x\right)^2-184\sin\left(2\,c+2\,d\,x\right)^2+\frac{\sin(2\,c+2\,d\,x)\,9867\mathrm{i}}{256}-18}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^6,x)
```

output
```
(a^8*sin(c + d*x)*(2*sin(c/2 + (d*x)/2)^2 - 1)*((sin(2*c + 2*d*x)*9867i)/2
56 + (sin(4*c + 4*d*x)*69069i)/1024 + (sin(6*c + 6*d*x)*42757i)/512 + (sin
(8*c + 8*d*x)*23023i)/256 + (sin(10*c + 10*d*x)*7007i)/512 + (sin(12*c + 1
2*d*x)*1001i)/1024 - 184*sin(2*c + 2*d*x)^2 - 184*sin(3*c + 3*d*x)^2 - 184
*sin(4*c + 4*d*x)^2 - 28*sin(5*c + 5*d*x)^2 - 2*sin(6*c + 6*d*x)^2 - 184*s
in(c + d*x)^2 + 429))/(429*d*(sin(c + d*x)^2 - 1)^7)
```

3.78. $\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.79 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.79.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}+\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

output

```
-1/5*I*(a+I*a*tan(d*x+c))^10/a^2/d+1/11*I*(a+I*a*tan(d*x+c))^11/a^3/d
```

3.79.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx=\frac{a^8(-i+\tan(c+dx))^{10}(6i+5\tan(c+dx))}{55d}$$

input

```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
(a^8*(-I + Tan[c + d*x])^10*(6*I + 5*Tan[c + d*x]))/(55*d)
```

3.79.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^8dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^9d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^9-(i\tan(c+dx)a+a)^{10}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{5}a(a+ia\tan(c+dx))^{10}-\frac{1}{11}(a+ia\tan(c+dx))^{11}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*((a*(a + I*a*Tan[c + d*x])^10)/5 - (a + I*a*Tan[c + d*x])^11/11))/(a
^3*d)
```

3.79.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.79.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(47) = 94$.

Time = 286.48 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

method	result
risch	$\frac{1024ia^8\left(55\,\mathrm{e}^{18i(dx+c)}+165\,\mathrm{e}^{16i(dx+c)}+330\,\mathrm{e}^{14i(dx+c)}+462\,\mathrm{e}^{12i(dx+c)}+462\,\mathrm{e}^{10i(dx+c)}+330\,\mathrm{e}^{8i(dx+c)}+165\,\mathrm{e}^{6i(dx+c)}+5}{55d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{11}}$
derivativedivides	$a^8\left(\frac{\sin^9(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^9(dx+c)\right)}{99\cos(dx+c)^9}\right)-56ia^8\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-28a^8\left(\frac{\sin^7(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^7(dx+c)\right)}{63\cos(dx+c)^7}\right)+$
default	$a^8\left(\frac{\sin^9(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^9(dx+c)\right)}{99\cos(dx+c)^9}\right)-56ia^8\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-28a^8\left(\frac{\sin^7(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^7(dx+c)\right)}{63\cos(dx+c)^7}\right)+$

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1024/55*I*a^8*(55*exp(18*I*(d*x+c))+165*exp(16*I*(d*x+c))+330*exp(14*I*(d*
x+c))+462*exp(12*I*(d*x+c))+462*exp(10*I*(d*x+c))+330*exp(8*I*(d*x+c))+165
*exp(6*I*(d*x+c))+55*exp(4*I*(d*x+c))+11*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d
*x+c))+1)^11
```

3.79.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(43) = 86$.

Time = 0.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.89

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{1024\,\big(-55i\,a^8e^{(18i\,dx+18i\,c)} - 165i\,a^8e^{(16i\,dx+16i\,c)} - 330i\,a^8e^{(14i\,dx+14i\,c)} - 462i\,a^8e^{(12i\,dx+12i\,c)} - 462i}{55\,\big(de^{(22i\,dx+22i\,c)} + 11\,de^{(20i\,dx+20i\,c)} + 55\,de^{(18i\,dx+18i\,c)} + 165\,de^{(16i\,dx+16i\,c)} + 330\,de^{(14i\,dx+14i\,c)} + 462\,de^{(}}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-1024/55*(-55*I*a^8*e^(18*I*d*x + 18*I*c) - 165*I*a^8*e^(16*I*d*x + 16*I*c
) - 330*I*a^8*e^(14*I*d*x + 14*I*c) - 462*I*a^8*e^(12*I*d*x + 12*I*c) - 46
2*I*a^8*e^(10*I*d*x + 10*I*c) - 330*I*a^8*e^(8*I*d*x + 8*I*c) - 165*I*a^8*
e^(6*I*d*x + 6*I*c) - 55*I*a^8*e^(4*I*d*x + 4*I*c) - 11*I*a^8*e^(2*I*d*x +
 2*I*c) - I*a^8)/(d*e^(22*I*d*x + 22*I*c) + 11*d*e^(20*I*d*x + 20*I*c) + 5
5*d*e^(18*I*d*x + 18*I*c) + 165*d*e^(16*I*d*x + 16*I*c) + 330*d*e^(14*I*d*
x + 14*I*c) + 462*d*e^(12*I*d*x + 12*I*c) + 462*d*e^(10*I*d*x + 10*I*c) +
330*d*e^(8*I*d*x + 8*I*c) + 165*d*e^(6*I*d*x + 6*I*c) + 55*d*e^(4*I*d*x +
4*I*c) + 11*d*e^(2*I*d*x + 2*I*c) + d)
```

3.79.6 Sympy [F]

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\bigg(\int \left(-28\tan^2{(c+dx)}\sec^4{(c+dx)}\right)\,dx \\ + \int 70\tan^4{(c+dx)}\sec^4{(c+dx)}\,dx \\ + \int \left(-28\tan^6{(c+dx)}\sec^4{(c+dx)}\right)\,dx \\ + \int \tan^8{(c+dx)}\sec^4{(c+dx)}\,dx \\ + \int 8i\tan{(c+dx)}\sec^4{(c+dx)}\,dx \\ + \int \left(-56i\tan^3{(c+dx)}\sec^4{(c+dx)}\right)\,dx \\ + \int 56i\tan^5{(c+dx)}\sec^4{(c+dx)}\,dx \\ + \int \left(-8i\tan^7{(c+dx)}\sec^4{(c+dx)}\right)\,dx \\ + \int \sec^4{(c+dx)}\,dx\bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**4, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**4, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**4, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**4, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**4, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**4, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

3.79.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(43) = 86$.

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{5\,a^8\tan{(dx+c)}^{11} - 44i\,a^8\tan{(dx+c)}^{10} - 165\,a^8\tan{(dx+c)}^9 + 330i\,a^8\tan{(dx+c)}^8 + 330\,a^8\tan{(dx}{}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
 c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*
x + c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*t
an(d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```

3.79.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(43) = 86.

Time = 1.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{5\,a^8\tan\left(dx+c\right)^{11}-44i\,a^8\tan\left(dx+c\right)^{10}-165\,a^8\tan\left(dx+c\right)^{9}+330i\,a^8\tan\left(dx+c\right)^{8}+330\,a^8\tan\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
 c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*
x + c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*t
an(d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```

3.79.9 Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\left(\frac{\sin(9\,c+9\,d\,x)}{10}+\frac{\sin(11\,c+11\,d\,x)}{110}+\frac{\cos(c+d\,x)\,63\mathrm{i}}{1280}+\frac{\cos(3\,c+3\,d\,x)\,9\mathrm{i}}{256}+\frac{\cos(5\,c+5\,d\,x)\,9\mathrm{i}}{512}+\frac{\cos(7\,c+7\,d\,x)\,3\mathrm{i}}{512}-\frac{\cos(9\,c+9\,d\,x)\,253\mathrm{i}}{2560}\right.}{d\cos\left(c+d\,x\right)^{11}}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^4,x)
```

output
```
(a^8*((cos(c + d*x)*63i)/1280 + (cos(3*c + 3*d*x)*9i)/256 + (cos(5*c + 5*d
*x)*9i)/512 + (cos(7*c + 7*d*x)*3i)/512 - (cos(9*c + 9*d*x)*253i)/2560 - (
cos(11*c + 11*d*x)*23i)/2560 + sin(9*c + 9*d*x)/10 + sin(11*c + 11*d*x)/11
0))/(d*cos(c + d*x)^11)
```

3.79. $\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.80 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.80.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

output
```
-1/9*I*(a+I*a*tan(d*x+c))^9/a/d
```

3.80.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 102 vs. 2(27) = 54.

Time = 0.67 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan(c+dx)\left(9+36i\tan(c+dx)-84\tan^2(c+dx)-126i\tan^3(c+dx)+126\tan^4(c+dx)+84i\tan^5(\right.}{9d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Tan[c + d*x]*(9 + (36*I)*Tan[c + d*x] - 84*Tan[c + d*x]^2 - (126*I)*T
an[c + d*x]^3 + 126*Tan[c + d*x]^4 + (84*I)*Tan[c + d*x]^5 - 36*Tan[c + d*
x]^6 - (9*I)*Tan[c + d*x]^7 + Tan[c + d*x]^8))/(9*d)
```

3.80.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^8dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^8d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/9*I)*(a + I*a*Tan[c + d*x])^9)/(a*d)
```

3.80.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.80.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(23) = 46$.

Time = 111.63 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.19

method	result
risch	$\frac{512ia^8 (9\,\mathrm{e}^{16i(dx+c)}+36\,\mathrm{e}^{14i(dx+c)}+84\,\mathrm{e}^{12i(dx+c)}+126\,\mathrm{e}^{10i(dx+c)}+126\,\mathrm{e}^{8i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+36\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{2i(dx+}}{9d(\mathrm{e}^{2i(dx+c)}+1)^9}$
derivativedivides	$\frac{\frac{a^8(\sin^9(dx+c))}{9\cos(dx+c)^9}+\frac{28ia^8(\sin^6(dx+c))}{3\cos(dx+c)^6}-\frac{4a^8(\sin^7(dx+c))}{\cos(dx+c)^7}+\frac{4ia^8}{\cos(dx+c)^2}+\frac{14a^8(\sin^5(dx+c))}{\cos(dx+c)^5}-\frac{14ia^8(\sin^4(dx+c))}{\cos(dx+c)^4}-\frac{28a^8(\sin^3(dx}{3\cos(dx+c}}{d}$
default	$\frac{\frac{a^8(\sin^9(dx+c))}{9\cos(dx+c)^9}+\frac{28ia^8(\sin^6(dx+c))}{3\cos(dx+c)^6}-\frac{4a^8(\sin^7(dx+c))}{\cos(dx+c)^7}+\frac{4ia^8}{\cos(dx+c)^2}+\frac{14a^8(\sin^5(dx+c))}{\cos(dx+c)^5}-\frac{14ia^8(\sin^4(dx+c))}{\cos(dx+c)^4}-\frac{28a^8(\sin^3(dx}{3\cos(dx+c}}{d}$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
512/9*I*a^8*(9*exp(16*I*(d*x+c))+36*exp(14*I*(d*x+c))+84*exp(12*I*(d*x+c))
+126*exp(10*I*(d*x+c))+126*exp(8*I*(d*x+c))+84*exp(6*I*(d*x+c))+36*exp(4*I
*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^9
```

3.80.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 8.56

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{512\,(-9i\,a^8e^{(16i\,dx+16i\,c)}-36i\,a^8e^{(14i\,dx+14i\,c)}-84i\,a^8e^{(12i\,dx+12i\,c)}-126i\,a^8e^{(10i\,dx+10i\,c)}-126i\,a^8e^{(8i\,dx+8}}{9\,(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8}}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-512/9*(-9*I*a^8*e^(16*I*d*x + 16*I*c) - 36*I*a^8*e^(14*I*d*x + 14*I*c) -
84*I*a^8*e^(12*I*d*x + 12*I*c) - 126*I*a^8*e^(10*I*d*x + 10*I*c) - 126*I*a
^8*e^(8*I*d*x + 8*I*c) - 84*I*a^8*e^(6*I*d*x + 6*I*c) - 36*I*a^8*e^(4*I*d*
x + 4*I*c) - 9*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(18*I*d*x + 18*I*c)
 + 9*d*e^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d
*x + 12*I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 8
4*d*e^(6*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*
c) + d)
```

3.80.6 Sympy [F]

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\Bigg(\int \left(-28\tan^2{(c+dx)}\sec^2{(c+dx)}\right)\,dx \\ + \int 70\tan^4{(c+dx)}\sec^2{(c+dx)}\,dx \\ + \int \left(-28\tan^6{(c+dx)}\sec^2{(c+dx)}\right)\,dx \\ + \int \tan^8{(c+dx)}\sec^2{(c+dx)}\,dx \\ + \int 8i\tan{(c+dx)}\sec^2{(c+dx)}\,dx \\ + \int \left(-56i\tan^3{(c+dx)}\sec^2{(c+dx)}\right)\,dx \\ + \int 56i\tan^5{(c+dx)}\sec^2{(c+dx)}\,dx \\ + \int \left(-8i\tan^7{(c+dx)}\sec^2{(c+dx)}\right)\,dx \\ + \int \sec^2{(c+dx)}\,dx\Bigg)\end{aligned}$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(70*tan(c
 + d*x)**4*sec(c + d*x)**2, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)
**2, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**2, x) + Integral(8*I*tan(
c + d*x)*sec(c + d*x)**2, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)
**2, x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**2, x) + Integral(-8*
I*tan(c + d*x)**7*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\left(i\,a\tan\left(dx+c\right)+a\right)^9}{9\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/9*I*(I*a*tan(d*x + c) + a)^9/(a*d)
```

3.80.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.

Time = 1.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.44

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan\left(dx+c\right)^9 - 9i\,a^8\tan\left(dx+c\right)^8 - 36\,a^8\tan\left(dx+c\right)^7 + 84i\,a^8\tan\left(dx+c\right)^6 + 126\,a^8\tan\left(dx+c\right)^5 -}{9\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/9*(a^8*tan(d*x + c)^9 - 9*I*a^8*tan(d*x + c)^8 - 36*a^8*tan(d*x + c)^7 +
 84*I*a^8*tan(d*x + c)^6 + 126*a^8*tan(d*x + c)^5 - 126*I*a^8*tan(d*x + c)
^4 - 84*a^8*tan(d*x + c)^3 + 36*I*a^8*tan(d*x + c)^2 + 9*a^8*tan(d*x + c))
/d
```

3.80.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(\sin\left(9\,c+9\,d\,x\right)+\frac{\cos(c+d\,x)\,63\mathrm{i}}{128}+\frac{\cos(3\,c+3\,d\,x)\,21\mathrm{i}}{64}+\frac{\cos(5\,c+5\,d\,x)\,9\mathrm{i}}{64}+\frac{\cos(7\,c+7\,d\,x)\,9\mathrm{i}}{256}-\frac{\cos(9\,c+9\,d\,x)\,255\mathrm{i}}{256}\right)}{9\,d\cos\left(c+d\,x\right)^9}$$

input
```
int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^2,x)
```

output
```
(a^8*((cos(c + d*x)*63i)/128 + (cos(3*c + 3*d*x)*21i)/64 + (cos(5*c + 5*d*
x)*9i)/64 + (cos(7*c + 7*d*x)*9i)/256 - (cos(9*c + 9*d*x)*255i)/256 + sin(
9*c + 9*d*x)))/(9*d*cos(c + d*x)^9)
```

3.80. $\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.81 $\int (a+ia\tan(c+dx))^8\,dx$

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3.81.1 Optimal result

Integrand size = 15, antiderivative size = 200

$$\begin{aligned}\int (a+ia\tan(c+dx))^8\,dx = {} & 128a^8x-\frac{128ia^8\log(\cos(c+dx))}{d}-\frac{64a^8\tan(c+dx)}{d}\\ &+\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d}\\ &+\frac{ia(a+ia\tan(c+dx))^7}{7d}+\frac{16ia^2\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}\\ &+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)^4}{d}+\frac{16i\left(a^4+ia^4\tan(c+dx)\right)^2}{d}\end{aligned}$$

output
```
128*a^8*x-128*I*a^8*ln(cos(d*x+c))/d-64*a^8*tan(d*x+c)/d+4/5*I*a^3*(a+I*a*
tan(d*x+c))^5/d+1/3*I*a^2*(a+I*a*tan(d*x+c))^6/d+1/7*I*a*(a+I*a*tan(d*x+c)
)^7/d+16/3*I*a^2*(a^2+I*a^2*tan(d*x+c))^3/d+2*I*(a^2+I*a^2*tan(d*x+c))^4/d
+16*I*(a^4+I*a^4*tan(d*x+c))^2/d
```

3.81.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.50

$$\begin{aligned}&\int (a+ia\tan(c+dx))^8\,dx\\ &=\frac{a^8(13440i\log(i+\tan(c+dx))-13335\tan(c+dx)-6300i\tan^2(c+dx)+3465\tan^3(c+dx)+1680i\,\mathrm{ta}}{105d}\end{aligned}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*((13440*I)*Log[I + Tan[c + d*x]] - 13335*Tan[c + d*x] - (6300*I)*Tan[
c + d*x]^2 + 3465*Tan[c + d*x]^3 + (1680*I)*Tan[c + d*x]^4 - 609*Tan[c + d
*x]^5 - (140*I)*Tan[c + d*x]^6 + 15*Tan[c + d*x]^7))/(105*d)
```

3.81.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.067$, Rules used = {3042, 3959, 3042, 3959, 3042, 3959, 3042, 3959, 3042, 3959, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(c+dx))^8dx$$

$$\Big\downarrow \; 3959$$

$$2a\int (i\tan(c+dx)a+a)^7dx+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \; 3042$$

$$2a\int (i\tan(c+dx)a+a)^7dx+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \; 3959$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^6dx+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \; 3042$$

$$2a\left(2a\int (i\tan(c+dx)a+a)^6dx+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \; 3959$$

$$2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^5dx+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow\ 3042$$

$$2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^5dx+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\frac{ia(a+ia\tan(c+dx))^6}{6d}\right)+\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow\ 3959$$

$$2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow\ 3042$$

$$2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^4dx+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\frac{ia(a+ia\tan(c+dx))^5}{5d}\right)+\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow\ 3959$$

$$2a\left(2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\right.\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow\ 3042$$

$$2a\left(2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{ia(a+ia\tan(c+dx))^4}{4d}\right)+\right.\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow\ 3959$$

$$2a\left(2a\left(2a\left(2a\left(2a\left(2a\int(i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\right.\right.\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow\ 3042$$

3.81. $\int(a+ia\tan(c+dx))^8\,dx$

$$2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\int(i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\Bigg)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\Bigg)+ia(a$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow 3958$$

$$2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2ia^2\int\tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\Bigg)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\Bigg)+ia(a+i$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow 3042$$

$$2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2ia^2\int\tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\Bigg)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\Bigg)+ia(a+i$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow 3956$$

$$2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(2a\Bigg(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\Bigg)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\Bigg)+ia(a+$$

$$\frac{ia(a+ia\tan(c+dx))^7}{7d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/7)*a*(a + I*a*Tan[c + d*x])^7)/d + 2*a*(((I/6)*a*(a + I*a*Tan[c + d*x]
)^6)/d + 2*a*(((I/5)*a*(a + I*a*Tan[c + d*x])^5)/d + 2*a*(((I/4)*a*(a + I*
a*Tan[c + d*x])^4)/d + 2*a*(((I/3)*a*(a + I*a*Tan[c + d*x])^3)/d + 2*a*(((
I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)*a^2*Log[Cos[c +
 d*x]])/d - (a^2*Tan[c + d*x])/d))))))
```

3.81. $\int(a+ia\tan(c+dx))^8\,dx$

3.81.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

3.81.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{a^8\left(-127\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{4i\left(\tan^6(dx+c)\right)}{3}-\frac{29\left(\tan^5(dx+c)\right)}{5}+16i\left(\tan^4(dx+c)\right)+33\left(\tan^3(dx+c)\right)-60i\left(\tan^2(\right.\right.}{d}$
default	$\frac{a^8\left(-127\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{4i\left(\tan^6(dx+c)\right)}{3}-\frac{29\left(\tan^5(dx+c)\right)}{5}+16i\left(\tan^4(dx+c)\right)+33\left(\tan^3(dx+c)\right)-60i\left(\tan^2(\right.\right.}{d}$
risch	$-\frac{256a^8c}{d}-\frac{32ia^8\left(2940\,\mathrm{e}^{12i(dx+c)}+13230\,\mathrm{e}^{10i(dx+c)}+26950\,\mathrm{e}^{8i(dx+c)}+30625\,\mathrm{e}^{6i(dx+c)}+20139\,\mathrm{e}^{4i(dx+c)}+7203\,\mathrm{e}^{2i(d\right.}{105d\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}$
parallelrisch	$\frac{-140ia^8\left(\tan^6(dx+c)\right)+15\left(\tan^7(dx+c)\right)a^8+1680ia^8\left(\tan^4(dx+c)\right)-609\left(\tan^5(dx+c)\right)a^8-6300ia^8\left(\tan^2(dx+c)\right)+3465}{105d}$
norman	$128a^8x-\frac{127a^8\tan(dx+c)}{d}+\frac{33a^8\left(\tan^3(dx+c)\right)}{d}-\frac{29a^8\left(\tan^5(dx+c)\right)}{5d}+\frac{a^8\left(\tan^7(dx+c)\right)}{7d}-\frac{60ia^8\left(\tan^2(dx+\right.}{d}$
parts	$a^8x+\frac{a^8\left(\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}-\frac{56ia^8\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/d*a^8*(-127*tan(d*x+c)+1/7*tan(d*x+c)^7-4/3*I*tan(d*x+c)^6-29/5*tan(d*x+
c)^5+16*I*tan(d*x+c)^4+33*tan(d*x+c)^3-60*I*tan(d*x+c)^2+64*I*ln(1+tan(d*x
+c)^2)+128*arctan(tan(d*x+c)))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int (a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{32\left(2940i\,a^8e^{(12i\,dx+12i\,c)}+13230i\,a^8e^{(10i\,dx+10i\,c)}+26950i\,a^8e^{(8i\,dx+8i\,c)}+30625i\,a^8e^{(6i\,dx+6i\,c)}+20139i\,a\right.}{105\left(de^{(14i}\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-32/105*(2940*I*a^8*e^(12*I*d*x + 12*I*c) + 13230*I*a^8*e^(10*I*d*x + 10*I
*c) + 26950*I*a^8*e^(8*I*d*x + 8*I*c) + 30625*I*a^8*e^(6*I*d*x + 6*I*c) +
20139*I*a^8*e^(4*I*d*x + 4*I*c) + 7203*I*a^8*e^(2*I*d*x + 2*I*c) + 1089*I*
a^8 + 420*(I*a^8*e^(14*I*d*x + 14*I*c) + 7*I*a^8*e^(12*I*d*x + 12*I*c) + 2
1*I*a^8*e^(10*I*d*x + 10*I*c) + 35*I*a^8*e^(8*I*d*x + 8*I*c) + 35*I*a^8*e^
(6*I*d*x + 6*I*c) + 21*I*a^8*e^(4*I*d*x + 4*I*c) + 7*I*a^8*e^(2*I*d*x + 2*
I*c) + I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(14*I*d*x + 14*I*c) + 7*d
*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*
I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*
x + 2*I*c) + d)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.50

$$\int (a+ia\tan(c+dx))^8\,dx = -\frac{128ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

$$+\frac{-94080ia^8e^{12ic}e^{12idx}-423360ia^8e^{10ic}e^{10idx}-862400ia^8e^{8ic}e^{8idx}-980000ia^8e^{6ic}e^{6idx}-644448ia^8e^{4ic}e^{4}}{105de^{14ic}e^{14idx}+735de^{12ic}e^{12idx}+2205de^{10ic}e^{10idx}+3675de^{8ic}e^{8idx}+3675de^{6ic}e^{6idx}+2205de^{4ic}}$$

input
```
integrate((a+I*a*tan(d*x+c))**8,x)
```

output
```
-128*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-94080*I*a**8*exp(12*I*c)
*exp(12*I*d*x) - 423360*I*a**8*exp(10*I*c)*exp(10*I*d*x) - 862400*I*a**8*e
xp(8*I*c)*exp(8*I*d*x) - 980000*I*a**8*exp(6*I*c)*exp(6*I*d*x) - 644448*I*
a**8*exp(4*I*c)*exp(4*I*d*x) - 230496*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 348
48*I*a**8)/(105*d*exp(14*I*c)*exp(14*I*d*x) + 735*d*exp(12*I*c)*exp(12*I*d
*x) + 2205*d*exp(10*I*c)*exp(10*I*d*x) + 3675*d*exp(8*I*c)*exp(8*I*d*x) +
3675*d*exp(6*I*c)*exp(6*I*d*x) + 2205*d*exp(4*I*c)*exp(4*I*d*x) + 735*d*ex
p(2*I*c)*exp(2*I*d*x) + 105*d)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int (a+ia\tan(c+dx))^8\,dx = \frac{15\,a^8\tan\left(dx+c\right)^7 - 140i\,a^8\tan\left(dx+c\right)^6 - 609\,a^8\tan\left(dx+c\right)^5 + 1680i\,a^8\tan\left(dx+c\right)^4 + 3465\,a^8\tan\left(\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/105*(15*a^8*tan(d*x + c)^7 - 140*I*a^8*tan(d*x + c)^6 - 609*a^8*tan(d*x
+ c)^5 + 1680*I*a^8*tan(d*x + c)^4 + 3465*a^8*tan(d*x + c)^3 - 6300*I*a^8*
tan(d*x + c)^2 + 13440*(d*x + c)*a^8 + 6720*I*a^8*log(tan(d*x + c)^2 + 1)
- 13335*a^8*tan(d*x + c))/d
```

3.81.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(166) = 332.

Time = 0.54 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.89

$$\int (a+ia\tan(c+dx))^8\,dx = -\frac{32\left(420i\,a^8e^{(14i\,dx+14i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + 2940i\,a^8e^{(12i\,dx+12i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + 8820i\,a^8e^{(10i\,dx}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-32/105*(420*I*a^8*e^(14*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 29
40*I*a^8*e^(12*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 8820*I*a^8*e
^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 14700*I*a^8*e^(8*I*d*x
 + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 14700*I*a^8*e^(6*I*d*x + 6*I*c)*l
og(e^(2*I*d*x + 2*I*c) + 1) + 8820*I*a^8*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) + 2940*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) +
 1) + 2940*I*a^8*e^(12*I*d*x + 12*I*c) + 13230*I*a^8*e^(10*I*d*x + 10*I*c)
 + 26950*I*a^8*e^(8*I*d*x + 8*I*c) + 30625*I*a^8*e^(6*I*d*x + 6*I*c) + 201
39*I*a^8*e^(4*I*d*x + 4*I*c) + 7203*I*a^8*e^(2*I*d*x + 2*I*c) + 420*I*a^8*
log(e^(2*I*d*x + 2*I*c) + 1) + 1089*I*a^8)/(d*e^(14*I*d*x + 14*I*c) + 7*d*
e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I
*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x
 + 2*I*c) + d)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

$$\int (a+ia\tan(c+dx))^8\,dx$$
$$=\frac{33\,a^8\tan(c+d\,x)^3-127\,a^8\tan(c+d\,x)-\frac{29\,a^8\tan(c+d\,x)^5}{5}+\frac{a^8\tan(c+d\,x)^7}{7}+a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,128\mathrm{i}-}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*log(tan(c + d*x) + 1i)*128i - 127*a^8*tan(c + d*x) - a^8*tan(c + d*x)
^2*60i + 33*a^8*tan(c + d*x)^3 + a^8*tan(c + d*x)^4*16i - (29*a^8*tan(c +
d*x)^5)/5 - (a^8*tan(c + d*x)^6*4i)/3 + (a^8*tan(c + d*x)^7)/7)/d
```

3.82 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.82.1 Optimal result

Integrand size = 24, antiderivative size = 133

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = {} & -192a^8x+\frac{192ia^8\log(\cos(c+dx))}{d}\\ & +\frac{129a^8\tan(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}\\ & -\frac{10a^8\tan^3(c+dx)}{d}-\frac{2ia^8\tan^4(c+dx)}{d}\\ & +\frac{a^8\tan^5(c+dx)}{5d}-\frac{64ia^9}{d(a-ia\tan(c+dx))}\end{aligned}$$

output
```
-192*a^8*x+192*I*a^8*ln(cos(d*x+c))/d+129*a^8*tan(d*x+c)/d+36*I*a^8*tan(d*
x+c)^2/d-10*a^8*tan(d*x+c)^3/d-2*I*a^8*tan(d*x+c)^4/d+1/5*a^8*tan(d*x+c)^5
/d-64*I*a^9/d/(a-I*a*tan(d*x+c))
```

3.82.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{ia^8\Big(960\log(i+\tan(c+dx))+645i\tan(c+dx)-180\tan^2(c+dx)-50i\tan^3(c+dx)+10\tan^4(c+}{5d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/5*I)*a^8*(960*Log[I + Tan[c + d*x]] + (645*I)*Tan[c + d*x] - 180*Tan[
c + d*x]^2 - (50*I)*Tan[c + d*x]^3 + 10*Tan[c + d*x]^4 + I*Tan[c + d*x]^5
+ (320*I)/(I + Tan[c + d*x])))/d
```

3.82.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^6}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^3\int \left(\frac{64a^6}{(a-ia\tan(c+dx))^2}-\frac{192a^5}{a-ia\tan(c+dx)}+\tan^4(c+dx)a^4-8i\tan^3(c+dx)a^4-30\tan^2(c+dx)a^4+72i\tan(c+}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{64a^6}{a-ia\tan(c+dx)}+\frac{1}{5}ia^5\tan^5(c+dx)+2a^5\tan^4(c+dx)-10ia^5\tan^3(c+dx)-36a^5\tan^2(c+dx)+129ia^5\,\text{ta}}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-I)*a^3*(192*a^5*Log[a - I*a*Tan[c + d*x]] + (129*I)*a^5*Tan[c + d*x] -
36*a^5*Tan[c + d*x]^2 - (10*I)*a^5*Tan[c + d*x]^3 + 2*a^5*Tan[c + d*x]^4 +
 (I/5)*a^5*Tan[c + d*x]^5 + (64*a^6)/(a - I*a*Tan[c + d*x])))/d
```

3.82.3.1 Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.82.4 Maple [A] (verified)

Time = 84.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{32 i a^{8} {\mathrm e}^{2 i \left(d x +c \right)}}{d}+\frac{384 a^{8} c}{d}+\frac{16 i a^{8} \left(150 \,{\mathrm e}^{8 i \left(d x +c \right)}+500 \,{\mathrm e}^{6 i \left(d x +c \right)}+650 \,{\mathrm e}^{4 i \left(d x +c \right)}+385 \,{\mathrm e}^{2 i \left(d x +c \right)}+87\right)}{5 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{5}}+ 192 i a^{8} \ln($
derivativedivides	$a^{8} \left(\frac{\sin^{9}\left(d x +c \right)}{5 \cos\left(d x +c \right)^{5}}-\frac{4 \left(\sin^{9}\left(d x +c \right)\right)}{15 \cos\left(d x +c \right)^{3}}+\frac{8 \left(\sin^{9}\left(d x +c \right)\right)}{5 \cos\left(d x +c \right)}+\frac{8 \left(\sin^{7}\left(d x +c \right)+\frac{7 \left(\sin^{5}\left(d x +c \right)\right)}{6}+\frac{35 \left(\sin^{3}\left(d x +c \right)\right)}{24}+\frac{35 \sin\left(d x +c \right)}{16}\right) \cos(d x +c}{5}\right.$
default	$a^{8} \left(\frac{\sin^{9}\left(d x +c \right)}{5 \cos\left(d x +c \right)^{5}}-\frac{4 \left(\sin^{9}\left(d x +c \right)\right)}{15 \cos\left(d x +c \right)^{3}}+\frac{8 \left(\sin^{9}\left(d x +c \right)\right)}{5 \cos\left(d x +c \right)}+\frac{8 \left(\sin^{7}\left(d x +c \right)+\frac{7 \left(\sin^{5}\left(d x +c \right)\right)}{6}+\frac{35 \left(\sin^{3}\left(d x +c \right)\right)}{24}+\frac{35 \sin\left(d x +c \right)}{16}\right) \cos(d x +c}{5}\right.$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-32*I/d*a^8*exp(2*I*(d*x+c))+384/d*a^8*c+16/5*I*a^8*(150*exp(8*I*(d*x+c))+
500*exp(6*I*(d*x+c))+650*exp(4*I*(d*x+c))+385*exp(2*I*(d*x+c))+87)/d/(exp(
2*I*(d*x+c))+1)^5+192*I/d*a^8*ln(exp(2*I*(d*x+c))+1)
```

3.82.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.

Time = 0.26 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{16\left(10i\,a^8e^{(12i\,dx+12i\,c)}+50i\,a^8e^{(10i\,dx+10i\,c)}-50i\,a^8e^{(8i\,dx+8i\,c)}-400i\,a^8e^{(6i\,dx+6i\,c)}-600i\,a^8e^{(4i\,dx+4i\,c)}-\right.}{5\left(de^{(10i\,dx+10i\,c)}+5\right.}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-16/5*(10*I*a^8*e^(12*I*d*x + 12*I*c) + 50*I*a^8*e^(10*I*d*x + 10*I*c) - 5
0*I*a^8*e^(8*I*d*x + 8*I*c) - 400*I*a^8*e^(6*I*d*x + 6*I*c) - 600*I*a^8*e^
(4*I*d*x + 4*I*c) - 375*I*a^8*e^(2*I*d*x + 2*I*c) - 87*I*a^8 + 60*(-I*a^8*
e^(10*I*d*x + 10*I*c) - 5*I*a^8*e^(8*I*d*x + 8*I*c) - 10*I*a^8*e^(6*I*d*x
+ 6*I*c) - 10*I*a^8*e^(4*I*d*x + 4*I*c) - 5*I*a^8*e^(2*I*d*x + 2*I*c) - I*
a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d
*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^
(2*I*d*x + 2*I*c) + d)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{192ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{2400ia^8e^{8ic}e^{8idx}+8000ia^8e^{6ic}e^{6idx}+10400ia^8e^{4ic}e^{4idx}+6160ia^8e^{2ic}e^{2idx}+1392ia^8}{5de^{10ic}e^{10idx}+25de^{8ic}e^{8idx}+50de^{6ic}e^{6idx}+50de^{4ic}e^{4idx}+25de^{2ic}e^{2idx}+5d} + \begin{cases} -\frac{32ia^8e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 64a^8xe^{2ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

output
```
192*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (2400*I*a**8*exp(8*I*c)*exp
(8*I*d*x) + 8000*I*a**8*exp(6*I*c)*exp(6*I*d*x) + 10400*I*a**8*exp(4*I*c)*
exp(4*I*d*x) + 6160*I*a**8*exp(2*I*c)*exp(2*I*d*x) + 1392*I*a**8)/(5*d*exp
(10*I*c)*exp(10*I*d*x) + 25*d*exp(8*I*c)*exp(8*I*d*x) + 50*d*exp(6*I*c)*ex
p(6*I*d*x) + 50*d*exp(4*I*c)*exp(4*I*d*x) + 25*d*exp(2*I*c)*exp(2*I*d*x) +
 5*d) + Piecewise((-32*I*a**8*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (64*a*
*8*x*exp(2*I*c), True))
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan(dx+c)^5 - 10i\,a^8\tan(dx+c)^4 - 50\,a^8\tan(dx+c)^3 + 180i\,a^8\tan(dx+c)^2 - 960\,(dx+c)a^8 - 48}{5\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/5*(a^8*tan(d*x + c)^5 - 10*I*a^8*tan(d*x + c)^4 - 50*a^8*tan(d*x + c)^3
+ 180*I*a^8*tan(d*x + c)^2 - 960*(d*x + c)*a^8 - 480*I*a^8*log(tan(d*x + c
)^2 + 1) + 645*a^8*tan(d*x + c) + 320*(a^8*tan(d*x + c) - I*a^8)/(tan(d*x
+ c)^2 + 1))/d
```

3.82.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(121) = 242$.

Time = 0.91 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.27

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{16\left(-60i\,a^8e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-300i\,a^8e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-600i\,a^8e^{(6i\,dx+6i\,c)}}{}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-16/5*(-60*I*a^8*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 300*
I*a^8*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 600*I*a^8*e^(6*I*
d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 600*I*a^8*e^(4*I*d*x + 4*I*c)*
log(e^(2*I*d*x + 2*I*c) + 1) - 300*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) + 10*I*a^8*e^(12*I*d*x + 12*I*c) + 50*I*a^8*e^(10*I*d*x +
10*I*c) - 50*I*a^8*e^(8*I*d*x + 8*I*c) - 400*I*a^8*e^(6*I*d*x + 6*I*c) - 6
00*I*a^8*e^(4*I*d*x + 4*I*c) - 375*I*a^8*e^(2*I*d*x + 2*I*c) - 60*I*a^8*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 87*I*a^8)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8
*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*
d*e^(2*I*d*x + 2*I*c) + d)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{\frac{64\,a^8}{\tan(c+d\,x)+1\mathrm{i}}+129\,a^8\tan(c+d\,x)-10\,a^8\tan(c+d\,x)^3+\frac{a^8\tan(c+d\,x)^5}{5}-a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,192\mathrm{i}+a}{d}$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
((64*a^8)/(tan(c + d*x) + 1i) - a^8*log(tan(c + d*x) + 1i)*192i + 129*a^8*
tan(c + d*x) + a^8*tan(c + d*x)^2*36i - 10*a^8*tan(c + d*x)^3 - a^8*tan(c
+ d*x)^4*2i + (a^8*tan(c + d*x)^5)/5)/d
```

3.83 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.83.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = 80a^8x - \frac{80ia^8\log(\cos(c+dx))}{d} - \frac{31a^8\tan(c+dx)}{d} - \frac{4ia^8\tan^2(c+dx)}{d} + \frac{a^8\tan^3(c+dx)}{3d} - \frac{16ia^{10}}{d(a-ia\tan(c+dx))^2} + \frac{80ia^9}{d(a-ia\tan(c+dx))}$$

output

```
80*a^8*x-80*I*a^8*ln(cos(d*x+c))/d-31*a^8*tan(d*x+c)/d-4*I*a^8*tan(d*x+c)^
2/d+1/3*a^8*tan(d*x+c)^3/d-16*I*a^10/d/(a-I*a*tan(d*x+c))^2+80*I*a^9/d/(a-
I*a*tan(d*x+c))
```

3.83.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^8\Big(-93i\tan(c+dx)+12\tan^2(c+dx)+i\tan^3(c+dx)+48\Big(-5\log(i+\tan(c+dx))+\frac{4-5i\tan(c+dx)}{(i+\tan(c+dx))^2}}{3d}$$

input

```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/3*I)*a^8*((-93*I)*Tan[c + d*x] + 12*Tan[c + d*x]^2 + I*Tan[c + d*x]^3
 + 48*(-5*Log[I + Tan[c + d*x]] + (4 - (5*I)*Tan[c + d*x])/(I + Tan[c + d*
x])^2)))/d
```

3.83.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{(i\tan(c+dx)a+a)^5}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^5\int \Big(\frac{32a^5}{(a-ia\tan(c+dx))^3}-\frac{80a^4}{(a-ia\tan(c+dx))^2}+\frac{80a^3}{a-ia\tan(c+dx)}+\tan^2(c+dx)a^2-8i\tan(c+dx)a^2-31a^2\Big)d(ia\tan(}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\Big(\frac{16a^5}{(a-ia\tan(c+dx))^2}-\frac{80a^4}{a-ia\tan(c+dx)}+\frac{1}{3}ia^3\tan^3(c+dx)+4a^3\tan^2(c+dx)-31ia^3\tan(c+dx)-80a^3\log(a-}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^5*(-80*a^3*Log[a - I*a*Tan[c + d*x]] - (31*I)*a^3*Tan[c + d*x] + 4
*a^3*Tan[c + d*x]^2 + (I/3)*a^3*Tan[c + d*x]^3 + (16*a^5)/(a - I*a*Tan[c +
 d*x])^2 - (80*a^4)/(a - I*a*Tan[c + d*x])))/d
```

3.83.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.83.4 Maple [A] (verified)

Time = 219.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{4 i a^{8} {\mathrm e}^{4 i (d x +c)}}{d}+\frac{32 i a^{8} {\mathrm e}^{2 i (d x +c)}}{d}-\frac{160 a^{8} c}{d}-\frac{4 i a^{8} \left(60 \,{\mathrm e}^{4 i (d x +c)}+105 \,{\mathrm e}^{2 i (d x +c)}+47\right)}{3 d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{3}}-\frac{80 i a^{8} \ln \left({\mathrm e}^{2 i (d x +c)}+1\right)}{d}$
derivativedivides	$a^{8} \left(\frac{\sin^{9}(d x +c)}{3 \cos(d x +c)^{3}}-\frac{2 \left(\sin^{9}(d x +c)\right)}{\cos(d x +c)}-2 \left(\sin^{7}(d x +c)+\frac{7 \left(\sin^{5}(d x +c)\right)}{6}+\frac{35 \left(\sin^{3}(d x +c)\right)}{24}+\frac{35 \sin(d x +c)}{16}\right) \cos(d x +c)+\frac{35 d x}{8}+$
default	$a^{8} \left(\frac{\sin^{9}(d x +c)}{3 \cos(d x +c)^{3}}-\frac{2 \left(\sin^{9}(d x +c)\right)}{\cos(d x +c)}-2 \left(\sin^{7}(d x +c)+\frac{7 \left(\sin^{5}(d x +c)\right)}{6}+\frac{35 \left(\sin^{3}(d x +c)\right)}{24}+\frac{35 \sin(d x +c)}{16}\right) \cos(d x +c)+\frac{35 d x}{8}+$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-4*I/d*a^8*exp(4*I*(d*x+c))+32*I/d*a^8*exp(2*I*(d*x+c))-160/d*a^8*c-4/3*I*
a^8*(60*exp(4*I*(d*x+c))+105*exp(2*I*(d*x+c))+47)/d/(exp(2*I*(d*x+c))+1)^3
-80*I/d*a^8*ln(exp(2*I*(d*x+c))+1)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.44

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4\left(3i\,a^8e^{(10i\,dx+10i\,c)} - 15i\,a^8e^{(8i\,dx+8i\,c)} - 63i\,a^8e^{(6i\,dx+6i\,c)} - 9i\,a^8e^{(4i\,dx+4i\,c)} + 81i\,a^8e^{(2i\,dx+2i\,c)} + 47i\,a^8 +\right.}{3\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,de\right.}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-4/3*(3*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 63*I*
a^8*e^(6*I*d*x + 6*I*c) - 9*I*a^8*e^(4*I*d*x + 4*I*c) + 81*I*a^8*e^(2*I*d*
x + 2*I*c) + 47*I*a^8 + 60*(I*a^8*e^(6*I*d*x + 6*I*c) + 3*I*a^8*e^(4*I*d*x
 + 4*I*c) + 3*I*a^8*e^(2*I*d*x + 2*I*c) + I*a^8)*log(e^(2*I*d*x + 2*I*c) +
 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2
*I*c) + d)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{80ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-240ia^8e^{4ic}e^{4idx} - 420ia^8e^{2ic}e^{2idx} - 188ia^8}{3de^{6ic}e^{6idx} + 9de^{4ic}e^{4idx} + 9de^{2ic}e^{2idx} + 3d} + \begin{cases} \frac{-4ia^8de^{4ic}e^{4idx}+32ia^8de^{2ic}e^{2idx}}{d^2} & \text{for } d^2 \neq 0 \\ x\left(16a^8e^{4ic} - 64a^8e^{2ic}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

output
```
-80*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-240*I*a**8*exp(4*I*c)*exp
(4*I*d*x) - 420*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 188*I*a**8)/(3*d*exp(6*I*
c)*exp(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x
) + 3*d) + Piecewise(((-4*I*a**8*d*exp(4*I*c)*exp(4*I*d*x) + 32*I*a**8*d*e
xp(2*I*c)*exp(2*I*d*x))/d**2, Ne(d**2, 0)), (x*(16*a**8*exp(4*I*c) - 64*a*
*8*exp(2*I*c)), True))
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$=\frac{a^8\tan\left(dx+c\right)^3-12i\,a^8\tan\left(dx+c\right)^2+240\,(dx+c)a^8+120i\,a^8\log\left(\tan\left(dx+c\right)^2+1\right)-93\,a^8\tan\left(dx\right.}{3\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/3*(a^8*tan(d*x + c)^3 - 12*I*a^8*tan(d*x + c)^2 + 240*(d*x + c)*a^8 + 12
0*I*a^8*log(tan(d*x + c)^2 + 1) - 93*a^8*tan(d*x + c) - 48*(5*a^8*tan(d*x
+ c)^3 - 6*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 4*I*a^8)/(tan(d*x +
 c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

3.83.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(110) = 220.

Time = 1.04 (sec) , antiderivative size = 785, normalized size of antiderivative = 6.33

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output

```
-4/3*(60*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 840*I*
a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 5460*I*a^8*e^(24*
I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 21840*I*a^8*e^(22*I*d*x + 8
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 60060*I*a^8*e^(20*I*d*x + 6*I*c)*log(
e^(2*I*d*x + 2*I*c) + 1) + 120120*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) + 180180*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) + 180180*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) +
120120*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 60060*I*a
^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 21840*I*a^8*e^(6*I*d
*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 5460*I*a^8*e^(4*I*d*x - 10*I*c)
*log(e^(2*I*d*x + 2*I*c) + 1) + 840*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) + 205920*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1)
 + 60*I*a^8*e^(-14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^8*e^(32*I*d*x
 + 18*I*c) + 18*I*a^8*e^(30*I*d*x + 16*I*c) - 63*I*a^8*e^(28*I*d*x + 14*I*
c) - 1032*I*a^8*e^(26*I*d*x + 12*I*c) - 4968*I*a^8*e^(24*I*d*x + 10*I*c) -
 13516*I*a^8*e^(22*I*d*x + 8*I*c) - 22847*I*a^8*e^(20*I*d*x + 6*I*c) - 220
66*I*a^8*e^(18*I*d*x + 4*I*c) - 3234*I*a^8*e^(16*I*d*x + 2*I*c) + 44979*I*
a^8*e^(12*I*d*x - 2*I*c) + 43332*I*a^8*e^(10*I*d*x - 4*I*c) + 27672*I*a^8*
e^(8*I*d*x - 6*I*c) + 12048*I*a^8*e^(6*I*d*x - 8*I*c) + 3467*I*a^8*e^(4*I*
d*x - 10*I*c) + 598*I*a^8*e^(2*I*d*x - 12*I*c) + 25674*I*a^8*e^(14*I*d*...
```

3.83.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\tan(c+d\,x)^3}{3\,d} - \frac{80\,a^8\tan(c+d\,x)+a^8\,64\mathrm{i}}{d\,\left(\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)} - \frac{31\,a^8\tan(c+d\,x)}{d} + \frac{a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,80\mathrm{i}}{d} - \frac{a^8\tan(c+d\,x)^2\,4\mathrm{i}}{d}$$

input

```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*log(tan(c + d*x) + 1i)*80i)/d - (31*a^8*tan(c + d*x))/d - (80*a^8*tan
(c + d*x) + a^8*64i)/(d*(tan(c + d*x)*2i + tan(c + d*x)^2 - 1)) - (a^8*tan
(c + d*x)^2*4i)/d + (a^8*tan(c + d*x)^3)/(3*d)
```

3.83. $\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.84 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.84.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\begin{aligned}\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx &= -8a^8x+\frac{8ia^8\log(\cos(c+dx))}{d}\\ &\quad+\frac{a^8\tan(c+dx)}{d}-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}\\ &\quad+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{24ia^9}{d(a-ia\tan(c+dx))}\end{aligned}$$

output

```
-8*a^8*x+8*I*a^8*ln(cos(d*x+c))/d+a^8*tan(d*x+c)/d-16/3*I*a^11/d/(a-I*a*ta
n(d*x+c))^3+16*I*a^10/d/(a-I*a*tan(d*x+c))^2-24*I*a^9/d/(a-I*a*tan(d*x+c))
```

3.84.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^7\left(8a\log(i+\tan(c+dx))+ia\tan(c+dx)+\frac{8ia\left(-5+12i\tan(c+dx)+9\tan^2(c+dx)\right)}{3(i+\tan(c+dx))^3}\right)}{d}$$

input

```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-I)*a^7*(8*a*Log[I + Tan[c + d*x]] + I*a*Tan[c + d*x] + (((8*I)/3)*a*(-5
 + (12*I)*Tan[c + d*x] + 9*Tan[c + d*x]^2))/(I + Tan[c + d*x])^3))/d
```

3.84.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{(i\tan(c+dx)a+a)^4}{(a-ia\tan(c+dx))^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{ia^7\int \left(\frac{16a^4}{(a-ia\tan(c+dx))^4}-\frac{32a^3}{(a-ia\tan(c+dx))^3}+\frac{24a^2}{(a-ia\tan(c+dx))^2}-\frac{8a}{a-ia\tan(c+dx)}+1\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^7\left(\frac{16a^4}{3(a-ia\tan(c+dx))^3}-\frac{16a^3}{(a-ia\tan(c+dx))^2}+\frac{24a^2}{a-ia\tan(c+dx)}+ia\tan(c+dx)+8a\log(a-ia\tan(c+dx))\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^7*(8*a*Log[a - I*a*Tan[c + d*x]] + I*a*Tan[c + d*x] + (16*a^4)/(3*
(a - I*a*Tan[c + d*x])^3) - (16*a^3)/(a - I*a*Tan[c + d*x])^2 + (24*a^2)/(
a - I*a*Tan[c + d*x])))/d
```

3.84.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.84.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(105) = 210$.

Time = 0.84 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.80

$$\frac{32ia^8(\sin^6(dx+c))}{3d}+\frac{14ia^8(\cos^4(dx+c))}{3d}+\frac{28ia^8(\sin^2(dx+c))(\cos^4(dx+c))}{3d}+\frac{8ia^8\ln(\cos(dx+c))}{d}+a^8($$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

output
```
32/3*I/d*a^8*sin(d*x+c)^6+14/3*I/d*a^8*cos(d*x+c)^4+28/3*I/d*a^8*sin(d*x+c
)^2*cos(d*x+c)^4+8*I*a^8*ln(cos(d*x+c))/d+1/d*a^8*sin(d*x+c)^7*cos(d*x+c)+
2*I/d*a^8*sin(d*x+c)^4-4/3*I/d*a^8*cos(d*x+c)^6-35/3/d*a^8*sin(d*x+c)^3*co
s(d*x+c)^3-233/24/d*a^8*sin(d*x+c)*cos(d*x+c)^3+29/6/d*a^8*cos(d*x+c)^5*si
n(d*x+c)+4*I/d*a^8*sin(d*x+c)^2-8*a^8*x+1/d*a^8*sin(d*x+c)^9/cos(d*x+c)+35
/6/d*a^8*cos(d*x+c)*sin(d*x+c)^5+175/24/d*a^8*cos(d*x+c)*sin(d*x+c)^3+111/
8/d*a^8*sin(d*x+c)*cos(d*x+c)-8/d*a^8*c
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2\left(i\,a^8e^{(8i\,dx+8i\,c)} - 2i\,a^8e^{(6i\,dx+6i\,c)} + 6i\,a^8e^{(4i\,dx+4i\,c)} + 9i\,a^8e^{(2i\,dx+2i\,c)} - 3i\,a^8 + 12\left(-i\,a^8e^{(2i\,dx+2i\,c)} - i\,a\right.\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input

```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output

```
-2/3*(I*a^8*e^(8*I*d*x + 8*I*c) - 2*I*a^8*e^(6*I*d*x + 6*I*c) + 6*I*a^8*e^
(4*I*d*x + 4*I*c) + 9*I*a^8*e^(2*I*d*x + 2*I*c) - 3*I*a^8 + 12*(-I*a^8*e^(
2*I*d*x + 2*I*c) - I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*
I*c) + d)
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.51

$$\begin{aligned}&\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx\\&= \frac{2ia^8}{de^{2ic}e^{2idx}+d} + \frac{8ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d}\\&\quad+ \begin{cases}\frac{-2ia^8d^2e^{6ic}e^{6idx}+6ia^8d^2e^{4ic}e^{4idx}-18ia^8d^2e^{2ic}e^{2idx}}{3d^3} & \text{for } d^3\neq 0\\ x(4a^8e^{6ic}-8a^8e^{4ic}+12a^8e^{2ic}) & \text{otherwise}\end{cases}\end{aligned}$$

input

```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

output

```
2*I*a**8/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 8*I*a**8*log(exp(2*I*d*x) + exp
(-2*I*c))/d + Piecewise(((-2*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x) + 6*I*a**
8*d**2*exp(4*I*c)*exp(4*I*d*x) - 18*I*a**8*d**2*exp(2*I*c)*exp(2*I*d*x))/(
3*d**3), Ne(d**3, 0)), (x*(4*a**8*exp(6*I*c) - 8*a**8*exp(4*I*c) + 12*a**8
*exp(2*I*c)), True))
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.28

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{24\,(dx+c)a^8 + 12i\,a^8\log\left(\tan\left(dx+c\right)^2+1\right) - 3\,a^8\tan\left(dx+c\right) - \frac{8\,\Big(9\,a^8\tan(dx+c)^5 - 15i\,a^8\tan(dx+c)^4 + 4\,a^8\tan}{\tan(dx+c)^6 + 3\,\tan(}}{3\,d}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/3*(24*(d*x + c)*a^8 + 12*I*a^8*log(tan(d*x + c)^2 + 1) - 3*a^8*tan(d*x
+ c) - 8*(9*a^8*tan(d*x + c)^5 - 15*I*a^8*tan(d*x + c)^4 + 4*a^8*tan(d*x +
 c)^3 - 12*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 5*I*a^8)/(tan(d*x +
 c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

3.84.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(98) = 196.

Time = 1.09 (sec) , antiderivative size = 799, normalized size of antiderivative = 7.01

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2/3*(-12*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 168*I
*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1092*I*a^8*e^(24
*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4368*I*a^8*e^(22*I*d*x + 8
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12012*I*a^8*e^(20*I*d*x + 6*I*c)*log(
e^(2*I*d*x + 2*I*c) + 1) - 24024*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - 36036*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c)
+ 1) - 36036*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 240
24*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12012*I*a^8*e
^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4368*I*a^8*e^(6*I*d*x -
8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1092*I*a^8*e^(4*I*d*x - 10*I*c)*log(
e^(2*I*d*x + 2*I*c) + 1) - 168*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x +
 2*I*c) + 1) - 41184*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*
I*a^8*e^(-14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + I*a^8*e^(34*I*d*x + 20*I*
c) + 11*I*a^8*e^(32*I*d*x + 18*I*c) + 58*I*a^8*e^(30*I*d*x + 16*I*c) + 217
*I*a^8*e^(28*I*d*x + 14*I*c) + 725*I*a^8*e^(26*I*d*x + 12*I*c) + 2236*I*a^
8*e^(24*I*d*x + 10*I*c) + 5772*I*a^8*e^(22*I*d*x + 8*I*c) + 11583*I*a^8*e^
(20*I*d*x + 6*I*c) + 17589*I*a^8*e^(18*I*d*x + 4*I*c) + 20020*I*a^8*e^(16*
I*d*x + 2*I*c) + 10231*I*a^8*e^(12*I*d*x - 2*I*c) + 4147*I*a^8*e^(10*I*d*x
 - 4*I*c) + 872*I*a^8*e^(8*I*d*x - 6*I*c) - 80*I*a^8*e^(6*I*d*x - 8*I*c) -
 111*I*a^8*e^(4*I*d*x - 10*I*c) - 30*I*a^8*e^(2*I*d*x - 12*I*c) + 16874...
```

3.84.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\tan(c+dx)}{d} - \frac{24\,a^8\tan(c+dx)^2 + a^8\tan(c+dx)\;32\mathrm{i} - \frac{40\,a^8}{3}}{d\,\left(-\tan\left(c+dx\right)^3 - \tan\left(c+dx\right)^2 3\mathrm{i} + 3\tan\left(c+dx\right) + 1\mathrm{i}\right)} - \frac{a^8\,\ln\left(\tan(c+dx)+1\mathrm{i}\right)\,8\mathrm{i}}{d}$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*tan(c + d*x))/d - (a^8*log(tan(c + d*x) + 1i)*8i)/d - (a^8*tan(c + d*
x)*32i - (40*a^8)/3 + 24*a^8*tan(c + d*x)^2)/(d*(3*tan(c + d*x) - tan(c +
d*x)^2*3i - tan(c + d*x)^3 + 1i))
```

3.85 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.85.1 Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a^3+ia^3\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

output
```
-1/8*I*(a^3+I*a^3*tan(d*x+c))^4/d/(a-I*a*tan(d*x+c))^4
```

3.85.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^8(\cos(c+dx)+i\sin(c+dx))^8}{8d}$$

input
```
Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/8*I)*a^8*(Cos[c + d*x] + I*Sin[c + d*x])^8)/d
```

3.85.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^8}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^9\int \frac{(i\tan(c+dx)a+a)^3}{(a-ia\tan(c+dx))^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 48$$

$$-\frac{ia^8(a+ia\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/8*I)*a^8*(a + I*a*Tan[c + d*x])^4)/(d*(a - I*a*Tan[c + d*x])^4)
```

3.85.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.85.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(38) = 76$.

Time = 1.40 (sec) , antiderivative size = 451, normalized size of antiderivative = 10.49

$$a^{8}\left(-\frac{\left(\sin^{7}(dx+c)+\frac{7\left(\sin^{5}(dx+c)\right)}{6}+\frac{35\left(\sin^{3}(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-ia^{8}\left(\cos^{8}\left(dx+c\right)\right)-28a^{8}\Big(-$$

input
```
int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin
(d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c)-I*a^8*cos(d*x+c)^8-28*a^8*(-1/8*s
in(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/64*cos(d*x+c)^3*
sin(d*x+c)+5/128*sin(d*x+c)*cos(d*x+c)+5/128*d*x+5/128*c)-56*I*a^8*(-1/8*c
os(d*x+c)^6*sin(d*x+c)^2-1/24*cos(d*x+c)^6)+70*a^8*(-1/8*sin(d*x+c)^3*cos(
d*x+c)^5-1/16*cos(d*x+c)^5*sin(d*x+c)+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+3/128*d*x+3/128*c)-I*a^8*sin(d*x+c)^8-28*a^8*(-1/8*sin(d*x+c)*co
s(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)
+5/128*d*x+5/128*c)+56*I*a^8*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*cos(d*x+
c)^4*sin(d*x+c)^2-1/24*cos(d*x+c)^4)+a^8*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)
^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \cos^{8}(c+dx)(a+ia\tan(c+dx))^{8}\,dx=-\frac{i\,a^{8}e^{(8i\,dx+8i\,c)}}{8\,d}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-1/8*I*a^8*e^(8*I*d*x + 8*I*c)/d
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} -\frac{ia^8e^{8ic}e^{8idx}}{8d} & \text{for}\ d \neq 0 \\ a^8xe^{8ic} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-I*a**8*exp(8*I*c)*exp(8*I*d*x)/(8*d), Ne(d, 0)), (a**8*x*exp(8
*I*c), True))
```

3.85.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(35) = 70.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{a^8\tan(dx+c)^7 - 4i\,a^8\tan(dx+c)^6 - 7\,a^8\tan(dx+c)^5 + 8i\,a^8\tan(dx+c)^4 + 7\,a^8\tan(dx+c)^3 - 4}{(\tan(dx+c)^8 + 4\,\tan(dx+c)^6 + 6\,\tan(dx+c)^4 + 4\,\tan(dx+c)^2 +}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-(a^8*tan(d*x + c)^7 - 4*I*a^8*tan(d*x + c)^6 - 7*a^8*tan(d*x + c)^5 + 8*I
*a^8*tan(d*x + c)^4 + 7*a^8*tan(d*x + c)^3 - 4*I*a^8*tan(d*x + c)^2 - a^8*
tan(d*x + c))/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*t
an(d*x + c)^2 + 1)*d)
```

3.85.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(35) = 70$.

Time = 1.12 (sec) , antiderivative size = 381, normalized size of antiderivative = 8.86

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{i\,a^8e^{(36i\,dx+22i\,c)} + 14i\,a^8e^{(34i\,dx+20i\,c)} + 91i\,a^8e^{(32i\,dx+18i\,c)} + 364i\,a^8e^{(30i\,dx+16i\,c)} + 1001i\,a^8e^{(28i\,dx+14i\,c)} +}{8\,(de^{(28i\,dx+14i\,c)} + 14\,de^{(26i\,dx+12i\,c)} + 91\,de^{(24i\,dx+10i\,c)} + 364\,de^{(22i\,dx+8i\,c)} + 1001\,de^{(20i\,dx}}$$

input
```
integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/8*(I*a^8*e^(36*I*d*x + 22*I*c) + 14*I*a^8*e^(34*I*d*x + 20*I*c) + 91*I*
a^8*e^(32*I*d*x + 18*I*c) + 364*I*a^8*e^(30*I*d*x + 16*I*c) + 1001*I*a^8*e
^(28*I*d*x + 14*I*c) + 2002*I*a^8*e^(26*I*d*x + 12*I*c) + 3003*I*a^8*e^(24
*I*d*x + 10*I*c) + 3432*I*a^8*e^(22*I*d*x + 8*I*c) + 3003*I*a^8*e^(20*I*d*
x + 6*I*c) + 2002*I*a^8*e^(18*I*d*x + 4*I*c) + 1001*I*a^8*e^(16*I*d*x + 2*
I*c) + 91*I*a^8*e^(12*I*d*x - 2*I*c) + 14*I*a^8*e^(10*I*d*x - 4*I*c) + I*a
^8*e^(8*I*d*x - 6*I*c) + 364*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c)
+ 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*
d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) +
 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*
I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) +
91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x
) + d*e^(-14*I*c))
```

3.85.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= -\frac{a^8\tan(c+d\,x)\,\left(\tan(c+d\,x)^2-1\right)}{d\,\left(\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,4\mathrm{i}-6\tan\left(c+d\,x\right)^2-\tan\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$

input
```
int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(a^8*tan(c + d*x)*(tan(c + d*x)^2 - 1))/(d*(tan(c + d*x)^3*4i - 6*tan(c +
 d*x)^2 - tan(c + d*x)*4i + tan(c + d*x)^4 + 1))
```

3.86 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.86.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5} + \frac{ia^{12}}{d(a-ia\tan(c+dx))^4} - \frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

output
```
-4/5*I*a^13/d/(a-I*a*tan(d*x+c))^5+I*a^12/d/(a-I*a*tan(d*x+c))^4-1/3*I*a^1
1/d/(a-I*a*tan(d*x+c))^3
```

3.86.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.55

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8(-2-5i\tan(c+dx)+5\tan^2(c+dx))}{15d(i+\tan(c+dx))^5}$$

input
```
Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/15*(a^8*(-2 - (5*I)*Tan[c + d*x] + 5*Tan[c + d*x]^2))/(d*(I + Tan[c + d
*x])^5)
```

3.86.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3042, 3968, 53, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{10}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{11}\int \frac{(i\tan(c+dx)a+a)^2}{(a-ia\tan(c+dx))^6}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{ia^{11}\int \left(\frac{4a^2}{(a-ia\tan(c+dx))^6}-\frac{4a}{(a-ia\tan(c+dx))^5}+\frac{1}{(a-ia\tan(c+dx))^4}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{11}\left(\frac{4a^2}{5(a-ia\tan(c+dx))^5}-\frac{a}{(a-ia\tan(c+dx))^4}+\frac{1}{3(a-ia\tan(c+dx))^3}\right)}{d}$$

input

```
Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-I)*a^11*((4*a^2)/(5*(a - I*a*Tan[c + d*x])^5) - a/(a - I*a*Tan[c + d*x]
)^4 + 1/(3*(a - I*a*Tan[c + d*x])^3)))/d
```

3.86.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.86.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(70) = 140$.

Time = 1.18 (sec) , antiderivative size = 588, normalized size of antiderivative = 7.35

$$a^8\left(-\frac{(\sin^7(dx+c))(\cos^3(dx+c))}{10}-\frac{7(\sin^5(dx+c))(\cos^3(dx+c))}{80}-\frac{7(\sin^3(dx+c))(\cos^3(dx+c))}{96}-\frac{7(\cos^3(dx+c))\sin(dx+c)}{128}+\frac{7\sin(dx+c)}{25}\right.$$

input
```
int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x)
```

output

```
1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7
/96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*cos(d*x+c)^3*sin(d*x+c)+7/256*sin(d*x+
c)*cos(d*x+c)+7/256*d*x+7/256*c)-8*I*a^8*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-
3/40*sin(d*x+c)^4*cos(d*x+c)^4-1/20*cos(d*x+c)^4*sin(d*x+c)^2-1/40*cos(d*x
+c)^4)-28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c
)^5-1/32*cos(d*x+c)^5*sin(d*x+c)+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/256*d*x+3/256*c)-56*I*a^8*(-1/10*cos(d*x+c)^8*sin(d*x+c)^2-1/40*co
s(d*x+c)^8)+70*a^8*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*
x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/
256*d*x+3/256*c)-4/5*I*a^8*cos(d*x+c)^10-28*a^8*(-1/10*sin(d*x+c)*cos(d*x+
c)^9+1/80*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+
c))*sin(d*x+c)+7/256*d*x+7/256*c)+56*I*a^8*(-1/10*sin(d*x+c)^4*cos(d*x+c)^
6-1/20*cos(d*x+c)^6*sin(d*x+c)^2-1/60*cos(d*x+c)^6)+a^8*(1/10*(cos(d*x+c)^
9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+
c))*sin(d*x+c)+63/256*d*x+63/256*c))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-6i\,a^8e^{(10i\,dx+10i\,c)}-15i\,a^8e^{(8i\,dx+8i\,c)}-10i\,a^8e^{(6i\,dx+6i\,c)}}{240\,d}$$

input

```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output

```
1/240*(-6*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 10*
I*a^8*e^(6*I*d*x + 6*I*c))/d
```

3.86.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-384ia^8d^2e^{10ic}e^{10idx}-960ia^8d^2e^{8ic}e^{8idx}-640ia^8d^2e^{6ic}e^{6idx}}{15360d^3} & \text{for } d^3\neq 0 \\ x\left(\frac{a^8e^{10ic}}{4}+\frac{a^8e^{8ic}}{2}+\frac{a^8e^{6ic}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-384*I*a**8*d**2*exp(10*I*c)*exp(10*I*d*x) - 960*I*a**8*d**2*e
xp(8*I*c)*exp(8*I*d*x) - 640*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x))/(15360*d
**3), Ne(d**3, 0)), (x*(a**8*exp(10*I*c)/4 + a**8*exp(8*I*c)/2 + a**8*exp(
6*I*c)/4), True))
```

3.86.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.90

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{5\,a^8\tan\left(dx+c\right)^7-30i\,a^8\tan\left(dx+c\right)^6-77\,a^8\tan\left(dx+c\right)^5+110i\,a^8\tan\left(dx+c\right)^4+95\,a^8\tan\left(dx+\right.}{15\left(\tan\left(dx+c\right)^{10}+5\,\tan\left(dx+c\right)^8+10\,\tan\left(dx+c\right)^6+10\,\tan\left(dx+c\right)^4\right.}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(5*a^8*tan(d*x + c)^7 - 30*I*a^8*tan(d*x + c)^6 - 77*a^8*tan(d*x + c
)^5 + 110*I*a^8*tan(d*x + c)^4 + 95*a^8*tan(d*x + c)^3 - 50*I*a^8*tan(d*x
+ c)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^10 + 5*tan(d*x + c)
^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)*d)
```

3.86.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(64) = 128.

Time = 1.23 (sec) , antiderivative size = 409, normalized size of antiderivative = 5.11

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{6i\,a^8e^{(38i\,dx+24i\,c)}+99i\,a^8e^{(36i\,dx+22i\,c)}+766i\,a^8e^{(34i\,dx+20i\,c)}+3689i\,a^8e^{(32i\,dx+18i\,c)}+12376i\,a^8e^{(30i\,dx+16i}}{240\left(de^{(28i\,dx+14i\,c)}+14\,de^{(26i\,dx+12i\,c)}+91\,de^{(24i\,dx+10i\,c)}\right.}$$

input
```
integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/240*(6*I*a^8*e^(38*I*d*x + 24*I*c) + 99*I*a^8*e^(36*I*d*x + 22*I*c) + 7
66*I*a^8*e^(34*I*d*x + 20*I*c) + 3689*I*a^8*e^(32*I*d*x + 18*I*c) + 12376*
I*a^8*e^(30*I*d*x + 16*I*c) + 30667*I*a^8*e^(28*I*d*x + 14*I*c) + 58058*I*
a^8*e^(26*I*d*x + 12*I*c) + 85657*I*a^8*e^(24*I*d*x + 10*I*c) + 99528*I*a^
8*e^(22*I*d*x + 8*I*c) + 91377*I*a^8*e^(20*I*d*x + 6*I*c) + 66066*I*a^8*e^
(18*I*d*x + 4*I*c) + 37219*I*a^8*e^(16*I*d*x + 2*I*c) + 5089*I*a^8*e^(12*I
*d*x - 2*I*c) + 1126*I*a^8*e^(10*I*d*x - 4*I*c) + 155*I*a^8*e^(8*I*d*x - 6
*I*c) + 10*I*a^8*e^(6*I*d*x - 8*I*c) + 16016*I*a^8*e^(14*I*d*x))/(d*e^(28*
I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c)
+ 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*
I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c)
 + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I
*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 34
32*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(-5\tan(c+d\,x)^2+\tan(c+d\,x)\,5\mathrm{i}+2\right)}{15\,d\,\left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,5\mathrm{i}-10\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,10\mathrm{i}+5\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*(tan(c + d*x)*5i - 5*tan(c + d*x)^2 + 2))/(15*d*(5*tan(c + d*x) - tan
(c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + tan(c + d*x)^5 +
 1i))
```

3.87 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.87.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}+\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}$$

output
```
-1/3*I*a^14/d/(a-I*a*tan(d*x+c))^6+1/5*I*a^13/d/(a-I*a*tan(d*x+c))^5
```

3.87.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{a^8(-2i+3\tan(c+dx))}{15d(i+\tan(c+dx))^6}$$

input
```
Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/15*(a^8*(-2*I + 3*Tan[c + d*x]))/(d*(I + Tan[c + d*x])^6)
```

3.87.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{12}}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{ia^{13}\int \frac{i\tan(c+dx)a+a}{(a-ia\tan(c+dx))^7}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 53$$

$$-\frac{ia^{13}\int \left(\frac{2a}{(a-ia\tan(c+dx))^7}-\frac{1}{(a-ia\tan(c+dx))^6}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{ia^{13}\left(\frac{a}{3(a-ia\tan(c+dx))^6}-\frac{1}{5(a-ia\tan(c+dx))^5}\right)}{d}$$

input

```
Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-I)*a^13*(a/(3*(a - I*a*Tan[c + d*x])^6) - 1/(5*(a - I*a*Tan[c + d*x])^5
)))/d
```

3.87.3.1 Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.87.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(47) = 94$.

Time = 2.81 (sec) , antiderivative size = 639, normalized size of antiderivative = 11.62

Expression too large to display

input

```
int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x)
```

output

```
1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-
7/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*cos(d*x+c)^5*sin(d*x+c)+7/1536*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-8*I*a^8*(-1/12*si
n(d*x+c)^6*cos(d*x+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*cos(d*x+c)^6*s
in(d*x+c)^2-1/120*cos(d*x+c)^6)-28*a^8*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/
24*sin(d*x+c)^3*cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c
)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+56*I
*a^8*(-1/12*cos(d*x+c)^8*sin(d*x+c)^4-1/30*cos(d*x+c)^8*sin(d*x+c)^2-1/120
*cos(d*x+c)^8)+70*a^8*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*sin(d*x+c)*cos
(d*x+c)^9+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*co
s(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-2/3*I*a^8*cos(d*x+c)^12-28*a^8*(
-1/12*sin(d*x+c)*cos(d*x+c)^11+1/120*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*
cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*
x+21/1024*c)-56*I*a^8*(-1/12*cos(d*x+c)^10*sin(d*x+c)^2-1/60*cos(d*x+c)^10
)+a^8*(1/12*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*c
os(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d
*x+231/1024*c))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-5i\,a^8e^{(12i\,dx+12i\,c)} - 24i\,a^8e^{(10i\,dx+10i\,c)} - 45i\,a^8e^{(8i\,dx+8i\,c)} - 40i\,a^8e^{(6i\,dx+6i\,c)} - 15i\,a^8e^{(4i\,dx+4i\,c)}}{960\,d}$$

input

```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output

```
1/960*(-5*I*a^8*e^(12*I*d*x + 12*I*c) - 24*I*a^8*e^(10*I*d*x + 10*I*c) - 4
5*I*a^8*e^(8*I*d*x + 8*I*c) - 40*I*a^8*e^(6*I*d*x + 6*I*c) - 15*I*a^8*e^(4
*I*d*x + 4*I*c))/d
```

3.87.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(42) = 84$.

Time $= 0.52$ (sec) , antiderivative size $= 197$, normalized size of antiderivative $= 3.58$

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \begin{cases} \frac{-3932160ia^8d^4e^{12ic}e^{12idx}-18874368ia^8d^4e^{10ic}e^{10idx}-35389440ia^8d^4e^{8ic}e^{8idx}-31457280ia^8d^4e^{6ic}e^{6idx}-11796480ia^8d^4e^{4ic}e^{4idx}}{754974720d^5} & \text{for} \\ x\left(\frac{a^8e^{12ic}}{16}+\frac{a^8e^{10ic}}{4}+\frac{3a^8e^{8ic}}{8}+\frac{a^8e^{6ic}}{4}+\frac{a^8e^{4ic}}{16}\right) & \text{othe} \end{cases}$$

input
```
integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-3932160*I*a**8*d**4*exp(12*I*c)*exp(12*I*d*x) - 18874368*I*a*
*8*d**4*exp(10*I*c)*exp(10*I*d*x) - 35389440*I*a**8*d**4*exp(8*I*c)*exp(8*
I*d*x) - 31457280*I*a**8*d**4*exp(6*I*c)*exp(6*I*d*x) - 11796480*I*a**8*d*
*4*exp(4*I*c)*exp(4*I*d*x))/(754974720*d**5), Ne(d**5, 0)), (x*(a**8*exp(1
2*I*c)/16 + a**8*exp(10*I*c)/4 + 3*a**8*exp(8*I*c)/8 + a**8*exp(6*I*c)/4 +
 a**8*exp(4*I*c)/16), True))
```

3.87.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(43) = 86$.

Time $= 0.33$ (sec) , antiderivative size $= 162$, normalized size of antiderivative $= 2.95$

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{3\,a^8\tan(dx+c)^7-20i\,a^8\tan(dx+c)^6-57\,a^8\tan(dx+c)^5+90i\,a^8\tan(dx+c)^4+85\,a^8\tan(dx+c}{15\left(\tan(dx+c)^{12}+6\,\tan(dx+c)^{10}+15\,\tan(dx+c)^8+20\,\tan(dx+c)^6+15\,\tan\right.}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(3*a^8*tan(d*x + c)^7 - 20*I*a^8*tan(d*x + c)^6 - 57*a^8*tan(d*x + c
)^5 + 90*I*a^8*tan(d*x + c)^4 + 85*a^8*tan(d*x + c)^3 - 48*I*a^8*tan(d*x +
 c)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^12 + 6*tan(d*x + c)^
10 + 15*tan(d*x + c)^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x
 + c)^2 + 1)*d)
```

3.87.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(43) = 86$.

Time = 1.27 (sec) , antiderivative size = 437, normalized size of antiderivative = 7.95

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{5i\,a^8e^{(40i\,dx+26i\,c)}+94i\,a^8e^{(38i\,dx+24i\,c)}+836i\,a^8e^{(36i\,dx+22i\,c)}+4674i\,a^8e^{(34i\,dx+20i\,c)}+18411i\,a^8e^{(32i\,dx+18i}}{960\,(de^{(28i\,dx+14i\,c)}+14\,de^{(26}}$$

input
```
integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/960*(5*I*a^8*e^(40*I*d*x + 26*I*c) + 94*I*a^8*e^(38*I*d*x + 24*I*c) + 8
36*I*a^8*e^(36*I*d*x + 22*I*c) + 4674*I*a^8*e^(34*I*d*x + 20*I*c) + 18411*
I*a^8*e^(32*I*d*x + 18*I*c) + 54264*I*a^8*e^(30*I*d*x + 16*I*c) + 124033*I
*a^8*e^(28*I*d*x + 14*I*c) + 224822*I*a^8*e^(26*I*d*x + 12*I*c) + 327613*I
*a^8*e^(24*I*d*x + 10*I*c) + 386672*I*a^8*e^(22*I*d*x + 8*I*c) + 370513*I*
a^8*e^(20*I*d*x + 6*I*c) + 287534*I*a^8*e^(18*I*d*x + 4*I*c) + 179361*I*a^
8*e^(16*I*d*x + 2*I*c) + 34011*I*a^8*e^(12*I*d*x - 2*I*c) + 9754*I*a^8*e^(
10*I*d*x - 4*I*c) + 1970*I*a^8*e^(8*I*d*x - 6*I*c) + 250*I*a^8*e^(6*I*d*x
- 8*I*c) + 15*I*a^8*e^(4*I*d*x - 10*I*c) + 88704*I*a^8*e^(14*I*d*x))/(d*e^
(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I
*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^
(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*
I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^
(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c)
+ 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.87.9 Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\,(3\tan(c+d\,x)-2\mathrm{i})}{15\,d\,\left(\tan(c+d\,x)^6+\tan(c+d\,x)^5\,6\mathrm{i}-15\tan(c+d\,x)^4-\tan(c+d\,x)^3\,20\mathrm{i}+15\tan(c+d\,x)^2+\tan\right.}$$

input
```
int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8,x)
```

output

```
-(a^8*(3*tan(c + d*x) - 2i))/(15*d*(tan(c + d*x)*6i + 15*tan(c + d*x)^2 -
tan(c + d*x)^3*20i - 15*tan(c + d*x)^4 + tan(c + d*x)^5*6i + tan(c + d*x)^
6 - 1))
```

3.87. $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.88 $\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.88.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

output
```
-1/7*I*a^15/d/(a-I*a*tan(d*x+c))^7
```

3.88.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 116 vs. 2(27) = 54.

Time = 2.93 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{a^8(35+56\cos(2(c+dx))+28\cos(4(c+dx))+8\cos(6(c+dx))-14i\sin(2(c+dx))-14i\sin(4(c+dx}{896d(\cos(dx)+i\sin(dx))^8}$$

input
```
Integrate[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*(35 + 56*Cos[2*(c + d*x)] + 28*Cos[4*(c + d*x)] + 8*Cos[6*(c + d*x)]
- (14*I)*Sin[2*(c + d*x)] - (14*I)*Sin[4*(c + d*x)] - (6*I)*Sin[6*(c + d*x
)])*((-I)*Cos[8*(c + 2*d*x)] + Sin[8*(c + 2*d*x)]))/(896*d*(Cos[d*x] + I*S
in[d*x])^8)
```

3.88.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{14}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{15}\int \frac{1}{(a-ia\tan(c+dx))^8}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

input
```
Int[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/7*I)*a^15)/(d*(a - I*a*Tan[c + d*x])^7)
```

3.88.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.88.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(23) = 46.

Time = 1.40 (sec) , antiderivative size = 689, normalized size of antiderivative = 25.52

Expression too large to display

input
```
int(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/14*sin(d*x+c)^7*cos(d*x+c)^7-1/24*sin(d*x+c)^5*cos(d*x+c)^7-1
/48*sin(d*x+c)^3*cos(d*x+c)^7-1/128*sin(d*x+c)*cos(d*x+c)^7+1/768*(cos(d*x
+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2048*d*x+5/2048*c)-8*
I*a^8*(-1/14*sin(d*x+c)^6*cos(d*x+c)^8-1/28*cos(d*x+c)^8*sin(d*x+c)^4-1/70
*cos(d*x+c)^8*sin(d*x+c)^2-1/280*cos(d*x+c)^8)-28*a^8*(-1/14*sin(d*x+c)^5*
cos(d*x+c)^9-5/168*sin(d*x+c)^3*cos(d*x+c)^9-1/112*sin(d*x+c)*cos(d*x+c)^9
+1/896*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))
*sin(d*x+c)+5/2048*d*x+5/2048*c)-4/7*I*a^8*cos(d*x+c)^14+70*a^8*(-1/14*sin
(d*x+c)^3*cos(d*x+c)^11-1/56*sin(d*x+c)*cos(d*x+c)^11+1/560*(cos(d*x+c)^9+
9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c)
)*sin(d*x+c)+9/2048*d*x+9/2048*c)-56*I*a^8*(-1/14*sin(d*x+c)^2*cos(d*x+c)^
12-1/84*cos(d*x+c)^12)-28*a^8*(-1/14*sin(d*x+c)*cos(d*x+c)^13+1/168*(cos(d
*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/12
8*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+33/2048*d*x+33/2048*c)+56*I*
a^8*(-1/14*sin(d*x+c)^4*cos(d*x+c)^10-1/42*cos(d*x+c)^10*sin(d*x+c)^2-1/21
0*cos(d*x+c)^10)+a^8*(1/14*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(
d*x+c)^9+429/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+
3003/1024*cos(d*x+c))*sin(d*x+c)+429/2048*d*x+429/2048*c))
```

3.88.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(21) = 42$.

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.85

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-i\,a^8e^{(14i\,dx+14i\,c)} - 7i\,a^8e^{(12i\,dx+12i\,c)} - 21i\,a^8e^{(10i\,dx+10i\,c)} - 35i\,a^8e^{(8i\,dx+8i\,c)} - 35i\,a^8e^{(6i\,dx+6i\,c)} - 21i\,a^8e^{(4}}{896\,d}$$

input
```
integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/896*(-I*a^8*e^(14*I*d*x + 14*I*c) - 7*I*a^8*e^(12*I*d*x + 12*I*c) - 21*I
*a^8*e^(10*I*d*x + 10*I*c) - 35*I*a^8*e^(8*I*d*x + 8*I*c) - 35*I*a^8*e^(6*
I*d*x + 6*I*c) - 21*I*a^8*e^(4*I*d*x + 4*I*c) - 7*I*a^8*e^(2*I*d*x + 2*I*c
))/d
```

3.88.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(22) = 44$.

Time = 0.63 (sec) , antiderivative size = 279, normalized size of antiderivative = 10.33

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \begin{cases} \frac{-4398046511104ia^8d^6e^{14ic}e^{14idx}-30786325577728ia^8d^6e^{12ic}e^{12idx}-92358976733184ia^8d^6e^{10ic}e^{10idx}-153931627888640ia^8d^6e^{8ic}e^{8idx}-15}{3940649673949184d^7} \\ x\left(\frac{a^8e^{14ic}}{64}+\frac{3a^8e^{12ic}}{32}+\frac{15a^8e^{10ic}}{64}+\frac{5a^8e^{8ic}}{16}+\frac{15a^8e^{6ic}}{64}+\frac{3a^8e^{4ic}}{32}+\frac{a^8e^{2ic}}{64}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**14*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-4398046511104*I*a**8*d**6*exp(14*I*c)*exp(14*I*d*x) - 3078632
5577728*I*a**8*d**6*exp(12*I*c)*exp(12*I*d*x) - 92358976733184*I*a**8*d**6
*exp(10*I*c)*exp(10*I*d*x) - 153931627888640*I*a**8*d**6*exp(8*I*c)*exp(8*
I*d*x) - 153931627888640*I*a**8*d**6*exp(6*I*c)*exp(6*I*d*x) - 92358976733
184*I*a**8*d**6*exp(4*I*c)*exp(4*I*d*x) - 30786325577728*I*a**8*d**6*exp(2
*I*c)*exp(2*I*d*x))/(3940649673949184*d**7), Ne(d**7, 0)), (x*(a**8*exp(14
*I*c)/64 + 3*a**8*exp(12*I*c)/32 + 15*a**8*exp(10*I*c)/64 + 5*a**8*exp(8*I
*c)/16 + 15*a**8*exp(6*I*c)/64 + 3*a**8*exp(4*I*c)/32 + a**8*exp(2*I*c)/64
), True))
```

3.88.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(21) = 42$.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{a^8\tan\left(dx+c\right)^7-7i\,a^8\tan\left(dx+c\right)^6-21\,a^8\tan\left(dx+c\right)^5+35i\,a^8\tan\left(dx+c\right)^4+35\,a^8\tan\left(dx+c\right)^3+}{7\left(\tan\left(dx+c\right)^{14}+7\,\tan\left(dx+c\right)^{12}+21\,\tan\left(dx+c\right)^{10}+35\,\tan\left(dx+c\right)^8+35\,\tan\left(dx+c\right)^6+\right.}$$

input
```
integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/7*(a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5
+ 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^
2 - 7*a^8*tan(d*x + c) + I*a^8)/((tan(d*x + c)^14 + 7*tan(d*x + c)^12 + 21
*tan(d*x + c)^10 + 35*tan(d*x + c)^8 + 35*tan(d*x + c)^6 + 21*tan(d*x + c)
^4 + 7*tan(d*x + c)^2 + 1)*d)
```

3.88.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(21) = 42$.

Time = 1.31 (sec) , antiderivative size = 465, normalized size of antiderivative = 17.22

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{i\,a^8e^{(42i\,dx+28i\,c)}+21i\,a^8e^{(40i\,dx+26i\,c)}+210i\,a^8e^{(38i\,dx+24i\,c)}+1330i\,a^8e^{(36i\,dx+22i\,c)}+5985i\,a^8e^{(34i\,dx+20i\,c)}}{896\,(d}$$

input
```
integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/896*(I*a^8*e^(42*I*d*x + 28*I*c) + 21*I*a^8*e^(40*I*d*x + 26*I*c) + 210
*I*a^8*e^(38*I*d*x + 24*I*c) + 1330*I*a^8*e^(36*I*d*x + 22*I*c) + 5985*I*a
^8*e^(34*I*d*x + 20*I*c) + 20349*I*a^8*e^(32*I*d*x + 18*I*c) + 54264*I*a^8
*e^(30*I*d*x + 16*I*c) + 116279*I*a^8*e^(28*I*d*x + 14*I*c) + 203476*I*a^8
*e^(26*I*d*x + 12*I*c) + 293839*I*a^8*e^(24*I*d*x + 10*I*c) + 352352*I*a^8
*e^(22*I*d*x + 8*I*c) + 351715*I*a^8*e^(20*I*d*x + 6*I*c) + 291928*I*a^8*e
^(18*I*d*x + 4*I*c) + 200487*I*a^8*e^(16*I*d*x + 2*I*c) + 51261*I*a^8*e^(1
2*I*d*x - 2*I*c) + 18347*I*a^8*e^(10*I*d*x - 4*I*c) + 4984*I*a^8*e^(8*I*d*
x - 6*I*c) + 966*I*a^8*e^(6*I*d*x - 8*I*c) + 119*I*a^8*e^(4*I*d*x - 10*I*c
) + 7*I*a^8*e^(2*I*d*x - 12*I*c) + 112848*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d
*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 3
64*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d
*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) +
2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*
x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*
d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.88.9 Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\cos\left(c+d\,x\right)^8\left(\tan(c+d\,x)-7\mathrm{i}\right)}{7\,d} + \frac{64\,a^8\cos\left(c+d\,x\right)^{14}\left(\tan(c+d\,x)-\mathrm{i}\right)}{7\,d} + \frac{8\,a^8\cos\left(c+d\,x\right)^{10}\left(3\tan(c+d\,x)-7\mathrm{i}\right)}{7\,d} - \frac{16\,a^8\cos\left(c+d\,x\right)^{12}\left(5\tan(c+d\,x)-7\mathrm{i}\right)}{7\,d}$$

input
```
int(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(64*a^8*cos(c + d*x)^14*(tan(c + d*x) - 1i))/(7*d) - (a^8*cos(c + d*x)^8*(
tan(c + d*x) - 7i))/(7*d) + (8*a^8*cos(c + d*x)^10*(3*tan(c + d*x) - 7i))/
(7*d) - (16*a^8*cos(c + d*x)^12*(5*tan(c + d*x) - 7i))/(7*d)
```

3.89 $\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.89.1 Optimal result

Integrand size = 24, antiderivative size = 225

$$\begin{aligned}\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = {} & \frac{a^8x}{256}-\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}\\ & -\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}\\ & -\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}\\ & -\frac{ia^{13}}{80d(a-ia\tan(c+dx))^5}\\ & -\frac{ia^{12}}{128d(a-ia\tan(c+dx))^4}\\ & -\frac{ia^{11}}{192d(a-ia\tan(c+dx))^3}\\ & -\frac{ia^{10}}{256d(a-ia\tan(c+dx))^2}\\ & -\frac{ia^9}{256d(a-ia\tan(c+dx))}\end{aligned}$$

output
```
1/256*a^8*x-1/16*I*a^16/d/(a-I*a*tan(d*x+c))^8-1/28*I*a^15/d/(a-I*a*tan(d*
x+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c))^6-1/80*I*a^13/d/(a-I*a*tan(d*x+c)
)^5-1/128*I*a^12/d/(a-I*a*tan(d*x+c))^4-1/192*I*a^11/d/(a-I*a*tan(d*x+c))^
3-1/256*I*a^10/d/(a-I*a*tan(d*x+c))^2-1/256*I*a^9/d/(a-I*a*tan(d*x+c))
```

3.89.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{ia^8\sec^8(c+dx)(7350+12544\cos(2(c+dx))+7840\cos(4(c+dx))+3840\cos(6(c+dx))+1194\cos(}{}$$

input
```
Integrate[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/215040*I)*a^8*Sec[c + d*x]^8*(7350 + 12544*Cos[2*(c + d*x)] + 7840*Co
s[4*(c + d*x)] + 3840*Cos[6*(c + d*x)] + 1194*Cos[8*(c + d*x)] - (3136*I)*
Sin[2*(c + d*x)] - (3920*I)*Sin[4*(c + d*x)] - (2880*I)*Sin[6*(c + d*x)] -
 (1089*I)*Sin[8*(c + d*x)] + 840*ArcTan[Tan[c + d*x]]*(I*Cos[8*(c + d*x)]
+ Sin[8*(c + d*x)])))/(d*(I + Tan[c + d*x])^8)
```

3.89.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{16}}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{ia^{17}\int \frac{1}{(a-ia\tan(c+dx))^9(i\tan(c+dx)a+a)}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \text{54}$$
$$-\frac{ia^{17}\int \left(\frac{1}{2(a-ia\tan(c+dx))^9a}+\frac{1}{4(a-ia\tan(c+dx))^8a^2}+\frac{1}{8(a-ia\tan(c+dx))^7a^3}+\frac{1}{16(a-ia\tan(c+dx))^6a^4}+\frac{1}{32(a-ia\tan(c+dx))^5a^5}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{17}\left(\frac{i\arctan(\tan(c+dx))}{256a^9}+\frac{1}{256a^8(a-ia\tan(c+dx))}+\frac{1}{256a^7(a-ia\tan(c+dx))^2}+\frac{1}{192a^6(a-ia\tan(c+dx))^3}+\frac{1}{128a^5(a-ia\tan(c+dx))^4}\right.}{d}$$

input
```
Int[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^17*(((I/256)*ArcTan[Tan[c + d*x]])/a^9 + 1/(16*a*(a - I*a*Tan[c +
d*x])^8) + 1/(28*a^2*(a - I*a*Tan[c + d*x])^7) + 1/(48*a^3*(a - I*a*Tan[c
+ d*x])^6) + 1/(80*a^4*(a - I*a*Tan[c + d*x])^5) + 1/(128*a^5*(a - I*a*Tan
[c + d*x])^4) + 1/(192*a^6*(a - I*a*Tan[c + d*x])^3) + 1/(256*a^7*(a - I*a
*Tan[c + d*x])^2) + 1/(256*a^8*(a - I*a*Tan[c + d*x]))))/d
```

3.89.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.89.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(191) = 382$.

Time = 2.59 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.28

Expression too large to display

input
```
int(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/16*sin(d*x+c)^7*cos(d*x+c)^9-1/32*sin(d*x+c)^5*cos(d*x+c)^9-5
/384*sin(d*x+c)^3*cos(d*x+c)^9-1/256*sin(d*x+c)*cos(d*x+c)^9+1/2048*(cos(d
*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+3
5/32768*d*x+35/32768*c)+56*I*a^8*(-1/16*sin(d*x+c)^4*cos(d*x+c)^12-1/56*si
n(d*x+c)^2*cos(d*x+c)^12-1/336*cos(d*x+c)^12)-28*a^8*(-1/16*sin(d*x+c)^5*c
os(d*x+c)^11-5/224*sin(d*x+c)^3*cos(d*x+c)^11-5/896*sin(d*x+c)*cos(d*x+c)^
11+1/1792*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x
+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+45/32768*d*x+45/32768*c)-8*I*a^8*(-1/
16*sin(d*x+c)^6*cos(d*x+c)^10-3/112*sin(d*x+c)^4*cos(d*x+c)^10-1/112*cos(d
*x+c)^10*sin(d*x+c)^2-1/560*cos(d*x+c)^10)+70*a^8*(-1/16*sin(d*x+c)^3*cos(
d*x+c)^13-3/224*sin(d*x+c)*cos(d*x+c)^13+1/896*(cos(d*x+c)^11+11/10*cos(d*
x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/25
6*cos(d*x+c))*sin(d*x+c)+99/32768*d*x+99/32768*c)-1/2*I*a^8*cos(d*x+c)^16-
28*a^8*(-1/16*sin(d*x+c)*cos(d*x+c)^15+1/224*(cos(d*x+c)^13+13/12*cos(d*x+
c)^11+143/120*cos(d*x+c)^9+429/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001
/512*cos(d*x+c)^3+3003/1024*cos(d*x+c))*sin(d*x+c)+429/32768*d*x+429/32768
*c)-56*I*a^8*(-1/16*cos(d*x+c)^14*sin(d*x+c)^2-1/112*cos(d*x+c)^14)+a^8*(1
/16*(cos(d*x+c)^15+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*cos(d*x
+c)^9+1287/896*cos(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^3+64
35/2048*cos(d*x+c))*sin(d*x+c)+6435/32768*d*x+6435/32768*c))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.56

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{1680\,a^8dx - 105i\,a^8e^{(16i\,dx+16i\,c)} - 960i\,a^8e^{(14i\,dx+14i\,c)} - 3920i\,a^8e^{(12i\,dx+12i\,c)} - 9408i\,a^8e^{(10i\,dx+10i\,c)} - 1470}{430080\,d}$$

input
```
integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/430080*(1680*a^8*d*x - 105*I*a^8*e^(16*I*d*x + 16*I*c) - 960*I*a^8*e^(14
*I*d*x + 14*I*c) - 3920*I*a^8*e^(12*I*d*x + 12*I*c) - 9408*I*a^8*e^(10*I*d
*x + 10*I*c) - 14700*I*a^8*e^(8*I*d*x + 8*I*c) - 15680*I*a^8*e^(6*I*d*x +
6*I*c) - 11760*I*a^8*e^(4*I*d*x + 4*I*c) - 6720*I*a^8*e^(2*I*d*x + 2*I*c))
/d
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.44

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8x}{256}$$
$$+\begin{cases} \frac{-354658470655426560ia^8d^7e^{16ic}e^{16idx}-3242591731706757120ia^8d^7e^{14ic}e^{14idx}-13240582904469258240ia^8d^7e^{12ic}e^{12idx}-317773989707}{} \\ x\left(\frac{a^8e^{16ic}}{256}+\frac{a^8e^{14ic}}{32}+\frac{7a^8e^{12ic}}{64}+\frac{7a^8e^{10ic}}{32}+\frac{35a^8e^{8ic}}{128}+\frac{7a^8e^{6ic}}{32}+\frac{7a^8e^{4ic}}{64}+\frac{a^8e^{2ic}}{32}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**16*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*x/256 + Piecewise(((-354658470655426560*I*a**8*d**7*exp(16*I*c)*exp(1
6*I*d*x) - 3242591731706757120*I*a**8*d**7*exp(14*I*c)*exp(14*I*d*x) - 132
40582904469258240*I*a**8*d**7*exp(12*I*c)*exp(12*I*d*x) - 3177739897072621
9776*I*a**8*d**7*exp(10*I*c)*exp(10*I*d*x) - 49652185891759718400*I*a**8*d
**7*exp(8*I*c)*exp(8*I*d*x) - 52962331617877032960*I*a**8*d**7*exp(6*I*c)*
exp(6*I*d*x) - 39721748713407774720*I*a**8*d**7*exp(4*I*c)*exp(4*I*d*x) -
22698142121947299840*I*a**8*d**7*exp(2*I*c)*exp(2*I*d*x))/(145268109580462
7189760*d**8), Ne(d**8, 0)), (x*(a**8*exp(16*I*c)/256 + a**8*exp(14*I*c)/3
2 + 7*a**8*exp(12*I*c)/64 + 7*a**8*exp(10*I*c)/32 + 35*a**8*exp(8*I*c)/128
 + 7*a**8*exp(6*I*c)/32 + 7*a**8*exp(4*I*c)/64 + a**8*exp(2*I*c)/32), True
))
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.09

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{105\,(dx+c)a^8 + \frac{105\,a^8\tan(dx+c)^{15}+805\,a^8\tan(dx+c)^{13}+2681\,a^8\tan(dx+c)^{11}+5053\,a^8\tan(dx+c)^{9}+2883\,a^8\tan(dx+c)^{7}+21504i\,a^8}{\tan(dx+c)^{16}+8\,\tan(dx+c)^{14}+28\,\tan(dx+c)^{12}+56\,\tan(dx}}{26880}$$

input
```
integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/26880*(105*(d*x + c)*a^8 + (105*a^8*tan(d*x + c)^15 + 805*a^8*tan(d*x +
c)^13 + 2681*a^8*tan(d*x + c)^11 + 5053*a^8*tan(d*x + c)^9 + 2883*a^8*tan(
d*x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 + 70791*a^8*tan(d*x + c)^5 - 11468
8*I*a^8*tan(d*x + c)^4 - 117285*a^8*tan(d*x + c)^3 + 74752*I*a^8*tan(d*x +
 c)^2 + 26775*a^8*tan(d*x + c) - 4096*I*a^8)/(tan(d*x + c)^16 + 8*tan(d*x
+ c)^14 + 28*tan(d*x + c)^12 + 56*tan(d*x + c)^10 + 70*tan(d*x + c)^8 + 56
*tan(d*x + c)^6 + 28*tan(d*x + c)^4 + 8*tan(d*x + c)^2 + 1))/d
```

3.89.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(175) = 350$.

Time = 1.53 (sec) , antiderivative size = 1457, normalized size of antiderivative = 6.48

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/13762560*(53760*a^8*d*x*e^(28*I*d*x + 14*I*c) + 752640*a^8*d*x*e^(26*I*d
*x + 12*I*c) + 4892160*a^8*d*x*e^(24*I*d*x + 10*I*c) + 19568640*a^8*d*x*e^
(22*I*d*x + 8*I*c) + 53813760*a^8*d*x*e^(20*I*d*x + 6*I*c) + 107627520*a^8
*d*x*e^(18*I*d*x + 4*I*c) + 161441280*a^8*d*x*e^(16*I*d*x + 2*I*c) + 16144
1280*a^8*d*x*e^(12*I*d*x - 2*I*c) + 107627520*a^8*d*x*e^(10*I*d*x - 4*I*c)
 + 53813760*a^8*d*x*e^(8*I*d*x - 6*I*c) + 19568640*a^8*d*x*e^(6*I*d*x - 8*
I*c) + 4892160*a^8*d*x*e^(4*I*d*x - 10*I*c) + 752640*a^8*d*x*e^(2*I*d*x -
12*I*c) + 184504320*a^8*d*x*e^(14*I*d*x) + 53760*a^8*d*x*e^(-14*I*c) - 259
35*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 363090*I*a^8
*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2360085*I*a^8*e^(24*
I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440340*I*a^8*e^(22*I*d*x +
 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 25960935*I*a^8*e^(20*I*d*x + 6*I*c)
*log(e^(2*I*d*x + 2*I*c) + 1) - 51921870*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^
(2*I*d*x + 2*I*c) + 1) - 77882805*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) - 77882805*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) - 51921870*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1
) - 25960935*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440
340*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2360085*I*a^8
*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 363090*I*a^8*e^(2*I*d
*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 89008920*I*a^8*e^(14*I*d*x)...
```

3.89.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.87

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\,x}{256} - \frac{-\frac{a^8\,\tan(c+d\,x)^7}{256}-\frac{a^8\,\tan(c+d\,x)^6\,1\mathrm{i}}{32}+\frac{85\,a^8\,\tan(c+d\,x)^5}{768}+\frac{a^8\,\tan(c+d\,x)^4\,11\mathrm{i}}{48}-\frac{1193\,a^8\,\tan(c+d\,x)^3}{3840}-a^8\,\mathrm{t}}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^8+{\mathrm{tan}\left(c+d\,x\right)}^7\,8\mathrm{i}-28\,{\mathrm{tan}\left(c+d\,x\right)}^6-{\mathrm{tan}\left(c+d\,x\right)}^5\,56\mathrm{i}+70\,{\mathrm{tan}\left(c+d\,x\right)}^4+\mathrm{tan}\,(c\right.}$$

input
```
int(cos(c + d*x)^16*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*x)/256 - ((5993*a^8*tan(c + d*x))/26880 + (a^8*16i)/105 - (a^8*tan(c
+ d*x)^2*143i)/480 - (1193*a^8*tan(c + d*x)^3)/3840 + (a^8*tan(c + d*x)^4*
11i)/48 + (85*a^8*tan(c + d*x)^5)/768 - (a^8*tan(c + d*x)^6*1i)/32 - (a^8*
tan(c + d*x)^7)/256)/(d*(tan(c + d*x)^3*56i - 28*tan(c + d*x)^2 - tan(c +
d*x)*8i + 70*tan(c + d*x)^4 - tan(c + d*x)^5*56i - 28*tan(c + d*x)^6 + tan
(c + d*x)^7*8i + tan(c + d*x)^8 + 1))
```

3.90 $\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.90.1 Optimal result

Integrand size = 24, antiderivative size = 279

$$\begin{aligned}\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{5a^8x}{512}-\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9}-\frac{ia^{16}}{32d(a-ia\tan(c+dx))^8}\\ &\quad-\frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{ia^{13}}{64d(a-ia\tan(c+dx))^5}\\ &\quad-\frac{3ia^{12}}{256d(a-ia\tan(c+dx))^4}-\frac{7ia^{11}}{768d(a-ia\tan(c+dx))^3}-\frac{ia^{10}}{128d(a-ia\tan(c+dx))^2}\\ &\quad-\frac{9ia^9}{1024d(a-ia\tan(c+dx))}+\frac{ia^9}{1024d(a+ia\tan(c+dx))}\end{aligned}$$

output

```
5/512*a^8*x-1/36*I*a^17/d/(a-I*a*tan(d*x+c))^9-1/32*I*a^16/d/(a-I*a*tan(d*
x+c))^8-3/112*I*a^15/d/(a-I*a*tan(d*x+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c
))^6-1/64*I*a^13/d/(a-I*a*tan(d*x+c))^5-3/256*I*a^12/d/(a-I*a*tan(d*x+c))^
4-7/768*I*a^11/d/(a-I*a*tan(d*x+c))^3-1/128*I*a^10/d/(a-I*a*tan(d*x+c))^2-
9/1024*I*a^9/d/(a-I*a*tan(d*x+c))+1/1024*I*a^9/d/(a+I*a*tan(d*x+c))
```

3.90.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{ia^8\sec^{10}(c+dx)(7938+14112\cos(2(c+dx))+10080\cos(4(c+dx))+6480\cos(6(c+dx))+2462\,\mathrm{co}}{}$$

input
```
Integrate[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/258048*I)*a^8*Sec[c + d*x]^10*(7938 + 14112*Cos[2*(c + d*x)] + 10080*
Cos[4*(c + d*x)] + 6480*Cos[6*(c + d*x)] + 2462*Cos[8*(c + d*x)] - 112*Cos
[10*(c + d*x)] - (3528*I)*Sin[2*(c + d*x)] - (5040*I)*Sin[4*(c + d*x)] - (
4860*I)*Sin[6*(c + d*x)] - (2147*I)*Sin[8*(c + d*x)] + 2520*ArcTan[Tan[c +
 d*x]]*(I*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]) + (140*I)*Sin[10*(c + d*x)]
))/(d*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^9)
```

3.90.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{18}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^{19}\int \frac{1}{(a-ia\tan(c+dx))^{10}(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^{19}\int\left(\frac{9}{1024a^{10}(a-ia\tan(c+dx))^2}+\frac{1}{1024a^{10}(i\tan(c+dx)a+a)^2}+\frac{1}{64a^9(a-ia\tan(c+dx))^3}+\frac{7}{256a^8(a-ia\tan(c+dx))^4}+\frac{3}{64a^7(a-ia\,\mathrm{t}}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^{19}\left(\frac{5i\arctan(\tan(c+dx))}{512a^{11}}+\frac{9}{1024a^{10}(a-ia\tan(c+dx))}-\frac{1}{1024a^{10}(a+ia\tan(c+dx))}+\frac{1}{128a^9(a-ia\tan(c+dx))^2}+\frac{7}{768a^8(a-ia\tan(c+}\right.}{}$$

input
```
Int[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*a^19*((((5*I)/512)*ArcTan[Tan[c + d*x]])/a^11 + 1/(36*a^2*(a - I*a*T
an[c + d*x])^9) + 1/(32*a^3*(a - I*a*Tan[c + d*x])^8) + 3/(112*a^4*(a - I*
a*Tan[c + d*x])^7) + 1/(48*a^5*(a - I*a*Tan[c + d*x])^6) + 1/(64*a^6*(a -
I*a*Tan[c + d*x])^5) + 3/(256*a^7*(a - I*a*Tan[c + d*x])^4) + 7/(768*a^8*(
a - I*a*Tan[c + d*x])^3) + 1/(128*a^9*(a - I*a*Tan[c + d*x])^2) + 9/(1024*
a^10*(a - I*a*Tan[c + d*x])) - 1/(1024*a^10*(a + I*a*Tan[c + d*x]))))/d
```

3.90.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.90.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(237) = 474$.

Time = 2.28 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.83

Expression too large to display

input
```
int(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/18*sin(d*x+c)^7*cos(d*x+c)^11-7/288*sin(d*x+c)^5*cos(d*x+c)^1
1-5/576*sin(d*x+c)^3*cos(d*x+c)^11-5/2304*sin(d*x+c)*cos(d*x+c)^11+1/4608*
(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/
128*cos(d*x+c))*sin(d*x+c)+35/65536*d*x+35/65536*c)-4/9*I*a^8*cos(d*x+c)^1
8-28*a^8*(-1/18*sin(d*x+c)^5*cos(d*x+c)^13-5/288*sin(d*x+c)^3*cos(d*x+c)^1
3-5/1344*sin(d*x+c)*cos(d*x+c)^13+5/16128*(cos(d*x+c)^11+11/10*cos(d*x+c)^
9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos
(d*x+c))*sin(d*x+c)+55/65536*d*x+55/65536*c)-8*I*a^8*(-1/18*cos(d*x+c)^12*
sin(d*x+c)^6-1/48*sin(d*x+c)^4*cos(d*x+c)^12-1/168*sin(d*x+c)^2*cos(d*x+c)
^12-1/1008*cos(d*x+c)^12)+70*a^8*(-1/18*sin(d*x+c)^3*cos(d*x+c)^15-1/96*si
n(d*x+c)*cos(d*x+c)^15+1/1344*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*c
os(d*x+c)^9+429/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)
^3+3003/1024*cos(d*x+c))*sin(d*x+c)+143/65536*d*x+143/65536*c)-56*I*a^8*(-
1/18*cos(d*x+c)^16*sin(d*x+c)^2-1/144*cos(d*x+c)^16)-28*a^8*(-1/18*sin(d*x
+c)*cos(d*x+c)^17+1/288*(cos(d*x+c)^15+15/14*cos(d*x+c)^13+65/56*cos(d*x+c
)^11+143/112*cos(d*x+c)^9+1287/896*cos(d*x+c)^7+429/256*cos(d*x+c)^5+2145/
1024*cos(d*x+c)^3+6435/2048*cos(d*x+c))*sin(d*x+c)+715/65536*d*x+715/65536
*c)+56*I*a^8*(-1/18*cos(d*x+c)^14*sin(d*x+c)^4-1/72*cos(d*x+c)^14*sin(d*x+
c)^2-1/504*cos(d*x+c)^14)+a^8*(1/18*(cos(d*x+c)^17+17/16*cos(d*x+c)^15+255
/224*cos(d*x+c)^13+1105/896*cos(d*x+c)^11+2431/1792*cos(d*x+c)^9+21879/...
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.58

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{(5040\,a^8dxe^{(2i\,dx+2i\,c)} - 28i\,a^8e^{(20i\,dx+20i\,c)} - 315i\,a^8e^{(18i\,dx+18i\,c)} - 1620i\,a^8e^{(16i\,dx+16i\,c)} - 5040i\,a^8e^{(14i\,dx+1}}{}$$

input
```
integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output

```
1/516096*(5040*a^8*d*x*e^(2*I*d*x + 2*I*c) - 28*I*a^8*e^(20*I*d*x + 20*I*c
) - 315*I*a^8*e^(18*I*d*x + 18*I*c) - 1620*I*a^8*e^(16*I*d*x + 16*I*c) - 5
040*I*a^8*e^(14*I*d*x + 14*I*c) - 10584*I*a^8*e^(12*I*d*x + 12*I*c) - 1587
6*I*a^8*e^(10*I*d*x + 10*I*c) - 17640*I*a^8*e^(8*I*d*x + 8*I*c) - 15120*I*
a^8*e^(6*I*d*x + 6*I*c) - 11340*I*a^8*e^(4*I*d*x + 4*I*c) + 252*I*a^8)*e^(
-2*I*d*x - 2*I*c)/d
```

3.90.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.48

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{5a^8x}{512} + \begin{cases} \frac{(-277298568799925181577403826176ia^8d^9e^{20ic}e^{18idx} - 311960889899915829274579304480ia^8d^9e^{18ic}e^{16idx} - 1604370290913852836269 2}{} \\ x\left(-\frac{5a^8}{512} + \frac{(a^8e^{20ic}+10a^8e^{18ic}+45a^8e^{16ic}+120a^8e^{14ic}+210a^8e^{12ic}+252a^8e^{10ic}+210a^8e^{8ic}+120a^8e^{6ic}+45a^8e^{4ic}+10a^8e^{2ic}+a^8)e^{-2i}}{1024}\right. \end{cases}$$

input

```
integrate(cos(d*x+c)**18*(a+I*a*tan(d*x+c))**8,x)
```

output

```
5*a**8*x/512 + Piecewise(((-277298568799925181577403826176*I*a**8*d**9*exp
(20*I*c)*exp(18*I*d*x) - 311960889899915829274579304448 0*I*a**8*d**9*exp(1
8*I*c)*exp(16*I*d*x) - 1604370290913852836269264994304 0*I*a**8*d**9*exp(16
*I*c)*exp(14*I*d*x) - 4991374238398653268393268871168 0*I*a**8*d**9*exp(14*
I*c)*exp(12*I*d*x) - 1048188590063717186362586462945 28*I*a**8*d**9*exp(12*
I*c)*exp(10*I*d*x) - 1572282885095575779543879694417 92*I*a**8*d**9*exp(10*
I*c)*exp(8*I*d*x) - 1746980983439528643937644104908 80*I*a**8*d**9*exp(8*I*
c)*exp(6*I*d*x) - 1497412271519595980517980661350 40*I*a**8*d**9*exp(6*I*c)
*exp(4*I*d*x) - 1123059203639696985388485496012 80*I*a**8*d**9*exp(4*I*c)*e
xp(2*I*d*x) + 2495687119199326634196634435584*I*a**8*d**9*exp(-2*I*d*x))*e
xp(-2*I*c)/(5111167220120220946834707324076032*d**10), Ne(d**10*exp(2*I*c)
, 0)), (x*(-5*a**8/512 + (a**8*exp(20*I*c) + 10*a**8*exp(18*I*c) + 45*a**8
*exp(16*I*c) + 120*a**8*exp(14*I*c) + 210*a**8*exp(12*I*c) + 252*a**8*exp(
10*I*c) + 210*a**8*exp(8*I*c) + 120*a**8*exp(6*I*c) + 45*a**8*exp(4*I*c) +
 10*a**8*exp(2*I*c) + a**8)*exp(-2*I*c)/1024), True))
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{315\,(dx+c)a^8 + \frac{315\,a^8\tan(dx+c)^{17}+2730\,a^8\tan(dx+c)^{15}+10458\,a^8\tan(dx+c)^{13}+23202\,a^8\tan(dx+c)^{11}+32768\,a^8\tan(dx+c)^{9}+27486}{\tan(dx+c)^{18}+9\,\tan(dx+c)^{16}+36\,\tan(dx+c)^{14}+84\,\tan}}{}$$

input
```
integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/32256*(315*(d*x + c)*a^8 + (315*a^8*tan(d*x + c)^17 + 2730*a^8*tan(d*x +
 c)^15 + 10458*a^8*tan(d*x + c)^13 + 23202*a^8*tan(d*x + c)^11 + 32768*a^8
*tan(d*x + c)^9 + 27486*a^8*tan(d*x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 +
86310*a^8*tan(d*x + c)^5 - 119808*I*a^8*tan(d*x + c)^4 - 121002*a^8*tan(d*
x + c)^3 + 82944*I*a^8*tan(d*x + c)^2 + 31941*a^8*tan(d*x + c) - 5120*I*a^
8)/(tan(d*x + c)^18 + 9*tan(d*x + c)^16 + 36*tan(d*x + c)^14 + 84*tan(d*x
+ c)^12 + 126*tan(d*x + c)^10 + 126*tan(d*x + c)^8 + 84*tan(d*x + c)^6 + 3
6*tan(d*x + c)^4 + 9*tan(d*x + c)^2 + 1))/d
```

3.90.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1514 vs. 2(217) = 434.

Time = 1.63 (sec) , antiderivative size = 1514, normalized size of antiderivative = 5.43

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/16515072*(161280*a^8*d*x*e^(30*I*d*x + 16*I*c) + 2257920*a^8*d*x*e^(28*I
*d*x + 14*I*c) + 14676480*a^8*d*x*e^(26*I*d*x + 12*I*c) + 58705920*a^8*d*x
*e^(24*I*d*x + 10*I*c) + 161441280*a^8*d*x*e^(22*I*d*x + 8*I*c) + 32288256
0*a^8*d*x*e^(20*I*d*x + 6*I*c) + 484323840*a^8*d*x*e^(18*I*d*x + 4*I*c) +
553512960*a^8*d*x*e^(16*I*d*x + 2*I*c) + 322882560*a^8*d*x*e^(12*I*d*x - 2
*I*c) + 161441280*a^8*d*x*e^(10*I*d*x - 4*I*c) + 58705920*a^8*d*x*e^(8*I*d
*x - 6*I*c) + 14676480*a^8*d*x*e^(6*I*d*x - 8*I*c) + 2257920*a^8*d*x*e^(4*
I*d*x - 10*I*c) + 161280*a^8*d*x*e^(2*I*d*x - 12*I*c) + 484323840*a^8*d*x*
e^(14*I*d*x) - 75789*I*a^8*e^(30*I*d*x + 16*I*c)*log(e^(2*I*d*x + 2*I*c) +
 1) - 1061046*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 6
896799*I*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 27587196
*I*a^8*e^(24*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 75864789*I*a^8
*e^(22*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 151729578*I*a^8*e^(20
*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 227594367*I*a^8*e^(18*I*d*x
 + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 260107848*I*a^8*e^(16*I*d*x + 2*I
*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 151729578*I*a^8*e^(12*I*d*x - 2*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 75864789*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) - 27587196*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x +
2*I*c) + 1) - 6896799*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) +
1) - 1061046*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - ...
```

3.90.9 Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.83

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{5\,a^8\,x}{512} + \frac{\frac{5\,a^8\,\tan(c+d\,x)^9}{512}+\frac{a^8\,\tan(c+d\,x)^8\,5\mathrm{i}}{64}-\frac{205\,a^8\,\tan(c+d\,x)^7}{768}-\frac{a^8\,\tan(c+d\,x)^6\,95\mathrm{i}}{192}+\frac{a^8\,\tan(c+d\,x)^5}{2}+\frac{a^8\,\tan(c+d\,x)^4}{64}}{d\,\left({\tan\left(c+d\,x\right)}^{10}+{\tan\left(c+d\,x\right)}^9\,8\mathrm{i}-27\,{\tan\left(c+d\,x\right)}^8-{\tan\left(c+d\,x\right)}^7\,48\mathrm{i}+42\,{\tan\left(c+d\,x\right)}^6+42\,\tan\right.}$$

input
```
int(cos(c + d*x)^18*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(5*a^8*x)/512 + ((a^8*tan(c + d*x)^2*163i)/448 - (a^8*10i)/63 - (9019*a^8*
tan(c + d*x))/32256 + (393*a^8*tan(c + d*x)^3)/1792 + (a^8*tan(c + d*x)^4*
11i)/64 + (a^8*tan(c + d*x)^5)/2 - (a^8*tan(c + d*x)^6*95i)/192 - (205*a^8
*tan(c + d*x)^7)/768 + (a^8*tan(c + d*x)^8*5i)/64 + (5*a^8*tan(c + d*x)^9)
/512)/(d*(tan(c + d*x)^3*48i - 27*tan(c + d*x)^2 - tan(c + d*x)*8i + 42*ta
n(c + d*x)^4 + 42*tan(c + d*x)^6 - tan(c + d*x)^7*48i - 27*tan(c + d*x)^8
+ tan(c + d*x)^9*8i + tan(c + d*x)^10 + 1))
```

3.91 $\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.91.1 Optimal result

Integrand size = 22, antiderivative size = 235

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = &-\frac{3003a^8\operatorname{arctanh}(\sin(c+dx))}{16d}\\ &-\frac{3003ia^8\sec(c+dx)}{16d}\\ &-\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}\\ &-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}\\ &-\frac{429ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{40d}\\ &-\frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d}\\ &-\frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d}\\ &-\frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d}\end{aligned}$$

output

```
-3003/16*a^8*arctanh(sin(d*x+c))/d-3003/16*I*a^8*sec(d*x+c)/d-13/6*I*a^3*s
ec(d*x+c)*(a+I*a*tan(d*x+c))^5/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^7/d-4
29/40*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-143/30*I*sec(d*x+c)*(a^2
+I*a^2*tan(d*x+c))^4/d-1001/40*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))^2/d-100
1/16*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

3.91.2 Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\cos^2(c+dx)(\cos(8c)-i\sin(8c))\,\big(-658944i\cos(c+dx)+720720\cos^6(c+dx)\,\big(\log\big(\cos\big(\tfrac{1}{2}(c+dx)\big)-}{}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Cos[c + d*x]^2*(Cos[8*c] - I*Sin[8*c])*((-658944*I)*Cos[c + d*x] + 72
0720*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-73216*I)*Cos[3*(c + d*x)] - (19968*
I)*Cos[5*(c + d*x)] - (1536*I)*Cos[7*(c + d*x)] + 12870*Sin[c + d*x] + 221
65*Sin[3*(c + d*x)] + 10959*Sin[5*(c + d*x)] + 1536*Sin[7*(c + d*x)]))*(-I
 + Tan[c + d*x])^8)/(3840*d*(Cos[d*x] + I*Sin[d*x])^8)
```

3.91.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3977, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3977}$$

$$-13a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3042}$$

$$-13a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \ 3979$$

$$-13a^2\left(\frac{11}{6}a\int \sec(c+dx)(i\tan(c+dx)a+a)^5dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \ 3042$$

$$-13a^2\left(\frac{11}{6}a\int \sec(c+dx)(i\tan(c+dx)a+a)^5dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}\right)-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \ 3979$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)+ia\sec(c+dx)(a\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \ 3042$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)+ia\sec(c+dx)(a\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \ 3979$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)+ia\sec(c+\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \ 3042$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)+ia\sec(c+\right.\right.$$
$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \ 3979$$

3.91. $\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int\sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+ia\sec(\right.\right.\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\int\sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+ia\sec(\right.\right.\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3979}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int\sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+i\right.\right.\right.\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int\sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+i\right.\right.\right.\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3967}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int\sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\right.\right.\right.\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{3042}$$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx\right.}{2d}\right.\right.\right.\right.\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

$$\Big\downarrow \text{4257}$$

3.91. $\int\cos(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$-13a^2\left(\frac{11}{6}a\left(\frac{9}{5}a\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right.\right.\right.\right.\right.$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^7)/d - 13*a^2*(((I/6)*a*Sec[
c + d*x]*(a + I*a*Tan[c + d*x])^5)/d + (11*a*(((I/5)*a*Sec[c + d*x]*(a + I
*a*Tan[c + d*x])^4)/d + (9*a*(((I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])
^3)/d + (7*a*(((I/3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (5*a*((3
*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c
+ d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d))/3))/4))/5))/6)
```

3.91.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.91.4 Maple [A] (verified)

Time = 90.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{128 i a^{8} {\mathrm e}^{i (d x +c)}}{d}-\frac{i a^{8} \left(62475 \,{\mathrm e}^{11 i (d x +c)}+246505 \,{\mathrm e}^{9 i (d x +c)}+416094 \,{\mathrm e}^{7 i (d x +c)}+364194 \,{\mathrm e}^{5 i (d x +c)}+163095 \,{\mathrm e}^{3 i (d x +c)}+\right.}{120 d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{6}}$
derivativedivides	$\frac{a^{8} \left(\frac{\sin^{9}(d x +c)}{6 \cos(d x +c)^{6}}-\frac{\sin^{9}(d x +c)}{8 \cos(d x +c)^{4}}+\frac{5 \left(\sin^{9}(d x +c)\right)}{16 \cos(d x +c)^{2}}+\frac{5 \left(\sin^{7}(d x +c)\right)}{16}+\frac{7 \left(\sin^{5}(d x +c)\right)}{16}+\frac{35 \left(\sin^{3}(d x +c)\right)}{48}+\frac{35 \sin(d x +c)}{16}-\frac{35 \ln(\sec(}{}\right.}{}$
default	$\frac{a^{8} \left(\frac{\sin^{9}(d x +c)}{6 \cos(d x +c)^{6}}-\frac{\sin^{9}(d x +c)}{8 \cos(d x +c)^{4}}+\frac{5 \left(\sin^{9}(d x +c)\right)}{16 \cos(d x +c)^{2}}+\frac{5 \left(\sin^{7}(d x +c)\right)}{16}+\frac{7 \left(\sin^{5}(d x +c)\right)}{16}+\frac{35 \left(\sin^{3}(d x +c)\right)}{48}+\frac{35 \sin(d x +c)}{16}-\frac{35 \ln(\sec(}{}\right.}{}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-128*I/d*a^8*exp(I*(d*x+c))-1/120*I*a^8/d/(exp(2*I*(d*x+c))+1)^6*(62475*ex
p(11*I*(d*x+c))+246505*exp(9*I*(d*x+c))+416094*exp(7*I*(d*x+c))+364194*exp
(5*I*(d*x+c))+163095*exp(3*I*(d*x+c))+29685*exp(I*(d*x+c)))-3003/16/d*a^8*
ln(exp(I*(d*x+c))+I)+3003/16/d*a^8*ln(exp(I*(d*x+c))-I)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.61

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{-30720i\,a^8e^{(13i\,dx+13i\,c)}-309270i\,a^8e^{(11i\,dx+11i\,c)}-953810i\,a^8e^{(9i\,dx+9i\,c)}-1446588i\,a^8e^{(7i\,dx+7i\,c)}-118918}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/240*(-30720*I*a^8*e^(13*I*d*x + 13*I*c) - 309270*I*a^8*e^(11*I*d*x + 11*
I*c) - 953810*I*a^8*e^(9*I*d*x + 9*I*c) - 1446588*I*a^8*e^(7*I*d*x + 7*I*c
) - 1189188*I*a^8*e^(5*I*d*x + 5*I*c) - 510510*I*a^8*e^(3*I*d*x + 3*I*c) -
 90090*I*a^8*e^(I*d*x + I*c) - 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^
(10*I*d*x + 10*I*c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I
*c) + 15*a^8*e^(4*I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^
(I*d*x + I*c) + I) + 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*I*d*x
+ 10*I*c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) + 15*a
^8*e^(4*I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I
*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8
*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*
d*e^(2*I*d*x + 2*I*c) + d)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.36

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx=\frac{3003a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{16}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{16}\right)}{d}$$
$$+\frac{-62475ia^8e^{11ic}e^{11idx}-246505ia^8e^{9ic}e^{9idx}-416094ia^8e^{7ic}e^{7idx}-364194ia^8e^{5ic}e^{5idx}-163095ia^8e^{3ic}e^{3id}}{120de^{12ic}e^{12idx}+720de^{10ic}e^{10idx}+1800de^{8ic}e^{8idx}+2400de^{6ic}e^{6idx}+1800de^{4ic}e^{4idx}+720de^{2ic}e}$$
$$+\begin{cases}-\frac{128ia^8e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 128a^8xe^{ic} & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**8,x)
```

output
```
3003*a**8*(log(exp(I*d*x) - I*exp(-I*c))/16 - log(exp(I*d*x) + I*exp(-I*c)
)/16)/d + (-62475*I*a**8*exp(11*I*c)*exp(11*I*d*x) - 246505*I*a**8*exp(9*I
*c)*exp(9*I*d*x) - 416094*I*a**8*exp(7*I*c)*exp(7*I*d*x) - 364194*I*a**8*e
xp(5*I*c)*exp(5*I*d*x) - 163095*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 29685*I*a
**8*exp(I*c)*exp(I*d*x))/(120*d*exp(12*I*c)*exp(12*I*d*x) + 720*d*exp(10*I
*c)*exp(10*I*d*x) + 1800*d*exp(8*I*c)*exp(8*I*d*x) + 2400*d*exp(6*I*c)*exp
(6*I*d*x) + 1800*d*exp(4*I*c)*exp(4*I*d*x) + 720*d*exp(2*I*c)*exp(2*I*d*x)
 + 120*d) + Piecewise((-128*I*a**8*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (128*
a**8*x*exp(I*c), True))
```

3.91.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(195) = 390.

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.69

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{5\,a^8\left(\frac{2\left(87\,\sin(dx+c)^5-136\,\sin(dx+c)^3+57\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}+105\,\log\left(\sin\left(dx+c\right)+1\right)-105\,\log\left(\sin\left(dx+c\right)-1\right)-\right.}{}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/480*(5*a^8*(2*(87*sin(d*x + c)^5 - 136*sin(d*x + c)^3 + 57*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 105*log(sin
(d*x + c) + 1) - 105*log(sin(d*x + c) - 1) - 96*sin(d*x + c)) + 840*a^8*(2
*(9*sin(d*x + c)^3 - 7*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1) + 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1) - 16*sin(d*x + c)
) + 8400*a^8*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 26880*I*a^8*(1/cos(d*x + c
) + cos(d*x + c)) + 8960*I*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*
cos(d*x + c)) + 768*I*a^8*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(
d*x + c)^5 + 5*cos(d*x + c)) + 6720*a^8*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1) - 2*sin(d*x + c)) + 3840*I*a^8*cos(d*x + c) - 480*a^8*sin(d*x
 + c))/d
```

3.91.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(195) = 390$.

Time = 1.44 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.93

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output

```
1/61440*(11512215*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6
9073290*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a
^8*e^(8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 230244300*a^8*e^(6*I*d
*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8*e^(4*I*d*x + 4*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*
d*x + I*c) + 1) - 19305*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) -
1) - 115830*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) - 289575*
a^8*e^(8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 386100*a^8*e^(6*I*d*x
 + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(4*I*d*x + 4*I*c)*log(
I*e^(I*d*x + I*c) - 1) - 115830*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I
*c) - 1) - 11512215*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 69073290*a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1726832
25*a^8*e^(8*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 230244300*a^8*e^(
6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(4*I*d*x +
4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(
-I*e^(I*d*x + I*c) + 1) + 19305*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) + 115830*a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1)
 + 289575*a^8*e^(8*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 386100*a^8
*e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(4*I*d*x +
 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 115830*a^8*e^(2*I*d*x + 2*I*c)*lo...
```

3.91.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.70

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\frac{3019\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{8}+\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}2891\mathrm{i}}{8}-\frac{52795\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{24}-\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}45115\mathrm{i}}{24}+\frac{22415\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{4}+}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}1\mathrm{i}-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}6\mathrm{i}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\tan\right.}$$
$$-\frac{3003\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
((a^8*tan(c/2 + (d*x)/2)^3*160729i)/120 - (127113*a^8*tan(c/2 + (d*x)/2)^2
)/40 + (167237*a^8*tan(c/2 + (d*x)/2)^4)/24 - (a^8*tan(c/2 + (d*x)/2)^5*12
977i)/4 - (97811*a^8*tan(c/2 + (d*x)/2)^6)/12 + (a^8*tan(c/2 + (d*x)/2)^7*
43757i)/12 + (22415*a^8*tan(c/2 + (d*x)/2)^8)/4 - (a^8*tan(c/2 + (d*x)/2)^
9*45115i)/24 - (52795*a^8*tan(c/2 + (d*x)/2)^10)/24 + (a^8*tan(c/2 + (d*x)
/2)^11*2891i)/8 + (3019*a^8*tan(c/2 + (d*x)/2)^12)/8 + (8848*a^8)/15 - (a^
8*tan(c/2 + (d*x)/2)*25499i)/120)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)
/2)^2*6i - 6*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*15i + 15*tan(c/2
+ (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*20i - 20*tan(c/2 + (d*x)/2)^7 + tan(c/
2 + (d*x)/2)^8*15i + 15*tan(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*6i -
6*tan(c/2 + (d*x)/2)^11 + tan(c/2 + (d*x)/2)^12*1i + tan(c/2 + (d*x)/2)^13
 + 1i)) - (3003*a^8*atanh(tan(c/2 + (d*x)/2)))/(8*d)
```

3.92 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.92.1 Optimal result 823
3.92.2 Mathematica [B] (warning: unable to verify) 824
3.92.3 Rubi [A] (verified) 824
3.92.4 Maple [A] (verified) 828
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3.92.1 Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{1155a^8\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{1155ia^8\sec(c+dx)}{8d}\\ &+\frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}\\ &-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}\\ &+\frac{33ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{4d}\\ &+\frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d}\\ &+\frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d}\end{aligned}$$

output
```
1155/8*a^8*arctanh(sin(d*x+c))/d+1155/8*I*a^8*sec(d*x+c)/d+22/3*I*a^3*cos(
d*x+c)*(a+I*a*tan(d*x+c))^5/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^7/d+
33/4*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d+77/4*I*sec(d*x+c)*(a^4+I*
a^4*tan(d*x+c))^2/d+385/8*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

3.92.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1540 vs. $2(205) = 410$.

Time = 8.30 (sec) , antiderivative size = 1540, normalized size of antiderivative = 7.51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input

```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
(-1155*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
]*(a + I*a*Tan[c + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (1155*Cos[8*
c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Ta
n[c + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^
8*(((-32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(C
os[d*x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*x]^8*((160*I)*Cos[7*c] + 16
0*Sin[7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((1
155*I)/8)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[
8*c]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((1155*I)/
8)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a
 + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Se
c[c]*(((236*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(a + I*a*Tan[c + d*x])^8)/(
d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(-160*Cos[7*c] + (160*I)*Si
n[7*c])*Sin[d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) +
 (Cos[c + d*x]^8*((32*Cos[5*c])/3 - ((32*I)/3)*Sin[5*c])*Sin[3*d*x]*(a + I
*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8
*c]/16 - (I/16)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d
*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) - (I*Cos[c + d*x]^8*((
4*Cos[8*c])/3 - ((4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)
/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2]...
```

3.92.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3977, 3042, 3977, 3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{11}{3}a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{11}{3}a^2\left(-9a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^5}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^5}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3979}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)-\frac{2ia\cos(c+da}{}\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\int \sec(c+dx)(i\tan(c+dx)a+a)^3dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}\right)-\frac{2ia\cos(c+da}{}\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{3979}$$

3.92. $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+}{}\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}\right)+\frac{ia\sec(c+}{}\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{ 3979}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\,\mathrm{se}}{}\right.\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia\,\mathrm{se}}{}\right.\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{ 3967}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)+\frac{ia}{}\right.\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)\right.\right.\right.$$
$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

$$\Big\downarrow \text{ 4257}$$

3.92. $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$-\frac{11}{3}a^2\left(-9a^2\left(\frac{7}{4}a\left(\frac{5}{3}a\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)\right.\right.\right.$$

$$\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^7)/d - (11*a^2*(((-2*I
)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - 9*a^2*(((I/4)*a*Sec[c + d*x
]*(a + I*a*Tan[c + d*x])^3)/d + (7*a*(((I/3)*a*Sec[c + d*x]*(a + I*a*Tan[c
 + d*x])^2)/d + (5*a*((3*a*((a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x
])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d))/3))/4)))/3
```

3.92.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.92.4 Maple [A] (verified)

Time = 253.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{32 i a^{8} {\mathrm e}^{3 i (d x +c)}}{3 d}+\frac{160 i a^{8} {\mathrm e}^{i (d x +c)}}{d}+\frac{i a^{8} \left(2295 \,{\mathrm e}^{7 i (d x +c)}+5855 \,{\mathrm e}^{5 i (d x +c)}+5153 \,{\mathrm e}^{3 i (d x +c)}+1545 \,{\mathrm e}^{i (d x +c)}\right)}{12 d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{4}}+ 1155$
derivativedivides	$a^{8} (\frac{\sin^{9}(d x +c)}{4 \cos(d x +c)^{4}}-\frac{5 \left(\sin^{9}(d x +c)\right)}{8 \cos(d x +c)^{2}}-\frac{5 \left(\sin^{7}(d x +c)\right)}{8}-\frac{7 \left(\sin^{5}(d x +c)\right)}{8}-\frac{35 \left(\sin^{3}(d x +c)\right)}{24}-\frac{35 \sin(d x +c)}{8}+\frac{35 \ln(\sec(d x +c)+\tan(d x +}{8}$
default	$a^{8} (\frac{\sin^{9}(d x +c)}{4 \cos(d x +c)^{4}}-\frac{5 \left(\sin^{9}(d x +c)\right)}{8 \cos(d x +c)^{2}}-\frac{5 \left(\sin^{7}(d x +c)\right)}{8}-\frac{7 \left(\sin^{5}(d x +c)\right)}{8}-\frac{35 \left(\sin^{3}(d x +c)\right)}{24}-\frac{35 \sin(d x +c)}{8}+\frac{35 \ln(\sec(d x +c)+\tan(d x +}{8}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-32/3*I/d*a^8*exp(3*I*(d*x+c))+160*I/d*a^8*exp(I*(d*x+c))+1/12*I*a^8/d/(ex
p(2*I*(d*x+c))+1)^4*(2295*exp(7*I*(d*x+c))+5855*exp(5*I*(d*x+c))+5153*exp(
3*I*(d*x+c))+1545*exp(I*(d*x+c)))+1155/8/d*a^8*ln(exp(I*(d*x+c))+I)-1155/8
/d*a^8*ln(exp(I*(d*x+c))-I)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.39

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{-256i\,a^8e^{(11i\,dx+11i\,c)}+2816i\,a^8e^{(9i\,dx+9i\,c)}+18414i\,a^8e^{(7i\,dx+7i\,c)}+33726i\,a^8e^{(5i\,dx+5i\,c)}+25410i\,a^8e^{(3i\,dx+3}}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/24*(-256*I*a^8*e^(11*I*d*x + 11*I*c) + 2816*I*a^8*e^(9*I*d*x + 9*I*c) +
18414*I*a^8*e^(7*I*d*x + 7*I*c) + 33726*I*a^8*e^(5*I*d*x + 5*I*c) + 25410*
I*a^8*e^(3*I*d*x + 3*I*c) + 6930*I*a^8*e^(I*d*x + I*c) + 3465*(a^8*e^(8*I*
d*x + 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a
^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) + I) - 3465*(a^8*e^(8*I*
d*x + 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a
^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*
I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x
+ 2*I*c) + d)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.35

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{1155a^8\left(-\frac{\log\left(e^{idx}-ie^{-ic}\right)}{8}+\frac{\log\left(e^{idx}+ie^{-ic}\right)}{8}\right)}{d}$$
$$+\frac{2295ia^8e^{7ic}e^{7idx}+5855ia^8e^{5ic}e^{5idx}+5153ia^8e^{3ic}e^{3idx}+1545ia^8e^{ic}e^{idx}}{12de^{8ic}e^{8idx}+48de^{6ic}e^{6idx}+72de^{4ic}e^{4idx}+48de^{2ic}e^{2idx}+12d}$$
$$+\begin{cases}\frac{-32ia^8de^{3ic}e^{3idx}+480ia^8de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0\\ x\left(32a^8e^{3ic}-160a^8e^{ic}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**8,x)
```

output
```
1155*a**8*(-log(exp(I*d*x) - I*exp(-I*c))/8 + log(exp(I*d*x) + I*exp(-I*c)
)/8)/d + (2295*I*a**8*exp(7*I*c)*exp(7*I*d*x) + 5855*I*a**8*exp(5*I*c)*exp
(5*I*d*x) + 5153*I*a**8*exp(3*I*c)*exp(3*I*d*x) + 1545*I*a**8*exp(I*c)*exp
(I*d*x))/(12*d*exp(8*I*c)*exp(8*I*d*x) + 48*d*exp(6*I*c)*exp(6*I*d*x) + 72
*d*exp(4*I*c)*exp(4*I*d*x) + 48*d*exp(2*I*c)*exp(2*I*d*x) + 12*d) + Piecew
ise(((-32*I*a**8*d*exp(3*I*c)*exp(3*I*d*x) + 480*I*a**8*d*exp(I*c)*exp(I*d
*x))/(3*d**2), Ne(d**2, 0)), (x*(32*a**8*exp(3*I*c) - 160*a**8*exp(I*c)),
True))
```

3.92.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(169) = 338$.

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.72

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{128i\,a^8\cos\left(dx+c\right)^3+448\,a^8\sin\left(dx+c\right)^3+896i\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^8+128i}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/48*(128*I*a^8*cos(d*x + c)^3 + 448*a^8*sin(d*x + c)^3 + 896*I*(cos(d*x
+ c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 + 128*I*(cos(d*x + c)^3 - (9
*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + 896*I*(cos(d*x
 + c)^3 - 3*cos(d*x + c))*a^8 + (16*sin(d*x + c)^3 - 6*(13*sin(d*x + c)^3
- 11*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 105*log(sin(d
*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 144*sin(d*x + c))*a^8 + 112*(4*
sin(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c)
 + 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^8 + 560*(2*sin(d*x +
 c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c)
)*a^8 + 16*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^8)/d
```

3.92.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2835 vs. $2(169) = 338$.

Time = 1.35 (sec) , antiderivative size = 2835, normalized size of antiderivative = 13.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output

```
1/98304*(763587*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 106
90218*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*
e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^(22*I*d
*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 764350587*a^8*e^(20*I*d*x + 6*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 1528701174*a^8*e^(18*I*d*x + 4*I*c)*log(I*e
^(I*d*x + I*c) + 1) + 2293051761*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 2293051761*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 1528701174*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 76435
0587*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^
(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*e^(4*I*d*x - 1
0*I*c)*log(I*e^(I*d*x + I*c) + 1) + 10690218*a^8*e^(2*I*d*x - 12*I*c)*log(
I*e^(I*d*x + I*c) + 1) + 2620630584*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c)
 + 1) + 763587*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 14956128*a^8*e
^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 209385792*a^8*e^(26*I*d*
x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1361007648*a^8*e^(24*I*d*x + 10*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 5444030592*a^8*e^(22*I*d*x + 8*I*c)*log(I
*e^(I*d*x + I*c) - 1) + 14971084128*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 29942168256*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c)
 - 1) + 44913252384*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) +
44913252384*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29942...
```

3.92.9 Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.67

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{\frac{1147\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{4}+\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\,3505\mathrm{i}}{4}-\frac{5639\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}-a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}3585\mathrm{i}+\frac{25993\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{6}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}3\mathrm{i}+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}13\mathrm{i}-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-\tan\right.}$$
$$+\frac{1155\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
((27565*a^8*tan(c/2 + (d*x)/2)^2)/12 - (a^8*tan(c/2 + (d*x)/2)^3*12041i)/3
 - 4575*a^8*tan(c/2 + (d*x)/2)^4 + (a^8*tan(c/2 + (d*x)/2)^5*33847i)/6 + (
25993*a^8*tan(c/2 + (d*x)/2)^6)/6 - a^8*tan(c/2 + (d*x)/2)^7*3585i - (5639
*a^8*tan(c/2 + (d*x)/2)^8)/3 + (a^8*tan(c/2 + (d*x)/2)^9*3505i)/4 + (1147*
a^8*tan(c/2 + (d*x)/2)^10)/4 - (1360*a^8)/3 + (a^8*tan(c/2 + (d*x)/2)*4293
i)/4)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*7i - 13*tan(c/2 + (d
*x)/2)^3 + tan(c/2 + (d*x)/2)^4*18i + 22*tan(c/2 + (d*x)/2)^5 - tan(c/2 +
(d*x)/2)^6*22i - 18*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*13i + 7*ta
n(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*3i - tan(c/2 + (d*x)/2)^11 + 1i
)) + (1155*a^8*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.93 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.93.1 Optimal result 833
3.93.2 Mathematica [B] (warning: unable to verify) 834
3.93.3 Rubi [A] (verified) 835
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3.93.9 Mupad [B] (verification not implemented) 841

3.93.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\begin{aligned}\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = &-\frac{63a^8\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{63ia^8\sec(c+dx)}{2d}\\ &+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d}\\ &-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}\\ &-\frac{42ia^2\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{5d}\\ &-\frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d}\end{aligned}$$

output

```
-63/2*a^8*arctanh(sin(d*x+c))/d-63/2*I*a^8*sec(d*x+c)/d+6/5*I*a^3*cos(d*x+
c)^3*(a+I*a*tan(d*x+c))^5/d-2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^7/d-42
/5*I*a^2*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-21/2*I*sec(d*x+c)*(a^8+I*a^
8*tan(d*x+c))/d
```

3.93.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1162 vs. $2(173) = 346$.

Time = 8.06 (sec) , antiderivative size = 1162, normalized size of antiderivative = 6.72

$$\begin{aligned}
&\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx \\
&= \frac{63\cos(8c)\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&\quad - \frac{63\cos(8c)\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos(5dx)\cos^8(c+dx)\left(-\frac{8}{5}i\cos(3c)-\frac{8}{5}\sin(3c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos(3dx)\cos^8(c+dx)(8i\cos(5c)+8\sin(5c))(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos(dx)\cos^8(c+dx)(-48i\cos(7c)-48\sin(7c))(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos^8(c+dx)\sec(c)(-8i\cos(8c)-8\sin(8c))(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&\quad - \frac{63i\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(8c)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{63i\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(8c)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos^8(c+dx)(48\cos(7c)-48i\sin(7c))\sin(dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos^8(c+dx)(-8\cos(5c)+8i\sin(5c))\sin(3dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos^8(c+dx)\left(\frac{8}{5}\cos(3c)-\frac{8}{5}i\sin(3c)\right)\sin(5dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&\quad + \frac{\cos^8(c+dx)\left(\frac{1}{4}\cos(8c)-\frac{1}{4}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad - \frac{i\cos^8(c+dx)(8\cos(8c)-8i\sin(8c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&\quad + \frac{\cos^8(c+dx)\left(-\frac{1}{4}\cos(8c)+\frac{1}{4}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&\quad + \frac{i\cos^8(c+dx)(8\cos(8c)-8i\sin(8c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(63*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(
a + I*a*Tan[c + d*x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) - (63*Cos[8*c]*Co
s[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c +
 d*x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[5*d*x]*Cos[c + d*x]^8*(((
-8*I)/5)*Cos[3*c] - (8*Sin[3*c])/5)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x]
 + I*Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((8*I)*Cos[5*c] + 8*Sin[5*c
])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos
[c + d*x]^8*((-48*I)*Cos[7*c] - 48*Sin[7*c])*(a + I*a*Tan[c + d*x])^8)/(d*
(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*((-8*I)*Cos[8*c] - 8*S
in[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((63*I
)/2)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*
(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((63*I)/2)*Cos[
c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*
Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(48*Cos[7
*c] - (48*I)*Sin[7*c])*Sin[d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I
*Sin[d*x])^8) + (Cos[c + d*x]^8*(-8*Cos[5*c] + (8*I)*Sin[5*c])*Sin[3*d*x]*
(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*
((8*Cos[3*c])/5 - ((8*I)/5)*Sin[3*c])*Sin[5*d*x]*(a + I*a*Tan[c + d*x])^8)
/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/4 - (I/4)*Sin[8
*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + ...
```

3.93.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3977, 3042, 3977, 3042, 3977, 3042, 3979, 3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{9}{5}a^2\int \cos^3(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{9}{5}a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^3}dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{ 3977}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^5}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^5}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{ 3977}$$

$$-\frac{9}{5}a^2\bigg(-\frac{7}{3}a^2\bigg(-5a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}\bigg)-2ia\cos^3(c$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{9}{5}a^2\bigg(-\frac{7}{3}a^2\bigg(-5a^2\int \sec(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}\bigg)-2ia\cos^3(c$$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{ 3979}$$

$$-\frac{9}{5}a^2\Bigg(-\frac{7}{3}a^2\Bigg(-5a^2\Bigg(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\Bigg)-2ia\cos($$
$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \text{ 3042}$$

3.93. $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{3}{2}a\int \sec(c+dx)(i\tan(c+dx)a+a)dx+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos(}{}\right.\right.$$

$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \; 3967$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{3}{2}a\left(a\int \sec(c+dx)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\frac{2ia\cos}{}\right.\right.$$

$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{3}{2}a\left(a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{ia\sec(c+dx)}{d}\right)+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\right)-\right.\right.$$

$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

$$\Big\downarrow \; 4257$$

$$-\frac{9}{5}a^2\left(-\frac{7}{3}a^2\left(-5a^2\left(\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{3}{2}a\left(\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{ia\sec(c+dx)}{d}\right)\right)-\frac{2}{}\right.\right.$$

$$\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^7)/d - (9*a^2*((((-2*I
)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5)/d - (7*a^2*(((-2*I)*a*Cos[
c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - 5*a^2*((3*a*((a*ArcTanh[Sin[c + d*x
]])/d + (I*a*Sec[c + d*x])/d))/2 + ((I/2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c
+ d*x]))/d)))/3))/5
```

3.93. $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.93.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.93.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(152) = 304$.

Time = 2.38 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.86

$$\frac{a^8(\sin^9(dx+c))}{2d\cos(dx+c)^2}+\frac{a^8(\sin^7(dx+c))}{2d}+\frac{203a^8(\sin^5(dx+c))}{10d}+\frac{21a^8(\sin^3(dx+c))}{2d}+\frac{283a^8\sin(dx+c)}{10d}-63a^8$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/2/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2+1/2*a^8*sin(d*x+c)^7/d+203/10*a^8*sin(
d*x+c)^5/d+21/2*a^8*sin(d*x+c)^3/d+283/10*a^8*sin(d*x+c)/d-63/2/d*a^8*ln(s
ec(d*x+c)+tan(d*x+c))-416/15*I/d*a^8*cos(d*x+c)*sin(d*x+c)^2+56/5*I/d*a^8*
cos(d*x+c)^3*sin(d*x+c)^2+112/15*I/d*a^8*cos(d*x+c)^3-8*I/d*a^8*sin(d*x+c)
^8/cos(d*x+c)-8/5*I/d*a^8*cos(d*x+c)^5-104/5*I/d*a^8*cos(d*x+c)*sin(d*x+c)
^4-832/15*I/d*a^8*cos(d*x+c)+29/5/d*a^8*cos(d*x+c)^4*sin(d*x+c)-8/5/d*a^8*
cos(d*x+c)^2*sin(d*x+c)-8*I/d*a^8*cos(d*x+c)*sin(d*x+c)^6
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-16i\,a^8e^{(9i\,dx+9i\,c)}+48i\,a^8e^{(7i\,dx+7i\,c)}-336i\,a^8e^{(5i\,dx+5i\,c)}-1050i\,a^8e^{(3i\,dx+3i\,c)}-630i\,a^8e^{(i\,dx+i\,c)}-315\,(a^8}{10\,(de^{(4i\,dx+4i\,c)}}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/10*(-16*I*a^8*e^(9*I*d*x + 9*I*c) + 48*I*a^8*e^(7*I*d*x + 7*I*c) - 336*I
*a^8*e^(5*I*d*x + 5*I*c) - 1050*I*a^8*e^(3*I*d*x + 3*I*c) - 630*I*a^8*e^(I
*d*x + I*c) - 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x + 2*I*c) + a
^8)*log(e^(I*d*x + I*c) + I) + 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I
*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.36

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{63a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}+\frac{-17ia^8e^{3ic}e^{3idx}-15ia^8e^{ic}e^{idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$
$$+\begin{cases}\frac{-8ia^8d^2e^{5ic}e^{5idx}+40ia^8d^2e^{3ic}e^{3idx}-240ia^8d^2e^{ic}e^{idx}}{5d^3} & \text{for } d^3\neq 0\\ x(8a^8e^{5ic}-24a^8e^{3ic}+48a^8e^{ic}) & \text{otherwise}\end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**8,x)
```

output
```
63*a**8*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2
)/d + (-17*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 15*I*a**8*exp(I*c)*exp(I*d*x))
/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise
(((-8*I*a**8*d**2*exp(5*I*c)*exp(5*I*d*x) + 40*I*a**8*d**2*exp(3*I*c)*exp(
3*I*d*x) - 240*I*a**8*d**2*exp(I*c)*exp(I*d*x))/(5*d**3), Ne(d**3, 0)), (x
*(8*a**8*exp(5*I*c) - 24*a**8*exp(3*I*c) + 48*a**8*exp(I*c)), True))
```

3.93.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(143) = 286$.

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{96i\,a^8\cos(dx+c)^5 - 840\,a^8\sin(dx+c)^5 + 224i\,(3\,\cos(dx+c)^5 - 5\,\cos(dx+c)^3)a^8 + 224i\,(3\,\cos(d}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/60*(96*I*a^8*cos(d*x + c)^5 - 840*a^8*sin(d*x + c)^5 + 224*I*(3*cos(d*x
 + c)^5 - 5*cos(d*x + c)^3)*a^8 + 224*I*(3*cos(d*x + c)^5 - 10*cos(d*x + c
)^3 + 15*cos(d*x + c))*a^8 + 96*I*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/c
os(d*x + c) + 15*cos(d*x + c))*a^8 - (12*sin(d*x + c)^5 + 40*sin(d*x + c)^
3 - 30*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105
*log(sin(d*x + c) - 1) + 180*sin(d*x + c))*a^8 - 56*(6*sin(d*x + c)^5 + 10
*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30
*sin(d*x + c))*a^8 - 112*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^8 - 4*(3*
sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^8)/d
```

3.93.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2849 vs. $2(143) = 286$.

Time = 1.42 (sec) , antiderivative size = 2849, normalized size of antiderivative = 16.47

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output

```
1/655360*(42021645*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) +
588303030*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 382396969
5*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 15295878780*a^8*e
^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 42063666645*a^8*e^(20*I*d
*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 84127333290*a^8*e^(18*I*d*x + 4*I
*c)*log(I*e^(I*d*x + I*c) + 1) + 126190999935*a^8*e^(16*I*d*x + 2*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 126190999935*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I
*d*x + I*c) + 1) + 84127333290*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 42063666645*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 15295878780*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3823969
695*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 588303030*a^8*e^
(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 144218285640*a^8*e^(14*I*d
*x)*log(I*e^(I*d*x + I*c) + 1) + 42021645*a^8*e^(-14*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 21376575*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 299272050*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 194526
8325*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7781073300*a^8
*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 21397951575*a^8*e^(20*I
*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 42795903150*a^8*e^(18*I*d*x + 4
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 64193854725*a^8*e^(16*I*d*x + 2*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 64193854725*a^8*e^(12*I*d*x - 2*I*c)*log(I*e...
```

3.93.9 Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.62

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{63\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

$$+\frac{65\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,309{}\mathrm{i}-761\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,1109{}\mathrm{i}+\frac{7351\,a^8\,\tan(}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,5{}\mathrm{i}-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,20{}\mathrm{i}+26\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\mathrm{ta}\right.}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(a^8*tan(c/2 + (d*x)/2)^3*1223i - (4407*a^8*tan(c/2 + (d*x)/2)^2)/5 + (735
1*a^8*tan(c/2 + (d*x)/2)^4)/5 - a^8*tan(c/2 + (d*x)/2)^5*1109i - 761*a^8*t
an(c/2 + (d*x)/2)^6 + a^8*tan(c/2 + (d*x)/2)^7*309i + 65*a^8*tan(c/2 + (d*
x)/2)^8 + (496*a^8)/5 - a^8*tan(c/2 + (d*x)/2)*431i)/(d*(5*tan(c/2 + (d*x)
/2) - tan(c/2 + (d*x)/2)^2*12i - 20*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)
/2)^4*26i + 26*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*20i - 12*tan(c/
2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*5i + tan(c/2 + (d*x)/2)^9 + 1i)) - (
63*a^8*atanh(tan(c/2 + (d*x)/2)))/d
```

3.93. $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.94 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.94.1 Optimal result 843
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3.94.1 Optimal result

Integrand size = 24, antiderivative size = 152

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{a^8\text{arctanh}(\sin(c+dx))}{d}\\ &+ \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}\\ &- \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}\\ &- \frac{2ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}\\ &+ \frac{2i\cos(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{d}\end{aligned}$$

output
```
a^8*arctanh(sin(d*x+c))/d+2/5*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/
7*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-2/3*I*a^2*cos(d*x+c)^3*(a^2+I*a^
2*tan(d*x+c))^3/d+2*I*cos(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

3.94.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 305 vs. $2(152) = 304$.

Time = 3.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.01

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-70i\cos\left(\frac{1}{2}(c+dx)\right)+42i\cos\left(\frac{3}{2}(c+dx)\right)+210i\cos\left(\frac{5}{2}(c+dx)\right)-30i\cos\left(\frac{7}{2}(c+dx)\right)-105\cos\left(\frac{7}{2}(\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*((-70*I)*Cos[(c + d*x)/2] + (42*I)*Cos[(3*(c + d*x))/2] + (210*I)*Cos
[(5*(c + d*x))/2] - (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c + d*x))/2]
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2]*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(3*(c +
 d*x))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] + (105*I)*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] - (105*I)*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(7*c + 23
*d*x)/2] + I*Sin[(7*c + 23*d*x)/2]))/(105*d*(Cos[d*x] + I*Sin[d*x])^8)
```

3.94.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3977, 3042, 3977, 3042, 3977, 3042, 3977, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^7}dx$$
$$\Big\downarrow \text{3977}$$
$$-a^2\int \cos^5(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^5}dx-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3977}$$

$$-a^2\left(-a^2\int \cos^3(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(-a^2\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^3}dx-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}\right)-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3977}$$

$$-a^2\left(-a^2\left(-a^2\int \cos(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}\right)-2ia\cos^5(c+d\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(-a^2\left(-a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}\right)-\frac{2ia\cos^5(c+dx)(a+ia\,\mathrm{ta}}{5d}\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3977}$$

$$-a^2\left(-a^2\left(-a^2\left(a^2(-\int \sec(c+dx)dx)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\tan(}{3d}\right.\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-a^2\left(-a^2\left(-a^2\left(a^2\left(-\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a-}{3d}\right.\right.$$
$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

$$\Big\downarrow \text{4257}$$

3.94. $\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$-a^2\left(-a^2\left(-a^2\left(-\frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\right)-\frac{2ia\cos^3(c+dx)(a+ia\,\mathrm{ta}}{3d}\right.\right.$$

$$\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - a^2*((((-2*I)/5
)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d - a^2*((((-2*I)/3)*a*Cos[c
+ d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - a^2*(-((a^2*ArcTanh[Sin[c + d*x]])/
d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d)))
```

3.94.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.94.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(138) = 276$.

Time = 1.56 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.53

$$-\frac{29a^8(\sin^7(dx+c))}{7d}-\frac{a^8(\sin^5(dx+c))}{5d}-\frac{a^8(\sin^3(dx+c))}{3d}+\frac{139a^8\sin(dx+c)}{105d}+\frac{a^8\ln(\sec(dx+c)+\tan(}{d}$$

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x)
```

output
```
-29/7*a^8*sin(d*x+c)^7/d-1/5*a^8*sin(d*x+c)^5/d-1/3*a^8*sin(d*x+c)^3/d+139
/105*a^8*sin(d*x+c)/d+1/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+128/35*I/d*a^8*cos
(d*x+c)-8*I/d*a^8*sin(d*x+c)^4*cos(d*x+c)^3-10/d*a^8*sin(d*x+c)^3*cos(d*x+
c)^4-232/35/d*a^8*cos(d*x+c)^4*sin(d*x+c)+122/105/d*a^8*cos(d*x+c)^2*sin(d
*x+c)-64/15*I/d*a^8*cos(d*x+c)^3+48/35*I/d*a^8*cos(d*x+c)*sin(d*x+c)^4+29/
7/d*a^8*cos(d*x+c)^6*sin(d*x+c)-32/5*I/d*a^8*cos(d*x+c)^3*sin(d*x+c)^2+8/7
*I/d*a^8*cos(d*x+c)*sin(d*x+c)^6+64/35*I/d*a^8*cos(d*x+c)*sin(d*x+c)^2-8/7
*I/d*a^8*cos(d*x+c)^7+8*I/d*a^8*cos(d*x+c)^5*sin(d*x+c)^2+16/5*I/d*a^8*cos
(d*x+c)^5
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-30i\,a^8e^{(7i\,dx+7i\,c)}+42i\,a^8e^{(5i\,dx+5i\,c)}-70i\,a^8e^{(3i\,dx+3i\,c)}+210i\,a^8e^{(i\,dx+i\,c)}+105\,a^8\log\left(e^{(i\,dx+i\,c)}+i\right)-10}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/105*(-30*I*a^8*e^(7*I*d*x + 7*I*c) + 42*I*a^8*e^(5*I*d*x + 5*I*c) - 70*I
*a^8*e^(3*I*d*x + 3*I*c) + 210*I*a^8*e^(I*d*x + I*c) + 105*a^8*log(e^(I*d*
x + I*c) + I) - 105*a^8*log(e^(I*d*x + I*c) - I))/d
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.23

$$\begin{aligned}&\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx\\&=\frac{a^8\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}\\&+\begin{cases}\frac{-30ia^8d^3e^{7ic}e^{7idx}+42ia^8d^3e^{5ic}e^{5idx}-70ia^8d^3e^{3ic}e^{3idx}+210ia^8d^3e^{ic}e^{idx}}{105d^4} & \text{for } d^4\neq 0\\ x\left(2a^8e^{7ic}-2a^8e^{5ic}+2a^8e^{3ic}-2a^8e^{ic}\right) & \text{otherwise}\end{cases}\end{aligned}$$

input
```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**8,x)
```

output
```
a**8*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d +
Piecewise(((-30*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) + 42*I*a**8*d**3*exp(5
*I*c)*exp(5*I*d*x) - 70*I*a**8*d**3*exp(3*I*c)*exp(3*I*d*x) + 210*I*a**8*d
**3*exp(I*c)*exp(I*d*x))/(105*d**4), Ne(d**4, 0)), (x*(2*a**8*exp(7*I*c) -
 2*a**8*exp(5*I*c) + 2*a**8*exp(3*I*c) - 2*a**8*exp(I*c)), True))
```

3.94.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(130) = 260.

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.03

$$\begin{aligned}&\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx =\\&-\frac{240i\,a^8\cos(dx+c)^7+840\,a^8\sin(dx+c)^7+112i\,\big(15\,\cos(dx+c)^7-42\,\cos(dx+c)^5+35\,\cos(dx+c}{}\end{aligned}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/210*(240*I*a^8*cos(d*x + c)^7 + 840*a^8*sin(d*x + c)^7 + 112*I*(15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 + 336*I*(5*cos(d*x
 + c)^7 - 7*cos(d*x + c)^5)*a^8 + 48*I*(5*cos(d*x + c)^7 - 21*cos(d*x + c)
^5 + 35*cos(d*x + c)^3 - 35*cos(d*x + c))*a^8 + (30*sin(d*x + c)^7 + 42*si
n(d*x + c)^5 + 70*sin(d*x + c)^3 - 105*log(sin(d*x + c) + 1) + 105*log(sin
(d*x + c) - 1) + 210*sin(d*x + c))*a^8 + 56*(15*sin(d*x + c)^7 - 42*sin(d*
x + c)^5 + 35*sin(d*x + c)^3)*a^8 + 420*(5*sin(d*x + c)^7 - 7*sin(d*x + c)
^5)*a^8 + 6*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*a^8)/d
```

3.94.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. 2(130) = 260.

Time = 1.54 (sec) , antiderivative size = 2863, normalized size of antiderivative = 18.84

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/55050240*(1635552135*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 148
835244285*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 595340977
140*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1637187687135*a^
8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(1
8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(16*I*d*
x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(12*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(10*I*d*x - 4*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 1637187687135*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 595340977140*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 148835244285*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 22897729890*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 56
13214927320*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1635552135*a^8*e
^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1690450650*a^8*e^(28*I*d*x + 14*I*
c)*log(I*e^(I*d*x + I*c) - 1) + 23666309100*a^8*e^(26*I*d*x + 12*I*c)*log(
I*e^(I*d*x + I*c) - 1) + 153831009150*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I
*d*x + I*c) - 1) + 615324036600*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 1692141100650*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 3384282201300*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) +
5076423301950*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 507...
```

3.94.9 Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.36

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{2\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,16\mathrm{i} - \frac{80\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3} - \frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,224\mathrm{i}}{3} + \frac{224\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5} + \frac{a^8\tan}{}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,7\mathrm{i} + 21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,35\mathrm{i} - 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - \mathrm{ta}\right.}$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(2*a^8*atanh(tan(c/2 + (d*x)/2)))/d + ((224*a^8*tan(c/2 + (d*x)/2)^2)/5 -
(a^8*tan(c/2 + (d*x)/2)^3*224i)/3 - (80*a^8*tan(c/2 + (d*x)/2)^4)/3 + a^8*
tan(c/2 + (d*x)/2)^5*16i - (304*a^8)/105 + (a^8*tan(c/2 + (d*x)/2)*304i)/1
5)/(d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*21i - 35*tan(c/2 + (d*x
)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d
*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.95 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.95.1 Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

output
```
-1/63*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-1/9*I*cos(d*x+c)^9*(a+I*a*ta
n(d*x+c))^8/d
```

3.95.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(66) = 132.

Time = 0.81 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\sec(c+dx)(-i\cos(5(c+dx))+\sin(5(c+dx)))\left(9\cos(c+dx)+16\cos(3(c+dx))+7\cos(5(c+dx))\right.}{}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Sec[c + d*x]*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)])*(9*Cos[c + d*
x] + 16*Cos[3*(c + d*x)] + 7*Cos[5*(c + d*x)] + 192*Sqrt[Cos[c + d*x]^2]*C
os[5*(c + d*x)] + (9*I)*Sin[c + d*x] + (16*I)*Sin[3*(c + d*x)] + (7*I)*Sin
[5*(c + d*x)] - (192*I)*Sqrt[Cos[c + d*x]^2]*Sin[5*(c + d*x)]))/(252*d)
```

3.95.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{1}{9}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^7dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}a\int \frac{(i\tan(c+dx)a+a)^7}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

$$\Big\downarrow \text{3969}$$

$$-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}-\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/63*I)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - ((I/9)*Cos[c + d
*x]^9*(a + I*a*Tan[c + d*x])^8)/d
```

3.95.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.95.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(58) = 116$.

Time = 1.59 (sec) , antiderivative size = 447, normalized size of antiderivative = 6.77

$$\frac{a^8(\sin^9(dx+c))}{9} - \frac{8ia^8(\cos^9(dx+c))}{9} - 28a^8\Bigg(-\frac{(\cos^4(dx+c))(\sin^5(dx+c))}{9} - \frac{5(\sin^3(dx+c))(\cos^4(dx+c))}{63} - \frac{\sin(dx+c)(\cos^4(dx+c))}{21} +$$

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x)
```

output

```
1/d*(1/9*a^8*sin(d*x+c)^9-8/9*I*a^8*cos(d*x+c)^9-28*a^8*(-1/9*cos(d*x+c)^4
*sin(d*x+c)^5-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*sin(d*x+c)*cos(d*x+c)^4+
1/63*(2+cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/9*cos(d*x+c)^3*sin(d*x+c)^6-
2/21*cos(d*x+c)^3*sin(d*x+c)^4-8/105*cos(d*x+c)^3*sin(d*x+c)^2-16/315*cos(
d*x+c)^3)+70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c
)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/9*c
os(d*x+c)^7*sin(d*x+c)^2-2/63*cos(d*x+c)^7)-28*a^8*(-1/9*cos(d*x+c)^8*sin(
d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c))+56*I*a^8*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*cos(d*x+c)^5*sin(d*x+c)^
2-8/315*cos(d*x+c)^5)+1/9*a^8*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*
cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-7i\,a^8e^{(9i\,dx+9i\,c)} - 9i\,a^8e^{(7i\,dx+7i\,c)}}{126\,d}$$

input

```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output

```
1/126*(-7*I*a^8*e^(9*I*d*x + 9*I*c) - 9*I*a^8*e^(7*I*d*x + 7*I*c))/d
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-14ia^8de^{9ic}e^{9idx}-18ia^8de^{7ic}e^{7idx}}{252d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^8e^{9ic}}{2}+\frac{a^8e^{7ic}}{2}\right) & \text{otherwise} \end{cases}$$

input

```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**8,x)
```

output

```
Piecewise(((-14*I*a**8*d*exp(9*I*c)*exp(9*I*d*x) - 18*I*a**8*d*exp(7*I*c)*
exp(7*I*d*x))/(252*d**2), Ne(d**2, 0)), (x*(a**8*exp(9*I*c)/2 + a**8*exp(7
*I*c)/2), True))
```

3.95.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(54) = 108$.

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.58

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{280i\,a^8\cos(dx+c)^9 - 35\,a^8\sin(dx+c)^9 + 56i\,\left(35\,\cos(dx+c)^9 - 90\,\cos(dx+c)^7 + 63\,\cos(dx+c)^5\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/315*(280*I*a^8*cos(d*x + c)^9 - 35*a^8*sin(d*x + c)^9 + 56*I*(35*cos(d*
x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 + 8*I*(35*cos(d*x +
c)^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 +
 280*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^8 - 70*(35*sin(d*x + c)^9 -
 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^8 - 28*(35*sin(d*x + c)^9 - 135*
sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^8 - (35*sin(d*
x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 +
315*sin(d*x + c))*a^8 - 140*(7*sin(d*x + c)^9 - 9*sin(d*x + c)^7)*a^8)/d
```

3.95.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2451 vs. $2(54) = 108$.

Time = 1.62 (sec) , antiderivative size = 2451, normalized size of antiderivative = 37.14

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/66060288*(1419343317*a^8*e^(24*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 17032119804*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 936
76658922*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 31225552974
0*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e
^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(14*I
*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(10*I*d*x -
 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e^(8*I*d*x - 4*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 312255529740*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 93676658922*a^8*e^(4*I*d*x - 8*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 17032119804*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 1311473224908*a^8*e^(12*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1419343317
*a^8*e^(-12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1419097050*a^8*e^(24*I*d*x +
 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17029164600*a^8*e^(22*I*d*x + 10*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 93660405300*a^8*e^(20*I*d*x + 8*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 312201351000*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 702453039750*a^8*e^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c
) - 1) + 1123924863600*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 1123924863600*a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 702
453039750*a^8*e^(8*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 31220135100
0*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 93660405300*a^8*...
```

3.95.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2\,a^8\,\left(\frac{\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,9\mathrm{i}}{4}+\frac{\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,7\mathrm{i}}{4}\right)}{63\,d}$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(2*a^8*((exp(c*7i + d*x*7i)*9i)/4 + (exp(c*9i + d*x*9i)*7i)/4))/(63*d)
```

3.96 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.96.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\begin{aligned}\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx =& -\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d}\\ &-\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d}\\ &-\frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d}\\ &-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}\end{aligned}$$

output
```
-2/1155*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/231*I*a^2*cos(d*x+c)^7
*(a+I*a*tan(d*x+c))^6/d-1/33*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^7/d-1/11*
I*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8/d
```

3.96.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{a^8\sec(c+dx)(-i\cos(6(c+dx))+\sin(6(c+dx)))\,\big(726+1111\cos(2(c+dx))+490\cos(4(c+dx))+10}{}$$

input
```
Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Sec[c + d*x]*((-I)*Cos[6*(c + d*x)] + Sin[6*(c + d*x)])*(726 + 1111*C
os[2*(c + d*x)] + 490*Cos[4*(c + d*x)] + 105*Cos[6*(c + d*x)] + 11008*Sqrt
[Cos[c + d*x]^2]*Cos[6*(c + d*x)] + (649*I)*Sin[2*(c + d*x)] + (490*I)*Sin
[4*(c + d*x)] + (105*I)*Sin[6*(c + d*x)] - (11008*I)*Sqrt[Cos[c + d*x]^2]*
Sin[6*(c + d*x)]))/(18480*d)
```

3.96.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{11}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{3}{11}a\int \cos^9(c+dx)(i\tan(c+dx)a+a)^7dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a\int \frac{(i\tan(c+dx)a+a)^7}{\sec(c+dx)^9}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{3}{11}a\left(\frac{2}{9}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^7}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{11}a\left(\frac{2}{9}a\int\frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^7}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow\ 3978$$

$$\frac{3}{11}a\left(\frac{2}{9}a\left(\frac{1}{7}a\int\cos^5(c+dx)(i\tan(c+dx)a+a)^5dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^6}{7d}\right)-\frac{i\cos^9(c+dx)(a+}{9}\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow\ 3042$$

$$\frac{3}{11}a\left(\frac{2}{9}a\left(\frac{1}{7}a\int\frac{(i\tan(c+dx)a+a)^5}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^6}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan(c+}{9d}\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

$$\Big\downarrow\ 3969$$

$$\frac{3}{11}a\left(\frac{2}{9}a\left(-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^6}{7d}-\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^5}{35d}\right)-\frac{i\cos^9(c+dx)(a+ia\,t}{9d}\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

input

```
Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-1/11*I)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8)/d + (3*a*(((-1/9*I)*C
os[c + d*x]^9*(a + I*a*Tan[c + d*x])^7)/d + (2*a*(((-1/35*I)*a*Cos[c + d*x
]^5*(a + I*a*Tan[c + d*x])^5)/d - ((I/7)*Cos[c + d*x]^7*(a + I*a*Tan[c + d
*x])^6)/d))/9))/11
```

3.96. $\int\cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.96.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.96.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(120) = 240$.

Time = 1.88 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.17

$$a^8\left(-\frac{(\sin^7(dx+c))(\cos^4(dx+c))}{11}-\frac{7(\cos^4(dx+c))(\sin^5(dx+c))}{99}-\frac{5(\sin^3(dx+c))(\cos^4(dx+c))}{99}-\frac{\sin(dx+c)(\cos^4(dx+c))}{33}+\frac{(2+\cos^2(dx+}{9}\right.$$

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*cos(d*x+c)^4*sin(d*x+c)^5-5
/99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x
+c)^2)*sin(d*x+c))-56*I*a^8*(-1/11*cos(d*x+c)^9*sin(d*x+c)^2-2/99*cos(d*x+
c)^9)-28*a^8*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)
^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c))-8/11*I*a^8*cos(d*x+c)^11+70*a^8*(-1/11*sin(d*x+c)^3*cos(d*x+c
)^8-1/33*cos(d*x+c)^8*sin(d*x+c)+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4
+8/5*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/11*cos(d*x+c)^5*sin(d*x+c)^6-2/
33*sin(d*x+c)^4*cos(d*x+c)^5-8/231*cos(d*x+c)^5*sin(d*x+c)^2-16/1155*cos(d
*x+c)^5)-28*a^8*(-1/11*sin(d*x+c)*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+
8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))+56*I*a
^8*(-1/11*cos(d*x+c)^7*sin(d*x+c)^4-4/99*cos(d*x+c)^7*sin(d*x+c)^2-8/693*c
os(d*x+c)^7)+1/11*a^8*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*
x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-105i\,a^8e^{(11i\,dx+11i\,c)} - 385i\,a^8e^{(9i\,dx+9i\,c)} - 495i\,a^8e^{(7i\,dx+7i\,c)} - 231i\,a^8e^{(5i\,dx+5i\,c)}}{9240\,d}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/9240*(-105*I*a^8*e^(11*I*d*x + 11*I*c) - 385*I*a^8*e^(9*I*d*x + 9*I*c) -
 495*I*a^8*e^(7*I*d*x + 7*I*c) - 231*I*a^8*e^(5*I*d*x + 5*I*c))/d
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-53760ia^8d^3e^{11ic}e^{11idx}-197120ia^8d^3e^{9ic}e^{9idx}-253440ia^8d^3e^{7ic}e^{7idx}-118272ia^8d^3e^{5ic}e^{5idx}}{4730880d^4} & \text{for } d^4 \neq 0 \\ x\left(\frac{a^8e^{11ic}}{8}+\frac{3a^8e^{9ic}}{8}+\frac{3a^8e^{7ic}}{8}+\frac{a^8e^{5ic}}{8}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-53760*I*a**8*d**3*exp(11*I*c)*exp(11*I*d*x) - 197120*I*a**8*d
**3*exp(9*I*c)*exp(9*I*d*x) - 253440*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) -
 118272*I*a**8*d**3*exp(5*I*c)*exp(5*I*d*x))/(4730880*d**4), Ne(d**4, 0)),
 (x*(a**8*exp(11*I*c)/8 + 3*a**8*exp(9*I*c)/8 + 3*a**8*exp(7*I*c)/8 + a**8
*exp(5*I*c)/8), True))
```

3.96.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(112) = 224.

Time = 0.43 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.61

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{2520i\,a^8\cos{(dx+c)}^{11} + 24i\,\left(105\,\cos{(dx+c)}^{11} - 385\,\cos{(dx+c)}^{9} + 495\,\cos{(dx+c)}^{7} - 231\,\cos{(dx}{}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/3465*(2520*I*a^8*cos(d*x + c)^11 + 24*I*(105*cos(d*x + c)^11 - 385*cos(
d*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 280*I*(63*cos(
d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*a^8 + 1960*I*(9*cos(
d*x + c)^11 - 11*cos(d*x + c)^9)*a^8 + 28*(315*sin(d*x + c)^11 - 1540*sin(
d*x + c)^9 + 2970*sin(d*x + c)^7 - 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)
^3)*a^8 + 210*(105*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 495*sin(d*x + c)
^7 - 231*sin(d*x + c)^5)*a^8 + 140*(63*sin(d*x + c)^11 - 154*sin(d*x + c)^
9 + 99*sin(d*x + c)^7)*a^8 + 5*(63*sin(d*x + c)^11 - 385*sin(d*x + c)^9 +
990*sin(d*x + c)^7 - 1386*sin(d*x + c)^5 + 1155*sin(d*x + c)^3 - 693*sin(d
*x + c))*a^8 + 35*(9*sin(d*x + c)^11 - 11*sin(d*x + c)^9)*a^8)/d
```

3.96.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. $2(112) = 224$.

Time = 1.73 (sec) , antiderivative size = 2863, normalized size of antiderivative = 21.05

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/4844421120*(82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 298
58174165820*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82109978
956005*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1642199579120
10*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a
^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e
^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^8*e^(10
*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82109978956005*a^8*e^(8*I*d*x
 - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 29858174165820*a^8*e^(6*I*d*x - 8*I
*c)*log(I*e^(I*d*x + I*c) + 1) + 7464543541455*a^8*e^(4*I*d*x - 10*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 281519927849160*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c
) + 1) + 82027951005*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82004266
575*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a
^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325*a^8*e^
(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29849553033300*a^8*e^(22*
I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 82086270841575*a^8*e^(20*I*d*x
 + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 164172541683150*a^8*e^(18*I*d*x + 4
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 246258812524725*a^8*e^(16*I*d*x + 2*...
```

3.96.9 Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.48

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= -\frac{a^8\left(\frac{\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,1\mathrm{i}}{40}+\frac{\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,3\mathrm{i}}{56}+\frac{\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,1\mathrm{i}}{24}+\frac{\mathrm{e}^{c\,11\mathrm{i}+d\,x\,11\mathrm{i}}\,1\mathrm{i}}{88}\right)}{d}$$

input
```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(a^8*((exp(c*5i + d*x*5i)*1i)/40 + (exp(c*7i + d*x*7i)*3i)/56 + (exp(c*9i
 + d*x*9i)*1i)/24 + (exp(c*11i + d*x*11i)*1i)/88))/d
```

3.96. $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.97 $\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.97.1 Optimal result

Integrand size = 24, antiderivative size = 211

$$\begin{aligned}\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = & -\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d} \\ & -\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d} \\ & -\frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} \\ & -\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} \\ & -\frac{8ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{9009d} \\ & -\frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{3003d}\end{aligned}$$

output
```
-20/3003*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d-20/1287*I*a^2*cos(d*x+c
)^9*(a+I*a*tan(d*x+c))^6/d-5/143*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^7/d-
1/13*I*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8/d-8/9009*I*a^2*cos(d*x+c)^3*(a^2
+I*a^2*tan(d*x+c))^3/d-8/3003*I*cos(d*x+c)^5*(a^2+I*a^2*tan(d*x+c))^4/d
```

3.97.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{a^8\sec(c+dx)(-i\cos(7(c+dx))+\sin(7(c+dx)))\left(44759\cos(c+dx)+26117\cos(3(c+dx))+7791\cos\right.}{}$$

input

```
Integrate[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

output

```
(a^8*Sec[c + d*x]*((-I)*Cos[7*(c + d*x)] + Sin[7*(c + d*x)])*(44759*Cos[c
+ d*x] + 26117*Cos[3*(c + d*x)] + 7791*Cos[5*(c + d*x)] + 693*Cos[7*(c + d
*x)] + 275456*Sqrt[Cos[c + d*x]^2]*Cos[7*(c + d*x)] + (1001*I)*Sin[c + d*x
] + (2093*I)*Sin[3*(c + d*x)] + (1785*I)*Sin[5*(c + d*x)] + (693*I)*Sin[7*
(c + d*x)] - (275456*I)*Sqrt[Cos[c + d*x]^2]*Sin[7*(c + d*x)]))/(576576*d)
```

3.97.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{13}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{5}{13}a\int \cos^{11}(c+dx)(i\tan(c+dx)a+a)^7dx-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{13}a\int \frac{(i\tan(c+dx)a+a)^7}{\sec(c+dx)^{11}}dx-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\int \cos^9(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{11d}\right)-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^9}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{11d}\right)-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^5dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^6}{9d}\right)-\frac{i\cos^{11}(c+dx)(a}{1}\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\int \frac{(i\tan(c+dx)a+a)^5}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^6}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c}{11d}\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}\right)-\frac{i\cos^9(c+d}{}\right.\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan}{9d}\right.\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\left(\frac{1}{5}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^3dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(c}{}\right.\right.\right.$$
$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

3.97. $\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\left(\frac{1}{5}a\int\frac{(i\tan(c+dx)a+a)^3}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\right)-\frac{i\cos^7(c+dx)(a+i}{7d}\right.\right.\right.\right.$$

$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

$$\Big\downarrow \text{3969}$$

$$\frac{5}{13}a\left(\frac{4}{11}a\left(\frac{1}{3}a\left(\frac{2}{7}a\left(-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}\right)-\frac{i\cos^7(c+d}{}\right.\right.\right.\right.$$

$$\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$

input
```
Int[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/13*I)*Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8)/d + (5*a*(((-1/11*I)*
Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^7)/d + (4*a*(((-1/9*I)*Cos[c + d*x]
^9*(a + I*a*Tan[c + d*x])^6)/d + (a*(((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan
[c + d*x])^5)/d + (2*a*(((-1/15*I)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])
^3)/d - ((I/5)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4/d))/7))/3))/11))/1
3
```

3.97.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.97.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(187) = 374$.

Time = 1.81 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.92

Expression too large to display

input
```
int(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-
35/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3
+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-8/13*I*a^8*cos(d*x+c)^13-28*a^
8*(-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/429*c
os(d*x+c)^8*sin(d*x+c)+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(
d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/13*cos(d*x+c)^7*sin(d*x+c)^6-6/143*cos(d
*x+c)^7*sin(d*x+c)^4-8/429*cos(d*x+c)^7*sin(d*x+c)^2-16/3003*cos(d*x+c)^7)
+70*a^8*(-1/13*sin(d*x+c)^3*cos(d*x+c)^10-3/143*sin(d*x+c)*cos(d*x+c)^10+1
/429*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*
x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/13*cos(d*x+c)^9*sin(d*x+c)^4-4/143*cos(d*
x+c)^9*sin(d*x+c)^2-8/1287*cos(d*x+c)^9)-28*a^8*(-1/13*sin(d*x+c)*cos(d*x+
c)^12+1/143*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/
21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/13*cos(d*x+c
)^11*sin(d*x+c)^2-2/143*cos(d*x+c)^11)+1/13*a^8*(1024/231+cos(d*x+c)^12+12
/11*cos(d*x+c)^10+40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c
)^4+512/231*cos(d*x+c)^2)*sin(d*x+c))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-693i\,a^8e^{(13i\,dx+13i\,c)} - 4095i\,a^8e^{(11i\,dx+11i\,c)} - 10010i\,a^8e^{(9i\,dx+9i\,c)} - 12870i\,a^8e^{(7i\,dx+7i\,c)} - 9009i\,a^8e^{(5i\,dx+}}{288288\,d}$$

input
```
integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/288288*(-693*I*a^8*e^(13*I*d*x + 13*I*c) - 4095*I*a^8*e^(11*I*d*x + 11*I
*c) - 10010*I*a^8*e^(9*I*d*x + 9*I*c) - 12870*I*a^8*e^(7*I*d*x + 7*I*c) -
9009*I*a^8*e^(5*I*d*x + 5*I*c) - 3003*I*a^8*e^(3*I*d*x + 3*I*c))/d
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.14

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\begin{cases}\frac{-17439916032ia^8d^5e^{13ic}e^{13idx}-103054049280ia^8d^5e^{11ic}e^{11idx}-251909898240ia^8d^5e^{9ic}e^{9idx}-323884154880ia^8d^5e^{7ic}e^{7idx}-226718908416}{7255005069312d^6} \\ x\left(\frac{a^8e^{13ic}}{32}+\frac{5a^8e^{11ic}}{32}+\frac{5a^8e^{9ic}}{16}+\frac{5a^8e^{7ic}}{16}+\frac{5a^8e^{5ic}}{32}+\frac{a^8e^{3ic}}{32}\right)\end{cases}$$

input
```
integrate(cos(d*x+c)**13*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-17439916032*I*a**8*d**5*exp(13*I*c)*exp(13*I*d*x) - 103054049
280*I*a**8*d**5*exp(11*I*c)*exp(11*I*d*x) - 251909898240*I*a**8*d**5*exp(9
*I*c)*exp(9*I*d*x) - 323884154880*I*a**8*d**5*exp(7*I*c)*exp(7*I*d*x) - 22
6718908416*I*a**8*d**5*exp(5*I*c)*exp(5*I*d*x) - 75572969472*I*a**8*d**5*e
xp(3*I*c)*exp(3*I*d*x))/(7255005069312*d**6), Ne(d**6, 0)), (x*(a**8*exp(1
3*I*c)/32 + 5*a**8*exp(11*I*c)/32 + 5*a**8*exp(9*I*c)/16 + 5*a**8*exp(7*I*
c)/16 + 5*a**8*exp(5*I*c)/32 + a**8*exp(3*I*c)/32), True))
```

3.97.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(175) = 350.

Time = 0.32 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.92

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{5544i\,a^8\cos(dx+c)^{13}+24i\,\left(231\,\cos(dx+c)^{13}-819\,\cos(dx+c)^{11}+1001\,\cos(dx+c)^9-429\,\cos(}{}$$

input
```
integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output

```
-1/9009*(5544*I*a^8*cos(d*x + c)^13 + 24*I*(231*cos(d*x + c)^13 - 819*cos(
d*x + c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*a^8 + 392*I*(99*co
s(d*x + c)^13 - 234*cos(d*x + c)^11 + 143*cos(d*x + c)^9)*a^8 + 3528*I*(11
*cos(d*x + c)^13 - 13*cos(d*x + c)^11)*a^8 - 42*(1155*sin(d*x + c)^13 - 54
60*sin(d*x + c)^11 + 10010*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 3003*sin
(d*x + c)^5)*a^8 - 28*(693*sin(d*x + c)^13 - 4095*sin(d*x + c)^11 + 10010*
sin(d*x + c)^9 - 12870*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 3003*sin(d*x
 + c)^3)*a^8 - 84*(231*sin(d*x + c)^13 - 819*sin(d*x + c)^11 + 1001*sin(d*
x + c)^9 - 429*sin(d*x + c)^7)*a^8 - 3*(231*sin(d*x + c)^13 - 1638*sin(d*x
 + c)^11 + 5005*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 9009*sin(d*x + c)^5
 - 6006*sin(d*x + c)^3 + 3003*sin(d*x + c))*a^8 - 7*(99*sin(d*x + c)^13 -
234*sin(d*x + c)^11 + 143*sin(d*x + c)^9)*a^8)/d
```

3.97.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2891 vs. $2(175) = 350$.

Time = 1.84 (sec) , antiderivative size = 2891, normalized size of antiderivative = 13.70

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output

```
1/151145938944*(1945052766657*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 27230738733198*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 176999801765787*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 707999207063148*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
1946997819423657*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 389
3995638847314*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 584099
3458270971*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 584099345
8270971*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 389399563884
7314*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 194699781942365
7*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 707999207063148*a^8
*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 176999801765787*a^8*e^(4
*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 27230738733198*a^8*e^(2*I*d*
x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6675421095166824*a^8*e^(14*I*d*x)
*log(I*e^(I*d*x + I*c) + 1) + 1945052766657*a^8*e^(-14*I*c)*log(I*e^(I*d*x
 + I*c) + 1) + 1944080407269*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 27217125701766*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 176911317061479*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 707645268245916*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1
946024487676269*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3892
048975352538*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5838...
```

3.97.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.44

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\left(\frac{e^{c\,3i+d\,x\,3i}\,1i}{96}+\frac{e^{c\,5i+d\,x\,5i}\,1i}{32}+\frac{e^{c\,7i+d\,x\,7i}\,5i}{112}+\frac{e^{c\,9i+d\,x\,9i}\,5i}{144}+\frac{e^{c\,11i+d\,x\,11i}\,5i}{352}+\frac{e^{c\,13i+d\,x\,13i}\,1i}{416}\right)}{d}$$

input

```
int(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8,x)
```

output

```
-(a^8*((exp(c*3i + d*x*3i)*1i)/96 + (exp(c*5i + d*x*5i)*1i)/32 + (exp(c*7i
 + d*x*7i)*5i)/112 + (exp(c*9i + d*x*9i)*5i)/144 + (exp(c*11i + d*x*11i)*5
i)/352 + (exp(c*13i + d*x*13i)*1i)/416))/d
```

3.98 $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$

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3.98.1 Optimal result

Integrand size = 24, antiderivative size = 212

$$\begin{aligned}\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{7a^8\sin(c+dx)}{1287d}-\frac{7a^8\sin^3(c+dx)}{1287d}\\&\quad+\frac{7a^8\sin^5(c+dx)}{2145d}-\frac{a^8\sin^7(c+dx)}{1287d}\\&\quad-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d}\\&\quad-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}\\&\quad-\frac{2ia^2\cos^{11}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{715d}\\&\quad-\frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d}\end{aligned}$$

output
```
7/1287*a^8*sin(d*x+c)/d-7/1287*a^8*sin(d*x+c)^3/d+7/2145*a^8*sin(d*x+c)^5/
d-1/1287*a^8*sin(d*x+c)^7/d-2/195*I*a^3*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^5
/d-2/15*I*a*cos(d*x+c)^15*(a+I*a*tan(d*x+c))^7/d-2/715*I*a^2*cos(d*x+c)^11
*(a^2+I*a^2*tan(d*x+c))^3/d-2/1287*I*cos(d*x+c)^9*(a^8+I*a^8*tan(d*x+c))/d
```

3.98.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\sec(c+dx)(-i\cos(8(c+dx))+\sin(8(c+dx)))\left(28600+48256\cos(2(c+dx))+28896\cos(4(c+dx)\right.}{}$$

input
```
Integrate[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(a^8*Sec[c + d*x]*((-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)])*(28600 + 4825
6*Cos[2*(c + d*x)] + 28896*Cos[4*(c + d*x)] + 12672*Cos[6*(c + d*x)] + 343
2*Cos[8*(c + d*x)] + 317440*Sqrt[Cos[c + d*x]^2]*Cos[8*(c + d*x)] - (10946
*I)*Sin[2*(c + d*x)] - (13146*I)*Sin[4*(c + d*x)] - (8778*I)*Sin[6*(c + d*
x)] - (3003*I)*Sin[8*(c + d*x)] - (317440*I)*Sqrt[Cos[c + d*x]^2]*Sin[8*(c
 + d*x)]))/(823680*d)
```

3.98.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {3042, 3977, 3042, 3977, 3042, 3977, 3042, 3977, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^8}{\sec(c+dx)^{15}}dx$$

$$\Big\downarrow \ 3977$$

$$\frac{1}{15}a^2\int \cos^{13}(c+dx)(i\tan(c+dx)a+a)^6dx-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{15}a^2\int \frac{(i\tan(c+dx)a+a)^6}{\sec(c+dx)^{13}}dx-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\int \cos^{11}(c+dx)(i\tan(c+dx)a+a)^4dx-\frac{2ia\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{13d}\right)-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\int \frac{(i\tan(c+dx)a+a)^4}{\sec(c+dx)^{11}}dx-\frac{2ia\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{13d}\right)-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\int \cos^9(c+dx)(i\tan(c+dx)a+a)^2dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{11d}\right)-2ia\cos^{13}(\right.$$

$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sec(c+dx)^9}dx-\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{11d}\right)-\frac{2ia\cos^{13}(c+dx)(a+}{13d}\right.$$

$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3977}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(\frac{7}{9}a^2\int \cos^7(c+dx)dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(a+ia}{11d}\right.\right.$$

$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(\frac{7}{9}a^2\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}\right)-\frac{2ia\cos^{11}(c+dx)(}{1}\right.\right.$$

$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{3113}$$

3.98. $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(-\frac{7a^2\int\left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9d}-\frac{2i\cos^9(c+}{}\right.\right.\right.$$

$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{15}a^2\left(\frac{3}{13}a^2\left(\frac{5}{11}a^2\left(-\frac{7a^2\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+\right.}{9d}\right.\right.\right.$$

$$\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

input
```
Int[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((-2*I)/15)*a*Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^7)/d + (a^2*((((-2*I
)/13)*a*Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^5)/d + (3*a^2*((((-2*I)/11)
*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^3)/d + (5*a^2*((-7*a^2*(-Sin[c +
 d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/(9*d) -
 (((2*I)/9)*Cos[c + d*x]^9*(a^2 + I*a^2*Tan[c + d*x]))/d))/11))/13))/15
```

3.98.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.98.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(188) = 376$.

Time = 1.80 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.15

Expression too large to display

input
```
int(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x)
```

output
```
1/d*(a^8*(-1/15*sin(d*x+c)^7*cos(d*x+c)^8-7/195*sin(d*x+c)^5*cos(d*x+c)^8-
7/429*sin(d*x+c)^3*cos(d*x+c)^8-7/1287*cos(d*x+c)^8*sin(d*x+c)+1/1287*(16/
5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-8/15*I*a^8*c
os(d*x+c)^15-28*a^8*(-1/15*sin(d*x+c)^5*cos(d*x+c)^10-1/39*sin(d*x+c)^3*co
s(d*x+c)^10-1/143*sin(d*x+c)*cos(d*x+c)^10+1/1287*(128/35+cos(d*x+c)^8+8/7
*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(
-1/15*cos(d*x+c)^9*sin(d*x+c)^6-2/65*cos(d*x+c)^9*sin(d*x+c)^4-8/715*cos(d
*x+c)^9*sin(d*x+c)^2-16/6435*cos(d*x+c)^9)+70*a^8*(-1/15*sin(d*x+c)^3*cos(
d*x+c)^12-1/65*sin(d*x+c)*cos(d*x+c)^12+1/715*(256/63+cos(d*x+c)^10+10/9*c
os(d*x+c)^8+80/63*cos(d*x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin
(d*x+c))+56*I*a^8*(-1/15*cos(d*x+c)^11*sin(d*x+c)^4-4/195*cos(d*x+c)^11*si
n(d*x+c)^2-8/2145*cos(d*x+c)^11)-28*a^8*(-1/15*sin(d*x+c)*cos(d*x+c)^14+1/
195*(1024/231+cos(d*x+c)^12+12/11*cos(d*x+c)^10+40/33*cos(d*x+c)^8+320/231
*cos(d*x+c)^6+128/77*cos(d*x+c)^4+512/231*cos(d*x+c)^2)*sin(d*x+c))-56*I*a
^8*(-1/15*cos(d*x+c)^13*sin(d*x+c)^2-2/195*cos(d*x+c)^13)+1/15*a^8*(2048/4
29+cos(d*x+c)^14+14/13*cos(d*x+c)^12+168/143*cos(d*x+c)^10+560/429*cos(d*x
+c)^8+640/429*cos(d*x+c)^6+256/143*cos(d*x+c)^4+1024/429*cos(d*x+c)^2)*sin
(d*x+c))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.56

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-429i\,a^8e^{(15i\,dx+15i\,c)} - 3465i\,a^8e^{(13i\,dx+13i\,c)} - 12285i\,a^8e^{(11i\,dx+11i\,c)} - 25025i\,a^8e^{(9i\,dx+9i\,c)} - 32175i\,a^8e^{(7i}}{823680\,d}$$

input
```
integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/823680*(-429*I*a^8*e^(15*I*d*x + 15*I*c) - 3465*I*a^8*e^(13*I*d*x + 13*I
*c) - 12285*I*a^8*e^(11*I*d*x + 11*I*c) - 25025*I*a^8*e^(9*I*d*x + 9*I*c)
- 32175*I*a^8*e^(7*I*d*x + 7*I*c) - 27027*I*a^8*e^(5*I*d*x + 5*I*c) - 1501
5*I*a^8*e^(3*I*d*x + 3*I*c) - 6435*I*a^8*e^(I*d*x + I*c))/d
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.48

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \begin{cases} \frac{-10867748850798428160ia^8d^7e^{15ic}e^{15idx} - 87777971487218073600ia^8d^7e^{13ic}e^{13idx} - 311212808000136806400ia^8d^7e^{11ic}e^{11idx} - 63395201629}{} \\ x\left(\frac{a^8e^{15ic}}{128} + \frac{7a^8e^{13ic}}{128} + \frac{21a^8e^{11ic}}{128} + \frac{35a^8e^{9ic}}{128} + \frac{35a^8e^{7ic}}{128} + \frac{21a^8e^{5ic}}{128} + \frac{7a^8e^{3ic}}{128} + \frac{a^8e^{ic}}{128}\right) \end{cases}$$

input
```
integrate(cos(d*x+c)**15*(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-10867748850798428160*I*a**8*d**7*exp(15*I*c)*exp(15*I*d*x) -
87777971487218073600*I*a**8*d**7*exp(13*I*c)*exp(13*I*d*x) - 3112128080001
36806400*I*a**8*d**7*exp(11*I*c)*exp(11*I*d*x) - 633952016296574976000*I*a
**8*d**7*exp(9*I*c)*exp(9*I*d*x) - 815081163809882112000*I*a**8*d**7*exp(7
*I*c)*exp(7*I*d*x) - 684668177600300974080*I*a**8*d**7*exp(5*I*c)*exp(5*I*
d*x) - 380371209777944985600*I*a**8*d**7*exp(3*I*c)*exp(3*I*d*x) - 1630162
32761976422400*I*a**8*d**7*exp(I*c)*exp(I*d*x))/(20866077793532982067200*d
**8), Ne(d**8, 0)), (x*(a**8*exp(15*I*c)/128 + 7*a**8*exp(13*I*c)/128 + 21
*a**8*exp(11*I*c)/128 + 35*a**8*exp(9*I*c)/128 + 35*a**8*exp(7*I*c)/128 +
21*a**8*exp(5*I*c)/128 + 7*a**8*exp(3*I*c)/128 + a**8*exp(I*c)/128), True)
)
```

3.98.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(180) = 360$.

Time = 0.50 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.14

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{3432i\,a^8\cos\left(dx+c\right)^{15} + 8i\left(429\cos\left(dx+c\right)^{15} - 1485\cos\left(dx+c\right)^{13} + 1755\cos\left(dx+c\right)^{11} - 715\cos\left(\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/6435*(3432*I*a^8*cos(d*x + c)^15 + 8*I*(429*cos(d*x + c)^15 - 1485*cos(
d*x + c)^13 + 1755*cos(d*x + c)^11 - 715*cos(d*x + c)^9)*a^8 + 168*I*(143*
cos(d*x + c)^15 - 330*cos(d*x + c)^13 + 195*cos(d*x + c)^11)*a^8 + 1848*I*
(13*cos(d*x + c)^15 - 15*cos(d*x + c)^13)*a^8 + 4*(3003*sin(d*x + c)^15 -
13860*sin(d*x + c)^13 + 24570*sin(d*x + c)^11 - 20020*sin(d*x + c)^9 + 643
5*sin(d*x + c)^7)*a^8 + 10*(3003*sin(d*x + c)^15 - 17325*sin(d*x + c)^13 +
 40950*sin(d*x + c)^11 - 50050*sin(d*x + c)^9 + 32175*sin(d*x + c)^7 - 900
9*sin(d*x + c)^5)*a^8 + 4*(3003*sin(d*x + c)^15 - 20790*sin(d*x + c)^13 +
61425*sin(d*x + c)^11 - 100100*sin(d*x + c)^9 + 96525*sin(d*x + c)^7 - 540
54*sin(d*x + c)^5 + 15015*sin(d*x + c)^3)*a^8 + (429*sin(d*x + c)^15 - 148
5*sin(d*x + c)^13 + 1755*sin(d*x + c)^11 - 715*sin(d*x + c)^9)*a^8 + (429*
sin(d*x + c)^15 - 3465*sin(d*x + c)^13 + 12285*sin(d*x + c)^11 - 25025*sin
(d*x + c)^9 + 32175*sin(d*x + c)^7 - 27027*sin(d*x + c)^5 + 15015*sin(d*x
+ c)^3 - 6435*sin(d*x + c))*a^8)/d
```

3.98.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(180) = 360$.

Time = 1.89 (sec) , antiderivative size = 2919, normalized size of antiderivative = 13.77

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/863691079680*(5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 79549418828880*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 517071222387720*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 2068284889550880*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 5687783446264920*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11
375566892529840*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1706
3350338794760*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 170633
50338794760*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11375566
892529840*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5687783446
264920*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 20682848895508
80*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 517071222387720*a^
8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 79549418828880*a^8*e^(
2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 19500971815765440*a^8*e^(14
*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 5682101344920*a^8*e^(-14*I*c)*log(I*e
^(I*d*x + I*c) + 1) + 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x +
 I*c) - 1) + 516344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 2065378254079140*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c)
- 1) + 5679790198717635*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1
) + 11359580397435270*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - ...
```

3.98.9 Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{2\,a^8\left(2\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)\Bigg(-\frac{44779\sin(c+d\,x)^2}{32}+\frac{\sin(c+d\,x)\,32175\mathrm{i}}{128}-\frac{26075\sin(2\,c+2\,d\,x)^2}{16}-\frac{\sin(2\,c+2\,d\,x)\,3575\mathrm{i}}{8}+\frac{114583}{}}{}$$

input
```
int(cos(c + d*x)^15*(a + a*tan(c + d*x)*1i)^8,x)
```

output

```
(2*a^8*(2*sin(c/4 + (d*x)/4)^2 - 1)*((sin(c + d*x)*32175i)/128 - (sin(2*c
+ 2*d*x)*3575i)/8 + (sin(3*c + 3*d*x)*84227i)/128 - sin(4*c + 4*d*x)*754i
+ (sin(5*c + 5*d*x)*111527i)/128 - (sin(6*c + 6*d*x)*7187i)/8 + (sin(7*c +
 7*d*x)*121427i)/128 - (26075*sin(2*c + 2*d*x)^2)/16 + (114583*sin(c/2 + (
d*x)/2)^2)/64 - (57925*sin(3*c + 3*d*x)^2)/32 + (116585*sin((3*c)/2 + (3*d
*x)/2)^2)/64 + (119315*sin((5*c)/2 + (5*d*x)/2)^2)/64 + (122285*sin((7*c)/
2 + (7*d*x)/2)^2)/64 - (44779*sin(c + d*x)^2)/32 - 952))/(6435*d*(sin((15*
c)/2 + (15*d*x)/2) - sin((15*c)/4 + (15*d*x)/4)^2*2i + 1i))
```

3.98. $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$

3.99 $\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.99.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{8i(a-ia\tan(c+dx))^5}{5a^6d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{i(a-ia\tan(c+dx))^8}{8a^9d}$$

output
```
8/5*I*(a-I*a*tan(d*x+c))^5/a^6/d-2*I*(a-I*a*tan(d*x+c))^6/a^7/d+6/7*I*(a-I
*a*tan(d*x+c))^7/a^8/d-1/8*I*(a-I*a*tan(d*x+c))^8/a^9/d
```

3.99.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.52

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{(i+\tan(c+dx))^5\left(93+185i\tan(c+dx)-135\tan^2(c+dx)-35i\tan^3(c+dx)\right)}{280ad}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

output
```
((I + Tan[c + d*x])^5*(93 + (185*I)*Tan[c + d*x] - 135*Tan[c + d*x]^2 - (3
5*I)*Tan[c + d*x]^3))/(280*a*d)
```

3.99.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(-(a-ia\tan(c+dx))^7+6a(a-ia\tan(c+dx))^6-12a^2(a-ia\tan(c+dx))^5+8a^3(a-ia\tan(c+dx))^4\right)}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{8}{5}a^3(a-ia\tan(c+dx))^5+2a^2(a-ia\tan(c+dx))^6+\frac{1}{8}(a-ia\tan(c+dx))^8-\frac{6}{7}a(a-ia\tan(c+dx))^7\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*((-8*a^3*(a - I*a*Tan[c + d*x])^5)/5 + 2*a^2*(a - I*a*Tan[c + d*x])^
6 - (6*a*(a - I*a*Tan[c + d*x])^7)/7 + (a - I*a*Tan[c + d*x])^8/8))/(a^9*d
)
```

3.99.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.99.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result
risch	$\frac{32i\left(56\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{35da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{8}}$
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{8}(dx+c)\right)}{8}-\frac{\left(\tan^{7}(dx+c)\right)}{7}+\frac{i\left(\tan^{6}(dx+c)\right)}{2}-\frac{3\left(\tan^{5}(dx+c)\right)}{5}+\frac{3i\left(\tan^{4}(dx+c)\right)}{4}-\left(\tan^{3}(dx+c)\right)+}{ad}$
default	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{8}(dx+c)\right)}{8}-\frac{\left(\tan^{7}(dx+c)\right)}{7}+\frac{i\left(\tan^{6}(dx+c)\right)}{2}-\frac{3\left(\tan^{5}(dx+c)\right)}{5}+\frac{3i\left(\tan^{4}(dx+c)\right)}{4}-\left(\tan^{3}(dx+c)\right)+}{ad}$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
32/35*I*(56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/a
/(exp(2*I*(d*x+c))+1)^8
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{32\,\big(-56i\,e^{(6i\,dx+6i\,c)}-28i\,e^{(4i\,dx+4i\,c)}-8i\,e^{(2i\,dx+2i\,c)}-}{35\,\big(ade^{(16i\,dx+16i\,c)}+8\,ade^{(14i\,dx+14i\,c)}+28\,ade^{(12i\,dx+12i\,c)}+56\,ade^{(10i\,dx+10i\,c)}+70\,ade^{(8i\,dx+8i\,c)}+56\,ade}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-32/35*(-56*I*e^(6*I*d*x + 6*I*c) - 28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*
d*x + 2*I*c) - I)/(a*d*e^(16*I*d*x + 16*I*c) + 8*a*d*e^(14*I*d*x + 14*I*c)
 + 28*a*d*e^(12*I*d*x + 12*I*c) + 56*a*d*e^(10*I*d*x + 10*I*c) + 70*a*d*e^
(8*I*d*x + 8*I*c) + 56*a*d*e^(6*I*d*x + 6*I*c) + 28*a*d*e^(4*I*d*x + 4*I*c
) + 8*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.99.6 Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^{10}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**10/(tan(c + d*x) - I), x)/a
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{35i\,\tan(dx+c)^8-40\,\tan(dx+c)^7+140i\,\tan(dx+c)^6-168\,\tan(dx+c)^5+210i\,\tan(dx+c)^4-}{280\,ad}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(
d*x + c)^2 - 280*tan(d*x + c))/(a*d)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{35i\,\tan\left(dx+c\right)^{8}-40\,\tan\left(dx+c\right)^{7}+140i\,\tan\left(dx+c\right)^{6}-168\,\tan\left(dx+c\right)^{5}+210i\,\tan\left(dx+c\right)^{4}-2}{280\,ad}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(
d*x + c)^2 - 280*tan(d*x + c))/(a*d)
```

3.99.9 Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$$

$$=\frac{\cos\left(c+d\,x\right)^{8}\,35\mathrm{i}+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{7}+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{5}+48\,\sin\left(c+d\,x\right)\,\cos}{280\,a\,d\cos\left(c+d\,x\right)^{8}}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(40*cos(c + d*x)*sin(c + d*x) + 48*cos(c + d*x)^3*sin(c + d*x) + 64*cos(c
+ d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + cos(c + d*x)^8*3
5i - 35i)/(280*a*d*cos(c + d*x)^8)
```

3.100 $\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.100.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{i(a-ia\tan(c+dx))^4}{a^5d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^6}{6a^7d}$$

output
```
I*(a-I*a*tan(d*x+c))^4/a^5/d-4/5*I*(a-I*a*tan(d*x+c))^5/a^6/d+1/6*I*(a-I*a
*tan(d*x+c))^6/a^7/d
```

3.100.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx=-\frac{i(i+\tan(c+dx))^4\left(-11-14i\tan(c+dx)+5\tan^2(c+dx)\right)}{30ad}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-1/30*I)*(I + Tan[c + d*x])^4*(-11 - (14*I)*Tan[c + d*x] + 5*Tan[c + d*x
]^2))/(a*d)
```

3.100.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left((a-ia\tan(c+dx))^5-4a(a-ia\tan(c+dx))^4+4a^2(a-ia\tan(c+dx))^3\right)d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-a^2(a-ia\tan(c+dx))^4-\frac{1}{6}(a-ia\tan(c+dx))^6+\frac{4}{5}a(a-ia\tan(c+dx))^5\right)}{a^7d}$$

input

```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

output

```
((-I)*(-(a^2*(a - I*a*Tan[c + d*x])^4) + (4*a*(a - I*a*Tan[c + d*x])^5)/5
- (a - I*a*Tan[c + d*x])^6/6))/(a^7*d)
```

3.100.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.100.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{16i\left(15\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{6}}$	47
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{6}(dx+c)\right)}{6}-\frac{\left(\tan^{5}(dx+c)\right)}{5}+\frac{i\left(\tan^{4}(dx+c)\right)}{2}-\frac{2\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}}{ad}$	71
default	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{6}(dx+c)\right)}{6}-\frac{\left(\tan^{5}(dx+c)\right)}{5}+\frac{i\left(\tan^{4}(dx+c)\right)}{2}-\frac{2\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}}{ad}$	71

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
16/15*I*(15*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1
)^6
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{16\left(-15i\,e^{(4i\,dx+4i\,c)}-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx}\right.}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-16/15*(-15*I*e^(4*I*d*x + 4*I*c) - 6*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(1
2*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*
c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*
I*d*x + 2*I*c) + a*d)
```

3.100.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^8(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**8/(tan(c + d*x) - I), x)/a
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$$

$$= \frac{-5i\,\tan\left(dx+c\right)^6+6\,\tan\left(dx+c\right)^5-15i\,\tan\left(dx+c\right)^4+20\,\tan\left(dx+c\right)^3-15i\,\tan\left(dx+c\right)^2+30\,\tan}{30\,ad}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(-5*I*tan(d*x + c)^6 + 6*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 + 20*ta
n(d*x + c)^3 - 15*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a*d)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{5i\,\tan\left(dx+c\right)^6 - 6\,\tan\left(dx+c\right)^5 + 15i\,\tan\left(dx+c\right)^4 - 20\,\tan\left(dx+c\right)^3 + 15i\,\tan\left(dx+c\right)^2 - 30\,\tan}{30\,ad}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*ta
n(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\sin\left(c+d\,x\right)\,\left(30\cos\left(c+d\,x\right)^5 - \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,15\mathrm{i} + 20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2 - \cos\left(c+\right.\right.}{30\,a\,d\cos\left(c+d\,x\right)^6}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d*x)
*15i + 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c + d*x)
^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a*d*cos(c + d*x)^6)
```

3.101 $\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.101.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2i(a-ia\tan(c+dx))^3}{3a^4d} - \frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

output
```
2/3*I*(a-I*a*tan(d*x+c))^3/a^4/d-1/4*I*(a-I*a*tan(d*x+c))^4/a^5/d
```

3.101.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\tan(c+dx)\left(12-6i\tan(c+dx)+4\tan^2(c+dx)-3i\tan^3(c+dx)\right)}{12ad}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]*(12 - (6*I)*Tan[c + d*x] + 4*Tan[c + d*x]^2 - (3*I)*Tan[c + 
d*x]^3))/(12*a*d)
```

3.101.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^2-(a-ia\tan(c+dx))^3\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{4}(a-ia\tan(c+dx))^4-\frac{2}{3}a(a-ia\tan(c+dx))^3\right)}{a^5d}$$

input

```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

output

```
((-I)*((-2*a*(a - I*a*Tan[c + d*x])^3)/3 + (a - I*a*Tan[c + d*x])^4/4))/(a
^5*d)
```

3.101.3.1 Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.101.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{4i\left(4\,\mathrm{e}^{2i(dx+c)}+1\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}$	36
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{4}(dx+c)\right)}{4}-\frac{\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}}{ad}$	50
default	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{4}(dx+c)\right)}{4}-\frac{\left(\tan^{3}(dx+c)\right)}{3}+\frac{i\left(\tan^{2}(dx+c)\right)}{2}}{ad}$	50

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(4*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1)^4
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{4\left(-4i\,e^{(2i\,dx+2i\,c)}-i\right)}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-4/3*(-4*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*
I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a
*d)
```

3.101.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int\frac{\sec^6(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**6/(tan(c + d*x) - I), x)/a
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{-3i\tan(dx+c)^4+4\tan(dx+c)^3-6i\tan(dx+c)^2+12\tan(dx+c)}{12\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(-3*I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 - 6*I*tan(d*x + c)^2 + 12*tan
(d*x + c))/(a*d)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{3i\tan(dx+c)^4-4\tan(dx+c)^3+6i\tan(dx+c)^2-12\tan(dx+c)}{12\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan
(d*x + c))/(a*d)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\sin\left(c+d\,x\right)\ \left(12\cos\left(c+d\,x\right)^3-\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,6\mathrm{i}+4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^2-\sin\left(c+d\right.}{12\,a\,d\cos\left(c+d\,x\right)^4}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 - cos(c + d*x)^2*sin(c + d*x)
*6i + 12*cos(c + d*x)^3 - sin(c + d*x)^3*3i))/(12*a*d*cos(c + d*x)^4)
```

3.102 $\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.102.6 Sympy [F] 900
3.102.7 Maxima [A] (verification not implemented) 900
3.102.8 Giac [A] (verification not implemented) 900
3.102.9 Mupad [B] (verification not implemented) 901

3.102.1 Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\tan(c+dx)}{ad}-\frac{i\tan^2(c+dx)}{2ad}$$

output
```
tan(d*x+c)/a/d-1/2*I*tan(d*x+c)^2/a/d
```

3.102.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\tan(c+dx)}{ad}-\frac{i\tan^2(c+dx)}{2ad}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
Tan[c + d*x]/(a*d) - ((I/2)*Tan[c + d*x]^2)/(a*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^4}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{i(a-ia\tan(c+dx))^2}{2a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
((I/2)*(a - I*a*Tan[c + d*x])^2)/(a^3*d)
```

3.102.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.102.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}$	23
derivativedivides	$-\frac{i\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+i\tan(dx+c)\right)}{ad}$	30
default	$-\frac{i\left(\frac{\left(\tan^{2}(dx+c)\right)}{2}+i\tan(dx+c)\right)}{ad}$	30

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a/(exp(2*I*(d*x+c))+1)^2
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2i}{ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.102.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^4(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**4/(tan(c + d*x) - I), x)/a
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\tan(dx+c)^2 - 2\tan(dx+c)}{2\,ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\tan(dx+c)^2 - 2\tan(dx+c)}{2\,ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\tan(c+d\,x)\,\left(-2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,a\,d}$$

input

```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)),x)
```

output

```
-(tan(c + d*x)*(tan(c + d*x)*1i - 2))/(2*a*d)
```

3.103 $\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

3.103.1 Optimal result 902
3.103.2 Mathematica [A] (verified) 902
3.103.3 Rubi [A] (verified) 903
3.103.4 Maple [A] (verified) 904
3.103.5 Fricas [A] (verification not implemented) 904
3.103.6 Sympy [F] 905
3.103.7 Maxima [A] (verification not implemented) 905
3.103.8 Giac [B] (verification not implemented) 905
3.103.9 Mupad [B] (verification not implemented) 906

3.103.1 Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

output
```
x/a+I*ln(cos(d*x+c))/a/d
```

3.103.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx=-\frac{i\log(i-\tan(c+dx))}{ad}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*Log[I - Tan[c + d*x]])/(a*d)
```

3.103.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{1}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{16}$$

$$-\frac{i\log(a+ia\tan(c+dx))}{ad}$$

input

```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output

```
((-I)*Log[a + I*a*Tan[c + d*x]])/(a*d)
```

3.103.3.1 Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.103.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{i\ln(a+ia\tan(dx+c))}{ad}$	23
default	$-\frac{i\ln(a+ia\tan(dx+c))}{ad}$	23
risch	$\frac{2x}{a}+\frac{2c}{ad}+\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{ad}$	38

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/a/d*ln(a+I*a*tan(d*x+c))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\,dx+i\,\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
(2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

3.103.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^2(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**2/(tan(c + d*x) - I), x)/a
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\log\left(i\,a\tan\left(dx+c\right)+a\right)}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-I*log(I*a*tan(d*x + c) + a)/(a*d)
```

3.103.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{-\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}+\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a}-\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-(-I*log(tan(1/2*d*x + 1/2*c) + 1)/a + 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
 - I*log(tan(1/2*d*x + 1/2*c) - 1)/a)/d
```

3.103.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)),x)
```

output
```
-(log(tan(c + d*x) - 1i)*1i)/(a*d)
```

3.104 $\int \frac{1}{a+ia\tan(c+dx)}\,dx$

3.104.1 Optimal result 907
3.104.2 Mathematica [A] (verified) 907
3.104.3 Rubi [A] (verified) 908
3.104.4 Maple [A] (verified) 909
3.104.5 Fricas [A] (verification not implemented) 909
3.104.6 Sympy [A] (verification not implemented) 909
3.104.7 Maxima [F(-2)] 910
3.104.8 Giac [B] (verification not implemented) 910
3.104.9 Mupad [B] (verification not implemented) 911

3.104.1 Optimal result

Integrand size = 15, antiderivative size = 33

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

output
```
1/2*x/a+1/2*I/d/(a+I*a*tan(d*x+c))
```

3.104.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{\frac{\arctan(\tan(c+dx))}{a}+\frac{1}{-ia+a\tan(c+dx)}}{2d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-1),x]
```

output
```
(ArcTan[Tan[c + d*x]]/a + ((-I)*a + a*Tan[c + d*x])^(-1))/(2*d)
```

3.104.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \; 3960$$

$$\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 24$$

$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(-1),x]
```

output

```
x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

3.104.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.104.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4ad}$	26
derivativedivides	$\frac{\arctan(\tan(dx+c))}{2da}+\frac{1}{2da(\tan(dx+c)-i)}$	36
default	$\frac{\arctan(\tan(dx+c))}{2da}+\frac{1}{2da(\tan(dx+c)-i)}$	36
norman	$\frac{\frac{x}{2a}+\frac{i}{2ad}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{\tan(dx+c)}{2ad}}{1+\tan^2(dx+c)}$	58

input
```
int(1/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/4*I/a/d*exp(-2*I*(d*x+c))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

3.104.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0\\ x\left(\frac{(e^{2ic}+1)e^{-2ic}}{2a}-\frac{1}{2a}\right) & \text{otherwise}\end{cases}+\frac{x}{2a}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x
*((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

3.104.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.104.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(25) = 50.

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = -\frac{-\frac{i\log(\tan(dx+c)+i)}{a} + \frac{i\log(\tan(dx+c)-i)}{a} + \frac{-i\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4\,d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(-I*log(tan(d*x + c) + I)/a + I*log(tan(d*x + c) - I)/a + (-I*tan(d*x
 + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

3.104.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{x}{2\,a}+\frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(a + a*tan(c + d*x)*1i),x)
```

output
```
x/(2*a) + 1i/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.105 $\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

3.105.1 Optimal result 912
3.105.2 Mathematica [A] (verified) 912
3.105.3 Rubi [A] (verified) 913
3.105.4 Maple [A] (verified) 914
3.105.5 Fricas [A] (verification not implemented) 915
3.105.6 Sympy [A] (verification not implemented) 915
3.105.7 Maxima [F(-2)] 916
3.105.8 Giac [A] (verification not implemented) 916
3.105.9 Mupad [B] (verification not implemented) 916

3.105.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3x}{8a} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia}{8d(a+ia\tan(c+dx))^2} + \frac{i}{4d(a+ia\tan(c+dx))}$$

output
```
3/8*x/a-1/8*I/d/(a-I*a*tan(d*x+c))+1/8*I*a/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/
(a+I*a*tan(d*x+c))
```

3.105.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2-3i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))^2(i+\tan(c+dx))}{8ad(-i+\tan(c+dx))^2(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2 - (3*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]*(-I +
Tan[c + d*x])^2*(I + Tan[c + d*x]))/(8*a*d*(-I + Tan[c + d*x])^2*(I + Tan[
c + d*x]))
```

3.105.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3042, 3968, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{ia^3\int \left(\frac{1}{8a^3(a-ia\tan(c+dx))^2}+\frac{1}{4a^3(i\tan(c+dx)a+a)^2}+\frac{1}{4a^2(i\tan(c+dx)a+a)^3}+\frac{3}{8a^3(\tan^2(c+dx)a^2+a^2)}\right)d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ia^3\left(\frac{3i\arctan(\tan(c+dx))}{8a^4}+\frac{1}{8a^3(a-ia\tan(c+dx))}-\frac{1}{4a^3(a+ia\tan(c+dx))}-\frac{1}{8a^2(a+ia\tan(c+dx))^2}\right)}{d}$$

input

```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

output

```
((-I)*a^3*((((3*I)/8)*ArcTan[Tan[c + d*x]])/a^4 + 1/(8*a^3*(a - I*a*Tan[c
+ d*x])) - 1/(8*a^2*(a + I*a*Tan[c + d*x])^2) - 1/(4*a^3*(a + I*a*Tan[c +
d*x]))))/d
```

3.105.3.1 Defintions of rubi rules used

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.105.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{3x}{8a}+\frac{i\mathrm{e}^{-4i(dx+c)}}{32ad}+\frac{i\cos(2dx+2c)}{8ad}+\frac{\sin(2dx+2c)}{4ad}$	61
derivativedivides	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75
default	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75

input

```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
3/8*x/a+1/32*I/a/d*exp(-4*I*(d*x+c))+1/8*I/a/d*cos(2*d*x+2*c)+1/4/a/d*sin(
2*d*x+2*c)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(12\,dxe^{(4i\,dx+4i\,c)} - 2i\,e^{(6i\,dx+6i\,c)} + 6i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*
x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

3.105.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \begin{cases} \frac{\left(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx}\right)e^{-6ic}}{8192a^3d^3} & \text{for } a^3d^3e^{6ic} \neq 0 \\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-4ic}}{8a} - \frac{3}{8a}\right) & \text{otherwise} \end{cases} + \frac{3x}{8a}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*ex
p(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*
I*c)/(8192*a**3*d**3), Ne(a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3*e
xp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/(
8*a)
```

3.105.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input

```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output

```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a}+\frac{6i\log(\tan(dx+c)-i)}{a}+\frac{2\,(3\tan(dx+c)+5i)}{a(-i\tan(dx+c)+1)}+\frac{-9i\tan(dx+c)^2-26\tan(dx+c)+21i}{a(\tan(dx+c)-i)^2}}{32\,d}$$

input

```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output

```
-1/32*(-6*I*log(tan(d*x + c) + I)/a + 6*I*log(tan(d*x + c) - I)/a + 2*(3*t
an(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*t
an(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

3.105.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\,x}{8\,a}-\frac{\frac{3\tan(c+d\,x)^2}{8}-\frac{\tan(c+d\,x)\,\mathrm{3i}}{8}+\frac{1}{4}}{a\,d\,\left(1+\tan\left(c+d\,x\right)\,\mathrm{1i}\right)^2\,\left(\tan\left(c+d\,x\right)+\mathrm{1i}\right)}$$

input

```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i),x)
```

output

```
(3*x)/(8*a) - ((3*tan(c + d*x)^2)/8 - (tan(c + d*x)*3i)/8 + 1/4)/(a*d*(tan
(c + d*x)*1i + 1)^2*(tan(c + d*x) + 1i))
```

3.106 $\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.106.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{5x}{16a}-\frac{ia}{32d(a-ia\tan(c+dx))^2} -\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia^2}{24d(a+ia\tan(c+dx))^3} +\frac{3ia}{32d(a+ia\tan(c+dx))^2}+\frac{3i}{16d(a+ia\tan(c+dx))}$$

output
```
5/16*x/a-1/32*I*a/d/(a-I*a*tan(d*x+c))^2-1/8*I/d/(a-I*a*tan(d*x+c))+1/24*I
*a^2/d/(a+I*a*tan(d*x+c))^3+3/32*I*a/d/(a+I*a*tan(d*x+c))^2+3/16*I/d/(a+I*
a*tan(d*x+c))
```

3.106.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec^5(c+dx)(-80\cos(c+dx)+15\cos(3(c+dx))+\cos(5(c+dx))+120i\arctan(\tan(c+dx))(\cos(c+}{384ad(-i+\tan(c+dx))^3(i+\tan}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/384*(Sec[c + d*x]^5*(-80*Cos[c + d*x] + 15*Cos[3*(c + d*x)] + Cos[5*(c
+ d*x)] + (120*I)*ArcTan[Tan[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x]) + (
40*I)*Sin[c + d*x] + (45*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*(c + d*x)]))/(a
*d*(-I + Tan[c + d*x])^3*(I + Tan[c + d*x])^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^5\int \left(\frac{1}{8a^5(a-ia\tan(c+dx))^2}+\frac{3}{16a^5(i\tan(c+dx)a+a)^2}+\frac{1}{16a^4(a-ia\tan(c+dx))^3}+\frac{3}{16a^4(i\tan(c+dx)a+a)^3}+\frac{1}{8a^3(i\tan(c+dx)a+a)^4}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{5i\arctan(\tan(c+dx))}{16a^6}+\frac{1}{8a^5(a-ia\tan(c+dx))}-\frac{3}{16a^5(a+ia\tan(c+dx))}+\frac{1}{32a^4(a-ia\tan(c+dx))^2}-\frac{3}{32a^4(a+ia\tan(c+dx))^2}-24\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*a^5*((((5*I)/16)*ArcTan[Tan[c + d*x]])/a^6 + 1/(32*a^4*(a - I*a*Tan[
c + d*x])^2) + 1/(8*a^5*(a - I*a*Tan[c + d*x])) - 1/(24*a^3*(a + I*a*Tan[c
 + d*x])^3) - 3/(32*a^4*(a + I*a*Tan[c + d*x])^2) - 3/(16*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

3.106.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.106.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

method	result
risch	$\frac{5x}{16a}+\frac{i\,{\mathrm e}^{-6i(dx+c)}}{192ad}+\frac{i\cos(4dx+4c)}{32ad}+\frac{3\sin(4dx+4c)}{64ad}+\frac{5i\cos(2dx+2c)}{64ad}+\frac{15\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+}{da}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+}{da}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
5/16*x/a+1/192*I/a/d*exp(-6*I*(d*x+c))+1/32*I/a/d*cos(4*d*x+4*c)+3/64/a/d*
sin(4*d*x+4*c)+5/64*I/a/d*cos(2*d*x+2*c)+15/64/a/d*sin(2*d*x+2*c)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.57

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(120\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(10i\,dx+10i\,c)}-30i\,e^{(8i\,dx+8i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8
*I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*
I)*e^(-6*I*d*x - 6*I*c)/(a*d)
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.63

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{(-50331648ia^4d^4e^{16ic}e^{4idx}-503316480ia^4d^4e^{14ic}e^{2idx}+1006632960ia^4d^4e^{10ic}e^{-2idx}+251658240ia^4d^4e^{8ic}e^{-4idx}+33554432ia^4d^4e^{6ic}e^{-6i}}{6442450944a^5d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-6ic}}{32a}-\frac{5}{16a}\right) \end{cases} + \frac{5x}{16a}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a
**4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp
(-2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a
**4*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne
(a**5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*
c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a)), Tru
e)) + 5*x/(16*a)
```

3.106.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{-\frac{30i\,\log(\tan(dx+c)+i)}{a} + \frac{30i\,\log(\tan(dx+c)-i)}{a} + \frac{3\left(-15i\,\tan(dx+c)^2+38\,\tan(dx+c)+25i\right)}{a(-i\,\tan(dx+c)+1)^2} - \frac{55i\,\tan(dx+c)^3+201\,\tan(dx+c)^2-25}{a(\tan(dx+c)-i)^3}}{192\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(
-15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2)
 - (55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(
a*(tan(d*x + c) - I)^3))/d
```

3.106.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{5\,x}{16\,a}$$
$$+\frac{\frac{25\tan(c+d\,x)}{48\,a} + \frac{1\mathrm{i}}{6\,a} + \frac{\tan(c+d\,x)^2\,25\mathrm{i}}{48\,a} + \frac{5\tan(c+d\,x)^3}{16\,a} + \frac{\tan(c+d\,x)^4\,5\mathrm{i}}{16\,a}}{d\,\left(\tan\left(c+d\,x\right)^5\,1\mathrm{i} + \tan\left(c+d\,x\right)^4 + \tan\left(c+d\,x\right)^3\,2\mathrm{i} + 2\tan\left(c+d\,x\right)^2 + \tan\left(c+d\,x\right)\,1\mathrm{i} + 1\right)}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i),x)
```

output
```
(5*x)/(16*a) + ((25*tan(c + d*x))/(48*a) + 1i/(6*a) + (tan(c + d*x)^2*25i)
/(48*a) + (5*tan(c + d*x)^3)/(16*a) + (tan(c + d*x)^4*5i)/(16*a))/(d*(tan(
c + d*x)*1i + 2*tan(c + d*x)^2 + tan(c + d*x)^3*2i + tan(c + d*x)^4 + tan(
c + d*x)^5*1i + 1))
```

3.106. $\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

3.107 $\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.107.1 Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{8ad} - \frac{i\sec^5(c+dx)}{5ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}$$

output
```
3/8*arctanh(sin(d*x+c))/a/d-1/5*I*sec(d*x+c)^5/a/d+3/8*sec(d*x+c)*tan(d*x+
c)/a/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a/d
```

3.107.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{240\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(-64i+70\sin(2(c+dx))+15\sin(4(c+dx)))}{320ad}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

output
```
(240*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5*(-64*I + 70*Si
n[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(320*a*d)
```

3.107.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3982, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^7}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3982}$$

$$\frac{\int \sec^5(c+dx)dx}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}$$

input

```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

output

```
((-1/5*I)*Sec[c + d*x]^5)/(a*d) + ((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)/a
```

3.107.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3982

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.107.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}+128\,\mathrm{e}^{5i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{20da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8ad}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8ad}$
derivativedivides	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{a}$
default	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{a}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/20*I/d/a/(exp(2*I*(d*x+c))+1)^5*(15*exp(9*I*(d*x+c))+70*exp(7*I*(d*x+c)
)+128*exp(5*I*(d*x+c))-70*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+3/8/a/d*ln(e
xp(I*(d*x+c))+I)-3/8/a/d*ln(exp(I*(d*x+c))-I)
```

3.107.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(74) = 148.

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int\frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$=\frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\,(}{40\,(ade^{(10i}}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e
^(I*d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c)
 - 256*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x +
 I*c))/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(
6*I*d*x + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c)
+ a*d)
```

3.107.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^7(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**7/(tan(c + d*x) - I), x)/a
```

3.107.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.43

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{3\left(\frac{16\left(\frac{25i\sin(dx+c)}{\cos(dx+c)+1}-\frac{10i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{40\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{25i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+8\right)}{-120ia+\frac{600ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\right.}{8d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-3/8*(16*(25*I*sin(d*x + c)/(cos(d*x + c) + 1) - 10*I*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*I*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*I*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 8)/(-120*I*a + 600*I*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12
00*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d
*x + c) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - log(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a)/d
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.64

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2\left(25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+40i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+80i\,\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5a}}{40\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a - 15*log(tan(1/2*d*x + 1/2*c) - 1
)/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 - 10*tan(
1/2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)
^3 - 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a))/d
```

3.107.9 Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.30

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}$$
$$+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2\,a}+\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4\,a}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}}{a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,2\mathrm{i}}{a}+\frac{2\mathrm{i}}{5\,a}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) + (tan(c/2 + (d*x)/2)^3/(2*a) + (tan
(c/2 + (d*x)/2)^4*4i)/a - tan(c/2 + (d*x)/2)^7/(2*a) + (tan(c/2 + (d*x)/2)
^8*2i)/a + (5*tan(c/2 + (d*x)/2)^9)/(4*a) + 2i/(5*a) - (5*tan(c/2 + (d*x)/
2))/(4*a))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c
/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.107. $\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$

3.108 $\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.108.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

output
```
1/2*arctanh(sin(d*x+c))/a/d-1/3*I*sec(d*x+c)^3/a/d+1/2*sec(d*x+c)*tan(d*x+
c)/a/d
```

3.108.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(-4i+3\sin(2(c+dx)))}{12ad}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output
```
(12*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-4*I + 3*Sin[2
*(c + d*x)]))/(12*a*d)
```

3.108.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3982, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{\int \sec^3(c+dx)dx}{a}-\frac{i\sec^3(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\sec^3(c+dx)}{3ad}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}$$

input

```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output

```
((-1/3*I)*Sec[c + d*x]^3)/(a*d) + (ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c +
d*x]*Tan[c + d*x])/(2*d))/a
```

3.108.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.108.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(dx+c)}+8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{ad}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{ad}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d/a/(exp(2*I*(d*x+c))+1)^3*(3*exp(5*I*(d*x+c))+8*exp(3*I*(d*x+c))-3
*exp(I*(d*x+c)))-1/2/a/d*ln(exp(I*(d*x+c))-I)+1/2/a/d*ln(exp(I*(d*x+c))+I)
```

3.108.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(52) = 104.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.90

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+}}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx}}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*
d*x + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I
*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*
d)
```

3.108.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^5(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**5/(tan(c + d*x) - I), x)/a
```

3.108.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(52) = 104$.

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.10

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{4\left(\frac{3i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6i\,a-\frac{18i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a}}{6\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a
 + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*
x + 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.108.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} + \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 2\mathrm{i}}{a} + \frac{2\mathrm{i}}{3\,a}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)),x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/(a*d) + ((tan(c/2 + (d*x)/2)^4*2i)/a + tan(c/2 +
 (d*x)/2)^5/a + 2i/(3*a) - tan(c/2 + (d*x)/2)/a)/(d*(3*tan(c/2 + (d*x)/2)^
2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.109 $\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.109.1 Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

output
```
arctanh(sin(d*x+c))/a/d-I*sec(d*x+c)/a/d
```

3.109.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{2\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)-i\sec(c+dx)}{ad}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x])/(a*d)
```

3.109.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3982, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{\int \sec(c+dx)dx}{a}-\frac{i\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{i\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.109.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.

Time = 1.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

method	result	size
derivativedivides	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	70
default	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	70
risch	$-\frac{2ie^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	74

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/a*(-1/2*I/(tan(1/2*d*x+1/2*c)+1)+1/2*ln(tan(1/2*d*x+1/2*c)+1)+1/2*I/(t
an(1/2*d*x+1/2*c)-1)-1/2*ln(tan(1/2*d*x+1/2*c)-1))
```

3.109.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time $= 0.24$ (sec) , antiderivative size $= 80$, normalized size of antiderivative $= 2.58$

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*
I*c) + a*d)
```

3.109.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^3(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sec(c + d*x)**3/(tan(c + d*x) - I), x)/a
```

3.109.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.

Time $= 0.37$ (sec) , antiderivative size $= 83$, normalized size of antiderivative $= 2.68$

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2}{-i\,a+\frac{i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a} - \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a} + \frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
(log(tan(1/2*d*x + 1/2*c) + 1)/a - log(tan(1/2*d*x + 1/2*c) - 1)/a + 2*I/(
(tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.109.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} + \frac{2\mathrm{i}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + 2i/(a*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.110 $\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.110.9 Mupad [B] (verification not implemented) 944

3.110.1 Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

output
```
I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))
```

3.110.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\sec(c+dx)}{ad(-i+\tan(c+dx))}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output
```
Sec[c + d*x]/(a*d*(-I + Tan[c + d*x]))
```

3.110.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(I*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

3.110.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.110.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{ad}$	19
derivativedivides	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
default	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I/a/d*exp(-I*(d*x+c))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
I*e^(-I*d*x - I*c)/(a*d)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\sec{(c+dx)}}{ad\tan{(c+dx)}-iad} & \text{for } d\neq 0 \\ \frac{x\sec{(c)}}{ia\tan{(c)}+a} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise((sec(c + d*x)/(a*d*tan(c + d*x) - I*a*d), Ne(d, 0)), (x*sec(c)/(
I*a*tan(c) + a), True))
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

3.110.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

3.111 $\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.111.8 Giac [A] (verification not implemented) 948
3.111.9 Mupad [B] (verification not implemented) 949

3.111.1 Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

output

```
2/3*sin(d*x+c)/a/d+1/3*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))
```

3.111.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx=-\frac{\sec(c+dx)(-3+\cos(2(c+dx))+2i\sin(2(c+dx)))}{6ad(-i+\tan(c+dx))}$$

input

```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output

```
-1/6*(Sec[c + d*x]*(-3 + Cos[2*(c + d*x)] + (2*I)*Sin[2*(c + d*x)]))/(a*d*
(-I + Tan[c + d*x]))
```

3.111.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3983, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{2\int \cos(c+dx)dx}{3a}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{3a}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*Sin[c + d*x])/(3*a*d) + ((I/3)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

3.111.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.111.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{12ad}+\frac{i\cos(dx+c)}{4ad}+\frac{3\sin(dx+c)}{4ad}$
derivativedivides	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$
default	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{2\tan(dx+c)}{3ad}-\frac{2i\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2i\left(\tan^{2}(dx+c)\right)}{3ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^{2}(dx+c)\right)}{3ad}-\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan(dx+c)}{3ad}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(1+\tan^{2}(dx+c))}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*I/a/d*exp(-3*I*(d*x+c))+1/4*I/a/d*cos(d*x+c)+3/4*sin(d*x+c)/a/d
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x
- 3*I*c)/(a*d)
```

3.111.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(36) = 72$.

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.68

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\left(-24ia^2d^2e^{5ic}e^{idx}+48ia^2d^2e^{3ic}e^{-idx}+8ia^2d^2e^{ic}e^{-3idx}\right)e^{-4ic}}{96a^3d^3} & \text{for } a^3d^3e^{4ic}\neq 0 \\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I
*c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a*
*3*d**3), Ne(a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)
*exp(-3*I*c)/(4*a), True))
```

3.111.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*t
an(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

3.111.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 3\mathrm{i}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)2\mathrm{i}}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^3}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i),x)
```

output
```
((tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 +
1i)*2i)/(3*a*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^3)
```

3.112 $\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.112.1 Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{4\sin(c+dx)}{5ad}-\frac{4\sin^3(c+dx)}{15ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

output
```
4/5*sin(d*x+c)/a/d-4/15*sin(d*x+c)^3/a/d+1/5*I*cos(d*x+c)^3/d/(a+I*a*tan(d
*x+c))
```

3.112.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec(c+dx)(-45+20\cos(2(c+dx))+\cos(4(c+dx))+40i\sin(2(c+dx))+4i\sin(4(c+dx)))}{120ad(-i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/120*(Sec[c + d*x]*(-45 + 20*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + (40*I
)*Sin[2*(c + d*x)] + (4*I)*Sin[4*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

3.112.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3983, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \cos^3(c+dx)dx}{5a}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{4\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

output
```
(-4*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*a*d) + ((I/5)*Cos[c + d*x]^3)/(
d*(a + I*a*Tan[c + d*x]))
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.112.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.25

method	result
risch	$\frac{i\mathrm{e}^{-5i(dx+c)}}{80ad}+\frac{i\cos(dx+c)}{8ad}+\frac{5\sin(dx+c)}{8ad}+\frac{i\cos(3dx+3c)}{16ad}+\frac{5\sin(3dx+3c)}{48ad}$
derivativedivides	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{3i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{ad}$
default	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{3i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{ad}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/80*I/a/d*exp(-5*I*(d*x+c))+1/8*I/a/d*cos(d*x+c)+5/8*sin(d*x+c)/a/d+1/16*
I/a/d*cos(3*d*x+3*c)+5/48/a/d*sin(3*d*x+3*c)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d
*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

3.112.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.93

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \begin{cases} \frac{\left(-30720ia^4d^4e^{12ic}e^{3idx}-368640ia^4d^4e^{10ic}e^{idx}+552960ia^4d^4e^{8ic}e^{-idx}+122880ia^4d^4e^{6ic}e^{-3idx}+18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for } a^5d^5 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherwi} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d*
*4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 12
2880*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*e
xp(-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(a**5*d**5*exp(9*I*c), 0)
), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-5
*I*c)/(16*a), True))
```

3.112.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.112.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{120\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(
tan(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2
*d*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c)
+ 113)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.112.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{\left(-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 15\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 25\mathrm{i}-13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\right.}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^5}$$

input

```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i),x)
```

output

```
-((9*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*21i - 13*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*25i - 5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*15i - 15*tan(c/2 + (d*x)/2)^7 + 3i)*2i)/(15*a*d*(tan(c/2 + (d*x)/2) +
1i)^3*(tan(c/2 + (d*x)/2)*1i + 1)^5)
```

3.113 $\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

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3.113.1 Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{6\sin(c+dx)}{7ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin^5(c+dx)}{35ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

output
```
6/7*sin(d*x+c)/a/d-4/7*sin(d*x+c)^3/a/d+6/35*sin(d*x+c)^5/a/d+1/7*I*cos(d*
x+c)^5/d/(a+I*a*tan(d*x+c))
```

3.113.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec(c+dx)(-350+175\cos(2(c+dx))+14\cos(4(c+dx))+\cos(6(c+dx))+350i\sin(2(c+dx))+5}{1120ad(-i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/1120*(Sec[c + d*x]*(-350 + 175*Cos[2*(c + d*x)] + 14*Cos[4*(c + d*x)] +
 Cos[6*(c + d*x)] + (350*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)] + (
6*I)*Sin[6*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

3.113.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3983, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\int \cos^5(c+dx)dx}{7a}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{6\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

input
```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

output
```
(-6*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a*d) + (
(I/7)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x]))
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.113.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

method	result
risch	$\frac{i\mathrm{e}^{-7i(dx+c)}}{448ad}+\frac{5i\cos(dx+c)}{64ad}+\frac{35\sin(dx+c)}{64ad}+\frac{i\cos(5dx+5c)}{64ad}+\frac{7\sin(5dx+5c)}{320ad}+\frac{3i\cos(3dx+3c)}{64ad}+$
derivativedivides	$-\frac{2}{7(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{7}}+\frac{i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{6}}+\frac{15i}{8(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{2}}-\frac{11i}{4(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{4}}+\frac{21}{10(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{5}}-$
default	$-\frac{2}{7(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{7}}+\frac{i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{6}}+\frac{15i}{8(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{2}}-\frac{11i}{4(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{4}}+\frac{21}{10(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^{5}}-$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/448*I/a/d*exp(-7*I*(d*x+c))+5/64*I/a/d*cos(d*x+c)+35/64*sin(d*x+c)/a/d+1
/64*I/a/d*cos(5*d*x+5*c)+7/320/a/d*sin(5*d*x+5*c)+3/64*I/a/d*cos(3*d*x+3*c
)+7/64/a/d*sin(3*d*x+3*c)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-7i\,e^{(12i\,dx+12i\,c)} - 70i\,e^{(10i\,dx+10i\,c)} - 525i\,e^{(8i\,dx+8i\,c)} + 700i\,e^{(6i\,dx+6i\,c)} + 175i\,e^{(4i\,dx+4i\,c)} + 42i\,e^{(2i\,dx+2i\,c)} +}{2240\,ad}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2240*(-7*I*e^(12*I*d*x + 12*I*c) - 70*I*e^(10*I*d*x + 10*I*c) - 525*I*e^
(8*I*d*x + 8*I*c) + 700*I*e^(6*I*d*x + 6*I*c) + 175*I*e^(4*I*d*x + 4*I*c)
+ 42*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a*d)
```

3.113.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(68) = 136.

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.11

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{(-150323855360ia^6d^6e^{21ic}e^{5idx}-1503238553600ia^6d^6e^{19ic}e^{3idx}-11274289152000ia^6d^6e^{17ic}e^{idx}+15032385536000ia^6d^6e^{15ic}e^{-idx}+375809}{48103633715200a^7d^7} \\ \frac{x\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-7ic}}{64a} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((-150323855360*I*a**6*d**6*exp(21*I*c)*exp(5*I*d*x) - 150323855
3600*I*a**6*d**6*exp(19*I*c)*exp(3*I*d*x) - 11274289152000*I*a**6*d**6*exp
(17*I*c)*exp(I*d*x) + 15032385536000*I*a**6*d**6*exp(15*I*c)*exp(-I*d*x) +
 3758096384000*I*a**6*d**6*exp(13*I*c)*exp(-3*I*d*x) + 901943132160*I*a**6
*d**6*exp(11*I*c)*exp(-5*I*d*x) + 107374182400*I*a**6*d**6*exp(9*I*c)*exp(
-7*I*d*x))*exp(-16*I*c)/(48103633715200*a**7*d**7), Ne(a**7*d**7*exp(16*I*
c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 20*exp(6*I*c) +
 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-7*I*c)/(64*a), True))
```

3.113.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.113.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(73) = 146$.

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.01

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{7\left(55\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+180i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-160i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+43\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5} + 735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-3360i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{560\,d}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/560*(7*(55*tan(1/2*d*x + 1/2*c)^4 + 180*I*tan(1/2*d*x + 1/2*c)^3 - 250*t
an(1/2*d*x + 1/2*c)^2 - 160*I*tan(1/2*d*x + 1/2*c) + 43)/(a*(tan(1/2*d*x +
 1/2*c) + I)^5) + (735*tan(1/2*d*x + 1/2*c)^6 - 3360*I*tan(1/2*d*x + 1/2*c
)^5 - 7315*tan(1/2*d*x + 1/2*c)^4 + 8820*I*tan(1/2*d*x + 1/2*c)^3 + 6321*t
an(1/2*d*x + 1/2*c)^2 - 2492*I*tan(1/2*d*x + 1/2*c) - 461)/(a*(tan(1/2*d*x
 + 1/2*c) - I)^7))/d
```

3.113.9 Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.21

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\left(-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}105\mathrm{i}-126\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\mathrm{t}\right.}{35\,a\,d\left(\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i),x)
```

output
```
((25*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*55i - 15*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*130i + 26*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)
/2)^6*182i - 126*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*105i - 35*tan
(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10*35i - 35*tan(c/2 + (d*x)/2)^11 +
 5i)*2i)/(35*a*d*(tan(c/2 + (d*x)/2) + 1i)^5*(tan(c/2 + (d*x)/2)*1i + 1)^7
)
```

3.114 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.114.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{4i(a-ia\tan(c+dx))^5}{5a^7d} - \frac{2i(a-ia\tan(c+dx))^6}{3a^8d} + \frac{i(a-ia\tan(c+dx))^7}{7a^9d}$$

output
```
4/5*I*(a-I*a*tan(d*x+c))^5/a^7/d-2/3*I*(a-I*a*tan(d*x+c))^6/a^8/d+1/7*I*(a
-I*a*tan(d*x+c))^7/a^9/d
```

3.114.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(i+\tan(c+dx))^5\left(-29-40i\tan(c+dx)+15\tan^2(c+dx)\right)}{105a^2d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/105*((I + Tan[c + d*x])^5*(-29 - (40*I)*Tan[c + d*x] + 15*Tan[c + d*x]^
2))/(a^2*d)
```

3.114.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left((a-ia\tan(c+dx))^6-4a(a-ia\tan(c+dx))^5+4a^2(a-ia\tan(c+dx))^4\right)d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{4}{5}a^2(a-ia\tan(c+dx))^5-\frac{1}{7}(a-ia\tan(c+dx))^7+\frac{2}{3}a(a-ia\tan(c+dx))^6\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*((-4*a^2*(a - I*a*Tan[c + d*x])^5)/5 + (2*a*(a - I*a*Tan[c + d*x])^6
)/3 - (a - I*a*Tan[c + d*x])^7/7))/(a^9*d)
```

3.114.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.114.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{128i\left(21\,\mathrm{e}^{4i(dx+c)}+7\,\mathrm{e}^{2i(dx+c)}+1\right)}{105d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{7}}$	47
derivativedivides	$\frac{\tan(dx+c)-\frac{\left(\tan^{7}(dx+c)\right)}{7}-\frac{i\left(\tan^{6}(dx+c)\right)}{3}-\frac{\left(\tan^{5}(dx+c)\right)}{5}-i\left(\tan^{4}(dx+c)\right)+\frac{\left(\tan^{3}(dx+c)\right)}{3}-i\left(\tan^{2}(dx+c)\right)}{a^{2}d}$	78
default	$\frac{\tan(dx+c)-\frac{\left(\tan^{7}(dx+c)\right)}{7}-\frac{i\left(\tan^{6}(dx+c)\right)}{3}-\frac{\left(\tan^{5}(dx+c)\right)}{5}-i\left(\tan^{4}(dx+c)\right)+\frac{\left(\tan^{3}(dx+c)\right)}{3}-i\left(\tan^{2}(dx+c)\right)}{a^{2}d}$	78

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*(21*exp(4*I*(d*x+c))+7*exp(2*I*(d*x+c))+1)/d/a^2/(exp(2*I*(d*x+c
))+1)^7
```

3.114.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(64) = 128.

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{128\left(-21i\,e^{(4i\,dx+4i\,c)}-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{105\left(a^2de^{(14i\,dx+14i\,c)}+7\,a^2de^{(12i\,dx+12i\,c)}+21\,a^2de^{(10i\,dx+10i\,c)}+35\,a^2de^{(8i\,dx+8i\,c)}+35\,a^2de^{(6i\,dx+6i\,c)}+2\right.}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-128/105*(-21*I*e^(4*I*d*x + 4*I*c) - 7*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*
e^(14*I*d*x + 14*I*c) + 7*a^2*d*e^(12*I*d*x + 12*I*c) + 21*a^2*d*e^(10*I*d
*x + 10*I*c) + 35*a^2*d*e^(8*I*d*x + 8*I*c) + 35*a^2*d*e^(6*I*d*x + 6*I*c)
 + 21*a^2*d*e^(4*I*d*x + 4*I*c) + 7*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.114.6 Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^{10}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**10/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a*
*2
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{15\,\tan(dx+c)^7+35i\,\tan(dx+c)^6+21\,\tan(dx+c)^5+105i\,\tan(dx+c)^4-35\,\tan(dx+c)^3+105}{105\,a^2d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*
I*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x
+ c))/(a^2*d)
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{15\,\tan\left(dx+c\right)^{7}+35i\,\tan\left(dx+c\right)^{6}+21\,\tan\left(dx+c\right)^{5}+105i\,\tan\left(dx+c\right)^{4}-35\,\tan\left(dx+c\right)^{3}+105}{105\,a^{2}d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*
I*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x
+ c))/(a^2*d)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\cos\left(c+d\,x\right)^{7}\,35\mathrm{i}+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{6}+32\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{4}+24\,\sin\left(c+d\,x\right)\,\cos\left(\right.}{105\,a^{2}\,d\cos\left(c+d\,x\right)^{7}}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(24*cos(c + d*x)^2*sin(c + d*x) - 15*sin(c + d*x) - cos(c + d*x)*35i + 32*
cos(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + cos(c + d*x
)^7*35i)/(105*a^2*d*cos(c + d*x)^7)
```

3.115 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.115.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^4}{2a^6d} - \frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

output
```
1/2*I*(a-I*a*tan(d*x+c))^4/a^6/d-1/5*I*(a-I*a*tan(d*x+c))^5/a^7/d
```

3.115.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{\tan(c+dx)\left(-10+10i\tan(c+dx)+5i\tan^3(c+dx)+2\tan^4(c+dx)\right)}{10a^2d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/10*(Tan[c + d*x]*(-10 + (10*I)*Tan[c + d*x] + (5*I)*Tan[c + d*x]^3 + 2*
Tan[c + d*x]^4))/(a^2*d)
```

3.115.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^3-(a-ia\tan(c+dx))^4\right)d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{5}(a-ia\tan(c+dx))^5-\frac{1}{2}a(a-ia\tan(c+dx))^4\right)}{a^7d}$$

input

```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

output

```
((-I)*(-1/2*(a*(a - I*a*Tan[c + d*x])^4) + (a - I*a*Tan[c + d*x])^5/5))/(a
^7*d)
```

3.115.3.1 Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.115.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{8i\left(5\,\mathrm{e}^{2i(dx+c)}+1\right)}{5d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$	36
derivativedivides	$\frac{\tan(dx+c)-\frac{\left(\tan^{5}(dx+c)\right)}{5}-\frac{i\left(\tan^{4}(dx+c)\right)}{2}-i\left(\tan^{2}(dx+c)\right)}{a^{2}d}$	47
default	$\frac{\tan(dx+c)-\frac{\left(\tan^{5}(dx+c)\right)}{5}-\frac{i\left(\tan^{4}(dx+c)\right)}{2}-i\left(\tan^{2}(dx+c)\right)}{a^{2}d}$	47

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
8/5*I*(5*exp(2*I*(d*x+c))+1)/d/a^2/(exp(2*I*(d*x+c))+1)^5
```

3.115.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int \frac{\sec^{8}(c+dx)}{(a+ia\tan(c+dx))^{2}}\,dx = -\frac{8\left(-5i\,e^{(2i\,dx+2i\,c)}-i\right)}{5\left(a^{2}de^{(10i\,dx+10i\,c)}+5\,a^{2}de^{(8i\,dx+8i\,c)}+10\,a^{2}de^{(6i\,dx+6i\,c)}+10\,a^{2}de^{(4i\,dx+4i\,c)}+5\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-8/5*(-5*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d
*e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x
+ 4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.115.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^8(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**8/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*ta
n(d*x + c))/(a^2*d)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*ta
n(d*x + c))/(a^2*d)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sin(c+d\,x)\,\Big(-10\cos(c+d\,x)^4+\cos(c+d\,x)^3\,\sin(c+d\,x)\,10\mathrm{i}+\cos(c+d\,x)\,\sin(c+d\,x)^3\,5\mathrm{i}+2\sin}{10\,a^2\,d\cos(c+d\,x)^5}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i + cos(c + d*x)^3*sin(c + d*
x)*10i - 10*cos(c + d*x)^4 + 2*sin(c + d*x)^4))/(10*a^2*d*cos(c + d*x)^5)
```

3.116 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.116.9 Mupad [B] (verification not implemented) 976

3.116.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

output
```
1/3*I*(a-I*a*tan(d*x+c))^3/a^5/d
```

3.116.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\tan(c+dx)}{a^2d} - \frac{i\tan^2(c+dx)}{a^2d} - \frac{\tan^3(c+dx)}{3a^2d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
Tan[c + d*x]/(a^2*d) - (I*Tan[c + d*x]^2)/(a^2*d) - Tan[c + d*x]^3/(3*a^2*
d)
```

3.116.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{17}$$

$$\frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

input

```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

output

```
((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^5*d)
```

3.116.3.1 Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.116.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{(\tan(dx+c)+i)^3}{3a^2d}$	20
default	$-\frac{(\tan(dx+c)+i)^3}{3a^2d}$	20
risch	$\frac{8i}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	23

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/d*(tan(d*x+c)+I)^3
```

3.116.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e
^(2*I*d*x + 2*I*c) + a^2*d)
```

3.116.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^6(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**6/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(dx+c)^3 + 3i\,\tan(dx+c)^2 - 3\,\tan(dx+c)}{3\,a^2d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(dx+c)^3 + 3i\,\tan(dx+c)^2 - 3\,\tan(dx+c)}{3\,a^2d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}-3\right)}{3\,a^2\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(tan(c + d*x)*(tan(c + d*x)*3i + tan(c + d*x)^2 - 3))/(3*a^2*d)
```

3.117 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.117.8 Giac [B] (verification not implemented) 980
3.117.9 Mupad [B] (verification not implemented) 981

3.117.1 Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2x}{a^2} + \frac{2i\log(\cos(c+dx))}{a^2d} - \frac{\tan(c+dx)}{a^2d}$$

output
```
2*x/a^2+2*I*ln(cos(d*x+c))/a^2/d-tan(d*x+c)/a^2/d
```

3.117.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2i\log(i-\tan(c+dx))}{a^2d} - \frac{\tan(c+dx)}{a^2d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-2*I)*Log[I - Tan[c + d*x]])/(a^2*d) - Tan[c + d*x]/(a^2*d)
```

3.117.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{2a}{i\tan(c+dx)a+a}-1\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(2a\log(a+ia\tan(c+dx))-ia\tan(c+dx))}{a^3d}$$

input

```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output

```
((-I)*(2*a*Log[a + I*a*Tan[c + d*x]] - I*a*Tan[c + d*x]))/(a^3*d)
```

3.117.3.1 Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.117.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$	30
default	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$	30
risch	$\frac{4x}{a^2}+\frac{4c}{a^2d}-\frac{2i}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{2i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^2d}$	60

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-tan(d*x+c)-2*I*ln(tan(d*x+c)-I))
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(2\,dxe^{(2i\,dx+2i\,c)}+2\,dx-\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
2*(2*d*x*e^(2*I*d*x + 2*I*c) + 2*d*x - (-I*e^(2*I*d*x + 2*I*c) - I)*log(e^
(2*I*d*x + 2*I*c) + 1) - I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.117.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^4(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**4/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\frac{2i\log(i\tan(dx+c)+1)}{a^2} - \frac{\tan(dx+c)}{a^2}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
(-2*I*log(I*tan(d*x + c) + 1)/a^2 - tan(d*x + c)/a^2)/d
```

3.117.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(36) = 72.

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2} - \frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^2} + \frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2} + \frac{-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}\right)}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
2*(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)
/a^2 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^2 + (-I*tan(1/2*d*x + 1/2*c)^2 +
tan(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

3.117.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+d\,x)+\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,2\mathrm{i}}{a^2\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(log(tan(c + d*x) - 1i)*2i + tan(c + d*x))/(a^2*d)
```

3.118 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.118.9 Mupad [B] (verification not implemented) 986

3.118.1 Optimal result

Integrand size = 24, antiderivative size = 26

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
I/d/(a^2+I*a^2*tan(d*x+c))
```

3.118.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{1}{a^2d(-i+\tan(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
1/(a^2*d*(-I + Tan[c + d*x]))
```

3.118.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \; 17$$

$$\frac{i}{ad(a+ia\tan(c+dx))}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
I/(a*d*(a + I*a*Tan[c + d*x]))
```

3.118.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.118.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{1}{a^2 d(\tan(dx+c)-i)}$	19
default	$\frac{1}{a^2 d(\tan(dx+c)-i)}$	19
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{2a^2 d}$	19

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/d/(tan(d*x+c)-I)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

3.118.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(17) = 34$.

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} -\frac{i\sec^2(c+dx)}{2a^2d\tan^2(c+dx)-4ia^2d\tan(c+dx)-2a^2d} & \text{for } d \neq 0 \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

input

```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

output

```
Piecewise((-I*sec(c + d*x)**2/(2*a**2*d*tan(c + d*x)**2 - 4*I*a**2*d*tan(c
 + d*x) - 2*a**2*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**2, True))
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i}{(i\,a\tan(dx+c)+a)ad}$$

input

```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output

```
I/((I*a*tan(d*x + c) + a)*a*d)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

input

```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output

```
-2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

3.118.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{1\mathrm{i}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
1i/(a^2*d*(tan(c + d*x)*1i + 1))
```

3.119 $\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx$

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3.119.8 Giac [A] (verification not implemented) 991
3.119.9 Mupad [B] (verification not implemented) 991

3.119.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx=\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}$$

output

```
1/4*x/a^2+1/4*I/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/(a^2+I*a^2*tan(d*x+c))
```

3.119.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx=\frac{-2i+\tan(c+dx)+\arctan(\tan(c+dx))(-i+\tan(c+dx))^2}{4a^2d(-i+\tan(c+dx))^2}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^(-2),x]
```

output

```
(-2*I + Tan[c + d*x] + ArcTan[Tan[c + d*x]]*(-I + Tan[c + d*x])^2)/(4*a^2*
d*(-I + Tan[c + d*x])^2)
```

3.119.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(-2),x]
```

output
```
(I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c +
d*x])))/(2*a)
```

3.119.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.119.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}$	44
derivativedivides	$\frac{\arctan(\tan(dx+c))}{4d\,a^2}-\frac{i}{4d\,a^2(\tan(dx+c)-i)^2}+\frac{1}{4a^2d(\tan(dx+c)-i)}$	56
default	$\frac{\arctan(\tan(dx+c))}{4d\,a^2}-\frac{i}{4d\,a^2(\tan(dx+c)-i)^2}+\frac{1}{4a^2d(\tan(dx+c)-i)}$	56
norman	$\frac{\frac{x}{4a}+\frac{\tan^3(dx+c)}{4ad}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{x\left(\tan^4(dx+c)\right)}{4a}+\frac{i}{2ad}+\frac{3\tan(dx+c)}{4ad}}{a\left(1+\tan^2(dx+c)\right)^2}$	91

input
```
int(1/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a^2+1/4*I/a^2/d*exp(-2*I*(d*x+c))+1/16*I/a^2/d*exp(-4*I*(d*x+c))
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx=\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
 - 4*I*c)/(a^2*d)
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.92

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(\frac{(e^{4ic}+2e^{2ic}+1)e^{-4ic}}{4a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*e
xp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x
*((exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True
)) + x/(4*a**2)
```

3.119.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = -\frac{-\frac{2i\log(\tan(dx+c)+i)}{a^2} + \frac{2i\log(\tan(dx+c)-i)}{a^2} + \frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/16*(-2*I*log(tan(d*x + c) + I)/a^2 + 2*I*log(tan(d*x + c) - I)/a^2 + (-
3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/d
```

3.119.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4\,a^2} - \frac{\frac{\tan(c+d\,x)}{4} - \frac{1}{2}\mathrm{i}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
x/(4*a^2) - (tan(c + d*x)/4 - 1i/2)/(a^2*d*(tan(c + d*x)*1i + 1)^2)
```

3.120 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.120.8 Giac [A] (verification not implemented) 996
3.120.9 Mupad [B] (verification not implemented) 996

3.120.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2} - \frac{i}{16d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{3i}{16d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
1/4*x/a^2+1/12*I*a/d/(a+I*a*tan(d*x+c))^3+1/8*I/d/(a+I*a*tan(d*x+c))^2-1/1
6*I/d/(a^2-I*a^2*tan(d*x+c))+3/16*I/d/(a^2+I*a^2*tan(d*x+c))
```

3.120.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4i+\tan(c+dx)+6i\tan^2(c+dx)-3\tan^3(c+dx)-3\arctan(\tan(c+dx))(-i+\tan(c+dx))^3(i+\mathrm{ta}}{12a^2d(-i+\tan(c+dx))^3(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/12*(4*I + Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 - 3*Tan[c + d*x]^3 - 3*Ar
cTan[Tan[c + d*x]]*(-I + Tan[c + d*x])^3*(I + Tan[c + d*x]))/(a^2*d*(-I +
Tan[c + d*x])^3*(I + Tan[c + d*x]))
```

3.120.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \left(\frac{1}{16a^4(a-ia\tan(c+dx))^2}+\frac{3}{16a^4(i\tan(c+dx)a+a)^2}+\frac{1}{4a^3(i\tan(c+dx)a+a)^3}+\frac{1}{4a^2(i\tan(c+dx)a+a)^4}+\frac{1}{4a^4(\tan^2(c+dx)a^2+a^2)}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{i\arctan(\tan(c+dx))}{4a^5}+\frac{1}{16a^4(a-ia\tan(c+dx))}-\frac{3}{16a^4(a+ia\tan(c+dx))}-\frac{1}{8a^3(a+ia\tan(c+dx))^2}-\frac{1}{12a^2(a+ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^3*(((I/4)*ArcTan[Tan[c + d*x]])/a^5 + 1/(16*a^4*(a - I*a*Tan[c + d
*x])) - 1/(12*a^2*(a + I*a*Tan[c + d*x])^3) - 1/(8*a^3*(a + I*a*Tan[c + d*
x])^2) - 3/(16*a^4*(a + I*a*Tan[c + d*x]))))/d
```

3.120.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.120.4 Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{96a^2d}+\frac{5i\cos(2dx+2c)}{32a^2d}+\frac{7\sin(2dx+2c)}{32a^2d}$	79
derivativedivides	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}}{d\,a^2}$	88
default	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}}{d\,a^2}$	88

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a^2+1/16*I/a^2/d*exp(-4*I*(d*x+c))+1/96*I/a^2/d*exp(-6*I*(d*x+c))+5/
32*I/a^2/d*cos(2*d*x+2*c)+7/32/a^2/d*sin(2*d*x+2*c)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{\left(24\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(8i\,dx+8i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d
*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.66

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\begin{cases}\frac{\left(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx}\right)e^{-12ic}}{786432a^8d^4} & \text{for } a^8d^4e^{12ic}\neq 0\\ x\left(\frac{\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-6ic}}{16a^2}-\frac{1}{4a^2}\right) & \text{otherwise}\end{cases}$$
$$+\frac{x}{4a^2}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d*
*3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x)
+ 8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**
4), Ne(a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*exp(
4*I*c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) + x
/(4*a**2)
```

3.120.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^2}+\frac{6i\log(\tan(dx+c)-i)}{a^2}+\frac{3(2i\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)}+\frac{-11i\tan(dx+c)^3-42\tan(dx+c)^2+57i\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*
(2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 -
42*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

3.120.9 Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4\,a^2}-\frac{\frac{\tan(c+d\,x)^3\,1\mathrm{i}}{4}+\frac{\tan(c+d\,x)^2}{2}-\frac{\tan(c+d\,x)\,1\mathrm{i}}{12}+\frac{1}{3}}{a^2\,d\,(1+\tan(c+d\,x)\,1\mathrm{i})^3\,(\tan(c+d\,x)+1\mathrm{i})}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
x/(4*a^2) - (tan(c + d*x)^2/2 - (tan(c + d*x)*1i)/12 + (tan(c + d*x)^3*1i)
/4 + 1/3)/(a^2*d*(tan(c + d*x)*1i + 1)^3*(tan(c + d*x) + 1i))
```

3.121 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.121.1 Optimal result

Integrand size = 24, antiderivative size = 165

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{15x}{64a^2}-\frac{i}{64d(a-ia\tan(c+dx))^2}+\frac{ia^2}{32d(a+ia\tan(c+dx))^4}\\ &\quad+\frac{ia}{16d(a+ia\tan(c+dx))^3}+\frac{3i}{32d(a+ia\tan(c+dx))^2}\\ &\quad-\frac{5i}{64d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{5i}{32d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

output
```
15/64*x/a^2-1/64*I/d/(a-I*a*tan(d*x+c))^2+1/32*I*a^2/d/(a+I*a*tan(d*x+c))^
4+1/16*I*a/d/(a+I*a*tan(d*x+c))^3+3/32*I/d/(a+I*a*tan(d*x+c))^2-5/64*I/d/(
a^2-I*a^2*tan(d*x+c))+5/32*I/d/(a^2+I*a^2*tan(d*x+c))
```

3.121.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.86

$$\begin{aligned}&\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx\\ &= \frac{i\sec^6(c+dx)(-80-65\cos(2(c+dx))+16\cos(4(c+dx))+\cos(6(c+dx))+120i\arctan(\tan(c+dx))(}{512a^2d(-i+\tan(c+dx))^4(}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/512)*Sec[c + d*x]^6*(-80 - 65*Cos[2*(c + d*x)] + 16*Cos[4*(c + d*x)] +
 Cos[6*(c + d*x)] + (120*I)*ArcTan[Tan[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin
[2*(c + d*x)]) - (5*I)*Sin[2*(c + d*x)] + (32*I)*Sin[4*(c + d*x)] + (3*I)*
Sin[6*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^4*(I + Tan[c + d*x])^2)
```

3.121.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^5\int \left(\frac{5}{64a^6(a-ia\tan(c+dx))^2}+\frac{5}{32a^6(i\tan(c+dx)a+a)^2}+\frac{1}{32a^5(a-ia\tan(c+dx))^3}+\frac{3}{16a^5(i\tan(c+dx)a+a)^3}+\frac{3}{16a^4(i\tan(c+dx)a+a}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{15i\arctan(\tan(c+dx))}{64a^7}+\frac{5}{64a^6(a-ia\tan(c+dx))}-\frac{5}{32a^6(a+ia\tan(c+dx))}+\frac{1}{64a^5(a-ia\tan(c+dx))^2}-\frac{3}{32a^5(a+ia\tan(c+dx))^2}-\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*a^5*((((15*I)/64)*ArcTan[Tan[c + d*x]])/a^7 + 1/(64*a^5*(a - I*a*Tan
[c + d*x])^2) + 5/(64*a^6*(a - I*a*Tan[c + d*x])) - 1/(32*a^3*(a + I*a*Tan
[c + d*x])^4) - 1/(16*a^4*(a + I*a*Tan[c + d*x])^3) - 3/(32*a^5*(a + I*a*T
an[c + d*x])^2) - 5/(32*a^6*(a + I*a*Tan[c + d*x]))))/d
```

3.121.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.121.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

method	result
risch	$\frac{15x}{64a^2}+\frac{i\mathrm{e}^{-6i(dx+c)}}{64a^2d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{512a^2d}+\frac{7i\cos(4dx+4c)}{128a^2d}+\frac{\sin(4dx+4c)}{16a^2d}+\frac{7i\cos(2dx+2c)}{64a^2d}+\frac{13\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{-\frac{15i\ln(\tan(dx+c)-i)}{128}+\frac{i}{32(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{5}{32(\tan(dx+c)-i)}+\frac{i}{64(\tan(dx+c)+i)^2}+15i}{d\,a^2}$
default	$\frac{-\frac{15i\ln(\tan(dx+c)-i)}{128}+\frac{i}{32(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{5}{32(\tan(dx+c)-i)}+\frac{i}{64(\tan(dx+c)+i)^2}+15i}{d\,a^2}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
15/64*x/a^2+1/64*I/a^2/d*exp(-6*I*(d*x+c))+1/512*I/a^2/d*exp(-8*I*(d*x+c))
+7/128*I/a^2/d*cos(4*d*x+4*c)+1/16/a^2/d*sin(4*d*x+4*c)+7/64*I/a^2/d*cos(2
*d*x+2*c)+13/64/a^2/d*sin(2*d*x+2*c)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(120\,dxe^{(8i\,dx+8i\,c)} - 2i\,e^{(12i\,dx+12i\,c)} - 24i\,e^{(10i\,dx+10i\,c)} + 80i\,e^{(6i\,dx+6i\,c)} + 30i\,e^{(4i\,dx+4i\,c)} + 8i\,e^{(2i\,dx+2i\,c)} + i\right)}{512\,a^2d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/512*(120*d*x*e^(8*I*d*x + 8*I*c) - 2*I*e^(12*I*d*x + 12*I*c) - 24*I*e^(1
0*I*d*x + 10*I*c) + 80*I*e^(6*I*d*x + 6*I*c) + 30*I*e^(4*I*d*x + 4*I*c) +
8*I*e^(2*I*d*x + 2*I*c) + I)*e^(-8*I*d*x - 8*I*c)/(a^2*d)
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.56

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{(-17179869184ia^{10}d^5e^{24ic}e^{4idx}-206158430208ia^{10}d^5e^{22ic}e^{2idx}+687194767360ia^{10}d^5e^{18ic}e^{-2idx}+257698037760ia^{10}d^5e^{16ic}e^{-4idx}+68719}{4398046511104a^{12}d^6} \\ x\left(\frac{(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-8ic}}{64a^2}-\frac{15}{64a^2}\right) \end{cases}$$
$$+\frac{15x}{64a^2}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-17179869184*I*a**10*d**5*exp(24*I*c)*exp(4*I*d*x) - 206158430
208*I*a**10*d**5*exp(22*I*c)*exp(2*I*d*x) + 687194767360*I*a**10*d**5*exp(
18*I*c)*exp(-2*I*d*x) + 257698037760*I*a**10*d**5*exp(16*I*c)*exp(-4*I*d*x
) + 68719476736*I*a**10*d**5*exp(14*I*c)*exp(-6*I*d*x) + 8589934592*I*a**1
0*d**5*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(4398046511104*a**12*d**6),
 Ne(a**12*d**6*exp(20*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c) + 15*exp
(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-8*I*c)/(6
4*a**2) - 15/(64*a**2)), True)) + 15*x/(64*a**2)
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{-\frac{60i\log(\tan(dx+c)+i)}{a^2}+\frac{60i\log(\tan(dx+c)-i)}{a^2}+\frac{2\left(45i\tan(dx+c)^2-110\tan(dx+c)-69i\right)}{a^2(\tan(dx+c)+i)^2}+\frac{-125i\tan(dx+c)^4-580\tan(dx+c)^3+}{a^2(\tan(}}{512\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/512*(-60*I*log(tan(d*x + c) + I)/a^2 + 60*I*log(tan(d*x + c) - I)/a^2 +
 2*(45*I*tan(d*x + c)^2 - 110*tan(d*x + c) - 69*I)/(a^2*(tan(d*x + c) + I)
^2) + (-125*I*tan(d*x + c)^4 - 580*tan(d*x + c)^3 + 1038*I*tan(d*x + c)^2
+ 868*tan(d*x + c) - 301*I)/(a^2*(tan(d*x + c) - I)^4))/d
```

3.121.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{15\,x}{64\,a^2}$$
$$+\frac{\frac{1}{4\,a^2}-\frac{\tan(c+d\,x)\,17\mathrm{i}}{64\,a^2}+\frac{25\tan(c+d\,x)^2}{32\,a^2}+\frac{\tan(c+d\,x)^3\,5\mathrm{i}}{32\,a^2}+\frac{15\tan(c+d\,x)^4}{32\,a^2}+\frac{\tan(c+d\,x)^5\,15\mathrm{i}}{64\,a^2}}{d\,\left(\tan\left(c+d\,x\right)^6\,1\mathrm{i}+2\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,1\mathrm{i}+4\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c\right.\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(15*x)/(64*a^2) + (1/(4*a^2) - (tan(c + d*x)*17i)/(64*a^2) + (25*tan(c + d
*x)^2)/(32*a^2) + (tan(c + d*x)^3*5i)/(32*a^2) + (15*tan(c + d*x)^4)/(32*a
^2) + (tan(c + d*x)^5*15i)/(64*a^2))/(d*(2*tan(c + d*x) - tan(c + d*x)^2*1
i + 4*tan(c + d*x)^3 + tan(c + d*x)^4*1i + 2*tan(c + d*x)^5 + tan(c + d*x)
^6*1i - 1i))
```

3.122 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.122.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{7\text{arctanh}(\sin(c+dx))}{16a^2d}+\frac{7\sec(c+dx)\tan(c+dx)}{16a^2d}\\ &\quad+\frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d}\\ &\quad+\frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d}-\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

output
```
7/16*arctanh(sin(d*x+c))/a^2/d+7/16*sec(d*x+c)*tan(d*x+c)/a^2/d+7/24*sec(d
*x+c)^3*tan(d*x+c)/a^2/d+7/30*sec(d*x+c)^5*tan(d*x+c)/a^2/d-2/5*I*sec(d*x+
c)^7/d/(a^2+I*a^2*tan(d*x+c))
```

3.122.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 294 vs. 2(124) = 248.

Time = 2.42 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.37

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\sec^6(c+dx)\,\big(3072i\cos(c+dx)+5\big(210\log\big(\cos\big(\tfrac{1}{2}(c+dx)\big)-\sin\big(\tfrac{1}{2}(c+dx)\big)\big)+21\cos(6(c+dx))\log$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/7680*(Sec[c + d*x]^6*((3072*I)*Cos[c + d*x] + 5*(210*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 21*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 315*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 126*Cos[4*(c + d*x)]*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]) - 210*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 21*Cos[6*(c + d*x
)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*Sin[c + d*x] - 238*Sin[3*
(c + d*x)] - 42*Sin[5*(c + d*x)])))/(a^2*d)
```

3.122.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\int \sec^7(c+dx)dx}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \csc\left(c+dx+\frac{\pi}{2}\right)^7dx}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5}{6}\int \sec^5(c+dx)dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \ 4255$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \ 4255$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \ 4257$$

$$\frac{7\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{\tan(c+dx)\sec^5(c+dx)}{6d}\right)}{5a^2}-\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/5)*Sec[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (7*((Sec[c +
d*x]^5*Tan[c + d*x])/(6*d) + (5*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*
(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/6))
/(5*a^2)
```

3.122. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.122.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.122.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{11 i (dx+c)}+595\,\mathrm{e}^{9 i (dx+c)}+1386\,\mathrm{e}^{7 i (dx+c)}+1686\,\mathrm{e}^{5 i (dx+c)}-595\,\mathrm{e}^{3 i (dx+c)}-105\,\mathrm{e}^{i (dx+c)}\right)}{120 d\, a^{2} \left(\mathrm{e}^{2 i (dx+c)}+1\right)^{6}}-\frac{7 \ln\left(\mathrm{e}^{i (dx+c)}-i\right)}{16 a^{2} d}$
derivativedivides	$\frac{2\left(-\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{9}{32}+\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{1}{12}+\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{1}{4}+\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}-$
default	$\frac{2\left(-\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{2\left(\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{9}{32}+\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{1}{12}+\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{1}{4}+\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}-$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/120*I/d/a^2/(exp(2*I*(d*x+c))+1)^6*(105*exp(11*I*(d*x+c))+595*exp(9*I*(
d*x+c))+1386*exp(7*I*(d*x+c))+1686*exp(5*I*(d*x+c))-595*exp(3*I*(d*x+c))-1
05*exp(I*(d*x+c)))-7/16/a^2/d*ln(exp(I*(d*x+c))-I)+7/16/a^2/d*ln(exp(I*(d*
x+c))+I)
```

3.122.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(110) = 220$.

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.63

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{105\left(e^{(12i\,dx+12i\,c)}+6\,e^{(10i\,dx+10i\,c)}+15\,e^{(8i\,dx+8i\,c)}+20\,e^{(6i\,dx+6i\,c)}+15\,e^{(4i\,dx+4i\,c)}+6\,e^{(2i\,dx+2i\,c)}+1\right)\log (e}{}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/240*(105*(e^(12*I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*
x + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*
x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(12*I*d*x + 12*I*c) + 6*
e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) +
15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) -
I) - 210*I*e^(11*I*d*x + 11*I*c) - 1190*I*e^(9*I*d*x + 9*I*c) - 2772*I*e^(
7*I*d*x + 7*I*c) - 3372*I*e^(5*I*d*x + 5*I*c) + 1190*I*e^(3*I*d*x + 3*I*c)
 + 210*I*e^(I*d*x + I*c))/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a^2*d*e^(10*I*d
*x + 10*I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*I*d*x + 6*I*c)
 + 15*a^2*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.122.6 Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^9(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**9/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

3.122.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(110) = 220.

Time = 0.26 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.40

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(\frac{135\sin(dx+c)}{\cos(dx+c)+1}+\frac{96i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{445\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{960i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{330\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{960i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480i\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\right.}{a^2-\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+}}{240\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/240*(2*(135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*I*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 960*I*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960
*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 330*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 - 480*I*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 + 480*I*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 135*sin
(d*x + c)^11/(cos(d*x + c) + 1)^11 - 96*I)/(a^2 - 6*a^2*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^2*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 - 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(
cos(d*x + c) + 1)^12) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 -
 105*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2} - \frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2} + \frac{2\left(135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-445\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-480i\,\mathrm{ta}\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 105*log(tan(1/2*d*x + 1/2*c
) - 1)/a^2 + 2*(135*tan(1/2*d*x + 1/2*c)^11 + 480*I*tan(1/2*d*x + 1/2*c)^1
0 - 445*tan(1/2*d*x + 1/2*c)^9 - 480*I*tan(1/2*d*x + 1/2*c)^8 - 330*tan(1/
2*d*x + 1/2*c)^7 + 960*I*tan(1/2*d*x + 1/2*c)^6 - 330*tan(1/2*d*x + 1/2*c)
^5 - 960*I*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 + 96*I*tan(
1/2*d*x + 1/2*c)^2 + 135*tan(1/2*d*x + 1/2*c) - 96*I)/((tan(1/2*d*x + 1/2*
c)^2 - 1)^6*a^2))/d
```

3.122.9 Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.54

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}$$

$$-\frac{-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}4\mathrm{i} + \frac{89\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}4\mathrm{i} + \frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}8\mathrm{i}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 -\right.}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(8*a^2*d) - ((89*tan(c/2 + (d*x)/2)^3)/24 -
(tan(c/2 + (d*x)/2)^2*4i)/5 - (9*tan(c/2 + (d*x)/2))/8 + tan(c/2 + (d*x)/2
)^4*8i + (11*tan(c/2 + (d*x)/2)^5)/4 - tan(c/2 + (d*x)/2)^6*8i + (11*tan(c
/2 + (d*x)/2)^7)/4 + tan(c/2 + (d*x)/2)^8*4i + (89*tan(c/2 + (d*x)/2)^9)/2
4 - tan(c/2 + (d*x)/2)^10*4i - (9*tan(c/2 + (d*x)/2)^11)/8 + 4i/5)/(a^2*d*
(tan(c/2 + (d*x)/2)^2 - 1)^6)
```

3.123 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.123.1 Optimal result 1010
3.123.2 Mathematica [B] (verified) 1010
3.123.3 Rubi [A] (verified) 1011
3.123.4 Maple [A] (verified) 1013
3.123.5 Fricas [B] (verification not implemented) 1013
3.123.6 Sympy [F] 1014
3.123.7 Maxima [B] (verification not implemented) 1014
3.123.8 Giac [A] (verification not implemented) 1015
3.123.9 Mupad [B] (verification not implemented) 1015

3.123.1 Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{8a^2d} + \frac{5\sec(c+dx)\tan(c+dx)}{8a^2d} + \frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d} - \frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}$$

output
```
5/8*arctanh(sin(d*x+c))/a^2/d+5/8*sec(d*x+c)*tan(d*x+c)/a^2/d+5/12*sec(d*x
+c)^3*tan(d*x+c)/a^2/d-2/3*I*sec(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

3.123.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(100) = 200.

Time = 1.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sec^4(c+dx)\big(128i\cos(c+dx)+45\log\big(\cos\big(\tfrac{1}{2}(c+dx)\big)-\sin\big(\tfrac{1}{2}(c+dx)\big)\big)+60\cos(2(c+dx))\big(\log\big(\mathrm{c}}{}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/192*(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x]
 - 30*Sin[3*(c + d*x)]))/(a^2*d)
```

3.123.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5\int \sec^5(c+dx)dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{5\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((-2*I)/3)*Sec[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (5*((Sec[c +
d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*
x]*Tan[c + d*x])/(2*d)))/4))/(3*a^2)
```

3.123.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

3.123. $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.123.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{7i(dx+c)}+55\,\mathrm{e}^{5i(dx+c)}+73\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{12d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}+\frac{5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^{2}d}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^{2}d}$
derivativedivides	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{a^{2}d}$
default	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{a^{2}d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d/a^2/(exp(2*I*(d*x+c))+1)^4*(15*exp(7*I*(d*x+c))+55*exp(5*I*(d*x+
c))+73*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+5/8/a^2/d*ln(exp(I*(d*x+c))+I)-
5/8/a^2/d*ln(exp(I*(d*x+c))-I)
```

3.123.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(88) = 176.

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.30

$$\int \frac{\sec^{7}(c+dx)}{(a+ia\tan(c+dx))^{2}}\,dx = \frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,c}\right.}{24\left(a^{2}de^{(8i\,dx+8i\,c)}+4\,a^{2}de^{(6i\,d}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I
*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x
 + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x +
 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e
^(5*I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^
2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x
 + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.123.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^7(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**7/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

3.123.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(88) = 176.

Time = 0.25 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.95

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^2}}{24d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
16*I)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.51

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2} - \frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2} + \frac{2\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 15*log(tan(1/2*d*x + 1/2*c) -
 1)/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^6 - 33*t
an(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2
*c)^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

3.123.9 Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.36

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d}$$

$$+ \frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 4\mathrm{i}}{3}+\frac{3\tan}{}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^4}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(5*atanh(tan(c/2 + (d*x)/2)))/(4*a^2*d) + ((3*tan(c/2 + (d*x)/2))/4 + (tan
(c/2 + (d*x)/2)^2*4i)/3 - (11*tan(c/2 + (d*x)/2)^3)/4 - tan(c/2 + (d*x)/2)
^4*4i - (11*tan(c/2 + (d*x)/2)^5)/4 + tan(c/2 + (d*x)/2)^6*4i + (3*tan(c/2
 + (d*x)/2)^7)/4 - 4i/3)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

3.123. $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.124 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.124.1 Optimal result 1017
3.124.2 Mathematica [A] (verified) 1017
3.124.3 Rubi [A] (verified) 1018
3.124.4 Maple [A] (verified) 1020
3.124.5 Fricas [B] (verification not implemented) 1020
3.124.6 Sympy [F] 1021
3.124.7 Maxima [B] (verification not implemented) 1021
3.124.8 Giac [A] (verification not implemented) 1022
3.124.9 Mupad [B] (verification not implemented) 1022

3.124.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx=\frac{3\text{arctanh}(\sin(c+dx))}{2a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
3/2*arctanh(sin(d*x+c))/a^2/d+3/2*sec(d*x+c)*tan(d*x+c)/a^2/d-2*I*sec(d*x+
c)^3/d/(a^2+I*a^2*tan(d*x+c))
```

3.124.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.97

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx=$$

$$-\frac{\sec^2(c+dx)\left(8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/4*(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + 2*Sin[c + d*x]))/(a^2*d)
```

3.124.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3981, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{3\int \sec^3(c+dx)dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-2*I)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (3*(ArcTanh[Sin[c
 + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2
```

3.124.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.124.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^{2}d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^{2}d}$
derivativedivides	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^{2}d}$
default	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^{2}d}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-I/d/a^2/(exp(2*I*(d*x+c))+1)^2*(3*exp(3*I*(d*x+c))+5*exp(I*(d*x+c)))+3/2/
a^2/d*ln(exp(I*(d*x+c))+I)-3/2/a^2/d*ln(exp(I*(d*x+c))-I)
```

3.124.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(66) = 132.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*
c) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
 + I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4
*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.124.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^5(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**5/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

3.124.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(66) = 132$.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.26

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) -
3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^2)/d
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 3*log(tan(1/2*d*x + 1/2*c) - 1)
/a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*
x + 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.124.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}}{a^2}+\frac{4\mathrm{i}}{a^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3/a^2 - (tan(c
/2 + (d*x)/2)^2*4i)/a^2 + 4i/a^2 + tan(c/2 + (d*x)/2)/a^2)/(d*(tan(c/2 + (
d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.125 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.125.1 Optimal result 1023
3.125.2 Mathematica [B] (verified) 1023
3.125.3 Rubi [A] (verified) 1024
3.125.4 Maple [A] (verified) 1025
3.125.5 Fricas [A] (verification not implemented) 1026
3.125.6 Sympy [F] 1026
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3.125.8 Giac [A] (verification not implemented) 1027
3.125.9 Mupad [B] (verification not implemented) 1027

3.125.1 Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{2i\sec(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
-arctanh(sin(d*x+c))/a^2/d+2*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))
```

3.125.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. 2(48) = 96.

Time = 0.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.83

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sec^2(c+dx)\,(\cos(\frac{1}{2}(c+dx))\,(2i+\log(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))-\log(\cos(\frac{1}{2}(c+dx))+\mathrm{si}}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)*Sin[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*
d*(-I + Tan[c + d*x])^2))
```

3.125.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3981, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$-\frac{\int \sec(c+dx)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Ta
n[c + d*x]))
```

3.125.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.125.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{2}d}$	54
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{2}d}$	54
risch	$\frac{2i\,e^{-i(dx+c)}}{a^{2}d}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^{2}d}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{a^{2}d}$	61

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^2*(1/2*ln(tan(1/2*d*x+1/2*c)-1)+2/(-I+tan(1/2*d*x+1/2*c))-1/2*ln(tan
(1/2*d*x+1/2*c)+1))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x
+ I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

3.125.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^3(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sec(c + d*x)**3/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

3.125.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(44) = 88.

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{-2i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\cos\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
 c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x
+ c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin
(d*x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2} - \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2} - \frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - log(tan(1/2*d*x + 1/2*c) - 1)/a^2 -
4/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

3.125.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} + \frac{4\mathrm{i}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
4i/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a^
2*d)
```

3.126 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.126.1 Optimal result 1028
3.126.2 Mathematica [A] (verified) 1028
3.126.3 Rubi [A] (verified) 1029
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3.126.5 Fricas [A] (verification not implemented) 1031
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3.126.7 Maxima [A] (verification not implemented) 1032
3.126.8 Giac [A] (verification not implemented) 1032
3.126.9 Mupad [B] (verification not implemented) 1032

3.126.1 Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx=\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
1/3*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^2+1/3*I*sec(d*x+c)/d/(a^2+I*a^2*tan(
d*x+c))
```

3.126.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx=\frac{\sec(c+dx)(-2i+\tan(c+dx))}{3a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-2*I + Tan[c + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

3.126.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec(c+dx)}{3ad(a+ia\tan(c+dx))}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(
a*d*(a + I*a*Tan[c + d*x]))
```

3.126.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.126.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{2a^2d}+\frac{i\mathrm{e}^{-3i(dx+c)}}{6a^2d}$	38
derivativedivides	$\frac{\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^2d}$	57
default	$\frac{\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^2d}$	57

input

```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*I/a^2/d*exp(-I*(d*x+c))+1/6*I/a^2/d*exp(-3*I*(d*x+c))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.46

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

3.126.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(51) = 102.

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{\tan(c+dx)\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} - \frac{2i\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} & \text{for } d\neq 0 \\ \frac{x\sec(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise((tan(c + d*x)*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 - 6*I*a**2*
d*tan(c + d*x) - 3*a**2*d) - 2*I*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 -
6*I*a**2*d*tan(c + d*x) - 3*a**2*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)
**2, True))
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\cos(3\,dx+3\,c)+3i\cos(dx+c)+\sin(3\,dx+3\,c)+3\sin(dx+c)}{6\,a^2d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x
+ c))/(a^2*d)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right)}{3\,a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(
1/2*d*x + 1/2*c) - I)^3)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{3\,a^2\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
-(2*(3*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 2i))/(3*a^2*d*(tan(c
/2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.126. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.127 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.127.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx=\frac{3\sin(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
3/5*sin(d*x+c)/a^2/d-1/5*sin(d*x+c)^3/a^2/d+2/5*I*cos(d*x+c)^3/d/(a^2+I*a^
2*tan(d*x+c))
```

3.127.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)(-12i+4i\cos(2(c+dx))-3\sec(c+dx)\sin(3(c+dx))+5\tan(c+dx))}{20a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-12*I + (4*I)*Cos[2*(c + d*x)] - 3*Sec[c + d*x]*Sin[3*(c +
d*x)] + 5*Tan[c + d*x]))/(20*a^2*d*(-I + Tan[c + d*x])^2)
```

3.127.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{3\int \cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{3\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-3*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x
]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.127.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
risch	$\frac{i \mathrm{e}^{-3 i (d x+c)}}{8 a^{2} d}+\frac{i \mathrm{e}^{-5 i (d x+c)}}{40 a^{2} d}+\frac{i \cos(d x+c)}{4 a^{2} d}+\frac{\sin(d x+c)}{2 a^{2} d}$
derivativedivides	$\frac{-\frac{2 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{5 i}{2\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{4}{5\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{3}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{7}{4\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}+\frac{2}{8 \tan\left(\frac{d x}{2}\right.}}{a^{2} d}$
default	$\frac{-\frac{2 i}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{5 i}{2\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{4}{5\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{3}{\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{7}{4\left(-i+\tan\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}+\frac{2}{8 \tan\left(\frac{d x}{2}\right.}}{a^{2} d}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^2/d*exp(-3*I*(d*x+c))+1/40*I/a^2/d*exp(-5*I*(d*x+c))+1/4*I/a^2/d*c
os(d*x+c)+1/2*sin(d*x+c)/a^2/d
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

3.127.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(60) = 120.

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.30

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{\left(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx}\right)e^{-9ic}}{20480a^8d^4} & \text{for } a^8d^4e^{9ic}\neq 0 \\ \frac{x\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*ex
p(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a
**6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(a**8*
d**4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*ex
p(-5*I*c)/(8*a**2), True))
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}}{20\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90
*I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x
+ 1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.127.9 Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 10\mathrm{i}+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\mathrm{i}\right)}{5\,a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)^5\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
-(2*(3*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4
*10i - 5*tan(c/2 + (d*x)/2)^5 - 2i))/(5*a^2*d*(tan(c/2 + (d*x)/2) - 1i)^5*
(tan(c/2 + (d*x)/2) + 1i))
```

3.127. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.128 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.128.9 Mupad [B] (verification not implemented) 1044

3.128.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\sin(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{\sin^5(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}$$

output
```
5/7*sin(d*x+c)/a^2/d-10/21*sin(d*x+c)^3/a^2/d+1/7*sin(d*x+c)^5/a^2/d+2/7*I
*cos(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

3.128.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec^2(c+dx)(-140\cos(c+dx)+42\cos(3(c+dx))+2\cos(5(c+dx))-70i\sin(c+dx)+63i\sin(3(c+}{336a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/336)*Sec[c + d*x]^2*(-140*Cos[c + d*x] + 42*Cos[3*(c + d*x)] + 2*Cos[5
*(c + d*x)] - (70*I)*Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*
(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

3.128.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5\int \cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{5\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-5*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) +
 (((2*I)/7)*Cos[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.128.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

method	result
risch	$\frac{i\mathrm{e}^{-5i(dx+c)}}{32a^2d}+\frac{i\mathrm{e}^{-7i(dx+c)}}{224a^2d}+\frac{5i\cos(dx+c)}{32a^2d}+\frac{15\sin(dx+c)}{32a^2d}+\frac{3i\cos(3dx+3c)}{32a^2d}+\frac{11\sin(3dx+3c)}{96a^2d}$
derivativedivides	$\frac{-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{23i}{8\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{a^2d}$
default	$\frac{-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{23i}{8\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{a^2d}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*I/a^2/d*exp(-5*I*(d*x+c))+1/224*I/a^2/d*exp(-7*I*(d*x+c))+5/32*I/a^2/
d*cos(d*x+c)+15/32*sin(d*x+c)/a^2/d+3/32*I/a^2/d*cos(3*d*x+3*c)+11/96/a^2/
d*sin(3*d*x+3*c)
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-7i\,e^{(10i\,dx+10i\,c)}-105i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6
*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

3.128.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(76) = 152.

Time = 0.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.60

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-176160768ia^{10}d^5e^{19ic}e^{3idx}-2642411520ia^{10}d^5e^{17ic}e^{idx}+5284823040ia^{10}d^5e^{15ic}e^{-idx}+1761607680ia^{10}d^5e^{13ic}e^{-3idx}+528482304ia^{10}d^5}{16911433728a^{12}d^6} \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-7ic}}{32a^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*
I*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*
exp(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 52848230
4*I*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c
)*exp(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(a**12*d**6*exp(
16*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*
c) + 5*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

3.128.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{7\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^3} + \frac{273\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1155i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2450\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2870i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{168\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(
tan(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/
2*d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*
c)^3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^
2*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.128.9 Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(-21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 42\mathrm{i}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 56\mathrm{i}+42\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\right.\right.}{21\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
((3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*24i + 76*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*28i + 42*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*56i + 28*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*42i - 21*tan(c/2
+ (d*x)/2)^9 - 6i)*2i)/(21*a^2*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d
*x)/2)*1i + 1)^7)
```

3.128. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

3.129 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

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3.129.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\sin(c+dx)}{9a^2d} - \frac{7\sin^3(c+dx)}{9a^2d} + \frac{7\sin^5(c+dx)}{15a^2d} - \frac{\sin^7(c+dx)}{9a^2d} + \frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))}$$

output
```
7/9*sin(d*x+c)/a^2/d-7/9*sin(d*x+c)^3/a^2/d+7/15*sin(d*x+c)^5/a^2/d-1/9*si
n(d*x+c)^7/a^2/d+2/9*I*cos(d*x+c)^7/d/(a^2+I*a^2*tan(d*x+c))
```

3.129.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec^2(c+dx)(-1050\cos(c+dx)+378\cos(3(c+dx))+30\cos(5(c+dx))+2\cos(7(c+dx))-525i\sin(}{2880a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/2880)*Sec[c + d*x]^2*(-1050*Cos[c + d*x] + 378*Cos[3*(c + d*x)] + 30*C
os[5*(c + d*x)] + 2*Cos[7*(c + d*x)] - (525*I)*Sin[c + d*x] + (567*I)*Sin[
3*(c + d*x)] + (75*I)*Sin[5*(c + d*x)] + (7*I)*Sin[7*(c + d*x)]))/(a^2*d*(
-I + Tan[c + d*x])^2)
```

3.129.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\int \cos^7(c+dx)dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{7\int \left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-7*(-Sin[c + d*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^
7/7))/(9*a^2*d) + (((2*I)/9)*Cos[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x])
)
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.129.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28

method	result
risch	$\frac{i\mathrm{e}^{-7i(dx+c)}}{128a^2d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{1152a^2d}+\frac{7i\cos(dx+c)}{64a^2d}+\frac{7\sin(dx+c)}{16a^2d}+\frac{i\cos(5dx+5c)}{32a^2d}+\frac{11\sin(5dx+5c)}{320a^2d}+\frac{7i\cos(3dx+3c}{96a^2d}$
derivativedivides	$\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}-\frac{9i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{1}{20\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{13}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{29}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}$
default	$\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}-\frac{9i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{1}{20\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{13}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{29}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/128*I/a^2/d*exp(-7*I*(d*x+c))+1/1152*I/a^2/d*exp(-9*I*(d*x+c))+7/64*I/a^
2/d*cos(d*x+c)+7/16*sin(d*x+c)/a^2/d+1/32*I/a^2/d*cos(5*d*x+5*c)+11/320/a^
2/d*sin(5*d*x+5*c)+7/96*I/a^2/d*cos(3*d*x+3*c)+7/64/a^2/d*sin(3*d*x+3*c)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(-9i\,e^{(14i\,dx+14i\,c)} - 105i\,e^{(12i\,dx+12i\,c)} - 945i\,e^{(10i\,dx+10i\,c)} + 1575i\,e^{(8i\,dx+8i\,c)} + 525i\,e^{(6i\,dx+6i\,c)} + 189i\,e^{(4i\,dx}}{5760\,a^2d}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/5760*(-9*I*e^(14*I*d*x + 14*I*c) - 105*I*e^(12*I*d*x + 12*I*c) - 945*I*e
^(10*I*d*x + 10*I*c) + 1575*I*e^(8*I*d*x + 8*I*c) + 525*I*e^(6*I*d*x + 6*I
*c) + 189*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9*I*
d*x - 9*I*c)/(a^2*d)
```

3.129.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(94) = 188.

Time = 0.40 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.79

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \begin{cases} \frac{(-227994731135631360ia^{14}d^7e^{30ic}e^{5idx} - 2659938529915699200ia^{14}d^7e^{28ic}e^{3idx} - 23939446769241292800ia^{14}d^7e^{26ic}e^{idx} + 39899077948735}{} \\ \frac{x\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-9ic}}{128a^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-227994731135631360*I*a**14*d**7*exp(30*I*c)*exp(5*I*d*x) - 26
59938529915699200*I*a**14*d**7*exp(28*I*c)*exp(3*I*d*x) - 2393944676924129
2800*I*a**14*d**7*exp(26*I*c)*exp(I*d*x) + 39899077948735488000*I*a**14*d*
*7*exp(24*I*c)*exp(-I*d*x) + 13299692649578496000*I*a**14*d**7*exp(22*I*c)
*exp(-3*I*d*x) + 4787889353848258560*I*a**14*d**7*exp(20*I*c)*exp(-5*I*d*x
) + 1139973655678156800*I*a**14*d**7*exp(18*I*c)*exp(-7*I*d*x) + 126663739
519795200*I*a**14*d**7*exp(16*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(1459166279
26804070400*a**16*d**8), Ne(a**16*d**8*exp(25*I*c), 0)), (x*(exp(14*I*c) +
 7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4
*I*c) + 7*exp(2*I*c) + 1)*exp(-9*I*c)/(128*a**2), True))
```

3.129.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.129.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(93) = 186.

Time = 0.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.84

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{3\left(435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1470i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1330i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+353\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{4455\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-26460i\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2880*(3*(435*tan(1/2*d*x + 1/2*c)^4 + 1470*I*tan(1/2*d*x + 1/2*c)^3 - 20
60*tan(1/2*d*x + 1/2*c)^2 - 1330*I*tan(1/2*d*x + 1/2*c) + 353)/(a^2*(tan(1
/2*d*x + 1/2*c) + I)^5) + (4455*tan(1/2*d*x + 1/2*c)^8 - 26460*I*tan(1/2*d
*x + 1/2*c)^7 - 78120*tan(1/2*d*x + 1/2*c)^6 + 137340*I*tan(1/2*d*x + 1/2*
c)^5 + 157374*tan(1/2*d*x + 1/2*c)^4 - 118356*I*tan(1/2*d*x + 1/2*c)^3 - 5
7744*tan(1/2*d*x + 1/2*c)^2 + 16596*I*tan(1/2*d*x + 1/2*c) + 2339)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^9))/d
```

3.129.9 Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.02

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{191\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\frac{1289\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}+\frac{649\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}-\frac{41\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{32}+\frac{41\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{32}-\frac{7\,\sin(}{}\right.}{45\,a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\right.}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*525i)/32 - (cos((5*c)/2 + (
5*d*x)/2)*205i)/32 + (cos((7*c)/2 + (7*d*x)/2)*1i)/2 - (cos((9*c)/2 + (9*d
*x)/2)*1i)/2 + (cos((11*c)/2 + (11*d*x)/2)*1i)/32 - (cos((13*c)/2 + (13*d*
x)/2)*1i)/32 + (191*sin(c/2 + (d*x)/2))/16 - (1289*sin((3*c)/2 + (3*d*x)/2
))/64 + (649*sin((5*c)/2 + (5*d*x)/2))/64 - (41*sin((7*c)/2 + (7*d*x)/2))/
32 + (41*sin((9*c)/2 + (9*d*x)/2))/32 - (7*sin((11*c)/2 + (11*d*x)/2))/64
+ (7*sin((13*c)/2 + (13*d*x)/2))/64)*2i)/(45*a^2*d*(cos(c/2 + (d*x)/2) + s
in(c/2 + (d*x)/2)*1i)^9*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^5)
```

3.130 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.130.1 Optimal result 1052
3.130.2 Mathematica [A] (verified) 1052
3.130.3 Rubi [A] (verified) 1053
3.130.4 Maple [A] (verified) 1054
3.130.5 Fricas [B] (verification not implemented) 1055
3.130.6 Sympy [F] 1055
3.130.7 Maxima [A] (verification not implemented) 1056
3.130.8 Giac [A] (verification not implemented) 1056
3.130.9 Mupad [B] (verification not implemented) 1056

3.130.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d} - \frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d} + \frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d} - \frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}$$

output
```
8/7*I*(a-I*a*tan(d*x+c))^7/a^10/d-3/2*I*(a-I*a*tan(d*x+c))^8/a^11/d+2/3*I*
(a-I*a*tan(d*x+c))^9/a^12/d-1/10*I*(a-I*a*tan(d*x+c))^10/a^13/d
```

3.130.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(i+\tan(c+dx))^7\left(-44-98i\tan(c+dx)+77\tan^2(c+dx)+21i\tan^3(c+dx)\right)}{210a^3d}$$

input
```
Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I + Tan[c + d*x])^7*(-44 - (98*I)*Tan[c + d*x] + 77*Tan[c + d*x]^2 + (21
*I)*Tan[c + d*x]^3))/(210*a^3*d)
```

3.130.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sec(c+dx)^{14}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \textbf{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^6(i\tan(c+dx)a+a)^3d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \textbf{49}$$

$$-\frac{i\int \left(-(a-ia\tan(c+dx))^9+6a(a-ia\tan(c+dx))^8-12a^2(a-ia\tan(c+dx))^7+8a^3(a-ia\tan(c+dx))^6\right)}{a^{13}d}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{i\left(-\frac{8}{7}a^3(a-ia\tan(c+dx))^7+\frac{3}{2}a^2(a-ia\tan(c+dx))^8+\frac{1}{10}(a-ia\tan(c+dx))^{10}-\frac{2}{3}a(a-ia\tan(c+dx))^9\right)}{a^{13}d}$$

input

```
Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((-I)*((-8*a^3*(a - I*a*Tan[c + d*x])^7)/7 + (3*a^2*(a - I*a*Tan[c + d*x])
^8)/2 - (2*a*(a - I*a*Tan[c + d*x])^9)/3 + (a - I*a*Tan[c + d*x])^10/10))/
(a^13*d)
```

3.130.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.130.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.53

method	result
risch	$\frac{128 i \left(120 \,{\mathrm e}^{6 i \left(d x +c \right)}+45 \,{\mathrm e}^{4 i \left(d x +c \right)}+10 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{105 d \,a^{3} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{10}}$
derivativedivides	$-\frac{i \Bigg(i \tan \left(d x +c \right)-\frac{\left(\tan^{10}\left(d x +c \right)\right)}{10}-\frac{i \left(\tan^{9}\left(d x +c \right)\right)}{3}-\frac{8 i \left(\tan^{7}\left(d x +c \right)\right)}{7}+\tan^{6}\left(d x +c \right)-\frac{6 i \left(\tan^{5}\left(d x +c \right)\right)}{5}+2 \left(\tan^{4}\left(d x +c \right)\right)+}{a^{3} d}$
default	$-\frac{i \Bigg(i \tan \left(d x +c \right)-\frac{\left(\tan^{10}\left(d x +c \right)\right)}{10}-\frac{i \left(\tan^{9}\left(d x +c \right)\right)}{3}-\frac{8 i \left(\tan^{7}\left(d x +c \right)\right)}{7}+\tan^{6}\left(d x +c \right)-\frac{6 i \left(\tan^{5}\left(d x +c \right)\right)}{5}+2 \left(\tan^{4}\left(d x +c \right)\right)+}{a^{3} d}$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
128/105*I*(120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)
/d/a^3/(exp(2*I*(d*x+c))+1)^10
```

3.130.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.78

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{128\,(-120i\,e^{(6i\,dx+6i\,c)} - 4}{105\,(a^3de^{(20i\,dx+20i\,c)} + 10\,a^3de^{(18i\,dx+18i\,c)} + 45\,a^3de^{(16i\,dx+16i\,c)} + 120\,a^3de^{(14i\,dx+14i\,c)} + 210\,a^3de^{(12i\,dx+12}}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-128/105*(-120*I*e^(6*I*d*x + 6*I*c) - 45*I*e^(4*I*d*x + 4*I*c) - 10*I*e^(
2*I*d*x + 2*I*c) - I)/(a^3*d*e^(20*I*d*x + 20*I*c) + 10*a^3*d*e^(18*I*d*x
+ 18*I*c) + 45*a^3*d*e^(16*I*d*x + 16*I*c) + 120*a^3*d*e^(14*I*d*x + 14*I*
c) + 210*a^3*d*e^(12*I*d*x + 12*I*c) + 252*a^3*d*e^(10*I*d*x + 10*I*c) + 2
10*a^3*d*e^(8*I*d*x + 8*I*c) + 120*a^3*d*e^(6*I*d*x + 6*I*c) + 45*a^3*d*e^
(4*I*d*x + 4*I*c) + 10*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.130.6 Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{14}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**14/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan
(c + d*x) + I), x)/a**3
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan(dx+c)^{10}+70\,\tan(dx+c)^{9}+240\,\tan(dx+c)^{7}+210i\,\tan(dx+c)^{6}+252\,\tan(dx+c)^{5}+}{210\,a^3d}$$

input

```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
-1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 2
10*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*ta
n(d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan(dx+c)^{10}+70\,\tan(dx+c)^{9}+240\,\tan(dx+c)^{7}+210i\,\tan(dx+c)^{6}+252\,\tan(dx+c)^{5}+}{210\,a^3d}$$

input

```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 2
10*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*ta
n(d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\cos(c+d\,x)^{10}\,84\mathrm{i}+128\,\sin(c+d\,x)\,\cos(c+d\,x)^{9}+64\,\sin(c+d\,x)\,\cos(c+d\,x)^{7}+48\,\sin(c+d\,x)\,\cos}{210\,a^3\,d\cos(c}$$

input
```
int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(40*cos(c + d*x)^3*sin(c + d*x) - 70*cos(c + d*x)*sin(c + d*x) + 48*cos(c
+ d*x)^5*sin(c + d*x) + 64*cos(c + d*x)^7*sin(c + d*x) + 128*cos(c + d*x)^
9*sin(c + d*x) - cos(c + d*x)^2*105i + cos(c + d*x)^10*84i + 21i)/(210*a^3
*d*cos(c + d*x)^10)
```

3.130. $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.131 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.131.1 Optimal result 1058
3.131.2 Mathematica [A] (verified) 1058
3.131.3 Rubi [A] (verified) 1059
3.131.4 Maple [A] (verified) 1060
3.131.5 Fricas [B] (verification not implemented) 1061
3.131.6 Sympy [F] 1061
3.131.7 Maxima [A] (verification not implemented) 1062
3.131.8 Giac [A] (verification not implemented) 1062
3.131.9 Mupad [B] (verification not implemented) 1062

3.131.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i(a-ia\tan(c+dx))^6}{3a^9d} - \frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d} + \frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}$$

output
```
2/3*I*(a-I*a*tan(d*x+c))^6/a^9/d-4/7*I*(a-I*a*tan(d*x+c))^7/a^10/d+1/8*I*(
a-I*a*tan(d*x+c))^8/a^11/d
```

3.131.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(i+\tan(c+dx))^6\left(-37i+54\tan(c+dx)+21i\tan^2(c+dx)\right)}{168a^3d}$$

input
```
Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I + Tan[c + d*x])^6*(-37*I + 54*Tan[c + d*x] + (21*I)*Tan[c + d*x]^2))/(
168*a^3*d)
```

3.131.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^{12}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{i\int \left((a-ia\tan(c+dx))^7-4a(a-ia\tan(c+dx))^6+4a^2(a-ia\tan(c+dx))^5\right)d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(-\frac{2}{3}a^2(a-ia\tan(c+dx))^6-\frac{1}{8}(a-ia\tan(c+dx))^8+\frac{4}{7}a(a-ia\tan(c+dx))^7\right)}{a^{11}d}$$

input
```
Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((-2*a^2*(a - I*a*Tan[c + d*x])^6)/3 + (4*a*(a - I*a*Tan[c + d*x])^7
)/7 - (a - I*a*Tan[c + d*x])^8/8))/(a^11*d)
```

3.131.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.131.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result
risch	$\frac{32i\left(28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{21d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{8}}$
derivativedivides	$\frac{i\bigg(\frac{\left(\tan^{8}(dx+c)\right)}{8}-\frac{\left(\tan^{6}(dx+c)\right)}{6}+\frac{3i\left(\tan^{7}(dx+c)\right)}{7}-\frac{5\left(\tan^{4}(dx+c)\right)}{4}+i\left(\tan^{5}(dx+c)\right)-\frac{3\left(\tan^{2}(dx+c)\right)}{2}+\frac{i\left(\tan^{3}(dx+c)\right)}{3}-i\tan}{a^{3}d}$
default	$\frac{i\bigg(\frac{\left(\tan^{8}(dx+c)\right)}{8}-\frac{\left(\tan^{6}(dx+c)\right)}{6}+\frac{3i\left(\tan^{7}(dx+c)\right)}{7}-\frac{5\left(\tan^{4}(dx+c)\right)}{4}+i\left(\tan^{5}(dx+c)\right)-\frac{3\left(\tan^{2}(dx+c)\right)}{2}+\frac{i\left(\tan^{3}(dx+c)\right)}{3}-i\tan}{a^{3}d}$

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
32/21*I*(28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/a^3/(exp(2*I*(d*x+c))
+1)^8
```

3.131.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{32\left(-28i\,e^{(4i\,dx+4i\,c)} - 8i\,e^{(2i\,dx+2i\,c)} - i\right)}{21\,(a^3de^{(16i\,dx+16i\,c)} + 8\,a^3de^{(14i\,dx+14i\,c)} + 28\,a^3de^{(12i\,dx+12i\,c)} + 56\,a^3de^{(10i\,dx+10i\,c)} + 70\,a^3de^{(8i\,dx+8i\,c)} + 5}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-32/21*(-28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^
(16*I*d*x + 16*I*c) + 8*a^3*d*e^(14*I*d*x + 14*I*c) + 28*a^3*d*e^(12*I*d*x
 + 12*I*c) + 56*a^3*d*e^(10*I*d*x + 10*I*c) + 70*a^3*d*e^(8*I*d*x + 8*I*c)
 + 56*a^3*d*e^(6*I*d*x + 6*I*c) + 28*a^3*d*e^(4*I*d*x + 4*I*c) + 8*a^3*d*e
^(2*I*d*x + 2*I*c) + a^3*d)
```

3.131.6 Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{12}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**12/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan
(c + d*x) + I), x)/a**3
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan\left(dx+c\right)^8+72\,\tan\left(dx+c\right)^7+28i\,\tan\left(dx+c\right)^6+168\,\tan\left(dx+c\right)^5+210i\,\tan\left(dx+c\right)^4+}{168\,a^3d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d
*x + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan\left(dx+c\right)^8+72\,\tan\left(dx+c\right)^7+28i\,\tan\left(dx+c\right)^6+168\,\tan\left(dx+c\right)^5+210i\,\tan\left(dx+c\right)^4+}{168\,a^3d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 1
68*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d
*x + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\cos\left(c+d\,x\right)^8\,91\mathrm{i}+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^7+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^5+48\,\sin\left(c+d\,x\right)\,\cos}{168\,a^3\,d\cos\left(c+d\,x\right)^8}$$

input
```
int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(48*cos(c + d*x)^3*sin(c + d*x) - 72*cos(c + d*x)*sin(c + d*x) + 64*cos(c
+ d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) - cos(c + d*x)^2*1
12i + cos(c + d*x)^8*91i + 21i)/(168*a^3*d*cos(c + d*x)^8)
```

3.131. $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.132 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.132.1 Optimal result 1064
3.132.2 Mathematica [A] (verified) 1064
3.132.3 Rubi [A] (verified) 1065
3.132.4 Maple [A] (verified) 1066
3.132.5 Fricas [B] (verification not implemented) 1066
3.132.6 Sympy [F] 1067
3.132.7 Maxima [A] (verification not implemented) 1067
3.132.8 Giac [A] (verification not implemented) 1068
3.132.9 Mupad [B] (verification not implemented) 1068

3.132.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

output
```
2/5*I*(a-I*a*tan(d*x+c))^5/a^8/d-1/6*I*(a-I*a*tan(d*x+c))^6/a^9/d
```

3.132.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{(7+5i\tan(c+dx))(i+\tan(c+dx))^5}{30a^3d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((7 + (5*I)*Tan[c + d*x])*(I + Tan[c + d*x])^5)/(30*a^3*d)
```

3.132.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^4-(a-ia\tan(c+dx))^5\right)d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{6}(a-ia\tan(c+dx))^6-\frac{2}{5}a(a-ia\tan(c+dx))^5\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*((-2*a*(a - I*a*Tan[c + d*x])^5)/5 + (a - I*a*Tan[c + d*x])^6/6))/(a
^9*d)
```

3.132.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.132.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{32i\left(6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{6}}$	36
derivativedivides	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^{6}(dx+c)\right)}{6}-\frac{3i\left(\tan^{5}(dx+c)\right)}{5}+\frac{\left(\tan^{4}(dx+c)\right)}{2}-\frac{2i\left(\tan^{3}(dx+c)\right)}{3}+\frac{3\left(\tan^{2}(dx+c)\right)}{2}\right)}{a^{3}d}$	72
default	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^{6}(dx+c)\right)}{6}-\frac{3i\left(\tan^{5}(dx+c)\right)}{5}+\frac{\left(\tan^{4}(dx+c)\right)}{2}-\frac{2i\left(\tan^{3}(dx+c)\right)}{3}+\frac{3\left(\tan^{2}(dx+c)\right)}{2}\right)}{a^{3}d}$	72

input

```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output

```
32/15*I*(6*exp(2*I*(d*x+c))+1)/d/a^3/(exp(2*I*(d*x+c))+1)^6
```

3.132.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{3}}\,dx =$$

$$-\frac{32\left(-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(a^{3}de^{(12i\,dx+12i\,c)}+6\,a^{3}de^{(10i\,dx+10i\,c)}+15\,a^{3}de^{(8i\,dx+8i\,c)}+20\,a^{3}de^{(6i\,dx+6i\,c)}+15\,a^{3}de^{(4i\,dx+4i\,c)}+6\,a^{3}\right.}$$

input

```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-32/15*(-6*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^(12*I*d*x + 12*I*c) + 6*a^3
*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20*a^3*d*e^(6*I*
d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

3.132.6 Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{10}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**10/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan
(c + d*x) + I), x)/a**3
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{5i\,\tan(dx+c)^6 - 18\,\tan(dx+c)^5 - 15i\,\tan(dx+c)^4 - 20\,\tan(dx+c)^3 - 45i\,\tan(dx+c)^2 + 30\,\tan(}{30\,a^3d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/30*(5*I*tan(d*x + c)^6 - 18*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 - 20*ta
n(d*x + c)^3 - 45*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a^3*d)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-5i\tan(dx+c)^6+18\tan(dx+c)^5+15i\tan(dx+c)^4+20\tan(dx+c)^3+45i\tan(dx+c)^2-30}{30\,a^3d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/30*(-5*I*tan(d*x + c)^6 + 18*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 + 20*
tan(d*x + c)^3 + 45*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a^3*d)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sin(c+d\,x)\,\left(-30\cos(c+d\,x)^5+\cos(c+d\,x)^4\,\sin(c+d\,x)\,45\mathrm{i}+20\cos(c+d\,x)^3\sin(c+d\,x)^2+\cos\right.}{30\,a^3\,d\cos(c+d\,x)^6}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-(sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*
x)*45i - 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*
x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a^3*d*cos(c + d*x)^6)
```

3.133 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.133.1 Optimal result 1069
3.133.2 Mathematica [A] (verified) 1069
3.133.3 Rubi [A] (verified) 1070
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3.133.9 Mupad [B] (verification not implemented) 1073

3.133.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

output
```
1/4*I*(a-I*a*tan(d*x+c))^4/a^7/d
```

3.133.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\tan(c+dx)\left(4-6i\tan(c+dx)-4\tan^2(c+dx)+i\tan^3(c+dx)\right)}{4a^3d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Tan[c + d*x]*(4 - (6*I)*Tan[c + d*x] - 4*Tan[c + d*x]^2 + I*Tan[c + d*x]^
3))/(4*a^3*d)
```

3.133.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{17}$$

$$\frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/4)*(a - I*a*Tan[c + d*x])^4)/(a^7*d)
```

3.133.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.133.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{i(\tan(dx+c)+i)^4}{4a^3d}$	21
default	$\frac{i(\tan(dx+c)+i)^4}{4a^3d}$	21
risch	$\frac{4i}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$	23

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/a^3/d*(tan(d*x+c)+I)^4
```

3.133.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{4i}{a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(
4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.133.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^8(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**8/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

3.133.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-i\tan(dx+c)^4+4\tan(dx+c)^3+6i\tan(dx+c)^2-4\tan(dx+c)}{4a^3d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*
x + c))/(a^3*d)
```

3.133.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.60 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-i\tan(dx+c)^4+4\tan(dx+c)^3+6i\tan(dx+c)^2-4\tan(dx+c)}{4a^3d}$$

input

```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*
x + c))/(a^3*d)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{\sin(c+d\,x)\ \left(-4\cos(c+d\,x)^3+\cos(c+d\,x)^2\ \sin(c+d\,x)\ 6\mathrm{i}+4\ \cos(c+d\,x)\ \sin(c+d\,x)^2-\sin(c\right.}{4\,a^3\,d\cos(c+d\,x)^4}$$

input

```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3),x)
```

output

```
-(sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 + cos(c + d*x)^2*sin(c + d*x
)*6i - 4*cos(c + d*x)^3 - sin(c + d*x)^3*1i))/(4*a^3*d*cos(c + d*x)^4)
```

3.134 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.134.9 Mupad [B] (verification not implemented) 1078

3.134.1 Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{4x}{a^3}+\frac{4i\log(\cos(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}$$

output
```
4*x/a^3+4*I*ln(cos(d*x+c))/a^3/d-3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3
/d
```

3.134.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{-8i\log(i-\tan(c+dx))-6\tan(c+dx)+i\tan^2(c+dx)}{2a^3d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-8*I)*Log[I - Tan[c + d*x]] - 6*Tan[c + d*x] + I*Tan[c + d*x]^2)/(2*a^3*
d)
```

3.134.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{4a^2}{i\tan(c+dx)a+a}+i\tan(c+dx)a-3a\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(-\frac{1}{2}a^2\tan^2(c+dx)-3ia^2\tan(c+dx)+4a^2\log(a+ia\tan(c+dx)))}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*(4*a^2*Log[a + I*a*Tan[c + d*x]] - (3*I)*a^2*Tan[c + d*x] - (a^2*Tan
[c + d*x]^2)/2))/(a^5*d)
```

3.134.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.134.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\frac{3\tan(dx+c)}{a^3 d}+\frac{i(\tan^2(dx+c))}{2a^3 d}+\frac{4\arctan(\tan(dx+c))}{a^3 d}-\frac{2i\ln(1+\tan^2(dx+c))}{a^3 d}$	68
default	$-\frac{3\tan(dx+c)}{a^3 d}+\frac{i(\tan^2(dx+c))}{2a^3 d}+\frac{4\arctan(\tan(dx+c))}{a^3 d}-\frac{2i\ln(1+\tan^2(dx+c))}{a^3 d}$	68
risch	$\frac{8x}{a^3}+\frac{8c}{a^3 d}-\frac{2i(2\,\mathrm{e}^{2i(dx+c)}+3)}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{4i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3 d}$	73

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/d+4/a^3/d*arctan(tan(d*x+c))-2*
I/a^3/d*ln(1+tan(d*x+c)^2)
```

3.134.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(52) = 104$.

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.95

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+4\,dx+2\,(4\,dx-i)e^{(2i\,dx+2i\,c)}-2\left(-i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right.\right.}{a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
2*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*d*x + 2*(4*d*x - I)*e^(2*I*d*x + 2*I*c) -
 2*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x +
 2*I*c) + 1) - 3*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*
c) + a^3*d)
```

3.134.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^6(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**6/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{i\tan(dx+c)^2-6\tan(dx+c)}{a^3}-\frac{8i\log(i\tan(dx+c)+1)}{a^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((I*tan(d*x + c)^2 - 6*tan(d*x + c))/a^3 - 8*I*log(I*tan(d*x + c) + 1)
/a^3)/d
```

3.134.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(52) = 104$.

Time = 0.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.21

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(\frac{2i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3} - \frac{4i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^3} + \frac{2i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3} + \frac{-3i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7i\,\mathrm{ta}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}\right.}{d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
2*(2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 4*I*log(tan(1/2*d*x + 1/2*c) -
I)/a^3 + 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (-3*I*tan(1/2*d*x + 1/2*c
)^4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*
x + 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.134.9 Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,8\mathrm{i}+6\tan(c+d\,x)-\tan(c+d\,x)^2\,1\mathrm{i}}{2\,a^3\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-(log(tan(c + d*x) - 1i)*8i + 6*tan(c + d*x) - tan(c + d*x)^2*1i)/(2*a^3*d
)
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.135.1 Optimal result 1079
3.135.2 Mathematica [A] (verified) 1079
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3.135.8 Giac [B] (verification not implemented) 1082
3.135.9 Mupad [B] (verification not implemented) 1083

3.135.1 Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=-\frac{x}{a^3}-\frac{i\log(\cos(c+dx))}{a^3d}+\frac{2i}{d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
-x/a^3-I*ln(cos(d*x+c))/a^3/d+2*I/d/(a^3+I*a^3*tan(d*x+c))
```

3.135.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=-\frac{i\left(-\log(i-\tan(c+dx))-\frac{2a}{a+ia\tan(c+dx)}\right)}{a^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*(-Log[I - Tan[c + d*x]] - (2*a)/(a + I*a*Tan[c + d*x])))/(a^3*d)
```

3.135.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^2}+\frac{1}{-i\tan(c+dx)a-a}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{2a}{a+ia\tan(c+dx)}-\log(a+ia\tan(c+dx))\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*(-Log[a + I*a*Tan[c + d*x]] - (2*a)/(a + I*a*Tan[c + d*x])))/(a^3*d)
```

3.135.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.135.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{2}{a^3d(\tan(dx+c)-i)}+\frac{i\ln(1+\tan^2(dx+c))}{2a^3d}-\frac{\arctan(\tan(dx+c))}{a^3d}$	56
default	$\frac{2}{a^3d(\tan(dx+c)-i)}+\frac{i\ln(1+\tan^2(dx+c))}{2a^3d}-\frac{\arctan(\tan(dx+c))}{a^3d}$	56
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{a^3d}-\frac{2x}{a^3}-\frac{2c}{a^3d}-\frac{i\ln(\mathrm{e}^{2i(dx+c)}+1)}{a^3d}$	56

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/a^3/d/(tan(d*x+c)-I)+1/2*I/a^3/d*ln(1+tan(d*x+c)^2)-1/a^3/d*arctan(tan(d
*x+c))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

3.135.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^4(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**4/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{4(-i\tan(dx+c)-1)}{2i\,a^3\tan(dx+c)^2+4\,a^3\tan(dx+c)-2i\,a^3} - \frac{i\log(i\tan(dx+c)+1)}{a^3}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-(4*(-I*tan(d*x + c) - 1)/(2*I*a^3*tan(d*x + c)^2 + 4*a^3*tan(d*x + c) - 2
*I*a^3) - I*log(I*tan(d*x + c) + 1)/a^3)/d
```

3.135.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(44) = 88.

Time = 0.65 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.00

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3} - \frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^3} + \frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3} + \frac{3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3i}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/
a^3 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (3*I*tan(1/2*d*x + 1/2*c)^2 +
10*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))/d
```

3.135.9 Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a^3\,d}+\frac{2\mathrm{i}}{a^3\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(log(tan(c + d*x) - 1i)*1i)/(a^3*d) + 2i/(a^3*d*(tan(c + d*x)*1i + 1))
```

3.136 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.136.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{i}{2ad(a+ia\tan(c+dx))^2}$$

output
```
1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

3.136.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=-\frac{i}{2a^3d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-1/2*I)/(a^3*d*(-I + Tan[c + d*x])^2)
```

3.136.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$\frac{i}{2ad(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(I/2)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

3.136.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.136.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
default	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^3d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^3d}$	38

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^3d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

3.136.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(19) = 38$.

Time = 0.82 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.67

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} -\frac{i\tan(c+dx)\sec^2(c+dx)}{8a^3d\tan^3(c+dx)-24ia^3d\tan^2(c+dx)-24a^3d\tan(c+dx)+8ia^3d} - \frac{3\sec^2(c+dx)}{8a^3d\tan^3(c+dx)-24ia^3d\tan^2(c+dx)-24a^3d\tan(c+dx)+8ia^3d} & \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^3} & \end{cases}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((-I*tan(c + d*x)*sec(c + d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*
I*a**3*d*tan(c + d*x)**2 - 24*a**3*d*tan(c + d*x) + 8*I*a**3*d) - 3*sec(c
+ d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*I*a**3*d*tan(c + d*x)**2 - 24*a**
3*d*tan(c + d*x) + 8*I*a**3*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**
3, True))
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i}{2\left(i\,a\tan\left(dx+c\right)+a\right)^2ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*I/((I*a*tan(d*x + c) + a)^2*a*d)
```

3.136.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*
c))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

3.136.9 Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{1\mathrm{i}}{2\,a^3\,d\,(\tan(c+d\,x)-\mathrm{i})^2}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-1i/(2*a^3*d*(tan(c + d*x) - 1i)^2)
```

3.137 $\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx$

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3.137.1 Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
1/8*x/a^3+1/6*I/d/(a+I*a*tan(d*x+c))^3+1/8*I/a/d/(a+I*a*tan(d*x+c))^2+1/8*
I/d/(a^3+I*a^3*tan(d*x+c))
```

3.137.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{-10-9i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))^3}{24a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-3),x]
```

output
```
(-10 - (9*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]*(-I
+ Tan[c + d*x])^3)/(24*a^3*d*(-I + Tan[c + d*x])^3)
```

3.137.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(-3),x]
```

output
```
(I/6)/(d*(a + I*a*Tan[c + d*x])^3) + ((I/4)/(d*(a + I*a*Tan[c + d*x])^2) +
 (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x])))/(2*a))/(2*a)
```

3.137.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.137.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x}{8a^3}+\frac{3i\,\mathrm{e}^{-2i(dx+c)}}{16a^3d}+\frac{3i\,\mathrm{e}^{-4i(dx+c)}}{32a^3d}+\frac{i\,\mathrm{e}^{-6i(dx+c)}}{48a^3d}$
derivativedivides	$\frac{\arctan(\tan(dx+c))}{8a^3d}-\frac{i}{8d\,a^3(\tan(dx+c)-i)^2}-\frac{1}{6d\,a^3(\tan(dx+c)-i)^3}+\frac{1}{8a^3d(\tan(dx+c)-i)}$
default	$\frac{\arctan(\tan(dx+c))}{8a^3d}-\frac{i}{8d\,a^3(\tan(dx+c)-i)^2}-\frac{1}{6d\,a^3(\tan(dx+c)-i)^3}+\frac{1}{8a^3d(\tan(dx+c)-i)}$
norman	$\frac{\frac{x}{8a}+\frac{\tan^3(dx+c)}{3ad}+\frac{\tan^5(dx+c)}{8ad}+\frac{3x\left(\tan^2(dx+c)\right)}{8a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{8a}+\frac{5i}{12ad}+\frac{7\tan(dx+c)}{8ad}-\frac{i\left(\tan^2(dx+c)\right)}{4ad}}{a^2(1+\tan^2(dx+c))^3}$

input
```
int(1/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x/a^3+3/16*I/a^3/d*exp(-2*I*(d*x+c))+3/32*I/a^3/d*exp(-4*I*(d*x+c))+1/
48*I/a^3/d*exp(-6*I*(d*x+c))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(12\,dxe^{(6i\,dx+6i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+9i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d
*x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0 \\ x\left(\frac{(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x}{8a^3}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*
exp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-
12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*c)
+ 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), Tru
e)) + x/(8*a**3)
```

3.137.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^3}+\frac{6i\log(\tan(dx+c)-i)}{a^3}+\frac{-11i\tan(dx+c)^3-45\tan(dx+c)^2+69i\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/96*(-6*I*log(tan(d*x + c) + I)/a^3 + 6*I*log(tan(d*x + c) - I)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(ta
n(d*x + c) - I)^3))/d
```

3.137.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{x}{8\,a^3}-\frac{\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{8}+\frac{3\tan(c+d\,x)}{8}-\frac{5}{12}\mathrm{i}}{a^3\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^3}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
x/(8*a^3) - ((3*tan(c + d*x))/8 + (tan(c + d*x)^2*1i)/8 - 5i/12)/(a^3*d*(t
an(c + d*x)*1i + 1)^3)
```

3.138 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.138.1 Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5x}{32a^3} + \frac{ia}{16d(a+ia\tan(c+dx))^4} + \frac{i}{12d(a+ia\tan(c+dx))^3} + \frac{3i}{32ad(a+ia\tan(c+dx))^2} - \frac{i}{32d\left(a^3-ia^3\tan(c+dx)\right)} + \frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
5/32*x/a^3+1/16*I*a/d/(a+I*a*tan(d*x+c))^4+1/12*I/d/(a+I*a*tan(d*x+c))^3+3
/32*I/a/d/(a+I*a*tan(d*x+c))^2-1/32*I/d/(a^3-I*a^3*tan(d*x+c))+1/8*I/d/(a^
3+I*a^3*tan(d*x+c))
```

3.138.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{ia^3\Bigg(\frac{5i\arctan(\tan(c+dx))}{32a^6} + \frac{1}{32a^5(a-ia\tan(c+dx))} - \frac{1}{16a^2(a+ia\tan(c+dx))^4} - \frac{1}{12a^3(a+ia\tan(c+dx))^3} - \frac{3}{32a^4(a+ia\tan(c+dx))^2}}{d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(32*a^5*(a - I*a*Tan[
c + d*x])) - 1/(16*a^2*(a + I*a*Tan[c + d*x])^4) - 1/(12*a^3*(a + I*a*Tan[
c + d*x])^3) - 3/(32*a^4*(a + I*a*Tan[c + d*x])^2) - 1/(8*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

3.138.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^5}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \left(\frac{1}{32a^5(a-ia\tan(c+dx))^2}+\frac{1}{8a^5(i\tan(c+dx)a+a)^2}+\frac{3}{16a^4(i\tan(c+dx)a+a)^3}+\frac{1}{4a^3(i\tan(c+dx)a+a)^4}+\frac{1}{4a^2(i\tan(c+dx)a+a)^5}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{32a^5(a-ia\tan(c+dx))}-\frac{1}{8a^5(a+ia\tan(c+dx))}-\frac{3}{32a^4(a+ia\tan(c+dx))^2}-\frac{1}{12a^3(a+ia\tan(c+dx))^3}-\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(32*a^5*(a - I*a*Tan[
c + d*x])) - 1/(16*a^2*(a + I*a*Tan[c + d*x])^4) - 1/(12*a^3*(a + I*a*Tan[
c + d*x])^3) - 3/(32*a^4*(a + I*a*Tan[c + d*x])^2) - 1/(8*a^5*(a + I*a*Tan
[c + d*x]))))/d
```

3.138.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.138.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

method	result
risch	$\frac{5x}{32a^3}+\frac{5i\mathrm{e}^{-4i(dx+c)}}{64a^3d}+\frac{5i\mathrm{e}^{-6i(dx+c)}}{192a^3d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{256a^3d}+\frac{9i\cos(2dx+2c)}{64a^3d}+\frac{11\sin(2dx+2c)}{64a^3d}$
derivativedivides	$-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+$ $d\,a^3$
default	$-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+$ $d\,a^3$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
5/32*x/a^3+5/64*I/a^3/d*exp(-4*I*(d*x+c))+5/192*I/a^3/d*exp(-6*I*(d*x+c))+
1/256*I/a^3/d*exp(-8*I*(d*x+c))+9/64*I/a^3/d*cos(2*d*x+2*c)+11/64/a^3/d*si
n(2*d*x+2*c)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^
(6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) +
3*I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d^4}{6442450944a^{15}d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) \end{cases} + \frac{5x}{32a^3}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*
I*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c
)*exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165
824*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15
*d**5), Ne(a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) +
10*exp(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**3) -
5/(32*a**3)), True)) + 5*x/(32*a**3)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{-\frac{60i\,\log(\tan(dx+c)+i)}{a^3}+\frac{60i\,\log(\tan(dx+c)-i)}{a^3}-\frac{12\,(5\,\tan(dx+c)+7i)}{a^3(i\,\tan(dx+c)-1)}+\frac{-125i\,\tan(dx+c)^4-596\,\tan(dx+c)^3+1110i\,\tan(dx+c)^2+9}{a^3(\tan(dx+c)-i)^4}}{768\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/768*(-60*I*log(tan(d*x + c) + I)/a^3 + 60*I*log(tan(d*x + c) - I)/a^3 -
 12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*x +
c)^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) - 405
*I)/(a^3*(tan(d*x + c) - I)^4))/d
```

3.138.9 Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\,x}{32\,a^3} + \frac{\frac{1}{3\,a^3}+\frac{35\tan(c+d\,x)^2}{96\,a^3}-\frac{5\tan(c+d\,x)^4}{32\,a^3}+\frac{\tan(c+d\,x)\,5\mathrm{i}}{32\,a^3}+\frac{\tan(c+d\,x)^3\,15\mathrm{i}}{32\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4 3\mathrm{i}+2\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2 2\mathrm{i}+3\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(5*x)/(32*a^3) + ((tan(c + d*x)*5i)/(32*a^3) + 1/(3*a^3) + (35*tan(c + d*x
)^2)/(96*a^3) + (tan(c + d*x)^3*15i)/(32*a^3) - (5*tan(c + d*x)^4)/(32*a^3
))/(d*(3*tan(c + d*x) + tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x
)^4*3i - tan(c + d*x)^5 - 1i))
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.139.2 Mathematica [A] (verified) . . . 1101
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3.139.1 Optimal result

Integrand size = 24, antiderivative size = 195

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{21x}{128a^3}-\frac{i}{128ad(a-ia\tan(c+dx))^2}\\ &\quad+\frac{ia^2}{40d(a+ia\tan(c+dx))^5}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}\\ &\quad+\frac{i}{16d(a+ia\tan(c+dx))^3}+\frac{5i}{64ad(a+ia\tan(c+dx))^2}\\ &\quad-\frac{3i}{64d\,(a^3-ia^3\tan(c+dx))}+\frac{15i}{128d\,(a^3+ia^3\tan(c+dx))}\end{aligned}$$

output
```
21/128*x/a^3-1/128*I/a/d/(a-I*a*tan(d*x+c))^2+1/40*I*a^2/d/(a+I*a*tan(d*x+
c))^5+3/64*I*a/d/(a+I*a*tan(d*x+c))^4+1/16*I/d/(a+I*a*tan(d*x+c))^3+5/64*I
/a/d/(a+I*a*tan(d*x+c))^2-3/64*I/d/(a^3-I*a^3*tan(d*x+c))+15/128*I/d/(a^3+
I*a^3*tan(d*x+c))
```

3.139.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^7(c+dx)(-1050\cos(c+dx)-469\cos(3(c+dx))+105\cos(5(c+dx))+6\cos(7(c+dx))-350i\sin(}{5120a^3d}$$

input

```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(Sec[c + d*x]^7*(-1050*Cos[c + d*x] - 469*Cos[3*(c + d*x)] + 105*Cos[5*(c
+ d*x)] + 6*Cos[7*(c + d*x)] - (350*I)*Sin[c + d*x] + (840*I)*ArcTan[Tan[c
 + d*x]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (189*I)*Sin[3*(c + d*x)
] + (175*I)*Sin[5*(c + d*x)] + (14*I)*Sin[7*(c + d*x)]))/(5120*a^3*d*(-I +
 Tan[c + d*x])^5*(I + Tan[c + d*x])^2)
```

3.139.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^6}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 54$$

$$-\frac{ia^5\int \Big(\frac{3}{64a^7(a-ia\tan(c+dx))^2}+\frac{15}{128a^7(i\tan(c+dx)a+a)^2}+\frac{1}{64a^6(a-ia\tan(c+dx))^3}+\frac{5}{32a^6(i\tan(c+dx)a+a)^3}+\frac{3}{16a^5(i\tan(c+dx)a+}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\bigg(\frac{21i\arctan(\tan(c+dx))}{128a^8}+\frac{3}{64a^7(a-ia\tan(c+dx))}-\frac{15}{128a^7(a+ia\tan(c+dx))}+\frac{1}{128a^6(a-ia\tan(c+dx))^2}-\frac{5}{64a^6(a+ia\tan(c+dx))^2}-}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^5*((((21*I)/128)*ArcTan[Tan[c + d*x]])/a^8 + 1/(128*a^6*(a - I*a*T
an[c + d*x])^2) + 3/(64*a^7*(a - I*a*Tan[c + d*x])) - 1/(40*a^3*(a + I*a*T
an[c + d*x])^5) - 3/(64*a^4*(a + I*a*Tan[c + d*x])^4) - 1/(16*a^5*(a + I*a
*Tan[c + d*x])^3) - 5/(64*a^6*(a + I*a*Tan[c + d*x])^2) - 15/(128*a^7*(a +
 I*a*Tan[c + d*x]))))/d
```

3.139.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.139.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{21 i \ln(\tan(dx+c)-i)}{256}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{5i}{64(\tan(dx+c)-i)^2}+\frac{1}{40(\tan(dx+c)-i)^5}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{15}{128(\tan(dx+c)-i)}+}{d\,a^3}$
default	$\frac{-\frac{21 i \ln(\tan(dx+c)-i)}{256}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{5i}{64(\tan(dx+c)-i)^2}+\frac{1}{40(\tan(dx+c)-i)^5}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{15}{128(\tan(dx+c)-i)}+}{d\,a^3}$
risch	$\frac{21x}{128a^3}+\frac{7i\mathrm{e}^{-6i(dx+c)}}{256a^3d}+\frac{7i\mathrm{e}^{-8i(dx+c)}}{1024a^3d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{1280a^3d}+\frac{17i\cos(4dx+4c)}{256a^3d}+\frac{9\sin(4dx+4c)}{128a^3d}+\frac{7i\cos(2dx+2c)}{64a^3d}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-21/256*I*ln(tan(d*x+c)-I)+3/64*I/(tan(d*x+c)-I)^4-5/64*I/(tan(d*
x+c)-I)^2+1/40/(tan(d*x+c)-I)^5-1/16/(tan(d*x+c)-I)^3+15/128/(tan(d*x+c)-I
)+1/128*I/(tan(d*x+c)+I)^2+21/256*I*ln(tan(d*x+c)+I)+3/64/(tan(d*x+c)+I))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.50

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\left(840\,dxe^{(10i\,dx+10i\,c)} - 10i\,e^{(14i\,dx+14i\,c)} - 140i\,e^{(12i\,dx+12i\,c)} + 700i\,e^{(8i\,dx+8i\,c)} + 350i\,e^{(6i\,dx+6i\,c)} + 140i\,e^{(4i\,dx}}{5120\,a^3d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/5120*(840*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(14*I*d*x + 14*I*c) - 140*I
*e^(12*I*d*x + 12*I*c) + 700*I*e^(8*I*d*x + 8*I*c) + 350*I*e^(6*I*d*x + 6*
I*c) + 140*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 4*I)*e^(-10*
I*d*x - 10*I*c)/(a^3*d)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.50

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-11258999068426240ia^{18}d^6e^{34ic}e^{4idx}-157625986957967360ia^{18}d^6e^{32ic}e^{2idx}+788129934789836800ia^{18}d^6e^{28ic}e^{-2idx}+3940649673949184}{576460752} \\ x\left(\frac{(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1)e^{-10ic}}{128a^3}-\frac{21}{128a^3}\right) \end{cases}$$
$$+\frac{21x}{128a^3}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-11258999068426240*I*a**18*d**6*exp(34*I*c)*exp(4*I*d*x) - 157
625986957967360*I*a**18*d**6*exp(32*I*c)*exp(2*I*d*x) + 788129934789836800
*I*a**18*d**6*exp(28*I*c)*exp(-2*I*d*x) + 394064967394918400*I*a**18*d**6*
exp(26*I*c)*exp(-4*I*d*x) + 157625986957967360*I*a**18*d**6*exp(24*I*c)*ex
p(-6*I*d*x) + 39406496739491840*I*a**18*d**6*exp(22*I*c)*exp(-8*I*d*x) + 4
503599627370496*I*a**18*d**6*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(576
4607523034234880*a**21*d**7), Ne(a**21*d**7*exp(30*I*c), 0)), (x*((exp(14*
I*c) + 7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21
*exp(4*I*c) + 7*exp(2*I*c) + 1)*exp(-10*I*c)/(128*a**3) - 21/(128*a**3)),
True)) + 21*x/(128*a**3)
```

3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{-\frac{420i\log(\tan(dx+c)+i)}{a^3}+\frac{420i\log(\tan(dx+c)-i)}{a^3}+\frac{10\left(-63i\tan(dx+c)^2+150\tan(dx+c)+91i\right)}{a^3(i\tan(dx+c)-1)^2}-\frac{959i\tan(dx+c)^5+5395\tan(dx+}{}}{5120\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/5120*(-420*I*log(tan(d*x + c) + I)/a^3 + 420*I*log(tan(d*x + c) - I)/a^
3 + 10*(-63*I*tan(d*x + c)^2 + 150*tan(d*x + c) + 91*I)/(a^3*(I*tan(d*x +
c) - 1)^2) - (959*I*tan(d*x + c)^5 + 5395*tan(d*x + c)^4 - 12390*I*tan(d*x
 + c)^3 - 14710*tan(d*x + c)^2 + 9275*I*tan(d*x + c) + 2647)/(a^3*(tan(d*x
 + c) - I)^5))/d
```

3.139.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{21\,x}{128\,a^3}$$

$$+\frac{\frac{7\tan(c+dx)}{640\,a^3}+\frac{11\mathrm{i}}{40\,a^3}+\frac{\tan(c+dx)^2\,469\mathrm{i}}{640\,a^3}-\frac{21\tan(c+dx)^3}{32\,a^3}+\frac{\tan(c+dx)^4\,7\mathrm{i}}{32\,a^3}-\frac{63\tan(c+dx)^5}{128\,a^3}-\tan}{d\,\left(-\tan\left(c+d\,x\right)^7\,1\mathrm{i}-3\tan\left(c+d\,x\right)^6+\tan\left(c+d\,x\right)^5\,1\mathrm{i}-5\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,5\mathrm{i}-\tan\left(c\right.\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(21*x)/(128*a^3) + ((7*tan(c + d*x))/(640*a^3) + 11i/(40*a^3) + (tan(c + d
*x)^2*469i)/(640*a^3) - (21*tan(c + d*x)^3)/(32*a^3) + (tan(c + d*x)^4*7i)
/(32*a^3) - (63*tan(c + d*x)^5)/(128*a^3) - (tan(c + d*x)^6*21i)/(128*a^3)
)/(d*(tan(c + d*x)*3i - tan(c + d*x)^2 + tan(c + d*x)^3*5i - 5*tan(c + d*x
)^4 + tan(c + d*x)^5*1i - 3*tan(c + d*x)^6 - tan(c + d*x)^7*1i + 1))
```

3.140 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.140.1 Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

output
```
7/8*arctanh(sin(d*x+c))/a^3/d-7/15*I*sec(d*x+c)^5/a^3/d+7/8*sec(d*x+c)*tan
(d*x+c)/a^3/d+7/12*sec(d*x+c)^3*tan(d*x+c)/a^3/d-2/3*I*sec(d*x+c)^7/a/d/(a
+I*a*tan(d*x+c))^2
```

3.140.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^8(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))\big(448+1680i\text{arctanh}\big(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\big)\cos^5(c+d}{960a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^8*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(448 + (1680*I)*Ar
cTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c + d*x)]
- (150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(960*a^3*d*(-I + T
an[c + d*x])^3)
```

3.140.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\int \frac{\sec^7(c+dx)}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{\sec(c+dx)^7}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{7\left(\frac{\int \sec^5(c+dx)dx}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$\frac{7\left(\frac{\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow 4255$$

$$\frac{7\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow 3042$$

$$\frac{7\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow 4257$$

$$\frac{7\left(\frac{\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}}{a}-\frac{i\sec^5(c+dx)}{5ad}\right)}{3a^2}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

input

```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(((-2*I)/3)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^2) + (7*(((-1/5*I)
*Sec[c + d*x]^5)/(a*d) + ((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTan
h[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)/a))/(3*a^2)
```

3.140. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.140.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.140. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.140.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{i(105\,\mathrm{e}^{9i(dx+c)}+490\,\mathrm{e}^{7i(dx+c)}+896\,\mathrm{e}^{5i(dx+c)}+790\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)})}{60d\,a^3(\mathrm{e}^{2i(dx+c)}+1)^5}+\frac{7\ln(\mathrm{e}^{i(dx+c)}+i)}{8a^3d}-\frac{7\ln(\mathrm{e}^{i(dx+c)}-i}{8a^3d}$
derivativedivides	$-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}$
default	$-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I/d/a^3/(exp(2*I*(d*x+c))+1)^5*(105*exp(9*I*(d*x+c))+490*exp(7*I*(d*
x+c))+896*exp(5*I*(d*x+c))+790*exp(3*I*(d*x+c))-105*exp(I*(d*x+c)))+7/8/a^
3/d*ln(exp(I*(d*x+c))+I)-7/8/a^3/d*ln(exp(I*(d*x+c))-I)
```

3.140.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(103) = 206.

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.34

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$= \frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-10}{120\,(a^3de^(}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x
+ 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*
x + I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^
(6*I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*lo
g(e^(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*
I*c) - 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(
I*d*x + I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) +
 10*a^3*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(
2*I*d*x + 2*I*c) + a^3*d)
```

3.140.6 Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^9(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**9/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

3.140.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(103) = 206.

Time = 0.23 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.87

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136\right)}{-120i\,a^3+\frac{600i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600i\,a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120i\,a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}}{8\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390
*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600
*I*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I
*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.38

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{105\,\log(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+1)}{a^3} - \frac{105\,\log(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-1)}{a^3} + \frac{2\,(15\,\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9+360i\,\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^8-390\,\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-960i\,\tan(}{}}{120\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 105*log(tan(1/2*d*x + 1/2*c
) - 1)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/2*c)^8 -
 390*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*tan(1/2
*d*x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1/2*c)^
2 - 15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^3))
/d
```

3.140.9 Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}$$

$$+ \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,6\mathrm{i} - \frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,16\mathrm{i} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,20\mathrm{i}}{3} + \frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2} - \frac{\tan(}{}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^5}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((13*tan(c/2 + (d*x)/2)^3)/2 - (
tan(c/2 + (d*x)/2)^2*16i)/3 - tan(c/2 + (d*x)/2)/4 + (tan(c/2 + (d*x)/2)^4
*20i)/3 - tan(c/2 + (d*x)/2)^6*16i - (13*tan(c/2 + (d*x)/2)^7)/2 + tan(c/2
 + (d*x)/2)^8*6i + tan(c/2 + (d*x)/2)^9/4 + 34i/15)/(a^3*d*(tan(c/2 + (d*x
)/2)^2 - 1)^5)
```

3.140. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.141 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.141.1 Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5i\sec^3(c+dx)}{3a^3d} + \frac{5\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

output
```
5/2*arctanh(sin(d*x+c))/a^3/d-5/3*I*sec(d*x+c)^3/a^3/d+5/2*sec(d*x+c)*tan(
d*x+c)/a^3/d-2*I*sec(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^2
```

3.141.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{60\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right) - i\sec^3(c+dx)(20+24\cos(2(c+dx))-9i\sin(2(c+dx)))}{12a^3d}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x]^3*(20 + 24*Cos[
2*(c + d*x)] - (9*I)*Sin[2*(c + d*x)]))/(12*a^3*d)
```

3.141.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5\int \frac{\sec^5(c+dx)}{i\tan(c+dx)a+a}dx}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\sec(c+dx)^5}{i\tan(c+dx)a+a}dx}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{5\left(\frac{\int \sec^3(c+dx)dx}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5\left(\frac{\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{4257}$$

$$\frac{5\left(\frac{\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{a}-\frac{i\sec^3(c+dx)}{3ad}\right)}{a^2}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^2) + (5*(((-1/3*I)*Sec
[c + d*x]^3)/(a*d) + (ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c +
d*x])/(2*d))/a))/a^2
```

3.141.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

3.141. $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.141.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{5i(dx+c)}+40\,\mathrm{e}^{3i(dx+c)}+33\,\mathrm{e}^{i(dx+c)}\right)}{3d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}+\frac{5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^{3}d}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^{3}d}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{2\left(-\frac{3}{4}-\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}}{a^{3}d}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{2\left(-\frac{3}{4}-\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}}{a^{3}d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d/a^3/(exp(2*I*(d*x+c))+1)^3*(15*exp(5*I*(d*x+c))+40*exp(3*I*(d*x+c
))+33*exp(I*(d*x+c)))+5/2/a^3/d*ln(exp(I*(d*x+c))+I)-5/2/a^3/d*ln(exp(I*(d
*x+c))-I)
```

3.141.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(81) = 162.

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.96

$$\int \frac{\sec^{7}(c+dx)}{(a+ia\tan(c+dx))^{3}}\,dx$$
$$=\frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e}{6\left(a^{3}de^{(6i\,dx+6i\,c)}+3\,a^{3}de^{(4i\,dx+4i\,c)}+3\,a^{3}de\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*
c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x
+ 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5
*I*d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*
e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2
*I*c) + a^3*d)
```

3.141.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^7(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**7/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

3.141.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(81) = 162$.

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.31

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6i\,a^3-\frac{18i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/
a^3 - 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3} - \frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3} - \frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4 + 48*I*
tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/
2*c)^2 - 1)^3*a^3))/d
```

3.141.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} + \frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^3} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 16\mathrm{i}}{a^3} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 6\mathrm{i}}{a^3} + \frac{22\mathrm{i}}{3\,a^3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(5*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) + ((tan(c/2 + (d*x)/2)^4*6i)/a^3 - (
tan(c/2 + (d*x)/2)^2*16i)/a^3 - (3*tan(c/2 + (d*x)/2)^5)/a^3 + 22i/(3*a^3)
 + (3*tan(c/2 + (d*x)/2))/a^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d
*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.142 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.142.1 Optimal result 1120
3.142.2 Mathematica [A] (verified) 1120
3.142.3 Rubi [A] (verified) 1121
3.142.4 Maple [A] (verified) 1122
3.142.5 Fricas [A] (verification not implemented) 1123
3.142.6 Sympy [F] 1123
3.142.7 Maxima [B] (verification not implemented) 1124
3.142.8 Giac [A] (verification not implemented) 1124
3.142.9 Mupad [B] (verification not implemented) 1125

3.142.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d} + \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

output
```
-3*arctanh(sin(d*x+c))/a^3/d+3*I*sec(d*x+c)/a^3/d+2*I*sec(d*x+c)^3/a/d/(a+
I*a*tan(d*x+c))^2
```

3.142.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(i\cos(dx)-\sin(dx))^3 (6\text{arctanh}(\sin(c)+\cos(c)\tan(\frac{dx}{2}))(\cos(3c)+i\sin(3c))+(\cos(2c-d}{a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(I*Cos[d*x] - Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(
d*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*(-5
*I + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

3.142.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3981, 3042, 3982, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\int \frac{\sec^3(c+dx)}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\int \frac{\sec(c+dx)^3}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\left(\frac{\int \sec(c+dx)dx}{a}-\frac{i\sec(c+dx)}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{i\sec(c+dx)}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}-\frac{3\left(\frac{\text{arctanh}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-3*(ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)))/a^2 + ((2*I)*S
ec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

3.142. $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}dx$

3.142.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.142.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{3}d}$	86
default	$\frac{\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^{3}d}$	86
risch	$\frac{4i\mathrm{e}^{-i(dx+c)}}{a^{3}d}+\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{3}d}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{3}d}$	93

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^3*(4/(-I+tan(1/2*d*x+1/2*c))-1/2*I/(tan(1/2*d*x+1/2*c)-1)+3/2*ln(tan
(1/2*d*x+1/2*c)-1)+1/2*I/(tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(1/2*d*x+1/2*c)+
1))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c}}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(
e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2
*I*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

3.142.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^5(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sec(c + d*x)**5/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

3.142.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(59) = 118$.

Time = 0.41 (sec) , antiderivative size = 319, normalized size of antiderivative = 4.91

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{6\left(\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(dx+c\right)\right)\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
(6*(cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))
*arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6*(cos(3*d*x + 3*c) + cos(d*x +
 c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x
+ c) + 1) + 3*(I*cos(3*d*x + 3*c) + I*cos(d*x + c) - sin(3*d*x + 3*c) - si
n(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*
(-I*cos(3*d*x + 3*c) - I*cos(d*x + c) + sin(3*d*x + 3*c) + sin(d*x + c))*l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 12*cos(2*d*x +
2*c) + 12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3*c) - 2*I*a^3*co
s(d*x + c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= -\frac{\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}-\frac{2\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)a^3}}{d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a^
3 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((tan(1/2*d*
x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)*a^3))/
d
```

3.142.9 Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= -\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}$$
$$-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}}{a^3}-\frac{10\mathrm{i}}{a^3}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
- (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - ((tan(c/2 + (d*x)/2)^2*8i)/a^3 -
 10i/a^3 + (2*tan(c/2 + (d*x)/2))/a^3)/(d*(tan(c/2 + (d*x)/2)*1i - tan(c/2
 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.143 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.143.1 Optimal result 1126
3.143.2 Mathematica [A] (verified) 1126
3.143.3 Rubi [A] (verified) 1127
3.143.4 Maple [A] (verified) 1128
3.143.5 Fricas [A] (verification not implemented) 1128
3.143.6 Sympy [B] (verification not implemented) 1128
3.143.7 Maxima [A] (verification not implemented) 1129
3.143.8 Giac [A] (verification not implemented) 1129
3.143.9 Mupad [B] (verification not implemented) 1130

3.143.1 Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

output
```
1/3*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3
```

3.143.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

3.143.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

input

```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

3.143.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.143.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{3a^3d}$	19
derivativedivides	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57
default	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/a^3/d*exp(-3*I*(d*x+c))
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

3.143.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.

Time = 0.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} -\frac{\sec^3(c+dx)}{3a^3d\tan^3(c+dx)-9ia^3d\tan^2(c+dx)-9a^3d\tan(c+dx)+3ia^3d} & \text{for } d\neq 0 \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((-sec(c + d*x)**3/(3*a**3*d*tan(c + d*x)**3 - 9*I*a**3*d*tan(c +
 d*x)**2 - 9*a**3*d*tan(c + d*x) + 3*I*a**3*d), Ne(d, 0)), (x*sec(c)**3/(I
*a*tan(c) + a)**3, True))
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\cos(3\,dx+3\,c)+\sin(3\,dx+3\,c)}{3\,a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= -\frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
-(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*ta
n(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.144 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.144.1 Optimal result 1131
3.144.2 Mathematica [A] (verified) 1131
3.144.3 Rubi [A] (verified) 1132
3.144.4 Maple [A] (verified) 1133
3.144.5 Fricas [A] (verification not implemented) 1134
3.144.6 Sympy [B] (verification not implemented) 1134
3.144.7 Maxima [A] (verification not implemented) 1135
3.144.8 Giac [A] (verification not implemented) 1135
3.144.9 Mupad [B] (verification not implemented) 1135

3.144.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2} + \frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}$$

output

```
1/5*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^3+2/15*I*sec(d*x+c)/a/d/(a+I*a*tan(d
*x+c))^2+2/15*I*sec(d*x+c)/d/(a^3+I*a^3*tan(d*x+c))
```

3.144.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)(5+9\cos(2(c+dx))+6i\sin(2(c+dx)))}{30a^3d(-i+\tan(c+dx))^3}$$

input

```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
-1/30*(Sec[c + d*x]^3*(5 + 9*Cos[2*(c + d*x)] + (6*I)*Sin[2*(c + d*x)]))/(
a^3*d*(-I + Tan[c + d*x])^3)
```

3.144.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\left(\frac{i\sec(c+dx)}{3ad(a+ia\tan(c+dx))}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/5)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (2*(((I/3)*Sec[c + d*x
])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c
 + d*x]))))/(5*a)
```

3.144.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.144.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{4a^3d}+\frac{i\mathrm{e}^{-3i(dx+c)}}{6a^3d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{20a^3d}$	56
derivativedivides	$\frac{\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^3d}$	90
default	$\frac{\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^3d}$	90

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/a^3/d*exp(-I*(d*x+c))+1/6*I/a^3/d*exp(-3*I*(d*x+c))+1/20*I/a^3/d*exp
(-5*I*(d*x+c))
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d
*x - 5*I*c)/(a^3*d)
```

3.144.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(82) = 164.

Time = 0.85 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.23

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{2\tan^2(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d} - \frac{6i\tan(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d} - \\ \frac{x\sec(c)}{(ia\tan(c)+a)^3} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((2*tan(c + d*x)**2*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*
I*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 6*I*tan
(c + d*x)*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*I*a**3*d*tan(c + d*
x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 7*sec(c + d*x)/(15*a**3*d*
tan(c + d*x)**3 - 45*I*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 1
5*I*a**3*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)**3, True))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{3i\,\cos\left(5\,dx+5\,c\right)+10i\,\cos\left(3\,dx+3\,c\right)+15i\,\cos\left(dx+c\right)+3\,\sin\left(5\,dx+5\,c\right)+10\,\sin\left(3\,dx+3\,c\right)+15}{60\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3
*sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2
*d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2
*c) - I)^5)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.36

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,15\mathrm{i}+30\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,40\mathrm{i}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+7\mathrm{i}\right)}{15\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,1\mathrm{i}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,10\mathrm{i}-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5\mathrm{i}+1\right.}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
(2*(30*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*40i - 20*tan(c/2 + (d*x
)/2) + tan(c/2 + (d*x)/2)^4*15i + 7i))/(15*a^3*d*(tan(c/2 + (d*x)/2)*5i -
10*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^
4 + tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.144. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.145 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.145.1 Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{12\sin(c+dx)}{35a^3d} - \frac{4\sin^3(c+dx)}{35a^3d} + \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}$$

output
```
12/35*sin(d*x+c)/a^3/d-4/35*sin(d*x+c)^3/a^3/d+1/7*I*cos(d*x+c)/d/(a+I*a*t
an(d*x+c))^3+8/35*I*cos(d*x+c)^3/d/(a^3+I*a^3*tan(d*x+c))
```

3.145.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)(35+84\cos(2(c+dx))-15\cos(4(c+dx))+56i\sin(2(c+dx))-20i\sin(4(c+dx)))}{280a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/280*(Sec[c + d*x]^3*(35 + 84*Cos[2*(c + d*x)] - 15*Cos[4*(c + d*x)] + (
56*I)*Sin[2*(c + d*x)] - (20*I)*Sin[4*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*
x])^3)
```

3.145.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4\left(\frac{3\int \cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3113}$$

$$\frac{4\left(-\frac{3\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/7)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (4*((-3*(-Sin[c + d*x]
 + Sin[c + d*x]^3/3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x]^3)/(d*(a^2 + I*a
^2*Tan[c + d*x]))))/(7*a)
```

3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.145.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{8a^3d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{20a^3d}+\frac{i\mathrm{e}^{-7i(dx+c)}}{112a^3d}+\frac{3i\cos(dx+c)}{16a^3d}+\frac{5\sin(dx+c)}{16a^3d}$
derivativedivides	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\right.\right.}}{a^3d}$
default	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\right.\right.}}{a^3d}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^3/d*exp(-3*I*(d*x+c))+1/20*I/a^3/d*exp(-5*I*(d*x+c))+1/112*I/a^3/d
*exp(-7*I*(d*x+c))+3/16*I/a^3/d*cos(d*x+c)+5/16*sin(d*x+c)/a^3/d
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I
*d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*
d)
```

3.145.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(87) = 174$.

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx})e^{-16ic}}{1146880a^{15}d^5} & \text{fo} \\ \frac{x(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-7ic}}{16a^3} & \text{o} \end{cases}$$

input

```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

output

```
Piecewise(((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d*
*4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x)
 + 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9
*I*c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(a**15*d**5*exp(
16*I*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c)
+ 1)*exp(-7*I*c)/(16*a**3), True))
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input

```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 1960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 4480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 -}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{280\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
 1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(
1/2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/
2*c) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.145.9 Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\Big(35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 105\mathrm{i} - 175\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 105\mathrm{i} - 7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \mathrm{ta}}{35\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^7}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
-((43*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*77i - 7*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*105i - 175*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x
)/2)^6*105i + 35*tan(c/2 + (d*x)/2)^7 - 13i)*2i)/(35*a^3*d*(tan(c/2 + (d*x
)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

3.146 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

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3.146.2 Mathematica [A] (verified) . . . 1143
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3.146.1 Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{10\sin(c+dx)}{21a^3d} - \frac{20\sin^3(c+dx)}{63a^3d} + \frac{2\sin^5(c+dx)}{21a^3d} + \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{21d\,(a^3+ia^3\tan(c+dx))}$$

output
```
10/21*sin(d*x+c)/a^3/d-20/63*sin(d*x+c)^3/a^3/d+2/21*sin(d*x+c)^5/a^3/d+1/
9*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3+4/21*I*cos(d*x+c)^5/d/(a^3+I*a^3*t
an(d*x+c))
```

3.146.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(-210-567\cos(2(c+dx))+162\cos(4(c+dx))+7\cos(6(c+dx))-378i\sin(2(c+dx))+}{2016a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(-210 - 567*Cos[2*(c + d*x)] + 162*Cos[4*(c + d*x)] + 7*Co
s[6*(c + d*x)] - (378*I)*Sin[2*(c + d*x)] + (216*I)*Sin[4*(c + d*x)] + (14
*I)*Sin[6*(c + d*x)]))/(2016*a^3*d*(-I + Tan[c + d*x])^3)
```

3.146.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2\left(\frac{5\int \cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3113}$$

$$\frac{2\left(-\frac{5\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d(a^2+ia^2\tan(c+dx))}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) + (2*((-5*(-Sin[c + d*
x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) + (((2*I)/7)*Cos[
c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(3*a)
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.146.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
risch	$\frac{3i\mathrm{e}^{-5i(dx+c)}}{64a^3d} + \frac{3i\mathrm{e}^{-7i(dx+c)}}{224a^3d} + \frac{i\mathrm{e}^{-9i(dx+c)}}{576a^3d} + \frac{9i\cos(dx+c)}{64a^3d} + \frac{21\sin(dx+c)}{64a^3d} + \frac{19i\cos(3dx+3c)}{192a^3d} + \frac{7\sin(3dx+3}{64a^3d}$
derivativedivides	$\frac{\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} - \frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8} + \frac{9i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{59i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} + \frac{8}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9} - \frac{}{7\left(-i+\tan\right.}}{}$
default	$\frac{\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} - \frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8} + \frac{9i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{59i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} + \frac{8}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9} - \frac{}{7\left(-i+\tan\right.}}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/64*I/a^3/d*exp(-5*I*(d*x+c))+3/224*I/a^3/d*exp(-7*I*(d*x+c))+1/576*I/a^3
/d*exp(-9*I*(d*x+c))+9/64*I/a^3/d*cos(d*x+c)+21/64*sin(d*x+c)/a^3/d+19/192
*I/a^3/d*cos(3*d*x+3*c)+7/64/a^3/d*sin(3*d*x+3*c)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(-21i\,e^{(12i\,dx+12i\,c)} - 378i\,e^{(10i\,dx+10i\,c)} + 945i\,e^{(8i\,dx+8i\,c)} + 420i\,e^{(6i\,dx+6i\,c)} + 189i\,e^{(4i\,dx+4i\,c)} + 54i\,e^{(2i\,dx+2i}\right.}{4032\,a^3d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4032*(-21*I*e^(12*I*d*x + 12*I*c) - 378*I*e^(10*I*d*x + 10*I*c) + 945*I*
e^(8*I*d*x + 8*I*c) + 420*I*e^(6*I*d*x + 6*I*c) + 189*I*e^(4*I*d*x + 4*I*c
) + 54*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

3.146.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(105) = 210$.

Time = 0.36 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.19

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-811748818944ia^{18}d^6e^{28ic}e^{3idx} - 14611478740992ia^{18}d^6e^{26ic}e^{idx} + 36528696852480ia^{18}d^6e^{24ic}e^{-idx} + 16234976378880ia^{18}d^6e^{22ic}e^{-3idx} +}{155855773237248a^{21}d^7} \\ \frac{x(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-9ic}}{64a^3} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-811748818944*I*a**18*d**6*exp(28*I*c)*exp(3*I*d*x) - 14611478
740992*I*a**18*d**6*exp(26*I*c)*exp(I*d*x) + 36528696852480*I*a**18*d**6*e
xp(24*I*c)*exp(-I*d*x) + 16234976378880*I*a**18*d**6*exp(22*I*c)*exp(-3*I*
d*x) + 7305739370496*I*a**18*d**6*exp(20*I*c)*exp(-5*I*d*x) + 208735410585
6*I*a**18*d**6*exp(18*I*c)*exp(-7*I*d*x) + 270582939648*I*a**18*d**6*exp(1
6*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(155855773237248*a**21*d**7), Ne(a**21*
d**7*exp(25*I*c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 2
0*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-9*I*c)/(64*a**3), Tr
ue))
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{21\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-19\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+3591\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-19656i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-56196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+95760i\tan\left(\frac{1}{2}dx\right.}{2016\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 + 36*I*tan(1/2*d*x + 1/2*c) - 19)/(a
^3*(tan(1/2*d*x + 1/2*c) + I)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 - 19656*I*
tan(1/2*d*x + 1/2*c)^7 - 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*I*tan(1/2*d*
x + 1/2*c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 - 79464*I*tan(1/2*d*x + 1/2*c
)^3 - 38484*tan(1/2*d*x + 1/2*c)^2 + 10944*I*tan(1/2*d*x + 1/2*c) + 1615)/
(a^3*(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

3.146.9 Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.55

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\left(63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}189\mathrm{i}-273\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}63\mathrm{i}-378\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\mathrm{ta}\right.}{63\,a^3\,d\left(\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((51*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*39i + 235*tan(c/2 + (d*x)/2
)^3 + tan(c/2 + (d*x)/2)^4*450i - 306*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*
x)/2)^6*294i - 378*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*63i - 273*t
an(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*189i + 63*tan(c/2 + (d*x)/2)^1
1 - 19i)*2i)/(63*a^3*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/2)*1i
+ 1)^9)
```

3.147 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

3.147.1 Optimal result 1149
3.147.2 Mathematica [A] (verified) 1149
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3.147.8 Giac [A] (verification not implemented) 1154
3.147.9 Mupad [B] (verification not implemented) 1155

3.147.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{56\sin(c+dx)}{99a^3d} - \frac{56\sin^3(c+dx)}{99a^3d} + \frac{56\sin^5(c+dx)}{165a^3d} - \frac{8\sin^7(c+dx)}{99a^3d} + \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))}$$

output
```
56/99*sin(d*x+c)/a^3/d-56/99*sin(d*x+c)^3/a^3/d+56/165*sin(d*x+c)^5/a^3/d-
8/99*sin(d*x+c)^7/a^3/d+1/11*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^3+16/99*I
*cos(d*x+c)^7/d/(a^3+I*a^3*tan(d*x+c))
```

3.147.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(-5775-16632\cos(2(c+dx))+5940\cos(4(c+dx))+440\cos(6(c+dx))+27\cos(8(c+d}{63360a^3d(-i+\tan(}$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(-5775 - 16632*Cos[2*(c + d*x)] + 5940*Cos[4*(c + d*x)] +
440*Cos[6*(c + d*x)] + 27*Cos[8*(c + d*x)] - (11088*I)*Sin[2*(c + d*x)] +
(7920*I)*Sin[4*(c + d*x)] + (880*I)*Sin[6*(c + d*x)] + (72*I)*Sin[8*(c + d
*x)]))/(63360*a^3*d*(-I + Tan[c + d*x])^3)
```

3.147.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\int \frac{\cos^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\int \frac{1}{\sec(c+dx)^5(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{8\left(\frac{7\int \cos^7(c+dx)dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{7\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3113}$$

$$\frac{8\left(-\frac{7\int\left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{8\left(-\frac{7\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d(a^2+ia^2\tan(c+dx))}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

input

```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((I/11)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^3) + (8*((-7*(-Sin[c + d
*x] + Sin[c + d*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/(9*a^2*d)
 + (((2*I)/9)*Cos[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(11*a)
```

3.147.3.1 Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.147.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

method	result
risch	$\frac{i\mathrm{e}^{-7i(dx+c)}}{64a^3d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{288a^3d}+\frac{i\mathrm{e}^{-11i(dx+c)}}{2816a^3d}+\frac{7i\cos(dx+c)}{64a^3d}+\frac{21\sin(dx+c)}{64a^3d}+\frac{11i\cos(5dx+5c)}{256a^3d}+\frac{57\sin(5dx+5}{1280a^3d}$
derivativedivides	$\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{217i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{1}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{7}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{37}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{23}{(-i+\tan(}$
default	$\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{217i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{1}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{7}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{37}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{23}{(-i+\tan(}$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*I/a^3/d*exp(-7*I*(d*x+c))+1/288*I/a^3/d*exp(-9*I*(d*x+c))+1/2816*I/a^
3/d*exp(-11*I*(d*x+c))+7/64*I/a^3/d*cos(d*x+c)+21/64*sin(d*x+c)/a^3/d+11/2
56*I/a^3/d*cos(5*d*x+5*c)+57/1280/a^3/d*sin(5*d*x+5*c)+31/384*I/a^3/d*cos(
3*d*x+3*c)+13/128/a^3/d*sin(3*d*x+3*c)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.77

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{\left(-99i\,e^{(16i\,dx+16i\,c)}-1320i\,e^{(14i\,dx+14i\,c)}-13860i\,e^{(12i\,dx+12i\,c)}+27720i\,e^{(10i\,dx+10i\,c)}+11550i\,e^{(8i\,dx+8i\,c)}+}{126720\,a^3d}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/126720*(-99*I*e^(16*I*d*x + 16*I*c) - 1320*I*e^(14*I*d*x + 14*I*c) - 138
60*I*e^(12*I*d*x + 12*I*c) + 27720*I*e^(10*I*d*x + 10*I*c) + 11550*I*e^(8*
I*d*x + 8*I*c) + 5544*I*e^(6*I*d*x + 6*I*c) + 1980*I*e^(4*I*d*x + 4*I*c) +
 440*I*e^(2*I*d*x + 2*I*c) + 45*I)*e^(-11*I*d*x - 11*I*c)/(a^3*d)
```

3.147.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(122) = 244$.

Time = 0.45 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.40

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-626985510622986240ia^{24}d^8e^{41ic}e^{5idx} - 8359806808306483200ia^{24}d^8e^{39ic}e^{3idx} - 87777971487218073600ia^{24}d^8e^{37ic}e^{idx} + 175555942974436}{} \\ \frac{x(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-11ic}}{256a^3} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise(((-626985510622986240*I*a**24*d**8*exp(41*I*c)*exp(5*I*d*x) - 83
59806808306483200*I*a**24*d**8*exp(39*I*c)*exp(3*I*d*x) - 8777797148721807
3600*I*a**24*d**8*exp(37*I*c)*exp(I*d*x) + 175555942974436147200*I*a**24*d
**8*exp(35*I*c)*exp(-I*d*x) + 73148309572681728000*I*a**24*d**8*exp(33*I*c
)*exp(-3*I*d*x) + 35111188594887229440*I*a**24*d**8*exp(31*I*c)*exp(-5*I*d
*x) + 12539710212459724800*I*a**24*d**8*exp(29*I*c)*exp(-7*I*d*x) + 278660
2269435494400*I*a**24*d**8*exp(27*I*c)*exp(-9*I*d*x) + 284993413919539200*
I*a**24*d**8*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(8025414535974223872
00*a**27*d**9), Ne(a**27*d**9*exp(36*I*c), 0)), (x*(exp(16*I*c) + 8*exp(14
*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*exp(6*I*c) +
28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-11*I*c)/(256*a**3), True))
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.60

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{33\left(555\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1920i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2710\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1760i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+463\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{108405\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-784080i\,\text{ta}}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/63360*(33*(555*tan(1/2*d*x + 1/2*c)^4 + 1920*I*tan(1/2*d*x + 1/2*c)^3 -
2710*tan(1/2*d*x + 1/2*c)^2 - 1760*I*tan(1/2*d*x + 1/2*c) + 463)/(a^3*(tan
(1/2*d*x + 1/2*c) + I)^5) + (108405*tan(1/2*d*x + 1/2*c)^10 - 784080*I*tan
(1/2*d*x + 1/2*c)^9 - 2901195*tan(1/2*d*x + 1/2*c)^8 + 6652800*I*tan(1/2*d
*x + 1/2*c)^7 + 10407474*tan(1/2*d*x + 1/2*c)^6 - 11435424*I*tan(1/2*d*x +
 1/2*c)^5 - 8949270*tan(1/2*d*x + 1/2*c)^4 + 4899840*I*tan(1/2*d*x + 1/2*c
)^3 + 1816265*tan(1/2*d*x + 1/2*c)^2 - 411664*I*tan(1/2*d*x + 1/2*c) - 472
79)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^11))/d
```

3.147.9 Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$= \frac{\left(\frac{\cos(7\,c+7\,d\,x)}{64} + \frac{\cos(9\,c+9\,d\,x)}{288} + \frac{\cos(11\,c+11\,d\,x)}{2816} - \frac{\sin(7\,c+7\,d\,x)\,1\mathrm{i}}{64} - \frac{\sin(9\,c+9\,d\,x)\,1\mathrm{i}}{288} - \frac{\sin(11\,c+11\,d\,x)\,1\mathrm{i}}{2816} + \frac{\sqrt{224}\,\cos\left(5\,c+5\,d\,x-\right.}{128}\right.}{a^3\,d}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((cos(7*c + 7*d*x)/64 + cos(9*c + 9*d*x)/288 + cos(11*c + 11*d*x)/2816 - (
sin(7*c + 7*d*x)*1i)/64 - (sin(9*c + 9*d*x)*1i)/288 - (sin(11*c + 11*d*x)*
1i)/2816 + (224^(1/2)*cos(5*c + atanh(57/55)*1i + 5*d*x)*1i)/1280 + (560^(
1/2)*cos(3*c + atanh(39/31)*1i + 3*d*x)*1i)/384 + (2^(1/2)*cos(c + atanh(3
)*1i + d*x)*7i)/32)*1i)/(a^3*d)
```

3.148 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.148.1 Optimal result 1156
3.148.2 Mathematica [A] (verified) 1156
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3.148.9 Mupad [B] (verification not implemented) 1160

3.148.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}$$

output
```
4/7*I*(a-I*a*tan(d*x+c))^7/a^11/d-1/2*I*(a-I*a*tan(d*x+c))^8/a^12/d+1/9*I*
(a-I*a*tan(d*x+c))^9/a^13/d
```

3.148.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{(i+\tan(c+dx))^7\left(-23-35i\tan(c+dx)+14\tan^2(c+dx)\right)}{126a^4d}$$

input
```
Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I + Tan[c + d*x])^7*(-23 - (35*I)*Tan[c + d*x] + 14*Tan[c + d*x]^2))/(12
6*a^4*d)
```

3.148.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{14}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^6(i\tan(c+dx)a+a)^2d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left((a-ia\tan(c+dx))^8-4a(a-ia\tan(c+dx))^7+4a^2(a-ia\tan(c+dx))^6\right)d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{4}{7}a^2(a-ia\tan(c+dx))^7-\frac{1}{9}(a-ia\tan(c+dx))^9+\frac{1}{2}a(a-ia\tan(c+dx))^8\right)}{a^{13}d}$$

input
```
Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*((-4*a^2*(a - I*a*Tan[c + d*x])^7)/7 + (a*(a - I*a*Tan[c + d*x])^8)/
2 - (a - I*a*Tan[c + d*x])^9/9))/(a^13*d)
```

3.148.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.148.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result
risch	$\frac{128 i \left(36 \,{\mathrm e}^{4 i \left(d x +c \right)}+9 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{63 d \,a^{4} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{9}}$
derivativedivides	$-\frac{-\tan \left(d x +c \right)-\frac{\left(\tan^{9}\left(d x +c \right)\right)}{9}-\frac{i \left(\tan^{8}\left(d x +c \right)\right)}{2}+\frac{4 \left(\tan^{7}\left(d x +c \right)\right)}{7}-\frac{2 i \left(\tan^{6}\left(d x +c \right)\right)}{3}+2 \left(\tan^{5}\left(d x +c \right)\right)+i \left(\tan^{4}\left(d x +c \right)\right)+}{a^{4} d}$
default	$-\frac{-\tan \left(d x +c \right)-\frac{\left(\tan^{9}\left(d x +c \right)\right)}{9}-\frac{i \left(\tan^{8}\left(d x +c \right)\right)}{2}+\frac{4 \left(\tan^{7}\left(d x +c \right)\right)}{7}-\frac{2 i \left(\tan^{6}\left(d x +c \right)\right)}{3}+2 \left(\tan^{5}\left(d x +c \right)\right)+i \left(\tan^{4}\left(d x +c \right)\right)+}{a^{4} d}$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
128/63*I*(36*exp(4*I*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/a^4/(exp(2*I*(d*x+c)
)+1)^9
```

3.148.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(64) = 128$.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{128\,(-36i\,e^{(4i\,dx+4i\,c)}-9i\,e^{(2i\,dx}}{63\,(a^4de^{(18i\,dx+18i\,c)}+9\,a^4de^{(16i\,dx+16i\,c)}+36\,a^4de^{(14i\,dx+14i\,c)}+84\,a^4de^{(12i\,dx+12i\,c)}+126\,a^4de^{(10i\,dx+10i\,c)}}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-128/63*(-36*I*e^(4*I*d*x + 4*I*c) - 9*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e
^(18*I*d*x + 18*I*c) + 9*a^4*d*e^(16*I*d*x + 16*I*c) + 36*a^4*d*e^(14*I*d*
x + 14*I*c) + 84*a^4*d*e^(12*I*d*x + 12*I*c) + 126*a^4*d*e^(10*I*d*x + 10*
I*c) + 126*a^4*d*e^(8*I*d*x + 8*I*c) + 84*a^4*d*e^(6*I*d*x + 6*I*c) + 36*a
^4*d*e^(4*I*d*x + 4*I*c) + 9*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

3.148.6 Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{14}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**14/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
 + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{14\tan{(dx+c)}^9 + 63i\tan{(dx+c)}^8 - 72\tan{(dx+c)}^7 + 84i\tan{(dx+c)}^6 - 252\tan{(dx+c)}^5 - 126i\ t}{126\,a^4d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*
tan(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x +
 c)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{14\tan{(dx+c)}^9 + 63i\tan{(dx+c)}^8 - 72\tan{(dx+c)}^7 + 84i\tan{(dx+c)}^6 - 252\tan{(dx+c)}^5 - 126i\ t}{126\,a^4d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*
tan(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x +
 c)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.46

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\cos{(c+d\,x)}^9\,105\mathrm{i} + 128\sin{(c+d\,x)}\cos{(c+d\,x)}^8 + 64\sin{(c+d\,x)}\cos{(c+d\,x)}^6 + 48\sin{(c+d\,x)}\cos}{126\,a^4\,d\cos(}$$

input
```
int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(cos(c + d*x)*63i + 14*sin(c + d*x) - 128*cos(c + d*x)^2*sin(c + d*x) + 48
*cos(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + 128*cos(c
+ d*x)^8*sin(c + d*x) - cos(c + d*x)^3*168i + cos(c + d*x)^9*105i)/(126*a^
4*d*cos(c + d*x)^9)
```

3.148. $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.149 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.149.1 Optimal result 1162
3.149.2 Mathematica [A] (verified) 1162
3.149.3 Rubi [A] (verified) 1163
3.149.4 Maple [A] (verified) 1164
3.149.5 Fricas [B] (verification not implemented) 1164
3.149.6 Sympy [F] 1165
3.149.7 Maxima [A] (verification not implemented) 1165
3.149.8 Giac [A] (verification not implemented) 1166
3.149.9 Mupad [B] (verification not implemented) 1166

3.149.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

output
```
1/3*I*(a-I*a*tan(d*x+c))^6/a^10/d-1/7*I*(a-I*a*tan(d*x+c))^7/a^11/d
```

3.149.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{(i+\tan(c+dx))^6(-4i+3\tan(c+dx))}{21a^4d}$$

input
```
Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I + Tan[c + d*x])^6*(-4*I + 3*Tan[c + d*x]))/(21*a^4*d)
```

3.149.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{12}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^5(i\tan(c+dx)a+a)d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(2a(a-ia\tan(c+dx))^5-(a-ia\tan(c+dx))^6\right)d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{7}(a-ia\tan(c+dx))^7-\frac{1}{3}a(a-ia\tan(c+dx))^6\right)}{a^{11}d}$$

input
```
Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*(-1/3*(a*(a - I*a*Tan[c + d*x])^6) + (a - I*a*Tan[c + d*x])^7/7))/(a
^11*d)
```

3.149.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.149.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{64i\left(7\,\mathrm{e}^{2i(dx+c)}+1\right)}{21d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}$	36
derivativedivides	$-\frac{-\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{2i\left(\tan^6(dx+c)\right)}{3}+\tan^5(dx+c)+\frac{5\left(\tan^3(dx+c)\right)}{3}+2i\left(\tan^2(dx+c)\right)}{a^4d}$	68
default	$-\frac{-\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{2i\left(\tan^6(dx+c)\right)}{3}+\tan^5(dx+c)+\frac{5\left(\tan^3(dx+c)\right)}{3}+2i\left(\tan^2(dx+c)\right)}{a^4d}$	68

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
64/21*I*(7*exp(2*I*(d*x+c))+1)/d/a^4/(exp(2*I*(d*x+c))+1)^7
```

3.149.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.31

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{64\left(-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{21\,(a^4de^{(14i\,dx+14i\,c)}+7\,a^4de^{(12i\,dx+12i\,c)}+21\,a^4de^{(10i\,dx+10i\,c)}+35\,a^4de^{(8i\,dx+8i\,c)}+35\,a^4de^{(6i\,dx+6i\,c)}+21}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-64/21*(-7*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e^(14*I*d*x + 14*I*c) + 7*a^4
*d*e^(12*I*d*x + 12*I*c) + 21*a^4*d*e^(10*I*d*x + 10*I*c) + 35*a^4*d*e^(8*
I*d*x + 8*I*c) + 35*a^4*d*e^(6*I*d*x + 6*I*c) + 21*a^4*d*e^(4*I*d*x + 4*I*
c) + 7*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

3.149.6 Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{12}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**12/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
 + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3\tan(dx+c)^7 + 14i\tan(dx+c)^6 - 21\tan(dx+c)^5 - 35\tan(dx+c)^3 - 42i\tan(dx+c)^2 + 21\tan(}{21\,a^4d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(
d*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{3\,\tan\left(dx+c\right)^{7}+14i\,\tan\left(dx+c\right)^{6}-21\,\tan\left(dx+c\right)^{5}-35\,\tan\left(dx+c\right)^{3}-42i\,\tan\left(dx+c\right)^{2}+21\,\tan\left(c\right.}{21\,a^{4}d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(
d*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

3.149.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sin\left(c+d\,x\right)\,\left(21\cos\left(c+d\,x\right)^{6}-\cos\left(c+d\,x\right)^{5}\,\sin\left(c+d\,x\right)\,42\mathrm{i}-35\cos\left(c+d\,x\right)^{4}\sin\left(c+d\,x\right)^{2}-21\cos\left(\right.\right.}{21\,a^{4}\,d\cos\left(c+d\,x\right)^{7}}$$

input
```
int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*14i - cos(c + d*x)^5*sin(c + d*
x)*42i + 21*cos(c + d*x)^6 + 3*sin(c + d*x)^6 - 21*cos(c + d*x)^2*sin(c +
d*x)^4 - 35*cos(c + d*x)^4*sin(c + d*x)^2))/(21*a^4*d*cos(c + d*x)^7)
```

3.150 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.150.1 Optimal result 1167
3.150.2 Mathematica [B] (verified) 1167
3.150.3 Rubi [A] (verified) 1168
3.150.4 Maple [A] (verified) 1169
3.150.5 Fricas [B] (verification not implemented) 1169
3.150.6 Sympy [F] 1170
3.150.7 Maxima [B] (verification not implemented) 1170
3.150.8 Giac [B] (verification not implemented) 1170
3.150.9 Mupad [B] (verification not implemented) 1171

3.150.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

output
```
1/5*I*(a-I*a*tan(d*x+c))^5/a^9/d
```

3.150.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(c+dx)\left(5-10i\tan(c+dx)-10\tan^2(c+dx)+5i\tan^3(c+dx)+\tan^4(c+dx)\right)}{5a^4d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Tan[c + d*x]*(5 - (10*I)*Tan[c + d*x] - 10*Tan[c + d*x]^2 + (5*I)*Tan[c +
 d*x]^3 + Tan[c + d*x]^4))/(5*a^4*d)
```

3.150.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{17}$$

$$\frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/5)*(a - I*a*Tan[c + d*x])^5)/(a^9*d)
```

3.150.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.150.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{(\tan(dx+c)+i)^5}{5a^4d}$	20
default	$\frac{(\tan(dx+c)+i)^5}{5a^4d}$	20
risch	$\frac{32i}{5d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$	23

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/5/a^4/d*(tan(d*x+c)+I)^5
```

3.150.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{32i}{5\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
32/5*I/(a^4*d*e^(10*I*d*x + 10*I*c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4
*d*e^(6*I*d*x + 6*I*c) + 10*a^4*d*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x
 + 2*I*c) + a^4*d)
```

3.150.6 Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{10}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**10/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
 + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.150.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(dx+c)^5 + 5i\tan(dx+c)^4 - 10\tan(dx+c)^3 - 10i\tan(dx+c)^2 + 5\tan(dx+c)}{5a^4d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*
x + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

3.150.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(dx+c)^5 + 5i\tan(dx+c)^4 - 10\tan(dx+c)^3 - 10i\tan(dx+c)^2 + 5\tan(dx+c)}{5a^4d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*
x + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sin(c+d\,x)\,\left(5\cos(c+d\,x)^4-\cos(c+d\,x)^3\,\sin(c+d\,x)\,10\mathrm{i}-10\cos(c+d\,x)^2\sin(c+d\,x)^2+\cos(c+\right.}{5\,a^4\,d\cos(c+d\,x)^5}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i - cos(c + d*x)^3*sin(c + d*x
)*10i + 5*cos(c + d*x)^4 + sin(c + d*x)^4 - 10*cos(c + d*x)^2*sin(c + d*x)
^2))/(5*a^4*d*cos(c + d*x)^5)
```

3.151 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.151.1 Optimal result . . . 1172
3.151.2 Mathematica [A] (verified) . . . 1172
3.151.3 Rubi [A] (verified) . . . 1173
3.151.4 Maple [A] (verified) . . . 1174
3.151.5 Fricas [A] (verification not implemented) . . . 1175
3.151.6 Sympy [F] . . . 1175
3.151.7 Maxima [A] (verification not implemented) . . . 1175
3.151.8 Giac [A] (verification not implemented) . . . 1176
3.151.9 Mupad [B] (verification not implemented) . . . 1176

3.151.1 Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{8x}{a^4}+\frac{8i\log(\cos(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}$$

output
```
8*x/a^4+8*I*ln(cos(d*x+c))/a^4/d-4*tan(d*x+c)/a^4/d-I*(a-I*a*tan(d*x+c))^2
/a^6/d-1/3*I*(a-I*a*tan(d*x+c))^3/a^7/d
```

3.151.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{-24i\log(i-\tan(c+dx))-21\tan(c+dx)+6i\tan^2(c+dx)+\tan^3(c+dx)}{3a^4d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-24*I)*Log[I - Tan[c + d*x]] - 21*Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 +
Tan[c + d*x]^3)/(3*a^4*d)
```

3.151.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^3}{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{8a^3}{i\tan(c+dx)a+a}-4a^2-2(a-ia\tan(c+dx))a-(a-ia\tan(c+dx))^2\right)d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-4ia^3\tan(c+dx)+8a^3\log(a+ia\tan(c+dx))+a(a-ia\tan(c+dx))^2+\frac{1}{3}(a-ia\tan(c+dx))^3\right)}{a^7d}$$

input

```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

output

```
((-I)*(8*a^3*Log[a + I*a*Tan[c + d*x]] - (4*I)*a^3*Tan[c + d*x] + a*(a - I
*a*Tan[c + d*x])^2 + (a - I*a*Tan[c + d*x])^3/3))/(a^7*d)
```

3.151.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.151.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{7\tan(dx+c)}{a^4d}+\frac{\tan^3(dx+c)}{3a^4d}+\frac{2i(\tan^2(dx+c))}{a^4d}+\frac{8\arctan(\tan(dx+c))}{a^4d}-\frac{4i\ln(1+\tan^2(dx+c))}{a^4d}$	84
default	$-\frac{7\tan(dx+c)}{a^4d}+\frac{\tan^3(dx+c)}{3a^4d}+\frac{2i(\tan^2(dx+c))}{a^4d}+\frac{8\arctan(\tan(dx+c))}{a^4d}-\frac{4i\ln(1+\tan^2(dx+c))}{a^4d}$	84
risch	$\frac{16x}{a^4}+\frac{16c}{a^4d}-\frac{4i(6\,\mathrm{e}^{4i(dx+c)}+15\,\mathrm{e}^{2i(dx+c)}+11)}{3d\,a^4(\mathrm{e}^{2i(dx+c)}+1)^3}+\frac{8i\ln(\mathrm{e}^{2i(dx+c)}+1)}{a^4d}$	84

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-7*tan(d*x+c)/a^4/d+1/3/a^4/d*tan(d*x+c)^3+2*I/a^4/d*tan(d*x+c)^2+8/a^4/d*
arctan(tan(d*x+c))-4*I/a^4/d*ln(1+tan(d*x+c)^2)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.73

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4\,(12\,dxe^{(6i\,dx+6i\,c)} + 12\,dx + 6\,(6\,dx-i)e^{(4i\,dx+4i\,c)} + 3\,(12\,dx-5i)e^{(2i\,dx+2i\,c)} - 6\,(-i\,e^{(6i\,dx+6i\,c)} - 3i\,e^{(4i\,d}}{3\,(a^4de^{(6i\,dx+6i\,c)} + 3\,a^4de^{(4i\,dx+4i\,c)} + 3\,a^4de^{(2i\,dx+2i\,c)} + a}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
4/3*(12*d*x*e^(6*I*d*x + 6*I*c) + 12*d*x + 6*(6*d*x - I)*e^(4*I*d*x + 4*I*
c) + 3*(12*d*x - 5*I)*e^(2*I*d*x + 2*I*c) - 6*(-I*e^(6*I*d*x + 6*I*c) - 3*
I*e^(4*I*d*x + 4*I*c) - 3*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x + 2*I*
c) + 1) - 11*I)/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) +
 3*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

3.151.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^8(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**8/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{\tan(dx+c)^3+6i\tan(dx+c)^2-21\tan(dx+c)}{a^4} - \frac{24i\log(i\tan(dx+c)+1)}{a^4}}{3\,d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 21*tan(d*x + c))/a^4 - 24*I*lo
g(I*tan(d*x + c) + 1)/a^4)/d
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.71

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(-\frac{12i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^4}+\frac{24i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^4}-\frac{12i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4}+\frac{22i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-21\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}\right.}{3d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-2/3*(-12*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 + 24*I*log(tan(1/2*d*x + 1/2
*c) - I)/a^4 - 12*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^4 + (22*I*tan(1/2*d*x
+ 1/2*c)^6 - 21*tan(1/2*d*x + 1/2*c)^5 - 78*I*tan(1/2*d*x + 1/2*c)^4 + 46*
tan(1/2*d*x + 1/2*c)^3 + 78*I*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x + 1/
2*c) - 22*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4))/d
```

3.151.9 Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\frac{7\tan(c+dx)}{a^4}-\frac{\tan(c+dx)^3}{3a^4}+\frac{\ln(\tan(c+dx)-\mathrm{i})\,8\mathrm{i}}{a^4}-\frac{\tan(c+dx)^2\,2\mathrm{i}}{a^4}}{d}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
-((log(tan(c + d*x) - 1i)*8i)/a^4 + (7*tan(c + d*x))/a^4 - (tan(c + d*x)^2
*2i)/a^4 - tan(c + d*x)^3/(3*a^4))/d
```

3.152 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.152.1 Optimal result 1177
3.152.2 Mathematica [A] (verified) 1177
3.152.3 Rubi [A] (verified) 1178
3.152.4 Maple [A] (verified) 1179
3.152.5 Fricas [A] (verification not implemented) 1180
3.152.6 Sympy [F] 1180
3.152.7 Maxima [A] (verification not implemented) 1180
3.152.8 Giac [B] (verification not implemented) 1181
3.152.9 Mupad [B] (verification not implemented) 1181

3.152.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{4x}{a^4}-\frac{4i\log(\cos(c+dx))}{a^4d}+\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
-4*x/a^4-4*I*ln(cos(d*x+c))/a^4/d+tan(d*x+c)/a^4/d+4*I/d/(a^4+I*a^4*tan(d*
x+c))
```

3.152.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\left(-4\log(i-\tan(c+dx))+i\tan(c+dx)+\frac{4i}{-i+\tan(c+dx)}\right)}{a^4d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*(-4*Log[I - Tan[c + d*x]] + I*Tan[c + d*x] + (4*I)/(-I + Tan[c + d*x
])))/(a^4*d)
```

3.152.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{49}$$

$$-\frac{i\int \left(\frac{4a^2}{(i\tan(c+dx)a+a)^2}-\frac{4a}{i\tan(c+dx)a+a}+1\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{4a^2}{a+ia\tan(c+dx)}+ia\tan(c+dx)-4a\log(a+ia\tan(c+dx))\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*(-4*a*Log[a + I*a*Tan[c + d*x]] + I*a*Tan[c + d*x] - (4*a^2)/(a + I*
a*Tan[c + d*x])))/(a^5*d)
```

3.152.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.152.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{a^4d}-\frac{4\arctan(\tan(dx+c))}{a^4d}+\frac{2i\ln(1+\tan^2(dx+c))}{a^4d}+\frac{4}{a^4d(\tan(dx+c)-i)}$	69
default	$\frac{\tan(dx+c)}{a^4d}-\frac{4\arctan(\tan(dx+c))}{a^4d}+\frac{2i\ln(1+\tan^2(dx+c))}{a^4d}+\frac{4}{a^4d(\tan(dx+c)-i)}$	69
risch	$\frac{2i\mathrm{e}^{-2i(dx+c)}}{a^4d}-\frac{8x}{a^4}-\frac{8c}{a^4d}+\frac{2i}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{4i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^4d}$	78

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
tan(d*x+c)/a^4/d-4/a^4/d*arctan(tan(d*x+c))+2*I/a^4/d*ln(1+tan(d*x+c)^2)+4
/a^4/d/(tan(d*x+c)-I)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(4\,dxe^{(4i\,dx+4i\,c)} + 2\,(2\,dx-i)e^{(2i\,dx+2i\,c)} + 2\left(i\,e^{(4i\,dx+4i\,c)} + i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right) - i\right)}{a^4de^{(4i\,dx+4i\,c)} + a^4de^{(2i\,dx+2i\,c)}}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2*(4*d*x*e^(4*I*d*x + 4*I*c) + 2*(2*d*x - I)*e^(2*I*d*x + 2*I*c) + 2*(I*e
^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) -
 I)/(a^4*d*e^(4*I*d*x + 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))
```

3.152.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**6/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{4\left(\tan(dx+c)^2-2i\tan(dx+c)-1\right)}{a^4\tan(dx+c)^3-3i\,a^4\tan(dx+c)^2-3\,a^4\tan(dx+c)+i\,a^4} + \frac{4i\log(i\tan(dx+c)+1)}{a^4} + \frac{\tan(dx+c)}{a^4}}{d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
(4*(tan(d*x + c)^2 - 2*I*tan(d*x + c) - 1)/(a^4*tan(d*x + c)^3 - 3*I*a^4*t
an(d*x + c)^2 - 3*a^4*tan(d*x + c) + I*a^4) + 4*I*log(I*tan(d*x + c) + 1)/
a^4 + tan(d*x + c)/a^4)/d
```

3.152.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(57) = 114$.

Time = 0.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.32

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^4}+\frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^4}-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4}+\frac{2i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4}-\right.}{d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
2*(-2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 + 4*I*log(tan(1/2*d*x + 1/2*c) -
 I)/a^4 - 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^4 + (2*I*tan(1/2*d*x + 1/2*c
)^2 - tan(1/2*d*x + 1/2*c) - 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4) - 2*(
3*I*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) - 3*I)/(a^4*(tan(1/2*d
*x + 1/2*c) - I)^2))/d
```

3.152.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,4\mathrm{i}}{a^4\,d}+\frac{\tan(c+d\,x)}{a^4\,d} + \frac{4\mathrm{i}}{a^4\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(log(tan(c + d*x) - 1i)*4i)/(a^4*d) + tan(c + d*x)/(a^4*d) + 4i/(a^4*d*(ta
n(c + d*x)*1i + 1))
```

3.152. $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.153 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.153.1 Optimal result 1183
3.153.2 Mathematica [A] (verified) 1183
3.153.3 Rubi [A] (verified) 1184
3.153.4 Maple [A] (verified) 1185
3.153.5 Fricas [A] (verification not implemented) 1185
3.153.6 Sympy [B] (verification not implemented) 1186
3.153.7 Maxima [B] (verification not implemented) 1186
3.153.8 Giac [A] (verification not implemented) 1187
3.153.9 Mupad [B] (verification not implemented) 1187

3.153.1 Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{\tan(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)^2}$$

output
```
tan(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2
```

3.153.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{i(i+\tan(c+dx))^2}{4a^4d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/4)*(I + Tan[c + d*x])^2)/(a^4*d*(-I + Tan[c + d*x])^2)
```

3.153.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \; 38$$

$$\frac{\tan(c+dx)}{a^2d(a+ia\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
Tan[c + d*x]/(a^2*d*(a + I*a*Tan[c + d*x])^2)
```

3.153.3.1 Defintions of rubi rules used

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.153.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{i\,\mathrm{e}^{-4i(dx+c)}}{4a^4d}$	19
derivativedivides	$\frac{-\frac{i}{(\tan(dx+c)-i)^2}-\frac{1}{\tan(dx+c)-i}}{a^4d}$	36
default	$\frac{-\frac{i}{(\tan(dx+c)-i)^2}-\frac{1}{\tan(dx+c)-i}}{a^4d}$	36

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/a^4/d*exp(-4*I*(d*x+c))
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\,e^{(-4i\,dx-4i\,c)}}{4\,a^4d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

3.153.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(24) = 48.

Time = 1.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{i\sec^4(c+dx)}{4a^4d\tan^4(c+dx)-16ia^4d\tan^3(c+dx)-24a^4d\tan^2(c+dx)+16ia^4d\tan(c+dx)+4a^4d} & \text{for } d\neq 0 \\ \frac{x\sec^4(c)}{(ia\tan(c)+a)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((I*sec(c + d*x)**4/(4*a**4*d*tan(c + d*x)**4 - 16*I*a**4*d*tan(c
 + d*x)**3 - 24*a**4*d*tan(c + d*x)**2 + 16*I*a**4*d*tan(c + d*x) + 4*a**4
*d), Ne(d, 0)), (x*sec(c)**4/(I*a*tan(c) + a)**4, True))
```

3.153.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= -\frac{\tan(dx+c)^2 - i\tan(dx+c)}{\left(a^4\tan(dx+c)^3 - 3i\,a^4\tan(dx+c)^2 - 3\,a^4\tan(dx+c) + i\,a^4\right)d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-(tan(d*x + c)^2 - I*tan(d*x + c))/((a^4*tan(d*x + c)^3 - 3*I*a^4*tan(d*x
+ c)^2 - 3*a^4*tan(d*x + c) + I*a^4)*d)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1
/2*c) - I)^4)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan(c+d\,x)}{a^4\,d\,\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)^2}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
-tan(c + d*x)/(a^4*d*(tan(c + d*x) - 1i)^2)
```

3.154 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.154.1 Optimal result 1188
3.154.2 Mathematica [A] (verified) 1188
3.154.3 Rubi [A] (verified) 1189
3.154.4 Maple [A] (verified) 1190
3.154.5 Fricas [A] (verification not implemented) 1190
3.154.6 Sympy [B] (verification not implemented) 1191
3.154.7 Maxima [A] (verification not implemented) 1191
3.154.8 Giac [B] (verification not implemented) 1192
3.154.9 Mupad [B] (verification not implemented) 1192

3.154.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{3ad(a+ia\tan(c+dx))^3}$$

output
```
1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

3.154.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{1}{3a^4d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/3*1/(a^4*d*(-I + Tan[c + d*x])^3)
```

3.154.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \; 17$$

$$\frac{i}{3ad(a+ia\tan(c+dx))^3}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(I/3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

3.154.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.154.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
default	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}$	56

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{\left(3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{24\,a^4d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(3*I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x -
 6*I*c)/(a^4*d)
```

3.154.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(19) = 38$.

Time = 1.20 (sec) , antiderivative size = 272, normalized size of antiderivative = 10.07

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{i\tan^2(c+dx)\sec^2(c+dx)}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144a^4d\tan^2(c+dx)+96ia^4d\tan(c+dx)+24a^4d} - \frac{4\tan(c+dx)}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144} \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^4} \end{cases}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**2/(24*a**4*d*tan(c + d*x)**4 -
 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a**4*d*ta
n(c + d*x) + 24*a**4*d) - 4*tan(c + d*x)*sec(c + d*x)**2/(24*a**4*d*tan(c
+ d*x)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*
I*a**4*d*tan(c + d*x) + 24*a**4*d) + 7*I*sec(c + d*x)**2/(24*a**4*d*tan(c
+ d*x)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*
I*a**4*d*tan(c + d*x) + 24*a**4*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) +
a)**4, True))
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{3\,(i\,a\tan(dx+c)+a)^3ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3*I/((I*a*tan(d*x + c) + a)^3*a*d)
```

3.154.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(21) = 42$.

Time = 0.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\tan\left(\frac{1}{2}\right.}{3\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^6}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-2/3*(3*tan(1/2*d*x + 1/2*c)^5 - 6*I*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d
*x + 1/2*c)^3 + 6*I*tan(1/2*d*x + 1/2*c)^2 + 3*tan(1/2*d*x + 1/2*c))/(a^4*
d*(tan(1/2*d*x + 1/2*c) - I)^6)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{1}{3\,a^4\,d\,(\tan(c+d\,x)-\mathrm{i})^3}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
-1/(3*a^4*d*(tan(c + d*x) - 1i)^3)
```

3.155 $\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx$

3.155.1 Optimal result 1193
3.155.2 Mathematica [A] (verified) 1193
3.155.3 Rubi [A] (verified) 1194
3.155.4 Maple [A] (verified) 1195
3.155.5 Fricas [A] (verification not implemented) 1196
3.155.6 Sympy [A] (verification not implemented) 1196
3.155.7 Maxima [F(-2)] 1197
3.155.8 Giac [A] (verification not implemented) 1197
3.155.9 Mupad [B] (verification not implemented) 1198

3.155.1 Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{x}{16a^4}+\frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
1/16*x/a^4+1/8*I/d/(a+I*a*tan(d*x+c))^4+1/12*I/a/d/(a+I*a*tan(d*x+c))^3+1/
16*I/d/(a^2+I*a^2*tan(d*x+c))^2+1/16*I/d/(a^4+I*a^4*tan(d*x+c))
```

3.155.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = -\frac{ia\left(\frac{i\arctan(\tan(c+dx))}{16a^5}-\frac{1}{8a(a+ia\tan(c+dx))^4}-\frac{1}{12a^2(a+ia\tan(c+dx))^3}-\frac{1}{16a^3(a+ia\tan(c+dx))^2}-\frac{1}{16a^4(a+ia\tan(c+dx))}\right)}{d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-4),x]
```

output
```
((-I)*a*(((I/16)*ArcTan[Tan[c + d*x]])/a^5 - 1/(8*a*(a + I*a*Tan[c + d*x])
^4) - 1/(12*a^2*(a + I*a*Tan[c + d*x])^3) - 1/(16*a^3*(a + I*a*Tan[c + d*x
])^2) - 1/(16*a^4*(a + I*a*Tan[c + d*x]))))/d
```

3.155.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3960}$$

$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(-4),x]
```

output

```
(I/8)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/6)/(d*(a + I*a*Tan[c + d*x])^3) +
 ((I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c
+ d*x])))/(2*a))/(2*a))/(2*a)
```

3.155.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.155.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x}{16a^4}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{32a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{i}{8d\,a^4(\tan(dx+c)-i)^4}+\frac{\arctan(\tan(dx+c))}{16a^4d}-\frac{i}{16d\,a^4(\tan(dx+c)-i)^2}-\frac{1}{12d\,a^4(\tan(dx+c)-i)^3}+\frac{1}{16a^4d(\tan(dx-}$
default	$\frac{i}{8d\,a^4(\tan(dx+c)-i)^4}+\frac{\arctan(\tan(dx+c))}{16a^4d}-\frac{i}{16d\,a^4(\tan(dx+c)-i)^2}-\frac{1}{12d\,a^4(\tan(dx+c)-i)^3}+\frac{1}{16a^4d(\tan(dx-}$
norman	$\frac{\frac{x}{16a}+\frac{5\left(\tan^3(dx+c)\right)}{48ad}+\frac{11\left(\tan^5(dx+c)\right)}{48ad}+\frac{\tan^7(dx+c)}{16ad}+\frac{x\left(\tan^2(dx+c)\right)}{4a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{4a}+\frac{x\left(\tan^8(dx+c)\right)}{16a}+}{a^3(1+\tan^2(dx+c))^4}$

input
```
int(1/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/16*x/a^4+1/8*I/a^4/d*exp(-2*I*(d*x+c))+3/32*I/a^4/d*exp(-4*I*(d*x+c))+1/
24*I/a^4/d*exp(-6*I*(d*x+c))+1/128*I/a^4/d*exp(-8*I*(d*x+c))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(24\,dxe^{(8i\,dx+8i\,c)}+48i\,e^{(6i\,dx+6i\,c)}+36i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/384*(24*d*x*e^(8*I*d*x + 8*I*c) + 48*I*e^(6*I*d*x + 6*I*c) + 36*I*e^(4*I
*d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-8*I*d*x - 8*I*c)/(a^4*
d)
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(98304ia^{12}d^3e^{18ic}e^{-2idx}+73728ia^{12}d^3e^{16ic}e^{-4idx}+32768ia^{12}d^3e^{14ic}e^{-6idx}+6144ia^{12}d^3e^{12ic}e^{-8idx})e^{-20ic}}{786432a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0 \\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-8ic}}{16a^4}-\frac{1}{16a^4}\right) & \text{otherwise} \end{cases} + \frac{x}{16a^4}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((98304*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 73728*I*a**12*d
**3*exp(16*I*c)*exp(-4*I*d*x) + 32768*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*
x) + 6144*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(786432*a**
16*d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) +
 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-8*I*c)/(16*a**4) - 1/(16*a**4)), Tr
ue)) + x/(16*a**4)
```

3.155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = -\frac{-\frac{12i\log(\tan(dx+c)+i)}{a^4} + \frac{12i\log(\tan(dx+c)-i)}{a^4} + \frac{-25i\tan(dx+c)^4-124\tan(dx+c)^3+246i\tan(dx+c)^2+252\tan(dx+c)-153i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/384*(-12*I*log(tan(d*x + c) + I)/a^4 + 12*I*log(tan(d*x + c) - I)/a^4 +
 (-25*I*tan(d*x + c)^4 - 124*tan(d*x + c)^3 + 246*I*tan(d*x + c)^2 + 252*t
an(d*x + c) - 153*I)/(a^4*(tan(d*x + c) - I)^4))/d
```

3.155.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{x}{16\,a^4} - \frac{-\frac{\tan(c+d\,x)^3}{16} + \frac{\tan(c+d\,x)^2\,1\mathrm{i}}{4} + \frac{19\tan(c+d\,x)}{48} - \frac{1}{3}\mathrm{i}}{a^4\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^4}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
x/(16*a^4) - ((19*tan(c + d*x))/48 + (tan(c + d*x)^2*1i)/4 - tan(c + d*x)^
3/16 - 1i/3)/(a^4*d*(tan(c + d*x)*1i + 1)^4)
```

3.156 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.156.1 Optimal result 1199
3.156.2 Mathematica [A] (verified) 1199
3.156.3 Rubi [A] (verified) 1200
3.156.4 Maple [A] (verified) 1201
3.156.5 Fricas [A] (verification not implemented) 1202
3.156.6 Sympy [A] (verification not implemented) 1202
3.156.7 Maxima [F(-2)] 1203
3.156.8 Giac [A] (verification not implemented) 1203
3.156.9 Mupad [B] (verification not implemented) 1204

3.156.1 Optimal result

Integrand size = 24, antiderivative size = 169

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = {} & \frac{3x}{32a^4}+\frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}\\ & +\frac{i}{16ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}\\ & -\frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
3/32*x/a^4+1/20*I*a/d/(a+I*a*tan(d*x+c))^5+1/16*I/d/(a+I*a*tan(d*x+c))^4+1
/16*I/a/d/(a+I*a*tan(d*x+c))^3+1/16*I/d/(a^2+I*a^2*tan(d*x+c))^2-1/64*I/d/
(a^4-I*a^4*tan(d*x+c))+5/64*I/d/(a^4+I*a^4*tan(d*x+c))
```

3.156.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$=\frac{\sec^6(c+dx)(50i+100i\cos(2(c+dx))+46i\cos(4(c+dx))-4i\cos(6(c+dx))-50\sin(2(c+dx))+60}{640a^4d(-i+\tan(c+dx))^5}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^6*(50*I + (100*I)*Cos[2*(c + d*x)] + (46*I)*Cos[4*(c + d*x)]
 - (4*I)*Cos[6*(c + d*x)] - 50*Sin[2*(c + d*x)] + 60*ArcTan[Tan[c + d*x]]*
(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) - 31*Sin[4*(c + d*x)] + 6*Sin[6*(c
 + d*x)]))/(640*a^4*d*(-I + Tan[c + d*x])^5*(I + Tan[c + d*x]))
```

3.156.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^6}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^3\int \Big(\frac{1}{64a^6(a-ia\tan(c+dx))^2}+\frac{5}{64a^6(i\tan(c+dx)a+a)^2}+\frac{1}{8a^5(i\tan(c+dx)a+a)^3}+\frac{3}{16a^4(i\tan(c+dx)a+a)^4}+\frac{1}{4a^3(i\tan(c+dx)a+a)^5}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\Big(\frac{3i\arctan(\tan(c+dx))}{32a^7}+\frac{1}{64a^6(a-ia\tan(c+dx))}-\frac{5}{64a^6(a+ia\tan(c+dx))}-\frac{1}{16a^5(a+ia\tan(c+dx))^2}-\frac{1}{16a^4(a+ia\tan(c+dx))^3}-}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*a^3*((((3*I)/32)*ArcTan[Tan[c + d*x]])/a^7 + 1/(64*a^6*(a - I*a*Tan[
c + d*x])) - 1/(20*a^2*(a + I*a*Tan[c + d*x])^5) - 1/(16*a^3*(a + I*a*Tan[
c + d*x])^4) - 1/(16*a^4*(a + I*a*Tan[c + d*x])^3) - 1/(16*a^5*(a + I*a*Ta
n[c + d*x])^2) - 5/(64*a^6*(a + I*a*Tan[c + d*x]))))/d
```

3.156.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.156.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{\frac{3 i \ln \left(\tan \left(d x +c \right)+i\right)}{64}+\frac{1}{64 \tan \left(d x +c \right)+64 i}-\frac{3 i \ln \left(\tan \left(d x +c \right)-i\right)}{64}+\frac{i}{16 \left(\tan \left(d x +c \right)-i\right)^{4}}-\frac{i}{16 \left(\tan \left(d x +c \right)-i\right)^{2}}+\frac{1}{20 \left(\tan \left(d x +c \right)-i\right)^{5}}-}{d \,a^{4}}$
default	$\frac{\frac{3 i \ln \left(\tan \left(d x +c \right)+i\right)}{64}+\frac{1}{64 \tan \left(d x +c \right)+64 i}-\frac{3 i \ln \left(\tan \left(d x +c \right)-i\right)}{64}+\frac{i}{16 \left(\tan \left(d x +c \right)-i\right)^{4}}-\frac{i}{16 \left(\tan \left(d x +c \right)-i\right)^{2}}+\frac{1}{20 \left(\tan \left(d x +c \right)-i\right)^{5}}-}{d \,a^{4}}$
risch	$\frac{3 x}{32 a^{4}}+\frac{5 i {\mathrm e}^{-4 i \left(d x +c \right)}}{64 a^{4} d}+\frac{5 i {\mathrm e}^{-6 i \left(d x +c \right)}}{128 a^{4} d}+\frac{3 i {\mathrm e}^{-8 i \left(d x +c \right)}}{256 a^{4} d}+\frac{i {\mathrm e}^{-10 i \left(d x +c \right)}}{640 a^{4} d}+\frac{7 i \cos \left(2 d x +2 c \right)}{64 a^{4} d}+\frac{\sin \left(2 d x +2 c \right)}{8 a^{4} d}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(3/64*I*ln(tan(d*x+c)+I)+1/64/(tan(d*x+c)+I)-3/64*I*ln(tan(d*x+c)-
I)+1/16*I/(tan(d*x+c)-I)^4-1/16*I/(tan(d*x+c)-I)^2+1/20/(tan(d*x+c)-I)^5-1
/16/(tan(d*x+c)-I)^3+5/64/(tan(d*x+c)-I))
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.51

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{(120\,dxe^{(10i\,dx+10i\,c)} - 10i\,e^{(12i\,dx+12i\,c)} + 150i\,e^{(8i\,dx+8i\,c)} + 100i\,e^{(6i\,dx+6i\,c)} + 50i\,e^{(4i\,dx+4i\,c)} + 15i\,e^{(2i\,dx+2i\,c)}}{1280\,a^4d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/1280*(120*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(12*I*d*x + 12*I*c) + 150*I
*e^(8*I*d*x + 8*I*c) + 100*I*e^(6*I*d*x + 6*I*c) + 50*I*e^(4*I*d*x + 4*I*c
) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-10*I*d*x - 10*I*c)/(a^4*d)
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-171798691840ia^{20}d^5e^{32ic}e^{2idx}+2576980377600ia^{20}d^5e^{28ic}e^{-2idx}+1717986918400ia^{20}d^5e^{26ic}e^{-4idx}+858993459200ia^{20}d^5e^{24ic}e^{-6idx}+2}{21990232555520a^{24}d^6} \\ x\left(\frac{(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-10ic}}{64a^4}-\frac{3}{32a^4}\right) \end{cases} + \frac{3x}{32a^4}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-171798691840*I*a**20*d**5*exp(32*I*c)*exp(2*I*d*x) + 25769803
77600*I*a**20*d**5*exp(28*I*c)*exp(-2*I*d*x) + 1717986918400*I*a**20*d**5*
exp(26*I*c)*exp(-4*I*d*x) + 858993459200*I*a**20*d**5*exp(24*I*c)*exp(-6*I
*d*x) + 257698037760*I*a**20*d**5*exp(22*I*c)*exp(-8*I*d*x) + 34359738368*
I*a**20*d**5*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(21990232555520*a**2
4*d**6), Ne(a**24*d**6*exp(30*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c)
+ 15*exp(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-1
0*I*c)/(64*a**4) - 3/(32*a**4)), True)) + 3*x/(32*a**4)
```

3.156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{-\frac{60i\log(\tan(dx+c)+i)}{a^4} + \frac{60i\log(\tan(dx+c)-i)}{a^4} + \frac{20\,(3i\tan(dx+c)-4)}{a^4(\tan(dx+c)+i)} + \frac{-137i\tan(dx+c)^5-785\tan(dx+c)^4+1850i\tan(dx+c)^3+2}{a^4(\tan(dx+c)-i)^5}}{1280\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/1280*(-60*I*log(tan(d*x + c) + I)/a^4 + 60*I*log(tan(d*x + c) - I)/a^4
+ 20*(3*I*tan(d*x + c) - 4)/(a^4*(tan(d*x + c) + I)) + (-137*I*tan(d*x + c
)^5 - 785*tan(d*x + c)^4 + 1850*I*tan(d*x + c)^3 + 2290*tan(d*x + c)^2 - 1
565*I*tan(d*x + c) - 541)/(a^4*(tan(d*x + c) - I)^5))/d
```

3.156.9 Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.53

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{3\,x}{32\,a^4} - \frac{-\frac{3\tan(c+d\,x)^5}{32} + \frac{\tan(c+d\,x)^4\,3\mathrm{i}}{8} + \frac{\tan(c+d\,x)^3}{2} - \frac{\tan(c+d\,x)^2\,1\mathrm{i}}{8} + \frac{47\tan(c+d\,x)}{160} - \frac{3}{10}\mathrm{i}}{a^4\,d\,(\tan(c+d\,x)-\mathrm{i})^5\,(\tan(c+d\,x)+1\mathrm{i})}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(3*x)/(32*a^4) - ((47*tan(c + d*x))/160 - (tan(c + d*x)^2*1i)/8 + tan(c +
d*x)^3/2 + (tan(c + d*x)^4*3i)/8 - (3*tan(c + d*x)^5)/32 - 3i/10)/(a^4*d*(
tan(c + d*x) - 1i)^5*(tan(c + d*x) + 1i))
```

3.157 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.157.1 Optimal result 1205
3.157.2 Mathematica [A] (verified) 1206
3.157.3 Rubi [A] (verified) 1206
3.157.4 Maple [A] (verified) 1208
3.157.5 Fricas [A] (verification not implemented) 1208
3.157.6 Sympy [A] (verification not implemented) 1209
3.157.7 Maxima [F(-2)] 1209
3.157.8 Giac [A] (verification not implemented) 1210
3.157.9 Mupad [B] (verification not implemented) 1210

3.157.1 Optimal result

Integrand size = 24, antiderivative size = 224

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =& \frac{7x}{64a^4}+\frac{ia^2}{48d(a+ia\tan(c+dx))^6}+\frac{3ia}{80d(a+ia\tan(c+dx))^5}\\ &+\frac{3i}{64d(a+ia\tan(c+dx))^4}+\frac{5i}{96ad(a+ia\tan(c+dx))^3}\\ &-\frac{i}{256d\left(a^2-ia^2\tan(c+dx)\right)^2}+\frac{15i}{256d\left(a^2+ia^2\tan(c+dx)\right)^2}\\ &-\frac{7i}{256d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{21i}{256d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
7/64*x/a^4+1/48*I*a^2/d/(a+I*a*tan(d*x+c))^6+3/80*I*a/d/(a+I*a*tan(d*x+c))
^5+3/64*I/d/(a+I*a*tan(d*x+c))^4+5/96*I/a/d/(a+I*a*tan(d*x+c))^3-1/256*I/d
/(a^2-I*a^2*tan(d*x+c))^2+15/256*I/d/(a^2+I*a^2*tan(d*x+c))^2-7/256*I/d/(a
^4-I*a^4*tan(d*x+c))+21/256*I/d/(a^4+I*a^4*tan(d*x+c))
```

3.157.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.73

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^8(c+dx)(525i+1120i\cos(2(c+dx))+504i\cos(4(c+dx))-96i\cos(6(c+dx))-5i\cos(8(c+dx))-}{768}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^8*(525*I + (1120*I)*Cos[2*(c + d*x)] + (504*I)*Cos[4*(c + d*
x)] - (96*I)*Cos[6*(c + d*x)] - (5*I)*Cos[8*(c + d*x)] - 560*Sin[2*(c + d*
x)] + 840*ArcTan[Tan[c + d*x]]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) - 2
94*Sin[4*(c + d*x)] + 144*Sin[6*(c + d*x)] + 10*Sin[8*(c + d*x)]))/(7680*a
^4*d*(-I + Tan[c + d*x])^6*(I + Tan[c + d*x])^2)
```

3.157.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^7}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{54}$$

$$-\frac{ia^5\int \left(\frac{7}{256a^8(a-ia\tan(c+dx))^2}+\frac{21}{256a^8(i\tan(c+dx)a+a)^2}+\frac{1}{128a^7(a-ia\tan(c+dx))^3}+\frac{15}{128a^7(i\tan(c+dx)a+a)^3}+\frac{5}{32a^6(i\tan(c+dx)}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^5\left(\frac{7i\arctan(\tan(c+dx))}{64a^9}+\frac{7}{256a^8(a-ia\tan(c+dx))}-\frac{21}{256a^8(a+ia\tan(c+dx))}+\frac{1}{256a^7(a-ia\tan(c+dx))^2}-\frac{15}{256a^7(a+ia\tan(c+dx))^2}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-I)*a^5*((((7*I)/64)*ArcTan[Tan[c + d*x]])/a^9 + 1/(256*a^7*(a - I*a*Tan
[c + d*x])^2) + 7/(256*a^8*(a - I*a*Tan[c + d*x])) - 1/(48*a^3*(a + I*a*Ta
n[c + d*x])^6) - 3/(80*a^4*(a + I*a*Tan[c + d*x])^5) - 3/(64*a^5*(a + I*a*
Tan[c + d*x])^4) - 5/(96*a^6*(a + I*a*Tan[c + d*x])^3) - 15/(256*a^7*(a +
I*a*Tan[c + d*x])^2) - 21/(256*a^8*(a + I*a*Tan[c + d*x]))))/d
```

3.157.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.157.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{7i\ln(\tan(dx+c)-i)}{128}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{15i}{256(\tan(dx+c)-i)^2}+\frac{3}{80(\tan(dx+c)-i)^5}-\frac{5}{96(\tan(dx+c)-i)^3}+}{d\,a^4}$
default	$\frac{-\frac{7i\ln(\tan(dx+c)-i)}{128}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{15i}{256(\tan(dx+c)-i)^2}+\frac{3}{80(\tan(dx+c)-i)^5}-\frac{5}{96(\tan(dx+c)-i)^3}+}{d\,a^4}$
risch	$\frac{7x}{64a^4}+\frac{7i\mathrm{e}^{-6i(dx+c)}}{192a^4d}+\frac{7i\mathrm{e}^{-8i(dx+c)}}{512a^4d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{320a^4d}+\frac{i\mathrm{e}^{-12i(dx+c)}}{3072a^4d}+\frac{69i\cos(4dx+4c)}{1024a^4d}+\frac{71\sin(4dx+4c)}{1024a^4d}+$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(-7/128*I*ln(tan(d*x+c)-I)+3/64*I/(tan(d*x+c)-I)^4-1/48*I/(tan(d*x
+c)-I)^6-15/256*I/(tan(d*x+c)-I)^2+3/80/(tan(d*x+c)-I)^5-5/96/(tan(d*x+c)-
I)^3+21/256/(tan(d*x+c)-I)+1/256*I/(tan(d*x+c)+I)^2+7/128*I*ln(tan(d*x+c)+
I)+7/256/(tan(d*x+c)+I))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(1680\,dxe^{(12i\,dx+12i\,c)}-15i\,e^{(16i\,dx+16i\,c)}-240i\,e^{(14i\,dx+14i\,c)}+1680i\,e^{(10i\,dx+10i\,c)}+1050i\,e^{(8i\,dx+8i\,c)}+560i\,e\right.}{15360\,a^4d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/15360*(1680*d*x*e^(12*I*d*x + 12*I*c) - 15*I*e^(16*I*d*x + 16*I*c) - 240
*I*e^(14*I*d*x + 14*I*c) + 1680*I*e^(10*I*d*x + 10*I*c) + 1050*I*e^(8*I*d*
x + 8*I*c) + 560*I*e^(6*I*d*x + 6*I*c) + 210*I*e^(4*I*d*x + 4*I*c) + 48*I*
e^(2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^4*d)
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.46

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{(-20266198323167232 0ia^{28}d^7e^{46ic}e^{4idx}-3242591731706757120ia^{28}d^7e^{44ic}e^{2idx}+2269814212194729 9840ia^{28}d^7e^{40ic}e^{-2idx}+141863388262}{} \\ x\left(\frac{(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-12ic}}{256a^4}-\frac{7}{64a^4}\right) \end{cases}$$
$$+\frac{7x}{64a^4}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-202661983231672320*I*a**28*d**7*exp(46*I*c)*exp(4*I*d*x) - 32
42591731706757120*I*a**28*d**7*exp(44*I*c)*exp(2*I*d*x) + 2269814212194729
9840*I*a**28*d**7*exp(40*I*c)*exp(-2*I*d*x) + 1418633882621706240 0*I*a**28
*d**7*exp(38*I*c)*exp(-4*I*d*x) + 756604737398243328 0*I*a**28*d**7*exp(36*
I*c)*exp(-6*I*d*x) + 283726776524341248 0*I*a**28*d**7*exp(34*I*c)*exp(-8*I
*d*x) + 648518346341351424*I*a**28*d**7*exp(32*I*c)*exp(-10*I*d*x) + 67553
994410557440*I*a**28*d**7*exp(30*I*c)*exp(-12*I*d*x))*exp(-42*I*c)/(207525
870829232455680*a**32*d**8), Ne(a**32*d**8*exp(42*I*c), 0)), (x*((exp(16*I
*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56
*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-12*I*c)/(256*a**4) -
7/(64*a**4)), True)) + 7*x/(64*a**4)
```

3.157.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{-\frac{420i\log(\tan(dx+c)+i)}{a^4} + \frac{420i\log(\tan(dx+c)-i)}{a^4} + \frac{30\left(21i\tan(dx+c)^2-49\tan(dx+c)-29i\right)}{a^4(\tan(dx+c)+i)^2} + \frac{-1029i\tan(dx+c)^6-6804\tan(dx+}{}}{7680\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/7680*(-420*I*log(tan(d*x + c) + I)/a^4 + 420*I*log(tan(d*x + c) - I)/a^
4 + 30*(21*I*tan(d*x + c)^2 - 49*tan(d*x + c) - 29*I)/(a^4*(tan(d*x + c) +
 I)^2) + (-1029*I*tan(d*x + c)^6 - 6804*tan(d*x + c)^5 + 19035*I*tan(d*x +
 c)^4 + 29080*tan(d*x + c)^3 - 25995*I*tan(d*x + c)^2 - 13332*tan(d*x + c)
 + 3317*I)/(a^4*(tan(d*x + c) - I)^6))/d
```

3.157.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{7\,x}{64\,a^4} + \frac{\frac{\tan(c+d\,x)\,169\mathrm{i}}{960\,a^4} + \frac{4}{15\,a^4} + \frac{119\tan(c+d\,x)^2}{240\,a^4} + \frac{\tan(c+d\,x)^3\,889\mathrm{i}}{960\,a^4} - \frac{7\tan(c+d\,x)^4}{24\,a^4} + \frac{\tan(c+d\,x)^5\,91\mathrm{i}}{192\,a^4}}{d\,\left(-\tan\left(c+d\,x\right)^8\,1\mathrm{i} - 4\tan\left(c+d\,x\right)^7 + \tan\left(c+d\,x\right)^6\,4\mathrm{i} - 4\tan\left(c+d\,x\right)^5 + \tan\left(c+d\,x\right)^4\,10\mathrm{i} + 4\tan\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(7*x)/(64*a^4) + ((tan(c + d*x)*169i)/(960*a^4) + 4/(15*a^4) + (119*tan(c
+ d*x)^2)/(240*a^4) + (tan(c + d*x)^3*889i)/(960*a^4) - (7*tan(c + d*x)^4)
/(24*a^4) + (tan(c + d*x)^5*91i)/(192*a^4) - (7*tan(c + d*x)^6)/(16*a^4) -
 (tan(c + d*x)^7*7i)/(64*a^4))/(d*(4*tan(c + d*x) + tan(c + d*x)^2*4i + 4*
tan(c + d*x)^3 + tan(c + d*x)^4*10i - 4*tan(c + d*x)^5 + tan(c + d*x)^6*4i
 - 4*tan(c + d*x)^7 - tan(c + d*x)^8*1i - 1i))
```

3.158 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.158.1 Optimal result 1211
3.158.2 Mathematica [A] (verified) 1211
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3.158.1 Optimal result

Integrand size = 24, antiderivative size = 133

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{35\text{arctanh}(\sin(c+dx))}{8a^4d}+\frac{35\sec(c+dx)\tan(c+dx)}{8a^4d}\\ &\quad+\frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}\\ &\quad-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
35/8*arctanh(sin(d*x+c))/a^4/d+35/8*sec(d*x+c)*tan(d*x+c)/a^4/d+35/12*sec(
d*x+c)^3*tan(d*x+c)/a^4/d-2*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^3-14/3*I
*sec(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

3.158.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{\sec^4(c+dx)\left(896i\cos(c+dx)+3\left(128i\cos(3(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

output

```
-1/192*(Sec[c + d*x]^4*((896*I)*Cos[c + d*x] + 3*((128*I)*Cos[3*(c + d*x)]
 + 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) -
105*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)])
))/(a^4*d)
```

3.158.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\left(\frac{5\int \sec^5(c+dx)dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-2*I)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^3) + (7*(((((-2*I)/3)*S
ec[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (5*((Sec[c + d*x]^3*Tan[c
+ d*x])/(4*d) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*
x])/(2*d)))/4))/(3*a^2)))/a^2
```

3.158. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.158.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.158.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{7i(dx+c)}+385\,\mathrm{e}^{5i(dx+c)}+511\,\mathrm{e}^{3i(dx+c)}+279\,\mathrm{e}^{i(dx+c)}\right)}{12d\,a^{4}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}+\frac{35\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^{4}d}-\frac{35\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^{4}d}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{27}{16}+3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}{a^{4}d}$
default	$\frac{\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{27}{16}+3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}{a^{4}d}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d/a^4/(exp(2*I*(d*x+c))+1)^4*(105*exp(7*I*(d*x+c))+385*exp(5*I*(d*
x+c))+511*exp(3*I*(d*x+c))+279*exp(I*(d*x+c)))+35/8/a^4/d*ln(exp(I*(d*x+c)
)+I)-35/8/a^4/d*ln(exp(I*(d*x+c))-I)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.73

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{105\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(8i\,dx+8i\,c)}+4\,e}{24\left(a^4de^{(8i\,dx+8i\,c)}+4\,a^4de}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(105*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*
I*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(8*I*d
*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x
 + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 210*I*e^(7*I*d*x + 7*I*c) - 770*
I*e^(5*I*d*x + 5*I*c) - 1022*I*e^(3*I*d*x + 3*I*c) - 558*I*e^(I*d*x + I*c)
)/(a^4*d*e^(8*I*d*x + 8*I*c) + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*
I*d*x + 4*I*c) + 4*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

3.158.6 Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^9(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**9/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.158.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(117) = 234$.

Time = 0.23 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.22

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{\frac{2\left(\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{544i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{105\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{480i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{105\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{96i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+160i\right)}{a^4-\frac{4a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-105\log}{24\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/24*(2*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 544*I*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 105*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 480*I*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 96*
I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + 160*I)/(a^4 - 4*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^4*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 105*log(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)/a^4 + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)
/d
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$=\frac{\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^4}-\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4}-\frac{2\left(81\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+480i\tan\right.}{\left(\tan\left(\frac{1}{2}dx\right.\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^4 - 2*(81*tan(1/2*d*x + 1/2*c)^7 - 96*I*tan(1/2*d*x + 1/2*c)^6 - 1
05*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan(1/2*d*x + 1/2*c)^4 - 105*tan(1/2*d*x
 + 1/2*c)^3 - 544*I*tan(1/2*d*x + 1/2*c)^2 + 81*tan(1/2*d*x + 1/2*c) + 160
*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^4))/d
```

3.158.9 Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{35\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^4\,d}$$
$$+\frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4\,a^4}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4\,a^4}-\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4\,a^4}-\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 136\mathrm{i}}{3\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 40\mathrm{i}}{a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{a^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(35*atanh(tan(c/2 + (d*x)/2)))/(4*a^4*d) + ((tan(c/2 + (d*x)/2)^2*136i)/(3
*a^4) + (35*tan(c/2 + (d*x)/2)^3)/(4*a^4) - (tan(c/2 + (d*x)/2)^4*40i)/a^4
 + (35*tan(c/2 + (d*x)/2)^5)/(4*a^4) + (tan(c/2 + (d*x)/2)^6*8i)/a^4 - (27
*tan(c/2 + (d*x)/2)^7)/(4*a^4) - 40i/(3*a^4) - (27*tan(c/2 + (d*x)/2))/(4*
a^4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.159 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

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3.159.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\text{arctanh}(\sin(c+dx))}{2a^4d} - \frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

output
```
-15/2*arctanh(sin(d*x+c))/a^4/d-15/2*sec(d*x+c)*tan(d*x+c)/a^4/d+2*I*sec(d
*x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+10*I*sec(d*x+c)^3/d/(a^4+I*a^4*tan(d*x+c)
)
```

3.159.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 988 vs. 2(107) = 214.

Time = 6.64 (sec) , antiderivative size = 988, normalized size of antiderivative = 9.23

$$
\begin{aligned}
&\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx \\
&= \frac{15\cos(4c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad - \frac{15\cos(4c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\cos(dx)\sec^4(c+dx)(8i\cos(3c)-8\sin(3c))(\cos(dx)+i\sin(dx))^4}{d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec(c)\sec^4(c+dx)(4i\cos(4c)-4\sin(4c))(\cos(dx)+i\sin(dx))^4}{d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{15i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)\sin(4c)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad - \frac{15i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)\sin(4c)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec^4(c+dx)(8\cos(3c)+8i\sin(3c))(\cos(dx)+i\sin(dx))^4\sin(dx)}{d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec^4(c+dx)\left(\frac{1}{4}\cos(4c)+\frac{1}{4}i\sin(4c)\right)(\cos(dx)+i\sin(dx))^4}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec^4(c+dx)\left(-\frac{1}{4}\cos(4c)-\frac{1}{4}i\sin(4c)\right)(\cos(dx)+i\sin(dx))^4}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^4} \\
&\quad + \frac{4\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\bigl(\frac{1}{2}\cos\bigl(4c-\frac{dx}{2}\bigr)-\frac{1}{2}\cos\bigl(4c+\frac{dx}{2}\bigr)+\frac{1}{2}i\sin\bigl(4c-\frac{dx}{2}\bigr)-\frac{1}{2}i\sin\bigl(4c+}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4} \\
&\quad + \frac{4\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\bigl(-\frac{1}{2}\cos\bigl(4c-\frac{dx}{2}\bigr)+\frac{1}{2}\cos\bigl(4c+\frac{dx}{2}\bigr)-\frac{1}{2}i\sin\bigl(4c-\frac{dx}{2}\bigr)+\frac{1}{2}i\sin\bigl(4c}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4}
\end{aligned}
$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

output

```
(15*Cos[4*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(
Cos[d*x] + I*Sin[d*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) - (15*Cos[4*c]*Lo
g[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(Cos[d*x] + I*Si
n[d*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) + (Cos[d*x]*Sec[c + d*x]^4*((8*I
)*Cos[3*c] - 8*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*Tan[c + d*
x])^4) + (Sec[c]*Sec[c + d*x]^4*((4*I)*Cos[4*c] - 4*Sin[4*c])*(Cos[d*x] +
I*Sin[d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4) + (((15*I)/2)*Log[Cos[c/2 + (d
*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x
])^4)/(d*(a + I*a*Tan[c + d*x])^4) - (((15*I)/2)*Log[Cos[c/2 + (d*x)/2] +
Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d*
(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(8*Cos[3*c] + (8*I)*Sin[3*c])*
(Cos[d*x] + I*Sin[d*x])^4*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (Sec[c
 + d*x]^4*(Cos[4*c]/4 + (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[
c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (Sec[c
 + d*x]^4*(-1/4*Cos[4*c] - (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(C
os[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (4*S
ec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(Cos[4*c - (d*x)/2]/2 - Cos[4*c +
(d*x)/2]/2 + (I/2)*Sin[4*c - (d*x)/2] - (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos
[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c
 + d*x])^4) + (4*Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-1/2*Cos[4*c...
```

3.159.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \; 3981$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int\sec^3(c+dx)dx}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^3) - (5*(((-2*I)*Sec[c
+ d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])) + (3*(ArcTanh[Sin[c + d*x]]/(2*d)
 + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/a^2))/a^2
```

3.159 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.159.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.159.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

method	result
risch	$\frac{8i\,\mathrm{e}^{-i(dx+c)}}{a^4 d}+\frac{i\left(7\,\mathrm{e}^{3i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{15\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^4 d}+\frac{15\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^4 d}$
derivativedivides	$\frac{\frac{16}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\left(\frac{1}{4}-2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(\frac{1}{4}+2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-}{a^4 d}$
default	$\frac{\frac{16}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\left(\frac{1}{4}-2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(\frac{1}{4}+2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-}{a^4 d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
8*I/a^4/d*exp(-I*(d*x+c))+I/d/a^4/(exp(2*I*(d*x+c))+1)^2*(7*exp(3*I*(d*x+c
))+9*exp(I*(d*x+c)))-15/2/a^4/d*ln(exp(I*(d*x+c))+I)+15/2/a^4/d*ln(exp(I*(
d*x+c))-I)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.50

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\mathrm{l}}{2\left(a^4de^{(5i\,dx+5i\,c)}+2\,a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/2*(15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*l
og(e^(I*d*x + I*c) + I) - 15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c)
+ e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 30*I*e^(4*I*d*x + 4*I*c) - 5
0*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*
I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

3.159.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^7(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**7/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.159.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(95) = 190$.

Time = 0.32 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.27

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{30\,(\cos(5\,dx+5\,c)+2\,\cos(3\,dx+3\,c)+\cos(dx+c)+i\,\sin(5\,dx+5\,c)+2i\,\sin(3\,dx+3\,c)+i\,\sin(dx}{}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
(30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(5*d*x +
5*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*
x + c) + 1) + 30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I
*sin(5*d*x + 5*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x
 + c), -sin(d*x + c) + 1) + 15*(I*cos(5*d*x + 5*c) + 2*I*cos(3*d*x + 3*c)
+ I*cos(d*x + c) - sin(5*d*x + 5*c) - 2*sin(3*d*x + 3*c) - sin(d*x + c))*l
og(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 15*(-I*cos(5*d*
x + 5*c) - 2*I*cos(3*d*x + 3*c) - I*cos(d*x + c) + sin(5*d*x + 5*c) + 2*si
n(3*d*x + 3*c) + sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin
(d*x + c) + 1) + 60*cos(4*d*x + 4*c) + 100*cos(2*d*x + 2*c) + 60*I*sin(4*d
*x + 4*c) + 100*I*sin(2*d*x + 2*c) + 32)/((-4*I*a^4*cos(5*d*x + 5*c) - 8*I
*a^4*cos(3*d*x + 3*c) - 4*I*a^4*cos(d*x + c) + 4*a^4*sin(5*d*x + 5*c) + 8*
a^4*sin(3*d*x + 3*c) + 4*a^4*sin(d*x + c))*d)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}-\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{32}{a^4(\tan(\frac{1}{2}\,dx}}{2\,d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/2*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 15*log(tan(1/2*d*x + 1/2*c) -
 1)/a^4 - 2*(tan(1/2*d*x + 1/2*c)^3 - 8*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2
*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - 32/(a^4*(tan(1
/2*d*x + 1/2*c) - I)))/d
```

3.159.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.51

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d} + \frac{\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a^4} - \frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 39\mathrm{i}}{a^4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 17\mathrm{i}}{a^4} + \frac{24\mathrm{i}}{a^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 2\mathrm{i} - 2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right) 1\mathrm{i} + 1\right)}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
((9*tan(c/2 + (d*x)/2)^3)/a^4 - (tan(c/2 + (d*x)/2)^2*39i)/a^4 + (tan(c/2
+ (d*x)/2)^4*17i)/a^4 + 24i/a^4 - (7*tan(c/2 + (d*x)/2))/a^4)/(d*(tan(c/2
+ (d*x)/2)*1i - 2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*2i + tan(c/2
 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*1i + 1)) - (15*atanh(tan(c/2 + (d*x)/
2)))/(a^4*d)
```

3.160 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.160.1 Optimal result 1226
3.160.2 Mathematica [B] (verified) 1226
3.160.3 Rubi [A] (verified) 1227
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3.160.8 Giac [A] (verification not implemented) 1230
3.160.9 Mupad [B] (verification not implemented) 1231

3.160.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

output
```
arctanh(sin(d*x+c))/a^4/d+2/3*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^3-2*I*
sec(d*x+c)/d/(a^4+I*a^4*tan(d*x+c))
```

3.160.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 247 vs. 2(82) = 164.

Time = 0.61 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.01

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(-3\cos(4c)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(\cos\right.\right.$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Sin[3*c] - (3*I)*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]]*Sin[4*c] + (3*I)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]]*Sin[4*c] + Cos[3*c]*((-6*I)*Cos[d*x] - 6*Sin[d*x]) - (6*I)*Sin[3*c]*S
in[d*x] + (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c]*(I*Cos[3*d*x] + Sin[3*d*x]))
/(3*a^4*d*(-I + Tan[c + d*x])^4)
```

3.160.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3981, 3042, 3981, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int \sec(c+dx)dx}{a^2}+\frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}+\frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}}{a^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}}{a^2}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3) - (-(ArcTanh[Sin
[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])))
/a^2
```

3.160.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.160.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^4d}$	71
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^4d}$	71
risch	$-\frac{2i\,\mathrm{e}^{-i(dx+c)}}{a^4d}+\frac{2i\,\mathrm{e}^{-3i(dx+c)}}{3a^4d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^4d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^4d}$	79

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^4*(1/2*ln(tan(1/2*d*x+1/2*c)+1)+4*I/(-I+tan(1/2*d*x+1/2*c))^2-8/3/(-
I+tan(1/2*d*x+1/2*c))^3-1/2*ln(tan(1/2*d*x+1/2*c)-1))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^4d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/3*(3*e^(3*I*d*x + 3*I*c)*log(e^(I*d*x + I*c) + I) - 3*e^(3*I*d*x + 3*I*c
)*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-3*I*d*x -
3*I*c)/(a^4*d)
```

3.160.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**5/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{-6i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-6i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+4i\cos\left(3\,dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/6*(-6*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 6*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) + 4*I*cos(3*d*x + 3*c) - 12*I*cos(d*x + c) + 3*log(
cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - 3*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*sin(3*d*x + 3*c) - 12*sin(d*
x + c))/(a^4*d)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}-\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}+\frac{8\,\left(3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}}{3\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 3*log(tan(1/2*d*x + 1/2*c) - 1)
/a^4 + 8*(3*I*tan(1/2*d*x + 1/2*c) + 1)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^3)
)/d
```

3.160.9 Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d} - \frac{-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}+\frac{8\mathrm{i}}{3\,a^4}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (8i/(3*a^4) - (8*tan(c/2 + (d*x)/2
))/a^4)/(d*(tan(c/2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*
x)/2)^3*1i + 1))
```

3.161 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.161.1 Optimal result 1232
3.161.2 Mathematica [A] (verified) 1232
3.161.3 Rubi [A] (verified) 1233
3.161.4 Maple [A] (verified) 1234
3.161.5 Fricas [A] (verification not implemented) 1235
3.161.6 Sympy [B] (verification not implemented) 1235
3.161.7 Maxima [A] (verification not implemented) 1236
3.161.8 Giac [A] (verification not implemented) 1236
3.161.9 Mupad [B] (verification not implemented) 1236

3.161.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4} + \frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}$$

output
```
1/5*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+1/15*I*sec(d*x+c)^3/a/d/(a+I*a*t
an(d*x+c))^3
```

3.161.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\sec^3(c+dx)(-4i+\tan(c+dx))}{15a^4d(-i+\tan(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/15*(Sec[c + d*x]^3*(-4*I + Tan[c + d*x]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

3.161.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec[c + d*x]
^3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

3.161.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.161.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{i\,\mathrm{e}^{-3i(dx+c)}}{6a^4d}+\frac{i\,\mathrm{e}^{-5i(dx+c)}}{10a^4d}$	38
derivativedivides	$\frac{-\frac{28}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{16}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^4d}$	90
default	$\frac{-\frac{28}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{16}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^4d}$	90

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*I/a^4/d*exp(-3*I*(d*x+c))+1/10*I/a^4/d*exp(-5*I*(d*x+c))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(5i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{30\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/30*(5*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a^4*d)
```

3.161.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(54) = 108.

Time = 1.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.68

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{\tan(c+dx)\sec^3(c+dx)}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a^4d\tan^2(c+dx)+60ia^4d\tan(c+dx)+15a^4d} + \frac{4i\sec^3(c+}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a^4} \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^4} \end{cases}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((-tan(c + d*x)*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I
*a**4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c +
d*x) + 15*a**4*d) + 4*I*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I*
a**4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c + d
*x) + 15*a**4*d), Ne(d, 0)), (x*sec(c)**3/(I*a*tan(c) + a)**4, True))
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3i\,\cos\left(5\,dx+5\,c\right)+5i\,\cos\left(3\,dx+3\,c\right)+3\,\sin\left(5\,dx+5\,c\right)+5\,\sin\left(3\,dx+3\,c\right)}{30\,a^4d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/30*(3*I*cos(5*d*x + 5*c) + 5*I*cos(3*d*x + 3*c) + 3*sin(5*d*x + 5*c) + 5
*sin(3*d*x + 3*c))/(a^4*d)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{15\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 15*I*tan(1/2*d*x + 1/2*c)^3 - 25*tan(1/2
*d*x + 1/2*c)^2 + 5*I*tan(1/2*d*x + 1/2*c) + 4)/(a^4*d*(tan(1/2*d*x + 1/2*
c) - I)^5)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.96

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15\mathrm{i}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 25\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4\mathrm{i}\right)}{15\,a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,5\mathrm{i}+1\right.}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
(2*(15*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*25i - 5*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*15i + 4i))/(15*a^4*d*(tan(c/2 + (d*x)/2)*5i - 1
0*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4
 + tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.161. $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.162 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.162.1 Optimal result 1238
3.162.2 Mathematica [A] (verified) 1238
3.162.3 Rubi [A] (verified) 1239
3.162.4 Maple [A] (verified) 1241
3.162.5 Fricas [A] (verification not implemented) 1241
3.162.6 Sympy [B] (verification not implemented) 1242
3.162.7 Maxima [A] (verification not implemented) 1242
3.162.8 Giac [A] (verification not implemented) 1243
3.162.9 Mupad [B] (verification not implemented) 1243

3.162.1 Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{2i\sec(c+dx)}{35d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
1/7*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^4+3/35*I*sec(d*x+c)/a/d/(a+I*a*tan(d
*x+c))^3+2/35*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2+2/35*I*sec(d*x+c)/d/
(a^4+I*a^4*tan(d*x+c))
```

3.162.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.55

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^4(c+dx)(28\cos(c+dx)+20\cos(3(c+dx))+7i\sin(c+dx)+15i\sin(3(c+dx)))}{140a^4d(-i+\tan(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/140)*Sec[c + d*x]^4*(28*Cos[c + d*x] + 20*Cos[3*(c + d*x)] + (7*I)*Sin
[c + d*x] + (15*I)*Sin[3*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

3.162.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow 3969$$

$$\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3\left(\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\left(\frac{i\sec(c+dx)}{3ad(a+ia\tan(c+dx))}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}\right)}{7a}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (3*(((I/5)*Sec[c + d*x
])/(d*(a + I*a*Tan[c + d*x])^3) + (2*(((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan
[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x]))))/(5*a))
)/(7*a)
```

3.162.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.162.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

method	result
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-3i(dx+c)}}{8a^4d}+\frac{3i\mathrm{e}^{-5i(dx+c)}}{40a^4d}+\frac{i\mathrm{e}^{-7i(dx+c)}}{56a^4d}$
derivativedivides	$\frac{-\frac{16i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}+\frac{8i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^6}+\frac{6i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^2}-\frac{12}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^3}+\frac{2}{-i+\tan(\frac{dx}{2}+\frac{c}{2})}+\frac{72}{5(-i+\tan(\frac{dx}{2}+}}{a^4d}$
default	$\frac{-\frac{16i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}+\frac{8i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^6}+\frac{6i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^2}-\frac{12}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^3}+\frac{2}{-i+\tan(\frac{dx}{2}+\frac{c}{2})}+\frac{72}{5(-i+\tan(\frac{dx}{2}+}}{a^4d}$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^4/d*exp(-I*(d*x+c))+1/8*I/a^4/d*exp(-3*I*(d*x+c))+3/40*I/a^4/d*exp
(-5*I*(d*x+c))+1/56*I/a^4/d*exp(-7*I*(d*x+c))
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(35i\,e^{(6i\,dx+6i\,c)}+35i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/280*(35*I*e^(6*I*d*x + 6*I*c) + 35*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d
*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

3.162.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(112) = 224$.

Time = 1.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.68

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{2\tan^3(c+dx)\sec(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a^4d\tan^2(c+dx)+140ia^4d\tan(c+dx)+35a^4d} - \frac{8i\tan^2(c+d}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210} \\ \frac{x\sec(c)}{(ia\tan(c)+a)^4} \end{cases}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise((2*tan(c + d*x)**3*sec(c + d*x)/(35*a**4*d*tan(c + d*x)**4 - 140
*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 140*I*a**4*d*tan(
c + d*x) + 35*a**4*d) - 8*I*tan(c + d*x)**2*sec(c + d*x)/(35*a**4*d*tan(c
+ d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 14
0*I*a**4*d*tan(c + d*x) + 35*a**4*d) - 13*tan(c + d*x)*sec(c + d*x)/(35*a*
*4*d*tan(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d
*x)**2 + 140*I*a**4*d*tan(c + d*x) + 35*a**4*d) + 12*I*sec(c + d*x)/(35*a*
*4*d*tan(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d
*x)**2 + 140*I*a**4*d*tan(c + d*x) + 35*a**4*d), Ne(d, 0)), (x*sec(c)/(I*a
*tan(c) + a)**4, True))
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{5i\cos(7\,dx+7\,c)+21i\cos(5\,dx+5\,c)+35i\cos(3\,dx+3\,c)+35i\cos(dx+c)+5\sin(7\,dx+7\,c)+21}{280\,a^4d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/280*(5*I*cos(7*d*x + 7*c) + 21*I*cos(5*d*x + 5*c) + 35*I*cos(3*d*x + 3*c
) + 35*I*cos(d*x + c) + 5*sin(7*d*x + 7*c) + 21*sin(5*d*x + 5*c) + 35*sin(
3*d*x + 3*c) + 35*sin(d*x + c))/(a^4*d)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{2\left(35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-105i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+210i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+147}{35\,a^4d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
2/35*(35*tan(1/2*d*x + 1/2*c)^6 - 105*I*tan(1/2*d*x + 1/2*c)^5 - 210*tan(1
/2*d*x + 1/2*c)^4 + 210*I*tan(1/2*d*x + 1/2*c)^3 + 147*tan(1/2*d*x + 1/2*c
)^2 - 49*I*tan(1/2*d*x + 1/2*c) - 12)/(a^4*d*(tan(1/2*d*x + 1/2*c) - I)^7)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.48

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{-c\,3\mathrm{i}-d\,x\,3\mathrm{i}}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,3\mathrm{i}}{40}+\frac{\mathrm{e}^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,1\mathrm{i}}{56}}{a^4\,d}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4),x)
```

output
```
((exp(- c*1i - d*x*1i)*1i)/8 + (exp(- c*3i - d*x*3i)*1i)/8 + (exp(- c*5i -
 d*x*5i)*3i)/40 + (exp(- c*7i - d*x*7i)*1i)/56)/(a^4*d)
```

3.163 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

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3.163.1 Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4\sin(c+dx)}{21a^4d} - \frac{4\sin^3(c+dx)}{63a^4d} + \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+dx))}$$

output

```
4/21*sin(d*x+c)/a^4/d-4/63*sin(d*x+c)^3/a^4/d+1/9*I*cos(d*x+c)/d/(a+I*a*ta
n(d*x+c))^4+5/63*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^3+8/63*I*cos(d*x+c)^3
/d/(a^4+I*a^4*tan(d*x+c))
```

3.163.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-168\cos(c+dx)-180\cos(3(c+dx))+28\cos(5(c+dx))-42i\sin(c+dx)-135i\sin(}{1008a^4d(-i+\tan(c+dx))^4}$$

input

```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output

```
((-1/1008*I)*Sec[c + d*x]^4*(-168*Cos[c + d*x] - 180*Cos[3*(c + d*x)] + 28
*Cos[5*(c + d*x)] - (42*I)*Sin[c + d*x] - (135*I)*Sin[3*(c + d*x)] + (35*I
)*Sin[5*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

3.163.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{5\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{5\left(\frac{4\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\left(\frac{4\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{5\left(\frac{4\left(\frac{3\int \cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5\left(\frac{4\left(\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3113}$$

$$\frac{5\left(\frac{4\left(-\frac{3\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(\frac{4\left(-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/9)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (5*(((I/7)*Cos[c + d*x
])/(d*(a + I*a*Tan[c + d*x])^3) + (4*((-3*(-Sin[c + d*x] + Sin[c + d*x]^3/
3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])))
)/(7*a)))/(9*a)
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.163. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.163.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

method	result
risch	$\frac{5i\mathrm{e}^{-3i(dx+c)}}{48a^4d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{16a^4d}+\frac{5i\mathrm{e}^{-7i(dx+c)}}{224a^4d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{288a^4d}+\frac{i\cos(dx+c)}{8a^4d}+\frac{3\sin(dx+c)}{16a^4d}$
derivativedivides	$\frac{\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-}{a^4d}$
default	$\frac{\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-}{a^4d}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
5/48*I/a^4/d*exp(-3*I*(d*x+c))+1/16*I/a^4/d*exp(-5*I*(d*x+c))+5/224*I/a^4/
d*exp(-7*I*(d*x+c))+1/288*I/a^4/d*exp(-9*I*(d*x+c))+1/8*I/a^4/d*cos(d*x+c)
+3/16*sin(d*x+c)/a^4/d
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.55

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-63i\,e^{(10i\,dx+10i\,c)}+315i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+126i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{2016\,a^4d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/2016*(-63*I*e^(10*I*d*x + 10*I*c) + 315*I*e^(8*I*d*x + 8*I*c) + 210*I*e^
(6*I*d*x + 6*I*c) + 126*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) +
 7*I)*e^(-9*I*d*x - 9*I*c)/(a^4*d)
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.72

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-1585446912ia^{20}d^5e^{26ic}e^{idx}+7927234560ia^{20}d^5e^{24ic}e^{-idx}+5284823040ia^{20}d^5e^{22ic}e^{-3idx}+3170893824ia^{20}d^5e^{20ic}e^{-5idx}+1132462080ia^{2}}{50734301184a^{24}d^6} \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-9ic}}{32a^4} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-1585446912*I*a**20*d**5*exp(26*I*c)*exp(I*d*x) + 7927234560*I
*a**20*d**5*exp(24*I*c)*exp(-I*d*x) + 5284823040*I*a**20*d**5*exp(22*I*c)*
exp(-3*I*d*x) + 3170893824*I*a**20*d**5*exp(20*I*c)*exp(-5*I*d*x) + 113246
2080*I*a**20*d**5*exp(18*I*c)*exp(-7*I*d*x) + 176160768*I*a**20*d**5*exp(1
6*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(50734301184*a**24*d**6), Ne(a**24*d**6
*exp(25*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp
(4*I*c) + 5*exp(2*I*c) + 1)*exp(-9*I*c)/(32*a**4), True))
```

3.163.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{63}{a^4(\tan(\frac{1}{2}dx+\frac{1}{2}c)+i)} + \frac{1953\tan(\frac{1}{2}dx+\frac{1}{2}c)^8 - 9450i\tan(\frac{1}{2}dx+\frac{1}{2}c)^7 - 25998\tan(\frac{1}{2}dx+\frac{1}{2}c)^6 + 42210i\tan(\frac{1}{2}dx+\frac{1}{2}c)^5 + 46368\tan(\frac{1}{2}dx+\frac{1}{2}}{a^4(\tan(\frac{1}{2}dx+\frac{1}{2}c)-i)^9}}{1008\,d}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/1008*(63/(a^4*(tan(1/2*d*x + 1/2*c) + I)) + (1953*tan(1/2*d*x + 1/2*c)^8
 - 9450*I*tan(1/2*d*x + 1/2*c)^7 - 25998*tan(1/2*d*x + 1/2*c)^6 + 42210*I*
tan(1/2*d*x + 1/2*c)^5 + 46368*tan(1/2*d*x + 1/2*c)^4 - 33054*I*tan(1/2*d*
x + 1/2*c)^3 - 15858*tan(1/2*d*x + 1/2*c)^2 + 4374*I*tan(1/2*d*x + 1/2*c)
+ 703)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

3.163.9 Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.20

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{(63\tan(\frac{c}{2}+\frac{dx}{2})^9 - \tan(\frac{c}{2}+\frac{dx}{2})^8\,252\mathrm{i} - 588\tan(\frac{c}{2}+\frac{dx}{2})^7 + \tan(\frac{c}{2}+\frac{dx}{2})^6\,672\mathrm{i} + 378\tan(\frac{c}{2}+\frac{dx}{2})^5 + \mathrm{ta}}{63\,a^4\,d\,(\tan(\frac{c}{2}+\frac{dx}{2})+1\mathrm{i})\,(1+\mathrm{ta}}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
((372*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*288i - 97*tan(c/2 + (d*x
)/2) + tan(c/2 + (d*x)/2)^4*168i + 378*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d
*x)/2)^6*672i - 588*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*252i + 63*
tan(c/2 + (d*x)/2)^9 + 20i)*2i)/(63*a^4*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c
/2 + (d*x)/2)*1i + 1)^9)
```

3.163. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.164 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.164.1 Optimal result 1251
3.164.2 Mathematica [A] (verified) 1251
3.164.3 Rubi [A] (verified) 1252
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3.164.5 Fricas [A] (verification not implemented) 1255
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3.164.8 Giac [A] (verification not implemented) 1256
3.164.9 Mupad [B] (verification not implemented) 1257

3.164.1 Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{10\sin(c+dx)}{33a^4d} - \frac{20\sin^3(c+dx)}{99a^4d} + \frac{2\sin^5(c+dx)}{33a^4d} + \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
10/33*sin(d*x+c)/a^4/d-20/99*sin(d*x+c)^3/a^4/d+2/33*sin(d*x+c)^5/a^4/d+1/
11*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+7/99*I*cos(d*x+c)^3/a/d/(a+I*a*ta
n(d*x+c))^3+4/33*I*cos(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

3.164.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-924\cos(c+dx)-1188\cos(3(c+dx))+308\cos(5(c+dx))+12\cos(7(c+dx))-231}{6336a^4d(-i+\tan(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/6336*I)*Sec[c + d*x]^4*(-924*Cos[c + d*x] - 1188*Cos[3*(c + d*x)] + 3
08*Cos[5*(c + d*x)] + 12*Cos[7*(c + d*x)] - (231*I)*Sin[c + d*x] - (891*I)
*Sin[3*(c + d*x)] + (385*I)*Sin[5*(c + d*x)] + (21*I)*Sin[7*(c + d*x)]))/(
a^4*d*(-I + Tan[c + d*x])^4)
```

3.164.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{2\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{2\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3981}$$

$$\frac{7\left(\frac{2\left(\frac{5\int \cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{2\left(\frac{5\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3113}$$

$$\frac{7\left(\frac{2\left(-\frac{5\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7\left(\frac{2\left(-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + (7*(((I/9)*Cos[c +
d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) + (2*((-5*(-Sin[c + d*x] + (2*Sin[c +
 d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) + (((2*I)/7)*Cos[c + d*x]^5)/(d*
(a^2 + I*a^2*Tan[c + d*x]))))/(3*a)))/(11*a)
```

3.164. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.164.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.164.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

method	result
risch	$\frac{7 i \mathrm{e}^{-5 i (dx+c)}}{128 a^4 d}+\frac{3 i \mathrm{e}^{-7 i (dx+c)}}{128 a^4 d}+\frac{7 i \mathrm{e}^{-9 i (dx+c)}}{1152 a^4 d}+\frac{i \mathrm{e}^{-11 i (dx+c)}}{1408 a^4 d}+\frac{7 i \cos(dx+c)}{64 a^4 d}+\frac{7 \sin(dx+c)}{32 a^4 d}+\frac{17 i \cos(3dx+3c}{192 a^4 d}$
derivativedivides	$\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{44i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}}+\frac{385i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{201i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{}{11(-i+t}$
default	$\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{44i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}}+\frac{385i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{201i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{}{11(-i+t}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
7/128*I/a^4/d*exp(-5*I*(d*x+c))+3/128*I/a^4/d*exp(-7*I*(d*x+c))+7/1152*I/a
^4/d*exp(-9*I*(d*x+c))+1/1408*I/a^4/d*exp(-11*I*(d*x+c))+7/64*I/a^4/d*cos(
d*x+c)+7/32*sin(d*x+c)/a^4/d+17/192*I/a^4/d*cos(3*d*x+3*c)+3/32/a^4/d*sin(
3*d*x+3*c)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-33i\,e^{(14i\,dx+14i\,c)} - 693i\,e^{(12i\,dx+12i\,c)} + 2079i\,e^{(10i\,dx+10i\,c)} + 1155i\,e^{(8i\,dx+8i\,c)} + 693i\,e^{(6i\,dx+6i\,c)} + 297i\,e^{(4i}{12672\,a^4d}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/12672*(-33*I*e^(14*I*d*x + 14*I*c) - 693*I*e^(12*I*d*x + 12*I*c) + 2079*
I*e^(10*I*d*x + 10*I*c) + 1155*I*e^(8*I*d*x + 8*I*c) + 693*I*e^(6*I*d*x +
6*I*c) + 297*I*e^(4*I*d*x + 4*I*c) + 77*I*e^(2*I*d*x + 2*I*c) + 9*I)*e^(-1
1*I*d*x - 11*I*c)/(a^4*d)
```

3.164.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(136) = 272.

Time = 0.42 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.92

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-167196136166129664ia^{28}d^7e^{39ic}e^{3idx}-3511118859488722944ia^{28}d^7e^{37ic}e^{idx}+10533356578466168832ia^{28}d^7e^{35ic}e^{-idx}+58518647658145}{} \\ \frac{x\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-11ic}}{128a^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-167196136166129664*I*a**28*d**7*exp(39*I*c)*exp(3*I*d*x) - 35
11118859488722944*I*a**28*d**7*exp(37*I*c)*exp(I*d*x) + 105333565784661688
32*I*a**28*d**7*exp(35*I*c)*exp(-I*d*x) + 5851864765814538240*I*a**28*d**7
*exp(33*I*c)*exp(-3*I*d*x) + 3511118859488722944*I*a**28*d**7*exp(31*I*c)*
exp(-5*I*d*x) + 1504765225495166976*I*a**28*d**7*exp(29*I*c)*exp(-7*I*d*x)
 + 390124317720969216*I*a**28*d**7*exp(27*I*c)*exp(-9*I*d*x) + 45598946227
126272*I*a**28*d**7*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(642033162877
93790976*a**32*d**8), Ne(a**32*d**8*exp(36*I*c), 0)), (x*(exp(14*I*c) + 7*
exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I*
c) + 7*exp(2*I*c) + 1)*exp(-11*I*c)/(128*a**4), True))
```

3.164.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{33\left(12\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+21i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-11\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^3}+\frac{5940\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-39501i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-141075\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+313236i\tan(}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3168*(33*(12*tan(1/2*d*x + 1/2*c)^2 + 21*I*tan(1/2*d*x + 1/2*c) - 11)/(a
^4*(tan(1/2*d*x + 1/2*c) + I)^3) + (5940*tan(1/2*d*x + 1/2*c)^10 - 39501*I
*tan(1/2*d*x + 1/2*c)^9 - 141075*tan(1/2*d*x + 1/2*c)^8 + 313236*I*tan(1/2
*d*x + 1/2*c)^7 + 479556*tan(1/2*d*x + 1/2*c)^6 - 516054*I*tan(1/2*d*x + 1
/2*c)^5 - 397914*tan(1/2*d*x + 1/2*c)^4 + 214500*I*tan(1/2*d*x + 1/2*c)^3
+ 79024*tan(1/2*d*x + 1/2*c)^2 - 17765*I*tan(1/2*d*x + 1/2*c) - 2155)/(a^4
*(tan(1/2*d*x + 1/2*c) - I)^11))/d
```

3.164.9 Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{269\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}-\frac{1307\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}+\frac{1307\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{1099\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{32}+\frac{203\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{32}-\right.}{99\,a^4\,d\,\left(\cos\left(\right.\right.}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
-(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*231i)/16 - (cos((5*c)/2 +
(5*d*x)/2)*231i)/16 + cos((7*c)/2 + (7*d*x)/2)*33i - cos((9*c)/2 + (9*d*x)
/2)*5i + (cos((11*c)/2 + (11*d*x)/2)*3i)/16 - (cos((13*c)/2 + (13*d*x)/2)*
3i)/16 + (269*sin(c/2 + (d*x)/2))/16 - (1307*sin((3*c)/2 + (3*d*x)/2))/64
+ (1307*sin((5*c)/2 + (5*d*x)/2))/64 - (1099*sin((7*c)/2 + (7*d*x)/2))/32
+ (203*sin((9*c)/2 + (9*d*x)/2))/32 - (21*sin((11*c)/2 + (11*d*x)/2))/64 +
 (21*sin((13*c)/2 + (13*d*x)/2))/64)*2i)/(99*a^4*d*(cos(c/2 + (d*x)/2) + s
in(c/2 + (d*x)/2)*1i)^11*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^3)
```

3.165 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.165.1 Optimal result 1258
3.165.2 Mathematica [A] (verified) 1258
3.165.3 Rubi [A] (verified) 1259
3.165.4 Maple [A] (verified) 1261
3.165.5 Fricas [A] (verification not implemented) 1262
3.165.6 Sympy [B] (verification not implemented) 1262
3.165.7 Maxima [F(-2)] 1263
3.165.8 Giac [A] (verification not implemented) 1263
3.165.9 Mupad [B] (verification not implemented) 1264

3.165.1 Optimal result

Integrand size = 24, antiderivative size = 174

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{56\sin(c+dx)}{143a^4d} - \frac{56\sin^3(c+dx)}{143a^4d} + \frac{168\sin^5(c+dx)}{715a^4d} \\ &\quad - \frac{8\sin^7(c+dx)}{143a^4d} + \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} \\ &\quad + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{143d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
56/143*sin(d*x+c)/a^4/d-56/143*sin(d*x+c)^3/a^4/d+168/715*sin(d*x+c)^5/a^4
/d-8/143*sin(d*x+c)^7/a^4/d+1/13*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^4+9/1
43*I*cos(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+16/143*I*cos(d*x+c)^7/d/(a^4+I*
a^4*tan(d*x+c))
```

3.165.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.80

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-24024\cos(c+dx)-34320\cos(3(c+dx))+11440\cos(5(c+dx))+780\cos(7(c+dx)}{}$$

input
```
Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/183040*I)*Sec[c + d*x]^4*(-24024*Cos[c + d*x] - 34320*Cos[3*(c + d*x)
] + 11440*Cos[5*(c + d*x)] + 780*Cos[7*(c + d*x)] + 44*Cos[9*(c + d*x)] -
(6006*I)*Sin[c + d*x] - (25740*I)*Sin[3*(c + d*x)] + (14300*I)*Sin[5*(c +
d*x)] + (1365*I)*Sin[7*(c + d*x)] + (99*I)*Sin[9*(c + d*x)]))/(a^4*d*(-I +
 Tan[c + d*x])^4)
```

3.165.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^5(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\int \frac{\cos^5(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{1}{\sec(c+dx)^5(i\tan(c+dx)a+a)^3}dx}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\int \frac{\cos^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{8\int \frac{1}{\sec(c+dx)^5(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3981}$$

$$\frac{9\left(\frac{8\left(\frac{7\int \cos^7(c+dx)dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{8\left(\frac{7\int \sin\left(c+dx+\frac{\pi}{2}\right)^7dx}{9a^2}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3113}$$

$$\frac{9\left(\frac{8\left(-\frac{7\int\left(-\sin^6(c+dx)+3\sin^4(c+dx)-3\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{9\left(\frac{8\left(-\frac{7\left(\frac{1}{7}\sin^7(c+dx)-\frac{3}{5}\sin^5(c+dx)+\sin^3(c+dx)-\sin(c+dx)\right)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{11a}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}\right)}{13a}+\frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

input
```
Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/13)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^4) + (9*(((I/11)*Cos[c +
 d*x]^5)/(d*(a + I*a*Tan[c + d*x])^3) + (8*((-7*(-Sin[c + d*x] + Sin[c + d
*x]^3 - (3*Sin[c + d*x]^5)/5 + Sin[c + d*x]^7/7))/(9*a^2*d) + (((2*I)/9)*C
os[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*x])))))/(11*a)))/(13*a)
```

3.165. $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.165.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

method	result
risch	$\frac{3i\mathrm{e}^{-7i(dx+c)}}{128a^4d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{128a^4d}+\frac{9i\mathrm{e}^{-11i(dx+c)}}{5632a^4d}+\frac{i\mathrm{e}^{-13i(dx+c)}}{6656a^4d}+\frac{3i\cos(dx+c)}{32a^4d}+\frac{15\sin(dx+c)}{64a^4d}+\frac{25i\cos(5dx+5}{512a^4d}$
derivativedivides	$-\frac{135i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{1}{80\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{5}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{23}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{1}{32(-i+\mathrm{ta}}$
default	$-\frac{135i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{1}{80\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{5}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{23}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{1}{32(-i+\mathrm{ta}}$

input
```
int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
3/128*I/a^4/d*exp(-7*I*(d*x+c))+1/128*I/a^4/d*exp(-9*I*(d*x+c))+9/5632*I/a
^4/d*exp(-11*I*(d*x+c))+1/6656*I/a^4/d*exp(-13*I*(d*x+c))+3/32*I/a^4/d*cos
(d*x+c)+15/64*sin(d*x+c)/a^4/d+25/512*I/a^4/d*cos(5*d*x+5*c)+127/2560/a^4/
d*sin(5*d*x+5*c)+39/512*I/a^4/d*cos(3*d*x+3*c)+45/512/a^4/d*sin(3*d*x+3*c)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-143i\,e^{(18i\,dx+18i\,c)} - 2145i\,e^{(16i\,dx+16i\,c)} - 25740i\,e^{(14i\,dx+14i\,c)} + 60060i\,e^{(12i\,dx+12i\,c)} + 30030i\,e^{(10i\,dx+10i\,c)}}{366080\,a^4 d}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/366080*(-143*I*e^(18*I*d*x + 18*I*c) - 2145*I*e^(16*I*d*x + 16*I*c) - 25
740*I*e^(14*I*d*x + 14*I*c) + 60060*I*e^(12*I*d*x + 12*I*c) + 30030*I*e^(1
0*I*d*x + 10*I*c) + 18018*I*e^(8*I*d*x + 8*I*c) + 8580*I*e^(6*I*d*x + 6*I*
c) + 2860*I*e^(4*I*d*x + 4*I*c) + 585*I*e^(2*I*d*x + 2*I*c) + 55*I)*e^(-13
*I*d*x - 13*I*c)/(a^4*d)
```

3.165.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(153) = 306.

Time = 0.50 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.11

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-1688246017625898163896320ia^{36}d^9e^{54ic}e^{5idx} - 25323690264388472458444800ia^{36}d^9e^{52ic}e^{3idx} - 303884283172661669501337600ia^{36}d^9e^{50}}{} \\ \frac{x\left(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1\right)e^{-13ic}}{512a^4} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((-1688246017625898163896320*I*a**36*d**9*exp(54*I*c)*exp(5*I*d*
x) - 2532369026438847245844800*I*a**36*d**9*exp(52*I*c)*exp(3*I*d*x) - 30
3884283172661669501337600*I*a**36*d**9*exp(50*I*c)*exp(I*d*x) + 7090633274
02877228836454400*I*a**36*d**9*exp(48*I*c)*exp(-I*d*x) + 35453166370143861
4418227200*I*a**36*d**9*exp(46*I*c)*exp(-3*I*d*x) + 2127189982208631686509
36320*I*a**36*d**9*exp(44*I*c)*exp(-5*I*d*x) + 101294761057553889833779200
*I*a**36*d**9*exp(42*I*c)*exp(-7*I*d*x) + 33764920352517963277926400*I*a**
36*d**9*exp(40*I*c)*exp(-9*I*d*x) + 6906460981196856125030400*I*a**36*d**9
*exp(38*I*c)*exp(-11*I*d*x) + 649325391394576216883200*I*a**36*d**9*exp(36
*I*c)*exp(-13*I*d*x))*exp(-49*I*c)/(4321909805122299299574579200*a**40*d**
10), Ne(a**40*d**10*exp(49*I*c), 0)), (x*(exp(18*I*c) + 9*exp(16*I*c) + 36
*exp(14*I*c) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp(8*I*c) + 84*exp(
6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-13*I*c)/(512*a**4), True))
```

3.165.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.43

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{143\left(115\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+405i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-575\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-375i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+98\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{166595\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-1409265i\tan}{}}{}$$

input
```
integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
1/91520*(143*(115*tan(1/2*d*x + 1/2*c)^4 + 405*I*tan(1/2*d*x + 1/2*c)^3 -
575*tan(1/2*d*x + 1/2*c)^2 - 375*I*tan(1/2*d*x + 1/2*c) + 98)/(a^4*(tan(1/
2*d*x + 1/2*c) + I)^5) + (166595*tan(1/2*d*x + 1/2*c)^12 - 1409265*I*tan(1
/2*d*x + 1/2*c)^11 - 6232655*tan(1/2*d*x + 1/2*c)^10 + 17535375*I*tan(1/2*
d*x + 1/2*c)^9 + 34610004*tan(1/2*d*x + 1/2*c)^8 - 49771722*I*tan(1/2*d*x
+ 1/2*c)^7 - 53349582*tan(1/2*d*x + 1/2*c)^6 + 42730974*I*tan(1/2*d*x + 1/
2*c)^5 + 25431835*tan(1/2*d*x + 1/2*c)^4 - 10954229*I*tan(1/2*d*x + 1/2*c)
^3 - 3278067*tan(1/2*d*x + 1/2*c)^2 + 614627*I*tan(1/2*d*x + 1/2*c) + 6009
4)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^13))/d
```

3.165.9 Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.51

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{15049\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}-\frac{4513\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{32}+\frac{4513\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{32}-\frac{15461\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{64}+\frac{3941\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{64}\right.}{}$$

input
```
int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*3003i)/32 - (cos((5*c)/2 +
(5*d*x)/2)*3003i)/32 + (cos((7*c)/2 + (7*d*x)/2)*7293i)/32 - (cos((9*c)/2
+ (9*d*x)/2)*1533i)/32 + (cos((11*c)/2 + (11*d*x)/2)*103i)/32 - (cos((13*c
)/2 + (13*d*x)/2)*103i)/32 + (cos((15*c)/2 + (15*d*x)/2)*11i)/64 - (cos((1
7*c)/2 + (17*d*x)/2)*11i)/64 + (15049*sin(c/2 + (d*x)/2))/128 - (4513*sin(
(3*c)/2 + (3*d*x)/2))/32 + (4513*sin((5*c)/2 + (5*d*x)/2))/32 - (15461*sin
((7*c)/2 + (7*d*x)/2))/64 + (3941*sin((9*c)/2 + (9*d*x)/2))/64 - (183*sin(
(11*c)/2 + (11*d*x)/2))/32 + (183*sin((13*c)/2 + (13*d*x)/2))/32 - (99*sin
((15*c)/2 + (15*d*x)/2))/256 + (99*sin((17*c)/2 + (17*d*x)/2))/256)*2i)/(7
15*a^4*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^13*(cos(c/2 + (d*x)/
2)*1i + sin(c/2 + (d*x)/2))^5)
```

3.166 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.166.1 Optimal result 1265
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3.166.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{192x}{a^8}-\frac{192i\log(\cos(c+dx))}{a^8d}+\frac{129\tan(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}-\frac{10\tan^3(c+dx)}{a^8d}+\frac{2i\tan^4(c+dx)}{a^8d}+\frac{\tan^5(c+dx)}{5a^8d}+\frac{64i}{d\,(a^8+ia^8\tan(c+dx))}$$

output
```
-192*x/a^8-192*I*ln(cos(d*x+c))/a^8/d+129*tan(d*x+c)/a^8/d-36*I*tan(d*x+c)
^2/a^8/d-10*tan(d*x+c)^3/a^8/d+2*I*tan(d*x+c)^4/a^8/d+1/5*tan(d*x+c)^5/a^8
/d+64*I/d/(a^8+I*a^8*tan(d*x+c))
```

3.166.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\left(-960\log(i-\tan(c+dx))+645i\tan(c+dx)+180\tan^2(c+dx)-50i\tan^3(c+dx)-10\tan^4(c+\right.}{5a^8d}$$

input
```
Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/5*I)*(-960*Log[I - Tan[c + d*x]] + (645*I)*Tan[c + d*x] + 180*Tan[c +
 d*x]^2 - (50*I)*Tan[c + d*x]^3 - 10*Tan[c + d*x]^4 + I*Tan[c + d*x]^5 + (
320*I)/(-I + Tan[c + d*x])))/(a^8*d)
```

3.166.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\int \frac{\sec(c+dx)^{14}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \textcolor{blue}{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^6}{(i\tan(c+dx)a+a)^2}d(ia\tan(c+dx))}{a^{13}d}$$

$$\Big\downarrow \textcolor{blue}{49}$$

$$-\frac{i\int \Big(\frac{64a^6}{(i\tan(c+dx)a+a)^2}-\frac{192a^5}{i\tan(c+dx)a+a}+\tan^4(c+dx)a^4+8i\tan^3(c+dx)a^4-30\tan^2(c+dx)a^4-72i\tan(c+dx}{a^{13}d}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$-\frac{i\Big(-\frac{64a^6}{a+ia\tan(c+dx)}+\frac{1}{5}ia^5\tan^5(c+dx)-2a^5\tan^4(c+dx)-10ia^5\tan^3(c+dx)+36a^5\tan^2(c+dx)+129ia^5\tan}{a^{13}d}$$

input
```
Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-192*a^5*Log[a + I*a*Tan[c + d*x]] + (129*I)*a^5*Tan[c + d*x] + 36*
a^5*Tan[c + d*x]^2 - (10*I)*a^5*Tan[c + d*x]^3 - 2*a^5*Tan[c + d*x]^4 + (I
/5)*a^5*Tan[c + d*x]^5 - (64*a^6)/(a + I*a*Tan[c + d*x])))/(a^13*d)
```

3.166.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.166.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
risch	$\frac{32i\mathrm{e}^{-2i(dx+c)}}{a^{8}d}-\frac{384x}{a^{8}}-\frac{384c}{a^{8}d}+\frac{16i\left(50\,\mathrm{e}^{8i(dx+c)}+220\,\mathrm{e}^{6i(dx+c)}+370\,\mathrm{e}^{4i(dx+c)}+285\,\mathrm{e}^{2i(dx+c)}+87\right)}{5d\,a^{8}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{192i\ln(\mathrm{e}^{2}}{a}$
derivativedivides	$\frac{129\tan(dx+c)}{a^{8}d}+\frac{\tan^{5}(dx+c)}{5a^{8}d}+\frac{2i\left(\tan^{4}(dx+c)\right)}{a^{8}d}-\frac{10\left(\tan^{3}(dx+c)\right)}{a^{8}d}-\frac{36i\left(\tan^{2}(dx+c)\right)}{a^{8}d}+\frac{64}{a^{8}d(\tan(dx+c)-i)}$
default	$\frac{129\tan(dx+c)}{a^{8}d}+\frac{\tan^{5}(dx+c)}{5a^{8}d}+\frac{2i\left(\tan^{4}(dx+c)\right)}{a^{8}d}-\frac{10\left(\tan^{3}(dx+c)\right)}{a^{8}d}-\frac{36i\left(\tan^{2}(dx+c)\right)}{a^{8}d}+\frac{64}{a^{8}d(\tan(dx+c)-i)}$

input
```
int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
32*I/a^8/d*exp(-2*I*(d*x+c))-384*x/a^8-384/a^8/d*c+16/5*I*(50*exp(8*I*(d*x
+c))+220*exp(6*I*(d*x+c))+370*exp(4*I*(d*x+c))+285*exp(2*I*(d*x+c))+87)/d/
a^8/(exp(2*I*(d*x+c))+1)^5-192*I/a^8/d*ln(exp(2*I*(d*x+c))+1)
```

3.166.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(122) = 244$.

Time = 0.30 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{16\,\left(120\,dxe^{(12i\,dx+12i\,c)}+60\,(10\,dx-i)e^{(10i\,dx+10i\,c)}+30\,(40\,dx-9i)e^{(8i\,dx+8i\,c)}+10\,(120\,dx-47i)e^{(6i\,d}\right.}{5\,\left(a^8de^{(12i\,dx}\right.}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-16/5*(120*d*x*e^(12*I*d*x + 12*I*c) + 60*(10*d*x - I)*e^(10*I*d*x + 10*I*
c) + 30*(40*d*x - 9*I)*e^(8*I*d*x + 8*I*c) + 10*(120*d*x - 47*I)*e^(6*I*d*
x + 6*I*c) + 5*(120*d*x - 77*I)*e^(4*I*d*x + 4*I*c) + (120*d*x - 137*I)*e^
(2*I*d*x + 2*I*c) + 60*(I*e^(12*I*d*x + 12*I*c) + 5*I*e^(10*I*d*x + 10*I*c
) + 10*I*e^(8*I*d*x + 8*I*c) + 10*I*e^(6*I*d*x + 6*I*c) + 5*I*e^(4*I*d*x +
 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 10*I)/(a^8
*d*e^(12*I*d*x + 12*I*c) + 5*a^8*d*e^(10*I*d*x + 10*I*c) + 10*a^8*d*e^(8*I
*d*x + 8*I*c) + 10*a^8*d*e^(6*I*d*x + 6*I*c) + 5*a^8*d*e^(4*I*d*x + 4*I*c)
 + a^8*d*e^(2*I*d*x + 2*I*c))
```

3.166.6 Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\sec^{14}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**14/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{320\left(\tan(dx+c)^6-6i\tan(dx+c)^5-15\tan(dx+c)^4+20i\tan(dx+c)^3+15\tan(dx+c)^2-6i\tan(dx+c)-1\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}+\frac{\tan(d}{}}{5\,d}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/5*(320*(tan(d*x + c)^6 - 6*I*tan(d*x + c)^5 - 15*tan(d*x + c)^4 + 20*I*t
an(d*x + c)^3 + 15*tan(d*x + c)^2 - 6*I*tan(d*x + c) - 1)/(a^8*tan(d*x + c
)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x +
c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c
) + I*a^8) + (tan(d*x + c)^5 + 10*I*tan(d*x + c)^4 - 50*tan(d*x + c)^3 - 1
80*I*tan(d*x + c)^2 + 645*tan(d*x + c))/a^8 + 960*I*log(I*tan(d*x + c) + 1
)/a^8)/d
```

3.166.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(122) = 244.

Time = 1.93 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.87

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{2\left(\frac{480i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}-\frac{960i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^8}+\frac{480i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^8}+\frac{160\left(9i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\tan\left(\frac{1}{2}\,d\right.\right.}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}\right.}{}$$

input
```
integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2/5*(480*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 960*I*log(tan(1/2*d*x + 1/
2*c) - I)/a^8 + 480*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 + 160*(9*I*tan(1/2
*d*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) - 9*I)/(a^8*(tan(1/2*d*x + 1/2*c
) - I)^2) + (-1096*I*tan(1/2*d*x + 1/2*c)^10 + 645*tan(1/2*d*x + 1/2*c)^9
+ 5840*I*tan(1/2*d*x + 1/2*c)^8 - 2780*tan(1/2*d*x + 1/2*c)^7 - 12120*I*ta
n(1/2*d*x + 1/2*c)^6 + 4286*tan(1/2*d*x + 1/2*c)^5 + 12120*I*tan(1/2*d*x +
 1/2*c)^4 - 2780*tan(1/2*d*x + 1/2*c)^3 - 5840*I*tan(1/2*d*x + 1/2*c)^2 +
645*tan(1/2*d*x + 1/2*c) + 1096*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^8))/d
```

3.166.9 Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{129\tan(c+d\,x)}{a^8} - \frac{10\tan(c+d\,x)^3}{a^8} + \frac{\tan(c+d\,x)^5}{5\,a^8} + \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,192\mathrm{i}}{a^8} + \frac{64\mathrm{i}}{a^8\,(1+\tan(c+d\,x)\,1\mathrm{i})} - \frac{\tan(c+d\,x)^2\,36\mathrm{i}}{a^8} + \frac{\tan(c+d\,x)^4\,2\mathrm{i}}{a^8}}{d}$$

input
```
int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((log(tan(c + d*x) - 1i)*192i)/a^8 + (129*tan(c + d*x))/a^8 + 64i/(a^8*(ta
n(c + d*x)*1i + 1)) - (tan(c + d*x)^2*36i)/a^8 - (10*tan(c + d*x)^3)/a^8 +
 (tan(c + d*x)^4*2i)/a^8 + tan(c + d*x)^5/(5*a^8))/d
```

3.167 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.167.1 Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{80x}{a^8}+\frac{80i\log(\cos(c+dx))}{a^8d}-\frac{31\tan(c+dx)}{a^8d} +\frac{4i\tan^2(c+dx)}{a^8d}+\frac{\tan^3(c+dx)}{3a^8d} +\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}-\frac{80i}{d\left(a^8+ia^8\tan(c+dx)\right)}$$

output
```
80*x/a^8+80*I*ln(cos(d*x+c))/a^8/d-31*tan(d*x+c)/a^8/d+4*I*tan(d*x+c)^2/a^
8/d+1/3*tan(d*x+c)^3/a^8/d+16*I/d/(a^4+I*a^4*tan(d*x+c))^2-80*I/d/(a^8+I*a
^8*tan(d*x+c))
```

3.167.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\left(-93i\tan(c+dx)-12\tan^2(c+dx)+i\tan^3(c+dx)+48\left(5\log(i-\tan(c+dx))+\frac{-4-5i\tan(c+dx)}{(-i+\tan(c+dx))^2}\right)\right)}{3a^8d}$$

input
```
Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/3*I)*((-93*I)*Tan[c + d*x] - 12*Tan[c + d*x]^2 + I*Tan[c + d*x]^3 + 4
8*(5*Log[I - Tan[c + d*x]] + (-4 - (5*I)*Tan[c + d*x])/(-I + Tan[c + d*x])
^2)))/(a^8*d)
```

3.167.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^{12}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^5}{(i\tan(c+dx)a+a)^3}d(ia\tan(c+dx))}{a^{11}d}$$

$$\Big\downarrow \; 49$$

$$-\frac{i\int \left(\frac{32a^5}{(i\tan(c+dx)a+a)^3}-\frac{80a^4}{(i\tan(c+dx)a+a)^2}+\frac{80a^3}{i\tan(c+dx)a+a}+\tan^2(c+dx)a^2+8i\tan(c+dx)a^2-31a^2\right)d(ia\tan(c}{a^{11}d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(-\frac{16a^5}{(a+ia\tan(c+dx))^2}+\frac{80a^4}{a+ia\tan(c+dx)}+\frac{1}{3}ia^3\tan^3(c+dx)-4a^3\tan^2(c+dx)-31ia^3\tan(c+dx)+80a^3\log(a+\right.}{a^{11}d}$$

input
```
Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(80*a^3*Log[a + I*a*Tan[c + d*x]] - (31*I)*a^3*Tan[c + d*x] - 4*a^3*
Tan[c + d*x]^2 + (I/3)*a^3*Tan[c + d*x]^3 - (16*a^5)/(a + I*a*Tan[c + d*x]
)^2 + (80*a^4)/(a + I*a*Tan[c + d*x])))/(a^11*d)
```

3.167.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.167.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{32 i \,{\mathrm e}^{-2 i \left(d x +c \right)}}{a^{8} d}+\frac{4 i \,{\mathrm e}^{-4 i \left(d x +c \right)}}{a^{8} d}+\frac{160 x}{a^{8}}+\frac{160 c}{a^{8} d}-\frac{4 i \left(36 \,{\mathrm e}^{4 i \left(d x +c \right)}+81 \,{\mathrm e}^{2 i \left(d x +c \right)}+47\right)}{3 d \,a^{8} \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{3}}+\frac{80 i \ln \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{a^{8} d}$
derivativedivides	$-\frac{31 \tan \left(d x +c \right)}{a^{8} d}+\frac{\tan^{3}\left(d x +c \right)}{3 a^{8} d}+\frac{4 i \left(\tan^{2}\left(d x +c \right)\right)}{a^{8} d}+\frac{80 \arctan \left(\tan \left(d x +c \right)\right)}{a^{8} d}-\frac{40 i \ln \left(1+\tan^{2}\left(d x +c \right)\right)}{a^{8} d}-$
default	$-\frac{31 \tan \left(d x +c \right)}{a^{8} d}+\frac{\tan^{3}\left(d x +c \right)}{3 a^{8} d}+\frac{4 i \left(\tan^{2}\left(d x +c \right)\right)}{a^{8} d}+\frac{80 \arctan \left(\tan \left(d x +c \right)\right)}{a^{8} d}-\frac{40 i \ln \left(1+\tan^{2}\left(d x +c \right)\right)}{a^{8} d}-$

input
```
int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-32*I/a^8/d*exp(-2*I*(d*x+c))+4*I/a^8/d*exp(-4*I*(d*x+c))+160*x/a^8+160/a^
8/d*c-4/3*I*(36*exp(4*I*(d*x+c))+81*exp(2*I*(d*x+c))+47)/d/a^8/(exp(2*I*(d
*x+c))+1)^3+80*I/a^8/d*ln(exp(2*I*(d*x+c))+1)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{4\,(120\,dxe^{(10i\,dx+10i\,c)}+60\,(6\,dx-i)e^{(8i\,dx+8i\,c)}+30\,(12\,dx-5i)e^{(6i\,dx+6i\,c)}+10\,(12\,dx-11i)e^{(4i\,dx+4i\,c)}-}{3\,(a^8de^{(10i\,dx+10i\,c)}+3\,a^8de^{(8i\,dx+8i}}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
4/3*(120*d*x*e^(10*I*d*x + 10*I*c) + 60*(6*d*x - I)*e^(8*I*d*x + 8*I*c) +
30*(12*d*x - 5*I)*e^(6*I*d*x + 6*I*c) + 10*(12*d*x - 11*I)*e^(4*I*d*x + 4*
I*c) - 60*(-I*e^(10*I*d*x + 10*I*c) - 3*I*e^(8*I*d*x + 8*I*c) - 3*I*e^(6*I
*d*x + 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 15*I
*e^(2*I*d*x + 2*I*c) + 3*I)/(a^8*d*e^(10*I*d*x + 10*I*c) + 3*a^8*d*e^(8*I*
d*x + 8*I*c) + 3*a^8*d*e^(6*I*d*x + 6*I*c) + a^8*d*e^(4*I*d*x + 4*I*c))
```

3.167.6 Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{12}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**12/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{\frac{48\left(5\tan(dx+c)^6-29i\tan(dx+c)^5-70\tan(dx+c)^4+90i\tan(dx+c)^3+65\tan(dx+c)^2-25i\tan(dx+c)-4\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}-}{3\,d}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/3*(48*(5*tan(d*x + c)^6 - 29*I*tan(d*x + c)^5 - 70*tan(d*x + c)^4 + 90*
I*tan(d*x + c)^3 + 65*tan(d*x + c)^2 - 25*I*tan(d*x + c) - 4)/(a^8*tan(d*x
 + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*
x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x
 + c) + I*a^8) - (tan(d*x + c)^3 + 12*I*tan(d*x + c)^2 - 93*tan(d*x + c))/
a^8 + 240*I*log(I*tan(d*x + c) + 1)/a^8)/d
```

3.167.8 Giac [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.78

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{2\left(-\frac{120i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}+\frac{240i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^8}-\frac{120i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^8}+ 220i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-93\tan\left(\frac{1}{2}\,d\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2/3*(-120*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 + 240*I*log(tan(1/2*d*x + 1
/2*c) - I)/a^8 - 120*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 + (220*I*tan(1/2*
d*x + 1/2*c)^6 - 93*tan(1/2*d*x + 1/2*c)^5 - 684*I*tan(1/2*d*x + 1/2*c)^4
+ 190*tan(1/2*d*x + 1/2*c)^3 + 684*I*tan(1/2*d*x + 1/2*c)^2 - 93*tan(1/2*d
*x + 1/2*c) - 220*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^8) + 4*(-125*I*tan(
1/2*d*x + 1/2*c)^4 - 536*tan(1/2*d*x + 1/2*c)^3 + 846*I*tan(1/2*d*x + 1/2*
c)^2 + 536*tan(1/2*d*x + 1/2*c) - 125*I)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^4
))/d
```

3.167.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\tan(c+d\,x)^3}{3\,a^8\,d} - \frac{31\tan(c+d\,x)}{a^8\,d} + \frac{\tan(c+d\,x)^2\,4\mathrm{i}}{a^8\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,80\mathrm{i}}{a^8\,d} - \frac{\frac{64}{a^8}+\frac{\tan(c+d\,x)\,80\mathrm{i}}{a^8}}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

input

```
int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8),x)
```

output

```
(tan(c + d*x)^2*4i)/(a^8*d) - (31*tan(c + d*x))/(a^8*d) - (log(tan(c + d*x
) - 1i)*80i)/(a^8*d) + tan(c + d*x)^3/(3*a^8*d) - ((tan(c + d*x)*80i)/a^8
+ 64/a^8)/(d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i))
```

3.168 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.168.1 Optimal result

Integrand size = 24, antiderivative size = 116

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = &-\frac{8x}{a^8}-\frac{8i\log(\cos(c+dx))}{a^8d}+\frac{\tan(c+dx)}{a^8d}\\ &+\frac{16i}{3a^5d(a+ia\tan(c+dx))^3}\\ &-\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output

```
-8*x/a^8-8*I*ln(cos(d*x+c))/a^8/d+tan(d*x+c)/a^8/d+16/3*I/a^5/d/(a+I*a*tan
(d*x+c))^3-16*I/d/(a^4+I*a^4*tan(d*x+c))^2+24*I/d/(a^8+I*a^8*tan(d*x+c))
```

3.168.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\begin{aligned}&\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx\\ &= -\frac{i\left(-8a\log(i-\tan(c+dx))+ia\tan(c+dx)+\frac{8a\left(-5i+12\tan(c+dx)+9i\tan^2(c+dx)\right)}{3(-i+\tan(c+dx))^3}\right)}{a^9d}\end{aligned}$$

input

```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-8*a*Log[I - Tan[c + d*x]] + I*a*Tan[c + d*x] + (8*a*(-5*I + 12*Tan
[c + d*x] + (9*I)*Tan[c + d*x]^2))/(3*(-I + Tan[c + d*x])^3)))/(a^9*d)
```

3.168.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3968, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3968}$$
$$-\frac{i\int \frac{(a-ia\tan(c+dx))^4}{(i\tan(c+dx)a+a)^4}d(ia\tan(c+dx))}{a^9d}$$
$$\Big\downarrow \text{49}$$
$$-\frac{i\int \left(\frac{16a^4}{(i\tan(c+dx)a+a)^4}-\frac{32a^3}{(i\tan(c+dx)a+a)^3}+\frac{24a^2}{(i\tan(c+dx)a+a)^2}-\frac{8a}{i\tan(c+dx)a+a}+1\right)d(ia\tan(c+dx))}{a^9d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\left(-\frac{16a^4}{3(a+ia\tan(c+dx))^3}+\frac{16a^3}{(a+ia\tan(c+dx))^2}-\frac{24a^2}{a+ia\tan(c+dx)}+ia\tan(c+dx)-8a\log(a+ia\tan(c+dx))\right)}{a^9d}$$

input
```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-8*a*Log[a + I*a*Tan[c + d*x]] + I*a*Tan[c + d*x] - (16*a^4)/(3*(a
+ I*a*Tan[c + d*x])^3) + (16*a^3)/(a + I*a*Tan[c + d*x])^2 - (24*a^2)/(a +
 I*a*Tan[c + d*x])))/(a^9*d)
```

3.168.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.168.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\tan(dx+c)}{a^8 d}+\frac{16i}{a^8 d(\tan(dx+c)-i)^2}-\frac{16}{3a^8 d(\tan(dx+c)-i)^3}-\frac{8\arctan(\tan(dx+c))}{a^8 d}+\frac{4i\ln(1+\tan^2(dx+c))}{a^8 d}+$
default	$\frac{\tan(dx+c)}{a^8 d}+\frac{16i}{a^8 d(\tan(dx+c)-i)^2}-\frac{16}{3a^8 d(\tan(dx+c)-i)^3}-\frac{8\arctan(\tan(dx+c))}{a^8 d}+\frac{4i\ln(1+\tan^2(dx+c))}{a^8 d}+$
risch	$\frac{6i\mathrm{e}^{-2i(dx+c)}}{a^8 d}-\frac{2i\mathrm{e}^{-4i(dx+c)}}{a^8 d}+\frac{2i\mathrm{e}^{-6i(dx+c)}}{3a^8 d}-\frac{16x}{a^8}-\frac{16c}{a^8 d}+\frac{2i}{d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{8i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^8 d}$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
tan(d*x+c)/a^8/d+16*I/a^8/d/(tan(d*x+c)-I)^2-16/3/a^8/d/(tan(d*x+c)-I)^3-8
/a^8/d*arctan(tan(d*x+c))+4*I/a^8/d*ln(1+tan(d*x+c)^2)+24/a^8/d/(tan(d*x+c
)-I)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(24\,dxe^{(8i\,dx+8i\,c)}+12\,(2\,dx-i)e^{(6i\,dx+6i\,c)}+12\left(i\,e^{(8i\,dx+8i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-6i\,e\right.}{3\left(a^8de^{(8i\,dx+8i\,c)}+a^8de^{(6i\,dx+6i\,c)}\right)}$$

input

```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output

```
-2/3*(24*d*x*e^(8*I*d*x + 8*I*c) + 12*(2*d*x - I)*e^(6*I*d*x + 6*I*c) + 12
*(I*e^(8*I*d*x + 8*I*c) + I*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c) +
 1) - 6*I*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) - I)/(a^8*d*e^(8*I
*d*x + 8*I*c) + a^8*d*e^(6*I*d*x + 6*I*c))
```

3.168.6 Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{10}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input

```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**8,x)
```

output

```
Integral(sec(c + d*x)**10/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.65

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{8\left(9\tan(dx+c)^6-48i\tan(dx+c)^5-107\tan(dx+c)^4+128i\tan(dx+c)^3+87\tan(dx+c)^2-32i\tan(dx+c)-5\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}+\frac{24i\,\mathrm{lo}}{}}{3\,d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/3*(8*(9*tan(d*x + c)^6 - 48*I*tan(d*x + c)^5 - 107*tan(d*x + c)^4 + 128*
I*tan(d*x + c)^3 + 87*tan(d*x + c)^2 - 32*I*tan(d*x + c) - 5)/(a^8*tan(d*x
 + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*
x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x
 + c) + I*a^8) + 24*I*log(I*tan(d*x + c) + 1)/a^8 + 3*tan(d*x + c)/a^8)/d
```

3.168.8 Giac [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.72

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{2\left(\frac{60i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}-\frac{120i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^8}+\frac{60i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^8}-\frac{15\left(4i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^8}\right.}{}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2/15*(60*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 120*I*log(tan(1/2*d*x + 1/
2*c) - I)/a^8 + 60*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 - 15*(4*I*tan(1/2*d
*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) - 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*
a^8) + 2*(147*I*tan(1/2*d*x + 1/2*c)^6 + 942*tan(1/2*d*x + 1/2*c)^5 - 2445
*I*tan(1/2*d*x + 1/2*c)^4 - 3460*tan(1/2*d*x + 1/2*c)^3 + 2445*I*tan(1/2*d
*x + 1/2*c)^2 + 942*tan(1/2*d*x + 1/2*c) - 147*I)/(a^8*(tan(1/2*d*x + 1/2*
c) - I)^6))/d
```

3.168.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\tan(c+d\,x)}{a^8\,d} - \frac{\frac{32\tan(c+d\,x)}{a^8} - \frac{40\mathrm{i}}{3\,a^8} + \frac{\tan(c+d\,x)^2\,24\mathrm{i}}{a^8}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i} - 3\tan\left(c+d\,x\right)^2 + \tan\left(c+d\,x\right)\,3\mathrm{i} + 1\right)}$$
$$+ \frac{\ln\left(\tan(c+d\,x) - \mathrm{i}\right)\,8\mathrm{i}}{a^8\,d}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(log(tan(c + d*x) - 1i)*8i)/(a^8*d) - ((32*tan(c + d*x))/a^8 - 40i/(3*a^8)
 + (tan(c + d*x)^2*24i)/a^8)/(d*(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(
c + d*x)^3*1i + 1)) + tan(c + d*x)/(a^8*d)
```

3.169 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.169.1 Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}$$

output
```
1/8*I*(a-I*a*tan(d*x+c))^4/d/(a^3+I*a^3*tan(d*x+c))^4
```

3.169.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i(i+\tan(c+dx))^4}{8a^8d(-i+\tan(c+dx))^4}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/8)*(I + Tan[c + d*x])^4)/(a^8*d*(-I + Tan[c + d*x])^4)
```

3.169.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^3}{(i\tan(c+dx)a+a)^5}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{i(a-ia\tan(c+dx))^4}{8a^8d(a+ia\tan(c+dx))^4}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/8)*(a - I*a*Tan[c + d*x])^4)/(a^8*d*(a + I*a*Tan[c + d*x])^4)
```

3.169.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.169.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{i\mathrm{e}^{-8i(dx+c)}}{8a^8d}$	19
derivativedivides	$-\frac{-\frac{2i}{(\tan(dx+c)-i)^4}+\frac{3i}{(\tan(dx+c)-i)^2}+\frac{1}{\tan(dx+c)-i}-\frac{4}{(\tan(dx+c)-i)^3}}{a^8d}$	62
default	$-\frac{-\frac{2i}{(\tan(dx+c)-i)^4}+\frac{3i}{(\tan(dx+c)-i)^2}+\frac{1}{\tan(dx+c)-i}-\frac{4}{(\tan(dx+c)-i)^3}}{a^8d}$	62

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^8/d*exp(-8*I*(d*x+c))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\,e^{(-8i\,dx-8i\,c)}}{8\,a^8d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/8*I*e^(-8*I*d*x - 8*I*c)/(a^8*d)
```

3.169.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(34) = 68$.

Time = 8.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.72

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{i\sec^8(c+dx)}{8a^8d\tan^8(c+dx)-64ia^8d\tan^7(c+dx)-224a^8d\tan^6(c+dx)+448ia^8d\tan^5(c+dx)+560a^8d\tan^4(c+dx)-448ia^8d\tan^3(c+dx)-224a^8d\tan^2(} \\ \frac{x\sec^8(c)}{(ia\tan(c)+a)^8} \end{cases}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((I*sec(c + d*x)**8/(8*a**8*d*tan(c + d*x)**8 - 64*I*a**8*d*tan(c
 + d*x)**7 - 224*a**8*d*tan(c + d*x)**6 + 448*I*a**8*d*tan(c + d*x)**5 + 5
60*a**8*d*tan(c + d*x)**4 - 448*I*a**8*d*tan(c + d*x)**3 - 224*a**8*d*tan(
c + d*x)**2 + 64*I*a**8*d*tan(c + d*x) + 8*a**8*d), Ne(d, 0)), (x*sec(c)**
8/(I*a*tan(c) + a)**8, True))
```

3.169.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(35) = 70$.

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{\tan(dx+c)^6-3i\tan(dx+c)^5-4\tan(dx+c)^4+4i\tan(dx+c)^3+3\tan(}{(a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-(tan(d*x + c)^6 - 3*I*tan(d*x + c)^5 - 4*tan(d*x + c)^4 + 4*I*tan(d*x + c
)^3 + 3*tan(d*x + c)^2 - I*tan(d*x + c))/((a^8*tan(d*x + c)^7 - 7*I*a^8*ta
n(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*ta
n(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

3.169.8 Giac [A] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^8d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^8}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2*(tan(1/2*d*x + 1/2*c)^7 - 7*tan(1/2*d*x + 1/2*c)^5 + 7*tan(1/2*d*x + 1/
2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^8)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2\,1\mathrm{i}-\mathrm{i}\right)}{a^8\,d\,\left(\tan\left(c+d\,x\right)^4\,1\mathrm{i}+4\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,6\mathrm{i}-4\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
-(tan(c + d*x)*(tan(c + d*x)^2*1i - 1i))/(a^8*d*(4*tan(c + d*x)^3 - tan(c
+ d*x)^2*6i - 4*tan(c + d*x) + tan(c + d*x)^4*1i + 1i))
```

3.170 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.170.1 Optimal result 1288
3.170.2 Mathematica [A] (verified) 1288
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3.170.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{4i}{5a^3d(a+ia\tan(c+dx))^5}+\frac{i}{3a^5d(a+ia\tan(c+dx))^3} - \frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}$$

output
```
4/5*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/3*I/a^5/d/(a+I*a*tan(d*x+c))^3-I/d/(a^2
+I*a^2*tan(d*x+c))^4
```

3.170.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2-5i\tan(c+dx)-5\tan^2(c+dx)}{15a^8d(-i+\tan(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(2 - (5*I)*Tan[c + d*x] - 5*Tan[c + d*x]^2)/(15*a^8*d*(-I + Tan[c + d*x])^
5)
```

3.170.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{(i\tan(c+dx)a+a)^6}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{4a^2}{(i\tan(c+dx)a+a)^6}-\frac{4a}{(i\tan(c+dx)a+a)^5}+\frac{1}{(i\tan(c+dx)a+a)^4}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{4a^2}{5(a+ia\tan(c+dx))^5}+\frac{a}{(a+ia\tan(c+dx))^4}-\frac{1}{3(a+ia\tan(c+dx))^3}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*((-4*a^2)/(5*(a + I*a*Tan[c + d*x])^5) + a/(a + I*a*Tan[c + d*x])^4
- 1/(3*(a + I*a*Tan[c + d*x])^3)))/(a^5*d)
```

3.170.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.170.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$-\frac{\frac{i}{(\tan(dx+c)-i)^4}-\frac{4}{5(\tan(dx+c)-i)^5}+\frac{1}{3(\tan(dx+c)-i)^3}}{a^8d}$	50
default	$-\frac{\frac{i}{(\tan(dx+c)-i)^4}-\frac{4}{5(\tan(dx+c)-i)^5}+\frac{1}{3(\tan(dx+c)-i)^3}}{a^8d}$	50
risch	$\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^8d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{16a^8d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{40a^8d}$	56

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-1/a^8/d*(I/(tan(d*x+c)-I)^4-4/5/(tan(d*x+c)-I)^5+1/3/(tan(d*x+c)-I)^3)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(10i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+6i\right)e^{(-10i\,dx-10i\,c)}}{240\,a^8d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/240*(10*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 6*I)*e^(-10*I
*d*x - 10*I*c)/(a^8*d)
```

3.170.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(65) = 130.

Time = 8.73 (sec) , antiderivative size = 466, normalized size of antiderivative = 5.75

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} -\frac{i\tan^2{(c+dx)}\sec^6{(c+dx)}}{240a^8d\tan^8{(c+dx)}-1920ia^8d\tan^7{(c+dx)}-6720a^8d\tan^6{(c+dx)}+13440ia^8d\tan^5{(c+dx)}+16800a^8d\tan^4{(c+dx)}-13440ia^8d\tan^3{(c+dx)}} \\ \frac{x\sec^6{(c)}}{(ia\tan{(c)}+a)^8} \end{cases}$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8
- 1920*I*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a*
*8*d*tan(c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c
 + d*x)**3 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 24
0*a**8*d) - 8*tan(c + d*x)*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1
920*I*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*
d*tan(c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c +
d*x)**3 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a
**8*d) + 31*I*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1920*I*a**8*d*
tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*d*tan(c + d*x
)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c + d*x)**3 - 672
0*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a**8*d), Ne(d,
 0)), (x*sec(c)**6/(I*a*tan(c) + a)**8, True))
```

3.170.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{5\tan(dx+c)^4 - 5i\tan(dx+c)^3 + 3\tan(dx+c)^2 - i\tan(}{15(a^8\tan(dx+c)^7 - 7i\,a^8\tan(dx+c)^6 - 21\,a^8\tan(dx+c)^5 + 35i\,a^8\tan(dx+c)^4 + 35\,a^8\tan(dx+}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(5*tan(d*x + c)^4 - 5*I*tan(d*x + c)^3 + 3*tan(d*x + c)^2 - I*tan(d*
x + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x
 + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d
*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

3.170.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(65) = 130$.

Time = 1.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.69

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\Big(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 - 30i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 - 140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 170i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 282}{15\,a^8d\Big(\mathrm{ta}}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2/15*(15*tan(1/2*d*x + 1/2*c)^9 - 30*I*tan(1/2*d*x + 1/2*c)^8 - 140*tan(1
/2*d*x + 1/2*c)^7 + 170*I*tan(1/2*d*x + 1/2*c)^6 + 282*tan(1/2*d*x + 1/2*c
)^5 - 170*I*tan(1/2*d*x + 1/2*c)^4 - 140*tan(1/2*d*x + 1/2*c)^3 + 30*I*tan
(1/2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c
) - I)^10)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{-\tan(c+d\,x)^2\,5\mathrm{i}+5\tan(c+d\,x)+2\mathrm{i}}{15\,a^8\,d\,\left(\tan\left(c+d\,x\right)^5\,1\mathrm{i}+5\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,10\mathrm{i}-10\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,5\mathrm{i}+1\right)}$$

input

```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8),x)
```

output

```
(5*tan(c + d*x) - tan(c + d*x)^2*5i + 2i)/(15*a^8*d*(tan(c + d*x)*5i - 10*
tan(c + d*x)^2 - tan(c + d*x)^3*10i + 5*tan(c + d*x)^4 + tan(c + d*x)^5*1i
 + 1))
```

3.171 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.171.9 Mupad [B] (verification not implemented) 1299

3.171.1 Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{3a^2d(a+ia\tan(c+dx))^6} - \frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

output
```
1/3*I/a^2/d/(a+I*a*tan(d*x+c))^6-1/5*I/a^3/d/(a+I*a*tan(d*x+c))^5
```

3.171.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2i+3\tan(c+dx)}{15a^8d(-i+\tan(c+dx))^6}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/15*(2*I + 3*Tan[c + d*x])/(a^8*d*(-I + Tan[c + d*x])^6)
```

3.171.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^7}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^7}-\frac{1}{(i\tan(c+dx)a+a)^6}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{1}{5(a+ia\tan(c+dx))^5}-\frac{a}{3(a+ia\tan(c+dx))^6}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-I)*(-1/3*a/(a + I*a*Tan[c + d*x])^6 + 1/(5*(a + I*a*Tan[c + d*x])^5)))/
(a^3*d)
```

3.171.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.171.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{-\frac{i}{3(\tan(dx+c)-i)^6}-\frac{1}{5(\tan(dx+c)-i)^5}}{a^8d}$	36
default	$\frac{-\frac{i}{3(\tan(dx+c)-i)^6}-\frac{1}{5(\tan(dx+c)-i)^5}}{a^8d}$	36
risch	$\frac{i\mathrm{e}^{-4i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^8d}+\frac{3i\mathrm{e}^{-8i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{40a^8d}+\frac{i\mathrm{e}^{-12i(dx+c)}}{192a^8d}$	92

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/a^8/d*(-1/3*I/(tan(d*x+c)-I)^6-1/5/(tan(d*x+c)-I)^5)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(15i\,e^{(8i\,dx+8i\,c)}+40i\,e^{(6i\,dx+6i\,c)}+45i\,e^{(4i\,dx+4i\,c)}+24i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-12i\,dx-12i\,c)}}{960\,a^8d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/960*(15*I*e^(8*I*d*x + 8*I*c) + 40*I*e^(6*I*d*x + 6*I*c) + 45*I*e^(4*I*d
*x + 4*I*c) + 24*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^8*
d)
```

3.171.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(42) = 84.

Time = 8.87 (sec) , antiderivative size = 774, normalized size of antiderivative = 14.07

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{i\tan^4(c+dx)\sec^4(c+dx)}{960a^8d\tan^8(c+dx)-7680ia^8d\tan^7(c+dx)-26880a^8d\tan^6(c+dx)+53760ia^8d\tan^5(c+dx)+67200a^8d\tan^4(c+dx)-53760ia^8d\tan^3(c+dx)-} \\ \frac{x\sec^4(c)}{(ia\tan(c)+a)^8} \end{cases}$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((I*tan(c + d*x)**4*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 -
 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a*
*8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c
 + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 9
60*a**8*d) + 8*tan(c + d*x)**3*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8
 - 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*
a**8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan
(c + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) +
 960*a**8*d) - 30*I*tan(c + d*x)**2*sec(c + d*x)**4/(960*a**8*d*tan(c + d*
x)**8 - 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 537
60*I*a**8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*
d*tan(c + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d
*x) + 960*a**8*d) - 72*tan(c + d*x)*sec(c + d*x)**4/(960*a**8*d*tan(c + d*
x)**8 - 7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 537
60*I*a**8*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*
d*tan(c + d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d
*x) + 960*a**8*d) + 129*I*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 76
80*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*
d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c +
d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 9...
```

3.171.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(43) = 86$.

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.20

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{3\,\tan(dx+c)^2 - i\,\tan(dx+c) + 2}{15\,\left(a^8\tan(dx+c)^7 - 7i\,a^8\tan(dx+c)^6 - 21\,a^8\tan(dx+c)^5 + 35i\,a^8\tan(dx+c)^4 + 35\,a^8\tan(dx+\right.}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-1/15*(3*tan(d*x + c)^2 - I*tan(d*x + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a
^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a
^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*
d)
```

3.171.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(43) = 86$.

Time = 1.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.96

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-60i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-235\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+82\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2/15*(15*tan(1/2*d*x + 1/2*c)^11 - 60*I*tan(1/2*d*x + 1/2*c)^10 - 235*tan
(1/2*d*x + 1/2*c)^9 + 480*I*tan(1/2*d*x + 1/2*c)^8 + 822*tan(1/2*d*x + 1/2
*c)^7 - 904*I*tan(1/2*d*x + 1/2*c)^6 - 822*tan(1/2*d*x + 1/2*c)^5 + 480*I*
tan(1/2*d*x + 1/2*c)^4 + 235*tan(1/2*d*x + 1/2*c)^3 - 60*I*tan(1/2*d*x + 1
/2*c)^2 - 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^12)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{-2+\tan(c+d\,x)\;3\mathrm{i}}{15\,a^8\,d\,\left(\tan\left(c+d\,x\right)^6\,1\mathrm{i}+6\tan\left(c+d\,x\right)^5-\tan\left(c+d\,x\right)^4\,15\mathrm{i}-20\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,15\mathrm{i}-\right.}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
-(tan(c + d*x)*3i - 2)/(15*a^8*d*(6*tan(c + d*x) + tan(c + d*x)^2*15i - 20
*tan(c + d*x)^3 - tan(c + d*x)^4*15i + 6*tan(c + d*x)^5 + tan(c + d*x)^6*1
i - 1i))
```

3.172 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.172.1 Optimal result 1300
3.172.2 Mathematica [A] (verified) 1300
3.172.3 Rubi [A] (verified) 1301
3.172.4 Maple [A] (verified) 1302
3.172.5 Fricas [B] (verification not implemented) 1302
3.172.6 Sympy [B] (verification not implemented) 1303
3.172.7 Maxima [A] (verification not implemented) 1303
3.172.8 Giac [B] (verification not implemented) 1304
3.172.9 Mupad [B] (verification not implemented) 1304

3.172.1 Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{7ad(a+ia\tan(c+dx))^7}$$

output
```
1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

3.172.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{1}{7a^8d(-i+\tan(c+dx))^7}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/7*1/(a^8*d*(-I + Tan[c + d*x])^7)
```

3.172.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^8}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$\frac{i}{7ad(a+ia\tan(c+dx))^7}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(I/7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

3.172.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.172.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{i}{7ad(a+ia\tan(dx+c))^{7}}$
default	$\frac{i}{7ad(a+ia\tan(dx+c))^{7}}$
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{128a^{8}d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{128a^{8}d}+\frac{5i\mathrm{e}^{-6i(dx+c)}}{128a^{8}d}+\frac{5i\mathrm{e}^{-8i(dx+c)}}{128a^{8}d}+\frac{3i\mathrm{e}^{-10i(dx+c)}}{128a^{8}d}+\frac{i\mathrm{e}^{-12i(dx+c)}}{128a^{8}d}+\frac{i\mathrm{e}^{-14i(dx+c)}}{896a^{8}d}$

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

3.172.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{\sec^{2}(c+dx)}{(a+ia\tan(c+dx))^{8}}\,dx$$
$$=\frac{\left(7i\,e^{(12i\,dx+12i\,c)}+21i\,e^{(10i\,dx+10i\,c)}+35i\,e^{(8i\,dx+8i\,c)}+35i\,e^{(6i\,dx+6i\,c)}+21i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-}}{896\,a^{8}d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/896*(7*I*e^(12*I*d*x + 12*I*c) + 21*I*e^(10*I*d*x + 10*I*c) + 35*I*e^(8*
I*d*x + 8*I*c) + 35*I*e^(6*I*d*x + 6*I*c) + 21*I*e^(4*I*d*x + 4*I*c) + 7*I
*e^(2*I*d*x + 2*I*c) + I)*e^(-14*I*d*x - 14*I*c)/(a^8*d)
```

3.172.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(19) = 38$.

Time = 8.89 (sec) , antiderivative size = 1081, normalized size of antiderivative = 40.04

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-I*tan(c + d*x)**6*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8
- 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a
**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(
c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) +
896*a**8*d) - 8*tan(c + d*x)**5*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**
8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I
*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*ta
n(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x)
+ 896*a**8*d) + 29*I*tan(c + d*x)**4*sec(c + d*x)**2/(896*a**8*d*tan(c + d
*x)**8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50
176*I*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8
*d*tan(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c +
d*x) + 896*a**8*d) + 64*tan(c + d*x)**3*sec(c + d*x)**2/(896*a**8*d*tan(c
+ d*x)**8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 +
 50176*I*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a
**8*d*tan(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c
 + d*x) + 896*a**8*d) - 99*I*tan(c + d*x)**2*sec(c + d*x)**2/(896*a**8*d*t
an(c + d*x)**8 - 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)
**6 + 50176*I*a**8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 5017
6*I*a**8*d*tan(c + d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8...
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{7\left(i\,a\tan\left(dx+c\right)+a\right)^7 ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/7*I/((I*a*tan(d*x + c) + a)^7*a*d)
```

3.172.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(21) = 42.

Time = 1.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 7.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{2\left(7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-42i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-182\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+490i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1\right.}{}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-2/7*(7*tan(1/2*d*x + 1/2*c)^13 - 42*I*tan(1/2*d*x + 1/2*c)^12 - 182*tan(1
/2*d*x + 1/2*c)^11 + 490*I*tan(1/2*d*x + 1/2*c)^10 + 1001*tan(1/2*d*x + 1/
2*c)^9 - 1484*I*tan(1/2*d*x + 1/2*c)^8 - 1716*tan(1/2*d*x + 1/2*c)^7 + 148
4*I*tan(1/2*d*x + 1/2*c)^6 + 1001*tan(1/2*d*x + 1/2*c)^5 - 490*I*tan(1/2*d
*x + 1/2*c)^4 - 182*tan(1/2*d*x + 1/2*c)^3 + 42*I*tan(1/2*d*x + 1/2*c)^2 +
 7*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^14)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{1}{7\,a^8\,d\,{\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)}^7}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
-1/(7*a^8*d*(tan(c + d*x) - 1i)^7)
```

3.173 $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$

3.173.1 Optimal result 1305
3.173.2 Mathematica [A] (verified) 1306
3.173.3 Rubi [A] (verified) 1306
3.173.4 Maple [A] (verified) 1310
3.173.5 Fricas [A] (verification not implemented) 1310
3.173.6 Sympy [A] (verification not implemented) 1311
3.173.7 Maxima [F(-2)] 1311
3.173.8 Giac [A] (verification not implemented) 1312
3.173.9 Mupad [B] (verification not implemented) 1312

3.173.1 Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx &= \frac{x}{256a^8}+\frac{i}{16d(a+ia\tan(c+dx))^8}\\ &\quad+\frac{i}{28ad(a+ia\tan(c+dx))^7}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}\\ &\quad+\frac{i}{80a^3d(a+ia\tan(c+dx))^5}+\frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4}\\ &\quad+\frac{i}{192a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}\\ &\quad+\frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output

```
1/256*x/a^8+1/16*I/d/(a+I*a*tan(d*x+c))^8+1/28*I/a/d/(a+I*a*tan(d*x+c))^7+
1/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/80*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/128*
I/d/(a^2+I*a^2*tan(d*x+c))^4+1/192*I/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+1/256*
I/d/(a^4+I*a^4*tan(d*x+c))^2+1/256*I/d/(a^8+I*a^8*tan(d*x+c))
```

3.173.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{i\sec^8(c+dx)(7350+12544\cos(2(c+dx))+7840\cos(4(c+dx))+3840\cos(6(c+dx))+1194\cos(8(c+}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(-8),x]
```

output
```
((I/215040)*Sec[c + d*x]^8*(7350 + 12544*Cos[2*(c + d*x)] + 7840*Cos[4*(c
+ d*x)] + 3840*Cos[6*(c + d*x)] + 1194*Cos[8*(c + d*x)] + (3136*I)*Sin[2*(
c + d*x)] + (3920*I)*Sin[4*(c + d*x)] + (2880*I)*Sin[6*(c + d*x)] + (1089*
I)*Sin[8*(c + d*x)] + 840*ArcTan[Tan[c + d*x]]*((-I)*Cos[8*(c + d*x)] + Si
n[8*(c + d*x)])))/(a^8*d*(-I + Tan[c + d*x])^8)
```

3.173.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.133$, Rules used = {3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3960}$$
$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^7}dx}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{(i\tan(c+dx)a+a)^7}dx}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^6}dx}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^6}dx}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^5}dx}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^5}dx}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^4}dx}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^4}dx}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3960}$$

$$\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

3.173. $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3960$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{i}{14d(a+ia\tan(c+dx))^7}}{2a}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3960$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{}{14d(a+ia\,\mathrm{t}}}{2a} \quad \frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+\frac{}{14d(a+ia\,\mathrm{t}}}{2a} \quad \frac{i}{16d(a+ia\tan(c+dx))^8}$$

3.173. $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \color{blue}{3960}$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+14d(}{2a}$$

$$\frac{i}{16d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \color{blue}{24}$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}}{2a}+\frac{i}{8d(a+ia\tan(c+dx))^4}}{2a}+\frac{i}{10d(a+ia\tan(c+dx))^5}}{2a}+\frac{i}{12d(a+ia\tan(c+dx))^6}}{2a}+14d(a+}{2a}$$

$$\frac{i}{16d(a+ia\tan(c+dx))^8}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(-8),x]
```

output

```
(I/16)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/14)/(d*(a + I*a*Tan[c + d*x])^7)
 + ((I/12)/(d*(a + I*a*Tan[c + d*x])^6) + ((I/10)/(d*(a + I*a*Tan[c + d*x]
)^5) + ((I/8)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/6)/(d*(a + I*a*Tan[c + d*
x])^3) + ((I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (x/(2*a) + (I/2)/(d*(a + I*
a*Tan[c + d*x])))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a)
```

3.173.3.1 Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.173.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

method	result
risch	$\frac{x}{256a^8}+\frac{i\mathrm{e}^{-2i(dx+c)}}{64a^8d}+\frac{7i\mathrm{e}^{-4i(dx+c)}}{256a^8d}+\frac{7i\mathrm{e}^{-6i(dx+c)}}{192a^8d}+\frac{35i\mathrm{e}^{-8i(dx+c)}}{1024a^8d}+\frac{7i\mathrm{e}^{-10i(dx+c)}}{320a^8d}+\frac{7i\mathrm{e}^{-12i(dx+c)}}{768a^8d}+$
derivativedivides	$\frac{\arctan(\tan(dx+c))}{256a^8d}+\frac{i}{16d\,a^8(\tan(dx+c)-i)^8}+\frac{i}{128d\,a^8(\tan(dx+c)-i)^4}-\frac{i}{48d\,a^8(\tan(dx+c)-i)^6}-$
default	$\frac{\arctan(\tan(dx+c))}{256a^8d}+\frac{i}{16d\,a^8(\tan(dx+c)-i)^8}+\frac{i}{128d\,a^8(\tan(dx+c)-i)^4}-\frac{i}{48d\,a^8(\tan(dx+c)-i)^6}-$
norman	$\frac{x}{256a}+\frac{961\left(\tan^7(dx+c)\right)}{8960ad}+\frac{7x\left(\tan^4(dx+c)\right)}{64a}-\frac{1117\left(\tan^3(dx+c)\right)}{256ad}+\frac{x\left(\tan^2(dx+c)\right)}{32a}+\frac{35x\left(\tan^8(dx+c)\right)}{128a}+\frac{7x\left(\tan^{10}(dx+c)\right)}{32a}+$

input
```
int(1/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/256*x/a^8+1/64*I/a^8/d*exp(-2*I*(d*x+c))+7/256*I/a^8/d*exp(-4*I*(d*x+c))
+7/192*I/a^8/d*exp(-6*I*(d*x+c))+35/1024*I/a^8/d*exp(-8*I*(d*x+c))+7/320*I
/a^8/d*exp(-10*I*(d*x+c))+7/768*I/a^8/d*exp(-12*I*(d*x+c))+1/448*I/a^8/d*e
xp(-14*I*(d*x+c))+1/4096*I/a^8/d*exp(-16*I*(d*x+c))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(1680\,dxe^{(16i\,dx+16i\,c)}+6720i\,e^{(14i\,dx+14i\,c)}+11760i\,e^{(12i\,dx+12i\,c)}+15680i\,e^{(10i\,dx+10i\,c)}+14700i\,e^{(8i\,dx+8i\,c)}\right.}{430080\,a^8d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/430080*(1680*d*x*e^(16*I*d*x + 16*I*c) + 6720*I*e^(14*I*d*x + 14*I*c) +
11760*I*e^(12*I*d*x + 12*I*c) + 15680*I*e^(10*I*d*x + 10*I*c) + 14700*I*e^
(8*I*d*x + 8*I*c) + 9408*I*e^(6*I*d*x + 6*I*c) + 3920*I*e^(4*I*d*x + 4*I*c
) + 960*I*e^(2*I*d*x + 2*I*c) + 105*I)*e^(-16*I*d*x - 16*I*c)/(a^8*d)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\begin{cases} \frac{(22698142121947299840ia^{56}d^7e^{70ic}e^{-2idx}+39721748713407774720ia^{56}d^7e^{68ic}e^{-4idx}+52962331617877032960ia^{56}d^7e^{66ic}e^{-6idx}+49652185}{} \\ x\left(\frac{(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-16ic}}{256a^8}-\frac{1}{256a^8}\right) \end{cases}$$
$$+\frac{x}{256a^8}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((22698142121947299840*I*a**56*d**7*exp(70*I*c)*exp(-2*I*d*x) +
39721748713407774720*I*a**56*d**7*exp(68*I*c)*exp(-4*I*d*x) + 529623316178
77032960*I*a**56*d**7*exp(66*I*c)*exp(-6*I*d*x) + 49652185891759718400*I*a
**56*d**7*exp(64*I*c)*exp(-8*I*d*x) + 31773989707262197776*I*a**56*d**7*ex
p(62*I*c)*exp(-10*I*d*x) + 13240582904469258240*I*a**56*d**7*exp(60*I*c)*e
xp(-12*I*d*x) + 3242591731706757120*I*a**56*d**7*exp(58*I*c)*exp(-14*I*d*x
) + 354658470655426560*I*a**56*d**7*exp(56*I*c)*exp(-16*I*d*x))*exp(-72*I*
c)/(1452681095804627189760*a**64*d**8), Ne(a**64*d**8*exp(72*I*c), 0)), (x
*((exp(16*I*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(
8*I*c) + 56*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-16*I*c)/(2
56*a**8) - 1/(256*a**8)), True)) + x/(256*a**8)
```

3.173.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{-\frac{840i\log(\tan(dx+c)+i)}{a^8}+\frac{840i\log(\tan(dx+c)-i)}{a^8}+\frac{-2283i\tan(dx+c)^8-19944\tan(dx+c)^7+77364i\tan(dx+c)^6+175448\tan(dx+c)^5}{a^8(}}{430080\,d}$$

input
```
integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/430080*(-840*I*log(tan(d*x + c) + I)/a^8 + 840*I*log(tan(d*x + c) - I)/
a^8 + (-2283*I*tan(d*x + c)^8 - 19944*tan(d*x + c)^7 + 77364*I*tan(d*x + c
)^6 + 175448*tan(d*x + c)^5 - 258370*I*tan(d*x + c)^4 - 261464*tan(d*x + c
)^3 + 192052*I*tan(d*x + c)^2 + 114152*tan(d*x + c) - 67819*I)/(a^8*(tan(d
*x + c) - I)^8))/d
```

3.173.9 Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{x}{256\,a^8}$$

$$-\frac{\frac{\tan(c+d\,x)\,5993\mathrm{i}}{26880\,a^8}+\frac{16}{105\,a^8}-\frac{143\tan(c+d\,x)^2}{480\,a^8}-\frac{\tan(c+d\,x)^3\,1193\mathrm{i}}{3840\,a^8}+\frac{11\tan(c+d\,x)^4}{48\,a^8}+\frac{\tan(c+d\,x)}{768\,a^8}}{d\,\left(\tan\left(c+d\,x\right)^8\,1\mathrm{i}+8\tan\left(c+d\,x\right)^7-\tan\left(c+d\,x\right)^6\,28\mathrm{i}-56\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,70\mathrm{i}+56\,\mathrm{t}\right.}$$

input
```
int(1/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
x/(256*a^8) - ((tan(c + d*x)*5993i)/(26880*a^8) + 16/(105*a^8) - (143*tan(
c + d*x)^2)/(480*a^8) - (tan(c + d*x)^3*1193i)/(3840*a^8) + (11*tan(c + d*
x)^4)/(48*a^8) + (tan(c + d*x)^5*85i)/(768*a^8) - tan(c + d*x)^6/(32*a^8)
- (tan(c + d*x)^7*1i)/(256*a^8))/(d*(56*tan(c + d*x)^3 - tan(c + d*x)^2*28
i - 8*tan(c + d*x) + tan(c + d*x)^4*70i - 56*tan(c + d*x)^5 - tan(c + d*x)
^6*28i + 8*tan(c + d*x)^7 + tan(c + d*x)^8*1i + 1i))
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.174.1 Optimal result

Integrand size = 24, antiderivative size = 278

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{5x}{512a^8} + \frac{ia}{36d(a+ia\tan(c+dx))^9} + \frac{i}{32d(a+ia\tan(c+dx))^8} \\
&\quad + \frac{3i}{112ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} \\
&\quad + \frac{i}{64a^3d(a+ia\tan(c+dx))^5} + \frac{7i}{768a^5d(a+ia\tan(c+dx))^3} \\
&\quad + \frac{3i}{256d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{i}{128d\left(a^4+ia^4\tan(c+dx)\right)^2} \\
&\quad - \frac{i}{1024d\left(a^8-ia^8\tan(c+dx)\right)} + \frac{9i}{1024d\left(a^8+ia^8\tan(c+dx)\right)}
\end{aligned}
$$

output

```
5/512*x/a^8+1/36*I*a/d/(a+I*a*tan(d*x+c))^9+1/32*I/d/(a+I*a*tan(d*x+c))^8+
3/112*I/a/d/(a+I*a*tan(d*x+c))^7+1/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/64*I/
a^3/d/(a+I*a*tan(d*x+c))^5+7/768*I/a^5/d/(a+I*a*tan(d*x+c))^3+3/256*I/d/(a
^2+I*a^2*tan(d*x+c))^4+1/128*I/d/(a^4+I*a^4*tan(d*x+c))^2-1/1024*I/d/(a^8-
I*a^8*tan(d*x+c))+9/1024*I/d/(a^8+I*a^8*tan(d*x+c))
```

3.174.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\sec^{10}(c+dx)(2520\arctan(\tan(c+dx))(\cos(8(c+dx))+i\sin(8(c+dx)))+i(7938+14112\cos(2(c+dx$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(Sec[c + d*x]^10*(2520*ArcTan[Tan[c + d*x]]*(Cos[8*(c + d*x)] + I*Sin[8*(c
 + d*x)]) + I*(7938 + 14112*Cos[2*(c + d*x)] + 10080*Cos[4*(c + d*x)] + 64
80*Cos[6*(c + d*x)] + 2462*Cos[8*(c + d*x)] - 112*Cos[10*(c + d*x)] + (352
8*I)*Sin[2*(c + d*x)] + (5040*I)*Sin[4*(c + d*x)] + (4860*I)*Sin[6*(c + d*
x)] + (2147*I)*Sin[8*(c + d*x)] - (140*I)*Sin[10*(c + d*x)])))/(258048*a^8
*d*(-I + Tan[c + d*x])^9*(I + Tan[c + d*x]))
```

3.174.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \; 3968$$
$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{10}}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \; 54$$

$$-\frac{ia^3\int\left(\frac{1}{1024a^{10}(a-ia\tan(c+dx))^2}+\frac{9}{1024a^{10}(i\tan(c+dx)a+a)^2}+\frac{1}{64a^9(i\tan(c+dx)a+a)^3}+\frac{7}{256a^8(i\tan(c+dx)a+a)^4}+\frac{3}{64a^7(i\tan(c+}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ia^3\left(\frac{5i\arctan(\tan(c+dx))}{512a^{11}}+\frac{1}{1024a^{10}(a-ia\tan(c+dx))}-\frac{9}{1024a^{10}(a+ia\tan(c+dx))}-\frac{1}{128a^9(a+ia\tan(c+dx))^2}-\frac{7}{768a^8(a+ia\tan(c+da}\right.}{}$$

input

```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-I)*a^3*((((5*I)/512)*ArcTan[Tan[c + d*x]])/a^11 + 1/(1024*a^10*(a - I*a
*Tan[c + d*x])) - 1/(36*a^2*(a + I*a*Tan[c + d*x])^9) - 1/(32*a^3*(a + I*a
*Tan[c + d*x])^8) - 3/(112*a^4*(a + I*a*Tan[c + d*x])^7) - 1/(48*a^5*(a +
I*a*Tan[c + d*x])^6) - 1/(64*a^6*(a + I*a*Tan[c + d*x])^5) - 3/(256*a^7*(a
 + I*a*Tan[c + d*x])^4) - 7/(768*a^8*(a + I*a*Tan[c + d*x])^3) - 1/(128*a^
9*(a + I*a*Tan[c + d*x])^2) - 9/(1024*a^10*(a + I*a*Tan[c + d*x]))))/d
```

3.174.3.1 Defintions of rubi rules used

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.174.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{1024}+\frac{3i}{256(\tan(dx+c)-i)^4}+\frac{i}{32(\tan(dx+c)-i)^8}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{i}{128(\tan(dx+c)-i)^2}+\frac{1}{36(\tan(dx+c)-i)^9}-}{d\,a}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{1024}+\frac{3i}{256(\tan(dx+c)-i)^4}+\frac{i}{32(\tan(dx+c)-i)^8}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{i}{128(\tan(dx+c)-i)^2}+\frac{1}{36(\tan(dx+c)-i)^9}-}{d\,a}$
risch	$\frac{5x}{512a^8}+\frac{15i\mathrm{e}^{-4i(dx+c)}}{512a^8d}+\frac{35i\mathrm{e}^{-6i(dx+c)}}{1024a^8d}+\frac{63i\mathrm{e}^{-8i(dx+c)}}{2048a^8d}+\frac{21i\mathrm{e}^{-10i(dx+c)}}{1024a^8d}+\frac{5i\mathrm{e}^{-12i(dx+c)}}{512a^8d}+\frac{45i\mathrm{e}^{-14i(dx+c}}{14336a^8d}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^8*(-5/1024*I*ln(tan(d*x+c)-I)+3/256*I/(tan(d*x+c)-I)^4+1/32*I/(tan(d
*x+c)-I)^8-1/48*I/(tan(d*x+c)-I)^6-1/128*I/(tan(d*x+c)-I)^2+1/36/(tan(d*x+
c)-I)^9-3/112/(tan(d*x+c)-I)^7+1/64/(tan(d*x+c)-I)^5-7/768/(tan(d*x+c)-I)^
3+9/1024/(tan(d*x+c)-I)+5/1024*I*ln(tan(d*x+c)+I)+1/1024/(tan(d*x+c)+I))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.47

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{(5040\,dxe^{(18i\,dx+18i\,c)}-252i\,e^{(20i\,dx+20i\,c)}+11340i\,e^{(16i\,dx+16i\,c)}+15120i\,e^{(14i\,dx+14i\,c)}+17640i\,e^{(12i\,dx+12i\,c)}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/516096*(5040*d*x*e^(18*I*d*x + 18*I*c) - 252*I*e^(20*I*d*x + 20*I*c) + 1
1340*I*e^(16*I*d*x + 16*I*c) + 15120*I*e^(14*I*d*x + 14*I*c) + 17640*I*e^(
12*I*d*x + 12*I*c) + 15876*I*e^(10*I*d*x + 10*I*c) + 10584*I*e^(8*I*d*x +
8*I*c) + 5040*I*e^(6*I*d*x + 6*I*c) + 1620*I*e^(4*I*d*x + 4*I*c) + 315*I*e
^(2*I*d*x + 2*I*c) + 28*I)*e^(-18*I*d*x - 18*I*c)/(a^8*d)
```

3.174.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.42

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{(-24956871191993266341966344355 84ia^{72}d^9e^{92ic}e^{2idx}+11230592036396969853884854960 1280ia^{72}d^9e^{88ic}e^{-2idx}+14974122715195959 8051}{} \\ x\left(\frac{(e^{20ic}+10e^{18ic}+45e^{16ic}+120e^{14ic}+210e^{12ic}+252e^{10ic}+210e^{8ic}+120e^{6ic}+45e^{4ic}+10e^{2ic}+1)e^{-18ic}}{1024a^8}-\frac{5}{512a^8}\right) \end{cases}$$
$$+\frac{5x}{512a^8}$$

input

```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

output

```
Piecewise(((-2495687119199326634196634435584*I*a**72*d**9*exp(92*I*c)*exp(
2*I*d*x) + 11230592036396969853884854960128O*I*a**72*d**9*exp(88*I*c)*exp(
-2*I*d*x) + 14974122715195959805179806613504O*I*a**72*d**9*exp(86*I*c)*exp
(-4*I*d*x) + 17469809834395286439376441049088O*I*a**72*d**9*exp(84*I*c)*ex
p(-6*I*d*x) + 15722828850955757795438796944179 2*I*a**72*d**9*exp(82*I*c)*e
xp(-8*I*d*x) + 1048188590063717186362586462945 28*I*a**72*d**9*exp(80*I*c)*
exp(-10*I*d*x) + 49913742383986532683932688711680*I*a**72*d**9*exp(78*I*c)
*exp(-12*I*d*x) + 16043702909138528362692649943040*I*a**72*d**9*exp(76*I*c
)*exp(-14*I*d*x) + 3119608898999158292745793044480*I*a**72*d**9*exp(74*I*c
)*exp(-16*I*d*x) + 277298568799925181577403826176*I*a**72*d**9*exp(72*I*c)
*exp(-18*I*d*x))*exp(-90*I*c)/(5111167220120220946834707324076032*a**80*d*
*10), Ne(a**80*d**10*exp(90*I*c), 0)), (x*((exp(20*I*c) + 10*exp(18*I*c) +
 45*exp(16*I*c) + 120*exp(14*I*c) + 210*exp(12*I*c) + 252*exp(10*I*c) + 21
0*exp(8*I*c) + 120*exp(6*I*c) + 45*exp(4*I*c) + 10*exp(2*I*c) + 1)*exp(-18
*I*c)/(1024*a**8) - 5/(512*a**8)), True)) + 5*x/(512*a**8)
```

3.174.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input

```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.174.8 Giac [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.59

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{-\frac{2520i\log(\tan(dx+c)+i)}{a^8}+\frac{2520i\log(\tan(dx+c)-i)}{a^8}+\frac{504\,(5i\tan(dx+c)-6)}{a^8(\tan(dx+c)+i)}+\frac{-7129i\tan(dx+c)^9-68697\tan(dx+c)^8+296964i\tan}{}}{5}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/516096*(-2520*I*log(tan(d*x + c) + I)/a^8 + 2520*I*log(tan(d*x + c) - I
)/a^8 + 504*(5*I*tan(d*x + c) - 6)/(a^8*(tan(d*x + c) + I)) + (-7129*I*tan
(d*x + c)^9 - 68697*tan(d*x + c)^8 + 296964*I*tan(d*x + c)^7 + 758772*tan(
d*x + c)^6 - 1271214*I*tan(d*x + c)^5 - 1465758*tan(d*x + c)^4 + 1191540*I
*tan(d*x + c)^3 + 693828*tan(d*x + c)^2 - 295425*I*tan(d*x + c) - 89553)/(
a^8*(tan(d*x + c) - I)^9))/d
```

3.174.9 Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{5\,x}{512\,a^8}$$

$$+\frac{\frac{163\tan(c+d\,x)^2}{448\,a^8}-\frac{10}{63\,a^8}-\frac{\tan(c+d\,x)\,9019\mathrm{i}}{32256\,a^8}+\frac{\tan(c+d\,x)^3\,393\mathrm{i}}{1792\,a^8}+\frac{11\tan(c+d\,x)^4}{64\,a^8}+\frac{\tan(c+d\,x)^5\,1\mathrm{i}}{2\,a^8}-}{d\,\left(\tan\left(c+d\,x\right)^{10}\,1\mathrm{i}+8\tan\left(c+d\,x\right)^9-\tan\left(c+d\,x\right)^8\,27\mathrm{i}-48\tan\left(c+d\,x\right)^7+\tan\left(c+d\,x\right)^6\,42\mathrm{i}+\tan\right.}$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(5*x)/(512*a^8) + ((163*tan(c + d*x)^2)/(448*a^8) - 10/(63*a^8) - (tan(c +
 d*x)*9019i)/(32256*a^8) + (tan(c + d*x)^3*393i)/(1792*a^8) + (11*tan(c +
d*x)^4)/(64*a^8) + (tan(c + d*x)^5*1i)/(2*a^8) - (95*tan(c + d*x)^6)/(192*
a^8) - (tan(c + d*x)^7*205i)/(768*a^8) + (5*tan(c + d*x)^8)/(64*a^8) + (ta
n(c + d*x)^9*5i)/(512*a^8))/(d*(48*tan(c + d*x)^3 - tan(c + d*x)^2*27i - 8
*tan(c + d*x) + tan(c + d*x)^4*42i + tan(c + d*x)^6*42i - 48*tan(c + d*x)^
7 - tan(c + d*x)^8*27i + 8*tan(c + d*x)^9 + tan(c + d*x)^10*1i + 1i))
```

3.174. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.175 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.175.9 Mupad [B] (verification not implemented) 1326

3.175.1 Optimal result

Integrand size = 24, antiderivative size = 333

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{33x}{2048a^8}+\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} \\ &\quad+\frac{ia}{48d(a+ia\tan(c+dx))^9}+\frac{3i}{128d(a+ia\tan(c+dx))^8} \\ &\quad+\frac{5i}{224ad(a+ia\tan(c+dx))^7}+\frac{5i}{256a^2d(a+ia\tan(c+dx))^6} \\ &\quad+\frac{21i}{1280a^3d(a+ia\tan(c+dx))^5} \\ &\quad+\frac{3i}{256a^5d(a+ia\tan(c+dx))^3}+\frac{7i}{512d\left(a^2+ia^2\tan(c+dx)\right)^4} \\ &\quad-\frac{i}{4096d\left(a^4-ia^4\tan(c+dx)\right)^2} \\ &\quad+\frac{45i}{4096d\left(a^4+ia^4\tan(c+dx)\right)^2} \\ &\quad-\frac{11i}{4096d\left(a^8-ia^8\tan(c+dx)\right)}+\frac{55i}{4096d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output
```
33/2048*x/a^8+1/80*I*a^2/d/(a+I*a*tan(d*x+c))^10+1/48*I*a/d/(a+I*a*tan(d*x
+c))^9+3/128*I/d/(a+I*a*tan(d*x+c))^8+5/224*I/a/d/(a+I*a*tan(d*x+c))^7+5/2
56*I/a^2/d/(a+I*a*tan(d*x+c))^6+21/1280*I/a^3/d/(a+I*a*tan(d*x+c))^5+3/256
*I/a^5/d/(a+I*a*tan(d*x+c))^3+7/512*I/d/(a^2+I*a^2*tan(d*x+c))^4-1/4096*I/
d/(a^4-I*a^4*tan(d*x+c))^2+45/4096*I/d/(a^4+I*a^4*tan(d*x+c))^2-11/4096*I/
d/(a^8-I*a^8*tan(d*x+c))+55/4096*I/d/(a^8+I*a^8*tan(d*x+c))
```

3.175.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.62

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\sec^{12}(c+dx)(48510i+88704i\cos(2(c+dx))+69300i\cos(4(c+dx))+52800i\cos(6(c+dx))+21538i\,c}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(Sec[c + d*x]^12*(48510*I + (88704*I)*Cos[2*(c + d*x)] + (69300*I)*Cos[4*(
c + d*x)] + (52800*I)*Cos[6*(c + d*x)] + (21538*I)*Cos[8*(c + d*x)] - (224
0*I)*Cos[10*(c + d*x)] - (84*I)*Cos[12*(c + d*x)] - 22176*Sin[2*(c + d*x)]
 - 34650*Sin[4*(c + d*x)] - 39600*Sin[6*(c + d*x)] + 27720*ArcTan[Tan[c +
d*x]]*(Cos[8*(c + d*x)] + I*Sin[8*(c + d*x)]) - 18073*Sin[8*(c + d*x)] + 2
800*Sin[10*(c + d*x)] + 126*Sin[12*(c + d*x)]))/(1720320*a^8*d*(-I + Tan[c
 + d*x])^10*(I + Tan[c + d*x])^2)
```

3.175.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \; 3968$$
$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11}}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \; 54$$

$$-\frac{ia^5\int\left(\frac{11}{4096a^{12}(a-ia\tan(c+dx))^2}+\frac{55}{4096a^{12}(i\tan(c+dx)a+a)^2}+\frac{1}{2048a^{11}(a-ia\tan(c+dx))^3}+\frac{45}{2048a^{11}(i\tan(c+dx)a+a)^3}+\frac{}{256a^{10}(}\right.}{}$$

$$\Big\downarrow 2009$$

$$-\frac{ia^5\left(\frac{33i\arctan(\tan(c+dx))}{2048a^{13}}+\frac{11}{4096a^{12}(a-ia\tan(c+dx))}-\frac{55}{4096a^{12}(a+ia\tan(c+dx))}+\frac{1}{4096a^{11}(a-ia\tan(c+dx))^2}-\frac{45}{4096a^{11}(a+ia\tan}\right.}{}$$

input

```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((-I)*a^5*((((33*I)/2048)*ArcTan[Tan[c + d*x]])/a^13 + 1/(4096*a^11*(a - I
*a*Tan[c + d*x])^2) + 11/(4096*a^12*(a - I*a*Tan[c + d*x])) - 1/(80*a^3*(a
 + I*a*Tan[c + d*x])^10) - 1/(48*a^4*(a + I*a*Tan[c + d*x])^9) - 3/(128*a^
5*(a + I*a*Tan[c + d*x])^8) - 5/(224*a^6*(a + I*a*Tan[c + d*x])^7) - 5/(25
6*a^7*(a + I*a*Tan[c + d*x])^6) - 21/(1280*a^8*(a + I*a*Tan[c + d*x])^5) -
 7/(512*a^9*(a + I*a*Tan[c + d*x])^4) - 3/(256*a^10*(a + I*a*Tan[c + d*x])
^3) - 45/(4096*a^11*(a + I*a*Tan[c + d*x])^2) - 55/(4096*a^12*(a + I*a*Tan
[c + d*x]))))/d
```

3.175.3.1 Defintions of rubi rules used

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.175.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\frac{i}{4096(\tan(dx+c)+i)^2}+\frac{33i\ln(\tan(dx+c)+i)}{4096}+\frac{11}{4096(\tan(dx+c)+i)}-\frac{33i\ln(\tan(dx+c)-i)}{4096}+\frac{7i}{512(\tan(dx+c)-i)^4}+\frac{3i}{128(\tan(dx+c)-i)^8}}{}$
default	$\frac{\frac{i}{4096(\tan(dx+c)+i)^2}+\frac{33i\ln(\tan(dx+c)+i)}{4096}+\frac{11}{4096(\tan(dx+c)+i)}-\frac{33i\ln(\tan(dx+c)-i)}{4096}+\frac{7i}{512(\tan(dx+c)-i)^4}+\frac{3i}{128(\tan(dx+c)-i)^8}}{}$
risch	$\frac{33x}{2048a^8}+\frac{33i\mathrm{e}^{-6i(dx+c)}}{1024a^8d}+\frac{231i\mathrm{e}^{-8i(dx+c)}}{8192a^8d}+\frac{99i\mathrm{e}^{-10i(dx+c)}}{5120a^8d}+\frac{165i\mathrm{e}^{-12i(dx+c)}}{16384a^8d}+\frac{55i\mathrm{e}^{-14i(dx+c)}}{14336a^8d}+\frac{33i\mathrm{e}^{-16i}}{32768}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^8*(1/4096*I/(tan(d*x+c)+I)^2+33/4096*I*ln(tan(d*x+c)+I)+11/4096/(tan
(d*x+c)+I)-33/4096*I*ln(tan(d*x+c)-I)+7/512*I/(tan(d*x+c)-I)^4+3/128*I/(ta
n(d*x+c)-I)^8-1/80*I/(tan(d*x+c)-I)^10-5/256*I/(tan(d*x+c)-I)^6-45/4096*I/
(tan(d*x+c)-I)^2+1/48/(tan(d*x+c)-I)^9-5/224/(tan(d*x+c)-I)^7+21/1280/(tan
(d*x+c)-I)^5-3/256/(tan(d*x+c)-I)^3+55/4096/(tan(d*x+c)-I))
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.46

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(55440\,dxe^{(20i\,dx+20i\,c)}-210i\,e^{(24i\,dx+24i\,c)}-5040i\,e^{(22i\,dx+22i\,c)}+92400i\,e^{(18i\,dx+18i\,c)}+103950i\,e^{(16i\,dx+16i\,c}\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/3440640*(55440*d*x*e^(20*I*d*x + 20*I*c) - 210*I*e^(24*I*d*x + 24*I*c) -
 5040*I*e^(22*I*d*x + 22*I*c) + 92400*I*e^(18*I*d*x + 18*I*c) + 103950*I*e
^(16*I*d*x + 16*I*c) + 110880*I*e^(14*I*d*x + 14*I*c) + 97020*I*e^(12*I*d*
x + 12*I*c) + 66528*I*e^(10*I*d*x + 10*I*c) + 34650*I*e^(8*I*d*x + 8*I*c)
+ 13200*I*e^(6*I*d*x + 6*I*c) + 3465*I*e^(4*I*d*x + 4*I*c) + 560*I*e^(2*I*
d*x + 2*I*c) + 42*I)*e^(-20*I*d*x - 20*I*c)/(a^8*d)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.39

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{(-11433487528543532372369386809707411904921600ia^{88}d^{11}e^{114ic}e^{4idx} - 274403700685044776936865283432977885718118400ia^{88}d^{11}e^{112ic}}{} \\ x\left(\frac{(e^{24ic}+12e^{22ic}+66e^{20ic}+220e^{18ic}+495e^{16ic}+792e^{14ic}+924e^{12ic}+792e^{10ic}+495e^{8ic}+220e^{6ic}+66e^{4ic}+12e^{2ic}+1)e^{-20ic}}{4096a^8} - \frac{33}{2048a^8}\right) \end{cases} + \frac{33x}{2048a^8}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-11433487528543532372369386809707411904921600*I*a**88*d**11*ex
p(114*I*c)*exp(4*I*d*x) - 274403700685044776936865283432977885718118400*I*
a**88*d**11*exp(112*I*c)*exp(2*I*d*x) + 5030734512559154243842530196271261
238165504000*I*a**88*d**11*exp(108*I*c)*exp(-2*I*d*x) + 565957632662904852
4322846470805168892936192000*I*a**88*d**11*exp(106*I*c)*exp(-4*I*d*x) + 60
36881415070985092611036235525513485798604800*I*a**88*d**11*exp(104*I*c)*ex
p(-6*I*d*x) + 5282271238187111956034656706084824300073779200*I*a**88*d**11
*exp(102*I*c)*exp(-8*I*d*x) + 36221288490425910555666217413153080914791628
80*I*a**88*d**11*exp(100*I*c)*exp(-10*I*d*x) + 188652544220968284144094882
3601722964312064000*I*a**88*d**11*exp(98*I*c)*exp(-12*I*d*x) + 71867635893
7022034834647170895894462595072000*I*a**88*d**11*exp(96*I*c)*exp(-14*I*d*x
) + 188652544220968284144094882360172296431206400*I*a**88*d**11*exp(94*I*c
)*exp(-16*I*d*x) + 30489300076116086326318364825886431746457600*I*a**88*d*
*11*exp(92*I*c)*exp(-18*I*d*x) + 22866975057087064744738773619414823809843
20*I*a**88*d**11*exp(90*I*c)*exp(-20*I*d*x))*exp(-110*I*c)/(18732625966765
7234388900033490246236650235494400*a**96*d**12), Ne(a**96*d**12*exp(110*I*
c), 0)), (x*((exp(24*I*c) + 12*exp(22*I*c) + 66*exp(20*I*c) + 220*exp(18*I
*c) + 495*exp(16*I*c) + 792*exp(14*I*c) + 924*exp(12*I*c) + 792*exp(10*I*c
) + 495*exp(8*I*c) + 220*exp(6*I*c) + 66*exp(4*I*c) + 12*exp(2*I*c) + 1)*e
xp(-20*I*c)/(4096*a**8) - 33/(2048*a**8)), True)) + 33*x/(2048*a**8)
```

3.175.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.175.8 Giac [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.55

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{-\frac{27720i\log(\tan(dx+c)+i)}{a^8}+\frac{27720i\log(\tan(dx+c)-i)}{a^8}+\frac{420\left(99i\tan(dx+c)^2-220\tan(dx+c)-123i\right)}{a^8(\tan(dx+c)+i)^2}-\frac{81191i\tan(dx+c)^{10}+85811}{}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/3440640*(-27720*I*log(tan(d*x + c) + I)/a^8 + 27720*I*log(tan(d*x + c)
- I)/a^8 + 420*(99*I*tan(d*x + c)^2 - 220*tan(d*x + c) - 123*I)/(a^8*(tan(
d*x + c) + I)^2) - (81191*I*tan(d*x + c)^10 + 858110*tan(d*x + c)^9 - 4107
195*I*tan(d*x + c)^8 - 11748840*tan(d*x + c)^7 + 22318590*I*tan(d*x + c)^6
 + 29583540*tan(d*x + c)^5 - 27983550*I*tan(d*x + c)^4 - 19002600*tan(d*x
+ c)^3 + 9206235*I*tan(d*x + c)^2 + 3108990*tan(d*x + c) - 648327*I)/(a^8*
(tan(d*x + c) - I)^10))/d
```

3.175.9 Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{33\,x}{2048\,a^8}$$

$$-\frac{\frac{\tan(c+d\,x)\,66953\mathrm{i}}{215040\,a^8}+\frac{17}{105\,a^8}-\frac{9097\tan(c+d\,x)^2}{26880\,a^8}+\frac{\tan(c+d\,x)^3\,4279\mathrm{i}}{43008\,a^8}-\frac{99\tan(c+d\,x)^4}{112\,a^8}-\mathrm{t}}{d\,\left(\tan\left(c+d\,x\right)^{12}\,1\mathrm{i}+8\tan\left(c+d\,x\right)^{11}-\tan\left(c+d\,x\right)^{10}\,26\mathrm{i}-40\tan\left(c+d\,x\right)^{9}+\tan\left(c+d\,x\right)^{8}\,15\mathrm{i}-4\right.}$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
(33*x)/(2048*a^8) - ((tan(c + d*x)*66953i)/(215040*a^8) + 17/(105*a^8) - (
9097*tan(c + d*x)^2)/(26880*a^8) + (tan(c + d*x)^3*4279i)/(43008*a^8) - (9
9*tan(c + d*x)^4)/(112*a^8) - (tan(c + d*x)^5*42537i)/(35840*a^8) + (341*t
an(c + d*x)^6)/(640*a^8) - (tan(c + d*x)^7*1969i)/(5120*a^8) + (11*tan(c +
 d*x)^8)/(16*a^8) + (tan(c + d*x)^9*869i)/(2048*a^8) - (33*tan(c + d*x)^10
)/(256*a^8) - (tan(c + d*x)^11*33i)/(2048*a^8))/(d*(40*tan(c + d*x)^3 - ta
n(c + d*x)^2*26i - 8*tan(c + d*x) + tan(c + d*x)^4*15i + 48*tan(c + d*x)^5
 + tan(c + d*x)^6*84i - 48*tan(c + d*x)^7 + tan(c + d*x)^8*15i - 40*tan(c
+ d*x)^9 - tan(c + d*x)^10*26i + 8*tan(c + d*x)^11 + tan(c + d*x)^12*1i +
1i))
```

3.176 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.176.1 Optimal result 1327
3.176.2 Mathematica [B] (warning: unable to verify) 1328
3.176.3 Rubi [A] (verified) 1328
3.176.4 Maple [A] (verified) 1333
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3.176.9 Mupad [B] (verification not implemented) 1336

3.176.1 Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{1155\text{arctanh}(\sin(c+dx))}{8a^8d}+\frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d}\\ &\quad+\frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d}\\ &\quad+\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5}\\ &\quad-\frac{66i\sec^7(c+dx)}{a^2d\,(a^2+ia^2\tan(c+dx))^3}-\frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}\end{aligned}$$

output
```
1155/8*arctanh(sin(d*x+c))/a^8/d+1155/8*sec(d*x+c)*tan(d*x+c)/a^8/d+385/4*
sec(d*x+c)^3*tan(d*x+c)/a^8/d+2/3*I*sec(d*x+c)^11/a/d/(a+I*a*tan(d*x+c))^7
-22/3*I*sec(d*x+c)^9/a^3/d/(a+I*a*tan(d*x+c))^5-66*I*sec(d*x+c)^7/a^2/d/(a
^2+I*a^2*tan(d*x+c))^3-154*I*sec(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

3.176.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1704 vs. $2(205) = 410$.

Time = 7.39 (sec) , antiderivative size = 1704, normalized size of antiderivative = 8.31

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input

```
Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

output

```
(-1155*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^
8*(Cos[d*x] + I*Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (1155*Cos[8*
c]*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] +
 I*Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^
8*(((32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a
 + I*a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*x]^8*((-160*I)*Cos[7*c] + 16
0*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((1
155*I)/8)*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[
8*c]*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((1155*I)/
8)*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(C
os[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*
x]^8*(((-236*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)
/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-160*Cos[7*c] - (160*I)*S
in[7*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8)
+ (Sec[c + d*x]^8*((32*Cos[5*c])/3 + ((32*I)/3)*Sin[5*c])*(Cos[d*x] + I*Si
n[d*x])^8*Sin[3*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(Cos[
8*c]/16 + (I/16)*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x)/
2] - Sin[c/2 + (d*x)/2])^4*(a + I*a*Tan[c + d*x])^8) - ((1/96 + I/96)*Sec[
c + d*x]^8*((-407*I)*Cos[(15*c)/2] + 343*Cos[(17*c)/2] + 407*Sin[(15*c)/2]
 + (343*I)*Sin[(17*c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] - Sin...
```

3.176.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 3981, 3042, 3981, 3042, 3981, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{13}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^6}dx}{3a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\int\frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\int\frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^4}dx}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\int\frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\int\frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^2}dx}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\int\sec^5(c+dx)dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\int\csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \ 4255$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\int\sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \ 3042$$

3.176. $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\right)}{a^2}\right)}{3a^2}$$

$$\Big\downarrow \; 4255$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(c+d}{ad(a+ia\tan(c+}\right.}{a^2}\right.}{3a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\sec^7(}{ad(a+ia\,\mathrm{ta}}\right.}{a^2}\right.}{3a^2}$$

3.176. $\int\frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \text{4257}$$

$$\frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}-\frac{11\left(\frac{2i\sec^9(c+dx)}{ad(a+ia\tan(c+dx))^5}-\frac{9\left(\frac{7\left(\frac{5\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{3a^2}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}-\frac{2i\,\mathrm{se}}{ad(a+ia}\right.}{a^2}\right.}{3a^2}$$

input
```
Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((2*I)/3)*Sec[c + d*x]^11)/(a*d*(a + I*a*Tan[c + d*x])^7) - (11*(((2*I)*S
ec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^5) - (9*(((-2*I)*Sec[c + d*x]^7
)/(a*d*(a + I*a*Tan[c + d*x])^3) + (7*((((-2*I)/3)*Sec[c + d*x]^5)/(d*(a^2
 + I*a^2*Tan[c + d*x])) + (5*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(Ar
cTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/(3*a^2
)))/a^2))/a^2))/(3*a^2)
```

3.176.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.176. $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.176.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{160 i \mathrm{e}^{-i(dx+c)}}{a^{8} d}+\frac{32 i \mathrm{e}^{-3 i(dx+c)}}{3 a^{8} d}-\frac{i\left(1545\,\mathrm{e}^{7 i(dx+c)}+5153\,\mathrm{e}^{5 i(dx+c)}+5855\,\mathrm{e}^{3 i(dx+c)}+2295\,\mathrm{e}^{i(dx+c)}\right)}{12 d\,a^{8}\left(\mathrm{e}^{2 i(dx+c)}+1\right)^{4}}+\frac{1155 \ln(\mathrm{e}}{8}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}-\frac{4i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{121}{16}-2i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{123}{16}+38i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$
default	$\frac{\frac{2\left(\frac{1}{4}-\frac{4i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{121}{16}-2i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{123}{16}+38i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-160*I/a^8/d*exp(-I*(d*x+c))+32/3*I/a^8/d*exp(-3*I*(d*x+c))-1/12*I/d/a^8/(
exp(2*I*(d*x+c))+1)^4*(1545*exp(7*I*(d*x+c))+5153*exp(5*I*(d*x+c))+5855*ex
p(3*I*(d*x+c))+2295*exp(I*(d*x+c)))+1155/8/a^8/d*ln(exp(I*(d*x+c))+I)-1155
/8/a^8/d*ln(exp(I*(d*x+c))-I)
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.30

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{8}}\,dx$$
$$=\frac{3465\left(e^{(11i\,dx+11i\,c)}+4\,e^{(9i\,dx+9i\,c)}+6\,e^{(7i\,dx+7i\,c)}+4\,e^{(5i\,dx+5i\,c)}+e^{(3i\,dx+3i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3465\,(e^{(11}}{24}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/24*(3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
 7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c)
 + I) - 3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x
 + 7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*
c) - I) - 6930*I*e^(10*I*d*x + 10*I*c) - 25410*I*e^(8*I*d*x + 8*I*c) - 337
26*I*e^(6*I*d*x + 6*I*c) - 18414*I*e^(4*I*d*x + 4*I*c) - 2816*I*e^(2*I*d*x
 + 2*I*c) + 256*I)/(a^8*d*e^(11*I*d*x + 11*I*c) + 4*a^8*d*e^(9*I*d*x + 9*I
*c) + 6*a^8*d*e^(7*I*d*x + 7*I*c) + 4*a^8*d*e^(5*I*d*x + 5*I*c) + a^8*d*e^
(3*I*d*x + 3*I*c))
```

3.176.6 Sympy [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\sec^{13}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**13/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

3.176.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(179) = 358$.

Time = 0.39 (sec) , antiderivative size = 786, normalized size of antiderivative = 3.83

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
-(6930*(cos(11*d*x + 11*c) + 4*cos(9*d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*c
os(5*d*x + 5*c) + cos(3*d*x + 3*c) + I*sin(11*d*x + 11*c) + 4*I*sin(9*d*x
+ 9*c) + 6*I*sin(7*d*x + 7*c) + 4*I*sin(5*d*x + 5*c) + I*sin(3*d*x + 3*c))
*arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6930*(cos(11*d*x + 11*c) + 4*co
s(9*d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*cos(5*d*x + 5*c) + cos(3*d*x + 3*c
) + I*sin(11*d*x + 11*c) + 4*I*sin(9*d*x + 9*c) + 6*I*sin(7*d*x + 7*c) + 4
*I*sin(5*d*x + 5*c) + I*sin(3*d*x + 3*c))*arctan2(cos(d*x + c), -sin(d*x +
 c) + 1) + 3465*(I*cos(11*d*x + 11*c) + 4*I*cos(9*d*x + 9*c) + 6*I*cos(7*d
*x + 7*c) + 4*I*cos(5*d*x + 5*c) + I*cos(3*d*x + 3*c) - sin(11*d*x + 11*c)
 - 4*sin(9*d*x + 9*c) - 6*sin(7*d*x + 7*c) - 4*sin(5*d*x + 5*c) - sin(3*d*
x + 3*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3465
*(-I*cos(11*d*x + 11*c) - 4*I*cos(9*d*x + 9*c) - 6*I*cos(7*d*x + 7*c) - 4*
I*cos(5*d*x + 5*c) - I*cos(3*d*x + 3*c) + sin(11*d*x + 11*c) + 4*sin(9*d*x
 + 9*c) + 6*sin(7*d*x + 7*c) + 4*sin(5*d*x + 5*c) + sin(3*d*x + 3*c))*log(
cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 13860*cos(10*d*x +
 10*c) + 50820*cos(8*d*x + 8*c) + 67452*cos(6*d*x + 6*c) + 36828*cos(4*d*x
 + 4*c) + 5632*cos(2*d*x + 2*c) + 13860*I*sin(10*d*x + 10*c) + 50820*I*sin
(8*d*x + 8*c) + 67452*I*sin(6*d*x + 6*c) + 36828*I*sin(4*d*x + 4*c) + 5632
*I*sin(2*d*x + 2*c) - 512)/((-48*I*a^8*cos(11*d*x + 11*c) - 192*I*a^8*cos(
9*d*x + 9*c) - 288*I*a^8*cos(7*d*x + 7*c) - 192*I*a^8*cos(5*d*x + 5*c) ...
```

3.176.8 Giac [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$= \frac{\frac{3465\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8} - \frac{3465\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^8} - \frac{1024\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\right)}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3} - \frac{2\left(369\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/24*(3465*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 3465*log(tan(1/2*d*x + 1/2*
c) - 1)/a^8 - 1024*(6*tan(1/2*d*x + 1/2*c)^2 - 15*I*tan(1/2*d*x + 1/2*c) -
 7)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^3) - 2*(369*tan(1/2*d*x + 1/2*c)^7 - 1
728*I*tan(1/2*d*x + 1/2*c)^6 - 393*tan(1/2*d*x + 1/2*c)^5 + 5568*I*tan(1/2
*d*x + 1/2*c)^4 - 393*tan(1/2*d*x + 1/2*c)^3 - 5696*I*tan(1/2*d*x + 1/2*c)
^2 + 369*tan(1/2*d*x + 1/2*c) + 1856*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^
8))/d
```

3.176.9 Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{33847\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{6\,a^8}-\frac{12041\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,a^8}-\frac{3585\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{a^8}+\frac{3505\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4\,a^8}+\frac{4293\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^8}-}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,7\mathrm{i}+13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,18\mathrm{i}-22\right.}$$
$$+\frac{1155\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^8\,d}$$

input
```
int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((tan(c/2 + (d*x)/2)^2*27565i)/(12*a^8) - (12041*tan(c/2 + (d*x)/2)^3)/(3*
a^8) - (tan(c/2 + (d*x)/2)^4*4575i)/a^8 + (33847*tan(c/2 + (d*x)/2)^5)/(6*
a^8) + (tan(c/2 + (d*x)/2)^6*25993i)/(6*a^8) - (3585*tan(c/2 + (d*x)/2)^7)
/a^8 - (tan(c/2 + (d*x)/2)^8*5639i)/(3*a^8) + (3505*tan(c/2 + (d*x)/2)^9)/
(4*a^8) + (tan(c/2 + (d*x)/2)^10*1147i)/(4*a^8) - 1360i/(3*a^8) + (4293*ta
n(c/2 + (d*x)/2))/(4*a^8))/(d*(tan(c/2 + (d*x)/2)*3i - 7*tan(c/2 + (d*x)/2
)^2 - tan(c/2 + (d*x)/2)^3*13i + 18*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)
/2)^5*22i - 22*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*18i + 13*tan(c/
2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^9*7i - 3*tan(c/2 + (d*x)/2)^10 - tan(c
/2 + (d*x)/2)^11*1i + 1)) + (1155*atanh(tan(c/2 + (d*x)/2)))/(4*a^8*d)
```

3.177 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.177.1 Optimal result 1337
3.177.2 Mathematica [B] (warning: unable to verify) 1337
3.177.3 Rubi [A] (verified) 1338
3.177.4 Maple [A] (verified) 1342
3.177.5 Fricas [A] (verification not implemented) 1342
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3.177.8 Giac [A] (verification not implemented) 1344
3.177.9 Mupad [B] (verification not implemented) 1345

3.177.1 Optimal result

Integrand size = 24, antiderivative size = 183

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = &-\frac{63\text{arctanh}(\sin(c+dx))}{2a^8d}-\frac{63\sec(c+dx)\tan(c+dx)}{2a^8d}\\ &+\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}\\ &+\frac{42i\sec^5(c+dx)}{5a^2d\,(a^2+ia^2\tan(c+dx))^3}+\frac{42i\sec^3(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}\end{aligned}$$

output
```
-63/2*arctanh(sin(d*x+c))/a^8/d-63/2*sec(d*x+c)*tan(d*x+c)/a^8/d+2/5*I*sec
(d*x+c)^9/a/d/(a+I*a*tan(d*x+c))^7-6/5*I*sec(d*x+c)^7/a^3/d/(a+I*a*tan(d*x
+c))^5+42/5*I*sec(d*x+c)^5/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+42*I*sec(d*x+c)^
3/d/(a^8+I*a^8*tan(d*x+c))
```

3.177.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1244 vs. $2(183) = 366$.

Time = 7.08 (sec) , antiderivative size = 1244, normalized size of antiderivative = 6.80

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(63*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(
Cos[d*x] + I*Sin[d*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) - (63*Cos[8*c]*Lo
g[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*Si
n[d*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) + (Cos[5*d*x]*Sec[c + d*x]^8*(((
8*I)/5)*Cos[3*c] - (8*Sin[3*c])/5)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*
Tan[c + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((-8*I)*Cos[5*c] + 8*Sin[5*c
])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Cos[d*x]*Sec
[c + d*x]^8*((48*I)*Cos[7*c] - 48*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*
(a + I*a*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*((8*I)*Cos[8*c] - 8*Sin
[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((63*I)/
2)*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(C
os[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((63*I)/2)*Log[Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] +
 I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(48*Cos[7*c
] + (48*I)*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c
 + d*x])^8) + (Sec[c + d*x]^8*(-8*Cos[5*c] - (8*I)*Sin[5*c])*(Cos[d*x] + I
*Sin[d*x])^8*Sin[3*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*((
8*Cos[3*c])/5 + ((8*I)/5)*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[5*d*x])/
(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/4 + (I/4)*Sin[8*c
])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/...
```

3.177.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3981, 3042, 3981, 3042, 3981, 3042, 3981, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{5a^2}$$

$$\Big\downarrow 3042$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^6}dx}{5a^2}$$

$$\Big\downarrow 3981$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow 3042$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^4}dx}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow 3981$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow 3042$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^2}dx}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow 3981$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int \sec^3(c+dx)dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}$$

3.177. $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \; 4255$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}$$

$$\Big\downarrow \; 4257$$

3.177. $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\left(\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^5}-\frac{7\left(\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{5\left(\frac{3\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{a^2}\right)}{3a^2}\right)}{5a^2}$$

input
```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((2*I)/5)*Sec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^7) - (9*((((2*I)/3)
*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^5) - (7*(((2*I)*Sec[c + d*x]^
5)/(a*d*(a + I*a*Tan[c + d*x])^3) - (5*(((-2*I)*Sec[c + d*x]^3)/(d*(a^2 +
I*a^2*Tan[c + d*x])) + (3*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan
[c + d*x])/(2*d)))/a^2))/a^2))/(3*a^2)))/(5*a^2)
```

3.177.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.177.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.78

method	result
risch	$\frac{48i\mathrm{e}^{-i(dx+c)}}{a^8d}-\frac{8i\mathrm{e}^{-3i(dx+c)}}{a^8d}+\frac{8i\mathrm{e}^{-5i(dx+c)}}{5a^8d}+\frac{i\left(15\,\mathrm{e}^{3i(dx+c)}+17\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{63\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^8d}+\frac{63\ln(\mathrm{e}^{i(dx}}{2a^8d}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}+4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(\frac{1}{4}-4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^8d}$
default	$\frac{\frac{2\left(\frac{1}{4}+4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(\frac{1}{4}-4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^8d}$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
48*I/a^8/d*exp(-I*(d*x+c))-8*I/a^8/d*exp(-3*I*(d*x+c))+8/5*I/a^8/d*exp(-5*
I*(d*x+c))+I/d/a^8/(exp(2*I*(d*x+c))+1)^2*(15*exp(3*I*(d*x+c))+17*exp(I*(d
*x+c)))-63/2/a^8/d*ln(exp(I*(d*x+c))+I)+63/2/a^8/d*ln(exp(I*(d*x+c))-I)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+}\right.}{10\left(a^8de^{(9i\,dx+9i\,c)}+2\,a^8de^{(7}\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
-1/10*(315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*I*c) + e^(5*I*d*x + 5*I
*c))*log(e^(I*d*x + I*c) + I) - 315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x +
7*I*c) + e^(5*I*d*x + 5*I*c))*log(e^(I*d*x + I*c) - I) - 630*I*e^(8*I*d*x
+ 8*I*c) - 1050*I*e^(6*I*d*x + 6*I*c) - 336*I*e^(4*I*d*x + 4*I*c) + 48*I*e
^(2*I*d*x + 2*I*c) - 16*I)/(a^8*d*e^(9*I*d*x + 9*I*c) + 2*a^8*d*e^(7*I*d*x
 + 7*I*c) + a^8*d*e^(5*I*d*x + 5*I*c))
```

3.177.6 Sympy [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{11}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**11/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(
c + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x
)**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

3.177.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(157) = 314$.

Time = 0.34 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.90

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{630\left(\cos\left(9\,dx+9\,c\right)+2\,\cos\left(7\,dx+7\,c\right)+\cos\left(5\,dx+5\,c\right)+i\,\sin\left(9\,dx+9\,c\right)+2i\,\sin\left(7\,dx+7\,c\right)+i\,\sin}{}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
(630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*d*x + 5*c) + I*sin(9*d
*x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c))*arctan2(cos(d*x + c
), sin(d*x + c) + 1) + 630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*
d*x + 5*c) + I*sin(9*d*x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c
))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) + 315*(I*cos(9*d*x + 9*c) + 2*
I*cos(7*d*x + 7*c) + I*cos(5*d*x + 5*c) - sin(9*d*x + 9*c) - 2*sin(7*d*x +
 7*c) - sin(5*d*x + 5*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
+ c) + 1) + 315*(-I*cos(9*d*x + 9*c) - 2*I*cos(7*d*x + 7*c) - I*cos(5*d*x
+ 5*c) + sin(9*d*x + 9*c) + 2*sin(7*d*x + 7*c) + sin(5*d*x + 5*c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 1260*cos(8*d*x + 8*c)
 + 2100*cos(6*d*x + 6*c) + 672*cos(4*d*x + 4*c) - 96*cos(2*d*x + 2*c) + 12
60*I*sin(8*d*x + 8*c) + 2100*I*sin(6*d*x + 6*c) + 672*I*sin(4*d*x + 4*c) -
 96*I*sin(2*d*x + 2*c) + 32)/((-20*I*a^8*cos(9*d*x + 9*c) - 40*I*a^8*cos(7
*d*x + 7*c) - 20*I*a^8*cos(5*d*x + 5*c) + 20*a^8*sin(9*d*x + 9*c) + 40*a^8
*sin(7*d*x + 7*c) + 20*a^8*sin(5*d*x + 5*c))*d)
```

3.177.8 Giac [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{\frac{315\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}-\frac{315\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8}-\frac{10\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-16i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+16i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^8}-64\,(1}{10\,d}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
-1/10*(315*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 315*log(tan(1/2*d*x + 1/2*c
) - 1)/a^8 - 10*(tan(1/2*d*x + 1/2*c)^3 - 16*I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + 16*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^8) - 64*(10*t
an(1/2*d*x + 1/2*c)^4 - 45*I*tan(1/2*d*x + 1/2*c)^3 - 85*tan(1/2*d*x + 1/2
*c)^2 + 55*I*tan(1/2*d*x + 1/2*c) + 13)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^5)
)/d
```

3.177.9 Mupad [B] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.55

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{63\,\mathrm{atanh}\bigl(\tan\bigl(\frac{c}{2}+\frac{d\,x}{2}\bigr)\bigr)}{a^8\,d}$$

$$+\frac{\frac{1223\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^8}-\frac{1109\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^8}+\frac{309\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{a^8}-\frac{431\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^8}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 4407\mathrm{i}}{5\,a^8}}{d\,\Bigl(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 12\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 26\mathrm{i}+26}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((1223*tan(c/2 + (d*x)/2)^3)/a^8 - (tan(c/2 + (d*x)/2)^2*4407i)/(5*a^8) +
(tan(c/2 + (d*x)/2)^4*7351i)/(5*a^8) - (1109*tan(c/2 + (d*x)/2)^5)/a^8 - (
tan(c/2 + (d*x)/2)^6*761i)/a^8 + (309*tan(c/2 + (d*x)/2)^7)/a^8 + (tan(c/2
 + (d*x)/2)^8*65i)/a^8 + 496i/(5*a^8) - (431*tan(c/2 + (d*x)/2))/a^8)/(d*(
tan(c/2 + (d*x)/2)*5i - 12*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*20i
 + 26*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*26i - 20*tan(c/2 + (d*x)
/2)^6 - tan(c/2 + (d*x)/2)^7*12i + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x
)/2)^9*1i + 1)) - (63*atanh(tan(c/2 + (d*x)/2)))/(a^8*d)
```

3.178 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.178.1 Optimal result

Integrand size = 24, antiderivative size = 156

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^8d} + \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} \\ &\quad - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} \\ &\quad + \frac{2i\sec^3(c+dx)}{3a^2d\,(a^2+ia^2\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}\end{aligned}$$

output
```
arctanh(sin(d*x+c))/a^8/d+2/7*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^7-2/5*
I*sec(d*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5+2/3*I*sec(d*x+c)^3/a^2/d/(a^2+I*
a^2*tan(d*x+c))^3-2*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c))
```

3.178.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\sec^8(c+dx)\left(70i\cos\left(\frac{1}{2}(c+dx)\right) - 42i\cos\left(\frac{3}{2}(c+dx)\right) - 210i\cos\left(\frac{5}{2}(c+dx)\right) + 30i\cos\left(\frac{7}{2}(c+dx)\right) - 10\right.}{}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(Sec[c + d*x]^8*((70*I)*Cos[(c + d*x)/2] - (42*I)*Cos[(3*(c + d*x))/2] - (
210*I)*Cos[(5*(c + d*x))/2] + (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c
+ d*x))/2]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x)
)/2]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*S
in[(3*(c + d*x))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] -
 (105*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + (
105*I)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos
[(9*(c + d*x))/2] + I*Sin[(9*(c + d*x))/2]))/(105*a^8*d*(-I + Tan[c + d*x]
)^8)
```

3.178.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3981, 3042, 3981, 3042, 3981, 3042, 3981, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^6}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{a^2}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^4}dx}{a^2}}{a^2}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int\frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int\frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^2}dx}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int\sec(c+dx)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{a^2}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}}{a^2}}{a^2}}{a^2}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}-\frac{\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^5}-\frac{\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{-\frac{\text{arctanh}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}}{a^2}}{a^2}}{a^2}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
(((2*I)/7)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^7) - ((((2*I)/5)*Se
c[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^5) - ((((2*I)/3)*Sec[c + d*x]^3)
/(a*d*(a + I*a*Tan[c + d*x])^3) - (-(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*
I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])))/a^2)/a^2)/a^2
```

3.178. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.178.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.178.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{2 i \mathrm{e}^{-i(dx+c)}}{a^8 d}+\frac{2 i \mathrm{e}^{-3i(dx+c)}}{3a^8 d}-\frac{2 i \mathrm{e}^{-5i(dx+c)}}{5a^8 d}+\frac{2 i \mathrm{e}^{-7i(dx+c)}}{7a^8 d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^8 d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^8 d}$
derivativedivides	$-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-$
default	$-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
-2*I/a^8/d*exp(-I*(d*x+c))+2/3*I/a^8/d*exp(-3*I*(d*x+c))-2/5*I/a^8/d*exp(-
5*I*(d*x+c))+2/7*I/a^8/d*exp(-7*I*(d*x+c))-1/a^8/d*ln(exp(I*(d*x+c))-I)+1/
a^8/d*ln(exp(I*(d*x+c))+I)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\left(105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}-}{105\,a^8d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/105*(105*e^(7*I*d*x + 7*I*c)*log(e^(I*d*x + I*c) + I) - 105*e^(7*I*d*x +
 7*I*c)*log(e^(I*d*x + I*c) - I) - 210*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I
*d*x + 4*I*c) - 42*I*e^(2*I*d*x + 2*I*c) + 30*I)*e^(-7*I*d*x - 7*I*c)/(a^8
*d)
```

3.178.6 Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\sec^9(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Integral(sec(c + d*x)**9/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)
**3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{-210i\ \arctan(\cos(dx+c), \sin(dx+c)+1) - 210i\ \arctan(\cos(dx+c), -\sin(dx+c)+1) + 60i\ \cos(7}{}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/210*(-210*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 210*I*arctan2(cos(
d*x + c), -sin(d*x + c) + 1) + 60*I*cos(7*d*x + 7*c) - 84*I*cos(5*d*x + 5*
c) + 140*I*cos(3*d*x + 3*c) - 420*I*cos(d*x + c) + 105*log(cos(d*x + c)^2
+ sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - 105*log(cos(d*x + c)^2 + sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1) + 60*sin(7*d*x + 7*c) - 84*sin(5*d*x + 5*c) +
 140*sin(3*d*x + 3*c) - 420*sin(d*x + c))/(a^8*d)
```

3.178.8 Giac [A] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{105\ \log(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+1)}{a^8} - \frac{105\ \log(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-1)}{a^8} - \frac{16\left(-105i\ \tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5 - 175\ \tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4 + 490i\ \tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3 + 294\ \mathrm{t}\right.}{a^8(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-i)^7}}{105\,d}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/105*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 105*log(tan(1/2*d*x + 1/2*c
) - 1)/a^8 - 16*(-105*I*tan(1/2*d*x + 1/2*c)^5 - 175*tan(1/2*d*x + 1/2*c)^
4 + 490*I*tan(1/2*d*x + 1/2*c)^3 + 294*tan(1/2*d*x + 1/2*c)^2 - 133*I*tan(
1/2*d*x + 1/2*c) - 19)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.178.9 Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.33

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^8\,d}$$

$$+\frac{\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^8}-\frac{224\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,a^8}+\frac{304\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15\,a^8}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 224\mathrm{i}}{5\,a^8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\,a^8}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 1\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 21\mathrm{i}+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 35\mathrm{i}-\right.}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a^8*d) + ((tan(c/2 + (d*x)/2)^2*224i)/(5*a^
8) - (224*tan(c/2 + (d*x)/2)^3)/(3*a^8) - (tan(c/2 + (d*x)/2)^4*80i)/(3*a^
8) + (16*tan(c/2 + (d*x)/2)^5)/a^8 - 304i/(105*a^8) + (304*tan(c/2 + (d*x)
/2))/(15*a^8))/(d*(tan(c/2 + (d*x)/2)*7i - 21*tan(c/2 + (d*x)/2)^2 - tan(c
/2 + (d*x)/2)^3*35i + 35*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*21i -
 7*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*1i + 1))
```

3.179 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.179.1 Optimal result 1353
3.179.2 Mathematica [A] (verified) 1353
3.179.3 Rubi [A] (verified) 1354
3.179.4 Maple [A] (verified) 1355
3.179.5 Fricas [A] (verification not implemented) 1356
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3.179.9 Mupad [B] (verification not implemented) 1358

3.179.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}+\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}$$

output
```
1/9*I*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^8+1/63*I*sec(d*x+c)^7/a/d/(a+I*a*t
an(d*x+c))^7
```

3.179.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=-\frac{\sec^7(c+dx)(-8i+\tan(c+dx))}{63a^8d(-i+\tan(c+dx))^8}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
-1/63*(Sec[c + d*x]^7*(-8*I + Tan[c + d*x]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

3.179.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{9a}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^7}dx}{9a}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/9)*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/63)*Sec[c + d*x]
^7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

3.179.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.179.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

method	result
risch	$\frac{i\mathrm{e}^{-7i(dx+c)}}{14a^8d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{18a^8d}$
derivativedivides	$\frac{-\frac{1856}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{152i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{172}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{992i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{14i}{(-i+\tan(}}{a^8d}$
default	$\frac{-\frac{1856}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}-\frac{152i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{172}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{992i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{14i}{(-i+\tan(}}{a^8d}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/14*I/a^8/d*exp(-7*I*(d*x+c))+1/18*I/a^8/d*exp(-9*I*(d*x+c))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(9i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{126\,a^8d}$$

input

```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output

```
1/126*(9*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^8*d)
```

3.179.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(54) = 108.

Time = 8.97 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.57

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} -\frac{\tan(c+dx)\sec^7(c+dx)}{63a^8d\tan^8(c+dx)-504ia^8d\tan^7(c+dx)-1764a^8d\tan^6(c+dx)+3528ia^8d\tan^5(c+dx)+4410a^8d\tan^4(c+dx)-3528ia^8d\tan^3(c+dx)-176} \\ \frac{x\sec^7(c)}{(ia\tan(c)+a)^8} \end{cases}$$

input

```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**8,x)
```

output

```
Piecewise((-tan(c + d*x)*sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*
I*a**8*d*tan(c + d*x)**7 - 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan
(c + d*x)**5 + 4410*a**8*d*tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3
 - 1764*a**8*d*tan(c + d*x)**2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d) +
8*I*sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*I*a**8*d*tan(c + d*x)
**7 - 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan(c + d*x)**5 + 4410*a
**8*d*tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3 - 1764*a**8*d*tan(c
+ d*x)**2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d), Ne(d, 0)), (x*sec(c)**
7/(I*a*tan(c) + a)**8, True))
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{7i\cos(9\,dx+9\,c)+9i\cos(7\,dx+7\,c)+7\sin(9\,dx+9\,c)+9\sin(7\,dx+7\,c)}{126\,a^8d}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/126*(7*I*cos(9*d*x + 9*c) + 9*I*cos(7*d*x + 7*c) + 7*sin(9*d*x + 9*c) +
9*sin(7*d*x + 7*c))/(a^8*d)
```

3.179.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.

Time = 1.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.84

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(63\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-63i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-483\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+315i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+693\,\mathrm{ta}\right.}{63\,a^8d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/63*(63*tan(1/2*d*x + 1/2*c)^8 - 63*I*tan(1/2*d*x + 1/2*c)^7 - 483*tan(1/
2*d*x + 1/2*c)^6 + 315*I*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)
^4 - 189*I*tan(1/2*d*x + 1/2*c)^3 - 225*tan(1/2*d*x + 1/2*c)^2 + 9*I*tan(1
/2*d*x + 1/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^9)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(\frac{\mathrm{e}^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,9\mathrm{i}}{4}+\frac{\mathrm{e}^{-c\,9\mathrm{i}-d\,x\,9\mathrm{i}}\,7\mathrm{i}}{4}\right)}{63\,a^8\,d}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(2*((exp(- c*7i - d*x*7i)*9i)/4 + (exp(- c*9i - d*x*9i)*7i)/4))/(63*a^8*d)
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.180.1 Optimal result 1359
3.180.2 Mathematica [A] (verified) 1359
3.180.3 Rubi [A] (verified) 1360
3.180.4 Maple [A] (verified) 1362
3.180.5 Fricas [A] (verification not implemented) 1362
3.180.6 Sympy [B] (verification not implemented) 1363
3.180.7 Maxima [A] (verification not implemented) 1364
3.180.8 Giac [A] (verification not implemented) 1364
3.180.9 Mupad [B] (verification not implemented) 1365

3.180.1 Optimal result

Integrand size = 24, antiderivative size = 138

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=&\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}\\&+\frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6}\\&+\frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}\end{aligned}$$

output
```
1/11*I*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^8+1/33*I*sec(d*x+c)^5/a/d/(a+I*a*
tan(d*x+c))^7+2/231*I*sec(d*x+c)^5/a^2/d/(a+I*a*tan(d*x+c))^6+2/1155*I*sec
(d*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5
```

3.180.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\begin{aligned}&\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx\\&=\frac{i\sec^8(c+dx)(440\cos(c+dx)+168\cos(3(c+dx))+55i\sin(c+dx)+63i\sin(3(c+dx)))}{4620a^8d(-i+\tan(c+dx))^8}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/4620)*Sec[c + d*x]^8*(440*Cos[c + d*x] + 168*Cos[3*(c + d*x)] + (55*I)
*Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

3.180.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^7}dx}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^6}dx}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{7a}+\frac{i\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^6}\right)}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^5}dx}{7a}+\frac{i\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^6}\right)}{9a}+\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}\right)}{11a}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{3\left(\frac{i\sec^5(c+dx)}{9d(a+ia\tan(c+dx))^7}+\frac{2\left(\frac{i\sec^5(c+dx)}{35ad(a+ia\tan(c+dx))^5}+\frac{i\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^6}\right)}{9a}\right)}{11a}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/11)*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^8) + (3*(((I/9)*Sec[c +
d*x]^5)/(d*(a + I*a*Tan[c + d*x])^7) + (2*(((I/7)*Sec[c + d*x]^5)/(d*(a +
I*a*Tan[c + d*x])^6) + ((I/35)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])
^5)))/(9*a)))/(11*a)
```

3.180.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.180.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

method	result
risch	$\frac{i\mathrm{e}^{-5i(dx+c)}}{40a^8d}+\frac{3i\mathrm{e}^{-7i(dx+c)}}{56a^8d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{24a^8d}+\frac{i\mathrm{e}^{-11i(dx+c)}}{88a^8d}$
derivativedivides	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{576i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{1024}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{176i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1864}{5\left(-i+\tan\left(\frac{dx}{2}+\right.\right.}}{a^8 c}$
default	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{576i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{1024}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{176i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1864}{5\left(-i+\tan\left(\frac{dx}{2}+\right.\right.}}{a^8 c}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/40*I/a^8/d*exp(-5*I*(d*x+c))+3/56*I/a^8/d*exp(-7*I*(d*x+c))+1/24*I/a^8/d
*exp(-9*I*(d*x+c))+1/88*I/a^8/d*exp(-11*I*(d*x+c))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.38

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(231i\,e^{(6i\,dx+6i\,c)}+495i\,e^{(4i\,dx+4i\,c)}+385i\,e^{(2i\,dx+2i\,c)}+105i\right)e^{(-11i\,dx-11i\,c)}}{9240\,a^8d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/9240*(231*I*e^(6*I*d*x + 6*I*c) + 495*I*e^(4*I*d*x + 4*I*c) + 385*I*e^(2
*I*d*x + 2*I*c) + 105*I)*e^(-11*I*d*x - 11*I*c)/(a^8*d)
```

3.180.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(119) = 238.

Time = 8.79 (sec) , antiderivative size = 620, normalized size of antiderivative = 4.49

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{2\tan^3(c+dx)\sec^5(c+dx)}{1155a^8d\tan^8(c+dx)-9240ia^8d\tan^7(c+dx)-32340a^8d\tan^6(c+dx)+64680ia^8d\tan^5(c+dx)+80850a^8d\tan^4(c+dx)-64680ia^8d\tan^3(c+dx)} \\ \frac{x\sec^5(c)}{(ia\tan(c)+a)^8} \end{cases}$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((2*tan(c + d*x)**3*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8
- 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a
**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(
c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) +
1155*a**8*d) - 16*I*tan(c + d*x)**2*sec(c + d*x)**5/(1155*a**8*d*tan(c + d
*x)**8 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64
680*I*a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8
*d*tan(c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c +
d*x) + 1155*a**8*d) - 61*tan(c + d*x)*sec(c + d*x)**5/(1155*a**8*d*tan(c +
 d*x)**8 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 +
64680*I*a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a*
*8*d*tan(c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c
+ d*x) + 1155*a**8*d) + 152*I*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8
 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*
a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan
(c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) +
 1155*a**8*d), Ne(d, 0)), (x*sec(c)**5/(I*a*tan(c) + a)**8, True))
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{105i\,\cos\left(11\,dx+11\,c\right)+385i\,\cos\left(9\,dx+9\,c\right)+495i\,\cos\left(7\,dx+7\,c\right)+231i\,\cos\left(5\,dx+5\,c\right)+105\,\sin(1}{9240\,a^8d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/9240*(105*I*cos(11*d*x + 11*c) + 385*I*cos(9*d*x + 9*c) + 495*I*cos(7*d*
x + 7*c) + 231*I*cos(5*d*x + 5*c) + 105*sin(11*d*x + 11*c) + 385*sin(9*d*x
 + 9*c) + 495*sin(7*d*x + 7*c) + 231*sin(5*d*x + 5*c))/(a^8*d)
```

3.180.8 Giac [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-3465i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-13860\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+23100i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 - 3465*I*tan(1/2*d*x + 1/2*c)^9 - 138
60*tan(1/2*d*x + 1/2*c)^8 + 23100*I*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2
*d*x + 1/2*c)^6 - 32802*I*tan(1/2*d*x + 1/2*c)^5 - 27060*tan(1/2*d*x + 1/2
*c)^4 + 11220*I*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 - 517
*I*tan(1/2*d*x + 1/2*c) - 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^11)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.46

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,1\mathrm{i}}{40}+\frac{\mathrm{e}^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,3\mathrm{i}}{56}+\frac{\mathrm{e}^{-c\,9\mathrm{i}-d\,x\,9\mathrm{i}}\,1\mathrm{i}}{24}+\frac{\mathrm{e}^{-c\,11\mathrm{i}-d\,x\,11\mathrm{i}}\,1\mathrm{i}}{88}}{a^8\,d}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((exp(- c*5i - d*x*5i)*1i)/40 + (exp(- c*7i - d*x*7i)*3i)/56 + (exp(- c*9i
 - d*x*9i)*1i)/24 + (exp(- c*11i - d*x*11i)*1i)/88)/(a^8*d)
```

3.181 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.181.1 Optimal result 1366
3.181.2 Mathematica [A] (verified) 1367
3.181.3 Rubi [A] (verified) 1367
3.181.4 Maple [A] (verified) 1371
3.181.5 Fricas [A] (verification not implemented) 1371
3.181.6 Sympy [B] (verification not implemented) 1372
3.181.7 Maxima [A] (verification not implemented) 1372
3.181.8 Giac [A] (verification not implemented) 1373
3.181.9 Mupad [B] (verification not implemented) 1373

3.181.1 Optimal result

Integrand size = 24, antiderivative size = 213

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} \\ &\quad + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6} \\ &\quad + \frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5} \\ &\quad + \frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4} \\ &\quad + \frac{8i\sec^3(c+dx)}{9009a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}\end{aligned}$$

output
```
1/13*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+5/143*I*sec(d*x+c)^3/a/d/(a+I*a
*tan(d*x+c))^7+20/1287*I*sec(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+20/3003*I
*sec(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+8/3003*I*sec(d*x+c)^3/d/(a^2+I*a^
2*tan(d*x+c))^4+8/9009*I*sec(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x+c))^3
```

3.181.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.45

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{i\sec^8(c+dx)(11440\cos(c+dx)+6552\cos(3(c+dx))+1848\cos(5(c+dx))+1430i\sin(c+dx)+2457}{144144a^8d(-i+\tan(c+dx))^8}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/144144)*Sec[c + d*x]^8*(11440*Cos[c + d*x] + 6552*Cos[3*(c + d*x)] + 1
848*Cos[5*(c + d*x)] + (1430*I)*Sin[c + d*x] + (2457*I)*Sin[3*(c + d*x)] +
 (1155*I)*Sin[5*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

3.181.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{5\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$
$$\Big\downarrow \text{3042}$$
$$\frac{5\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^7}dx}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$
$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{4\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{4\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^6}dx}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{4\left(\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{4\left(\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^5}dx}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

3.181. $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^4}dx}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\left(\frac{\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}\right)}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{4\left(\frac{\frac{2\left(\frac{\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^3}dx}{5a}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}\right)}{7a}+\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}}{3a}+\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}\right)}{11a}+\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}\right)}{13a}+\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

3.181. $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}+\frac{5\left(\frac{i\sec^3(c+dx)}{11d(a+ia\tan(c+dx))^7}+\frac{4\left(\frac{i\sec^3(c+dx)}{9d(a+ia\tan(c+dx))^6}+\frac{\frac{i\sec^3(c+dx)}{7d(a+ia\tan(c+dx))^5}+\frac{2\left(\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}\right)}{7a}}{3a}\right)}{11a}\right)}{13a}$$

input

```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output

```
((I/13)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (5*(((I/11)*Sec[c +
 d*x]^3)/(d*(a + I*a*Tan[c + d*x])^7) + (4*(((I/9)*Sec[c + d*x]^3)/(d*(a +
 I*a*Tan[c + d*x])^6) + (((I/7)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^
5) + (2*(((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec
[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3)))/(7*a))/(3*a)))/(11*a)))/(13*
a)
```

3.181.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.181. $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.181.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{96a^8d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{32a^8d}+\frac{5i\mathrm{e}^{-7i(dx+c)}}{112a^8d}+\frac{5i\mathrm{e}^{-9i(dx+c)}}{144a^8d}+\frac{5i\mathrm{e}^{-11i(dx+c)}}{352a^8d}+\frac{i\mathrm{e}^{-13i(dx+c)}}{416a^8d}$
derivativedivides	$-\frac{188}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{256}{13\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{13}}+\frac{480}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2672i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9056}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+$
default	$-\frac{188}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{256}{13\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{13}}+\frac{480}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2672i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9056}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/96*I/a^8/d*exp(-3*I*(d*x+c))+1/32*I/a^8/d*exp(-5*I*(d*x+c))+5/112*I/a^8/
d*exp(-7*I*(d*x+c))+5/144*I/a^8/d*exp(-9*I*(d*x+c))+5/352*I/a^8/d*exp(-11*
I*(d*x+c))+1/416*I/a^8/d*exp(-13*I*(d*x+c))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{\left(3003i\,e^{(10i\,dx+10i\,c)}+9009i\,e^{(8i\,dx+8i\,c)}+12870i\,e^{(6i\,dx+6i\,c)}+10010i\,e^{(4i\,dx+4i\,c)}+4095i\,e^{(2i\,dx+2i\,c)}+693i\right)e}{288288\,a^8d}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/288288*(3003*I*e^(10*I*d*x + 10*I*c) + 9009*I*e^(8*I*d*x + 8*I*c) + 1287
0*I*e^(6*I*d*x + 6*I*c) + 10010*I*e^(4*I*d*x + 4*I*c) + 4095*I*e^(2*I*d*x
+ 2*I*c) + 693*I)*e^(-13*I*d*x - 13*I*c)/(a^8*d)
```

3.181.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(189) = 378.

Time = 8.74 (sec) , antiderivative size = 928, normalized size of antiderivative = 4.36

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((-8*tan(c + d*x)**5*sec(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8
 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6 + 504504
*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*
d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*a**8*d*tan(c +
 d*x) + 9009*a**8*d) + 64*I*tan(c + d*x)**4*sec(c + d*x)**3/(9009*a**8*d*t
an(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*
x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 -
504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*
a**8*d*tan(c + d*x) + 9009*a**8*d) + 236*tan(c + d*x)**3*sec(c + d*x)**3/(
9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8
*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c
 + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)*
*2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) - 544*I*tan(c + d*x)**2*se
c(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**
7 - 252252*a**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 6306
30*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*
d*tan(c + d*x)**2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) - 911*tan(c
 + d*x)*sec(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(
c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)
**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 -...
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{693i\cos(13\,dx+13\,c)+4095i\cos(11\,dx+11\,c)+10010i\cos(9\,dx+9\,c)+12870i\cos(7\,dx+7\,c)+90}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/288288*(693*I*cos(13*d*x + 13*c) + 4095*I*cos(11*d*x + 11*c) + 10010*I*c
os(9*d*x + 9*c) + 12870*I*cos(7*d*x + 7*c) + 9009*I*cos(5*d*x + 5*c) + 300
3*I*cos(3*d*x + 3*c) + 693*sin(13*d*x + 13*c) + 4095*sin(11*d*x + 11*c) +
10010*sin(9*d*x + 9*c) + 12870*sin(7*d*x + 7*c) + 9009*sin(5*d*x + 5*c) +
3003*sin(3*d*x + 3*c))/(a^8*d)
```

3.181.8 Giac [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(9009\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} - 45045i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 183183\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} + 435435i\,\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 - 45045*I*tan(1/2*d*x + 1/2*c)^11 - 1
83183*tan(1/2*d*x + 1/2*c)^10 + 435435*I*tan(1/2*d*x + 1/2*c)^9 + 810810*t
an(1/2*d*x + 1/2*c)^8 - 1051050*I*tan(1/2*d*x + 1/2*c)^7 - 1076790*tan(1/2
*d*x + 1/2*c)^6 + 785070*I*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1
/2*c)^4 - 171457*I*tan(1/2*d*x + 1/2*c)^3 - 51675*tan(1/2*d*x + 1/2*c)^2 +
 7111*I*tan(1/2*d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^13)
```

3.181.9 Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.75

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{\cos(3\,c+3\,d\,x)^3\,5\mathrm{i}}{36} + \frac{5\,\sin(3\,c+3\,d\,x)\cos(3\,c+3\,d\,x)^2}{36} - \frac{\cos(3\,c+3\,d\,x)\,3\mathrm{i}}{32} + \frac{\cos(5\,c+5\,d\,x)\,1\mathrm{i}}{32} + \frac{\cos(7\,c+7\,d\,x)\,5\mathrm{i}}{112} + \frac{\cos(11\,c+11\,d\,x)\,5\mathrm{i}}{352} + \mathrm{c}}{a^8\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
((cos(5*c + 5*d*x)*1i)/32 - (cos(3*c + 3*d*x)*3i)/32 + (cos(7*c + 7*d*x)*5
i)/112 + (cos(11*c + 11*d*x)*5i)/352 + (cos(13*c + 13*d*x)*1i)/416 - (7*si
n(3*c + 3*d*x))/288 + sin(5*c + 5*d*x)/32 + (5*sin(7*c + 7*d*x))/112 + (5*
sin(11*c + 11*d*x))/352 + sin(13*c + 13*d*x)/416 + (cos(3*c + 3*d*x)^3*5i)
/36 + (5*cos(3*c + 3*d*x)^2*sin(3*c + 3*d*x))/36)/(a^8*d)
```

3.181. $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.182 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.182.1 Optimal result 1375
3.182.2 Mathematica [A] (verified) 1376
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3.182.4 Maple [A] (verified) 1386
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3.182.8 Giac [A] (verification not implemented) 1389
3.182.9 Mupad [B] (verification not implemented) 1389

3.182.1 Optimal result

Integrand size = 22, antiderivative size = 269

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}+\frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7}\\ &\quad+\frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6}\\ &\quad+\frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5}\\ &\quad+\frac{8i\sec(c+dx)}{1287d\,(a^2+ia^2\tan(c+dx))^4}\\ &\quad+\frac{8i\sec(c+dx)}{2145a^2d\,(a^2+ia^2\tan(c+dx))^3}\\ &\quad+\frac{16i\sec(c+dx)}{6435d\,(a^4+ia^4\tan(c+dx))^2}\\ &\quad+\frac{16i\sec(c+dx)}{6435d\,(a^8+ia^8\tan(c+dx))}\end{aligned}$$

output
```
1/15*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^8+7/195*I*sec(d*x+c)/a/d/(a+I*a*tan
(d*x+c))^7+14/715*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+14/1287*I*sec(d*
x+c)/a^3/d/(a+I*a*tan(d*x+c))^5+8/1287*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c
))^4+8/2145*I*sec(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+16/6435*I*sec(d*x+
c)/d/(a^4+I*a^4*tan(d*x+c))^2+16/6435*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c)
)
```

3.182.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.43

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{i\sec^8(c+dx)(28600\cos(c+dx)+19656\cos(3(c+dx))+9240\cos(5(c+dx))+3432\cos(7(c+dx))+3}{411840a^8d(-i+\tan(c+}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/411840)*Sec[c + d*x]^8*(28600*Cos[c + d*x] + 19656*Cos[3*(c + d*x)] +
9240*Cos[5*(c + d*x)] + 3432*Cos[7*(c + d*x)] + (3575*I)*Sin[c + d*x] + (7
371*I)*Sin[3*(c + d*x)] + (5775*I)*Sin[5*(c + d*x)] + (3003*I)*Sin[7*(c +
d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

3.182.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}dx$$
$$\Big\downarrow \text{ 3983}$$
$$\frac{7\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{7\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{6\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{6\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{6\left(\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{6\left(\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3983$$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}\right)}{15a}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\,s}{13d(a+}\right.}{15a}$$

$$\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}\right)}{13a}+\frac{i\,s}{13d(a+}\right.}{15a}$$

$$\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{ 3983}$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+}{11d(a+ia\tan(}\right)}{13a}\right.$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow 3042$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\sec(c+dx)}{i\tan(c+dx)a+a}dx}{3a}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}\right)}{5a}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}\right)}{7a}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}\right)}{9a}+\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}\right)}{11a}+\frac{i\sec(c+}{11d(a+ia\tan(}\right.}{13a}\right.$$

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \text{3969}$$

$$\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}+\frac{7\left(\frac{i\sec(c+dx)}{13d(a+ia\tan(c+dx))^7}+\frac{6\left(\frac{i\sec(c+dx)}{11d(a+ia\tan(c+dx))^6}+\frac{5\left(\frac{i\sec(c+dx)}{9d(a+ia\tan(c+dx))^5}+\frac{4\left(\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3\left(\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\left(\frac{i\sec(c+d}{3ad(a+ia\tan(}\right.}{}\right.}{7a}\right.}{9a}\right.}{11a}\right.}{13a}\right.}{15a}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

3.182. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

output
```
((I/15)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (7*(((I/13)*Sec[c + d
*x])/(d*(a + I*a*Tan[c + d*x])^7) + (6*(((I/11)*Sec[c + d*x])/(d*(a + I*a*
Tan[c + d*x])^6) + (5*(((I/9)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^5) +
 (4*(((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (3*(((I/5)*Sec[c
+ d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (2*(((I/3)*Sec[c + d*x])/(d*(a + I*
a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x]))))/(
5*a)))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15*a)
```

3.182.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.182.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.54

method	result
risch	$\frac{i e^{-i(dx+c)}}{128a^8d}+\frac{7i e^{-3i(dx+c)}}{384a^8d}+\frac{21i e^{-5i(dx+c)}}{640a^8d}+\frac{5i e^{-7i(dx+c)}}{128a^8d}+\frac{35i e^{-9i(dx+c)}}{1152a^8d}+\frac{21i e^{-11i(dx+c)}}{1408a^8d}+$
derivativedivides	$\frac{29792}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}+\frac{15008i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{196}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{224i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}-$
default	$\frac{29792}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}+\frac{15008i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{196}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{224i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}}-$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
1/128*I/a^8/d*exp(-I*(d*x+c))+7/384*I/a^8/d*exp(-3*I*(d*x+c))+21/640*I/a^8
/d*exp(-5*I*(d*x+c))+5/128*I/a^8/d*exp(-7*I*(d*x+c))+35/1152*I/a^8/d*exp(-
9*I*(d*x+c))+21/1408*I/a^8/d*exp(-11*I*(d*x+c))+7/1664*I/a^8/d*exp(-13*I*(
d*x+c))+1/1920*I/a^8/d*exp(-15*I*(d*x+c))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(6435i\,e^{(14i\,dx+14i\,c)}+15015i\,e^{(12i\,dx+12i\,c)}+27027i\,e^{(10i\,dx+10i\,c)}+32175i\,e^{(8i\,dx+8i\,c)}+25025i\,e^{(6i\,dx+6i\,c)}+\right.}{823680\,a^8d}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/823680*(6435*I*e^(14*I*d*x + 14*I*c) + 15015*I*e^(12*I*d*x + 12*I*c) + 2
7027*I*e^(10*I*d*x + 10*I*c) + 32175*I*e^(8*I*d*x + 8*I*c) + 25025*I*e^(6*
I*d*x + 6*I*c) + 12285*I*e^(4*I*d*x + 4*I*c) + 3465*I*e^(2*I*d*x + 2*I*c)
+ 429*I)*e^(-15*I*d*x - 15*I*c)/(a^8*d)
```

3.182.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1221 vs. 2(238) = 476.

Time = 8.86 (sec) , antiderivative size = 1221, normalized size of antiderivative = 4.54

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise((16*tan(c + d*x)**7*sec(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 -
51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6 + 360360*I*
a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360*I*a**8*d*t
an(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*tan(c + d*
x) + 6435*a**8*d) - 128*I*tan(c + d*x)**6*sec(c + d*x)/(6435*a**8*d*tan(c
+ d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6
 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 36036
0*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*
d*tan(c + d*x) + 6435*a**8*d) - 456*tan(c + d*x)**5*sec(c + d*x)/(6435*a**
8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c
 + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)*
*4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 514
80*I*a**8*d*tan(c + d*x) + 6435*a**8*d) + 960*I*tan(c + d*x)**4*sec(c + d*
x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*
a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*t
an(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d
*x)**2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) + 1350*tan(c + d*x)**3
*sec(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**
7 - 180180*a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 4504
50*a**8*d*tan(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a*...
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{429i\,\cos\left(15\,dx+15\,c\right)+3465i\,\cos\left(13\,dx+13\,c\right)+12285i\,\cos\left(11\,dx+11\,c\right)+25025i\,\cos\left(9\,dx+9\,c\right)+}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
1/823680*(429*I*cos(15*d*x + 15*c) + 3465*I*cos(13*d*x + 13*c) + 12285*I*c
os(11*d*x + 11*c) + 25025*I*cos(9*d*x + 9*c) + 32175*I*cos(7*d*x + 7*c) +
27027*I*cos(5*d*x + 5*c) + 15015*I*cos(3*d*x + 3*c) + 6435*I*cos(d*x + c)
+ 429*sin(15*d*x + 15*c) + 3465*sin(13*d*x + 13*c) + 12285*sin(11*d*x + 11
*c) + 25025*sin(9*d*x + 9*c) + 32175*sin(7*d*x + 7*c) + 27027*sin(5*d*x +
5*c) + 15015*sin(3*d*x + 3*c) + 6435*sin(d*x + c))/(a^8*d)
```

3.182.8 Giac [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(6435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}-45045i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-210210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+630630i\tan\left(\frac{1}{2}dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
2/6435*(6435*tan(1/2*d*x + 1/2*c)^14 - 45045*I*tan(1/2*d*x + 1/2*c)^13 - 2
10210*tan(1/2*d*x + 1/2*c)^12 + 630630*I*tan(1/2*d*x + 1/2*c)^11 + 1414413
*tan(1/2*d*x + 1/2*c)^10 - 2357355*I*tan(1/2*d*x + 1/2*c)^9 - 3063060*tan(
1/2*d*x + 1/2*c)^8 + 3063060*I*tan(1/2*d*x + 1/2*c)^7 + 2407405*tan(1/2*d*
x + 1/2*c)^6 - 1444443*I*tan(1/2*d*x + 1/2*c)^5 - 668850*tan(1/2*d*x + 1/2
*c)^4 + 222950*I*tan(1/2*d*x + 1/2*c)^3 + 54915*tan(1/2*d*x + 1/2*c)^2 - 7
845*I*tan(1/2*d*x + 1/2*c) - 952)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^15)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.83

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(2\sin\left(\frac{c}{4}+\frac{dx}{4}\right)^2-1\right)\left(-\frac{\sin(c+dx)^2\,44779i}{32}+\frac{32175\sin(c+dx)}{128}-\frac{\sin(2c+2dx)^2\,26075i}{16}-\frac{3575\sin(2c+2dx)}{8}+\right.}{}$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8),x)
```

output
```
(2*(2*sin(c/4 + (d*x)/4)^2 - 1)*((32175*sin(c + d*x))/128 - (3575*sin(2*c
+ 2*d*x))/8 + (84227*sin(3*c + 3*d*x))/128 - 754*sin(4*c + 4*d*x) + (11152
7*sin(5*c + 5*d*x))/128 - (7187*sin(6*c + 6*d*x))/8 + (121427*sin(7*c + 7*
d*x))/128 - (sin(2*c + 2*d*x)^2*26075i)/16 + (sin(c/2 + (d*x)/2)^2*114583i
)/64 - (sin(3*c + 3*d*x)^2*57925i)/32 + (sin((3*c)/2 + (3*d*x)/2)^2*116585
i)/64 + (sin((5*c)/2 + (5*d*x)/2)^2*119315i)/64 + (sin((7*c)/2 + (7*d*x)/2
)^2*122285i)/64 - (sin(c + d*x)^2*44779i)/32 - 952i))/(6435*a^8*d*(sin((15
*c)/2 + (15*d*x)/2)*1i - 2*sin((15*c)/4 + (15*d*x)/4)^2 + 1))
```

3.183 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.183.1 Optimal result 1390
3.183.2 Mathematica [A] (verified) 1391
3.183.3 Rubi [A] (verified) 1391
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3.183.1 Optimal result

Integrand size = 22, antiderivative size = 271

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{192\sin(c+dx)}{12155a^8d} - \frac{64\sin^3(c+dx)}{12155a^8d} + \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5} + \frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{16i\cos(c+dx)}{2431a^2d\left(a^2+ia^2\tan(c+dx)\right)^3} + \frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)}$$

output
```
192/12155*sin(d*x+c)/a^8/d-64/12155*sin(d*x+c)^3/a^8/d+1/17*I*cos(d*x+c)/d
/(a+I*a*tan(d*x+c))^8+3/85*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^7+24/1105*I
*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+168/12155*I*cos(d*x+c)/a^3/d/(a+I*a
*tan(d*x+c))^5+112/12155*I*cos(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^4+16/2431*I
*cos(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+128/12155*I*cos(d*x+c)^3/d/(a^8
+I*a^8*tan(d*x+c))
```

3.183.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.51

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\sec^8(c+dx)(-194480\cos(c+dx)-148512\cos(3(c+dx))-89760\cos(5(c+dx))-58344\cos(7(c+}{}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/3111680*I)*Sec[c + d*x]^8*(-194480*Cos[c + d*x] - 148512*Cos[3*(c + d
*x)] - 89760*Cos[5*(c + d*x)] - 58344*Cos[7*(c + d*x)] + 5720*Cos[9*(c + d
*x)] - (24310*I)*Sin[c + d*x] - (55692*I)*Sin[3*(c + d*x)] - (56100*I)*Sin
[5*(c + d*x)] - (51051*I)*Sin[7*(c + d*x)] + (6435*I)*Sin[9*(c + d*x)]))/(
a^8*d*(-I + Tan[c + d*x])^8)
```

3.183.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{9\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^7}dx}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{8\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^6}dx}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\left(\frac{7\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{8\left(\frac{7\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^5}dx}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^4}dx}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^3}dx}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i\cos(c+dx)}{15d(a+ia\tan(c+dx))^7}\right)}{17a}$$

$$\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow 3983$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}+\frac{i}{15d(a}\right.}{17a}$$

$$\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow 3042$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^2}dx}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+dx)}{13d(a+ia\tan(c+dx))^6}\right)}{15a}\right.}{17a}$$

$$\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3981$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(\frac{3\int\cos^3(c+dx)dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos(c+}{13d(a+ia\tan(}\right.}{15a}\right.$$

3.183. $\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \text{3042}$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\left(9\,\frac{\left(8\,\frac{\left(7\,\frac{\left(6\,\frac{\left(5\,\frac{\left(4\,\left(\frac{3\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{5a^2}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)\right.}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\frac{i\cos}{13d(a+ia}\right.}{15a}\right.$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Bigg\downarrow \text{3113}$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(-\frac{3\int\left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right.}{13a}\right.}{15a}\right.$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

↓ 2009

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$9\left(\frac{8\left(\frac{7\left(\frac{6\left(\frac{5\left(\frac{4\left(-\frac{3\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{7a}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}\right)}{9a}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\right)}{11a}+\frac{i\cos(c+dx)}{11d(a+ia\tan(c+dx))^5}\right)}{13a}+\right.}{15a}\right.$$

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/17)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (9*(((I/15)*Cos[c + d
*x])/(d*(a + I*a*Tan[c + d*x])^7) + (8*(((I/13)*Cos[c + d*x])/(d*(a + I*a*
Tan[c + d*x])^6) + (7*(((I/11)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^5)
+ (6*(((I/9)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (5*(((I/7)*Cos[c
 + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (4*((-3*(-Sin[c + d*x] + Sin[c + d
*x]^3/3))/(5*a^2*d) + (((2*I)/5)*Cos[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d
*x]))))/(7*a)))/(9*a)))/(11*a)))/(13*a)))/(15*a)))/(17*a)
```

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.183.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.65

method	result
risch	$\frac{3i\mathrm{e}^{-3i(dx+c)}}{128a^8d}+\frac{21i\mathrm{e}^{-5i(dx+c)}}{640a^8d}+\frac{9i\mathrm{e}^{-7i(dx+c)}}{256a^8d}+\frac{7i\mathrm{e}^{-9i(dx+c)}}{256a^8d}+\frac{21i\mathrm{e}^{-11i(dx+c)}}{1408a^8d}+\frac{9i\mathrm{e}^{-13i(dx+c)}}{1664a^8d}+\frac{3i\mathrm{e}^{-15i(d}}{2560a^8}$
derivativedivides	$\frac{2}{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+512i}+\frac{38218i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{1793i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}+$
default	$\frac{2}{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+512i}+\frac{38218i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{1793i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}+$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
3/128*I/a^8/d*exp(-3*I*(d*x+c))+21/640*I/a^8/d*exp(-5*I*(d*x+c))+9/256*I/a
^8/d*exp(-7*I*(d*x+c))+7/256*I/a^8/d*exp(-9*I*(d*x+c))+21/1408*I/a^8/d*exp
(-11*I*(d*x+c))+9/1664*I/a^8/d*exp(-13*I*(d*x+c))+3/2560*I/a^8/d*exp(-15*I
*(d*x+c))+1/8704*I/a^8/d*exp(-17*I*(d*x+c))+1/64*I/a^8/d*cos(d*x+c)+5/256*
sin(d*x+c)/a^8/d
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.44

$$\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{\left(-12155i\,e^{(18i\,dx+18i\,c)}+109395i\,e^{(16i\,dx+16i\,c)}+145860i\,e^{(14i\,dx+14i\,c)}+204204i\,e^{(12i\,dx+12i\,c)}+218790i\,e^{(10i}\right.}{6223}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/6223360*(-12155*I*e^(18*I*d*x + 18*I*c) + 109395*I*e^(16*I*d*x + 16*I*c)
 + 145860*I*e^(14*I*d*x + 14*I*c) + 204204*I*e^(12*I*d*x + 12*I*c) + 21879
0*I*e^(10*I*d*x + 10*I*c) + 170170*I*e^(8*I*d*x + 8*I*c) + 92820*I*e^(6*I*
d*x + 6*I*c) + 33660*I*e^(4*I*d*x + 4*I*c) + 7293*I*e^(2*I*d*x + 2*I*c) +
715*I)*e^(-17*I*d*x - 17*I*c)/(a^8*d)
```

3.183.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.35

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{(-143500911498201343931187200ia^{72}d^9e^{82ic}e^{idx}+1291508203483812095380684800ia^{72}d^9e^{80ic}e^{-idx}+1722010937978416127174246400ia^{72} \\ \frac{x(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1)e^{-17ic}}{512a^8} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-143500911498201343931187200*I*a**72*d**9*exp(82*I*c)*exp(I*d*
x) + 1291508203483812095380684800*I*a**72*d**9*exp(80*I*c)*exp(-I*d*x) + 1
722010937978416127174246400*I*a**72*d**9*exp(78*I*c)*exp(-3*I*d*x) + 24108
15313169782578043944960*I*a**72*d**9*exp(76*I*c)*exp(-5*I*d*x) + 258301640
6967624190761369600*I*a**72*d**9*exp(74*I*c)*exp(-7*I*d*x) + 2009012760974
818815036620800*I*a**72*d**9*exp(72*I*c)*exp(-9*I*d*x) + 10958251423499011
71838156800*I*a**72*d**9*exp(70*I*c)*exp(-11*I*d*x) + 39738713953348064473
2518400*I*a**72*d**9*exp(68*I*c)*exp(-13*I*d*x) + 861005468989208063587123
20*I*a**72*d**9*exp(66*I*c)*exp(-15*I*d*x) + 8441230088129490819481600*I*a
**72*d**9*exp(64*I*c)*exp(-17*I*d*x))*exp(-81*I*c)/(7347246668707908809276
7846400*a**80*d**10), Ne(a**80*d**10*exp(81*I*c), 0)), (x*(exp(18*I*c) + 9
*exp(16*I*c) + 36*exp(14*I*c) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp
(8*I*c) + 84*exp(6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-17*I*c)/(
512*a**8), True))
```

3.183.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.183.8 Giac [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.92

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{12155}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{6211205\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16} - 55791450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 303072770\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14} + 1091397450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) + I)) + (6211205*tan(1/2*d*x +
 1/2*c)^16 - 55791450*I*tan(1/2*d*x + 1/2*c)^15 - 303072770*tan(1/2*d*x +
1/2*c)^14 + 1091397450*I*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x
+ 1/2*c)^12 - 5901218466*I*tan(1/2*d*x + 1/2*c)^11 - 9405145178*tan(1/2*d*
x + 1/2*c)^10 + 11877161010*I*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2
*d*x + 1/2*c)^8 - 9710430158*I*tan(1/2*d*x + 1/2*c)^7 - 6263238566*tan(1/2
*d*x + 1/2*c)^6 + 3172666718*I*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2
*d*x + 1/2*c)^4 - 365303990*I*tan(1/2*d*x + 1/2*c)^3 - 77883902*tan(1/2*d*
x + 1/2*c)^2 + 10498214*I*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x
 + 1/2*c) - I)^17))/d
```

3.183.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{152329\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128} - \frac{41121\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{32} + \frac{41121\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32} - \frac{96165\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{64} + \frac{96165\,\sin\left(\frac{9\,c}{2}+\frac{9\,d}{2}\right)}{64}\right.}{}$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^8,x)
```

output

```
(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*12155i)/16 - (cos((5*c)/2 +
 (5*d*x)/2)*12155i)/16 + (cos((7*c)/2 + (7*d*x)/2)*21437i)/16 - (cos((9*c)
/2 + (9*d*x)/2)*21437i)/16 + (cos((11*c)/2 + (11*d*x)/2)*27047i)/16 - (cos
((13*c)/2 + (13*d*x)/2)*27047i)/16 + (cos((15*c)/2 + (15*d*x)/2)*61387i)/3
2 - (cos((17*c)/2 + (17*d*x)/2)*715i)/32 + (152329*sin(c/2 + (d*x)/2))/128
 - (41121*sin((3*c)/2 + (3*d*x)/2))/32 + (41121*sin((5*c)/2 + (5*d*x)/2))/
32 - (96165*sin((7*c)/2 + (7*d*x)/2))/64 + (96165*sin((9*c)/2 + (9*d*x)/2)
)/64 - (55095*sin((11*c)/2 + (11*d*x)/2))/32 + (55095*sin((13*c)/2 + (13*d
*x)/2))/32 - (491811*sin((15*c)/2 + (15*d*x)/2))/256 + (6435*sin((17*c)/2
+ (17*d*x)/2))/256)*2i)/(12155*a^8*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)
/2)*1i)^17*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2)))
```

3.183. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.184 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

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3.184.1 Optimal result

Integrand size = 24, antiderivative size = 301

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{160\sin(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}\\ &+\frac{32\sin^5(c+dx)}{4199a^8d}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}\\ &+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}\\ &+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5}\\ &+\frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4}\\ &+\frac{112i\cos^3(c+dx)}{12597a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}\\ &+\frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

output

```
160/4199*sin(d*x+c)/a^8/d-320/12597*sin(d*x+c)^3/a^8/d+32/4199*sin(d*x+c)^
5/a^8/d+1/19*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+11/323*I*cos(d*x+c)^3/a
/d/(a+I*a*tan(d*x+c))^7+22/969*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+6
6/4199*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+48/4199*I*cos(d*x+c)^3/d/
(a^2+I*a^2*tan(d*x+c))^4+112/12597*I*cos(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x
+c))^3+64/4199*I*cos(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

3.184.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.53

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{i\sec^8(c+dx)(-739024\cos(c+dx)-604656\cos(3(c+dx))-426360\cos(5(c+dx))-369512\cos(7(c}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((-1/12899328*I)*Sec[c + d*x]^8*(-739024*Cos[c + d*x] - 604656*Cos[3*(c +
d*x)] - 426360*Cos[5*(c + d*x)] - 369512*Cos[7*(c + d*x)] + 65208*Cos[9*(c
 + d*x)] + 1768*Cos[11*(c + d*x)] - (92378*I)*Sin[c + d*x] - (226746*I)*Si
n[3*(c + d*x)] - (266475*I)*Sin[5*(c + d*x)] - (323323*I)*Sin[7*(c + d*x)]
 + (73359*I)*Sin[9*(c + d*x)] + (2431*I)*Sin[11*(c + d*x)]))/(a^8*d*(-I +
Tan[c + d*x])^8)
```

3.184.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3981, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^8}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^7}dx}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^7}dx}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{10\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^6}dx}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{10\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^6}dx}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{10\left(\frac{3\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^5}dx}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{10\left(\frac{3\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^5}dx}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^4}dx}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3042}$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^4}dx}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3983$$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+dx))^7}\right)}{19a}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \ 3042$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^3}dx}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{i\cos^3(c+dx)}{17d(a+ia\tan(c+d}\right.}{19a}$$

$$\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow 3983$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}+\frac{}{17}}{19a}$$

$$\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \; 3042$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\frac{11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3(c+dx)}{15d(a+ia\tan(c+dx))^6}\right)}{17a}\right)}{19a}$$

$$\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

$$\Big\downarrow \text{3981}$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(\frac{5\int\cos^5(c+dx)dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i\cos^3}{15d(a+ia}\right.}{17a}\right.$$

3.184. $\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \text{3042}$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(\frac{5\int\sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{7a^2}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\cos^3(c+dx)}{13d(a+ia\tan(c+dx))^5}\right)}{5a}+\frac{i}{15d(a}\right.}{17a}\right.$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$\Big\downarrow \text{3113}$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(-\frac{5\int\left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{}{13d}\right.}{5a}\right.}{17a}\right.$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$\downarrow$ 2009

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

$$11\left(\frac{10\left(\frac{3\left(\frac{8\left(\frac{7\left(\frac{2\left(-\frac{5\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\right)}{3a}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}\right)}{11a}+\frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}\right)}{13a}+\frac{i\,c}{13d(a+}}{5a}}{17a}\right.$$

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

output
```
((I/19)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (11*(((I/17)*Cos[c
+ d*x]^3)/(d*(a + I*a*Tan[c + d*x])^7) + (10*(((I/15)*Cos[c + d*x]^3)/(d*(
a + I*a*Tan[c + d*x])^6) + (3*(((I/13)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
 d*x])^5) + (8*(((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + (7*
(((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) + (2*((-5*(-Sin[c + d
*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]^5/5))/(7*a^2*d) + (((2*I)/7)*Cos
[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(3*a)))/(11*a)))/(13*a)))/(5
*a)))/(17*a)))/(19*a)
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.184.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.70

method	result
risch	$\frac{33i\mathrm{e}^{-5i(dx+c)}}{1024a^8d}+\frac{33i\mathrm{e}^{-7i(dx+c)}}{1024a^8d}+\frac{77i\mathrm{e}^{-9i(dx+c)}}{3072a^8d}+\frac{15i\mathrm{e}^{-11i(dx+c)}}{1024a^8d}+\frac{165i\mathrm{e}^{-13i(dx+c)}}{26624a^8d}+\frac{11i\mathrm{e}^{-15i(dx+c)}}{6144a^8d}+$
derivativedivides	$-\frac{1984i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{32525i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-$
default	$-\frac{1984i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{32525i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

output
```
33/1024*I/a^8/d*exp(-5*I*(d*x+c))+33/1024*I/a^8/d*exp(-7*I*(d*x+c))+77/307
2*I/a^8/d*exp(-9*I*(d*x+c))+15/1024*I/a^8/d*exp(-11*I*(d*x+c))+165/26624*I
/a^8/d*exp(-13*I*(d*x+c))+11/6144*I/a^8/d*exp(-15*I*(d*x+c))+11/34816*I/a^
8/d*exp(-17*I*(d*x+c))+1/38912*I/a^8/d*exp(-19*I*(d*x+c))+11/512*I/a^8/d*c
os(d*x+c)+33/1024*sin(d*x+c)/a^8/d+41/1536*I/a^8/d*cos(3*d*x+3*c)+83/3072/
a^8/d*sin(3*d*x+3*c)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.47

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \left(-4199i\,e^{(22i\,dx+22i\,c)}-138567i\,e^{(20i\,dx+20i\,c)}+692835i\,e^{(18i\,dx+18i\,c)}+692835i\,e^{(16i\,dx+16i\,c)}+831402i\,e^{(14i}\right.$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

output
```
1/25798656*(-4199*I*e^(22*I*d*x + 22*I*c) - 138567*I*e^(20*I*d*x + 20*I*c)
 + 692835*I*e^(18*I*d*x + 18*I*c) + 692835*I*e^(16*I*d*x + 16*I*c) + 83140
2*I*e^(14*I*d*x + 14*I*c) + 831402*I*e^(12*I*d*x + 12*I*c) + 646646*I*e^(1
0*I*d*x + 10*I*c) + 377910*I*e^(8*I*d*x + 8*I*c) + 159885*I*e^(6*I*d*x + 6
*I*c) + 46189*I*e^(4*I*d*x + 4*I*c) + 8151*I*e^(2*I*d*x + 2*I*c) + 663*I)*
e^(-19*I*d*x - 19*I*c)/(a^8*d)
```

3.184.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.45

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{(-6279106898588469469113471576881812733952ia^{88}d^{11}e^{103ic}e^{3idx}-207210527653419492480744562037099820220416ia^{88}d^{11}e^{101ic}e^{idx}+10}{} \\ \frac{x\left(e^{22ic}+11e^{20ic}+55e^{18ic}+165e^{16ic}+330e^{14ic}+462e^{12ic}+462e^{10ic}+330e^{8ic}+165e^{6ic}+55e^{4ic}+11e^{2ic}+1\right)e^{-19ic}}{2048a^8} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

output
```
Piecewise(((-6279106898588469469113471576881812733952*I*a**88*d**11*exp(10
3*I*c)*exp(3*I*d*x) - 207210527653419492480744562037099820220416*I*a**88*d
**11*exp(101*I*c)*exp(I*d*x) + 1036052638267097462403722810185499101102080
*I*a**88*d**11*exp(99*I*c)*exp(-I*d*x) + 103605263826709746240372281018549
9101102080*I*a**88*d**11*exp(97*I*c)*exp(-3*I*d*x) + 124326316592051695488
4467372222598921322496*I*a**88*d**11*exp(95*I*c)*exp(-5*I*d*x) + 124326316
5920516954884467372222598921322496*I*a**88*d**11*exp(93*I*c)*exp(-7*I*d*x)
 + 966982462382624298243474622839799161028608*I*a**88*d**11*exp(91*I*c)*ex
p(-9*I*d*x) + 565119620872962252220212441919363146055680*I*a**88*d**11*exp
(89*I*c)*exp(-11*I*d*x) + 239089070369330183631628340812038254100480*I*a**
88*d**11*exp(87*I*c)*exp(-13*I*d*x) + 690701758844731641602481873456999400
73472*I*a**88*d**11*exp(85*I*c)*exp(-15*I*d*x) + 1218885456784820544004379
7766888224718848*I*a**88*d**11*exp(83*I*c)*exp(-17*I*d*x) + 99143793135607
4126702127091086602010624*I*a**88*d**11*exp(81*I*c)*exp(-19*I*d*x))*exp(-1
00*I*c)/(38578832784927556418233169368361857437401088*a**96*d**12), Ne(a**
96*d**12*exp(100*I*c), 0)), (x*(exp(22*I*c) + 11*exp(20*I*c) + 55*exp(18*I
*c) + 165*exp(16*I*c) + 330*exp(14*I*c) + 462*exp(12*I*c) + 462*exp(10*I*c
) + 330*exp(8*I*c) + 165*exp(6*I*c) + 55*exp(4*I*c) + 11*exp(2*I*c) + 1)*e
xp(-19*I*c)/(2048*a**8), True))
```

3.184.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.184.8 Giac [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$

$$= \frac{\frac{4199\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+33i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-17\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{12823746\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{18}-140368371i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}-879644311\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

output
```
1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 + 33*I*tan(1/2*d*x + 1/2*c) - 1
7)/(a^8*(tan(1/2*d*x + 1/2*c) + I)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18
- 140368371*I*tan(1/2*d*x + 1/2*c)^17 - 879644311*tan(1/2*d*x + 1/2*c)^16
+ 3693272440*I*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^
14 - 27403194676*I*tan(1/2*d*x + 1/2*c)^13 - 51919375300*tan(1/2*d*x + 1/2
*c)^12 + 79183835016*I*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x +
 1/2*c)^10 - 99750226290*I*tan(1/2*d*x + 1/2*c)^9 - 82860874122*tan(1/2*d*
x + 1/2*c)^8 + 56110430792*I*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*
d*x + 1/2*c)^6 - 13462452660*I*tan(1/2*d*x + 1/2*c)^5 - 4616712644*tan(1/2
*d*x + 1/2*c)^4 + 1197851960*I*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*
d*x + 1/2*c)^2 - 27911475*I*tan(1/2*d*x + 1/2*c) - 2143959)/(a^8*(tan(1/2*
d*x + 1/2*c) - I)^19))/d
```

3.184.9 Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{46189\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{46189\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}-\frac{20995\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{16}+\frac{20995\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{16}-221255\,\mathrm{co}\right.}{}$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^8,x)
```

output
```
-(2*cos(c/2 + (d*x)/2)*((46189*cos((5*c)/2 + (5*d*x)/2))/64 - (46189*cos((
3*c)/2 + (3*d*x)/2))/64 - (20995*cos((7*c)/2 + (7*d*x)/2))/16 + (20995*cos
((9*c)/2 + (9*d*x)/2))/16 - (221255*cos((11*c)/2 + (11*d*x)/2))/128 + (221
255*cos((13*c)/2 + (13*d*x)/2))/128 - (66861*cos((15*c)/2 + (15*d*x)/2))/3
2 + (2093*cos((17*c)/2 + (17*d*x)/2))/32 - (221*cos((19*c)/2 + (19*d*x)/2)
)/128 + (221*cos((21*c)/2 + (21*d*x)/2))/128 + (sin(c/2 + (d*x)/2)*309861i
)/256 - (sin((3*c)/2 + (3*d*x)/2)*665911i)/512 + (sin((5*c)/2 + (5*d*x)/2)
*665911i)/512 - (sin((7*c)/2 + (7*d*x)/2)*194821i)/128 + (sin((9*c)/2 + (9
*d*x)/2)*194821i)/128 - (sin((11*c)/2 + (11*d*x)/2)*1825043i)/1024 + (sin(
(13*c)/2 + (13*d*x)/2)*1825043i)/1024 - (sin((15*c)/2 + (15*d*x)/2)*107418
3i)/512 + (sin((17*c)/2 + (17*d*x)/2)*37895i)/512 - (sin((19*c)/2 + (19*d*
x)/2)*2431i)/1024 + (sin((21*c)/2 + (21*d*x)/2)*2431i)/1024))/(12597*a^8*d
*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^19*(cos(c/2 + (d*x)/2)*1i +
sin(c/2 + (d*x)/2))^3)
```

3.184. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

3.185 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

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3.185.1 Optimal result

Integrand size = 26, antiderivative size = 123

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = -\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

output
```
2/7*I*a*(e*sec(d*x+c))^(7/2)/d+2/5*a*e*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/d-6
/5*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+6/5*a*e^3*sin
(d*x+c)*(e*sec(d*x+c))^(1/2)/d
```

3.185.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{aee^{-idx}(e\sec(c+dx))^{5/2}(\cos(dx)-i\sin(dx))(\cos(c+3dx)+i\sin(c+3dx))\left(-36i-\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 3*d*x] + I*Si
n[c + 3*d*x])*(-36*I - (28*I)*Cos[2*(c + d*x)] + ((7*I)*(1 + E^((2*I)*(c +
 d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((
2*I)*(c + d*x)) + 7*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c + d*x]))/(70*
d*E^(I*d*x))
```

3.185.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (e\sec(c+dx))^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \; 4258$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \; 3119$$

$$a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}$$

input

```
Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output

```
((((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + a*((2*e*(e*Sec[c + d*x])^(5/2)*Si
n[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos
[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])
/d))/5)
```

3.185. $\int(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

3.185.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.185.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(130) = 260.

Time = 27.13 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.53

method	result
default	$2a\sqrt{e\sec(dx+c)}\,e^{3}\left(3iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^{2}(dx+c))-3iE(i(\csc(dx+c)-\cot(dx+c)),i)\right.$
parts	$2a\sqrt{e\sec(dx+c)}\,e^{3}\left(3iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^{2}(dx+c))-3iE(i(\csc(dx+c)-\cot(dx+c)),i)\right.$

input
```
int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
2/5*a/d*(e*sec(d*x+c))^(1/2)*e^3/(cos(d*x+c)+1)*(3*I*EllipticF(I*(csc(d*x+
c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)^2-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+6*I*cos(d*x+c)*El
lipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)-6*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*I*EllipticF
(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*sin(d*x+c)+tan(d*x+c)+sec(d*x
+c)*tan(d*x+c))+2/7*I*a*(e*sec(d*x+c))^(7/2)/d
```

3.185.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.69

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(21i\,ae^3e^{(7i\,dx+7i\,c)}+77i\,ae^3e^{(5i\,dx+5i\,c)}+23i\,ae^3e^{(3i\,dx+3i\,c)}+7i\,ae^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{35\,(de^{(6i\,dx}}$$

input

```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output

```
-2/35*(sqrt(2)*(21*I*a*e^3*e^(7*I*d*x + 7*I*c) + 77*I*a*e^3*e^(5*I*d*x + 5
*I*c) + 23*I*a*e^3*e^(3*I*d*x + 3*I*c) + 7*I*a*e^3*e^(I*d*x + I*c))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*a*e^3*
e^(6*I*d*x + 6*I*c) + 3*I*a*e^3*e^(4*I*d*x + 4*I*c) + 3*I*a*e^3*e^(2*I*d*x
 + 2*I*c) + I*a*e^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, e^(I*d*x + I*c))))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) +
 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

3.185.7 Maxima [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

3.185.8 Giac [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{7/2}\,\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

3.186 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

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3.186.1 Optimal result

Integrand size = 26, antiderivative size = 94

$$\begin{aligned}\int (e\sec(c+dx))^{5/2}(a & \\ +ia\tan(c+dx))\,dx &= \frac{2ae^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} \\ &+ \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d}\end{aligned}$$

output
```
2/5*I*a*(e*sec(d*x+c))^(5/2)/d+2/3*a*e*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2
/3*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d
```

3.186.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\begin{aligned}\int (e\sec(c+dx))^{5/2}(a & \\ +ia\tan(c+dx))\,dx &= \frac{a(e\sec(c+dx))^{5/2}\left(6i+10\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))\right)}{15d}\end{aligned}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(e*Sec[c + d*x])^(5/2)*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)
/2, 2] + 5*Sin[2*(c + d*x)]))/(15*d)
```

3.186.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (e\sec(c+dx))^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

input

```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output

```
(((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + a*((2*e^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^
(3/2)*Sin[c + d*x])/(3*d))
```

3.186.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.186.4 Maple [A] (verified)

Time = 24.87 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.73

method	result
default	$-\frac{2a\sqrt{e\sec(dx+c)}\,e^{2}\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}\right.}{3d}$
parts	$-\frac{2a\sqrt{e\sec(dx+c)}\,e^{2}\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}\right.}{3d}$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a/d*(e*sec(d*x+c))^(1/2)*e^2*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+I*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(
1/(cos(d*x+c)+1))^(1/2)-tan(d*x+c))+2/5*I*a*(e*sec(d*x+c))^(5/2)/d
```

3.186.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.64

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(5i\,ae^{2}e^{(4i\,dx+4i\,c)}-12i\,ae^{2}e^{(2i\,dx+2i\,c)}-5i\,ae^{2}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,ae^{2}e^{(4i\,dx+4i\,c)}+2i\right.\right.}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(5*I*a*e^2*e^(4*I*d*x + 4*I*c) - 12*I*a*e^2*e^(2*I*d*x + 2*
I*c) - 5*I*a*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
) + 5*sqrt(2)*(I*a*e^2*e^(4*I*d*x + 4*I*c) + 2*I*a*e^2*e^(2*I*d*x + 2*I*c)
 + I*a*e^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(4*I
*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.186.6 Sympy [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i(e\sec{(c+dx)})^{\frac{5}{2}}\right)\,dx + \int (e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*sec(c + d*x))**(5/2), x) + Integral((e*sec(c + d*x))**
(5/2)*tan(c + d*x), x))
```

3.186.7 Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\sec{(dx+c)})^{\frac{5}{2}}\,(i\,a\tan{(dx+c)}+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

3.186.8 Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\sec{(dx+c)})^{\frac{5}{2}}\,(i\,a\tan{(dx+c)}+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(a+a\,\tan(c+d\,x)\,\,1\mathrm{i}\right)\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

3.187 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

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3.187.1 Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d}$$

output
```
2/3*I*a*(e*sec(d*x+c))^(3/2)/d-2*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*s
ec(d*x+c))^(1/2)+2*a*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d
```

3.187.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.01 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{2aee^{-2idx}\sqrt{e\sec(c+dx)}(\cos(c+3dx)+i\sin(c+3dx))\Big(-2i+i\sqrt{1+e^{2i(c+dx)}}\,\text{Hyper}}{3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*a*e*Sqrt[e*Sec[c + d*x]]*(Cos[c + 3*d*x] + I*Sin[c + 3*d*x])*(-2*I + I*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))] + Tan[c + d*x]))/(3*d*E^((2*I)*d*x))
```

3.187.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x)/2,
 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x
]]*Sin[c + d*x])/d)
```

3.187.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.187.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(105) = 210$.

Time = 3.33 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.57

method	result
default	$\frac{2a\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))-iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{}{\cos}}\right.}{}$
parts	$\frac{2a\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))-iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{}{\cos}}\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*a/d*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-I*EllipticE(I*(csc(d*x+c)-co
t(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)^2+2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*EllipticE(I*(csc(d*x
+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)
-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+sin(d*x
+c))*(e*sec(d*x+c))^(1/2)*e/(cos(d*x+c)+1)+2/3*I*a*(e*sec(d*x+c))^(3/2)/d
```

3.187.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(3i\,aee^{(3i\,dx+3i\,c)}+i\,aee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\sqrt{2}\left(i\,aee^{(2i\,dx+2i\,c)}+i\,ae\right)\sqrt{e}\mathrm{weierstras}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(3*I*a*e*e^(3*I*d*x + 3*I*c) + I*a*e*e^(I*d*x + I*c))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*sqrt(2)*(I*a*e*e^(
2*I*d*x + 2*I*c) + I*a*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, e^(I*d*x + I*c))))/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.187.6 Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i(e\sec{(c+dx)})^{\frac{3}{2}}\right)\,dx + \int (e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**
(3/2)*tan(c + d*x), x))
```

3.187.7 Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}(i\,a\tan{(dx+c)}+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

3.187.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}(i\,a\tan{(dx+c)}+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

3.188 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx$

3.188.1 Optimal result 1447
3.188.2 Mathematica [A] (verified) 1447
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3.188.9 Mupad [B] (verification not implemented) 1451

3.188.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2ia\sqrt{e \sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e \sec(c+dx)}}{d}$$

output
```
2*I*a*(e*sec(d*x+c))^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c
))^(1/2)/d
```

3.188.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2a\left(i+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{e \sec(c+dx)}}{d}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*a*(I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d*x
]])/d
```

3.188.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3967}$$

$$a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4258}$$

$$a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d
```

3.188.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.188.4 Maple [A] (verified)

Time = 7.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

method	result
parts	$-\frac{2ia(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)}{d}+\frac{2ia\sqrt{e\sec(dx+c)}}{d}$
default	$-\frac{2ia\left(F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)+F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{d}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*a/d*(cos(d*x+c)+1)*(e*sec(d*x+c))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+2*I*a
*(e*sec(d*x+c))^(1/2)/d
```

3.188.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = -\frac{2\left(-i\sqrt{2}a\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}a\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(2)*a*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
 + I*sqrt(2)*a*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/d
```

3.188.6 Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sqrt{e\sec{(c+dx)}}\right)\,dx + \int \sqrt{e\sec{(c+dx)}}\tan{(c+dx)}\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*
tan(c + d*x), x))
```

3.188.7 Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \int \sqrt{e\sec{(dx+c)}}(i\,a\tan{(dx+c)}+a)\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

3.188.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

3.188.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+1{}\mathrm{i}\right)\,\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}{d}$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
(2*a*(cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2) + 1i)*(e/cos(c + d*x)
)^(1/2))/d
```

3.189 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx$

3.189.1 Optimal result 1452
3.189.2 Mathematica [C] (verified) 1452
3.189.3 Rubi [A] (verified) 1453
3.189.4 Maple [B] (verified) 1454
3.189.5 Fricas [C] (verification not implemented) 1455
3.189.6 Sympy [F] 1455
3.189.7 Maxima [F] 1456
3.189.8 Giac [F] 1456
3.189.9 Mupad [F(-1)] 1456

3.189.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2ia}{d\sqrt{e\sec(c+dx)}}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

output

```
-2*I*a/d/(e*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*
x+c))^(1/2)
```

3.189.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{4iae^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{e\sec(c+dx)}}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

output

```
(((-4*I)/3)*a*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])
```

3.189.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{\sqrt{e\sec(c+dx)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*S
qrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

3.189.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.189.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(79) = 158$.

Time = 6.64 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.02

method	result
risch	$-\frac{2ia\sqrt{2}}{d\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\Big(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\Big(-2iE\Big(\sqrt{-i\left(e^{i(dx+c)}+i\right)},}{\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}}}{d\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)}$
parts	$\frac{2a\Big(i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}}{}$
default	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d*a*2^(1/2)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-I/d*(-2*(e*
exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*
(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d
*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE(
(-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))
+I))^(1/2),1/2*2^(1/2))))*a*2^(1/2)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1)
)^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)/(exp(I*(d*x+c))^2+1)
```

3.189.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2i\,\sqrt{2}a\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d\sqrt{e}}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x +
 I*c)))/(d*sqrt(e))
```

3.189.6 Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = ia\left(\int \left(-\frac{i}{\sqrt{e\sec{(c+dx)}}}\right)dx + \int \frac{\tan{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(1/2),x)
```

output
```
I*a*(Integral(-I/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*s
ec(c + d*x)), x))
```

3.189.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

3.189.8 Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2), x)
```

3.190 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$

3.190.1 Optimal result . . . 1457
3.190.2 Mathematica [A] (verified) . . . 1457
3.190.3 Rubi [A] (verified) . . . 1458
3.190.4 Maple [A] (verified) . . . 1460
3.190.5 Fricas [C] (verification not implemented) . . . 1460
3.190.6 Sympy [F] . . . 1461
3.190.7 Maxima [F] . . . 1461
3.190.8 Giac [F] . . . 1461
3.190.9 Mupad [F(-1)] . . . 1462

3.190.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2ia}{3d(e\sec(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} + \frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

output
```
-2/3*I*a/d/(e*sec(d*x+c))^(3/2)+2/3*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(1/2)+
2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^2
```

3.190.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \frac{2a\left(-i\cos(c+dx)+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\sqrt{\cos(c+dx)}}+\sin(c+dx)\right)}{3de\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(2*a*((-I)*Cos[c + d*x] + EllipticF[(c + d*x)/2, 2]/Sqrt[Cos[c + d*x]] + S
in[c + d*x]))/(3*d*e*Sqrt[e*Sec[c + d*x]])
```

3.190.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(e\sec(c+dx))^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*a)/(d*(e*Sec[c + d*x])^(3/2)) + a*((2*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3
*d*e*Sqrt[e*Sec[c + d*x]]))
```

3.190.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.190.4 Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.58

method	result
default	$\frac{2a\left(iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{3ed\sqrt{e\sec(dx+c)}}$
parts	$-\frac{2a\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)\right.}{3d\sqrt{e\sec(dx+c)}\,e}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}a\sqrt{2}}{3de\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a\sqrt{e\,\mathrm{e}^{i(dx+c)}\left(\mathrm{e}^{2i(dx+c)}+1\right)}}{3d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*a/e/d/(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)*El
lipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)-I*cos(d*x+c)+sin(d*x+c))
```

3.190.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx=\frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-2i\,\sqrt{2}a\sqrt{e}\text{weierstrassPInver}}{3\,de^{2}}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c) - 2*I*sqrt(2)*a*sqrt(e)*weierstrassPInverse(
-4, 0, e^(I*d*x + I*c)))/(d*e^2)
```

3.190.6 Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = ia\left(\int \left(-\frac{i}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx + \int \frac{\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(3/2),x)
```

output
```
I*a*(Integral(-I/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*se
c(c + d*x))**(3/2), x))
```

3.190.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

3.190.8 Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2), x)
```

3.191 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$

3.191.1 Optimal result . . . 1463
3.191.2 Mathematica [C] (verified) . . . 1463
3.191.3 Rubi [A] (verified) . . . 1464
3.191.4 Maple [B] (verified) . . . 1466
3.191.5 Fricas [C] (verification not implemented) . . . 1467
3.191.6 Sympy [F] . . . 1467
3.191.7 Maxima [F] . . . 1467
3.191.8 Giac [F] . . . 1468
3.191.9 Mupad [F(-1)] . . . 1468

3.191.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

output
```
-2/5*I*a/d/(e*sec(d*x+c))^(5/2)+2/5*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(3/2)+
6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)
```

3.191.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx=$$
$$-\frac{a\Big(2+2\cos(2(c+dx))-2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3i\sin(2(c+dx))\Big)(-}{5de^2\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

output
```
-1/5*(a*(2 + 2*Cos[2*(c + d*x)] - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - (3*I)*Sin[2*(c + d*x)])*(
-I + Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

3.191.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(e\sec(c+dx))^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*a)/(d*(e*Sec[c + d*x])^(5/2)) + a*((6*EllipticE[(c + d*x)/2, 2
])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5
*d*e*(e*Sec[c + d*x])^(3/2)))
```

3.191.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

3.191. $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.191.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(107) = 214.

Time = 6.19 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.33

method	result
risch	$-\frac{i\left(e^{2i(dx+c)}+7\right)a\sqrt{2}}{10d\,e^{2}\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\left(-2iE\left(\sqrt{-i\left(e^{i(dx+c)}\right.}\right.\right.}{\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}}\right.}{5d\,e^{2}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+}}}$
default	$\frac{2a\left(-3i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+3i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(d}{\cos(dx}}\right.}{}$
parts	$\frac{2a\left(-3i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+3i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(d}{\cos(dx}}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*I*(exp(I*(d*x+c))^2+7)/d*a*2^(1/2)/e^2/(e*exp(I*(d*x+c))/(exp(I*(d*x
+c))^2+1))^(1/2)-3/5*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*e
xp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(
I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*
(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))
+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*a*2^(1/2)/e^2/(e
xp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*
(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

3.191.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(12i\sqrt{2}a\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right.\right.}{10}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/10*(12*I*sqrt(2)*a*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c)
 + 4*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^
(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(d*e^3)
```

3.191.6 Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx + \int\frac{\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(5/2),x)
```

output
```
I*a*(Integral(-I/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)/(e*se
c(c + d*x))**(5/2), x))
```

3.191.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

3.191.8 Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2), x)
```

3.192 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$

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3.192.1 Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}}$$

output

```
-2/7*I*a/d/(e*sec(d*x+c))^(7/2)+2/7*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(5/2)+
10/21*a*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)+10/21*a*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4
```

3.192.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a\sqrt{e\sec(c+dx)}(\cos(c+dx)+i\sin(c+dx))\big(-14i\cos(c+dx)+2i\cos(3(c+}{}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(a*Sqrt[e*Sec[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x])*((-14*I)*Cos[c + d
*x] + (2*I)*Cos[3*(c + d*x)] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 5*Sin[c + d*x] + 5*Sin[3*(c + d*x)]
))/(42*d*e^4)
```

3.192.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(e\sec(c+dx))^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

3.192. $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$

output

```
(((-2*I)/7)*a)/(d*(e*Sec[c + d*x])^(7/2)) + a*((2*Sin[c + d*x])/(7*d*e*(e*
Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c +
d*x]])))/(7*e^2))
```

3.192.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.192.4 Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.43

method	result
default	$-\frac{2a\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)\right.}{21d\sqrt{e\sec(dx+c)}\,e^3}$
parts	$-\frac{2a\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)\right.}{21d\sqrt{e\sec(dx+c)}\,e^3}$

input
```
int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*a/d/(e*sec(d*x+c))^(1/2)/e^3*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+5*I*sec(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c))
,I)*(1/(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)^2*sin(d*x+c)-5*sin(d*x+c))-2/7*I
*a/d/(e*sec(d*x+c))^(7/2)
```

3.192.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-40i\,\sqrt{2}a\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,ae^{(6i\,dx+\right.\right.}{84}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/84*(-40*I*sqrt(2)*a*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4,
0, e^(I*d*x + I*c)) + sqrt(2)*(-3*I*a*e^(6*I*d*x + 6*I*c) - 19*I*a*e^(4*I*
d*x + 4*I*c) - 9*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^4)
```

3.192.6 Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\sec{(c+dx)})^{\frac{7}{2}}}\right)dx + \int\frac{\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{7}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(7/2),x)
```

output
```
I*a*(Integral(-I/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)/(e*se
c(c + d*x))**(7/2), x))
```

3.192.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \int\frac{i\,a\tan{(dx+c)}+a}{(e\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

3.192.8 Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \int\frac{i\,a\tan{(dx+c)}+a}{(e\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx=\int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2), x)
```

3.193 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx$

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3.193.1 Optimal result

Integrand size = 28, antiderivative size = 138

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = -\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

output
```
14/15*I*a^2*(e*sec(d*x+c))^(3/2)/d-14/5*a^2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(
1/2)/(e*sec(d*x+c))^(1/2)+14/5*a^2*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/5
*I*(e*sec(d*x+c))^(3/2)*(a^2+I*a^2*tan(d*x+c))/d
```

3.193.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.30 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.93

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \frac{(e\sec(c+dx))^{3/2}\left(-\frac{14i\sqrt{2}\left(3\sqrt{1+e^{2i(c+dx)}}-e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{\left(-1+e^{2ic}\right)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}}+\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((e*Sec[c + d*x])^(3/2)*(((-14*I)*Sqrt[2]*(3*Sqrt[1 + E^((2*I)*(c + d*x))]
 - E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))]))/((-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (Csc[c]*Sec[c + d*x]^(5/2)*(
Cos[2*c] - I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c + 3*d
*x] - (20*I)*Sin[d*x] + (20*I)*Sin[2*c + d*x]))/2)*(a + I*a*Tan[c + d*x])^
2)/(15*d*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

3.193.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow 3042$$

$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow 3979$$

$$\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow 3967$$

$$\frac{7}{5}a\left(a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{7}{5}a\left(a\int \left(e\csc \left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc \left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\sin \left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

3.193. $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx$

output
```
(7*a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d
*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c
 + d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I
*a^2*Tan[c + d*x]))/d
```

3.193.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.193.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(143) = 286$.

Time = 14.59 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.12

method	result
default	$\frac{2e\,a^{2}\sqrt{e\sec(dx+c)}\,\Big(21iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^{2}(dx+c))-21iE(i(\csc(dx+c)-\cot(dx+c)),}{}$
parts	Expression too large to display

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*e*a^2/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*I*EllipticF(I*(csc(d*
x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)^2-21*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+42*I*cos(d*x+c
)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)-42*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+
c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*I*Ell
ipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)-21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(c
sc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+10*I+21*sin(d*x+c)+10*I*
sec(d*x+c)-3*tan(d*x+c)-3*sec(d*x+c)*tan(d*x+c))
```

3.193.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{2}\,dx =$$

$$-\frac{2\,\Big(\sqrt{2}\big(21i\,a^{2}ee^{(5i\,dx+5i\,c)}+16i\,a^{2}ee^{(3i\,dx+3i\,c)}+7i\,a^{2}ee^{(i\,dx+i\,c)}\big)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\,\sqrt{2}\big(i\,a^{2}ee^{(4i\,dx}}{15\,\big(de^{(4i\,dx+4i\,c)}+2\,de}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(21*I*a^2*e*e^(5*I*d*x + 5*I*c) + 16*I*a^2*e*e^(3*I*d*x + 3
*I*c) + 7*I*a^2*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/
2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*a^2*e*e^(4*I*d*x + 4*I*c) + 2*I*a^2*e*e
^(2*I*d*x + 2*I*c) + I*a^2*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +
 2*I*c) + d)
```

3.193.6 Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \left(-(e\sec{(c+dx)})^{\frac{3}{2}}\right)\,dx + \int (e\sec{(c+dx)})^{\frac{3}{2}}\tan^2{(c+dx)}\,dx + \int \left(-2i(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**
(3/2)*tan(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**(3/2)*tan(c +
d*x), x))
```

3.193.7 Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^2\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

3.193.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx=\int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^2\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\;1\mathrm{i})^2\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.194 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$

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3.194.1 Optimal result

Integrand size = 28, antiderivative size = 106

$$\begin{aligned}&\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx\\&=\frac{10ia^2\sqrt{e\sec(c+dx)}}{3d}+\frac{10a^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}\\&\quad+\frac{2i\sqrt{e\sec(c+dx)}(a^2+ia^2\tan(c+dx))}{3d}\end{aligned}$$

output
```
10/3*I*a^2*(e*sec(d*x+c))^(1/2)/d+10/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e
*sec(d*x+c))^(1/2)/d+2/3*I*(e*sec(d*x+c))^(1/2)*(a^2+I*a^2*tan(d*x+c))/d
```

3.194.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\begin{aligned}&\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx\\&=\frac{2a^2(e\sec(c+dx))^{3/2}\left(6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\sin(c+dx)\right)}{3de}\end{aligned}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*a^2*(e*Sec[c + d*x])^(3/2)*((6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*E
llipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*d*e)
```

3.194.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{5}{3}a\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(5*a*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c + d
*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

3.194.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.194.4 Maple [A] (verified)

Time = 10.73 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36

method	result
default	$-\frac{2a^2\sqrt{e\sec(dx+c)}\left(-5i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}F(i(-\csc(dx+c)+\cot(dx+c)),i)\cos(dx+c)-5iF(i(-\csc(dx+c)+\cot(dx+c)\right.}{3d}$
parts	$-\frac{2ia^2(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)}{d}+\frac{4ia^2\sqrt{e\sec(dx+c)}}{d}-\frac{2a^2\sqrt{e}}{}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2/d*(e*sec(d*x+c))^(1/2)*(-5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*cos(d*x+c)-
5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I
*(-csc(d*x+c)+cot(d*x+c)),I)-6*I+tan(d*x+c))
```

3.194.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(-7i\,a^2e^{(2i\,dx+2i\,c)}-5i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\sqrt{e}\mathrm{weierstrassPI}\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(-7*I*a^2*e^(2*I*d*x + 2*I*c) - 5*I*a^2)*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I
*c) + I*a^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(2*
I*d*x + 2*I*c) + d)
```

3.194.6 Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \left(-\sqrt{e\sec{(c+dx)}}\right)\,dx + \int \sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}\,dx + \int \left(-2i\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*
tan(c + d*x)**2, x) + Integral(-2*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x))
```

3.194.7 Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \int \sqrt{e\sec{(dx+c)}}(i\,a\tan{(dx+c)}+a)^2\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2, x)
```

3.194.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx=\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\tan(c+d\,x)\,\,1{}\mathrm{i}\right)}^2\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.195 $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$

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3.195.1 Optimal result

Integrand size = 28, antiderivative size = 107

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx=\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

output
```
6*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-6*a^2*sin(d*x+c)
*(e*sec(d*x+c))^(1/2)/d/e-4*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(1/2
)
```

3.195.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.23

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2i\sqrt{2}a^2e^{2i(c+dx)}\left(-\sqrt{1+e^{2i(c+dx)}}+\left(1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{d\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[2]*a^2*E^((2*I)*(c + d*x))*(-Sqrt[1 + E^((2*I)*(c + d*x))] +
(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))]))/(d*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2
*I)*(c + d*x)))^(3/2))
```

3.195.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3977, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{3a^2\int(e\sec(c+dx))^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 4258$$

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(-3*a^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/e^2 - ((4*I)*(
a^2 + I*a^2*Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

3.195.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.195. $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.195.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(123) = 246.

Time = 10.34 (sec) , antiderivative size = 807, normalized size of antiderivative = 7.54

method	result
parts	$2a^{2}\left(i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}-i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c}{\cos(dx+c)}}\right.$
default	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*EllipticE(I*(csc(d*x+c)-cot
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos
(d*x+c)-I*cos(d*x+c)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-2
*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-I*sec(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)+sin(d*x+c))-4*I*a^2/(e*sec(d*x+c))^(1/2)/d+2*a^
2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(2*I*EllipticE(I*(csc(d*x+c)-cot(d
*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)-2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+4*I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-4
*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)+2*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-2*I*sec(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c
)),I)*(1/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)-tan(d*x+c))
```

3.195.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{(a + ia\tan(c + dx))^2}{\sqrt{e\sec(c + dx)}}\,dx = -\frac{2\left(-i\sqrt{2}a^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)} - 3i\sqrt{2}a^2\sqrt{e}\text{weierstrassZeta}\bigl(-4, 0, \text{weierstrassPInverse}\bigl(-4, 0, e^{(}\right.}{de}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(2)*a^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*
c) - 3*I*sqrt(2)*a^2*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, e^(I*d*x + I*c))))/(d*e)
```

3.195.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -a^2\left(\int \left(-\frac{1}{\sqrt{e\sec(c+dx)}}\right)dx + \int \frac{\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int \left(-\frac{2i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(1/2),x)
```

output
```
-a**2*(Integral(-1/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**2/sqr
t(e*sec(c + d*x)), x) + Integral(-2*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x
))
```

3.195.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{\sqrt{e\sec(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

3.195.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{\sqrt{e\sec(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2), x)
```

3.196 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx$

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3.196.7 Maxima [F] 1500
3.196.8 Giac [F] 1500
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3.196.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

output
```
-2/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*(a^
2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(3/2)
```

3.196.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\left(2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\sin(c+dx))\right)(c}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sec[c + d*x]^2*((2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]))*(Cos[c + 3*d*x] + I*Sin[c
 + 3*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

3.196.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3977, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\int \sqrt{e\sec(c+dx)}dx}{3e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3
/2))
```

3.196.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.196. $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx$

3.196.4 Maple [A] (verified)

Time = 9.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.84

method	result
default	$-\frac{2a^2\left(iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{3ed\sqrt{e\sec(dx+c)}}$
risch	$-\frac{2i\mathrm{e}^{i(dx+c)}a^2\sqrt{2}}{3de\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^2\sqrt{e\,\mathrm{e}^{i(dx+c)}\left(\mathrm{e}^{2i(dx+c)}+1\right)}}{3d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^2\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)\right.}{3d\sqrt{e\sec(dx+c)}\,e}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2/e/d/(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),
I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)
*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)+2*I*cos(d*x+c)-2*sin(d*x+c))
```

3.196.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(-i\sqrt{2}a^2\sqrt{e}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)}{3\,de^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(-I*sqrt(2)*a^2*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) +
 sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^2)
```

3.196.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -a^2\left(\int \left(-\frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx + \int \frac{\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx + \int \left(-\frac{2i\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(3/2),x)
```

output
```
-a**2*(Integral(-1/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**2/
(e*sec(c + d*x))**(3/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))*
*(3/2), x))
```

3.196.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

3.196.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2), x)
```

3.197 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx$

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3.197.2 Mathematica [C] (verified) 1502
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3.197.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx=\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

output

```
2/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/5*I*(a^2
+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(5/2)
```

3.197.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx=$$
$$-\frac{i\sqrt{2}a^2\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(3\sqrt{1+e^{2i(c+dx)}}+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{15de^4}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

output
```
((-1/15*I)*Sqrt[2]*a^2*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(3/
2)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(3*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(d*e^4)
```

3.197.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3977, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{a^2\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2a^2E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/
2))
```

3.197.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.197.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(99) = 198.

Time = 13.22 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.81

method	result
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}+2\right)a^{2}\sqrt{2}}{5d\,e^{2}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\left(-2iE\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+\right.\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}}}{5d\,e^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+}}}$
default	$\frac{2ia^{2}\left(\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)-\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\right.}}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I*(exp(I*(d*x+c))^2+2)/d*a^2*2^(1/2)/e^2/(e*exp(I*(d*x+c))/(exp(I*(d*
x+c))^2+1))^(1/2)-1/5*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*
exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp
(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I
*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2)
)+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*a^2*2^(1/2)/e^2
/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp
(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

3.197.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx=\frac{2i\sqrt{2}a^{2}\sqrt{e}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{}}{5\,de^{3}}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*I*sqrt(2)*a^2*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(3*I*d*x + 3*I*c) - I*a^2*e^(I*
d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*
e^3)
```

3.197.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = -a^2\left(\int \left(-\frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int \frac{\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{2i\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(5/2),x)
```

output
```
-a**2*(Integral(-1/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**2/
(e*sec(c + d*x))**(5/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))*
*(5/2), x))
```

3.197.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

3.197.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input

```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2),x)
```

output

```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2), x)
```

3.198 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx$

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3.198.1 Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7de^4} + \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

output
```
2/7*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)+2/7*a^2*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-4/7*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(
d*x+c))^(7/2)
```

3.198.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a^2\sqrt{e\sec(c+dx)}\Big(-2i-2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\big(\frac{1}{2}(c+}{7d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(a^2*Sqrt[e*Sec[c + d*x]]*(-2*I - (2*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) -
Sin[2*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(7*d*e^4*(C
os[d*x] + I*Sin[d*x])^2)
```

3.198.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{3a^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{3a^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3a^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3042$$

$$\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3120$$

$$\frac{3a^2\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(3*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2) -
(((4*I)/7)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))
```

3.198.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.198.4 Maple [A] (verified)

Time = 11.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.48

method	result
default	$\frac{2a^2 (-2i(\cos^3(dx+c))+iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2(\cos^2(dx+c))\sin(dx+c)+i\sec(dx+c)F(i(}{7e^3d\sqrt{e\sec(dx+c)}}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}(\mathrm{e}^{2i(dx+c)}+3)a^2\sqrt{2}}{14d\,e^3\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)}\sqrt{i(\mathrm{e}^{i(dx+c)}-i)}\sqrt{i\mathrm{e}^{i(dx+c)}}F(\sqrt{-i(\mathrm{e}^{i(dx+c)}+i)},\frac{\sqrt{2}}{2})a^2\sqrt{e\,\mathrm{e}^{i(dx+c)}(\mathrm{e}^{2i(d}}{7d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e^3(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^2(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}}{21d\sqrt{e\sec(dx+c)}\,e^3}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*a^2/e^3/d/(e*sec(d*x+c))^(1/2)*(-2*I*cos(d*x+c)^3+I*EllipticF(I*(-csc(
d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)+2*cos(d*x+c)^2*sin(d*x+c)+I*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin
(d*x+c))
```

3.198.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{-4i\,\sqrt{2}a^2\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-4i\right.}{14\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/14*(-4*I*sqrt(2)*a^2*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c) - 3*I
*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

3.198.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = -a^2\left(\int\left(-\frac{1}{(e\sec{(c+dx)})^{\frac{7}{2}}}\right)dx + \int\frac{\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{7}{2}}}\,dx + \int\left(-\frac{2i\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{7}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(7/2),x)
```

output
```
-a**2*(Integral(-1/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**2/
(e*sec(c + d*x))**(7/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))*
*(7/2), x))
```

3.198.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

3.198.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2), x)
```

3.199 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx$

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3.199.1 Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}} - \frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

output

```
2/9*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(3/2)+2/3*a^2*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^4/co
s(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/9*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(
d*x+c))^(9/2)
```

3.199.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{ia^2\left(9-4e^{2i(c+dx)}-e^{4i(c+dx)}-\frac{8e^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)}{18\sqrt{2}de^4\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

output
```
((I/18)*a^2*(9 - 4*E^((2*I)*(c + d*x)) - E^((4*I)*(c + d*x)) - (8*E^((2*I)
*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1
 + E^((2*I)*(c + d*x))]))/(Sqrt[2]*d*e^4*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^(
(2*I)*(c + d*x)))])
```

3.199.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{5a^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5a^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

output

```
(5*a^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^2)
- (((4*I)/9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))
```

3.199.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.199.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(126) = 252$.

Time = 19.86 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.91

method	result
risch	$-\frac{i(\mathrm{e}^{4i(dx+c)}+4\,\mathrm{e}^{2i(dx+c)}+15)a^2\sqrt{2}}{36d\,e^4\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\left(-2iE\left(\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e}}\right.}{3d\,e^4\left(\mathrm{e}^{2i(dx+c)}+1\right)}$
default	$-\frac{2ia^2\left(2\left(\cos^5(dx+c)\right)+3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)-3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\csc(dx+\right.}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/36*I*(exp(I*(d*x+c))^4+4*exp(I*(d*x+c))^2+15)/d*a^2*2^(1/2)/e^4/(e*exp(
I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-1/3*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/
e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^
(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp
(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+
I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/
2))))*a^2*2^(1/2)/e^4/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c
))^2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

3.199.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(24i\sqrt{2}a^2\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx}\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/36*(24*I*sqrt(2)*a^2*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(6*I*d*x + 6*
I*c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) + 5*I*a^2*e^(2*I*d*x + 2*I*c) + 9*I*a^2
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I
*c)/(d*e^5)
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.199.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

3.199.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^2}{\left(e\sec\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2), x)
```

3.200 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$

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3.200.1 Optimal result

Integrand size = 28, antiderivative size = 147

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx &= \frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^6}\\ &\quad+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\end{aligned}$$

output
```
2/11*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(5/2)+10/33*a^2*sin(d*x+c)/d/e^5/
(e*sec(d*x+c))^(1/2)+10/33*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))
^(1/2)/d/e^6-4/11*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(11/2)
```

3.200.2 Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^2\sqrt{e\sec(c+dx)}\Big(-28i-24i\cos(2(c+dx))+4i\cos(4(c+dx))+40\sqrt{\cos(c}}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(a^2*Sqrt[e*Sec[c + d*x]]*(-28*I - (24*I)*Cos[2*(c + d*x)] + (4*I)*Cos[4*(
c + d*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*
x)] - I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)])*(Cos[
2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(132*d*e^6*(Cos[d*x] + I*Sin[d*x])
^2)
```

3.200.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{7a^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7a^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{7a^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

3.200. $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$

output
```
(7*a^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*S
in[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2) - (((4*I)/1
1)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))
```

3.200.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.200. $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$

3.200.4 Maple [A] (verified)

Time = 20.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.29

method	result
default	$\frac{2a^2\left(-6i(\cos^5(dx+c))+6\sin(dx+c)(\cos^4(dx+c))+5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+5i\sec(dx+c)F}{33e^5d\sqrt{e\sec(dx+c)}}$
parts	$-\frac{2a^2\left(-7\sin(dx+c)(\cos^4(dx+c))+15iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F}{77d\sqrt{e\sec(dx+c)}\,e^5}$

input
```
int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/33*a^2/e^5/d/(e*sec(d*x+c))^(1/2)*(-6*I*cos(d*x+c)^5+6*sin(d*x+c)*cos(d*
x+c)^4+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2
)+3*cos(d*x+c)^2*sin(d*x+c)+5*sin(d*x+c))
```

3.200.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\left(-80i\,\sqrt{2}a^2\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,a^2e^{}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="fricas"
)
```

output
```
1/264*(-80*I*sqrt(2)*a^2*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-
4, 0, e^(I*d*x + I*c)) + sqrt(2)*(-3*I*a^2*e^(8*I*d*x + 8*I*c) - 18*I*a^2*
e^(6*I*d*x + 6*I*c) - 56*I*a^2*e^(4*I*d*x + 4*I*c) - 30*I*a^2*e^(2*I*d*x +
 2*I*c) + 11*I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))*e^(-2*I*d*x - 2*I*c)/(d*e^6)
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

3.200.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^2}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

3.200.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^2}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

3.200.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input

```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2),x)
```

output

```
int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2), x)
```

3.201 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

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3.201.2 Mathematica [C] (verified) 1528
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3.201.1 Optimal result

Integrand size = 28, antiderivative size = 202

$$\begin{aligned}\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx &= -\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &+\frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} \\ &+\frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d}+\frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\ &+\frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}\end{aligned}$$

output
```
10/21*I*a^3*(e*sec(d*x+c))^(7/2)/d+2/3*a^3*e*(e*sec(d*x+c))^(5/2)*sin(d*x+
c)/d-2*a^3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2*a^3*e
^3*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/11*I*a*(e*sec(d*x+c))^(7/2)*(a+I*a*
tan(d*x+c))^2/d+10/33*I*(e*sec(d*x+c))^(7/2)*(a^3+I*a^3*tan(d*x+c))/d
```

3.201.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.84 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{a^3e^3\sec^4(c+dx)\sqrt{e\sec(c+dx)}\Big(-908\cos(c+dx)-858\cos(3(c+dx))-154\cos(5(c+dx))+\frac{77}{2}e^{-5i(c+}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/1848*(a^3*e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-908*Cos[c + d*x] -
858*Cos[3*(c + d*x)] - 154*Cos[5*(c + d*x)] + (77*(1 + E^((2*I)*(c + d*x))
)^(11/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/(2*E^((5*
I)*(c + d*x))) - (38*I)*Sin[c + d*x] - (451*I)*Sin[3*(c + d*x)] - (77*I)*S
in[5*(c + d*x)])*(-I + Tan[c + d*x]))/d
```

3.201.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{15}{11}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{11}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{15}{11}a\left(\frac{11}{9}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{7/2}}{9d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{11}a\left(\frac{11}{9}a\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{7/2}}{9d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\int (e\sec(c+dx))^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{7/2}}{9d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\left(e\sec(c+dx)\right)^{7}}{9d}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)+2i\left(a^2\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{11}a\left(\frac{11}{9}a\left(a\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}\right)\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{4255}$$

3.201. $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

$$\frac{15}{11}a\Bigg(\frac{11}{9}a\Bigg(a\Bigg(\frac{3}{5}e^2\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\Bigg)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow 3042$$

$$\frac{15}{11}a\Bigg(\frac{11}{9}a\Bigg(a\Bigg(\frac{3}{5}e^2\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\Bigg)+\frac{2e\sin(c+dx)(e\sec(c+d}{5d}$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow 4258$$

$$\frac{15}{11}a\Bigg(\frac{11}{9}a\Bigg(a\Bigg(\frac{3}{5}e^2\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\Bigg)+\frac{2e\sin(c+dx)(e\sec(c+d}{5d}$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow 3042$$

$$\frac{15}{11}a\Bigg(\frac{11}{9}a\Bigg(a\Bigg(\frac{3}{5}e^2\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\Bigg)+\frac{2e\sin(c+dx)(e\sec(c+}{5d}$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow 3119$$

$$\frac{15}{11}a\Bigg(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}{9d}+\frac{11}{9}a\Bigg(a\Bigg(\frac{3}{5}e^2\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(\right.}{d\sqrt{\cos(c+dx)}}$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

3.201. $\int(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

output
```
(((2*I)/11)*a*(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2)/d + (15*a*(
(11*a*((((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + a*((2*e*(e*Sec[c + d*x])^(
5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*S
qrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c
+ d*x])/d))/5)))/9 + (((2*I)/9)*(e*Sec[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c
+ d*x]))/d))/11
```

3.201.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.201. $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

3.201.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(201) = 402$.

Time = 11.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.42

$$-\frac{2ie^3a^3\sqrt{e\sec\left(dx+c\right)}\left(231(\cos^2\left(dx+c\right))\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(\csc\left(dx+c\right)-\cot\left(dx+c\right)),i)-\right.}{}$$

input
```
int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
-2/231*I*e^3*a^3/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(231*cos(d*x+c)^2*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc
(d*x+c)-cot(d*x+c)),I)-231*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+462*cos(d*
x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(
I*(csc(d*x+c)-cot(d*x+c)),I)-462*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+231*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(
d*x+c)-cot(d*x+c)),I)-231*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-231*I*sin(d*x+c)*tan(d*x
+c)^2-77*I*tan(d*x+c)^3-308*I*tan(d*x+c)^3*sec(d*x+c)+231*I*tan(d*x+c)*sec
(d*x+c)^3-132*sec(d*x+c)^2-132*sec(d*x+c)^3+21*sec(d*x+c)^4+21*sec(d*x+c)^
5)
```

3.201.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.56

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(231i\,a^3e^3e^{(11i\,dx+11i\,c)}+1309i\,a^3e^3e^{(9i\,dx+9i\,c)}+946i\,a^3e^3e^{(7i\,dx+7i\,c)}+870i\,a^3e^3e^{(5i\,dx+5i\,c)}+407i\,a^3e\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
-2/231*(sqrt(2)*(231*I*a^3*e^3*e^(11*I*d*x + 11*I*c) + 1309*I*a^3*e^3*e^(9
*I*d*x + 9*I*c) + 946*I*a^3*e^3*e^(7*I*d*x + 7*I*c) + 870*I*a^3*e^3*e^(5*I
*d*x + 5*I*c) + 407*I*a^3*e^3*e^(3*I*d*x + 3*I*c) + 77*I*a^3*e^3*e^(I*d*x
+ I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sq
rt(2)*(I*a^3*e^3*e^(10*I*d*x + 10*I*c) + 5*I*a^3*e^3*e^(8*I*d*x + 8*I*c) +
 10*I*a^3*e^3*e^(6*I*d*x + 6*I*c) + 10*I*a^3*e^3*e^(4*I*d*x + 4*I*c) + 5*I
*a^3*e^3*e^(2*I*d*x + 2*I*c) + I*a^3*e^3)*sqrt(e)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(10*I*d*x + 10*I*c) + 5*
d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c
) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx=\text{Timed out}$$

input

```
integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**3,x)
```

output

```
Timed out
```

3.201.7 Maxima [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx=\int (e\sec{(dx+c)})^{\frac{7}{2}}\,(i\,a\tan{(dx+c)}+a)^3\,dx$$

input

```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.201.8 Giac [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx=\int (e\sec(dx+c))^{\frac{7}{2}}\left(ia\tan(dx+c)+a\right)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^3\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.202 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx$

3.202.1 Optimal result 1535
3.202.2 Mathematica [A] (verified) 1536
3.202.3 Rubi [A] (verified) 1536
3.202.4 Maple [A] (verified) 1539
3.202.5 Fricas [C] (verification not implemented) 1540
3.202.6 Sympy [F] 1540
3.202.7 Maxima [F] 1541
3.202.8 Giac [F] 1541
3.202.9 Mupad [F(-1)] 1541

3.202.1 Optimal result

Integrand size = 28, antiderivative size = 175

$$\begin{aligned}\int (e\sec(c+dx))^{5/2}(a&+ia\tan(c+dx))^3\,dx=\frac{26a^3e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21d}\\&+\frac{26ia^3(e\sec(c+dx))^{5/2}}{35d}+\frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d}\\&+\frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d}\\&+\frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d}\end{aligned}$$

output

```
26/35*I*a^3*(e*sec(d*x+c))^(5/2)/d+26/21*a^3*e*(e*sec(d*x+c))^(3/2)*sin(d*
x+c)/d+26/21*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d+2
/9*I*a*(e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^2/d+26/63*I*(e*sec(d*x+c))^
(5/2)*(a^3+I*a^3*tan(d*x+c))/d
```

3.202.2 Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.51

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^2(c+dx)(e\sec(c+dx))^{5/2}\left(728i+1008i\cos(2(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)\,\mathrm{El}\right.}{1260d}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(728*I + (1008*I)*Cos[2*(c + d*
x)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2] - 150*Sin[2*(c + d
*x)] + 195*Sin[4*(c + d*x)]))/(1260*d)
```

3.202.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \; 3979$$

$$\frac{13}{9}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{13}{9}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \; 3979$$

$$\frac{13}{9}a\left(\frac{9}{7}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\int (e\sec(c+dx))^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/}}{7d}\right. \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i\left(a^2+i\right.}{}\right. \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2i(}{}\right. \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \text{ 4258}$$

3.202. $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\,\mathrm{se}}{}\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{13}{9}a\left(\frac{9}{7}a\left(a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+2\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

$$\Big\downarrow \; 3120$$

$$\frac{13}{9}a\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}{7d}+\frac{9}{7}a\left(a\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+d}}{3d}\right.\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/9)*a*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2)/d + (13*a*((
9*a*((((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + a*((2*e^2*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(3*d))))/7 + (((2*I)/7)*(e*Sec[c + d*x])^(5/2)*(a
^2 + I*a^2*Tan[c + d*x]))/d))/9
```

3.202.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.202.4 Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$-\frac{2ie^2a^3\sqrt{e\sec\left(dx+c\right)}\,\Bigl(195F\left(i\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right),i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos\left(dx+c\right)+1}{}$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x)
```

output
```
-2/315*I*e^2*a^3/d*(e*sec(d*x+c))^(1/2)*(195*EllipticF(I*(csc(d*x+c)-cot(d
*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)+195*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+195*I*tan(d*x+c)-252*sec(d*x+c)^2-135*I*
sec(d*x+c)^2*tan(d*x+c)+35*sec(d*x+c)^4)
```

3.202.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}(195i\,a^3e^2e^{(8i\,dx+8i\,c)} - 1158i\,a^3e^2e^{(6i\,dx+6i\,c)} - 1456i\,a^3e^2e^{(4i\,dx+4i\,c)} - 858i\,a^3e^2e^{(2i\,dx+2i\,c)} - 195i\,a^3e^2}{315\,(d}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/315*(sqrt(2)*(195*I*a^3*e^2*e^(8*I*d*x + 8*I*c) - 1158*I*a^3*e^2*e^(6*I
*d*x + 6*I*c) - 1456*I*a^3*e^2*e^(4*I*d*x + 4*I*c) - 858*I*a^3*e^2*e^(2*I*
d*x + 2*I*c) - 195*I*a^3*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c) + 195*sqrt(2)*(I*a^3*e^2*e^(8*I*d*x + 8*I*c) + 4*I*a^3*e^2*e
^(6*I*d*x + 6*I*c) + 6*I*a^3*e^2*e^(4*I*d*x + 4*I*c) + 4*I*a^3*e^2*e^(2*I*
d*x + 2*I*c) + I*a^3*e^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*
c)))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4
*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.202.6 Sympy [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-ia^3\left(\int i(e\sec{(c+dx)})^{\frac{5}{2}}\,dx + \int \left(-3(e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}\right)\,dx\right.$$
$$\left. + \int (e\sec{(c+dx)})^{\frac{5}{2}}\tan^3{(c+dx)}\,dx + \int \left(-3i(e\sec{(c+dx)})^{\frac{5}{2}}\tan^2{(c+dx)}\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*(e*sec(c + d*x))**(5/2), x) + Integral(-3*(e*sec(c + d
*x))**(5/2)*tan(c + d*x), x) + Integral((e*sec(c + d*x))**(5/2)*tan(c + d*
x)**3, x) + Integral(-3*I*(e*sec(c + d*x))**(5/2)*tan(c + d*x)**2, x))
```

3.202.7 Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.202.8 Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\ 1\mathrm{i})^3\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.203 $\int (e\sec (c+dx))^{3/2}(a+ia\tan (c+dx))^3\,dx$

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3.203.1 Optimal result

Integrand size = 28, antiderivative size = 175

$$\begin{aligned}\int (e\sec (c+dx))^{3/2}(a+ia\tan (c+dx))^3\,dx= &-\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos (c+dx)}\sqrt{e\sec (c+dx)}}+\frac{22ia^3(e\sec (c+dx))^{3/2}}{15d}\\ &+\frac{22a^3e\sqrt{e\sec (c+dx)}\sin (c+dx)}{5d}+\frac{2ia(e\sec (c+dx))^{3/2}(a+ia\tan (c+dx))^2}{7d}\\ &+\frac{22i(e\sec (c+dx))^{3/2}\left(a^3+ia^3\tan (c+dx)\right)}{35d}\end{aligned}$$

output
```
22/15*I*a^3*(e*sec(d*x+c))^(3/2)/d-22/5*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(
1/2)/(e*sec(d*x+c))^(1/2)+22/5*a^3*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/7
*I*a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2/d+22/35*I*(e*sec(d*x+c))^(3
/2)*(a^3+I*a^3*tan(d*x+c))/d
```

3.203.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \frac{a^3(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))\big(-116i-308i\cos(2(c+dx))+77ie^{-2i(c+dx)}(}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])*(-116*I - (308*I)*Cos[2*(
c + d*x)] + ((77*I)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 77*Sec[c + d*x]*Si
n[3*(c + d*x)] + 17*Tan[c + d*x]))/(210*d)
```

3.203.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{3979}$$

$$\begin{gathered}\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)+\\ \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\end{gathered}$$

$$\Big\downarrow \text{3042}$$

$$\begin{gathered}\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)+\\ \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\end{gathered}$$

$$\Big\downarrow \text{3967}$$

$$\begin{gathered}\frac{11}{7}a\left(\frac{7}{5}a\left(a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)+\\ \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\end{gathered}$$

$$\Big\downarrow \text{3042}$$

$$\begin{gathered}\frac{11}{7}a\left(\frac{7}{5}a\left(a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right.\\ \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\end{gathered}$$

$$\Big\downarrow \text{4255}$$

$$\begin{gathered}\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+2i\left(a^2+ia^2\right.\right.\\ \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\end{gathered}$$

$$\Big\downarrow \text{3042}$$

$$\begin{gathered}\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+2i\left(a^2-\right.\right.\\ \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\end{gathered}$$

$$\Big\downarrow \text{4258}$$

3.203. $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx$

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{7}a\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{11}{7}a\left(\frac{2i(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\right.}{d\sqrt{\cos(c+dx)}\sqrt{e\sec}}$$

$$\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2)/d + (11*a*((
7*a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*
x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c
+ d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*
a^2*Tan[c + d*x]))/d))/7
```

3.203.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.203.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(174) = 348.

Time = 16.21 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.58

method	result
default	$-\frac{2ea^3\sqrt{e\sec(dx+c)}\left(231i(\cos^2(dx+c))F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-231i(\cos^2(dx+c))E(i(-\mathrm{cs}}{}$
parts	Expression too large to display

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/105*e*a^3/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(231*I*cos(d*x+c)^2*Ell
ipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)-231*I*cos(d*x+c)^2*EllipticE(I*(-csc(d*x+c)+cot(d*x+
c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+462*I*co
s(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-462*I*cos(d*x+c)*EllipticE(I*(-csc(d*x+c
)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)+231*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-231*I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-140*I-231*
sin(d*x+c)-140*I*sec(d*x+c)+63*tan(d*x+c)+15*I*sec(d*x+c)^2+63*sec(d*x+c)*
tan(d*x+c)+15*I*sec(d*x+c)^3)
```

3.203.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(231i\,a^3ee^{(7i\,dx+7i\,c)}+287i\,a^3ee^{(5i\,dx+5i\,c)}+253i\,a^3ee^{(3i\,dx+3i\,c)}+77i\,a^3ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{(\frac{1}{2}i\,dx}}{105\,(de^{(}}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/105*(sqrt(2)*(231*I*a^3*e*e^(7*I*d*x + 7*I*c) + 287*I*a^3*e*e^(5*I*d*x
+ 5*I*c) + 253*I*a^3*e*e^(3*I*d*x + 3*I*c) + 77*I*a^3*e*e^(I*d*x + I*c))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(I*
a^3*e*e^(6*I*d*x + 6*I*c) + 3*I*a^3*e*e^(4*I*d*x + 4*I*c) + 3*I*a^3*e*e^(2
*I*d*x + 2*I*c) + I*a^3*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, e^(I*d*x + I*c))))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*
I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.203.6 Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i(e\sec(c+dx))^{\frac{3}{2}}\,dx + \int\left(-3(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\right)\,dx + \int (e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)\,dx + \int\left(-3i(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*(e*sec(c + d*x))**(3/2), x) + Integral(-3*(e*sec(c + d
*x))**(3/2)*tan(c + d*x), x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*
x)**3, x) + Integral(-3*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**2, x))
```

3.203.7 Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.203.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx=\int\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\;1\mathrm{i})^3\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.204 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$

3.204.1 Optimal result 1550
3.204.2 Mathematica [A] (verified) 1550
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3.204.8 Giac [F(-2)] 1555
3.204.9 Mupad [F(-1)] 1556

3.204.1 Optimal result

Integrand size = 28, antiderivative size = 139

$$\begin{aligned}\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3\,dx &= \frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}\\&\quad+\frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d}+\frac{6i\sqrt{e\sec(c+dx)}(a^3+ia^3\tan(c+dx))}{5d}\end{aligned}$$

output
```
6*I*a^3*(e*sec(d*x+c))^(1/2)/d+6*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d
*x+c))^(1/2)/d+2/5*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2/d+6/5*I*(
e*sec(d*x+c))^(1/2)*(a^3+I*a^3*tan(d*x+c))/d
```

3.204.2 Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(18i+20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-5\sin\right.}{5d}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(a^3*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(18*I + (20*I)*Cos[2*(c + d*x)] +
 30*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5
*d)
```

3.204.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3979, 3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3967}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{}}{3d}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+d\right.}{3d}\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{9}{5}a\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2}{}\right.\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

3.204. $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$

output
```
(((2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2)/d + (9*a*((5*a
*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c + d*x]]
*(a^2 + I*a^2*Tan[c + d*x]))/d))/5
```

3.204.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.204.4 Maple [A] (verified)

Time = 14.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

method	result
default	$\frac{2a^3\sqrt{e\sec(dx+c)}\left(15i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15iF(i(-\csc(dx+c)+\cot(dx+c)),i\right.}{5d}$
parts	$-\frac{2ia^3(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)}{d}-\frac{ia^3\sqrt{e\sec(dx+c)}\left(5\cos(dx+c\right.}{}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*a^3/d*(e*sec(d*x+c))^(1/2)*(15*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)+c
ot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+1
5*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)+20*I-5*tan(d*x+c)-I*sec(d*x+c)^2)
```

3.204.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(-25i\,a^3e^{(4i\,dx+4i\,c)}-36i\,a^3e^{(2i\,dx+2i\,c)}-15i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(i\,a^3e^{(4i\,dx+4i\,c)}+\right.\right.}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/5*(sqrt(2)*(-25*I*a^3*e^(4*I*d*x + 4*I*c) - 36*I*a^3*e^(2*I*d*x + 2*I*c
) - 15*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) +
15*sqrt(2)*(I*a^3*e^(4*I*d*x + 4*I*c) + 2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^
3)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(4*I*d*x + 4*
I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.204.6 Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sqrt{e\sec(c+dx)}\,dx + \int\left(-3\sqrt{e\sec(c+dx)}\tan(c+dx)\right)\,dx + \int\sqrt{e\sec(c+dx)}\tan^3(c+dx)\,dx + \int\left(-3i\sqrt{e\sec(c+dx)}\tan^2(c+dx)\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*sqrt(e*sec(c + d*x)), x) + Integral(-3*sqrt(e*sec(c +
d*x))*tan(c + d*x), x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**3, x)
 + Integral(-3*I*sqrt(e*sec(c + d*x))*tan(c + d*x)**2, x))
```

3.204.7 Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \int \sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^3\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3, x)
```

3.204.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)}^3\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.205 $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$

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3.205.2 Mathematica [C] (verified) 1557
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3.205.1 Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{26ia^3}{3d\sqrt{e\sec(c+dx)}} + \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6a^3\tan(c+dx)}{d\sqrt{e\sec(c+dx)}} - \frac{2ia^3\tan^2(c+dx)}{3d\sqrt{e\sec(c+dx)}}$$

output

```
-26/3*I*a^3/d/(e*sec(d*x+c))^(1/2)+14*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(
e*sec(d*x+c))^(1/2)-6*a^3*tan(d*x+c)/d/(e*sec(d*x+c))^(1/2)-2/3*I*a^3*tan(
d*x+c)^2/d/(e*sec(d*x+c))^(1/2)
```

3.205.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$$

$$= \frac{2a^3\sqrt{e\sec(c+dx)}(\cos(c)+i\sin(c))(-i\cos(dx)+\sin(dx))\,(-8+7\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\,(}{3de}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(2*a^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*((-I)*Cos[d*x] + Sin[d*x])
*(-8 + 7*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))] - I*Tan[c + d*x]))/(3*d*e)
```

3.205.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3979, 3042, 3977, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{3}a\int \frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\int \frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{7}{3}a\left(-\frac{3a^2\int (e\sec(c+dx))^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\left(-\frac{3a^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7}{3}a\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

3.205. $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$

output
```
(((2*I)/3)*a*(a + I*a*Tan[c + d*x])^2)/(d*Sqrt[e*Sec[c + d*x]]) + (7*a*((-
3*a^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec
[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/e^2 - ((4*I)*(a^
2 + I*a^2*Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])))/3
```

3.205.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*(m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

3.205. $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.205.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1113 vs. 2(134) = 268.

Time = 12.23 (sec) , antiderivative size = 1114, normalized size of antiderivative = 8.98

method	result	size
parts	Expression too large to display	1114
default	Expression too large to display	1306

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*EllipticE(I*(csc(d*x+c)-cot
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos
(d*x+c)-I*cos(d*x+c)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-2
*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-I*sec(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)+sin(d*x+c))+1/6*I*a^3/d/(cos(d*x+c)+1)/(e*sec(d
*x+c))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(-12*cos(d*x+c)*(-cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)+3*ln((2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1
)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c
)+1))-3*ln(2*(2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-12*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)-4*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
-4*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))-6*I*a^3/(e*sec(d*x+c
))^(1/2)/d+6*a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(2*I*EllipticE(I*(c
sc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)-2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos...
```

3.205.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2\left(\sqrt{2}\left(-9i\,a^3e^{(3i\,dx+3i\,c)}-7i\,a^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\sqrt{2}\left(-i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\sqrt{e}\mathrm{we}}{3\left(dee^{(2i\,dx+2i\,c)}+de\right)}$$

input

```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
-2/3*(sqrt(2)*(-9*I*a^3*e^(3*I*d*x + 3*I*c) - 7*I*a^3*e^(I*d*x + I*c))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(-I*a^
3*e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, e^(I*d*x + I*c))))/(d*e*e^(2*I*d*x + 2*I*c) + d*e)
```

3.205.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -ia^3\left(\int \frac{i}{\sqrt{e\sec(c+dx)}}\,dx + \int \left(-\frac{3\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx + \int \frac{\tan^3(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int \left(-\frac{3i\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

input

```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(1/2),x)
```

output

```
-I*a**3*(Integral(I/sqrt(e*sec(c + d*x)), x) + Integral(-3*tan(c + d*x)/sq
rt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**3/sqrt(e*sec(c + d*x)), x)
 + Integral(-3*I*tan(c + d*x)**2/sqrt(e*sec(c + d*x)), x))
```

3.205.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

3.205.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^3}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2), x)
```

3.206 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx$

3.206.1 Optimal result 1564
3.206.2 Mathematica [A] (verified) 1564
3.206.3 Rubi [A] (verified) 1565
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3.206.7 Maxima [F] 1568
3.206.8 Giac [F] 1569
3.206.9 Mupad [F(-1)] 1569

3.206.1 Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

output
```
-10/3*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^2-10/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*tan(d*x+c))^2/d/(e*sec(d*x+c)
)^(3/2)
```

3.206.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{2a^3\sec^2(c+dx)\left(7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\sin(c+dx))+\right.}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-2*a^3*Sec[c + d*x]^2*((7*I)*Cos[c + d*x] + 5*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 3*Sin[c + d*x])*(Cos[c
 + 4*d*x] + I*Sin[c + 4*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*S
in[d*x])^3)
```

3.206.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3967}$$

$$-\frac{5a^2\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{5a^2\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{5a^2\left(\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(-5*a^2*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/(3*e^2) - (((4*I)/3)*a*(a +
I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(3/2))
```

3.206.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.206.4 Maple [A] (verified)

Time = 12.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.49

method	result
default	$-\frac{2a^3\left(5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+5i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+}}\right.}{3ed\sqrt{e\sec(dx+c)}}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^3/e/d/(e*sec(d*x+c))^(1/2)*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+5*I*sec(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(1/(cos(d*x+c)+1))^(1/2)+4*I*cos(d*x+c)-4*sin(d*x+c)+3*I*sec(d*x+c))
```

3.206.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(-5i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(2i\,a^3e^{(2i\,dx+2i\,c)}+5i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\right.}\right.}{3\,de^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(-5*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(2*I*a^3*e^(2*I*d*x + 2*I*c) + 5*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^2)
```

3.206.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx + \int \left(-\frac{3\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx + \int \frac{\tan^3{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx + \int \left(-\frac{3i\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(3/2),x)
```

output
```
-I*a**3*(Integral(I/(e*sec(c + d*x))**(3/2), x) + Integral(-3*tan(c + d*x)
/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**
(3/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(3/2), x))
```

3.206.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

3.206.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2), x)
```

3.207 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx$

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3.207.1 Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx=\frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

output
```
6/5*I*a^3/d/e^2/(e*sec(d*x+c))^(1/2)-6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^
(1/2)/(e*sec(d*x+c))^(1/2)-4/5*I*a*(a+I*a*tan(d*x+c))^2/d/(e*sec(d*x+c))^(
5/2)
```

3.207.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{4ia^3e^{2i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-4*I)/5)*a^3*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
)/(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

3.207.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3977}$$

$$-\frac{3a^2\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3a^2\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3967}$$

$$-\frac{3a^2\left(a\int \frac{1}{\sqrt{e\sec(c+dx)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3a^2\left(a\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{3a^2\left(\frac{a\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\left(\frac{a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3a^2\left(\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(-3*a^2*(((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2,
 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])))/(5*e^2) - (((4*I)/5)*a*
(a + I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(5/2))
```

3.207.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.207.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(120) = 240$.

Time = 14.92 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.92

method	result
risch	$-\frac{2i\left(\mathrm{e}^{2i(dx+c)}-3\right)a^{3}\sqrt{2}}{5d\,e^{2}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{3i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\left(-2iE\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c}\right.}\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}}\right.}{5d\,e^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c}}{\mathrm{e}^{2i(dx+c)}}}}$
default	Expression too large to display
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I*(exp(I*(d*x+c))^2-3)/d*a^3*2^(1/2)/e^2/(e*exp(I*(d*x+c))/(exp(I*(d*
x+c))^2+1))^(1/2)+3/5*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*
exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp
(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I
*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2)
)+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*a^3*2^(1/2)/e^2
/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp
(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

3.207.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(3i\sqrt{2}a^3\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(i\,a^3e^{(3i\,dx+3i\,c)}+i\,a^3e^{(i\,d}\right.\right.}{5\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(3*I*sqrt(2)*a^3*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, e^(I*d*x + I*c))) + sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) + I*a^3*e^(I*
d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*
e^3)
```

3.207.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{3\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int \frac{\tan^3{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{3i\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(5/2),x)
```

output
```
-I*a**3*(Integral(I/(e*sec(c + d*x))**(5/2), x) + Integral(-3*tan(c + d*x)
/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**
(5/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(5/2), x))
```

3.207.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

3.207.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2), x)
```

3.208 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx$

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3.208.1 Optimal result

Integrand size = 28, antiderivative size = 124

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx =&\\ &-\frac{2a^3\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4}\\ &-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}\end{aligned}$$

output
```
-2/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-2/7*I*(a
+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)-4/21*I*(a^3+I*a^3*tan(d*x+c))/d/
e^2/(e*sec(d*x+c))^(3/2)
```

3.208.2 Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx =$$

$$-\frac{a^3\sqrt{e\sec(c+dx)}\Big(5i+5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))-i\sin}{21de^4(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

output
```
-1/21*(a^3*Sqrt[e*Sec[c + d*x]]*(5*I + (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)
]) - Sin[2*(c + d*x)])*(Cos[2*c + 5*d*x] + I*Sin[2*c + 5*d*x]))/(d*e^4*(Co
s[d*x] + I*Sin[d*x])^3)
```

3.208.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3978, 3042, 3977, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a\left(-\frac{a^2\int \sqrt{e\sec(c+dx)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(-\frac{a^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a\left(-\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

output

```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)) + (a*((-2
*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3
*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3/2)
)))/(7*e^2)
```

3.208.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.208.4 Maple [A] (verified)

Time = 13.45 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2a^3\left(12i(\cos^3(dx+c))+iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}-12(\cos^2(dx+c))\sin(dx+c)+i\sec(dx+c)F\right.}{21e^3d\sqrt{e\sec(dx+c)}}$
risch	$-\frac{ie^{i(dx+c)}(3\,e^{2i(dx+c)}+2)a^3\sqrt{2}}{21d\,e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i(e^{i(dx+c)}+i)}\sqrt{i(e^{i(dx+c)}-i)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2}\right)a^3\sqrt{e\,e^{i(dx+c)}\,(e^{2i}}}{21d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^3\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}\right.}{21d\sqrt{e\sec(dx+c)}\,e^3}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*a^3/e^3/d/(e*sec(d*x+c))^(1/2)*(12*I*cos(d*x+c)^3+I*EllipticF(I*(-cs
c(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)-12*cos(d*x+c)^2*sin(d*x+c)+I*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+
cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-
7*I*cos(d*x+c)+sin(d*x+c))
```

3.208.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,a^3e^{(4i\,dx+4i\,c)}-5i}{21\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(2*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
+ sqrt(2)*(-3*I*a^3*e^(4*I*d*x + 4*I*c) - 5*I*a^3*e^(2*I*d*x + 2*I*c) - 2*
I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

3.208.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx + \int \left(-\frac{3\tan(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx + \int \frac{\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx + \int \left(-\frac{3i\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(7/2),x)
```

output
```
-I*a**3*(Integral(I/(e*sec(c + d*x))**(7/2), x) + Integral(-3*tan(c + d*x)
/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**
(7/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(7/2), x))
```

3.208.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

3.208.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2), x)
```

3.209 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx$

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3.209.1 Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx=\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

output

```
2/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-2/9*I*(a+
I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)-4/15*I*(a^3+I*a^3*tan(d*x+c))/d/e
^2/(e*sec(d*x+c))^(5/2)
```

3.209.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.95 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx=$$

$$-\frac{a^3e^{-2i(c+dx)}\Big(11+16e^{2i(c+dx)}+5e^{4i(c+dx)}+4\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\Big)\big(-i+}{90de^2(e\sec(c+dx))^{5/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

output
```
-1/90*(a^3*(11 + 16*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 4*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])*(-I + Tan[c + d*x])^3)/(d*e^2*E^((2*I)*(c + d*x))*(e*Sec[c + d*x])^(
5/2))
```

3.209.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3978, 3042, 3977, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a\left(\frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{a^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\left(\frac{a^2\int\sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{a^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\left(\frac{2a^2E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

output

```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2)) + (a*((2*
a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/2))
))/(3*e^2)
```

3.209.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.209. $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx$

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.209.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(132) = 264.

Time = 14.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.73

method	result
risch	$-\frac{i\left(5\,\mathrm{e}^{4i(dx+c)}+11\,\mathrm{e}^{2i(dx+c)}+12\right)a^{3}\sqrt{2}}{90d\,e^{4}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\left(-2iE\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}}}\right.}{15d\,e^{4}\left(\mathrm{e}^{2i(dx+c)}\right.}$
default	$\frac{2ia^{3}\left(-20i\sin(dx+c)\left(\cos^{4}(dx+c)\right)-20i\left(\cos^{3}(dx+c)\right)\sin(dx+c)-20\left(\cos^{5}(dx+c)\right)+3\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(\mathrm{cs}\right.}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/90*I*(5*exp(I*(d*x+c))^4+11*exp(I*(d*x+c))^2+12)/d*a^3*2^(1/2)/e^4/(e*e
xp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-1/15*I/d*(-2*(e*exp(I*(d*x+c))^2
+e)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+
I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e
*exp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+
c))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2
^(1/2))))*a^3*2^(1/2)/e^4/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d
*x+c))^2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

3.209.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \frac{12i\sqrt{2}a^3\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+}{9}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/90*(12*I*sqrt(2)*a^3*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-5*I*a^3*e^(5*I*d*x + 5*I*c) - 16*I*a^
3*e^(3*I*d*x + 3*I*c) - 11*I*a^3*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^5)
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.209.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{\left(i\,a\tan\left(dx+c\right)+a\right)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

3.209.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2), x)
```

3.210 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx$

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3.210.1 Optimal result

Integrand size = 28, antiderivative size = 155

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx &= \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6}\\ &\quad+\frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}-\frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}\end{aligned}$$

output
```
10/77*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)+10/77*a^3*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(
d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^6-2/11*I*(a+I*a*tan(d*x+c))^3/d/(e*s
ec(d*x+c))^(11/2)-20/77*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(7/2
)
```

3.210.2 Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}\left(-46i\cos(c+dx)-22i\cos(3(c+dx))-15\sin(c+dx)+\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

output

```
(a^3*Sqrt[e*Sec[c + d*x]]*((-46*I)*Cos[c + d*x] - (22*I)*Cos[3*(c + d*x)]
- 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3
*(c + d*x)] - I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)])*(Cos[3*(c + 2*d*x
)] + I*Sin[3*(c + 2*d*x)]))/(154*d*e^6*(Cos[d*x] + I*Sin[d*x])^3)
```

3.210.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3978, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{5a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{5a\left(\frac{3a^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5a\left(\frac{3a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5a\left(\frac{3a^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$\downarrow$ 3042

$$\frac{5a\left(\frac{3a^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$\downarrow$ 4258

$$\frac{5a\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$\downarrow$ 3042

$$\frac{5a\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$\downarrow$ 3120

$$\frac{5a\left(\frac{3a^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

3.210. $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx$

output
```
(((-2*I)/11)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) + (5*a*
((3*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*
x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2) -
 (((4*I)/7)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))))/(11*e
^2)
```

3.210.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.210.4 Maple [A] (verified)

Time = 20.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30

method	result
default	$-\frac{2a^3\left(28i(\cos^5(dx+c))-28\sin(dx+c)(\cos^4(dx+c))+5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-11i(\cos^3(dx+\right.}{77e^5d\sqrt{e\sec(d}}$
risch	$-\frac{ie^{i(dx+c)}(7\,e^{4i(dx+c)}+24\,e^{2i(dx+c)}+37)a^3\sqrt{2}}{308d\,e^5\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i(e^{i(dx+c)}+i)}\sqrt{i(e^{i(dx+c)}-i)}\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2}\right)a^3}{77d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^5\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+}}{e^{2i(dx+c)}}}}$
parts	$-\frac{2a^3\left(-7\sin(dx+c)(\cos^4(dx+c))+15iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F\right.}{77d\sqrt{e\sec(dx+c)}\,e^5}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*a^3/e^5/d/(e*sec(d*x+c))^(1/2)*(28*I*cos(d*x+c)^5-28*sin(d*x+c)*cos(
d*x+c)^4+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-11*I*cos(d*x+c)^3+5*I*sec(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(c
os(d*x+c)+1))^(1/2)-3*cos(d*x+c)^2*sin(d*x+c)-5*sin(d*x+c))
```

3.210.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{-40i\,\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-7i\,a^3e^{(6i\,dx+6i\,c)}-\right.}{308\,de^6}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="fricas"
)
```

output
```
1/308*(-40*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c
)) + sqrt(2)*(-7*I*a^3*e^(6*I*d*x + 6*I*c) - 31*I*a^3*e^(4*I*d*x + 4*I*c)
- 61*I*a^3*e^(2*I*d*x + 2*I*c) - 37*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

3.210.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

3.210.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2), x)
```

3.211 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$

3.211.1 Optimal result 1595
3.211.2 Mathematica [C] (verified) 1595
3.211.3 Rubi [A] (verified) 1596
3.211.4 Maple [B] (verified) 1599
3.211.5 Fricas [C] (verification not implemented) 1599
3.211.6 Sympy [F(-1)] 1600
3.211.7 Maxima [F] 1600
3.211.8 Giac [F] 1600
3.211.9 Mupad [F(-1)] 1601

3.211.1 Optimal result

Integrand size = 28, antiderivative size = 155

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = {} & \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ & + \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}\end{aligned}$$

output
```
14/117*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)+14/39*a^3*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e
^6/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-2/13*I*(a+I*a*tan(d*x+c))^3/d/(e*
sec(d*x+c))^(13/2)-28/117*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(9
/2)
```

3.211.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(62+8\cos(2(c+dx))\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

output

```
(a^3*Sqrt[e*Sec[c + d*x]]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 +
 8*Cos[2*(c + d*x)] - 54*Cos[4*(c + d*x)] + (56*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c +
d*x)) + (42*I)*Sin[2*(c + d*x)] + (63*I)*Sin[4*(c + d*x)]))/(468*d*e^7)
```

3.211.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3978, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}dx$$

$$\Big\downarrow \text{ 3978}$$

$$\frac{7a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{7a\left(\frac{5a^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7a\left(\frac{5a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7a\left(\frac{5a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

output
```
(((-2*I)/13)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) + (7*a*
((5*a^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*
Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^2)
 - (((4*I)/9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))))/(13
*e^2)
```

3.211. $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$

3.211.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.211. $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$

3.211.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(159) = 318$.

Time = 25.77 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{i(9\,e^{6i(dx+c)}+41\,e^{4i(dx+c)}+83\,e^{2i(dx+c)}+219)a^3\sqrt{2}}{936d\,e^6\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{7i\Bigg(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}}{}}{39}$
default	$-\frac{2ia^3\Big(36(\cos^7(dx+c))+36(\cos^6(dx+c))+7i(\cos^2(dx+c))\sin(dx+c)-13(\cos^5(dx+c))+5i\sin(dx+c)(\cos^4(dx+c))+21\sqrt{\frac{\cos(dx+}{\cos(dx+c)}}}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/936*I*(9*exp(I*(d*x+c))^6+41*exp(I*(d*x+c))^4+83*exp(I*(d*x+c))^2+219)/
d*a^3*2^(1/2)/e^6/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-7/39*I/d*(
-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+
I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*ex
p(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*Elli
pticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d
*x+c))+I))^(1/2),1/2*2^(1/2))))*a^3*2^(1/2)/e^6/(exp(I*(d*x+c))^2+1)/(e*ex
p(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))
^2+1))^(1/2)
```

3.211.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{\Big(336i\sqrt{2}a^3\sqrt{e}e^{(i\,dx+i\,c)}\mathrm{weierstrassZeta}\big(-4,0,\mathrm{weierstrassPInverse}\big(-4,0,e^{(i\,d}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="fricas"
)
```

output
```
1/936*(336*I*sqrt(2)*a^3*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^3*e^(8*I*d*x
+ 8*I*c) - 50*I*a^3*e^(6*I*d*x + 6*I*c) - 124*I*a^3*e^(4*I*d*x + 4*I*c) +
34*I*a^3*e^(2*I*d*x + 2*I*c) + 117*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(d*e^7)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(13/2),x)
```

output
```
Timed out
```

3.211.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

3.211.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{13/2}} \,d x$$

input
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2), x)
```

3.212 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$

3.212.1 Optimal result . . . 1602
3.212.2 Mathematica [A] (verified) . . . 1602
3.212.3 Rubi [A] (verified) . . . 1603
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3.212.1 Optimal result

Integrand size = 28, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx &= \frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11de^8} \\ &\quad+\frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}+\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} \\ &\quad-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}-\frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}\end{aligned}$$

output
```
6/55*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/11*a^3*sin(d*x+c)/d/e^7/(
e*sec(d*x+c))^(1/2)+2/11*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(
1/2)/d/e^8-2/15*I*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(15/2)-12/55*I*(a^
3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2)
```

3.212.2 Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}\left(-332i\cos(c+dx)-154i\cos(3(c+dx))+22i\cos(5(c+d\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

output
```
(a^3*Sqrt[e*Sec[c + d*x]]*((-332*I)*Cos[c + d*x] - (154*I)*Cos[3*(c + d*x)
] + (22*I)*Cos[5*(c + d*x)] - 114*Sin[c + d*x] + 240*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 81*Sin[3
*(c + d*x)] + 33*Sin[5*(c + d*x)])*(Cos[3*(c + 2*d*x)] + I*Sin[3*(c + 2*d*
x)]))/(1320*d*e^8*(Cos[d*x] + I*Sin[d*x])^3)
```

3.212.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3978, 3042, 3977, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{3a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{3a\left(\frac{7a^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{7a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \ 4256$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \ 4256$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \ 4258$$

3.212. $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3a\left(\frac{7a^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

3.212. $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$

output
```
(((-2*I)/15)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) + (3*a*
((7*a^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*
Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2) - (((4*I)/
11)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))))/(5*e^2)
```

3.212.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.212.4 Maple [A] (verified)

Time = 24.65 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

method	result
default	$\frac{2a^3(-44i(\cos^7(dx+c))+44\sin(dx+c)(\cos^6(dx+c))+15i(\cos^5(dx+c))+7\sin(dx+c)(\cos^4(dx+c))+15iF(i(-\csc(dx+c)+\cot(dx+c)}{}$
parts	$-\frac{2a^3(-77\sin(dx+c)(\cos^6(dx+c))-91\sin(dx+c)(\cos^4(dx+c))+195iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+}{1155d\sqrt{e\sec(d}}$

input
```
int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

output
```
2/165*a^3/e^7/d/(e*sec(d*x+c))^(1/2)*(-44*I*cos(d*x+c)^7+44*sin(d*x+c)*cos
(d*x+c)^6+15*I*cos(d*x+c)^5+7*sin(d*x+c)*cos(d*x+c)^4+15*I*EllipticF(I*(-c
sc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)+15*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+9*cos(d*x+c)^2*sin(d*x+
c)+15*sin(d*x+c))
```

3.212.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{(-480i\sqrt{2}a^3\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}(-4,0,e^{(i\,dx+i\,c)})+\sqrt{2}(-11i\,a}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="fricas"
)
```

output
```
1/2640*(-480*I*sqrt(2)*a^3*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse
(-4, 0, e^(I*d*x + I*c)) + sqrt(2)*(-11*I*a^3*e^(10*I*d*x + 10*I*c) - 73*I
*a^3*e^(8*I*d*x + 8*I*c) - 218*I*a^3*e^(6*I*d*x + 6*I*c) - 446*I*a^3*e^(4*
I*d*x + 4*I*c) - 235*I*a^3*e^(2*I*d*x + 2*I*c) + 55*I*a^3)*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^8)
```

3.212.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(15/2),x)
```

output
```
Timed out
```

3.212.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

3.212.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\int \frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{15/2}} \,d x$$

input

```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2),x)
```

output

```
int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2), x)
```

3.213 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$

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3.213.1 Optimal result

Integrand size = 28, antiderivative size = 215

$$
\begin{aligned}
\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \\
&-\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} \\
&+\frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} \\
&+\frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} \\
&+\frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d}
\end{aligned}
$$

output

```
22/9*I*a^4*(e*sec(d*x+c))^(3/2)/d-22/3*a^4*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1
/2)/(e*sec(d*x+c))^(1/2)+22/3*a^4*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/9*
I*a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3/d+10/21*I*(e*sec(d*x+c))^(3/
2)*(a^2+I*a^2*tan(d*x+c))^2/d+22/21*I*(e*sec(d*x+c))^(3/2)*(a^4+I*a^4*tan(
d*x+c))/d
```

3.213.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.21 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.52

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \frac{(e\sec(c+dx))^{3/2}\left(\frac{22i\sqrt{2}e^{-i(3c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}\left(-1+e^{2ic}\right)\text{Hype}\right.}{-1+e^{2ic}}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((e*Sec[c + d*x])^(3/2)*(((22*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))]))/(E^(I*(3*c + d*x))*(-1 + E^((2*I)*c))) + (Csc[c]*Se
c[c + d*x]^(9/2)*(Cos[4*c] - I*Sin[4*c])*(1260*Cos[d*x] + 1050*Cos[2*c + d
*x] + 742*Cos[2*c + 3*d*x] + 413*Cos[4*c + 3*d*x] + 231*Cos[4*c + 5*d*x] -
 (720*I)*Sin[d*x] + (720*I)*Sin[2*c + d*x] - (336*I)*Sin[2*c + 3*d*x] + (3
36*I)*Sin[4*c + 3*d*x]))/56)*(a + I*a*Tan[c + d*x])^4)/(9*d*Sec[c + d*x]^(
11/2)*(Cos[d*x] + I*Sin[d*x])^4)
```

3.213.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^4(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^4(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{3}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{3}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3979$$

$$\frac{5}{3}a\left(\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{3}a\left(\frac{11}{7}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}\right)+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3979$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)+2ia(a\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)+2ia(a\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3967$$

$$\frac{5}{3}a\left(\frac{11}{7}a\left(\frac{7}{5}a\left(a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3042$$

3.213. $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$

$$\frac{5}{3}a\Bigg(\frac{11}{7}a\Bigg(\frac{7}{5}a\Bigg(a\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\Bigg)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx}{5d}$$

$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5}{3}a\Bigg(\frac{11}{7}a\Bigg(\frac{7}{5}a\Bigg(a\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\Bigg)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\Bigg)+\frac{2i\left(a^2+\right.}{}$$

$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\Bigg(\frac{11}{7}a\Bigg(\frac{7}{5}a\Bigg(a\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\Bigg)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\Bigg)+\frac{2i}{}$$

$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{3}a\Bigg(\frac{11}{7}a\Bigg(\frac{7}{5}a\Bigg(a\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\Bigg)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\Bigg)+\frac{2i}{}$$

$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}a\Bigg(\frac{11}{7}a\Bigg(\frac{7}{5}a\Bigg(a\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\Bigg)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\Bigg)+\frac{2}{}$$

$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5}{3}a\Bigg(\frac{11}{7}a\Bigg(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\Bigg(a\Bigg(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+\right.}{d\sqrt{\cos(c+dx)}\sqrt{}}$$

$$\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

3.213. $\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/9)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3)/d + (5*a*(((
(2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2)/d + (11*a*((7*
a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x)
/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^
2*Tan[c + d*x]))/d))/7))/3
```

3.213.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.213.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(208) = 416.

Time = 24.52 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2ie\,a^4\sqrt{e\sec(dx+c)}\left(231\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(\csc(dx+c)-\cot(dx+c)),i)-231\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c}}\right.}{}$
parts	Expression too large to display

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/63*I*e*a^4/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(231*cos(d*x+c)^2*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*
x+c)-cot(d*x+c)),I)-231*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+462*cos(d*x+c
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
csc(d*x+c)-cot(d*x+c)),I)-462*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+231*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x
+c)-cot(d*x+c)),I)-231*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-168+231*I*sin(d*x+c)-168*se
c(d*x+c)-91*I*tan(d*x+c)+36*sec(d*x+c)^2-91*I*tan(d*x+c)*sec(d*x+c)+36*sec
(d*x+c)^3+7*I*tan(d*x+c)*sec(d*x+c)^2+7*I*tan(d*x+c)*sec(d*x+c)^3)
```

3.213.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.16

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{2\left(\sqrt{2}(231i\,a^4ee^{(9i\,dx+9i\,c)}+406i\,a^4ee^{(7i\,dx+7i\,c)}+540i\,a^4ee^{(5i\,dx+5i\,c)}+330i\,a^4ee^{(3i\,dx+3i\,c)}+77i\,a^4ee^{(i\,dx+i\,c)}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/63*(sqrt(2)*(231*I*a^4*e*e^(9*I*d*x + 9*I*c) + 406*I*a^4*e*e^(7*I*d*x +
 7*I*c) + 540*I*a^4*e*e^(5*I*d*x + 5*I*c) + 330*I*a^4*e*e^(3*I*d*x + 3*I*c
) + 77*I*a^4*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I
*d*x + 1/2*I*c) + 231*sqrt(2)*(I*a^4*e*e^(8*I*d*x + 8*I*c) + 4*I*a^4*e*e^(
6*I*d*x + 6*I*c) + 6*I*a^4*e*e^(4*I*d*x + 4*I*c) + 4*I*a^4*e*e^(2*I*d*x +
2*I*c) + I*a^4*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, e^(I*d*x + I*c))))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*
d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.213.6 Sympy [F]

$$\begin{aligned}\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = a^4\Bigg(\int (e\sec{(c+dx)})^{\frac{3}{2}}\,dx \\ +\int \left(-6(e\sec{(c+dx)})^{\frac{3}{2}}\tan^2{(c+dx)}\right)\,dx+\int (e\sec{(c+dx)})^{\frac{3}{2}}\tan^4{(c+dx)}\,dx \\ +\int 4i(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}\,dx+\int \left(-4i(e\sec{(c+dx)})^{\frac{3}{2}}\tan^3{(c+dx)}\right)\,dx\Bigg)\end{aligned}$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral((e*sec(c + d*x))**(3/2), x) + Integral(-6*(e*sec(c + d*x))*
*(3/2)*tan(c + d*x)**2, x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*x)
**4, x) + Integral(4*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x), x) + Integral
(-4*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**3, x))
```

3.213.7 Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx=\int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^4\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^4, x)
```

3.213.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx=\int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^4\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^4, x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^4\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4, x)
```

3.214 $\int \sqrt{e\sec(c+dx)}\,(a+ia\tan(c+dx))^4\,dx$

3.214.1 Optimal result . . . 1618
3.214.2 Mathematica [A] (verified) . . . 1619
3.214.3 Rubi [A] (verified) . . . 1619
3.214.4 Maple [A] (verified) . . . 1622
3.214.5 Fricas [C] (verification not implemented) . . . 1623
3.214.6 Sympy [F] . . . 1623
3.214.7 Maxima [F] . . . 1624
3.214.8 Giac [F(-2)] . . . 1624
3.214.9 Mupad [F(-1)] . . . 1624

3.214.1 Optimal result

Integrand size = 28, antiderivative size = 183

$$\begin{aligned}\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx &= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{78a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7d}\\ &\quad+\frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d}+\frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d}\\ &\quad+\frac{78i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{35d}\end{aligned}$$

output
```
78/7*I*a^4*(e*sec(d*x+c))^(1/2)/d+78/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e
*sec(d*x+c))^(1/2)/d+2/7*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3/d+2
6/35*I*(e*sec(d*x+c))^(1/2)*(a^2+I*a^2*tan(d*x+c))^2/d+78/35*I*(e*sec(d*x+
c))^(1/2)*(a^4+I*a^4*tan(d*x+c))/d
```

3.214.2 Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{a^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\Big(728i+1008i\cos(2(c+dx))+280i\cos(4(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)\,\mathrm{El}}{140d}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(a^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(728*I + (1008*I)*Cos[2*(c + d*x)
] + (280*I)*Cos[4*(c + d*x)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)
/2, 2] - 150*Sin[2*(c + d*x)] - 85*Sin[4*(c + d*x)]))/(140*d)
```

3.214.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^4\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))^4\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{13}{7}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{7}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^3dx+\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}\right)+$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}\right)+$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+2ia(a+ia\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+2ia(a+ia\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)+2ia(\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)\right.$$
$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{ 4258}$$

3.214. $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+d\right.}{3d}\right.\right.$$

$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan\right.}{}\right.\right.$$

$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{13}{7}a\left(\frac{9}{5}a\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx}}{d}\right.\right.\right.$$

$$\frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/7)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3)/d + (13*a*((((
2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2)/d + (9*a*((5*a*((
(2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c + d*x]]*(a
^2 + I*a^2*Tan[c + d*x]))/d))/5))/7
```

3.214.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.214.4 Maple [A] (verified)

Time = 19.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.94

method	result
default	$\frac{2ia^4\sqrt{e\sec(dx+c)}\,\big(195\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+195F(i(-\csc(dx+c)+\cot(dx+c)}{35d}$
parts	$-\frac{2ia^4(\cos(dx+c)+1)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{e\sec(dx+c)}}{d}+\frac{2ia^4\sqrt{e\sec(dx+c)}\,\big(-4F(i(\csc}{}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/35*I*a^4/d*(e*sec(d*x+c))^(1/2)*(195*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
+195*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+280+85*I*tan(d*x+c)-28*sec(d*x+c)^2-5*I*sec(
d*x+c)^2*tan(d*x+c))
```

3.214.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.03

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(-365i\,a^4e^{(6i\,dx+6i\,c)}-793i\,a^4e^{(4i\,dx+4i\,c)}-663i\,a^4e^{(2i\,dx+2i\,c)}-195i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i}\right.}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/35*(sqrt(2)*(-365*I*a^4*e^(6*I*d*x + 6*I*c) - 793*I*a^4*e^(4*I*d*x + 4*
I*c) - 663*I*a^4*e^(2*I*d*x + 2*I*c) - 195*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 195*sqrt(2)*(I*a^4*e^(6*I*d*x + 6*I*c)
 + 3*I*a^4*e^(4*I*d*x + 4*I*c) + 3*I*a^4*e^(2*I*d*x + 2*I*c) + I*a^4)*sqrt
(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(6*I*d*x + 6*I*c) +
3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.214.6 Sympy [F]

$$\begin{aligned}\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = a^4\Bigg(&\int \sqrt{e\sec{(c+dx)}}\,dx\\ &+\int\left(-6\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}\right)dx\\ &+\int \sqrt{e\sec{(c+dx)}}\tan^4{(c+dx)}\,dx\\ &+\int 4i\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}\,dx\\ &+\int\left(-4i\sqrt{e\sec{(c+dx)}}\tan^3{(c+dx)}\right)dx\Bigg)\end{aligned}$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**4,x)
```

output
```
a**4*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(-6*sqrt(e*sec(c + d*x))
*tan(c + d*x)**2, x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**4, x) +
 Integral(4*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x) + Integral(-4*I*sqrt(e
*sec(c + d*x))*tan(c + d*x)**3, x))
```

3.214.7 Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \int \sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^4\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^4, x)
```

3.214.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%
}+%%%{%%%
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4, x)
```

3.215 $\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$

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3.215.1 Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx=\frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2}-\frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}-\frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2}$$

output
```
-154/15*I*a^4*(e*sec(d*x+c))^(3/2)/d/e^2+154/5*a^4*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c
)^(1/2)/(e*sec(d*x+c))^(1/2)-154/5*a^4*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d/e
-4*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(1/2)-22/5*I*(e*sec(d*x+c))^(
3/2)*(a^4+I*a^4*tan(d*x+c))/d/e^2
```

3.215.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx=$$

$$-\frac{2ia^4e^{i(c+dx)}\Big(-77-176e^{2i(c+dx)}-111e^{4i(c+dx)}+77\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\Big(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c}}{15de\left(1+e^{2i(c+dx)}\right)^2}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

output

```
(((-2*I)/15)*a^4*E^(I*(c + d*x))*(-77 - 176*E^((2*I)*(c + d*x)) - 111*E^((
4*I)*(c + d*x)) + 77*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e*(1 + E^((2*I
)*(c + d*x)))^2)
```

3.215.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3977, 3042, 3979, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3977}$$

$$-\frac{11a^2\int(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{11a^2\int(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3979}$$

$$-\frac{11a^2\left(\frac{7}{5}a\int(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{11a^2\left(\frac{7}{5}a\int(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3967$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\int(e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 4255$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}\right)}{e^2}$$

$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+d}{5d}}{e^2}$$

$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 4258$$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3}}{5d}}{e^2}$$

$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

3.215. $\int\frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$

$$-\frac{11a^2\left(\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx}{5d}\right.}{e^2}$$

$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{11a^2\left(\frac{2i(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}{5d}+\frac{7}{5}a\left(a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3}}{3d}\right.\right.}{e^2}$$

$$\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-4*I)*a*(a + I*a*Tan[c + d*x])^3)/(d*Sqrt[e*Sec[c + d*x]]) - (11*a^2*((7
*a*((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x
)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c +
 d*x]]*Sin[c + d*x])/d)))/5 + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a
^2*Tan[c + d*x]))/d))/e^2
```

3.215.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.215. $\int\frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.215.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1377 vs. $2(179) = 358$.

Time = 19.09 (sec) , antiderivative size = 1378, normalized size of antiderivative = 7.74

method	result	size
default	Expression too large to display	1378
parts	Expression too large to display	1543

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*I*a^4/d/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)/(cos(d*x+c)+1)^3/(e*sec(
d*x+c))^(1/2)*(231*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)
^(3/2)*cos(d*x+c)^3-231*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+
1)^2)^(3/2)*cos(d*x+c)^3+924*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(3/2)*cos(d*x+c)^2-924*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(c
os(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2+1386*cos(d*x+c)*EllipticF(I*(-csc(d*x+c
)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-1386*cos(d*x+c)*EllipticE(I*(-csc(d
*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+99*I*sin(d*x+c)*(-cos(d*x+c)/(c
os(d*x+c)+1)^2)^(3/2)+924*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-
csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c
)+1)^2)^(3/2)-924*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+
c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^
(3/2)-120*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-120*I*cos(d*x+
c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+231*sec(d*x+c)*(co...
```

3.215.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(-111i\,a^4e^{(5i\,dx+5i\,c)}-176i\,a^4e^{(3i\,dx+3i\,c)}-77i\,a^4e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+231\sqrt{2}(-i}{15\left(dee^{(4i\,dx+4i\,c)}+2\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(-111*I*a^4*e^(5*I*d*x + 5*I*c) - 176*I*a^4*e^(3*I*d*x + 3*
I*c) - 77*I*a^4*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*
I*d*x + 1/2*I*c) + 231*sqrt(2)*(-I*a^4*e^(4*I*d*x + 4*I*c) - 2*I*a^4*e^(2*
I*d*x + 2*I*c) - I*a^4)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, e^(I*d*x + I*c))))/(d*e*e^(4*I*d*x + 4*I*c) + 2*d*e*e^(2*I*d*x + 2
*I*c) + d*e)
```

3.215.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = a^4\left(\int \frac{1}{\sqrt{e\sec{(c+dx)}}}\,dx + \int \left(-\frac{6\tan^2{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\right)dx + \int \frac{\tan^4{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\,dx + \int \frac{4i\tan{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\,dx + \int \left(-\frac{4i\tan^3{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(1/2),x)
```

output
```
a**4*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(-6*tan(c + d*x)**2/sq
rt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**4/sqrt(e*sec(c + d*x)), x)
 + Integral(4*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x) + Integral(-4*I*tan(
c + d*x)**3/sqrt(e*sec(c + d*x)), x))
```

3.215.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

3.215.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2), x)
```

3.216 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx$

3.216.1 Optimal result 1633
3.216.2 Mathematica [A] (verified) 1633
3.216.3 Rubi [A] (verified) 1634
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3.216.1 Optimal result

Integrand size = 28, antiderivative size = 146

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx &= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}\\ &\quad-\frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{de^2}\\ &\quad-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}-\frac{2i\sqrt{e\sec(c+dx)}(a^4+ia^4\tan(c+dx))}{de^2}\end{aligned}$$

output
```
-10*I*a^4*(e*sec(d*x+c))^(1/2)/d/e^2-10*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(
e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(3
/2)-2*I*(e*sec(d*x+c))^(1/2)*(a^4+I*a^4*tan(d*x+c))/d/e^2
```

3.216.2 Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \frac{a^4\sec^3(c+dx)\left(21+19\cos(2(c+dx))-30i\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+d\right.\right.}{3d(e\sec(c+d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(a^4*Sec[c + d*x]^3*(21 + 19*Cos[2*(c + d*x)] - (30*I)*Cos[c + d*x]^(3/2)*
EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) - (11*I)*Sin[2*(
c + d*x)])*((-I)*Cos[c + 5*d*x] + Sin[c + 5*d*x]))/(3*d*(e*Sec[c + d*x])^(
3/2)*(Cos[d*x] + I*Sin[d*x])^4)
```

3.216.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3977, 3042, 3979, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{3a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3979}$$

$$-\frac{3a^2\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a^2\left(\frac{5}{3}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx+\frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3967$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\int\sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4258$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3a^2\left(\frac{5}{3}a\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3120$$

$$-\frac{3a^2\left(\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{5}{3}a\left(\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}\right)\right)}{e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

3.216. $\int\frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx$

output
```
(((-4*I)/3)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(3/2)) - (3*a^
2*((5*a*(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d))/3 + (((2*I)/3)*Sqrt[e*Sec[c
+ d*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d))/e^2
```

3.216.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.216.4 Maple [A] (verified)

Time = 17.23 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

method	result
default	$-\frac{2a^4 (15iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+15i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c}}}{3ed\sqrt{e\sec(dx+c)}}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^4/e/d/(e*sec(d*x+c))^(1/2)*(15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c
)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+15*I*sec(
d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
F(I*(-csc(d*x+c)+cot(d*x+c)),I)+8*I*cos(d*x+c)-8*sin(d*x+c)+12*I*sec(d*x+c
)-sec(d*x+c)*tan(d*x+c))
```

3.216.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{2}(4i\,a^4e^{(4i\,dx+4i\,c)}+21i\,a^4e^{(2i\,dx+2i\,c)}+15i\,a^4)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}+15\sqrt{2}(-i\,a^4e^{(2i\,dx+2i\,c)}-i\,a\right.}{3\left(de^2e^{(2i\,dx+2i\,c)}+de^2\right)}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(4*I*a^4*e^(4*I*d*x + 4*I*c) + 21*I*a^4*e^(2*I*d*x + 2*I*c)
+ 15*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15
*sqrt(2)*(-I*a^4*e^(2*I*d*x + 2*I*c) - I*a^4)*sqrt(e)*weierstrassPInverse(
-4, 0, e^(I*d*x + I*c)))/(d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

3.216.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = a^4\left(\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx + \int \frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(3/2),x)
```

output
```
a**4*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(-6*tan(c + d*x)**2/
(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(
3/2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(3/2), x) + Integra
l(-4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(3/2), x))
```

3.216.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

3.216.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2), x)
```

3.217 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$

3.217.1 Optimal result 1640
3.217.2 Mathematica [C] (verified) 1640
3.217.3 Rubi [A] (verified) 1641
3.217.4 Maple [B] (verified) 1644
3.217.5 Fricas [C] (verification not implemented) 1645
3.217.6 Sympy [F] 1645
3.217.7 Maxima [F] 1646
3.217.8 Giac [F] 1646
3.217.9 Mupad [F(-1)] 1646

3.217.1 Optimal result

Integrand size = 28, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx &= -\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &+\frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3}\\ &-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}+\frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}\end{aligned}$$

output
```
-42/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+42/5*a^4
*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d/e^3-4/5*I*a*(a+I*a*tan(d*x+c))^3/d/(e*s
ec(d*x+c))^(5/2)+28/5*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(1/2)
```

3.217.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.01 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx &=\\ &-\frac{4ia^4e^{2i(c+dx)}\left(7+2e^{2i(c+dx)}-7\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}\end{aligned}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-4*I)/5)*a^4*E^((2*I)*(c + d*x))*(7 + 2*E^((2*I)*(c + d*x)) - 7*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))]))/(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

3.217.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 3977, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{7a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\int \frac{(i\tan(c+dx)a+a)^2}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{7a^2\left(-\frac{3a^2\int (e\sec(c+dx))^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\left(-\frac{3a^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{7a^2\left(-\frac{3a^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

3.217. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$

output
```
(((-4*I)/5)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(5/2)) - (7*a^
2*((-3*a^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[
e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/e^2 - ((4*I
)*(a^2 + I*a^2*Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)
```

3.217.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.217. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$

3.217.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(160) = 320$.

Time = 22.49 (sec) , antiderivative size = 1721, normalized size of antiderivative = 11.03

method	result	size
parts	Expression too large to display	1721
default	Expression too large to display	1949

input

```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/5*a^4/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(3*I*cos(d*x+c)*Ellipti
cF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)-3*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*I*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+
1))^(1/2)-6*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+3*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/
2)-3*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-cos(d*x+c)^2*sin(d*x+c)-sin(d*x+
c)*cos(d*x+c)-3*sin(d*x+c))+2/5*a^4/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/
e^2*(-12*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+12*I*cos(d*x+c)*EllipticF(I*
(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)-24*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+24*I*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+cos(
d*x+c)^2*sin(d*x+c)-12*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+12*I*sec(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c))...
```

3.217.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(21i\sqrt{2}a^4\sqrt{e}\,\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(2i\,a^4e^{(3i\,dx+3i\,c)}+7i\,a^4\right.\right.}{5\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(21*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(2*I*a^4*e^(3*I*d*x + 3*I*c) + 7*I*a^4*
e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
)/(d*e^3)
```

3.217.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = a^4\left(\int \frac{1}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx\right.$$
$$\left.+\int \frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int \frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(5/2),x)
```

output
```
a**4*(Integral((e*sec(c + d*x))**(-5/2), x) + Integral(-6*tan(c + d*x)**2/
(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(
5/2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(5/2), x) + Integra
l(-4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(5/2), x))
```

3.217.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

3.217.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2), x)
```

3.218 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx$

3.218.1 Optimal result . . . 1647
3.218.2 Mathematica [A] (verified) . . . 1647
3.218.3 Rubi [A] (verified) . . . 1648
3.218.4 Maple [A] (verified) . . . 1650
3.218.5 Fricas [C] (verification not implemented) . . . 1651
3.218.6 Sympy [F] . . . 1651
3.218.7 Maxima [F] . . . 1652
3.218.8 Giac [F] . . . 1652
3.218.9 Mupad [F(-1)] . . . 1652

3.218.1 Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

output
```
10/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-4/7*I*a*
(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)+20/21*I*(a^4+I*a^4*tan(d*x+c))
/d/e^2/(e*sec(d*x+c))^(3/2)
```

3.218.2 Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2a^4\sqrt{e\sec(c+dx)}\Big(2i+2i\cos(2(c+dx))+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{21}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(2*a^4*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 5*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) +
 8*Sin[2*(c + d*x)])*(Cos[2*(c + 3*d*x)] + I*Sin[2*(c + 3*d*x)]))/(21*d*e^
4*(Cos[d*x] + I*Sin[d*x])^4)
```

3.218.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3977, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{5a^2\left(-\frac{a^2\int \sqrt{e\sec(c+dx)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\left(-\frac{a^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{5a^2\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5a^2\left(-\frac{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{5a^2\left(-\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-4*I)/7)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)) - (5*a^
2*((-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])
^(3/2))))/(7*e^2)
```

3.218.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.218.4 Maple [A] (verified)

Time = 14.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.46

method	result
default	$\frac{2a^4\left(5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}-24i\left(\cos^3(dx+c)\right)+5i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c))}{21e^3d\sqrt{e\sec(dx+c)}}$
risch	$-\frac{2i\mathrm{e}^{i(dx+c)}\left(3\,\mathrm{e}^{2i(dx+c)}-5\right)a^4\sqrt{2}}{21d\,e^3\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^4\sqrt{e\,\mathrm{e}^{i(dx+c)}\,(\mathrm{e}}}{21d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,e^3\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^4\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)\right.}{21d\sqrt{e\sec(dx+c)}\,e^3}$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*a^4/e^3/d/(e*sec(d*x+c))^(1/2)*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-24*I*cos
(d*x+c)^3+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-c
sc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+24*cos(d*x+c)^2*sin(d*x+
c)+28*I*cos(d*x+c)-16*sin(d*x+c))
```

3.218.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}a^4\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(3i\,a^4e^{(4i\,dx+4i\,c)}-2i\,a^4e^{(2i\,dx+2i\,c)}-5i\,a^4\right)\sqrt{\frac{}{e^{(2i\,dx}}}}{21\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/21*(5*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(3*I*a^4*e^(4*I*d*x + 4*I*c) - 2*I*a^4*e^(2*I*d*x + 2*I*c) - 5*
I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

3.218.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = a^4\left(\int \frac{1}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int\left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx\right.$$
$$\left.+\int \frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int \frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int\left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(7/2),x)
```

output
```
a**4*(Integral((e*sec(c + d*x))**(-7/2), x) + Integral(-6*tan(c + d*x)**2/
(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(
7/2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(7/2), x) + Integra
l(-4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(7/2), x))
```

3.218.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

3.218.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2), x)
```

3.219 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx$

3.219.1 Optimal result 1653
3.219.2 Mathematica [C] (verified) 1653
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3.219.1 Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

output
```
-2/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/9*I*a*
(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)+4/15*I*(a^4+I*a^4*tan(d*x+c))/
d/e^2/(e*sec(d*x+c))^(5/2)
```

3.219.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx =$$

$$-\frac{ia^4e^{i(c+dx)}\left(2+7e^{2i(c+dx)}+5e^{4i(c+dx)}-2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c}}{45de^5}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

output
```
((-1/45*I)*a^4*E^(I*(c + d*x))*(2 + 7*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c
+ d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e^5)
```

3.219.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3977, 3042, 3977, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\left(\frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{a^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{a^2\left(\frac{a^2\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{a^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{a^2\left(\frac{2a^2E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(((-4*I)/9)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2)) - (a^2*
((2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5
/2))))/(3*e^2)
```

3.219.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.219. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.219.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(133) = 266$.

Time = 25.45 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.71

method	result
risch	$-\frac{i(5\,e^{4i(dx+c)}+2\,e^{2i(dx+c)}-6)a^4\sqrt{2}}{45d\,e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{i(-\frac{2(e\,e^{2i(dx+c)}+e)}{e\sqrt{e^{i(dx+c)}(e\,e^{2i(dx+c)}+e)}}+\frac{i\sqrt{-i(e^{i(dx+c)}+i)}\,\sqrt{2}\,\sqrt{i(e^{i(dx+c)}-i)}\,\sqrt{ie^{i(dx+c)}}\,(-2iE(}{\sqrt{e\,e^{3i(dx+c)}+}}}{15d\,e^4(e^{2i(dx+c)}+1}$
default	$-\frac{2ia^4(40(\cos^5(dx+c))+3\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(\csc(dx+c)-\cot(dx+c)),i)-3F(i(\csc(dx+c)-\cot(dx+c)),i)}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*I*(5*exp(I*(d*x+c))^4+2*exp(I*(d*x+c))^2-6)/d*a^4*2^(1/2)/e^4/(e*exp
(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)+1/15*I/d*(-2*(e*exp(I*(d*x+c))^2+e
)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I)
)^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*e
xp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c)
)+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(
1/2))))*a^4*2^(1/2)/e^4/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x
+c))^2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

3.219.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \frac{-6i\sqrt{2}a^4\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+}{45}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

output
```
1/45*(-6*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-5*I*a^4*e^(5*I*d*x + 5*I*c) - 7*I*a^4
*e^(3*I*d*x + 3*I*c) - 2*I*a^4*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^5)
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.219.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

3.219.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{\left(e\sec\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2), x)
```

3.220 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx$

3.220.1 Optimal result 1659
3.220.2 Mathematica [A] (verified) 1659
3.220.3 Rubi [A] (verified) 1660
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3.220.5 Fricas [C] (verification not implemented) 1663
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3.220.8 Giac [F] 1664
3.220.9 Mupad [F(-1)] 1665

3.220.1 Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} - \frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}$$

output
```
-2/77*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)-2/77*a^4*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^6-4/11*I*a*(a+I*a*tan(d*x+c))^3/d/(e*
sec(d*x+c))^(11/2)+4/77*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(7/2
)
```

3.220.2 Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{a^4\sqrt{e\sec(c+dx)}\Big(37i\cos(c+dx)+11i\cos(3(c+dx))-3\sin(c+dx)+4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.}{154de^6(\cos(dx)+i}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

output
```
-1/154*(a^4*Sqrt[e*Sec[c + d*x]]*((37*I)*Cos[c + d*x] + (11*I)*Cos[3*(c +
d*x)] - 3*Sin[c + d*x] + 4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(C
os[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 3*Sin[3*(c + d*x)])*(Cos[3*c + 7*d
*x] + I*Sin[3*c + 7*d*x]))/(d*e^6*(Cos[d*x] + I*Sin[d*x])^4)
```

3.220.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\left(\frac{3a^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\left(\frac{3a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{a^2\left(\frac{3a^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

3.220. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx$

output
```
(((-4*I)/11)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) - (a^
2*((3*a^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)
 - (((4*I)/7)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))))/(11
*e^2)
```

3.220.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.220.4 Maple [A] (verified)

Time = 22.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29

method	result
default	$\frac{2a^4\left(-56i(\cos^5(dx+c))+56\sin(dx+c)(\cos^4(dx+c))+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+44i(\cos^3(dx+c)\right.}{77e^5d\sqrt{e\sec(dx+}}$
risch	$-\frac{ie^{i(dx+c)}\left(7e^{4i(dx+c)}+13e^{2i(dx+c)}+4\right)a^4\sqrt{2}}{154d\,e^5\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^4\sqrt{e}}{77d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^5\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+}}}$
parts	$-\frac{2a^4\left(-7\sin(dx+c)(\cos^4(dx+c))+15iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F\right.}{77d\sqrt{e\sec(dx+c)}\,e^5}$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/77*a^4/e^5/d/(e*sec(d*x+c))^(1/2)*(-56*I*cos(d*x+c)^5+56*sin(d*x+c)*cos(
d*x+c)^4+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d
*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+44*I*cos(d*x+c)^3+I*sec(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d
*x+c)+1))^(1/2)-16*cos(d*x+c)^2*sin(d*x+c)-sin(d*x+c))
```

3.220.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx=\frac{4i\sqrt{2}a^4\sqrt{e}\,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-7i\,a^4e^{(6i\,dx+6i\,c)}-20i\right.}{154\,de^6}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="fricas"
)
```

output
```
1/154*(4*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
 + sqrt(2)*(-7*I*a^4*e^(6*I*d*x + 6*I*c) - 20*I*a^4*e^(4*I*d*x + 4*I*c) -
17*I*a^4*e^(2*I*d*x + 2*I*c) - 4*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
e^(1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

3.220.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^4}{(e\sec{(dx+c)})^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

3.220.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^4}{(e\sec{(dx+c)})^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2), x)
```

3.221 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$

3.221.1 Optimal result 1666
3.221.2 Mathematica [C] (verified) 1666
3.221.3 Rubi [A] (verified) 1667
3.221.4 Maple [B] (verified) 1670
3.221.5 Fricas [C] (verification not implemented) 1670
3.221.6 Sympy [F(-1)] 1671
3.221.7 Maxima [F] 1671
3.221.8 Giac [F] 1671
3.221.9 Mupad [F(-1)] 1672

3.221.1 Optimal result

Integrand size = 28, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx &= \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &+ \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}\end{aligned}$$

output
```
2/117*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)+2/39*a^4*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^6
/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/13*I*a*(a+I*a*tan(d*x+c))^3/d/(e*
sec(d*x+c))^(13/2)-4/117*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(9/
2)
```

3.221.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = -\frac{ia^4e^{i(c+dx)}\left(31+59e^{2i(c+dx)}+37e^{4i(c+dx)}+9e^{6i(c+dx)}+8\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c}\right.\right.}{468de^7}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

output

```
((-1/468*I)*a^4*E^(I*(c + d*x))*(31 + 59*E^((2*I)*(c + d*x)) + 37*E^((4*I)
*(c + d*x)) + 9*E^((6*I)*(c + d*x)) + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/
(d*e^7)
```

3.221.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3977, 3042, 3977, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{9/2}}dx}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\left(\frac{5a^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{5a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \; 4256$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

$$\Big\downarrow \; 3119$$

$$\frac{a^2\left(\frac{5a^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}\right)}{13e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

3.221. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$

output
```
(((-4*I)/13)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) + (a^
2*((5*a^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[
e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^
2) - (((4*I)/9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))))/(
13*e^2)
```

3.221.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.221.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(160) = 320$.

Time = 26.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.26

method	result
risch	$-\frac{i(9\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+31\,e^{2i(dx+c)}+24)a^4\sqrt{2}}{468d\,e^6\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}}{}\right.}{39d}$
default	$-\frac{2ia^4\left(72(\cos^7(dx+c))+72(\cos^6(dx+c))+i(\cos^2(dx+c))\sin(dx+c)-52(\cos^5(dx+c))-16i\sin(dx+c)(\cos^4(dx+c))+3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/468*I*(9*exp(I*(d*x+c))^6+28*exp(I*(d*x+c))^4+31*exp(I*(d*x+c))^2+24)/d
*a^4*2^(1/2)/e^6/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-1/39*I/d*(-
2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I
*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp
(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*Ellip
ticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*
x+c))+I))^(1/2),1/2*2^(1/2))))*a^4*2^(1/2)/e^6/(exp(I*(d*x+c))^2+1)/(e*exp
(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^
2+1))^(1/2)
```

3.221.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \frac{24i\sqrt{2}a^4\sqrt{e}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="fricas"
)
```

output
```
1/468*(24*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^4*e^(7*I*d*x + 7*I*c) - 37*I*a
^4*e^(5*I*d*x + 5*I*c) - 59*I*a^4*e^(3*I*d*x + 3*I*c) - 31*I*a^4*e^(I*d*x
+ I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^7)
```

3.221.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(13/2),x)
```

output
```
Timed out
```

3.221.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{\left(e\sec\left(dx+c\right)\right)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(13/2), x)
```

3.221.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{\left(e\sec\left(dx+c\right)\right)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(13/2), x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2), x)
```

3.222 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

3.222.1 Optimal result 1673
3.222.2 Mathematica [A] (verified) 1673
3.222.3 Rubi [A] (verified) 1674
3.222.4 Maple [A] (verified) 1678
3.222.5 Fricas [C] (verification not implemented) 1678
3.222.6 Sympy [F(-1)] 1679
3.222.7 Maxima [F] 1679
3.222.8 Giac [F] 1679
3.222.9 Mupad [F(-1)] 1680

3.222.1 Optimal result

Integrand size = 28, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx &= \frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^8}\\ &+\frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}+\frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}}\\ &-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}-\frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}\end{aligned}$$

output
```
2/55*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/33*a^4*sin(d*x+c)/d/e^7/(
e*sec(d*x+c))^(1/2)+2/33*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(
1/2)/d/e^8-4/15*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(15/2)-4/55*I*(a
^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2)
```

3.222.2 Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx =$$

$$-\frac{ia^4\sqrt{e\sec(c+dx)}\big(64+112\cos(2(c+dx))+48\cos(4(c+dx))-54i\sin(2(c+dx))-37i\sin(4(c+dx))}{660de^8(c}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

output
```
((-1/660*I)*a^4*Sqrt[e*Sec[c + d*x]]*(64 + 112*Cos[2*(c + d*x)] + 48*Cos[4
*(c + d*x)] - (54*I)*Sin[2*(c + d*x)] - (37*I)*Sin[4*(c + d*x)] + 40*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(I*Cos[4*(c + d*x)] + Sin[4*(c + d
*x)]))*(Cos[4*(c + 2*d*x)] + I*Sin[4*(c + 2*d*x)]))/(d*e^8*(Cos[d*x] + I*S
in[d*x])^4)
```

3.222.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3977, 3042, 3977, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{(i\tan(c+dx)a+a)^2}{(e\sec(c+dx))^{11/2}}dx}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3977}$$

$$\frac{a^2\left(\frac{7a^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{7a^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \; 4256$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \; 4256$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \; 4258$$

3.222. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a^2\left(\frac{7a^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}\right)}{5e^2}-\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

3.222. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

output
```
(((-4*I)/15)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) + (a^
2*((7*a^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (
2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2) - (((4*I
)/11)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))))/(5*e^2)
```

3.222.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.222. $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

3.222.4 Maple [A] (verified)

Time = 33.74 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.16

method	result
default	$\frac{2a^4\left(-88i\left(\cos^7(dx+c)\right)+88\sin(dx+c)\left(\cos^6(dx+c)\right)+60i\left(\cos^5(dx+c)\right)-16\sin(dx+c)\left(\cos^4(dx+c)\right)+5iF(i(-\csc(dx+c)+\cot(dx+c)}{1}$
risch	$-\frac{ie^{i(dx+c)}\left(11\,e^{6i(dx+c)}+47\,e^{4i(dx+c)}+81\,e^{2i(dx+c)}+85\right)a^4\sqrt{2}}{1320d\,e^7\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i\left(e^{i(dx+c}\right.\right.}{33d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^7\left(e^{2i(dx+c)}+\right.}$
parts	$-\frac{2a^4\left(-77\sin(dx+c)\left(\cos^6(dx+c)\right)-91\sin(dx+c)\left(\cos^4(dx+c)\right)+195iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+\right.}{1155d\sqrt{e\sec(d}}$

input
```
int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

output
```
2/165*a^4/e^7/d/(e*sec(d*x+c))^(1/2)*(-88*I*cos(d*x+c)^7+88*sin(d*x+c)*cos
(d*x+c)^6+60*I*cos(d*x+c)^5-16*sin(d*x+c)*cos(d*x+c)^4+5*I*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+
1))^(1/2)+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-c
sc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*sin(d*x+c
)+5*sin(d*x+c))
```

3.222.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \frac{-80i\,\sqrt{2}a^4\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-11i\,a^4e^{(8i\,dx+8i\,c)}-\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="fricas"
)
```

output
```
1/1320*(-80*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*
c)) + sqrt(2)*(-11*I*a^4*e^(8*I*d*x + 8*I*c) - 58*I*a^4*e^(6*I*d*x + 6*I*c
) - 128*I*a^4*e^(4*I*d*x + 4*I*c) - 166*I*a^4*e^(2*I*d*x + 2*I*c) - 85*I*a
^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^8)
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(15/2),x)
```

output
```
Timed out
```

3.222.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="maxima"
)
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(15/2), x)
```

3.222.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(15/2), x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2), x)
```

3.223 $\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx$

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3.223.1 Optimal result

Integrand size = 28, antiderivative size = 136

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = -\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad}$$

output
```
-2/7*I*e^2*(e*sec(d*x+c))^(7/2)/a/d+2/5*e^3*(e*sec(d*x+c))^(5/2)*sin(d*x+c
)/a/d-6/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+6/5*e^
5*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a/d
```

3.223.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \frac{e^4(e\sec(c+dx))^{3/2}\left(76+28\cos(2(c+dx))-7e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hype}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

output

```
(e^4*(e*Sec[c + d*x])^(3/2)*(76 + 28*Cos[2*(c + d*x)] - (7*(1 + E^((2*I)*(
c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E
^((2*I)*(c + d*x)) + (7*I)*Sec[c + d*x]*Sin[3*(c + d*x)] - (13*I)*Tan[c +
d*x])*(-I + Tan[c + d*x]))/(70*a*d)
```

3.223.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3982, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow 3982$$

$$\frac{e^2\int (e\sec(c+dx))^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow 3042$$

$$\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow 4255$$

$$\frac{e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow 3042$$

$$\frac{e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow 4255$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{3119}$$

$$\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

input
```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((-2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (e^2*((2*e*(e*Sec[c + d*x]
)^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin
[c + d*x])/d))/5))/a
```

3.223. $\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx$

3.223.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.223.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(143) = 286.

Time = 8.74 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.26

method	result
default	$\frac{2e^5\sqrt{e\sec(dx+c)}\left(21iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))-21iE(i(\csc(dx+c)-\cot(dx+c)),i)}{}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/35*e^5/a/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*I*EllipticF(I*(csc(d*
x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)^2-21*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+42*I*cos(d*x+c
)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)-42*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+
c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*I*Ell
ipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)-21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(c
sc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+21*sin(d*x+c)+7*tan(d*x+
c)-5*I*sec(d*x+c)^2+7*sec(d*x+c)*tan(d*x+c)-5*I*sec(d*x+c)^3)
```

3.223.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.51

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(21i\,e^5e^{(7i\,dx+7i\,c)}+77i\,e^5e^{(5i\,dx+5i\,c)}+103i\,e^5e^{(3i\,dx+3i\,c)}+7i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+2}{35\,(ade^{(6i\,dx+6i\,c)}}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/35*(sqrt(2)*(21*I*e^5*e^(7*I*d*x + 7*I*c) + 77*I*e^5*e^(5*I*d*x + 5*I*c
) + 103*I*e^5*e^(3*I*d*x + 3*I*c) + 7*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*e^5*e^(6*I*d*
x + 6*I*c) + 3*I*e^5*e^(4*I*d*x + 4*I*c) + 3*I*e^5*e^(2*I*d*x + 2*I*c) + I
*e^5)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x +
 I*c))))/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2
*I*d*x + 2*I*c) + a*d)
```

3.223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

3.223.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.223.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx=\int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i), x)
```

3.224 $\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx$

3.224.1 Optimal result 1688

3.224.2 Mathematica [A] (verified) 1688

3.224.3 Rubi [A] (verified) 1689

3.224.4 Maple [A] (verified) 1691

3.224.5 Fricas [C] (verification not implemented) 1691

3.224.6 Sympy [F(-1)] 1692

3.224.7 Maxima [F(-2)] 1692

3.224.8 Giac [F] 1692

3.224.9 Mupad [F(-1)] 1693

3.224.1 Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx=\frac{2e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad}$$

output
```
-2/5*I*e^2*(e*sec(d*x+c))^(5/2)/a/d+2/3*e^3*(e*sec(d*x+c))^(3/2)*sin(d*x+c
)/a/d+2/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d
```

3.224.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx=\frac{e^2(e\sec(c+dx))^{5/2}\left(-6i+10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c\right.}{15ad}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(e^2*(e*Sec[c + d*x])^(5/2)*(-6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d
*x)/2, 2] + 5*Sin[2*(c + d*x)]))/(15*a*d)
```

3.224.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3982, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{e^2\int (e\sec(c+dx))^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{4255}$$

$$\frac{e^2\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3120}$$

$$\frac{e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

input

```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

output

```
(((-2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (e^2*((2*e^2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a
```

3.224.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.224.4 Maple [A] (verified)

Time = 7.63 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.50

method	result
default	$\frac{2e^4\sqrt{e\sec(dx+c)}\,\left(5i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\right.}{15ad}$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/15*e^4/a/d*(e*sec(d*x+c))^(1/2)*(5*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)
+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(1/(cos(d*x+c)+1))^(1/2)+5*tan(d*x+c)-3*I*sec(d*x+c)^2)
```

3.224.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.45

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{2\left(\sqrt{2}(5i\,e^4e^{(4i\,dx+4i\,c)}+12i\,e^4e^{(2i\,dx+2i\,c)}-5i\,e^4)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}(i\,e^4e^{(4i\,dx+4i\,c)}+2i\,e^4e^{(2i\right.}}{15\,(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad)}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/15*(sqrt(2)*(5*I*e^4*e^(4*I*d*x + 4*I*c) + 12*I*e^4*e^(2*I*d*x + 2*I*c)
 - 5*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*
sqrt(2)*(I*e^4*e^(4*I*d*x + 4*I*c) + 2*I*e^4*e^(2*I*d*x + 2*I*c) + I*e^4)*
sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a*d*e^(4*I*d*x + 4*I
*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

3.224.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.224.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i), x)
```

3.225 $\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx$

3.225.1 Optimal result 1694
3.225.2 Mathematica [C] (verified) 1694
3.225.3 Rubi [A] (verified) 1695
3.225.4 Maple [B] (verified) 1697
3.225.5 Fricas [C] (verification not implemented) 1697
3.225.6 Sympy [F(-1)] 1698
3.225.7 Maxima [F(-2)] 1698
3.225.8 Giac [F] 1699
3.225.9 Mupad [F(-1)] 1699

3.225.1 Optimal result

Integrand size = 28, antiderivative size = 101

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx=-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}$$

output

```
-2/3*I*e^2*(e*sec(d*x+c))^(3/2)/a/d-2*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)
/(e*sec(d*x+c))^(1/2)+2*e^3*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a/d
```

3.225.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx=\frac{2ie^3\sqrt{e\sec(c+dx)}(\cos(c)+i\sin(c))(\cos(dx)+i\sin(dx))\Big(-4+\sqrt{1+e^{2i(c+dx)}}}{3ad}$$

input

```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((2*I)/3)*e^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*(Cos[d*x] + I*Sin[
d*x])*(-4 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))] + I*Tan[c + d*x]))/(a*d)
```

3.225.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3982, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{e^2\int (e\sec(c+dx))^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{4255}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3119}$$

$$\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((-2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (e^2*((-2*e^2*EllipticE[(c
 + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*S
ec[c + d*x]]*Sin[c + d*x])/d))/a
```

3.225.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

3.225. $\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.225.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(116) = 232$.

Time = 6.54 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.06

method	result
default	$\frac{2e^3\sqrt{e\sec(dx+c)}\,\left(3iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))-3iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}\right.}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/3*e^3/a/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(3*I*EllipticF(I*(csc(d*x+
c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)^2-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+6*I*cos(d*x+c)*El
lipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)-6*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*I*EllipticF
(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-I+3*sin(d*x+c)-I*sec(d*x+c))
```

3.225.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(3i\,e^3e^{(3i\,dx+3i\,c)}+5i\,e^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\sqrt{2}\left(i\,e^3e^{(2i\,dx+2i\,c)}+i\,e^3\right)\sqrt{e}\mathrm{weierstras}\right.}{3\left(ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(3*I*e^3*e^(3*I*d*x + 3*I*c) + 5*I*e^3*e^(I*d*x + I*c))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*sqrt(2)*(I*e^3*e
^(2*I*d*x + 2*I*c) + I*e^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, e^(I*d*x + I*c))))/(a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.225.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

3.225.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.225.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx=\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i), x)
```

3.226 $\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx$

3.226.1 Optimal result 1700
3.226.2 Mathematica [A] (verified) 1700
3.226.3 Rubi [A] (verified) 1701
3.226.4 Maple [A] (verified) 1702
3.226.5 Fricas [C] (verification not implemented) 1703
3.226.6 Sympy [F] 1703
3.226.7 Maxima [F(-2)] 1703
3.226.8 Giac [F] 1704
3.226.9 Mupad [F(-1)] 1704

3.226.1 Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad} + \frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}$$

output
```
-2*I*e^2*(e*sec(d*x+c))^(1/2)/a/d+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*se
c(d*x+c))^(1/2)/a/d
```

3.226.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \frac{2e^2\left(-i+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{e\sec(c+dx)}}{ad}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*e^2*(-I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c +
d*x]])/(a*d)
```

3.226.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3982, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

input

```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

output

```
((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)
```

3.226.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.226.4 Maple [A] (verified)

Time = 6.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2ie^2\left(F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)+F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{ad}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*e^2/a/d*(EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+EllipticF(I*(csc(d*x+c)-c
ot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1
)*(e*sec(d*x+c))^(1/2)
```

3.226.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(i\sqrt{2}e^{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}e^{\frac{5}{2}}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{ad}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*e^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
) + I*sqrt(2)*e^(5/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a*d)
```

3.226.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int\frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x) - I), x)/a
```

3.226.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.226.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx=\int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i), x)
```

3.227 $\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx$

3.227.1 Optimal result . . . 1705
3.227.2 Mathematica [C] (verified) . . . 1705
3.227.3 Rubi [A] (verified) . . . 1706
3.227.4 Maple [B] (warning: unable to verify) . . . 1707
3.227.5 Fricas [C] (verification not implemented) . . . 1708
3.227.6 Sympy [F] . . . 1709
3.227.7 Maxima [F(-2)] . . . 1709
3.227.8 Giac [F] . . . 1709
3.227.9 Mupad [F(-1)] . . . 1710

3.227.1 Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx=\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

output
```
2*I*e^2/a/d/(e*sec(d*x+c))^(1/2)+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e
*sec(d*x+c))^(1/2)
```

3.227.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx=\frac{2iee^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\sqrt{e\sec(c+}}{ad}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
((2*I)*e*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -
E^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])/(a*d*E^(I*(c + d*x)))
```

3.227.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3982, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\int \sqrt{\cos(c+dx)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

output
```
((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*x)/2, 2])
/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

3.227.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.227.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(89) = 178$.

Time = 4.51 (sec) , antiderivative size = 780, normalized size of antiderivative = 11.14

method	result	size
default	Expression too large to display	780

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e/a/d*(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+
c))^2-1))^(1/2)*(4*I*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(-csc(d*x+c)^
2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)
^2*(1-cos(d*x+c))^2-1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)-4*I*(
csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^
(1/2)*((csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)
)^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-I*ln(2*csc(d*x+c)^2*(1-cos(
d*x+c))^2+2*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2))*(csc(d*x+c)^4*(1-cos(
d*x+c))^4-1)^(1/2)*((csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)^2*(1-cos
(d*x+c))^2-1))^(1/2)+I*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)*((csc(d*x+c
)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1/2)*ln(csc(d*
x+c)^2*(1-cos(d*x+c))^2+(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2))+4*I*csc(d
*x+c)^2*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)*(1-cos(d*x+c))^2+4*csc(d*x
+c)^3*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)*(1-cos(d*x+c))^3-4*I*(csc(d*
x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)-4*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)
*(csc(d*x+c)-cot(d*x+c)))/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^4*
(1-cos(d*x+c))^4-1)^(1/2)
```

3.227.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{2\left(-i\sqrt{2}e^{\frac{3}{2}}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(-i\,ee^{(2i\,dx+2i\,c)}-\right.\right.}{ad}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(2)*e^(3/2)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*e*e^(2*I*d*x + 2*I*c) - I*e
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I
*c)/(a*d)
```

3.227.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x) - I), x)/a
```

3.227.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.227.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx=\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i), x)
```

3.228 $\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx$

3.228.1 Optimal result 1711
3.228.2 Mathematica [A] (verified) 1711
3.228.3 Rubi [A] (verified) 1712
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3.228.7 Maxima [F(-2)] 1715
3.228.8 Giac [F] 1715
3.228.9 Mupad [F(-1)] 1715

3.228.1 Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad} + \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

output
```
2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d+2/3*I*(e*sec(d*x
+c))^(1/2)/d/(a+I*a*tan(d*x+c))
```

3.228.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{2(e\sec(c+dx))^{3/2}\left(\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(-i\cos(c+dx)+\sin(c+dx))\right)}{3ade(-i+\tan(c+dx))}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*(e*Sec[c + d*x])^(3/2)*(Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*((-I)*Cos[c + d*x] + Sin[c + d*x])))/(3*a*d*e*(-I + Tan[c + d
*x]))
```

3.228.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3983, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{\int \sqrt{e\sec(c+dx)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a
*d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))
```

3.228.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.228.4 Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.00

method	result
default	$\frac{2\sqrt{e\sec(dx+c)}\,(i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(}{}}}{3ad}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/3/a/d*(e*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*cos(d*x+c)+I*Elli
pticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+I*cos(d*x+c)^2+sin(d*x+c)*cos(d*x+c))
```

3.228.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-2i\,\sqrt{2}\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\,ad}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)
*e^(1/2*I*d*x + 1/2*I*c) - 2*I*sqrt(2)*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierst
rassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

3.228.6 Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sqrt{e\sec(c+dx)}}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x) - I), x)/a
```

3.228.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.228.8 Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{i\,a\tan\left(dx+c\right)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i), x)
```

3.229 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx$

3.229.1 Optimal result 1716
3.229.2 Mathematica [C] (verified) 1716
3.229.3 Rubi [A] (verified) 1717
3.229.4 Maple [B] (verified) 1718
3.229.5 Fricas [C] (verification not implemented) 1719
3.229.6 Sympy [F] 1719
3.229.7 Maxima [F(-2)] 1720
3.229.8 Giac [F] 1720
3.229.9 Mupad [F(-1)] 1720

3.229.1 Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{6E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}$$

output
```
6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/5*I/d/(e*sec(d
*x+c))^(1/2)/(a+I*a*tan(d*x+c))
```

3.229.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{\left(4+4\cos(2(c+dx))-2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+3i\sin(2(c+d\right.}{5ad\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

output
```
((4 + 4*Cos[2*(c + d*x)] - 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*
(c + d*x)])*(I + Tan[c + d*x]))/(5*a*d*Sqrt[e*Sec[c + d*x]])
```

3.229.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3983, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3\int \sqrt{\cos(c+dx)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

output
```
(6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x
]]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

3.229.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.229.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(94) = 188$.

Time = 7.54 (sec) , antiderivative size = 440, normalized size of antiderivative = 5.50

method	result
default	$-\frac{2i\left(i\left(\cos^2(dx+c)\right)\sin(dx+c)+3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)-3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-}{}$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
-2/5*I/a/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*cos(d*x+c)^2*sin(d*x+c)+
3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)+I*sin(d*x+c)*cos(d*x+c)-cos(d*x+c)^3+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-6*Ellipt
icF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+3*I*sin(d*x+c)-cos(d*x+c)^2+3*sec(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)-3*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

3.229.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{\Big(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(7i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+12i\,\sqrt{2}\sqrt{e}e^{(3i\,dx+3i\,c)}\text{weierstrassZeta}(-4}{10\,ade}$$

input

```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output

```
1/10*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(7*I*e^(4*I*d*x + 4*I*c) +
 8*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c) + 12*I*sqrt(2)*sqrt(
e)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e
^(I*d*x + I*c))))*e^(-3*I*d*x - 3*I*c)/(a*d*e)
```

3.229.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}-i\sqrt{e\sec{(c+dx)}}}\,dx}{a}$$

input

```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x) - I*sqrt(e*sec(c + d*x)))
, x)/a
```

3.229.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.229.8 Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{e\sec(dx+c)}(ia\tan(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,(a+a\tan(c+d\,x)\;1\mathrm{i})}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.230 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx$

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3.230.1 Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21ade^2} + \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}$$

output
```
10/21*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*(e*sec(d*x+c))^(1/2)/a/d/e^2+2/7*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(
d*x+c))
```

3.230.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = -\frac{\sec^3(c+dx)\big(-14\cos(c+dx)+2\cos(3(c+dx))+20i\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx}{42ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
-1/42*(Sec[c + d*x]^3*(-14*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + (20*I)*Sqrt
[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) +
 (5*I)*Sin[c + d*x] + (5*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)
*(-I + Tan[c + d*x]))
```

3.230.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3983, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
(5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a) + ((2*I)
/7)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]))
```

3.230.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.230.4 Maple [A] (verified)

Time = 8.86 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.53

method	result
default	$\frac{\frac{10 i F\left(i\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right), i\right)\sqrt{\frac{1}{\cos\left(dx+c\right)+1}}\,\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}}{21}+\frac{2 i\left(\cos^{3}\left(dx+c\right)\right)}{7}+\frac{10 i \sec\left(dx+c\right) F\left(i\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right), i\right)\sqrt{\frac{1}{\cos\left(dx+c\right)+}}}{21}}{a d\sqrt{e\sec\left(dx+c\right)}\, e}$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/21/a/d/(e*sec(d*x+c))^(1/2)/e*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*I*cos(d*x+
c)^3+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*
x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*sin(d*x+c)+5*s
in(d*x+c))
```

3.230.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(6i\,dx+6i\,c)}+9i\,e^{(4i\,dx+4i\,c)}+19i\,e^{(2i\,dx}\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/84*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(6*I*d*x + 6*I*c)
+ 9*I*e^(4*I*d*x + 4*I*c) + 19*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(1/2*I*d*x +
 1/2*I*c) - 40*I*sqrt(2)*sqrt(e)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-
4, 0, e^(I*d*x + I*c)))*e^(-4*I*d*x - 4*I*c)/(a*d*e^2)
```

3.230.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}-i(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x) - I*(e*sec(c + d*x))**
(3/2)), x)/a
```

3.230.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.230.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.231 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx$

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3.231.1 Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}$$

output
```
14/45*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(3/2)+14/15*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/e^2/cos(
d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/9*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(
d*x+c))
```

3.231.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(106+104\cos(2(c+dx))-2\cos(4(c+dx))-56e^{2i(c+dx)}\sqrt{}\right.}{}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
((106 + 104*Cos[2*(c + d*x)] - 2*Cos[4*(c + d*x)] - 56*E^((2*I)*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))] + (70*I)*Sin[2*(c + d*x)] - (7*I)*Sin[4*(c + d*x)])*(I + Tan[c
 + d*x]))/(180*a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

3.231.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3983, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
(7*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a) + ((2*
I)/9)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]))
```

3.231.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.231.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(124) = 248$.

Time = 10.56 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.18

method	result
default	$-\frac{2i\left(5i\sin(dx+c)(\cos^4(dx+c))+5i(\cos^3(dx+c))\sin(dx+c)-5(\cos^5(dx+c))+7i(\cos^2(dx+c))\sin(dx+c)-5(\cos^4(dx+c))-21\sqrt{\frac{\cos}{\cos(}}\right.}{}$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/45*I/a/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(5*I*sin(d*x+c)*cos(d*
x+c)^4+5*I*cos(d*x+c)^3*sin(d*x+c)-5*cos(d*x+c)^5+7*I*cos(d*x+c)^2*sin(d*x
+c)-5*cos(d*x+c)^4-21*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(
d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+21*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)+7*I*cos(d*x+c)*sin(d*x+c)-42*EllipticF(I*(-csc(d*x+
c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+42*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c
)),I)*(1/(cos(d*x+c)+1))^(1/2)+21*I*sin(d*x+c)-21*sec(d*x+c)*EllipticF(I*(
-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+21*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-
csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
```

3.231.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(8i\,dx+8i\,c)}+174i\,e^{(6i\,dx+6i\,c)}+212i\,e^{(4}\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/360*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(8*I*d*x + 8*I*c)
 + 174*I*e^(6*I*d*x + 6*I*c) + 212*I*e^(4*I*d*x + 4*I*c) + 34*I*e^(2*I*d*x
 + 2*I*c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c) + 336*I*sqrt(2)*sqrt(e)*e^(5*I*d*
x + 5*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*
c))))*e^(-5*I*d*x - 5*I*c)/(a*d*e^3)
```

3.231.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int\frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}-i(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x) - I*(e*sec(c + d*x))**
(5/2)), x)/a
```

3.231.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.231.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec{(dx+c)})^{\frac{5}{2}}\,(i\,a\tan{(dx+c)}+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.232 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx$

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3.232.1 Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77ade^4} + \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}$$

output

```
18/77*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(5/2)+30/77*sin(d*x+c)/a/d/e^3/(e*se
c(d*x+c))^(1/2)+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/
d/e^4+2/11*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))
```

3.232.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = -\frac{(e\sec(c+dx))^{3/2}\left(-148\cos(c+dx)+34\cos(3(c+dx))+2\cos(5(c+dx))+240i\sqrt{\cos(c+dx)}\,\text{EllipticI}\right.}{616ade^5(-i+}$$

input

```
Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
-1/616*((e*Sec[c + d*x])^(3/2)*(-148*Cos[c + d*x] + 34*Cos[3*(c + d*x)] +
2*Cos[5*(c + d*x)] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*
(Cos[c + d*x] + I*Sin[c + d*x]) + (78*I)*Sin[c + d*x] + (87*I)*Sin[3*(c +
d*x)] + (9*I)*Sin[5*(c + d*x)]))/(a*d*e^5*(-I + Tan[c + d*x]))
```

3.232.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3983, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{9\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{9\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{9\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

input

```
Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

3.232. $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx$

output
```
(9*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c
 + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*a) + ((2*I)/11)/(d*(
e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]))
```

3.232.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.232.4 Maple [A] (verified)

Time = 8.92 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.31

method	result
default	$\frac{\frac{2i\left(\cos^{5}(dx+c)\right)}{11}+\frac{2\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{11}+\frac{30iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{77}+\frac{18\left(\cos^{2}(dx+c)\right)\sin(dx+c)}{77}+}{ad\sqrt{e\sec(dx+c)}\,e^{3}}$

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/77/a/d/(e*sec(d*x+c))^(1/2)/e^3*(7*I*cos(d*x+c)^5+7*sin(d*x+c)*cos(d*x+c
)^4+15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+9*cos(d*x+c)^2*sin(d*x+c)+15*I*sec(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
-csc(d*x+c)+cot(d*x+c)),I)+15*sin(d*x+c))
```

3.232.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx=\frac{\Big(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,\big(-11i\,e^{(10i\,dx+10i\,c)}-121i\,e^{(8i\,dx+8i\,c)}+70i}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/1232*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(10*I*d*x + 10*
I*c) - 121*I*e^(8*I*d*x + 8*I*c) + 70*I*e^(6*I*d*x + 6*I*c) + 226*I*e^(4*I
*d*x + 4*I*c) + 53*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c) -
480*I*sqrt(2)*sqrt(e)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0, e^(I*
d*x + I*c)))*e^(-6*I*d*x - 6*I*c)/(a*d*e^4)
```

3.232.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{7}{2}}\tan(c+dx)-i(e\sec(c+dx))^{\frac{7}{2}}}\,dx}{a}$$

input
```
integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral(1/((e*sec(c + d*x))**(7/2)*tan(c + d*x) - I*(e*sec(c + d*x))**
(7/2)), x)/a
```

3.232.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.232.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx=\int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.233 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$

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3.233.1 Optimal result

Integrand size = 28, antiderivative size = 183

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &\quad+\frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d}+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d}\\ &\quad+\frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

output
```
22/45*e^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+22/63*e^3*(e*sec(d*x+c))^(
9/2)*sin(d*x+c)/a^2/d-22/15*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(
d*x+c))^(1/2)+22/15*e^7*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^2/d-4/7*I*e^2*(e
*sec(d*x+c))^(11/2)/d/(a^2+I*a^2*tan(d*x+c))
```

3.233.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.88 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.65

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\sec(c+dx))^{15/2}(\cos(dx)+i\sin(dx))^2\Bigg(\frac{22i\sqrt{2}e^{3ic-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\big(-}{}}{}$$

input

```
Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output

```
((e*Sec[c + d*x])^(15/2)*(Cos[d*x] + I*Sin[d*x])^2*(((22*I)*Sqrt[2]*E^((3*
I)*c - I*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 +
E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(-1
+ E^((2*I)*c)) + (Csc[c]*Sec[c + d*x]^(9/2)*(Cos[2*c] + I*Sin[2*c])*(1260*
Cos[d*x] + 1050*Cos[2*c + d*x] + 1078*Cos[2*c + 3*d*x] + 77*Cos[4*c + 3*d*
x] + 231*Cos[4*c + 5*d*x] + (720*I)*Sin[d*x] - (720*I)*Sin[2*c + d*x]))/56
))/(45*d*Sec[c + d*x]^(11/2)*(a + I*a*Tan[c + d*x])^2)
```

3.233.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{11e^2\int (e\sec(c+dx))^{11/2}dx}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{11e^2\left(\frac{7}{9}e^2\int (e\sec(c+dx))^{7/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\left(\frac{7}{9}e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 4255$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\int(e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 3042$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}\right)}{7a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 4255$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{9/2}}{9d}}{7a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 3042$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+}{9d}}{7a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 4258$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx)}{9d}}{7a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 3042$$

3.233. $\int\frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+}{9d}}{7a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{11e^2\left(\frac{7}{9}e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx}{9d}}{7a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(11*e^2*((2*e*(e*Sec[c + d*x])^(9/2)*Sin[c + d*x])/(9*d) + (7*e^2*((2*e*(e
*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c +
d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[
c + d*x]]*Sin[c + d*x])/d))/5))/9))/(7*a^2) - (((4*I)/7)*e^2*(e*Sec[c + d*
x])^(11/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

3.233.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.233. $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.233.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(185) = 370$.

Time = 9.20 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.58

method	result
default	$-\frac{2ie^7\sqrt{e\sec(dx+c)}\left(-231\left(\cos^2(dx+c)\right)E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+231\left(\cos^2(dx+c)\right)F(i(-\mathrm{cs}\right.}{}$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/315*I*e^7/a^2/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(-231*cos(d*x+c)^2*
EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)+231*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x
+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-462*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+462*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2
31*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),
I)*(1/(cos(d*x+c)+1))^(1/2)+231*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+231*I*sin(d*x+c)+
77*I*tan(d*x+c)+77*I*tan(d*x+c)*sec(d*x+c)-35*I*tan(d*x+c)*sec(d*x+c)^2-35
*I*tan(d*x+c)*sec(d*x+c)^3+90*sec(d*x+c)^2+90*sec(d*x+c)^3)
```

3.233.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.40

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{2\left(\sqrt{2}(231i\,e^7e^{(9i\,dx+9i\,c)}+1078i\,e^7e^{(7i\,dx+7i\,c)}+1980i\,e^7e^{(5i\,dx+5i\,c)}+1770i\,e^7e^{(3i\,dx+3i\,c)}+77i\,e^7e^{(i\,dx+i\,c)}\right)}{315\,(a}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-2/315*(sqrt(2)*(231*I*e^7*e^(9*I*d*x + 9*I*c) + 1078*I*e^7*e^(7*I*d*x + 7
*I*c) + 1980*I*e^7*e^(5*I*d*x + 5*I*c) + 1770*I*e^7*e^(3*I*d*x + 3*I*c) +
77*I*e^7*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c) + 231*sqrt(2)*(I*e^7*e^(8*I*d*x + 8*I*c) + 4*I*e^7*e^(6*I*d*x +
6*I*c) + 6*I*e^7*e^(4*I*d*x + 4*I*c) + 4*I*e^7*e^(2*I*d*x + 2*I*c) + I*e^7
)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c
))))/(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^
(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.233.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.233.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.234 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx$

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3.234.1 Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}$$

output
```
6/7*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+18/35*e^3*(e*sec(d*x+c))^(7/
2)*sin(d*x+c)/a^2/d+6/7*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1
/2)/a^2/d-4/5*I*e^2*(e*sec(d*x+c))^(9/2)/d/(a^2+I*a^2*tan(d*x+c))
```

3.234.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{e^6\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-56i\cos(c+dx)+60\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\right.}{70a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e^6*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-56*I)*Cos[c + d*x] + 60*Cos[c
+ d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[c + d*x] + 15*Sin[3*(c + d*
x)]))/(70*a^2*d)
```

3.234.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{9e^2\int (e\sec(c+dx))^{9/2}dx}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\int (e\sec(c+dx))^{5/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 3042$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 4258$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 3042$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))}{7d}\right.}{5a^2}$$
$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow 3120$$

$$\frac{9e^2\left(\frac{5}{7}e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{7/2}}{7d}\right)}{5a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(9*e^2*((2*e*(e*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(7*d) + (5*e^2*((2*e^2*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) +
(2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/7))/(5*a^2) - (((4*I)/5)
*e^2*(e*Sec[c + d*x])^(9/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

3.234. $\int\frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.234.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.234.4 Maple [A] (verified)

Time = 8.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2 i e^{6} \sqrt{e \sec(dx+c)}\, \left(15 F(i(\csc(dx+c)-\cot(dx+c)),i) \sqrt{\frac{1}{\cos(dx+c)+1}}\, \sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\, \cos(dx+c)+15 F(i(\csc(dx+c)-\cot(dx+c)),i)\right.}{35 a^{2} d}$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/35*I*e^6/a^2/d*(e*sec(d*x+c))^(1/2)*(15*EllipticF(I*(csc(d*x+c)-cot(d*x
+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)+15*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+15*I*tan(d*x+c)+14*sec(d*x+c)^2-5*I*sec(d*x
+c)^2*tan(d*x+c))
```

3.234.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.32

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{2\left(\sqrt{2}(15i\,e^6e^{(6i\,dx+6i\,c)}+51i\,e^6e^{(4i\,dx+4i\,c)}+61i\,e^6e^{(2i\,dx+2i\,c)}-15i\,e^6)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}(i}{35\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}\right.}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-2/35*(sqrt(2)*(15*I*e^6*e^(6*I*d*x + 6*I*c) + 51*I*e^6*e^(4*I*d*x + 4*I*c
) + 61*I*e^6*e^(2*I*d*x + 2*I*c) - 15*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) + 15*sqrt(2)*(I*e^6*e^(6*I*d*x + 6*I*c) + 3*I
*e^6*e^(4*I*d*x + 4*I*c) + 3*I*e^6*e^(2*I*d*x + 2*I*c) + I*e^6)*sqrt(e)*we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*
a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.234.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.234.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{13}{2}}}{(i\,a\tan{(dx+c)}+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{(a+a\tan{(c+d\,x)}\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.235 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.235.1 Optimal result 1753
3.235.2 Mathematica [C] (verified) 1753
3.235.3 Rubi [A] (verified) 1754
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3.235.9 Mupad [F(-1)] 1759

3.235.1 Optimal result

Integrand size = 28, antiderivative size = 152

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = &-\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &+\frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} \\ &+\frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

output
```
14/15*e^3*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d-14/5*e^6*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d
/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+14/5*e^5*sin(d*x+c)*(e*sec(d*x+c))^
(1/2)/a^2/d-4/3*I*e^2*(e*sec(d*x+c))^(7/2)/d/(a^2+I*a^2*tan(d*x+c))
```

3.235.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.90 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2ie^5e^{i(c+dx)}\Big(-47-56e^{2i(c+dx)}-21e^{4i(c+dx)}+7\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeome}}{15a^2d\left(1+e^{2i(c+dx)}\right)^2}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(((2*I)/15)*e^5*E^(I*(c + d*x))*(-47 - 56*E^((2*I)*(c + d*x)) - 21*E^((4*I
)*(c + d*x)) + 7*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*(1 + E^((2*I)*
(c + d*x)))^2)
```

3.235.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7e^2\int (e\sec(c+dx))^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(7*e^2*((2*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*
EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (
2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/5))/(3*a^2) - (((4*I)/3)*e^2*(e
*Sec[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

3.235. $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.235.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.235.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(158) = 316$.

Time = 7.88 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.84

method	result
default	$\frac{2e^5\sqrt{e\sec(dx+c)}\left(21iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\left(\cos^2(dx+c)\right)-21iE(i(\csc(dx+c)-\cot(dx+c)),i)}{}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15*e^5/a^2/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*I*EllipticF(I*(csc(
d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)^2-21*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+42*I*cos(d*x
+c)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)-42*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*
x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*I*E
llipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)-21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*
(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-10*I+21*sin(d*x+c)-10*
I*sec(d*x+c)-3*tan(d*x+c)-3*sec(d*x+c)*tan(d*x+c))
```

3.235.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(21i\,e^5e^{(5i\,dx+5i\,c)}+56i\,e^5e^{(3i\,dx+3i\,c)}+47i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\,\sqrt{2}\big(i\,e^5e^{(4i\,dx+4i}}{15\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i}\right.}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-2/15*(sqrt(2)*(21*I*e^5*e^(5*I*d*x + 5*I*c) + 56*I*e^5*e^(3*I*d*x + 3*I*c
) + 47*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c) + 21*sqrt(2)*(I*e^5*e^(4*I*d*x + 4*I*c) + 2*I*e^5*e^(2*I*d*x
 + 2*I*c) + I*e^5)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, e^(I*d*x + I*c))))/(a^2*d*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*
I*c) + a^2*d)
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.235.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.235.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input

```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output

```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.236 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx$

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3.236.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d} - \frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}$$

output
```
10/3*e^3*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+10/3*e^4*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d-4*I*e^2*(e*sec(d*x+c))^(5/2)/d/(a^2+
I*a^2*tan(d*x+c))
```

3.236.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2e^3(e\sec(c+dx))^{3/2}\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.\right.}{3a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*e^3*(e*Sec[c + d*x])^(3/2)*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*
EllipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*a^2*d)
```

3.236.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3981, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5e^2\int (e\sec(c+dx))^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(5*e^2*((2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
 d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a^2 - ((
4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

3.236.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.236.4 Maple [A] (verified)

Time = 6.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

method	result
default	$\frac{2e^{4}\sqrt{e\sec(dx+c)}\,\left(5i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{}\right.}{3a^{2}d}$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*e^4/a^2/d*(e*sec(d*x+c))^(1/2)*(5*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c
)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c
)),I)*(1/(cos(d*x+c)+1))^(1/2)-6*I-tan(d*x+c))
```

3.236.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{2}}\,dx = -\frac{2\left(\sqrt{2}\left(5i\,e^{4}e^{(2i\,dx+2i\,c)}+7i\,e^{4}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\,\sqrt{2}\left(i\,e^{4}e^{(2i\,dx+2i\,c)}+i\,e^{4}\right)\sqrt{e}\mathrm{weierstrassPInvers}\right.}{3\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*(5*I*e^4*e^(2*I*d*x + 2*I*c) + 7*I*e^4)*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*sqrt(2)*(I*e^4*e^(2*I*d*x + 2*I*
c) + I*e^4)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.236.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.236.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.237 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

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3.237.1 Optimal result

Integrand size = 28, antiderivative size = 115

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
6*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-6*e^3*sin(d*
x+c)*(e*sec(d*x+c))^(1/2)/a^2/d+4*I*e^2*(e*sec(d*x+c))^(3/2)/d/(a^2+I*a^2*
tan(d*x+c))
```

3.237.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2ie^3e^{-i(c+dx)}\left(-1+3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((2*I)*e^3*(-1 + 3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*E^(I*(c + d*x
)))
```

3.237.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3981, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$-\frac{3e^2\int (e\sec(c+dx))^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4255}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

input

```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output

```
(-3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a^2 + ((4*I)*e
^2*(e*Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

3.237.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.237.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(131) = 262$.

Time = 8.13 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.75

method	result
default	$-\frac{2\left(3i\left(\cos^{2}(dx+c)\right)E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}-3i\left(\cos^{2}(dx+c)\right)F(i(-\csc(dx+c)+\cot(dx+c)),i}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2/d*(3*I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-3*I*EllipticF(I*(-csc(
d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)^2+6*I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-6*I*EllipticF(
I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)+3*I*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d
*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*(1/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)-2*I*cos(d*x+c)^2-2*I*cos(d*x+c)-2*sin(d*x+c)*cos(d*x+c)+sin(d*x+
c))*(e*sec(d*x+c))^(1/2)*e^3/(cos(d*x+c)+1)
```

3.237.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{2}}\,dx =$$

$$-\frac{2\left(-3i\sqrt{2}e^{\frac{7}{2}}e^{(i\,dx+i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(-3i\,e^{3}e^{(2i\,dx+2i\,c)}\right.\right.}{a^{2}d}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2*(-3*I*sqrt(2)*e^(7/2)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-3*I*e^3*e^(2*I*d*x + 2*I*c)
 - 2*I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(
-I*d*x - I*c)/(a^2*d)
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.237.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.237.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec\left(dx+c\right))^{\frac{7}{2}}}{\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.238 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.238.1 Optimal result 1772
3.238.2 Mathematica [A] (verified) 1772
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3.238.9 Mupad [F(-1)] 1776

3.238.1 Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
-2/3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d+4/3*I*e^2
*(e*sec(d*x+c))^(1/2)/d/(a^2+I*a^2*tan(d*x+c))
```

3.238.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2(e\sec(c+dx))^{5/2}\left(-2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{3a^2d(-i+\tan(c+d}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*(e*Sec[c + d*x])^(5/2)*((-2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]))*(Cos[c + d*x] + I*Sin
[c + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

3.238.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3981, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3981}$$

$$-\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c +
d*x]))
```

3.238.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.238.4 Maple [A] (verified)

Time = 7.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.82

method	result
default	$\frac{2e^2\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\right.}{3a^2d}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/a^2/d*e^2*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+I*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1
/2)+2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)
```

3.238.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(-i\sqrt{2}e^{\frac{5}{2}}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-i\,e^2e^{(2i\,dx+2i\,c)}-i\,e^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\right.}}{3\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(-I*sqrt(2)*e^(5/2)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e^
(I*d*x + I*c)) + sqrt(2)*(-I*e^2*e^(2*I*d*x + 2*I*c) - I*e^2)*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^2*d
)
```

3.238.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec{(c+dx)})^{\frac{5}{2}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1)
, x)/a**2
```

3.238.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.238.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.239 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.239.1 Optimal result 1777
3.239.2 Mathematica [C] (verified) 1777
3.239.3 Rubi [A] (verified) 1778
3.239.4 Maple [B] (verified) 1779
3.239.5 Fricas [C] (verification not implemented) 1780
3.239.6 Sympy [F] 1780
3.239.7 Maxima [F(-2)] 1781
3.239.8 Giac [F] 1781
3.239.9 Mupad [F(-1)] 1781

3.239.1 Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

output
```
2/5*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/5*I*e^2/
d/(e*sec(d*x+c))^(1/2)/(a^2+I*a^2*tan(d*x+c))
```

3.239.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{iee^{-3i(c+dx)}\Bigl(1+e^{2i(c+dx)}+2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\right.}{5a^2d}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I/5)*e*(1 + E^((2*I)*(c + d*x)) + 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])*Sqr
t[e*Sec[c + d*x]])/(a^2*d*E^((3*I)*(c + d*x)))
```

3.239.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3981, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^2}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a^2}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\int \sqrt{\cos(c+dx)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2e^2E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c + d
*x]))
```

3.239.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.239.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(104) = 208$.

Time = 7.07 (sec) , antiderivative size = 451, normalized size of antiderivative = 5.01

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\left(2i(\cos^3(dx+c))\sin(dx+c)+(\cos^2(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)-(\cos^2(d\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*I/a^2/d*(e*sec(d*x+c))^(1/2)*(2*I*cos(d*x+c)^3*sin(d*x+c)+cos(d*x+c)^
2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
csc(d*x+c)-cot(d*x+c)),I)-cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+2*I*cos(d*x
+c)^2*sin(d*x+c)-2*cos(d*x+c)^4+2*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*cos
(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cE(I*(csc(d*x+c)-cot(d*x+c)),I)+I*sin(d*x+c)*cos(d*x+c)-2*cos(d*x+c)^3+Ell
ipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)-(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I))*e/(cos(d*x+c)+1)
```

3.239.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\Big(2i\sqrt{2}e^{\frac{3}{2}}e^{(3i\,dx+3i\,c)}\text{weierstrassZeta}\big(-4,0,\text{weierstrassPInverse}\big(-4,0,e^{(i\,dx+i\,c}}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/5*(2*I*sqrt(2)*e^(3/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(2*I*e*e^(4*I*d*x + 4*I*c
) + 3*I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

3.239.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1)
, x)/a**2
```

3.239.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.239.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.240 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

3.240.1 Optimal result 1782
3.240.2 Mathematica [A] (verified) 1782
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3.240.1 Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/7*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(1/2)+2/7*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*(e*sec(d*x+c))^(1/2)/a^2/d+4/7*I*e^2/d/(e*sec(d*x+c))^(3/2)/(a^2+I*a^2*
tan(d*x+c))
```

3.240.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(2i+2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c -}{7a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/7*(Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] +
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2
*(c + d*x)]) - Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

3.240.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}$$

input

```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

output

```
(3*e^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) +
(((4*I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

3.240.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.240.4 Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\,\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}-2i\left(\cos^4(dx+c)\right)+iF(i(\csc(dx+c)-\cot(\right.}{7a^2d}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/7/a^2/d*(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*
cos(d*x+c)^4+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-c
ot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-2*cos(d*x+c)^3*sin(d*x+c)-sin(d*x+c
)*cos(d*x+c))
```

3.240.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}(3i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i)e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}-4i\,\sqrt{2}\sqrt{e}e^{(4i\,dx+4i\,c)}\text{weierstrassPInverse}(}{14\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/14*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(3*I*e^(4*I*d*x + 4*I*c) +
 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c) - 4*I*sqrt(2)*sqrt(e
)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-4*I
*d*x - 4*I*c)/(a^2*d)
```

3.240.6 Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sqrt{e\sec{(c+dx)}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x
)/a**2
```

3.240.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.240.8 Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx=\int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx=\int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.241 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

3.241.1 Optimal result 1788
3.241.2 Mathematica [C] (verified) 1788
3.241.3 Rubi [A] (verified) 1789
3.241.4 Maple [B] (verified) 1791
3.241.5 Fricas [C] (verification not implemented) 1792
3.241.6 Sympy [F] 1792
3.241.7 Maxima [F(-2)] 1793
3.241.8 Giac [F] 1793
3.241.9 Mupad [F(-1)] 1793

3.241.1 Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/9*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+
c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2/d/(e*sec(d*x+c))^(5/2)/(a^2+I*a^2*
tan(d*x+c))
```

3.241.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-\frac{8e^{4i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2(2+8\cos(2(c+dx))+7i\sin(2(c+dx)))\right)(i\cos(2(c+dx))}{18a^2d\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(((-8*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(2 + 8*Cos[2*(c + d*x)] + (7*I)*
Sin[2*(c + d*x)]))*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(18*a^2*d*Sqrt
[e*Sec[c + d*x]])
```

3.241.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{5e^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5e^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5e^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}$$

input

```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output

```
(5*e^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^2)
+ (((4*I)/9)*e^2)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

3.241.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.241. $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.241.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(126) = 252.

Time = 8.48 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.09

method	result
default	$-\frac{2i\left(2i\sin(dx+c)\left(\cos^{4}(dx+c)\right)+2i\left(\cos^{3}(dx+c)\right)\sin(dx+c)-2\left(\cos^{5}(dx+c)\right)+i\left(\cos^{2}(dx+c)\right)\sin(dx+c)-2\left(\cos^{4}(dx+c)\right)-3\sqrt{\frac{\cos(d}{\cos(dx}}\right.}{}$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/9*I/a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(2*I*sin(d*x+c)*cos(d*x+c
)^4+2*I*cos(d*x+c)^3*sin(d*x+c)-2*cos(d*x+c)^5+I*cos(d*x+c)^2*sin(d*x+c)-2
*cos(d*x+c)^4-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)
+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)+I*sin(d*x+c)*cos(d*x+c)-6*EllipticF(I*(-csc(d*x+c)+cot(d*x
+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(c
os(d*x+c)+1))^(1/2)+3*I*sin(d*x+c)-3*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+c
ot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3
*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
```

3.241.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(15i\,e^{(6i\,dx+6i\,c)}+19i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+24i\,\sqrt{2}\sqrt{e}e^{(5i\,dx+5i\,c)}\mathrm{we}\right.}{36\,a^2de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/36*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(15*I*e^(6*I*d*x + 6*I*c)
+ 19*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1
/2*I*c) + 24*I*sqrt(2)*sqrt(e)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-5*I*d*x - 5*I*c)/(a^2*d*
e)
```

3.241.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{\int \frac{1}{\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}-2i\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}-\sqrt{e\sec{(c+dx)}}}\,dx}{a^2}$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*sec(c + d*x
))*tan(c + d*x) - sqrt(e*sec(c + d*x))), x)/a**2
```

3.241.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.241.8 Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.242 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

3.242.1 Optimal result . . . 1794
3.242.2 Mathematica [A] (verified) . . . 1794
3.242.3 Rubi [A] (verified) . . . 1795
3.242.4 Maple [A] (verified) . . . 1798
3.242.5 Fricas [C] (verification not implemented) . . . 1798
3.242.6 Sympy [F] . . . 1799
3.242.7 Maxima [F(-2)] . . . 1799
3.242.8 Giac [F] . . . 1799
3.242.9 Mupad [F(-1)] . . . 1800

3.242.1 Optimal result

Integrand size = 28, antiderivative size = 150

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx &= \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2}\\ &+ \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}}\\ &+ \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

output
```
2/11*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(5/2)+10/33*sin(d*x+c)/a^2/d/e/(e*s
ec(d*x+c))^(1/2)+10/33*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a
^2/d/e^2+4/11*I*e^2/d/(e*sec(d*x+c))^(7/2)/(a^2+I*a^2*tan(d*x+c))
```

3.242.2 Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sec^4(c+dx)\left(28i+24i\cos(2(c+dx))-4i\cos(4(c+dx))+40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos}{132a^2d(e\sec(c+dx))^{3/2}(-i+\tan(c+}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
-1/132*(Sec[c + d*x]^4*(28*I + (24*I)*Cos[2*(c + d*x)] - (4*I)*Cos[4*(c +
d*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)]
+ I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)]))/(a^2*d*(
e*Sec[c + d*x])^(3/2)*(-I + Tan[c + d*x])^2)
```

3.242.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{7e^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7e^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{7e^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}$$

3.242. $\int\frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(7*e^2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*S
in[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*a^2) + (((4*I)/1
1)*e^2)/(d*(e*Sec[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

3.242.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.242.4 Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.27

method	result
default	$\frac{\frac{4i\left(\cos^{5}(dx+c)\right)}{11}+\frac{4\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{11}-\frac{10iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}}{33}+\frac{2\left(\cos^{2}(dx+c)\right)\sin(dx+c)}{11}-10i\,\mathrm{s}}{a^{2}d\sqrt{e\sec(dx+c)}\,e}$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/33/a^2/d/(e*sec(d*x+c))^(1/2)/e*(6*I*cos(d*x+c)^5+6*sin(d*x+c)*cos(d*x+c
)^4-5*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*sin(d*x+c)-5*I*sec(d*x+c)*E
llipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+5*sin(d*x+c))
```

3.242.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{2}}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(8i\,dx+8i\,c)}+30i\,e^{(6i\,dx+6i\,c)}+56i\,e^{(4}\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/264*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(8*I*d*x + 8*I*c
) + 30*I*e^(6*I*d*x + 6*I*c) + 56*I*e^(4*I*d*x + 4*I*c) + 18*I*e^(2*I*d*x
+ 2*I*c) + 3*I)*e^(1/2*I*d*x + 1/2*I*c) - 80*I*sqrt(2)*sqrt(e)*e^(6*I*d*x
+ 6*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-6*I*d*x - 6*I*c)
/(a^2*d*e^2)
```

3.242.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)-2i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)-(e\sec(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x)
)**(3/2)*tan(c + d*x) - (e*sec(c + d*x))**(3/2)), x)/a**2
```

3.242.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.242.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.243 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

3.243.1 Optimal result 1801
3.243.2 Mathematica [C] (verified) 1801
3.243.3 Rubi [A] (verified) 1802
3.243.4 Maple [B] (verified) 1805
3.243.5 Fricas [C] (verification not implemented) 1806
3.243.6 Sympy [F] 1806
3.243.7 Maxima [F(-2)] 1807
3.243.8 Giac [F] 1807
3.243.9 Mupad [F(-1)] 1807

3.243.1 Optimal result

Integrand size = 28, antiderivative size = 150

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx &= \frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &\quad+\frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}} \\ &\quad+\frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

output
```
2/13*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(7/2)+14/65*sin(d*x+c)/a^2/d/e/(e*s
ec(d*x+c))^(3/2)+42/65*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c
))^(1/2)+4/13*I*e^2/d/(e*sec(d*x+c))^(9/2)/(a^2+I*a^2*tan(d*x+c))
```

3.243.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{(\cos(2(c+dx))-i\sin(2(c+dx)))\Big(88i+416i\cos(2(c+dx}{}$$

input

```
Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output

```
((Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(88*I + (416*I)*Cos[2*(c + d*x)]
- (8*I)*Cos[4*(c + d*x)] - ((224*I)*E^((4*I)*(c + d*x))*Hypergeometric2F1[
1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - 356*
Sin[2*(c + d*x)] + 18*Sin[4*(c + d*x)]))/(520*a^2*d*e^2*Sqrt[e*Sec[c + d*x
]])
```

3.243.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{9e^2\int \frac{1}{(e\sec(c+dx))^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{9e^2\left(\frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\left(\frac{7\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \; 4256$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{9e^2\left(\frac{7\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \; 3119$$

3.243. $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

$$\frac{9e^2\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(9*e^2*((2*Sin[c + d*x])/(9*d*e*(e*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE
[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*S
in[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*e^2)))/(13*a^2) + (((4*I)
/13)*e^2)/(d*(e*Sec[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

3.243.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.243.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(156) = 312$.

Time = 11.16 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.41

method	result
default	$-\frac{2i\left(10i(\cos^{6}(dx+c)) \sin(dx+c)+10i(\cos^{5}(dx+c)) \sin(dx+c)-10(\cos^{7}(dx+c))+5i \sin(dx+c)(\cos^{4}(dx+c))-10(\cos^{6}(dx+c))+5i(\mathrm{c}\right.}{}$

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/65*I/a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(10*I*sin(d*x+c)*cos
(d*x+c)^6+10*I*cos(d*x+c)^5*sin(d*x+c)-10*cos(d*x+c)^7+5*I*sin(d*x+c)*cos(
d*x+c)^4-10*cos(d*x+c)^6+5*I*cos(d*x+c)^3*sin(d*x+c)+7*I*cos(d*x+c)^2*sin(
d*x+c)-21*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+21*EllipticF(I*(csc(d*x+c)-co
t(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)+7*I*cos(d*x+c)*sin(d*x+c)-42*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+42*EllipticF
(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+21*I*sin(d*x+c)-21*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+21*se
c(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x
+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
```

3.243.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(10i\,dx+10i\,c)}+373i\,e^{(8i\,dx+8i\,c)}+47\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/1040*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(10*I*d*x + 10*
I*c) + 373*I*e^(8*I*d*x + 8*I*c) + 474*I*e^(6*I*d*x + 6*I*c) + 118*I*e^(4*
I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c) +
 672*I*sqrt(2)*sqrt(e)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-7*I*d*x - 7*I*c)/(a^2*d*e^3)
```

3.243.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}\tan^2{(c+dx)}-2i(e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}-(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx}{a^2}$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x)
)**(5/2)*tan(c + d*x) - (e*sec(c + d*x))**(5/2)), x)/a**2
```

3.243.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.243.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.244 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

3.244.1 Optimal result . . . 1808
3.244.2 Mathematica [A] (verified) . . . 1808
3.244.3 Rubi [A] (verified) . . . 1809
3.244.4 Maple [A] (verified) . . . 1813
3.244.5 Fricas [C] (verification not implemented) . . . 1813
3.244.6 Sympy [F(-1)] . . . 1814
3.244.7 Maxima [F(-2)] . . . 1814
3.244.8 Giac [F] . . . 1815
3.244.9 Mupad [F(-1)] . . . 1815

3.244.1 Optimal result

Integrand size = 28, antiderivative size = 181

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4}$$
$$+\frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}}+\frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}}$$
$$+\frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/15*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(9/2)+6/35*sin(d*x+c)/a^2/d/e/(e*se
c(d*x+c))^(5/2)+2/7*sin(d*x+c)/a^2/d/e^3/(e*sec(d*x+c))^(1/2)+2/7*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d/e^4+4/15*I*e^2/d/(e*sec(d*
x+c))^(11/2)/(a^2+I*a^2*tan(d*x+c))
```

3.244.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=$$
$$-\frac{(e\sec(c+dx))^{5/2}\left(296i+228i\cos(2(c+dx))-72i\cos(4(c+dx))-4i\cos(6(c+dx))+480\sqrt{\cos(c+dx}\right.}{1680a}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
-1/1680*((e*Sec[c + d*x])^(5/2)*(296*I + (228*I)*Cos[2*(c + d*x)] - (72*I)
*Cos[4*(c + d*x)] - (4*I)*Cos[6*(c + d*x)] + 480*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - 17*Sin[2*(c
+ d*x)] + 128*Sin[4*(c + d*x)] + 11*Sin[6*(c + d*x)]))/(a^2*d*e^6*(-I + Ta
n[c + d*x])^2)
```

3.244.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3981, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{11e^2\int \frac{1}{(e\sec(c+dx))^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{11e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$\frac{11e^2\left(\frac{9\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{11e^2\left(\frac{9\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ 4256$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ 4256$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \ 3042$$

3.244. $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \; 3120$$

3.244. $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

$$\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(11*e^2*((2*Sin[c + d*x])/(11*d*e*(e*Sec[c + d*x])^(9/2)) + (9*((2*Sin[c +
 d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*
e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*e^2)))/(15*a^2) + (((4*I)/15)*e^2)
/(d*(e*Sec[c + d*x])^(11/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

3.244.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.244.4 Maple [A] (verified)

Time = 9.96 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{4i\left(\cos^{7}(dx+c)\right)}{15}+\frac{4\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{15}+\frac{2\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{15}+\frac{2iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{7}+\frac{6(c}{}}{a^{2}d\sqrt{e\sec(dx+c)}\,e^{3}}$

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/105/a^2/d/(e*sec(d*x+c))^(1/2)/e^3*(14*I*cos(d*x+c)^7+14*sin(d*x+c)*cos(
d*x+c)^6+7*sin(d*x+c)*cos(d*x+c)^4+15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c
)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+9*cos(d*x
+c)^2*sin(d*x+c)+15*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+15*sin(d*x+c))
```

3.244.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.82

$$\int\frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^{2}}\,dx=\frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(12i\,dx+12i\,c)}-200i\,e^{(10i\,dx+10i\,c)}+2\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/3360*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(12*I*d*x + 12*
I*c) - 200*I*e^(10*I*d*x + 10*I*c) + 245*I*e^(8*I*d*x + 8*I*c) + 592*I*e^(
6*I*d*x + 6*I*c) + 211*I*e^(4*I*d*x + 4*I*c) + 56*I*e^(2*I*d*x + 2*I*c) +
7*I)*e^(1/2*I*d*x + 1/2*I*c) - 960*I*sqrt(2)*sqrt(e)*e^(8*I*d*x + 8*I*c)*w
eierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-8*I*d*x - 8*I*c)/(a^2*d*e^
4)
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.244.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.244.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{7/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.245 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.245.1 Optimal result 1816
3.245.2 Mathematica [C] (verified) 1816
3.245.3 Rubi [A] (verified) 1817
3.245.4 Maple [B] (verified) 1820
3.245.5 Fricas [C] (verification not implemented) 1821
3.245.6 Sympy [F(-1)] 1822
3.245.7 Maxima [F(-2)] 1822
3.245.8 Giac [F] 1822
3.245.9 Mupad [F(-1)] 1823

3.245.1 Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

output
```
-22/21*I*e^4*(e*sec(d*x+c))^(7/2)/a^3/d+22/15*e^5*(e*sec(d*x+c))^(5/2)*sin
(d*x+c)/a^3/d-22/5*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(
1/2)+22/5*e^7*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^3/d-4/3*I*e^2*(e*sec(d*x+c
))^(11/2)/a/d/(a+I*a*tan(d*x+c))^2
```

3.245.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{e^6(e\sec(c+dx))^{3/2}\left(-556-868\cos(2(c+dx))+77e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.}{210a^3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/210*(e^6*(e*Sec[c + d*x])^(3/2)*(-556 - 868*Cos[2*(c + d*x)] + (77*(1 +
 E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
+ d*x))])/E^((2*I)*(c + d*x)) + (203*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (1
43*I)*Tan[c + d*x])*(-I + Tan[c + d*x]))/(a^3*d)
```

3.245.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{11e^2\int \frac{(e\sec(c+dx))^{11/2}}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\int \frac{(e\sec(c+dx))^{11/2}}{i\tan(c+dx)a+a}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{11e^2\left(\frac{e^2\int (e\sec(c+dx))^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11e^2\left(\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

3.245. $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{11e^2\left(\frac{e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

input
```
Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(11*e^2*((((-2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (e^2*((2*e*(e*Sec
[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)
/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/d))/5))/a))/(3*a^2) - (((4*I)/3)*e^2*(e*Sec[c + d*x])^
(11/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

3.245.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.245. $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.245.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(179) = 358.

Time = 9.28 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.55

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\,e^{7}\left(231iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^{2}(dx+c))-231iF(i(\csc(dx+c)-\cot(dx+\right.}{}$

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output

```
-2/105/a^3/d*(e*sec(d*x+c))^(1/2)*e^7/(cos(d*x+c)+1)*(231*I*cos(d*x+c)^2*E
llipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)-231*I*cos(d*x+c)^2*EllipticF(I*(csc(d*x+c)-cot(d*x+
c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+462*I*co
s(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-462*I*cos(d*x+c)*EllipticF(I*(csc(d*x+c)-
cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+
231*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)
),I)*(1/(cos(d*x+c)+1))^(1/2)-231*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+140*I-231*sin(
d*x+c)+140*I*sec(d*x+c)+63*tan(d*x+c)-15*I*sec(d*x+c)^2+63*sec(d*x+c)*tan(
d*x+c)-15*I*sec(d*x+c)^3)
```

3.245.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.20

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\sqrt{2}\left(231i\,e^7e^{(7i\,dx+7i\,c)}+847i\,e^7e^{(5i\,dx+5i\,c)}+1133i\,e^7e^{(3i\,dx+3i\,c)}+637i\,e^7e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}\right.}}{105\,(a^3de^{(6i\,dx+6i}}$$

input

```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output

```
-2/105*(sqrt(2)*(231*I*e^7*e^(7*I*d*x + 7*I*c) + 847*I*e^7*e^(5*I*d*x + 5*
I*c) + 1133*I*e^7*e^(3*I*d*x + 3*I*c) + 637*I*e^7*e^(I*d*x + I*c))*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(I*e^7*e^
(6*I*d*x + 6*I*c) + 3*I*e^7*e^(4*I*d*x + 4*I*c) + 3*I*e^7*e^(2*I*d*x + 2*I
*c) + I*e^7)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(
I*d*x + I*c))))/(a^3*d*e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) +
 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

3.245.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.245.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.246 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.246.1 Optimal result 1824
3.246.2 Mathematica [A] (verified) 1824
3.246.3 Rubi [A] (verified) 1825
3.246.4 Maple [A] (verified) 1828
3.246.5 Fricas [C] (verification not implemented) 1828
3.246.6 Sympy [F(-1)] 1829
3.246.7 Maxima [F(-2)] 1829
3.246.8 Giac [F] 1829
3.246.9 Mupad [F(-1)] 1830

3.246.1 Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^3d} - \frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

output
```
-18/5*I*e^4*(e*sec(d*x+c))^(5/2)/a^3/d+6*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+
c)/a^3/d+6*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d-4*I
*e^2*(e*sec(d*x+c))^(9/2)/a/d/(a+I*a*tan(d*x+c))^2
```

3.246.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{e^4(e\sec(c+dx))^{5/2}\big(-18i-20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\,(}{5a^3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(e^4*(e*Sec[c + d*x])^(5/2)*(-18*I - (20*I)*Cos[2*(c + d*x)] + 30*Cos[c +
d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*a^3*d)
```

3.246.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{9e^2\int \frac{(e\sec(c+dx))^{9/2}}{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\int \frac{(e\sec(c+dx))^{9/2}}{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{9e^2\left(\frac{e^2\int (e\sec(c+dx))^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\left(\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{9e^2\left(\frac{e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

input

```
Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(9*e^2*((((-2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (e^2*((2*e^2*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e
*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a))/a^2 - ((4*I)*e^2*(e*Sec[
c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

3.246. $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.246.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.246. $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.246.4 Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13

method	result
default	$\frac{2e^{6}\sqrt{e\sec(dx+c)}\left(15i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15iF(i(-\csc(dx+c)+\cot(dx+c)),i\right.}{5a^{3}d}$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*e^6/a^3/d*(e*sec(d*x+c))^(1/2)*(15*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+
c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)+15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-20*I-5*tan(d*x+c)+I*sec(d*x+c)^2)
```

3.246.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^{3}}\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(15i\,e^{6}e^{(4i\,dx+4i\,c)}+36i\,e^{6}e^{(2i\,dx+2i\,c)}+25i\,e^{6}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(i\,e^{6}e^{(4i\,dx+4i\,c)}+2i\,e^{6}\right.\right.}{5\left(a^{3}de^{(4i\,dx+4i\,c)}+2\,a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d\right)}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-2/5*(sqrt(2)*(15*I*e^6*e^(4*I*d*x + 4*I*c) + 36*I*e^6*e^(2*I*d*x + 2*I*c)
 + 25*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 1
5*sqrt(2)*(I*e^6*e^(4*I*d*x + 4*I*c) + 2*I*e^6*e^(2*I*d*x + 2*I*c) + I*e^6
)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^3*d*e^(4*I*d*x +
 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

3.246.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.246.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(ia\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.247 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.247.1 Optimal result 1831
3.247.2 Mathematica [C] (verified) 1831
3.247.3 Rubi [A] (verified) 1832
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3.247.8 Giac [F] 1837
3.247.9 Mupad [F(-1)] 1837

3.247.1 Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

output
```
14/3*I*e^4*(e*sec(d*x+c))^(3/2)/a^3/d+14*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^
(1/2)/(e*sec(d*x+c))^(1/2)-14*e^5*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^3/d+4*
I*e^2*(e*sec(d*x+c))^(7/2)/a/d/(a+I*a*tan(d*x+c))^2
```

3.247.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.85 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{ie^4(e\sec(c+dx))^{3/2}\left(35+33\cos(2(c+dx))-7\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeom}}{3a^3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/3)*e^4*(e*Sec[c + d*x])^(3/2)*(35 + 33*Cos[2*(c + d*x)] - 7*(1 + E^((2
*I)*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)] + (9*I)*Sin[2*(c + d*x)]))/(a^3*d)
```

3.247.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3982, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\int \frac{(e\sec(c+dx))^{7/2}}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\int \frac{(e\sec(c+dx))^{7/2}}{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int (e\sec(c+dx))^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \ 4258$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

$$\Big\downarrow \ 3119$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{7e^2\left(\frac{e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}\right)}{a^2}$$

input

```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(-7*e^2*((((-2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (e^2*((-2*e^2*Ell
ipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e
*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a))/a^2 + ((4*I)*e^2*(e*Sec[c + d*
x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

3.247. $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.247.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.247. $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.247.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(152) = 304.

Time = 10.35 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.15

method	result
default	$\frac{2\sqrt{e\sec(dx+c)}\, e^5 \left(21i(\cos^2(dx+c))F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-21i(\cos^2(dx+c))E(i(-\csc(dx+\right.}{}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/a^3/d*(e*sec(d*x+c))^(1/2)*e^5/(cos(d*x+c)+1)*(21*I*cos(d*x+c)^2*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*
x+c)+cot(d*x+c)),I)-21*I*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)+42*I*cos(d*
x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(
I*(-csc(d*x+c)+cot(d*x+c)),I)-42*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)+21*
I*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)-21*I*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc
(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+12*I*cos(d*x+c)^2
+12*I*cos(d*x+c)+12*sin(d*x+c)*cos(d*x+c)+I-9*sin(d*x+c)+I*sec(d*x+c))
```

3.247.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{2\left(\sqrt{2}(-21i\,e^5e^{(4i\,dx+4i\,c)}-35i\,e^5e^{(2i\,dx+2i\,c)}-12i\,e^5)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{(\frac{1}{2}i\,dx+\frac{1}{2}i\,c)}+21\sqrt{2}(-i\,e^5e^{(3i\,dx+3i\,c)}-\right.}{3\,(a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c}}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-2/3*(sqrt(2)*(-21*I*e^5*e^(4*I*d*x + 4*I*c) - 35*I*e^5*e^(2*I*d*x + 2*I*c
) - 12*I*e^5)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) +
21*sqrt(2)*(-I*e^5*e^(3*I*d*x + 3*I*c) - I*e^5*e^(I*d*x + I*c))*sqrt(e)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^3*d*
e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

3.247.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.247.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{(e\sec\left(dx+c\right))^{\frac{11}{2}}}{\left(i\,a\tan\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.248 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.248.1 Optimal result 1838
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3.248.1 Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

output
```
10/3*I*e^4*(e*sec(d*x+c))^(1/2)/a^3/d-10/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*(e*sec(d*x+c))^(1/2)/a^3/d+4/3*I*e^2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan
(d*x+c))^2
```

3.248.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2e^4\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\right.\right.}{3a}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(2*e^4*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-7*I)*Cos[c + d*x] + 5*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + 3
*Sin[c + d*x])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*a^3*d*(-I +
Tan[c + d*x])^3)
```

3.248.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3982, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{i\tan(c+dx)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{i\tan(c+dx)a+a}dx}{3a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

$$\Big\downarrow \textcolor{blue}{3120}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}\right)}{3a^2}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-5*e^2*(((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)))/(3*a^2) + (((4*
I)/3)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

3.248.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.248. $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx$

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.248.4 Maple [A] (verified)

Time = 9.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

method	result
default	$\frac{2\big(i(-5\cos(dx+c)-5)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)+i\big(4(\cos^2(dx+c))+3\big)+4\sin(dx+c)\cos(dx+c)}{3a^3d}$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/a^3/d*(I*(-5*cos(d*x+c)-5)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*(4*cos(d*x+c)^2+
3)+4*sin(d*x+c)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)*e^4
```

3.248.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{2\,\Big(-5i\,\sqrt{2}e^{\frac{9}{2}}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\big(-4,0,e^{(i\,dx+i\,c)}\big)+\sqrt{2}\big(-5i\,e^{4}e^{(2i\,dx+2i\,c)}-2i\,e^{4}\big)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}}{3\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-2/3*(-5*I*sqrt(2)*e^(9/2)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)) + sqrt(2)*(-5*I*e^4*e^(2*I*d*x + 2*I*c) - 2*I*e^4)*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/
(a^3*d)
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

3.248.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.248.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.249 $\int \frac{(e \sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.249.1 Optimal result 1844
3.249.2 Mathematica [C] (verified) 1844
3.249.3 Rubi [A] (verified) 1845
3.249.4 Maple [B] (verified) 1847
3.249.5 Fricas [C] (verification not implemented) 1848
3.249.6 Sympy [F(-1)] 1848
3.249.7 Maxima [F(-2)] 1848
3.249.8 Giac [F] 1849
3.249.9 Mupad [F(-1)] 1849

3.249.1 Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(e \sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}} - \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}$$

output
```
-6/5*I*e^4/a^3/d/(e*sec(d*x+c))^(1/2)-6/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)
^(1/2)/(e*sec(d*x+c))^(1/2)+4/5*I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(
d*x+c))^2
```

3.249.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{(e \sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2ee^{-idx}\left(-2+\frac{6e^{2i(c+dx)}\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)(e\sec(c+dx))^{5/2}(\cos}{5a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(2*e*(-2 + (6*E^((2*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2
*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))])*(e*Sec[c + d*x])^(5/2)*(Co
s[c + 2*d*x] + I*Sin[c + 2*d*x]))/(5*a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3
)
```

3.249.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3981, 3042, 3982, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\int \frac{(e\sec(c+dx))^{3/2}}{i\tan(c+dx)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\int \frac{(e\sec(c+dx))^{3/2}}{i\tan(c+dx)a+a}dx}{5a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{a}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int\sqrt{\cos(c+dx)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow 3042$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

$$\Big\downarrow 3119$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{3e^2\left(\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}\right)}{5a^2}$$

input

```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(-3*e^2*(((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*
x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])))/(5*a^2) + (((4*I
)/5)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

3.249.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.249.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(125) = 250$.

Time = 8.08 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.08

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\,(4i(\cos^{3}(dx+c))\sin(dx+c)-3(\cos^{2}(dx+c))E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+3(\cos}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*I/a^3/d*(e*sec(d*x+c))^(1/2)*(4*I*sin(d*x+c)*cos(d*x+c)^3-3*cos(d*x+c
)^2*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d
*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+4*I*c
os(d*x+c)^2*sin(d*x+c)-4*cos(d*x+c)^4-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c))
,I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*I*cos(d*x+c)*sin(d*x+c)-4*cos(d*
x+c)^3-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*
x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*cos(d*x+c)^
2+5*cos(d*x+c))*e^3/(cos(d*x+c)+1)
```

3.249.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{2\Big(3i\sqrt{2}e^{\frac{7}{2}}e^{(3i\,dx+3i\,c)}\text{weierstrassZeta}\big(-4,0,\text{weierstrassPInverse}\big(-4,0,e^{(i\,dx+i\,c)}\big)\big)+\sqrt{2}\big(3i\,e^{3}e^{(4i\,dx+4i\,c)}+}{5\,a^{3}d}$$

input

```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
-2/5*(3*I*sqrt(2)*e^(7/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(3*I*e^3*e^(4*I*d*x + 4*
I*c) + 2*I*e^3*e^(2*I*d*x + 2*I*c) - I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input

```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**3,x)
```

output

```
Timed out
```

3.249.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.249.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.250 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.250.1 Optimal result 1850
3.250.2 Mathematica [A] (verified) 1850
3.250.3 Rubi [A] (verified) 1851
3.250.4 Maple [A] (verified) 1853
3.250.5 Fricas [C] (verification not implemented) 1853
3.250.6 Sympy [F] 1854
3.250.7 Maxima [F(-2)] 1854
3.250.8 Giac [F] 1855
3.250.9 Mupad [F(-1)] 1855

3.250.1 Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^3d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\,(a^3+ia^3\tan(c+dx))}$$

output
```
-2/21*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d+4/7*I*e^
2*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^2-2/21*I*e^2*(e*sec(d*x+c))^
(1/2)/d/(a^3+I*a^3*tan(d*x+c))
```

3.250.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{(e\sec(c+dx))^{5/2}\Big(-5i-5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\big(\frac{1}{2}(c+}{21a^3d(-i+\tan(c+}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((e*Sec[c + d*x])^(5/2)*(-5*I - (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - Si
n[2*(c + d*x)]))/(21*a^3*d*(-I + Tan[c + d*x])^2)
```

3.250.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3983, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{i\tan(c+dx)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{i\tan(c+dx)a+a}dx}{7a^2}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}-\frac{e^2\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\right)}{7a^2}$$

input

```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
(((4*I)/7)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^2) - (e^2
*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3
*a*d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))))/(7*a
^2)
```

3.250.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.250. $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.250.4 Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

method	result
default	$\frac{2e^2\Big(12i(\cos^4(dx+c))-i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-iF(i(-\csc(dx+c)+\cot(dx+c)),i)}{21a^3d}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/21/a^3/d*e^2*(12*I*cos(d*x+c)^4-I*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*
(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+12*cos(d*x+c)^3*sin(d*x+c)-7*I*cos(d*x+c)^2-sin(d
*x+c)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)
```

3.250.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\Big(2i\sqrt{2}e^{\frac{5}{2}}e^{(4i\,dx+4i\,c)}\mathrm{weierstrassPInverse}\big(-4,0,e^{(i\,dx+i\,c)}\big)+\sqrt{2}\big(2i\,e^2e^{(4i\,dx+4i\,c)}}{21\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/21*(2*I*sqrt(2)*e^(5/2)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e
^(I*d*x + I*c)) + sqrt(2)*(2*I*e^2*e^(4*I*d*x + 4*I*c) + 5*I*e^2*e^(2*I*d*
x + 2*I*c) + 3*I*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c))*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

3.250.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2
- 3*tan(c + d*x) + I), x)/a**3
```

3.250.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.250.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec\left(dx+c\right))^{\frac{5}{2}}}{\left(i\,a\tan\left(dx+c\right)+a\right)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.251 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx$

3.251.1 Optimal result . . . 1856
3.251.2 Mathematica [C] (verified) . . . 1856
3.251.3 Rubi [A] (verified) . . . 1857
3.251.4 Maple [B] (verified) . . . 1859
3.251.5 Fricas [C] (verification not implemented) . . . 1860
3.251.6 Sympy [F] . . . 1861
3.251.7 Maxima [F(-2)] . . . 1861
3.251.8 Giac [F] . . . 1861
3.251.9 Mupad [F(-1)] . . . 1862

3.251.1 Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan(c+dx))}$$

output
```
2/15*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2
/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2/45*I*e^2/d/(e*sec(d*x+c))
^(1/2)/(a^3+I*a^3*tan(d*x+c))
```

3.251.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{e^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\left(8+8\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\right.}{45a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/45*(Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(8 +
8*Cos[2*(c + d*x)] + 6*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c +
d*x)]))/(a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

3.251.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3981, 3042, 3983, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)}dx}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)}dx}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{e^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{9a^2}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((4*I)/9)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2) + (e^2
*((6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))))/(9*a^2
)
```

3.251.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.251. $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx$

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.251.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(140) = 280.

Time = 7.50 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.84

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\,\big(20i(\cos^5(dx+c))\sin(dx+c)+20i\sin(dx+c)(\cos^4(dx+c))-20(\cos^6(dx+c))+i(\cos^3(dx+c))\sin(dx+c)-20(\cos^5(}{}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/45*I/a^3/d*(e*sec(d*x+c))^(1/2)*(20*I*cos(d*x+c)^5*sin(d*x+c)+20*I*cos(
d*x+c)^4*sin(d*x+c)-20*cos(d*x+c)^6+I*sin(d*x+c)*cos(d*x+c)^3-20*cos(d*x+c
)^5-3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*EllipticE(I*(-c
sc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)+I*cos(d*x+c)^2*sin(d*x+c)+9*cos(d*x+c)^4-6*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c
)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*I*sin(d*x+c)*cos(d*
x+c)+9*cos(d*x+c)^3-3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
*e/(cos(d*x+c)+1)
```

3.251.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(12i\sqrt{2}e^{\frac{3}{2}}e^{(5i\,dx+5i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+}\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/90*(12*I*sqrt(2)*e^(3/2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(12*I*e*e^(6*I*d*x + 6*
I*c) + 23*I*e*e^(4*I*d*x + 4*I*c) + 16*I*e*e^(2*I*d*x + 2*I*c) + 5*I*e)*sq
rt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-5*I*d*x - 5*I
*c)/(a^3*d)
```

3.251.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2
- 3*tan(c + d*x) + I), x)/a**3
```

3.251.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.251.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(ia\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

input

```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output

```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.252 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$

3.252.1 Optimal result . . . 1863
3.252.2 Mathematica [A] (verified) . . . 1863
3.252.3 Rubi [A] (verified) . . . 1864
3.252.4 Maple [A] (verified) . . . 1867
3.252.5 Fricas [C] (verification not implemented) . . . 1867
3.252.6 Sympy [F] . . . 1868
3.252.7 Maxima [F(-2)] . . . 1868
3.252.8 Giac [F] . . . 1868
3.252.9 Mupad [F(-1)] . . . 1869

3.252.1 Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^3d} + \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
10/77*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(1/2)+10/77*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d+2/11*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan
(d*x+c))^3+20/77*I*e^2/d/(e*sec(d*x+c))^(3/2)/(a^3+I*a^3*tan(d*x+c))
```

3.252.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec^3(c+dx)\sqrt{e\sec(c+dx)}\,(46i\cos(c+dx)+22i\cos(3(c+dx))-15\sin(c+dx)+20\sqrt{\cos(c+dx)}\,\mathrm{El}}{154a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I/154)*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((46*I)*Cos[c + d*x] + (22*I)
*Cos[3*(c + d*x)] - 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)])
)/(a^3*d*(-I + Tan[c + d*x])^3)
```

3.252.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3983, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3981}$$

$$\frac{5\left(\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4256}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5\left(\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

3.252. $\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(((2*I)/11)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (5*((3*e^
2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) + (((4*
I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(11*a)
```

3.252.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.252.4 Maple [A] (verified)

Time = 6.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2i\big((-5\cos(dx+c)-5)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)+i\sin(dx+c)\cos(dx+c)\big(28\big(\cos^4(dx+c)\big)+3\big(}{77a^3d}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/77*I/a^3/d*((-5*cos(d*x+c)-5)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*sin(d*x+c)*cos
(d*x+c)*(28*cos(d*x+c)^4+3*cos(d*x+c)^2+5)+cos(d*x+c)^4*(-28*cos(d*x+c)^2+
11))*(e*sec(d*x+c))^(1/2)
```

3.252.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{\Big(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(37i\,e^{(6i\,dx+6i\,c)}+61i\,e^{(4i\,dx+4i\,c)}+31i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-40i\,\sqrt{2}\sqrt{e}e^{(6i\,dx+6i\,c)}}{308\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/308*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(37*I*e^(6*I*d*x + 6*I*c)
 + 61*I*e^(4*I*d*x + 4*I*c) + 31*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x
 + 1/2*I*c) - 40*I*sqrt(2)*sqrt(e)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse
(-4, 0, e^(I*d*x + I*c)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

3.252.6 Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sqrt{e\sec(c+dx)}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3
*tan(c + d*x) + I), x)/a**3
```

3.252.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.252.8 Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\sqrt{e\sec(dx+c)}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^3, x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx=\int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.253 $\int \frac{1}{\sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

3.253.1 Optimal result 1870
3.253.2 Mathematica [C] (verified) 1871
3.253.3 Rubi [A] (verified) 1871
3.253.4 Maple [B] (verified) 1874
3.253.5 Fricas [C] (verification not implemented) 1875
3.253.6 Sympy [F] 1876
3.253.7 Maxima [F(-2)] 1876
3.253.8 Giac [F] 1876
3.253.9 Mupad [F(-1)] 1877

3.253.1 Optimal result

Integrand size = 28, antiderivative size = 152

$$\begin{aligned}\int \frac{1}{\sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx &= \frac{14E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &\quad+ \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} \\ &\quad+ \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} \\ &\quad+ \frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

output
```
14/117*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(3/2)+14/39*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos
(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/13*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*ta
n(d*x+c))^3+28/117*I*e^2/d/(e*sec(d*x+c))^(5/2)/(a^3+I*a^3*tan(d*x+c))
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{\sqrt{e\sec(c+dx)}(i\cos(3(c+dx))+\sin(3(c+dx)))\left(62+176\cos(2(c+dx))+114\cos(4(c+dx))-56e^{4i}\right.}{468a}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
(Sqrt[e*Sec[c + d*x]]*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 + 176*Co
s[2*(c + d*x)] + 114*Cos[4*(c + d*x)] - 56*E^((4*I)*(c + d*x))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
+ (126*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(468*a^3*d*e)
```

3.253.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3983, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{7\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 3981$$

$$\frac{7\left(\frac{5e^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{7\left(\frac{5e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 4256$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 4258$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

3.253. $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5e^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7\left(\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

input

```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

output

```
((2*I)/13)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (7*((5*e^2*
((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^2) + (((4*
I)/9)*e^2)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(13*a)
```

3.253.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.253. $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.253.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(156) = 312.

Time = 8.70 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.47

method	result
default	$-\frac{2i\left(7i(\cos^2(dx+c))\sin(dx+c)+5i\sin(dx+c)(\cos^4(dx+c))-36(\cos^7(dx+c))+36i(\cos^6(dx+c))\sin(dx+c)-36(\cos^6(dx+c))+5i(c}{}$

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/117*I/a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(7*I*cos(d*x+c)^2*sin(d
*x+c)+5*I*sin(d*x+c)*cos(d*x+c)^4-36*cos(d*x+c)^7+36*I*cos(d*x+c)^6*sin(d*
x+c)-36*cos(d*x+c)^6+5*I*cos(d*x+c)^3*sin(d*x+c)+13*cos(d*x+c)^5+21*I*sin(
d*x+c)+13*cos(d*x+c)^4+21*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-
csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-21*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)+7*I*cos(d*x+c)*sin(d*x+c)+42*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))
^(1/2)-42*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+36*I*cos(d*x+c)^5*sin(d*x+c)+21*sec(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c))
,I)*(1/(cos(d*x+c)+1))^(1/2)-21*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*
x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

3.253.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(219i\,e^{(8i\,dx+8i\,c)}+302i\,e^{(6i\,dx+6i\,c)}+124i\,e^{(4i\,dx+4i\,c)}+50i\,e^{(2i\,dx+2i\,c)}+9i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+}{936\,a^3de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas
")
```

output
```
1/936*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(219*I*e^(8*I*d*x + 8*I*c
) + 302*I*e^(6*I*d*x + 6*I*c) + 124*I*e^(4*I*d*x + 4*I*c) + 50*I*e^(2*I*d*
x + 2*I*c) + 9*I)*e^(1/2*I*d*x + 1/2*I*c) + 336*I*sqrt(2)*sqrt(e)*e^(7*I*d
*x + 7*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I
*c))))*e^(-7*I*d*x - 7*I*c)/(a^3*d*e)
```

3.253.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{1}{\sqrt{e\sec(c+dx)}\tan^3(c+dx)-3i\sqrt{e\sec(c+dx)}\tan^2(c+dx)-3\sqrt{e\sec(c+dx)}\tan(c+dx)+i\sqrt{e\sec(c+dx)}}\,dx}{a^3}$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**3 - 3*I*sqrt(e*sec(c + d*
x))*tan(c + d*x)**2 - 3*sqrt(e*sec(c + d*x))*tan(c + d*x) + I*sqrt(e*sec(c
 + d*x))), x)/a**3
```

3.253.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.253.8 Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.254 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$

3.254.1 Optimal result . . . 1878
3.254.2 Mathematica [A] (verified) . . . 1878
3.254.3 Rubi [A] (verified) . . . 1879
3.254.4 Maple [A] (verified) . . . 1883
3.254.5 Fricas [C] (verification not implemented) . . . 1884
3.254.6 Sympy [F] . . . 1884
3.254.7 Maxima [F(-2)] . . . 1885
3.254.8 Giac [F] . . . 1885
3.254.9 Mupad [F(-1)] . . . 1885

3.254.1 Optimal result

Integrand size = 28, antiderivative size = 186

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2}+\frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+\frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}}+\frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}+\frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
6/55*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(5/2)+2/11*sin(d*x+c)/a^3/d/e/(e*se
c(d*x+c))^(1/2)+2/11*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3
/d/e^2+2/15*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3+12/55*I*e^2/d/(e
*sec(d*x+c))^(7/2)/(a^3+I*a^3*tan(d*x+c))
```

3.254.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx=\frac{\sec^5(c+dx)\Big(-332\cos(c+dx)-154\cos(3(c+dx))+22\,\mathrm{c}}{}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
(Sec[c + d*x]^5*(-332*Cos[c + d*x] - 154*Cos[3*(c + d*x)] + 22*Cos[5*(c +
d*x)] - (114*I)*Sin[c + d*x] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (81*I)*Sin[3*(c + d*x)
] + (33*I)*Sin[5*(c + d*x)]))/(1320*a^3*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan
[c + d*x])^3)
```

3.254.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3983, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow 3983$$

$$\frac{3\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{3\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3981$$

$$\frac{3\left(\frac{7e^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{3\left(\frac{7e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.254. $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right)}{5a}+\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3120$$

3.254. $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$

$$\frac{3\left(\frac{7e^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{7/2}}\right.}{5a}$$

$$\frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
((2*I)/15)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3) + (3*((7*e^
2*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c
+ d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))/(11*a^2) + (((4*I)/11)*e^
2)/(d*(e*Sec[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(5*a)
```

3.254.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.254.4 Maple [A] (verified)

Time = 9.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

method	result
default	$\frac{\frac{8i\left(\cos^{7}(dx+c)\right)}{15}+\frac{8\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{15}-\frac{2i\left(\cos^{5}(dx+c)\right)}{11}+\frac{14\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{165}+\frac{2iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{}}{11}}{a^{3}d\sqrt{e\sec(dx+}}$

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/165/a^3/d/(e*sec(d*x+c))^(1/2)/e*(44*I*cos(d*x+c)^7+44*sin(d*x+c)*cos(d*
x+c)^6-15*I*cos(d*x+c)^5+7*sin(d*x+c)*cos(d*x+c)^4+15*I*EllipticF(I*(-csc(
d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)+9*cos(d*x+c)^2*sin(d*x+c)+15*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+
15*sin(d*x+c))
```

3.254.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-55i\,e^{(10i\,dx+10i\,c)}+235i\,e^{(8i\,dx+8i\,c)}+446\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas
")
```

output
```
1/2640*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-55*I*e^(10*I*d*x + 10*
I*c) + 235*I*e^(8*I*d*x + 8*I*c) + 446*I*e^(6*I*d*x + 6*I*c) + 218*I*e^(4*
I*d*x + 4*I*c) + 73*I*e^(2*I*d*x + 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c)
- 480*I*sqrt(2)*sqrt(e)*e^(8*I*d*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)))*e^(-8*I*d*x - 8*I*c)/(a^3*d*e^2)
```

3.254.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)-3i(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)-3(e\sec(c+dx))}}{a^3}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**3 - 3*I*(e*sec(c + d*x
))**(3/2)*tan(c + d*x)**2 - 3*(e*sec(c + d*x))**(3/2)*tan(c + d*x) + I*(e*
sec(c + d*x))**(3/2)), x)/a**3
```

3.254.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.254.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}\,(ia\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.255 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.255.1 Optimal result 1886
3.255.2 Mathematica [C] (verified) 1886
3.255.3 Rubi [A] (verified) 1887
3.255.4 Maple [B] (verified) 1890
3.255.5 Fricas [C] (verification not implemented) 1891
3.255.6 Sympy [F(-1)] 1892
3.255.7 Maxima [F(-2)] 1892
3.255.8 Giac [F] 1892
3.255.9 Mupad [F(-1)] 1893

3.255.1 Optimal result

Integrand size = 28, antiderivative size = 192

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx &= \frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &\quad-\frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d}-\frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d}\\ &\quad+\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}+\frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
-154/15*e^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^4/d+154/5*e^8*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^
4/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-154/5*e^7*sin(d*x+c)*(e*sec(d*x+
c))^(1/2)/a^4/d+4*I*e^2*(e*sec(d*x+c))^(11/2)/a/d/(a+I*a*tan(d*x+c))^3+44/
3*I*e^4*(e*sec(d*x+c))^(7/2)/d/(a^4+I*a^4*tan(d*x+c))
```

3.255.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{ie^5(e\sec(c+dx))^{5/2}\left(-1133\cos(c+dx)+77e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(}\right.\right.}{30a^4d}$$

input
```
Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((-1/30*I)*e^5*(e*Sec[c + d*x])^(5/2)*(-1133*Cos[c + d*x] + (77*(1 + E^((2
*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])/E^(I*(c + d*x)) - 3*(117*Cos[3*(c + d*x)] + (33*I)*Sin[c + d*x] + (37*
I)*Sin[3*(c + d*x)])))/(a^4*d)
```

3.255.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\int \frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\int \frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\int (e\sec(c+dx))^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\int(e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

3.255. $\int\frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{11e^2\left(\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

input
```
Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((4*I)*e^2*(e*Sec[c + d*x])^(11/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (11*e
^2*((7*e^2*((2*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*
e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
 + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/5))/(3*a^2) - (((4*I)/3)*e^
2*(e*Sec[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))))/a^2
```

3.255.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.255.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(194) = 388$.

Time = 11.66 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.43

method	result
default	$2\sqrt{e\sec(dx+c)}\,e^{7}(-231i(\cos^{2}(dx+c))E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+231i(\cos^{2}(dx+c))F(i(-\csc($

input
```
int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/15/a^4/d*(e*sec(d*x+c))^(1/2)*e^7/(cos(d*x+c)+1)*(-231*I*cos(d*x+c)^2*El
lipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)+231*I*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x
+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-462*I*c
os(d*x+c)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+462*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+
c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)-231*I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+231*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c
)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+120*I*cos
(d*x+c)^2+120*sin(d*x+c)*cos(d*x+c)+120*I*cos(d*x+c)-111*sin(d*x+c)+20*I+3
*tan(d*x+c)+20*I*sec(d*x+c)+3*sec(d*x+c)*tan(d*x+c))
```

3.255.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(\sqrt{2}\left(-231i\,e^7e^{(6i\,dx+6i\,c)}-616i\,e^7e^{(4i\,dx+4i\,c)}-517i\,e^7e^{(2i\,dx+2i\,c)}-120i\,e^7\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+23\right.}{15\left(a^4de^{(5i\,dx+5i\,c)}\right.}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-2/15*(sqrt(2)*(-231*I*e^7*e^(6*I*d*x + 6*I*c) - 616*I*e^7*e^(4*I*d*x + 4*
I*c) - 517*I*e^7*e^(2*I*d*x + 2*I*c) - 120*I*e^7)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(-I*e^7*e^(5*I*d*x + 5*I*c
) - 2*I*e^7*e^(3*I*d*x + 3*I*c) - I*e^7*e^(I*d*x + I*c))*sqrt(e)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^4*d*e^(5*I*
d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

3.255.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.255.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

input
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.256 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.256.1 Optimal result 1894
3.256.2 Mathematica [A] (verified) 1895
3.256.3 Rubi [A] (verified) 1895
3.256.4 Maple [A] (verified) 1898
3.256.5 Fricas [C] (verification not implemented) 1898
3.256.6 Sympy [F(-1)] 1899
3.256.7 Maxima [F(-2)] 1899
3.256.8 Giac [F] 1900
3.256.9 Mupad [F(-1)] 1900

3.256.1 Optimal result

Integrand size = 28, antiderivative size = 157

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = &-\frac{10e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^4d}\\ &-\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d}\\ &+\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output

```
-10*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^4/d-10*e^6*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c
)^(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/3*I*e^2*(e*sec(d*x+c))^(9/2)/a/d/(a+I
*a*tan(d*x+c))^3+12*I*e^4*(e*sec(d*x+c))^(5/2)/d/(a^4+I*a^4*tan(d*x+c))
```

3.256.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{ie^6\sec^5(c+dx)\sqrt{e\sec(c+dx)}\Big(21+19\cos(2(c+dx))+30i\cos^{\frac{3}{2}}(c+dx)\,\mathrm{Ell}}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/3)*e^6*Sec[c + d*x]^5*Sqrt[e*Sec[c + d*x]]*(21 + 19*Cos[2*(c + d*x)] +
 (30*I)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin
[c + d*x]) + (11*I)*Sin[2*(c + d*x)])*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x
)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

3.256.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\int(e\sec(c+dx))^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\int\sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

$$\Big\downarrow \text{3120}$$

3.256. $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$

$$\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{3e^2\left(\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{a^2}$$

input
```
Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/3)*e^2*(e*Sec[c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (3
*e^2*((5*e^2*((2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*S
ec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a^
2 - ((4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))))/a^
2
```

3.256.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

3.256. $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.256.4 Maple [A] (verified)

Time = 10.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
default	$\frac{2e^{6}\left(-15i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}-15iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)}}\right.}{3a^{4}d}$

input
```
int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/3/a^4/d*e^6*(-15*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d
*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-15*I*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+
c)+1))^(1/2)+8*sin(d*x+c)*cos(d*x+c)+8*I*cos(d*x+c)^2+tan(d*x+c)+12*I)*(e*
sec(d*x+c))^(1/2)
```

3.256.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^{4}}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-15i\,e^{6}e^{(4i\,dx+4i\,c)}-21i\,e^{6}e^{(2i\,dx+2i\,c)}-4i\,e^{6}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(-i\,e^{6}e^{(4i\,dx+4i\,c)}-i\right.\right.}{3\left(a^{4}de^{(4i\,dx+4i\,c)}+a^{4}de^{(2i\,dx+2i\,c)}\right)}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-2/3*(sqrt(2)*(-15*I*e^6*e^(4*I*d*x + 4*I*c) - 21*I*e^6*e^(2*I*d*x + 2*I*c
) - 4*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 1
5*sqrt(2)*(-I*e^6*e^(4*I*d*x + 4*I*c) - I*e^6*e^(2*I*d*x + 2*I*c))*sqrt(e)
*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^4*d*e^(4*I*d*x + 4*I*c) +
 a^4*d*e^(2*I*d*x + 2*I*c))
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

3.256.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.256.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{(e\sec\left(dx+c\right))^{\frac{13}{2}}}{\left(i\,a\tan\left(dx+c\right)+a\right)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

input
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.257 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.257.1 Optimal result 1901
3.257.2 Mathematica [C] (verified) 1901
3.257.3 Rubi [A] (verified) 1902
3.257.4 Maple [B] (verified) 1905
3.257.5 Fricas [C] (verification not implemented) 1905
3.257.6 Sympy [F(-1)] 1906
3.257.7 Maxima [F(-2)] 1906
3.257.8 Giac [F] 1906
3.257.9 Mupad [F(-1)] 1907

3.257.1 Optimal result

Integrand size = 28, antiderivative size = 163

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &+\frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d}\\ &+\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
-42/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+42/5*e^5
*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^4/d+4/5*I*e^2*(e*sec(d*x+c))^(7/2)/a/d/
(a+I*a*tan(d*x+c))^3-28/5*I*e^4*(e*sec(d*x+c))^(3/2)/d/(a^4+I*a^4*tan(d*x+
c))
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{2ie^5e^{-3i(c+dx)}\left(-2-7e^{2i(c+dx)}+21e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e\,s}}{5a^4d}$$

input
```
Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((-2*I)/5)*e^5*(-2 - 7*E^((2*I)*(c + d*x)) + 21*E^((2*I)*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))])*Sqrt[e*Sec[c + d*x]])/(a^4*d*E^((3*I)*(c + d*x)))
```

3.257.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3981, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\int \frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\int \frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{5a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\int (e\sec(c+dx))^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}-\frac{7e^2\left(-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}\right)}{5a^2}$$

input

```
Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

3.257. $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$

output
```
(((4*I)/5)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (7
*e^2*((-3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sq
rt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a^2 + ((
4*I)*e^2*(e*Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))))/(5*a^2)
```

3.257.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.257. $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.257.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(167) = 334$.

Time = 9.68 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.96

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\,(21(\cos^2(dx+c))F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}-21(\cos^2(dx+c))E(i(-\csc(dx+c}{}$

input
```
int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5*I/a^4/d*(e*sec(d*x+c))^(1/2)*(21*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c
)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)-21*cos(d*x+c)^2*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+8*I*sin(d*x+c)*cos(d*x+c)^3+42*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*
(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-42*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
-8*cos(d*x+c)^4+8*I*sin(d*x+c)*cos(d*x+c)^2+21*EllipticF(I*(-csc(d*x+c)+co
t(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)
*(1/(cos(d*x+c)+1))^(1/2)-8*cos(d*x+c)^3-16*I*sin(d*x+c)*cos(d*x+c)+20*cos
(d*x+c)^2+5*I*sin(d*x+c)+20*cos(d*x+c))*e^5/(cos(d*x+c)+1)
```

3.257.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{2\left(21i\sqrt{2}e^{\frac{11}{2}}e^{(3i\,dx+3i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}(21i\,e^5e^{(4i\,dx+4i\,c}}{5\,a^4d}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-2/5*(21*I*sqrt(2)*e^(11/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(21*I*e^5*e^(4*I*d*x +
 4*I*c) + 14*I*e^5*e^(2*I*d*x + 2*I*c) - 2*I*e^5)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

3.257.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.257.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{11}{2}}}{(i\,a\tan{(dx+c)}+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{11/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4} \,d x$$

input
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.258 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.258.1 Optimal result 1908

3.258.2 Mathematica [A] (verified) 1908

3.258.3 Rubi [A] (verified) 1909

3.258.4 Maple [A] (verified) 1911

3.258.5 Fricas [C] (verification not implemented) 1911

3.258.6 Sympy [F(-1)] 1912

3.258.7 Maxima [F(-2)] 1912

3.258.8 Giac [F] 1912

3.258.9 Mupad [F(-1)] 1913

3.258.1 Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^4d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
10/21*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/7*I*e^
2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan(d*x+c))^3-20/21*I*e^4*(e*sec(d*x+c))
^(1/2)/d/(a^4+I*a^4*tan(d*x+c))
```

3.258.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2e^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(2*e^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(5*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - (2*I)*(1 + Cos[
2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))*(Cos[2*(c + d*x)] + I*Sin[2*(c + d
*x)]))/(21*a^4*d*(-I + Tan[c + d*x])^4)
```

3.258.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3981, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{7a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{5e^2\left(-\frac{2e^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}\right)}{7a^2}$$

input

```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output

```
(((4*I)/7)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (5
*e^2*((-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
d*x]])/(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Ta
n[c + d*x]))))/(7*a^2)
```

3.258.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.258. $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.258.4 Maple [A] (verified)

Time = 7.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

method	result
default	$\frac{2e^4\left(24i(\cos^4(dx+c))+5i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+24(\cos^3(dx+c))\sin(dx+c)+5iF\right.}{21a^4d}$

input
```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/21/a^4/d*e^4*(24*I*cos(d*x+c)^4+5*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)+
cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
24*cos(d*x+c)^3*sin(d*x+c)+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-28*I*cos(d*x+c)^2-
16*sin(d*x+c)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)
```

3.258.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{2\left(5i\sqrt{2}e^{\frac{9}{2}}e^{(4i\,dx+4i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(5i\,e^4e^{(4i\,dx+4i\,c)}+2i\,e^4e^{(2i\,dx+2i\,c)}-3i\,e^4\right)\right.}{21\,a^4d}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
-2/21*(5*I*sqrt(2)*e^(9/2)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)) + sqrt(2)*(5*I*e^4*e^(4*I*d*x + 4*I*c) + 2*I*e^4*e^(2*I*d
*x + 2*I*c) - 3*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/
2*I*c))*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

3.258.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.258.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(ia\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.259 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.259.1 Optimal result 1914
3.259.2 Mathematica [C] (verified) 1914
3.259.3 Rubi [A] (verified) 1915
3.259.4 Maple [B] (verified) 1917
3.259.5 Fricas [C] (verification not implemented) 1918
3.259.6 Sympy [F(-1)] 1918
3.259.7 Maxima [F(-2)] 1918
3.259.8 Giac [F] 1919
3.259.9 Mupad [F(-1)] 1919

3.259.1 Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx=-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))}$$

output
```
-2/15*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^
2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^3-4/15*I*e^4/d/(e*sec(d*x+c)
)^(1/2)/(a^4+I*a^4*tan(d*x+c))
```

3.259.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.72 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.13

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx=\frac{e^3e^{-idx}\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(-7-7\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-7 - 7*Cos[2*(c + d*x)] + 6*E^((
2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x)])*((-I)*Cos[c + 2*d*x]
 + Sin[c + 2*d*x]))/(45*a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

3.259.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3981, 3042, 3981, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{3a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int\sqrt{\cos(c+dx)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow 3042$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

$$\Big\downarrow 3119$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}\right)}{3a^2}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/9)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (e
^2*((2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*S
ec[c + d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c
 + d*x]))))/(3*a^2)
```

3.259.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.259. $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.259.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(140) = 280$.

Time = 8.04 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.86

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\left(40i(\cos^5(dx+c))\sin(dx+c)+40i\sin(dx+c)(\cos^4(dx+c))-40(\cos^6(dx+c))-16i(\cos^3(dx+c))\sin(dx+c)-40(\right.}{}$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/45*I/a^4/d*(e*sec(d*x+c))^(1/2)*(40*I*sin(d*x+c)*cos(d*x+c)^5+40*I*cos(
d*x+c)^4*sin(d*x+c)-40*cos(d*x+c)^6-16*I*sin(d*x+c)*cos(d*x+c)^3-40*cos(d*
x+c)^5+3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)^2*EllipticE(I*
(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)-16*I*cos(d*x+c)^2*sin(d*x+c)+36*cos(d*x+c)^4+6*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-cs
c(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*I*cos(d*x+c)
*sin(d*x+c)+36*cos(d*x+c)^3+3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1)
)^(1/2))*e^3/(cos(d*x+c)+1)
```

3.259.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-6i\sqrt{2}e^{\frac{7}{2}}e^{(5i\,dx+5i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+}\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/45*(-6*I*sqrt(2)*e^(7/2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-6*I*e^3*e^(6*I*d*x +
6*I*c) - 4*I*e^3*e^(4*I*d*x + 4*I*c) + 7*I*e^3*e^(2*I*d*x + 2*I*c) + 5*I*e
^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-5*I*d*x
 - 5*I*c)/(a^4*d)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Timed out
```

3.259.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.259.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.260 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.260.1 Optimal result . . . 1920
3.260.2 Mathematica [A] (verified) . . . 1920
3.260.3 Rubi [A] (verified) . . . 1921
3.260.4 Maple [A] (verified) . . . 1924
3.260.5 Fricas [C] (verification not implemented) . . . 1924
3.260.6 Sympy [F] . . . 1925
3.260.7 Maxima [F(-2)] . . . 1925
3.260.8 Giac [F] . . . 1925
3.260.9 Mupad [F(-1)] . . . 1926

3.260.1 Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^4d} - \frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}$$

output

```
-2/77*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)-2/77*e^2*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/11*I*e^2*(e*sec(d*x+c))^(1/2)/a/d
/(a+I*a*tan(d*x+c))^3-4/77*I*e^4/d/(e*sec(d*x+c))^(3/2)/(a^4+I*a^4*tan(d*x
+c))
```

3.260.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \sec^2(c+dx)(e\sec(c+dx))^{5/2}(\cos(c+dx)+i\sin(c+dx))\Big(37i\cos(c+dx)+$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(Cos[c + d*x] + I*Sin[c + d*x])*((3
7*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d*x)] + 3*Sin[c + d*x] - 4*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]
) + 3*Sin[3*(c + d*x)]))/(154*a^4*d*(-I + Tan[c + d*x])^4)
```

3.260.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a^2}$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \text{4256}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}-\frac{e^2\left(\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}\right)}{11a^2}$$

3.260. $\int\frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}dx$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/11)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) - (e^
2*((3*e^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c +
d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2)
 + (((4*I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))))
/(11*a^2)
```

3.260.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.260. $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.260.4 Maple [A] (verified)

Time = 6.85 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

method	result
default	$-\frac{2i\left((\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)+i\sin(dx+c)\cos(dx+c)\left(56\left(\cos^4(dx+c)\right)-16(\mathrm{co}\right.\right.}{77a^4d}$

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-2/77*I/a^4/d*((cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*sin(d*x+c)*cos(d*
x+c)*(56*cos(d*x+c)^4-16*cos(d*x+c)^2-1)+cos(d*x+c)^4*(-56*cos(d*x+c)^2+44
))*(e*sec(d*x+c))^(1/2)*e^2
```

3.260.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(4i\sqrt{2}e^{\frac{5}{2}}e^{(6i\,dx+6i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(4i\,e^{2}e^{(6i\,dx+6i\,c)}\right.\right.}{1}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/154*(4*I*sqrt(2)*e^(5/2)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0,
e^(I*d*x + I*c)) + sqrt(2)*(4*I*e^2*e^(6*I*d*x + 6*I*c) + 17*I*e^2*e^(4*I*
d*x + 4*I*c) + 20*I*e^2*e^(2*I*d*x + 2*I*c) + 7*I*e^2)*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-6*I*d*x - 6*I*c)/(a^4*d)
```

3.260.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input

```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**4,x)
```

output

```
Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 -
6*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.260.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input

```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output

```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.260.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(ia\tan(dx+c)+a)^4}\,dx$$

input

```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output

```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.261 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx$

3.261.1 Optimal result 1927
3.261.2 Mathematica [C] (verified) 1927
3.261.3 Rubi [A] (verified) 1928
3.261.4 Maple [B] (verified) 1931
3.261.5 Fricas [C] (verification not implemented) 1932
3.261.6 Sympy [F] 1932
3.261.7 Maxima [F(-2)] 1932
3.261.8 Giac [F] 1933
3.261.9 Mupad [F(-1)] 1933

3.261.1 Optimal result

Integrand size = 28, antiderivative size = 163

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx &= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &+ \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} \\ &+ \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

output
```
2/117*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(3/2)+2/39*e^2*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d
/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/13*I*e^2/a/d/(e*sec(d*x+c))^(1/2)
/(a+I*a*tan(d*x+c))^3+4/117*I*e^4/d/(e*sec(d*x+c))^(5/2)/(a^4+I*a^4*tan(d*
x+c))
```

3.261.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{ie^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\Big(28+40\cos(2(c+}{234a^4d(-i+\tan(}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I/234)*Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(28
 + 40*Cos[2*(c + d*x)] + (24*E^((4*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + (22*I)*Sin
[2*(c + d*x)]))/(a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

3.261.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3981, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{e^2\left(\frac{5e^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{e^2\left(\frac{5e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 4256$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 4258$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{e^2\left(\frac{5e^2\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \; 3119$$

3.261. $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx$

$$\frac{e^2\left(\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{5/2}}\right)}{13a^2}+\frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((4*I)/13)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (e^
2*((5*e^2*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[
e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^
2) + (((4*I)/9)*e^2)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
))/(13*a^2)
```

3.261.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.261.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(167) = 334$.

Time = 8.18 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.32

method	result
default	$\frac{2i\sqrt{e\sec(dx+c)}\,(-i(\cos^2(dx+c))\sin(dx+c)+72(\cos^8(dx+c))-72i(\cos^7(dx+c))\sin(dx+c)+72(\cos^7(dx+c))+16i\sin(dx+c)(\cos^4(}{}$

input

```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output

```
2/117*I/a^4/d*(e*sec(d*x+c))^(1/2)*(-I*cos(d*x+c)^2*sin(d*x+c)+72*cos(d*x+
c)^8-72*I*cos(d*x+c)^7*sin(d*x+c)+72*cos(d*x+c)^7+16*I*cos(d*x+c)^4*sin(d*
x+c)-52*cos(d*x+c)^6-72*I*cos(d*x+c)^6*sin(d*x+c)-52*cos(d*x+c)^5-I*cos(d*
x+c)^3*sin(d*x+c)+3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)^2*E
llipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)-3*I*cos(d*x+c)*sin(d*x+c)+6*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)
+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+16*I*cos(d*x+c)^5*sin(
d*x+c)+3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))*e/(cos(d*x+c
)+1)
```

3.261.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\Big(24i\sqrt{2}e^{\frac{3}{2}}e^{(7i\,dx+7i\,c)}\mathrm{weierstrassZeta}\big(-4,0,\mathrm{weierstrassPInverse}\big(-4,0,e^{(i\,dx+}}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/468*(24*I*sqrt(2)*e^(3/2)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(24*I*e*e^(8*I*d*x + 8
*I*c) + 55*I*e*e^(6*I*d*x + 6*I*c) + 59*I*e*e^(4*I*d*x + 4*I*c) + 37*I*e*e
^(2*I*d*x + 2*I*c) + 9*I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

3.261.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{\tan^4{(c+dx)}-4i\tan^3{(c+dx)}-6\tan^2{(c+dx)}+4i\tan{(c+dx)}+1}\,dx}{a^4}$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 -
6*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.261.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.261.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.262 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$

3.262.1 Optimal result 1934
3.262.2 Mathematica [A] (verified) 1935
3.262.3 Rubi [A] (verified) 1935
3.262.4 Maple [A] (verified) 1939
3.262.5 Fricas [C] (verification not implemented) 1940
3.262.6 Sympy [F] 1940
3.262.7 Maxima [F(-2)] 1941
3.262.8 Giac [F] 1941
3.262.9 Mupad [F(-1)] 1941

3.262.1 Optimal result

Integrand size = 28, antiderivative size = 191

$$
\begin{aligned}
\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx &= \frac{2\sqrt{\cos(c+dx)}\,\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^4d} \\
&\quad + \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} \\
&\quad + \frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}
\end{aligned}
$$

output
```
2/33*e*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)+2/33*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+2/15*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d
*x+c))^4+14/165*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^3+4/33*I*e^2
/d/(e*sec(d*x+c))^(3/2)/(a^4+I*a^4*tan(d*x+c))
```

3.262.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(4(c+dx))+i\sin(4(c+dx)))\right.}{660a^4d(-i+\tan(c-}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) + I*(64 + 112*Cos[2*(c
 + d*x)] + 48*Cos[4*(c + d*x)] + (54*I)*Sin[2*(c + d*x)] + (37*I)*Sin[4*(c
 + d*x)])))/(660*a^4*d*(-I + Tan[c + d*x])^4)
```

3.262.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3983, 3042, 3983, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{7\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^3}dx}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{7\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^3}dx}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3983$$

$$\frac{7\left(\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\left(\frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3981$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 4256$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3042$$

3.262. $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 4258$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3120$$

3.262. $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$

$$\frac{7\left(\frac{5\left(\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}\right)}{11a}+\frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}\right)}{15a}$$

$$\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(((2*I)/15)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^4) + (7*((((2*
I)/11)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (5*((3*e^2*((2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e
^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) + (((4*I)/7)
*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))))/(11*a)))/(15
*a)
```

3.262.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.262.4 Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.80

method	result
default	$\frac{2i\left(\left(5\cos\left(dx+c\right)+5\right)\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\,\sqrt{\frac{1}{\cos\left(dx+c\right)+1}}\,F\left(i\left(-\csc\left(dx+c\right)+\cot\left(dx+c\right)\right),i\right)+i\sin\left(dx+c\right)\cos\left(dx+c\right)\left(-88\left(\cos^{6}\left(dx+c\right)\right)+16(c\right.\right.}{165a^{4}d}$

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
2/165*I/a^4/d*((5*cos(d*x+c)+5)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*sin(d*x+c)*cos(
d*x+c)*(-88*cos(d*x+c)^6+16*cos(d*x+c)^4-3*cos(d*x+c)^2-5)+cos(d*x+c)^6*(8
8*cos(d*x+c)^2-60))*(e*sec(d*x+c))^(1/2)
```

3.262.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(85i\,e^{(8i\,dx+8i\,c)}+166i\,e^{(6i\,dx+6i\,c)}+128i\,e^{(4i\,dx+4i\,c)}+58i\,e^{(2i\,dx+2i\,c)}+11i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\right.}{1320\,a^4d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/1320*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(85*I*e^(8*I*d*x + 8*I*c
) + 166*I*e^(6*I*d*x + 6*I*c) + 128*I*e^(4*I*d*x + 4*I*c) + 58*I*e^(2*I*d*
x + 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c) - 80*I*sqrt(2)*sqrt(e)*e^(8*I*d
*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-8*I*d*x - 8*I
*c)/(a^4*d)
```

3.262.6 Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sqrt{e\sec{(c+dx)}}}{\tan^4{(c+dx)}-4i\tan^3{(c+dx)}-6\tan^2{(c+dx)}+4i\tan{(c+dx)}+1}\,dx}{a^4}$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*t
an(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

3.262.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.262.8 Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^4}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^4, x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^4}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.263 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx$

3.263.1 Optimal result 1942
3.263.2 Mathematica [A] (verified) 1942
3.263.3 Rubi [A] (verified) 1943
3.263.4 Maple [F] 1945
3.263.5 Fricas [F] 1945
3.263.6 Sympy [F] 1945
3.263.7 Maxima [F] 1946
3.263.8 Giac [F] 1946
3.263.9 Mupad [F(-1)] 1946

3.263.1 Optimal result

Integrand size = 26, antiderivative size = 69

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\frac{6i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

output

```
6/5*I*2^(5/6)*a*hypergeom([-5/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(
f*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

3.263.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\frac{3ad(d\sec(e+fx))^{2/3}\left(i\sec(e+fx)+\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(\right.\right.}{5f}$$

input

```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

output

```
(3*a*d*(d*Sec[e + f*x])^(2/3)*(I*Sec[e + f*x] + Csc[e + f*x]*Hypergeometri
c2F1[1/2, 5/6, 11/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(5*f)
```

3.263.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{2^{5/6}a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)+1)^{5/6}}{2^{5/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)+1)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{6i2^{5/6}a(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

output
```
(((6*I)/5)*2^(5/6)*a*Hypergeometric2F1[-5/6, 5/6, 11/6, (1 - I*Tan[e + f*x
])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

3.263.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.263.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{5}{3}} (a+ia\tan (fx+e)) \, dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

3.263.5 Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx)) \, dx = \int (d\sec (fx+e))^{\frac{5}{3}} (i\,a\tan (fx+e)+a) \, dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
-1/10*(3*2^(2/3)*(5*I*a*d*e^(3*I*f*x + 3*I*e) + I*a*d*e^(I*f*x + I*e))*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 10*(f*e^(2*I*f*
x + 2*I*e) + f)*integral(1/2*I*2^(2/3)*a*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*e^(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

3.263.6 Sympy [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx)) \, dx = ia\left(\int \left(-i(d\sec (e+fx))^{\frac{5}{3}}\right) \, dx + \int (d\sec (e+fx))^{\frac{5}{3}} \tan (e+fx) \, dx\right)$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
I*a*(Integral(-I*(d*sec(e + f*x))**(5/3), x) + Integral((d*sec(e + f*x))**
(5/3)*tan(e + f*x), x))
```

3.263.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\int (d\sec\left(fx+e\right))^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

3.263.8 Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\int (d\sec\left(fx+e\right))^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/3}\,\left(a+a\tan(e+f\,x)\;1\mathrm{i}\right)\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i), x)
```

3.264 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx$

3.264.1 Optimal result 1947
3.264.2 Mathematica [A] (verified) 1947
3.264.3 Rubi [A] (verified) 1948
3.264.4 Maple [F] 1950
3.264.5 Fricas [F] 1950
3.264.6 Sympy [F] 1950
3.264.7 Maxima [F] 1951
3.264.8 Giac [F] 1951
3.264.9 Mupad [F(-1)] 1951

3.264.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{6i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

output
```
6*I*2^(1/6)*a*hypergeom([-1/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x
+e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

3.264.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{3a\sqrt[3]{d\sec(e+fx)}\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

output
```
(3*a*(d*Sec[e + f*x])^(1/3)*(I + Cot[e + f*x]*Hypergeometric2F1[1/6, 1/2,
7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

3.264.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{i\tan(e+fx)+1}}{\sqrt[6]{2}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{i\tan(e+fx)+1}}{(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{6i\sqrt[6]{2}a\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

output
```
((6*I)*2^(1/6)*a*Hypergeometric2F1[-1/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/2]
*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

3.264.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.264.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{1}{3}} (a+ia\tan (fx+e))\, dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

3.264.5 Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\, dx = \int (d\sec (fx+e))^{\frac{1}{3}} (i\, a\tan (fx+e)+a)\, dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
(3*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e)
 + f*integral(-I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f
*x - 2/3*I*e)/f, x))/f
```

3.264.6 Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\, dx = ia\left(\int \left(-i\sqrt[3]{d\sec (e+fx)}\right)\, dx + \int \sqrt[3]{d\sec (e+fx)}\tan (e+fx)\, dx\right)$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e)),x)
```

output
```
I*a*(Integral(-I*(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**
(1/3)*tan(e + f*x), x))
```

3.264.7 Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx=\int (d\sec{(fx+e)})^{\frac{1}{3}}\,(i\,a\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

3.264.8 Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx=\int (d\sec{(fx+e)})^{\frac{1}{3}}\,(i\,a\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^{1/3}\,(a+a\tan(e+f\,x)\;1\mathrm{i})\;dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i), x)
```

3.265 $\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

3.265.1 Optimal result 1952
3.265.2 Mathematica [A] (verified) 1952
3.265.3 Rubi [A] (verified) 1953
3.265.4 Maple [F] 1955
3.265.5 Fricas [F] 1955
3.265.6 Sympy [F] 1956
3.265.7 Maxima [F] 1956
3.265.8 Giac [F] 1956
3.265.9 Mupad [F(-1)] 1957

3.265.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{f\sqrt[3]{d\sec(e+fx)}}$$

output
```
-3*I*2^(5/6)*a*hypergeom([-1/6, 1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(
f*x+e))^(1/6)/f/(d*sec(f*x+e))^(1/3)
```

3.265.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3a\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*a*(I + Cot[e + f*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3))
```

3.265.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{7/6}\sqrt[6]{i\tan(e+fx)a+a}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{\sqrt[6]{2}}{\sqrt[6]{i\tan(e+fx)+1}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int\frac{1}{\sqrt[6]{i\tan(e+fx)+1}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 79$$

$$-\frac{3i2^{5/6}a\sqrt[6]{1+i\tan(e+fx)}\operatorname{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((-3*I)*2^(5/6)*a*Hypergeometric2F1[-1/6, 1/6, 5/6, (1 - I*Tan[e + f*x])/2
]*(1 + I*Tan[e + f*x])^(1/6))/(f*(d*Sec[e + f*x])^(1/3))
```

3.265.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.265. $\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.265.4 Maple [F]

$$\int \frac{a+ia\tan(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

3.265.5 Fricas [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
-(3*2^(2/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^
(2/3)*e^(2/3*I*f*x + 2/3*I*e) - (d*f*e^(I*f*x + I*e) - d*f)*integral(-2*2^
(2/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a*e^(I*f*x + I*e) + I*a)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2
*d*f*e^(2*I*f*x + 2*I*e) + d*f*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e)
- d*f)
```

3.265.6 Sympy [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=ia\left(\int\left(-\frac{i}{\sqrt[3]{d\sec\left(e+fx\right)}}\right)dx+\int\frac{\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
```

output
```
I*a*(Integral(-I/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)/(d*se
c(e + f*x))**(1/3), x))
```

3.265.7 Maxima [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int\frac{i\,a\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

3.265.8 Giac [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int\frac{i\,a\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{a+a\tan(e+f\,x)\;1\mathrm{i}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3), x)
```

3.266 $\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

3.266.1 Optimal result 1958
3.266.2 Mathematica [A] (verified) 1958
3.266.3 Rubi [A] (verified) 1959
3.266.4 Maple [F] 1961
3.266.5 Fricas [F] 1961
3.266.6 Sympy [F] 1961
3.266.7 Maxima [F] 1962
3.266.8 Giac [F] 1962
3.266.9 Mupad [F(-1)] 1962

3.266.1 Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{5f(d\sec(e+fx))^{5/3}}$$

output
```
-3/5*I*2^(1/6)*a*hypergeom([-5/6, 5/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*ta
n(f*x+e))^(5/6)/f/(d*sec(f*x+e))^(5/3)
```

3.266.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3a\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*a*(I + Cot[e + f*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3))
```

3.266.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{11/6}(i\tan(e+fx)a+a)^{5/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{2^{5/6}}{(i\tan(e+fx)+1)^{5/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{1}{(i\tan(e+fx)+1)^{5/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3i\sqrt[6]{2}a(1+i\tan(e+fx))^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(((-3*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, 5/6, 1/6, (1 - I*Tan[e + f*x
])/2]*(1 + I*Tan[e + f*x])^(5/6))/(f*(d*Sec[e + f*x])^(5/3))
```

3.266.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.266.4 Maple [F]

$$\int \frac{a+ia\tan (fx+e)}{(d\sec (fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

3.266.5 Fricas [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{i\,a\tan (fx+e)+a}{(d\sec (fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
1/10*(10*d^2*f*integral(-2/5*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/
3)*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a*e^(2*I*f*x + 2*I*
e) + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/(d^
2*f)
```

3.266.6 Sympy [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = ia\left(\int \left(-\frac{i}{(d\sec (e+fx))^{\frac{5}{3}}}\right)\,dx+\int \frac{\tan (e+fx)}{(d\sec (e+fx))^{\frac{5}{3}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

output
```
I*a*(Integral(-I/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)/(d*se
c(e + f*x))**(5/3), x))
```

3.266.7 Maxima [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{i\,a\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

3.266.8 Giac [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{i\,a\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{a+a\tan(e+f\,x)\ 1\mathrm{i}}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{5/3}}\,dx$$

input
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3), x)
```

3.267 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx$

3.267.1 Optimal result . . . 1963
3.267.2 Mathematica [A] (verified) . . . 1963
3.267.3 Rubi [A] (verified) . . . 1964
3.267.4 Maple [F] . . . 1966
3.267.5 Fricas [F] . . . 1966
3.267.6 Sympy [F(-1)] . . . 1966
3.267.7 Maxima [F] . . . 1967
3.267.8 Giac [F] . . . 1967
3.267.9 Mupad [F(-1)] . . . 1967

3.267.1 Optimal result

Integrand size = 28, antiderivative size = 71

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\frac{12i2^{5/6}a^2\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

output

```
12/5*I*2^(5/6)*a^2*hypergeom([-11/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*
sec(f*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

3.267.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\frac{3ia^2(d\sec(e+fx))^{5/3}\left(i\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e+fx)\right)\tan(e+fx)+\right.}{5f\sqrt{-\tan^2}}$$

input

```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((3*I)/5)*a^2*(d*Sec[e + f*x])^(5/3)*(I*Hypergeometric2F1[-1/2, 5/6, 11/6
, Sec[e + f*x]^2]*Tan[e + f*x] + I*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[e
 + f*x]^2]*Tan[e + f*x] + 2*Sqrt[-Tan[e + f*x]^2]))/(f*Sqrt[-Tan[e + f*x]^
2])
```

3.267.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)a+a)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2\ 2^{5/6}a^3(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)+1)^{11/6}}{2\ 2^{5/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3(d\sec(e+fx))^{5/3}\int \frac{(i\tan(e+fx)+1)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow 79$$

$$\frac{12i2^{5/6}a^2(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((12*I)/5)*2^(5/6)*a^2*Hypergeometric2F1[-11/6, 5/6, 11/6, (1 - I*Tan[e +
 f*x])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

3.267.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.267. $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.267.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{5}{3}} (a+ia\tan (fx+e))^{2} dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

3.267.5 Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \int (d\sec (fx+e))^{\frac{5}{3}} (i\,a\tan (fx+e)+a)^{2} dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/80*(3*2^(2/3)*(55*I*a^2*d*e^(5*I*f*x + 5*I*e) + 26*I*a^2*d*e^(3*I*f*x +
 3*I*e) + 11*I*a^2*d*e^(I*f*x + I*e))*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*
e^(2/3*I*f*x + 2/3*I*e) - 80*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I
*e) + f)*integral(11/16*I*2^(2/3)*a^2*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3
)*e^(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x +
 2*I*e) + f)
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e))**2,x)
```

output
```
Timed out
```

3.267.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

3.267.8 Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+a\tan(e+f\,x)\ 1\mathrm{i})^2\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

3.268 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$

3.268.1 Optimal result 1968
3.268.2 Mathematica [A] (verified) 1968
3.268.3 Rubi [A] (verified) 1969
3.268.4 Maple [F] 1971
3.268.5 Fricas [F] 1971
3.268.6 Sympy [F] 1971
3.268.7 Maxima [F] 1972
3.268.8 Giac [F] 1972
3.268.9 Mupad [F(-1)] 1972

3.268.1 Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \frac{12i\sqrt[6]{2}a^2\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

output
```
12*I*2^(1/6)*a^2*hypergeom([-7/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(
f*x+e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

3.268.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \frac{3a^2\sqrt[3]{d\sec(e+fx)}\Big(2i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}+\mathrm{co}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(3*a^2*(d*Sec[e + f*x])^(1/3)*(2*I + Cot[e + f*x]*Hypergeometric2F1[-1/2,
1/6, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + Cot[e + f*x]*Hypergeomet
ric2F1[1/6, 1/2, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

3.268.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int (a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \textbf{3986}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{4006}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{7/6}}{(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{80}$$

$$\frac{2\sqrt[6]{2}a^3\sqrt[3]{d\sec(e+fx)}\int \frac{(i\tan(e+fx)+1)^{7/6}}{2\sqrt[6]{2}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{a^3\sqrt[3]{d\sec(e+fx)}\int \frac{(i\tan(e+fx)+1)^{7/6}}{(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}}$$

$$\Big\downarrow \textbf{79}$$

$$\frac{12i\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

output
```
((12*I)*2^(1/6)*a^2*Hypergeometric2F1[-7/6, 1/6, 7/6, (1 - I*Tan[e + f*x])
/2]*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

3.268.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.268.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} (a+ia\tan(fx+e))^2\,dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

3.268.5 Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}} (i\,a\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/4*(3*2^(1/3)*(-9*I*a^2*e^(2*I*f*x + 2*I*e) - 7*I*a^2)*(d/(e^(2*I*f*x +
2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e) - 4*(f*e^(2*I*f*x + 2*I*e) + f)
*integral(-7/4*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I
*f*x - 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

3.268.6 Sympy [F]

$$\begin{aligned}\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = -a^2\Bigg(\int \left(-\sqrt[3]{d\sec(e+fx)}\right)dx \\ + \int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx \\ + \int \left(-2i\sqrt[3]{d\sec(e+fx)}\tan(e+fx)\right)dx\Bigg)\end{aligned}$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e))**2,x)
```

output
```
-a**2*(Integral(-(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**
(1/3)*tan(e + f*x)**2, x) + Integral(-2*I*(d*sec(e + f*x))**(1/3)*tan(e +
f*x), x))
```

3.268.7 Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}\,(ia\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

3.268.8 Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}\,(ia\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+a\tan(e+f\,x)\;1\mathrm{i})^2\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

3.269 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

3.269.1 Optimal result 1973
3.269.2 Mathematica [A] (verified) 1973
3.269.3 Rubi [A] (verified) 1974
3.269.4 Maple [F] 1976
3.269.5 Fricas [F] 1976
3.269.6 Sympy [F] 1977
3.269.7 Maxima [F] 1977
3.269.8 Giac [F] 1977
3.269.9 Mupad [F(-1)] 1978

3.269.1 Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{6i2^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}$$

output
```
-6*I*2^(5/6)*hypergeom([-5/6, -1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a^2
*tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)/(1+I*tan(f*x+e))^(5/6)
```

3.269.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{3a^2\Big(\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)+\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+\right.}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(3*a^2*(Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] +
Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] - (2*I)*Sqr
t[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])
```

3.269.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{(i\tan(e+fx)a+a)^{5/6}}{(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{5/6}a^2\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))\int \frac{(i\tan(e+fx)+1)^{5/6}}{2^{5/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))\int\frac{(i\tan(e+fx)+1)^{5/6}}{(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 79$$

$$-\frac{6i2^{5/6}a(a+ia\tan(e+fx))\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((-6*I)*2^(5/6)*a*Hypergeometric2F1[-5/6, -1/6, 5/6, (1 - I*Tan[e + f*x])/
2]*(a + I*a*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^
(5/6))
```

3.269.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.269. $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.269.4 Maple [F]

$$\int \frac{(a+ia\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

3.269.5 Fricas [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(3*2^(2/3)*(4*I*a^2*e^(2*I*f*x + 2*I*e) + I*a^2*e^(I*f*x + I*e) + 5*I
*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 2*(d*f
*e^(I*f*x + I*e) - d*f)*integral(-5*2^(2/3)*(I*a^2*e^(2*I*f*x + 2*I*e) + I
*a^2*e^(I*f*x + I*e) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I
*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2*d*f*e^(2*I*f*x + 2*I*e) + d*f
*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e) - d*f)
```

3.269.6 Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -a^2\left(\int \left(-\frac{1}{\sqrt[3]{d\sec(e+fx)}}\right)dx + \int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx + \int \left(-\frac{2i\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
```

output
```
-a**2*(Integral(-1/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)**2/
(d*sec(e + f*x))**(1/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))*
*(1/3), x))
```

3.269.7 Maxima [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

3.269.8 Giac [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3}} \,d x$$

input
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3), x)
```

3.270 $\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

3.270.1 Optimal result 1979

3.270.2 Mathematica [A] (verified) 1979

3.270.3 Rubi [A] (verified) 1980

3.270.4 Maple [F] 1982

3.270.5 Fricas [F] 1982

3.270.6 Sympy [F] 1982

3.270.7 Maxima [F] 1983

3.270.8 Giac [F] 1983

3.270.9 Mupad [F(-1)] 1983

3.270.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{6i\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{5f(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}$$

output
```
-6/5*I*2^(1/6)*hypergeom([-5/6, -1/6],[1/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a
^2*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)/(1+I*tan(f*x+e))^(1/6)
```

3.270.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{3a^2\left(\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\tan(e+fx)+\text{Hypergeor}\right.}{5f(d\sec(e+fx))^{5/3}\sqrt{}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(3*a^2*(Hypergeometric2F1[-5/6, -1/2, 1/6, Sec[e + f*x]^2]*Tan[e + f*x] +
Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*Tan[e + f*x] - (2*I)*Sqr
t[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3)*Sqrt[-Tan[e + f*x]^2])
```

3.270.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{(i\tan(e+fx)a+a)^{7/6}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{(i\tan(e+fx)a+a)^{7/6}}{(a-ia\tan(e+fx))^{5/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{\sqrt[6]{i\tan(e+fx)a+a}}{(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{2}a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))\int \frac{\sqrt[6]{i\tan(e+fx)+1}}{\sqrt[6]{2}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))\int \frac{\sqrt[6]{i\tan(e+fx)+1}}{(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{6i\sqrt[6]{2}a(a+ia\tan(e+fx))\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(((-6*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, -1/6, 1/6, (1 - I*Tan[e + f*
x])/2]*(a + I*a*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*
x])^(1/6))
```

3.270.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.270.4 Maple [F]

$$\int \frac{(a+ia\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

3.270.5 Fricas [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
1/5*(5*d^2*f*integral(1/5*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3
)*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a^2*e^(2*I*f*x + 2*I
*e) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/
(d^2*f)
```

3.270.6 Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -a^2\left(\int\left(-\frac{1}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx + \int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx + \int\left(-\frac{2i\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

output
```
-a**2*(Integral(-1/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)**2/
(d*sec(e + f*x))**(5/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))*
*(5/3), x))
```

3.270.7 Maxima [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(i\,a\tan\left(fx+e\right)+a)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

3.270.8 Giac [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(i\,a\tan\left(fx+e\right)+a)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{{\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/3}}\,dx$$

input
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3), x)
```

3.271 $\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx$

3.271.1 Optimal result 1984
3.271.2 Mathematica [A] (verified) 1984
3.271.3 Rubi [A] (verified) 1985
3.271.4 Maple [F] 1987
3.271.5 Fricas [F] 1987
3.271.6 Sympy [F] 1987
3.271.7 Maxima [F(-2)] 1988
3.271.8 Giac [F] 1988
3.271.9 Mupad [F(-1)] 1988

3.271.1 Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx=\frac{3i\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i}}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

output
```
3/10*I*hypergeom([5/6, 7/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(5
/3)*(1+I*tan(f*x+e))^(1/6)*2^(5/6)/f/(a+I*a*tan(f*x+e))
```

3.271.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx=\frac{6de^{i(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2i(e+fx)}\right)(d\sec(e+fx))^{2/3}}{a\sqrt[3]{1+e^{2i(e+fx)}}f(-i+\tan(e+fx))}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
(6*d*E^(I*(e + f*x))*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(e + f*x)
)]*(d*Sec[e + f*x])^(2/3))/(a*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*(-I + Tan[
e + f*x]))
```

3.271.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{\sqrt[6]{i\tan(e+fx)a+a}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{\sqrt[6]{i\tan(e+fx)a+a}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{2\sqrt[6]{2}}{(i\tan(e+fx)+1)^{7/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{2\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{1}{(i\tan(e+fx)+1)^{7/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{79}$$

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
(((3*I)/5)*Hypergeometric2F1[5/6, 7/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*Se
c[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a + I*a*Tan[e +
f*x]))
```

3.271.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.271.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{a+ia\tan(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

3.271.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
(a*f*e^(I*f*x + I*e)*integral(-I*2^(2/3)*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*e^(2/3*I*f*x + 2/3*I*e)/(a*f), x) - 3*2^(2/3)*(-I*d*e^(2*I*f*x + 2*I*
e) - I*d)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(
-I*f*x - I*e)/(a*f)
```

3.271.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = -\frac{i\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{\tan(e+fx)-i}\,dx}{a}$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x) - I), x)/a
```

3.271.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.271.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a), x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+a\tan(e+f\,x)\;1\mathrm{i}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i), x)
```

3.272 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx$

3.272.1 Optimal result 1989
3.272.2 Mathematica [A] (verified) 1989
3.272.3 Rubi [A] (verified) 1990
3.272.4 Maple [F] 1992
3.272.5 Fricas [F] 1992
3.272.6 Sympy [F] 1992
3.272.7 Maxima [F(-2)] 1993
3.272.8 Giac [F] 1993
3.272.9 Mupad [F(-1)] 1993

3.272.1 Optimal result

Integrand size = 28, antiderivative size = 81

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2^{5/6}f(a+ia\tan(e+fx))}$$

output
```
3/2*I*hypergeom([1/6, 11/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(1/
3)*(1+I*tan(f*x+e))^(5/6)*2^(1/6)/f/(a+I*a*tan(f*x+e))
```

3.272.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{3ie^{-2i(e+fx)}\left(-1-e^{2i(e+fx)}+4e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)\right)\sqrt[3]{d\sec}}{10af}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
(((-3*I)/10)*(-1 - E^((2*I)*(e + f*x)) + 4*E^((2*I)*(e + f*x))*(1 + E^((2*
I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))
])*(d*Sec[e + f*x])^(1/3))/(a*E^((2*I)*(e + f*x))*f)
```

3.272.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{5/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{5/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int \frac{2\ 2^{5/6}}{(i\tan(e+fx)+1)^{11/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{2\ 2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(i\tan(e+fx)+1)^{11/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$\Big\downarrow 79$$

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2^{5/6}f(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

output
```
((3*I)*Hypergeometric2F1[1/6, 11/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec[e
+ f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a + I*a*Tan[e + f*x]
))
```

3.272.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

---

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.272.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{a+ia\tan(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

3.272.5 Fricas [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/10*(10*a*f*e^(2*I*f*x + 2*I*e)*integral(-2/5*I*2^(1/3)*(d/(e^(2*I*f*x +
2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*f), x) - 3*2^(1/3)*(d/(e^(2
*I*f*x + 2*I*e) + 1))^(1/3)*(-I*e^(2*I*f*x + 2*I*e) - I)*e^(1/3*I*f*x + 1/
3*I*e))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

3.272.6 Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{i\int \frac{\sqrt[3]{d\sec(e+fx)}}{\tan(e+fx)-i}\,dx}{a}$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x) - I), x)/a
```

3.272.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.272.8 Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+a\tan(e+f\,x)\,1\mathrm{i}}\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i),x)
```

output
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i), x)
```

3.273 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx$

3.273.1 Optimal result 1994
3.273.2 Mathematica [A] (verified) 1994
3.273.3 Rubi [A] (verified) 1995
3.273.4 Maple [F] 1997
3.273.5 Fricas [F] 1997
3.273.6 Sympy [F] 1998
3.273.7 Maxima [F(-2)] 1998
3.273.8 Giac [F] 1998
3.273.9 Mupad [F(-1)] 1999

3.273.1 Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

output
```
-3/4*I*hypergeom([-1/6, 13/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(1/6)*2^(5/6)/a/f/(d*sec(f*x+e))^(1/3)
```

3.273.2 Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \frac{3\left(-8e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)+5(5+5\cos(2(e+fx))+4i\sin}{70af\sqrt[3]{d\sec(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(3*(-8*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2
F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] + 5*(5 + 5*Cos[2*(e + f*x)] + (4*
I)*Sin[2*(e + f*x)]))*(I + Tan[e + f*x]))/(70*a*f*(d*Sec[e + f*x])^(1/3))
```

3.273.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{7/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{7/6}(i\tan(e+fx)a+a)^{13/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{4\sqrt[6]{2}}{(i\tan(e+fx)+1)^{13/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{4\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{1}{(i\tan(e+fx)+1)^{13/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 79$$

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(((-3*I)/2)*Hypergeometric2F1[-1/6, 13/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(d*Sec[e + f*x])^(1/3))
```

3.273.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.273. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.273.4 Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x +e \right)\right)^{\frac{1}{3}} \left(a +i a \tan \left(f x +e \right)\right)}d x$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

3.273.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{\left(d \sec \left(f x + e\right)\right)^{\frac{1}{3}} \left(i \, a \tan \left(f x + e\right) + a\right)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
-1/28*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(7*I*e^(5*I*f*x + 5*I
*e) + 9*I*e^(4*I*f*x + 4*I*e) + 6*I*e^(3*I*f*x + 3*I*e) + 10*I*e^(2*I*f*x
+ 2*I*e) - I*e^(I*f*x + I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 28*(a*d*f*e^(4
*I*f*x + 4*I*e) - a*d*f*e^(3*I*f*x + 3*I*e))*integral(-8/7*2^(2/3)*(d/(e^(
2*I*f*x + 2*I*e) + 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) +
I)*e^(2/3*I*f*x + 2/3*I*e)/(a*d*f*e^(3*I*f*x + 3*I*e) - 2*a*d*f*e^(2*I*f*x
 + 2*I*e) + a*d*f*e^(I*f*x + I*e)), x))/(a*d*f*e^(4*I*f*x + 4*I*e) - a*d*f
*e^(3*I*f*x + 3*I*e))
```

3.273.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = -\frac{i\int \frac{1}{\sqrt[3]{d\sec{(e+fx)}}\tan{(e+fx)}-i\sqrt[3]{d\sec{(e+fx)}}}\,dx}{a}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x) - I*(d*sec(e + f*x))**
(1/3)), x)/a
```

3.273.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.273.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{\left(d\sec{(fx+e)}\right)^{\frac{1}{3}}\left(i\,a\tan{(fx+e)}+a\right)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)), x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx=\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3}\,\left(a+a\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)} \,d x$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)), x)
```

3.274 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx$

3.274.1 Optimal result . . . 2000
3.274.2 Mathematica [A] (verified) . . . 2000
3.274.3 Rubi [A] (verified) . . . 2001
3.274.4 Maple [F] . . . 2003
3.274.5 Fricas [F] . . . 2003
3.274.6 Sympy [F] . . . 2003
3.274.7 Maxima [F(-2)] . . . 2004
3.274.8 Giac [F] . . . 2004
3.274.9 Mupad [F(-1)] . . . 2004

3.274.1 Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{10\;2^{5/6}af(d\sec(e+fx))^{5/3}}$$

output
```
-3/20*I*hypergeom([-5/6, 17/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e)
)^(5/6)*2^(1/6)/a/f/(d*sec(f*x+e))^(5/3)
```

3.274.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = -\frac{3\sec^2(e+fx)\left(-26+6\cos(2(e+fx))+\frac{128e^{2i(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^{2/3}}+16i\sin(2(e+fx))\right)}{220af(d\sec(e+fx))^{5/3}(-i+\tan(e+fx))}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(-3*Sec[e + f*x]^2*(-26 + 6*Cos[2*(e + f*x)] + (128*E^((2*I)*(e + f*x))*Hy
pergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])/(1 + E^((2*I)*(e + f
*x)))^(2/3) + (16*I)*Sin[2*(e + f*x)]))/(220*a*f*(d*Sec[e + f*x])^(5/3)*(-
I + Tan[e + f*x]))
```

3.274.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{11/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{11/6}(i\tan(e+fx)a+a)^{17/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{4\ 2^{5/6}}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{4\ 2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{1}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow 79$$

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

output
```
(((-3*I)/10)*Hypergeometric2F1[-5/6, 17/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(d*Sec[e + f*x])^(5/3))
```

3.274.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.274.4 Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+ia\tan\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

3.274.5 Fricas [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx=\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/440*(440*a*d^2*f*e^(4*I*f*x + 4*I*e)*integral(-16/55*I*2^(1/3)*(d/(e^(2*
I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*d^2*f), x) - 3*2^(1
/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(11*I*e^(6*I*f*x + 6*I*e) - 15*I*e
^(4*I*f*x + 4*I*e) - 31*I*e^(2*I*f*x + 2*I*e) - 5*I)*e^(1/3*I*f*x + 1/3*I*
e))*e^(-4*I*f*x - 4*I*e)/(a*d^2*f)
```

3.274.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx=-\frac{i\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}\tan\left(e+fx\right)-i\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx}{a}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x) - I*(d*sec(e + f*x))**
(5/3)), x)/a
```

3.274.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.274.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}\,(i\,a\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,(a+a\tan(e+f\,x)\;1\mathrm{i})}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)), x)
```

3.275 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$

3.275.1 Optimal result 2005
3.275.2 Mathematica [A] (verified) 2005
3.275.3 Rubi [A] (verified) 2006
3.275.4 Maple [F] 2008
3.275.5 Fricas [F] 2008
3.275.6 Sympy [F] 2008
3.275.7 Maxima [F(-2)] 2009
3.275.8 Giac [F] 2009
3.275.9 Mupad [F(-1)] 2009

3.275.1 Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1}}{10\sqrt[6]{2}f\left(a^2+ia^2\tan(e+fx)\right)}$$

output
```
3/20*I*hypergeom([5/6, 13/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(
5/3)*(1+I*tan(f*x+e))^(1/6)*2^(5/6)/f/(a^2+I*a^2*tan(f*x+e))
```

3.275.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.47

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{3e^{-i(4e+5fx)}\left(1+e^{2i(e+fx)}\right)\left(1+e^{2i(e+fx)}+2e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2i(}\right.\right.}{28a^2f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(-3*(1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)) + 2*E^((2*I)*(e + f
*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^
((2*I)*(e + f*x))])*(d*Sec[e + f*x])^(5/3)*((-I)*Cos[f*x] + Sin[f*x]))/(28
*a^2*E^(I*(4*e + 5*f*x))*f)
```

3.275.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{(i\tan(e+fx)a+a)^{7/6}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{(i\tan(e+fx)a+a)^{7/6}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2(d\sec(e+fx))^{5/3}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{4\sqrt[6]{2}}{(i\tan(e+fx)+1)^{13/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{4\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\int \frac{1}{(i\tan(e+fx)+1)^{13/6}\sqrt[6]{a-ia\tan(e+fx)}}d\tan(e+fx)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\sqrt[6]{2}af(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((3*I)/10)*Hypergeometric2F1[5/6, 13/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*
Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(a + I*a*Tan[
e + f*x]))
```

3.275.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.275. $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.275.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(a+ia\tan(fx+e))^2}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

3.275.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(i\,a\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/14*(14*a^2*f*e^(3*I*f*x + 3*I*e)*integral(-1/7*I*2^(2/3)*d*(d/(e^(2*I*f*
x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(a^2*f), x) - 3*2^(2/3)*(-2
*I*d*e^(4*I*f*x + 4*I*e) - 3*I*d*e^(2*I*f*x + 2*I*e) - I*d)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-3*I*f*x - 3*I*e)/(a^2*f
)
```

3.275.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1)
, x)/a**2
```

3.275.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.275.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{5}{3}}}{(i\,a\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a)^2, x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{(a+a\tan{(e+f\,x)}\,1\mathrm{i})^2}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

3.275. $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$

3.276 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

3.276.1 Optimal result 2011
3.276.2 Mathematica [A] (verified) 2011
3.276.3 Rubi [A] (verified) 2012
3.276.4 Maple [F] 2014
3.276.5 Fricas [F] 2014
3.276.6 Sympy [F] 2015
3.276.7 Maxima [F(-2)] 2015
3.276.8 Giac [F] 2015
3.276.9 Mupad [F(-1)] 2016

3.276.1 Optimal result

Integrand size $=28$, antiderivative size $=87$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2\;2^{5/6}f\left(a^2+ia^2\tan(e+fx)\right)}$$

output
```
3/4*I*hypergeom([1/6, 17/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(1/
3)*(1+I*tan(f*x+e))^(5/6)*2^(1/6)/f/(a^2+I*a^2*tan(f*x+e))
```

3.276.2 Mathematica [A] (verified)

Time $=1.41$ (sec) , antiderivative size $=121$, normalized size of antiderivative $=1.39$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3\sec^2(e+fx)\sqrt[3]{d\sec(e+fx)}\Big(-2i-2i\cos(2(e+fx))+4ie^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\text{Hypergeometric2F1}\Big(}{22a^2f(-i+\tan(e+fx))^2}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(1/3)*(-2*I - (2*I)*Cos[2*(e + f*x)] +
(4*I)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F
1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] + Sin[2*(e + f*x)]))/(22*a^2*f*(-I
+ Tan[e + f*x])^2)
```

3.276.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{11/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^{11/6}}dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int \frac{4\ 2^{5/6}}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{4\ 2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\int\frac{1}{(i\tan(e+fx)+1)^{17/6}(a-ia\tan(e+fx))^{5/6}}d\tan(e+fx)}{f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$\Big\downarrow 79$$

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\,2^{5/6}af(a+ia\tan(e+fx))}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(((3*I)/2)*Hypergeometric2F1[1/6, 17/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Se
c[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(a + I*a*Tan[e
+ f*x]))
```

3.276.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.276. $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.276.4 Maple [F]

$$\int \frac{\left(d \sec \left(f x +e \right)\right)^{\frac{1}{3}}}{\left(a +i a \tan \left(f x +e \right)\right)^{2}}d x$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

3.276.5 Fricas [F]

$$\int \frac{\sqrt[3]{d \sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx=\int \frac{\left(d \sec \left(f x + e\right)\right)^{\frac{1}{3}}}{{\left(i \, a \tan \left(f x + e\right) + a\right)}^{2}}\,{d x}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/44*(44*a^2*f*e^(4*I*f*x + 4*I*e)*integral(-2/11*I*2^(1/3)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*f), x) - 3*2^(1/3)*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(-3*I*e^(4*I*f*x + 4*I*e) - 4*I*e^(2*I*f*
x + 2*I*e) - I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

3.276.6 Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{\sqrt[3]{d\sec{(e+fx)}}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx}{a^2}$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1)
, x)/a**2
```

3.276.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.276.8 Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{1}{3}}}{(i\,a\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a)^2, x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2}\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

3.277 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx$

3.277.1 Optimal result . . . 2017
3.277.2 Mathematica [A] (verified) . . . 2017
3.277.3 Rubi [A] (verified) . . . 2018
3.277.4 Maple [F] . . . 2020
3.277.5 Fricas [F] . . . 2020
3.277.6 Sympy [F] . . . 2021
3.277.7 Maxima [F(-2)] . . . 2021
3.277.8 Giac [F] . . . 2021
3.277.9 Mupad [F(-1)] . . . 2022

3.277.1 Optimal result

Integrand size $= 28$, antiderivative size $= 71$

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

output
```
-3/8*I*hypergeom([-1/6, 19/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(1/6)*2^(5/6)/a^2/f/(d*sec(f*x+e))^(1/3)
```

3.277.2 Mathematica [A] (verified)

Time $= 2.16$ (sec) , antiderivative size $= 141$, normalized size of antiderivative $= 1.99$

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{2/3}\left(16e^{3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)-10(7\cos(e+f}{260a^2df}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
((d*Sec[e + f*x])^(2/3)*(16*E^((3*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^
(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] - 10*(7*Cos[
e + f*x] + 5*Cos[3*(e + f*x)] + (18*I)*Cos[e + f*x]^2*Sin[e + f*x]))*((-3*
I)*Cos[2*(e + f*x)] - 3*Sin[2*(e + f*x)]))/(260*a^2*d*f)
```

3.277.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))^2\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(i\tan(e+fx)a+a)^{13/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\int \frac{1}{(a-ia\tan(e+fx))^{7/6}(i\tan(e+fx)a+a)^{19/6}}d\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int \frac{8\sqrt[6]{2}}{(i\tan(e+fx)+1)^{19/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{8\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[6]{1+i\tan(e+fx)}\sqrt[6]{a-ia\tan(e+fx)}\int\frac{1}{(i\tan(e+fx)+1)^{19/6}(a-ia\tan(e+fx))^{7/6}}d\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}}$$
$$\Big\downarrow 79$$
$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
(((-3*I)/4)*Hypergeometric2F1[-1/6, 19/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(1/6))/(2^(1/6)*a^2*f*(d*Sec[e + f*x])^(1/3))
```

3.277.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

---

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.277.4 Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x +e \right)\right)^{\frac{1}{3}} \left(a +i a \tan \left(f x +e \right)\right)^{2}}d x$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

3.277.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{d \sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx=\int \frac{1}{\left(d \sec \left(f x + e\right)\right)^{\frac{1}{3}} {\left(i \, a \tan \left(f x + e\right) + a\right)}^{2}}\,{d x}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas
")
```

output
```
-1/104*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(13*I*e^(7*I*f*x + 7
*I*e) + 19*I*e^(6*I*f*x + 6*I*e) + 9*I*e^(5*I*f*x + 5*I*e) + 23*I*e^(4*I*f
*x + 4*I*e) - 5*I*e^(3*I*f*x + 3*I*e) + 5*I*e^(2*I*f*x + 2*I*e) - I*e^(I*f
*x + I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 104*(a^2*d*f*e^(6*I*f*x + 6*I*e)
- a^2*d*f*e^(5*I*f*x + 5*I*e))*integral(-8/13*2^(2/3)*(d/(e^(2*I*f*x + 2*I
*e) + 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) + I)*e^(2/3*I*f
*x + 2/3*I*e)/(a^2*d*f*e^(3*I*f*x + 3*I*e) - 2*a^2*d*f*e^(2*I*f*x + 2*I*e)
 + a^2*d*f*e^(I*f*x + I*e)), x))/(a^2*d*f*e^(6*I*f*x + 6*I*e) - a^2*d*f*e^
(5*I*f*x + 5*I*e))
```

3.277.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)-2i\sqrt[3]{d\sec(e+fx)}\tan(e+fx)-\sqrt[3]{d\sec(e+fx)}}\,dx}{a^2}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x)
)**(1/3)*tan(e + f*x) - (d*sec(e + f*x))**(1/3)), x)/a**2
```

3.277.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.277.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(ia\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2), x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

3.278 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx$

3.278.1 Optimal result 2023
3.278.2 Mathematica [B] (verified) 2023
3.278.3 Rubi [A] (verified) 2024
3.278.4 Maple [F] 2026
3.278.5 Fricas [F] 2026
3.278.6 Sympy [F] 2027
3.278.7 Maxima [F(-2)] 2027
3.278.8 Giac [F] 2027
3.278.9 Mupad [F(-1)] 2028

3.278.1 Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{23}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{20\;2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

output

```
-3/40*I*hypergeom([-5/6, 23/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e)
)^(5/6)*2^(1/6)/a^2/f/(d*sec(f*x+e))^(5/3)
```

3.278.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 1.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.01

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \frac{3i\sec^4(e+fx)\left(-46-40\cos(2(e+fx))+6\cos(4(e+fx\right.}{}$$

input

```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
(((3*I)/680)*Sec[e + f*x]^4*(-46 - 40*Cos[2*(e + f*x)] + 6*Cos[4*(e + f*x)
] + 128*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric
2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] - (10*I)*Sin[2*(e + f*x)] + (11*I
)*Sin[4*(e + f*x)]))/(a^2*f*(d*Sec[e + f*x])^(5/3)*(-I + Tan[e + f*x])^2)
```

3.278.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(i\tan(e+fx)a+a)^{17/6}}dx}{(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\int \frac{1}{(a-ia\tan(e+fx))^{11/6}(i\tan(e+fx)a+a)^{23/6}}d\tan(e+fx)}{f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{8\ 2^{5/6}}{(i\tan(e+fx)+1)^{23/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{8\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(1+i\tan(e+fx))^{5/6}(a-ia\tan(e+fx))^{5/6}\int \frac{1}{(i\tan(e+fx)+1)^{23/6}(a-ia\tan(e+fx))^{11/6}}d\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow 79$$

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{23}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{20\;2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

output
```
(((-3*I)/20)*Hypergeometric2F1[-5/6, 23/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*a^2*f*(d*Sec[e + f*x])^(5/3))
```

3.278.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.278.4 Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(a+i a \tan \left(f x+e\right)\right)^{2}}dx$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

3.278.5 Fricas [F]

$$\int \frac{1}{(d \sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\, dx = \int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(i\, a \tan \left(f x+e\right)+a\right)^{2}}\, dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas
")
```

output
```
1/1360*(1360*a^2*d^2*f*e^(6*I*f*x + 6*I*e)*integral(-16/85*I*2^(1/3)*(d/(e
^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*d^2*f), x) -
3*2^(1/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(17*I*e^(8*I*f*x + 8*I*e) -
50*I*e^(6*I*f*x + 6*I*e) - 92*I*e^(4*I*f*x + 4*I*e) - 30*I*e^(2*I*f*x + 2*
I*e) - 5*I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-6*I*f*x - 6*I*e)/(a^2*d^2*f)
```

3.278.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{(d\sec{(e+fx)})^{\frac{5}{3}}\tan^2{(e+fx)}-2i(d\sec{(e+fx)})^{\frac{5}{3}}\tan{(e+fx)}-(d\sec{(e+fx)})^{\frac{5}{3}}}\,dx}{a^2}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x)
)**(5/3)*tan(e + f*x) - (d*sec(e + f*x))**(5/3)), x)/a**2
```

3.278.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.278.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{5}{3}}\,(ia\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx=\int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/3}\,{\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

3.279 $\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.279.1 Optimal result 2029
3.279.2 Mathematica [A] (verified) 2029
3.279.3 Rubi [A] (verified) 2030
3.279.4 Maple [A] (verified) 2031
3.279.5 Fricas [A] (verification not implemented) 2032
3.279.6 Sympy [F] 2032
3.279.7 Maxima [A] (verification not implemented) 2032
3.279.8 Giac [F] 2033
3.279.9 Mupad [B] (verification not implemented) 2034

3.279.1 Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d} + \frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d} - \frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d} + \frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d}$$

output
```
-16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d+24/11*I*(a+I*a*tan(d*x+c))^(11/2)/a
^5/d-12/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^6/d+2/15*I*(a+I*a*tan(d*x+c))^(15
/2)/a^7/d
```

3.279.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^4\sqrt{a+ia\tan(c+dx)}(-1241i-2367\tan(c+dx)+1683i\tan^2(c+dx)+429\tan^3(c}{6435d}$$

input
```
Integrate[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^4*Sqrt[a + I*a*Tan[c + d*x]]*(-1241*I - 2367*Tan[c
+ d*x] + (1683*I)*Tan[c + d*x]^2 + 429*Tan[c + d*x]^3))/(6435*d)
```

3.279.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^8\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \; 53$$

$$-\frac{i\int\left(-(i\tan(c+dx)a+a)^{13/2}+6a(i\tan(c+dx)a+a)^{11/2}-12a^2(i\tan(c+dx)a+a)^{9/2}+8a^3(i\tan(c+dx)a\right.}{a^7d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(\frac{16}{9}a^3(a+ia\tan(c+dx))^{9/2}-\frac{24}{11}a^2(a+ia\tan(c+dx))^{11/2}-\frac{2}{15}(a+ia\tan(c+dx))^{15/2}+\frac{12}{13}a(a+ia\tan(c+\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(9/2))/9 - (24*a^2*(a + I*a*Tan[c +
d*x])^(11/2))/11 + (12*a*(a + I*a*Tan[c + d*x])^(13/2))/13 - (2*(a + I*a*T
an[c + d*x])^(15/2))/15))/(a^7*d)
```

3.279.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.279.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{6a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{6a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^{7}}$	82

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/15*(a+I*a*tan(d*x+c))^(15/2)-6/13*a*(a+I*a*tan(d*x+c))^(13/2)
+12/11*a^2*(a+I*a*tan(d*x+c))^(11/2)-8/9*a^3*(a+I*a*tan(d*x+c))^(9/2))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(15i\,dx+15i\,c)}+120i\,e^{(13i\,dx+13i\,c)}+390i\,e^{(11i\,dx+11i\,c)}+715i\,e^{(9i}\right.}{6435\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4}\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(15*I*d*x + 15
*I*c) + 120*I*e^(13*I*d*x + 13*I*c) + 390*I*e^(11*I*d*x + 11*I*c) + 715*I*
e^(9*I*d*x + 9*I*c))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c)
+ 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x
+ 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

3.279.6 Sympy [F]

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^8\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**8, x)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$=\frac{2i\left(429\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{15}{2}}-2970\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}a+7020\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}a^2-5720\,(\right.}{6435\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2/6435*I*(429*(I*a*tan(d*x + c) + a)^(15/2) - 2970*(I*a*tan(d*x + c) + a)^
(13/2)*a + 7020*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 5720*(I*a*tan(d*x + c)
 + a)^(9/2)*a^3)/(a^7*d)
```

3.279.8 Giac [F]

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^8\,dx$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^8, x)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.05

$$
\begin{aligned}
\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = &-\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{6435\,d} \\
&-\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{6435\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} \\
&-\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{2145\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} \\
&-\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{1287\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} \\
&+\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,40960\mathrm{i}}{1287\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4} \\
&-\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,52736\mathrm{i}}{715\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5} \\
&+\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,11776\mathrm{i}}{195\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6} \\
&-\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{15\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^7}
\end{aligned}
$$

input

```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^8,x)
```

output

```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
40960i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(6435*d*(exp(c*2i + d
*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*512i)/(2145*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(1287*d*(
exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(6435*d) - ((a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*52736i)/(715*d*(exp(c*2i +
d*x*2i) + 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*11776i)/(195*d*(exp(c*2i + d*x*2i) + 1)^6) - ((a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(15*d
*(exp(c*2i + d*x*2i) + 1)^7)
```

3.280 $\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.280.1 Optimal result . . . 2035
3.280.2 Mathematica [A] (verified) . . . 2035
3.280.3 Rubi [A] (verified) . . . 2036
3.280.4 Maple [A] (verified) . . . 2037
3.280.5 Fricas [A] (verification not implemented) . . . 2038
3.280.6 Sympy [F] . . . 2038
3.280.7 Maxima [A] (verification not implemented) . . . 2038
3.280.8 Giac [F] . . . 2039
3.280.9 Mupad [B] (verification not implemented) . . . 2039

3.280.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d} + \frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d} - \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}$$

output
```
-8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d+8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d
-2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^5/d
```

3.280.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}(-151+182i\tan(c+dx)+63\tan^2(c+dx))}{693d}$$

input
```
Integrate[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]*(-151 + (182*I)*Tan[c
+ d*x] + 63*Tan[c + d*x]^2))/(693*d)
```

3.280.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^6\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{9/2}-4a(i\tan(c+dx)a+a)^{7/2}+4a^2(i\tan(c+dx)a+a)^{5/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{8}{7}a^2(a+ia\tan(c+dx))^{7/2}+\frac{2}{11}(a+ia\tan(c+dx))^{11/2}-\frac{8}{9}a(a+ia\tan(c+dx))^{9/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(7/2))/7 - (8*a*(a + I*a*Tan[c + d*x]
)^(9/2))/9 + (2*(a + I*a*Tan[c + d*x])^(11/2))/11))/(a^5*d)
```

3.280.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.280.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{5}}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{5}}$	63

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/11*(a+I*a*tan(d*x+c))^(11/2)+4/9*a*(a+I*a*tan(d*x+c))^(9/2)-
4/7*a^2*(a+I*a*tan(d*x+c))^(7/2))
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(11i\,dx+11i\,c)}+44i\,e^{(9i\,dx+9i\,c)}+99i\,e^{(7i\,dx+7i\,c)}\right)}{693\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(11*I*d*x + 11*I*
c) + 44*I*e^(9*I*d*x + 9*I*c) + 99*I*e^(7*I*d*x + 7*I*c))/(d*e^(10*I*d*x +
 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*
I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

3.280.6 Sympy [F]

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^6\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**6, x)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{2i\left(63\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-308\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a+396\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a^2\right)}{693\,a^5d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/693*I*(63*(I*a*tan(d*x + c) + a)^(11/2) - 308*(I*a*tan(d*x + c) + a)^(9
/2)*a + 396*(I*a*tan(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

3.280.8 Giac [F]

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\,\sec\left(dx+c\right)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^6, x)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.00

$$\begin{aligned}\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,d}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{693\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\ &+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,7232\mathrm{i}}{693\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1472\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\\ &+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}\end{aligned}$$

input

```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^6,x)
```

output

```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
7232i)/(693*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(693*d*(exp(c*2i + d*x*2
i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*64i)/(231*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(693*d) - ((a -
 (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1472i
)/(99*d*(exp(c*2i + d*x*2i) + 1)^4) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(c*2i + d*x*2i) + 1)^
5)
```

3.280. $\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.281 $\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.281.1 Optimal result 2041
3.281.2 Mathematica [A] (verified) 2041
3.281.3 Rubi [A] (verified) 2042
3.281.4 Maple [A] (verified) 2043
3.281.5 Fricas [A] (verification not implemented) 2043
3.281.6 Sympy [F] 2044
3.281.7 Maxima [A] (verification not implemented) 2044
3.281.8 Giac [F] 2044
3.281.9 Mupad [B] (verification not implemented) 2045

3.281.1 Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

output
```
-4/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^2/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d
```

3.281.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^2(9i+5\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{35d}$$

input
```
Integrate[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^2*(9*I + 5*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]
)/(35*d)
```

3.281.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{3/2}-(i\tan(c+dx)a+a)^{5/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{5}a(a+ia\tan(c+dx))^{5/2}-\frac{2}{7}(a+ia\tan(c+dx))^{7/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(5/2))/5 - (2*(a + I*a*Tan[c + d*x])^(7
/2))/7))/(a^3*d)
```

3.281.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.281.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/7*(a+I*a*tan(d*x+c))^(7/2)-2/5*a*(a+I*a*tan(d*x+c))^(5/2))
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=-\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(7i\,dx+7i\,c)}+7i\,e^{(5i\,dx+5i\,c)}\right)}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(7*I*d*x + 7*I*c)
+ 7*I*e^(5*I*d*x + 5*I*c))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c
) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.281.6 Sympy [F]

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^4\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**4, x)
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i\left(5\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}-14\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a\right)}{35\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 14*(I*a*tan(d*x + c) + a)^(5/2)*a
)/(a^3*d)
```

3.281.8 Giac [F]

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\,\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^4, x)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.90

$$\begin{aligned}\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{35\,d}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{35\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\ &+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{35\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^4,x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
128i)/(35*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(35*d*(exp(c*2i + d*x*2i) +
 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))
^(1/2)*32i)/(35*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.282 $\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.282.1 Optimal result 2046
3.282.2 Mathematica [A] (verified) 2046
3.282.3 Rubi [A] (verified) 2047
3.282.4 Maple [A] (verified) 2048
3.282.5 Fricas [B] (verification not implemented) 2048
3.282.6 Sympy [F] 2048
3.282.7 Maxima [A] (verification not implemented) 2049
3.282.8 Giac [B] (verification not implemented) 2049
3.282.9 Mupad [B] (verification not implemented) 2049

3.282.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

3.282.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

input
```
Integrate[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

3.282.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

input
```
Int[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

3.282.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.282.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3ad}$	24

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

3.282.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(3i\,dx+3i\,c)}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(d*e^
(2*I*d*x + 2*I*c) + d)
```

3.282.6 Sympy [F]

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**2, x)
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\,(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}{3\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/3*I*(I*a*tan(d*x + c) + a)^(3/2)/(a*d)
```

3.282.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)^{\frac{3}{2}}}{3\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2/3*I*((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1
/2*d*x + 1/2*c)^2 - 1))^(3/2)/(a*d)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.83

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= -\frac{\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,2\mathrm{i}}{3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^2,x)
```

output
```
-((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)*((a*(cos(2*c + 2*d*x) + sin
(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*2i)/(3*d*(cos(2*c + 2
*d*x) + 1))
```

3.282. $\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.283 $\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.283.1 Optimal result 2051
3.283.2 Mathematica [C] (verified) 2051
3.283.3 Rubi [A] (warning: unable to verify) 2052
3.283.4 Maple [B] (verified) 2054
3.283.5 Fricas [B] (verification not implemented) 2055
3.283.6 Sympy [F] 2055
3.283.7 Maxima [A] (verification not implemented) 2056
3.283.8 Giac [F] 2056
3.283.9 Mupad [F(-1)] 2056

3.283.1 Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{3i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

output
```
-3/8*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d*2^(
1/2)+3/4*I*a/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^2/d/(a+I*a*tan(d*x+c))^(1/
2)/(a-I*a*tan(d*x+c))
```

3.283.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{2d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/2)*a*Hypergeometric2F1[-1/2, 2, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqrt[
a + I*a*Tan[c + d*x]])
```

3.283.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^3\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \; 73$$

$$-\frac{ia^3\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$-\frac{ia^3\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]) + (3*
((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[
a + I*a*Tan[c + d*x]]))/(4*a)))/d
```

3.283.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.283.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(94) = 188$.

Time = 38.56 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.20

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(3i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+3i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\sqrt{-\right.\right.}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)+3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*sin(d*x+c)+3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*I*cos(d*x
+c)*sin(d*x+c)+3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(co
s(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c
)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))-2*cos(d*x+c)^2)
```

3.283.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(87) = 174$.

Time = 0.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.11

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{\Big(3\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(i\,dx+i\,c)}\log\Big(-4\Big(\sqrt{2}\sqrt{\frac{1}{2}}\big(i\,de^{(2i\,dx+2i\,c)}+i\,d\big)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\Big)e^{(-i\,dx-i\,c}}{}$$

input

```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
-1/8*(3*sqrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)
*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a
/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(1/2)*d*sqrt(-a/d^2)*
e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I
*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x +
4*I*c) + I*e^(2*I*d*x + 2*I*c) + 2*I))*e^(-I*d*x - I*c)/d
```

3.283.6 Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^2\left(c+dx\right)dx$$

input

```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

output

```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**2, x)
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\left(3\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\,(3\,(ia\tan(dx+c)+a)a^2-4\,a^3)}{(ia\tan(dx+c)+a)^{\frac{3}{2}}-2\,\sqrt{ia\tan(dx+c)+a}a}\right)}{16\,ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/16*I*(3*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x +
c) + a)*a^2 - 4*a^3)/((I*a*tan(d*x + c) + a)^(3/2) - 2*sqrt(I*a*tan(d*x +
c) + a)*a))/(a*d)
```

3.283.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.284 $\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.284.1 Optimal result 2057
3.284.2 Mathematica [C] (verified) 2058
3.284.3 Rubi [A] (warning: unable to verify) 2058
3.284.4 Maple [B] (verified) 2062
3.284.5 Fricas [A] (verification not implemented) 2062
3.284.6 Sympy [F] 2063
3.284.7 Maxima [A] (verification not implemented) 2063
3.284.8 Giac [F] 2064
3.284.9 Mupad [F(-1)] 2064

3.284.1 Optimal result

Integrand size = 26, antiderivative size = 193

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{35i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-35/128*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d*
2^(1/2)+35/64*I*a/d/(a+I*a*tan(d*x+c))^(1/2)+35/96*I*a^2/d/(a+I*a*tan(d*x+
c))^(3/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(3/2)-7/16*I
*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

3.284.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{12d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/12)*a^2*Hypergeometric2F1[-3/2, 3, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(3/2))
```

3.284.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3968, 52, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\int\frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\int\frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a(a-i}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\frac{\int\frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3}}\right.}{8a}\right.}{d}$$

$$\Big\downarrow 73$$

3.284. $\int\cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a}\right.}{d}$$

$$\Big\downarrow 219$$

$$-\frac{ia^5\left(\frac{7\left(\frac{5\left(\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a(a-ia\tan(c+}\right.}{d}$$

input

```
Int[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) +
 (7*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(-1/
3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/S
qrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a)))/(4*
a)))/(8*a)))/d
```

3.284. $\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.284.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.284.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(154) = 308$.

Time = 109.91 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.13

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(112i(\cos^3(dx+c))\sin(dx+c)+105i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+\right.}{}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(112*I*cos(d*x+c)^3*sin(d*x+c)+105*I
*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+105*I*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-16*cos(d*x
+c)^4+105*I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+210*I*cos(d*x+c)*sin(d*x+c)+105
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-105*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-105*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-70*c
os(d*x+c)^2)
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.42

$$\int\cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{\left(105\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,d}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/384*(105*sqrt(1/2)*d*sqrt(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 105*sqrt(1/2)*d*sqr
t(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x +
 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-6
*I*e^(8*I*d*x + 8*I*c) - 45*I*e^(6*I*d*x + 6*I*c) + 41*I*e^(4*I*d*x + 4*I*
c) + 88*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-3*I*d*x - 3*I*c)/d
```

3.284.6 Sympy [F]

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**4, x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\biggl(105\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\bigl(105\,(i\,a\tan(dx+c)+a)^3a^2-350\,(i\,a\tan(dx+c)+a)^2a^3+224\,(i\,a\tan(dx+c)+a)a^4}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+4\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2}}{768\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/768*I*(105*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(d
*x + c) + a)^3*a^2 - 350*(I*a*tan(d*x + c) + a)^2*a^3 + 224*(I*a*tan(d*x +
 c) + a)*a^4 + 64*a^5)/((I*a*tan(d*x + c) + a)^(7/2) - 4*(I*a*tan(d*x + c)
 + a)^(5/2)*a + 4*(I*a*tan(d*x + c) + a)^(3/2)*a^2))/(a*d)
```

3.284.8 Giac [F]

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^4, x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^4\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.285 $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.285.1 Optimal result 2065
3.285.2 Mathematica [C] (verified) 2066
3.285.3 Rubi [A] (warning: unable to verify) 2066
3.285.4 Maple [B] (verified) 2073
3.285.5 Fricas [A] (verification not implemented) 2073
3.285.6 Sympy [F] 2074
3.285.7 Maxima [A] (verification not implemented) 2074
3.285.8 Giac [F] 2075
3.285.9 Mupad [F(-1)] 2075

3.285.1 Optimal result

Integrand size = 26, antiderivative size = 266

$$\begin{aligned}\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = {} & -\frac{231i\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}d} \\ & + \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} \\ & - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} \\ & - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} \\ & - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} \\ & + \frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}} \\ & + \frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-231/1024*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/
d*2^(1/2)+231/512*I*a/d/(a+I*a*tan(d*x+c))^(1/2)+231/640*I*a^3/d/(a+I*a*ta
n(d*x+c))^(5/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(5/2)-
11/48*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(5/2)-33/64*I*a^4/d/
(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+77/256*I*a^2/d/(a+I*a*tan(d*x+
c))^(3/2)
```

3.285.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.20

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},4,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{40d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/40)*a^3*Hypergeometric2F1[-5/2, 4, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(5/2))
```

3.285.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^6}dx$$
$$\Big\downarrow \; 3968$$
$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{d}$$
$$\Big\downarrow \; 52$$
$$-\frac{ia^7\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$
$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan}{d}$$

$$\Big\downarrow 52$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx}}{12a}}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{}{4a(a-}}{12a}}{d}$$

$$\Big\downarrow 61$$

3.285. $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia}}{8a}}{12a}}{d}$$

$$\Big\downarrow \; 61$$

3.285. $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{}{2a(a-i}\right)}{8a}}{12a}\right)}{d}$$

$$\Big\downarrow \; 73$$

3.285. $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{\int\frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{}{2a(a-ia\tan(c+dx))}}{8a}}{12a}}{d}$$

$$\Big\downarrow 219$$

3.285. $\int\cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

$$-\frac{ia^7\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx}}{8a}}{12a}}{d}$$

input
```
Int[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

3.285. $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(5/2)) +
 (11*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) + (9*(
1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(-1/5*1/(
a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2))
 + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sq
rt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a)))/(4*a)))/(8*a)))/(12*a)))/d
```

3.285.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.285. $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.285.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(214) = 428$.

Time = 108.03 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.65

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(2816i(\cos^5(dx+c))\sin(dx+c)-256(\cos^6(dx+c))+3696i(\cos^3(dx+c))\sin(dx+c)+3465i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ a\right.}{}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15360*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(2816*I*cos(d*x+c)^5*sin(d*x+c)-25
6*cos(d*x+c)^6+3696*I*cos(d*x+c)^3*sin(d*x+c)+3465*I*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)+3465*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-528*cos(d*x+c)^4+3465*I*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))+6930*I*cos(d*x+c)*sin(d*x+c)+3465*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))*sin(d*x+c)-3465*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3465*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2310*cos(d*x+c)^2)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.12

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{\left(3465\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15360*(3465*sqrt(1/2)*d*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3465*sqrt(1/2)*d
*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d
*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I
*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*(-40*I*e^(12*I*d*x + 12*I*c) - 350*I*e^(10*I*d*x + 10*I*c) - 1645*I*e^(8*
I*d*x + 8*I*c) + 1433*I*e^(6*I*d*x + 6*I*c) + 3184*I*e^(4*I*d*x + 4*I*c) +
 464*I*e^(2*I*d*x + 2*I*c) + 48*I))*e^(-5*I*d*x - 5*I*c)/d
```

3.285.6 Sympy [F]

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^6\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**6, x)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.86

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\left(3465\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(3465\left(i\,a\tan(dx+c)+a\right)^5a^2-18480\left(i\,a\tan(dx+c)+a\right)^4a^3+30492\left(i\,a\tan(dx+c\right.\right.}{\left(i\,a\tan(dx+c)+a\right)^{\frac{11}{2}}-6\left(i\,a\tan(dx+c)+a\right)^{\frac{9}{2}}a+12\left(i\right.}\right.}{30720\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/30720*I*(3465*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x +
 c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3465*(I*a*t
an(d*x + c) + a)^5*a^2 - 18480*(I*a*tan(d*x + c) + a)^4*a^3 + 30492*(I*a*t
an(d*x + c) + a)^3*a^4 - 12672*(I*a*tan(d*x + c) + a)^2*a^5 - 2816*(I*a*ta
n(d*x + c) + a)*a^6 - 1536*a^7)/((I*a*tan(d*x + c) + a)^(11/2) - 6*(I*a*ta
n(d*x + c) + a)^(9/2)*a + 12*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 8*(I*a*tan
(d*x + c) + a)^(5/2)*a^3))/(a*d)
```

3.285.8 Giac [F]

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^6, x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^6\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.286 $\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.286.1 Optimal result . . . 2076
3.286.2 Mathematica [A] (verified) . . . 2076
3.286.3 Rubi [A] (verified) . . . 2077
3.286.4 Maple [A] (verified) . . . 2079
3.286.5 Fricas [A] (verification not implemented) . . . 2079
3.286.6 Sympy [F] . . . 2080
3.286.7 Maxima [F(-1)] . . . 2080
3.286.8 Giac [F] . . . 2080
3.286.9 Mupad [B] (verification not implemented) . . . 2081

3.286.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}} + \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} + \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2/13*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(1/2)+256/3003*I*a^4*sec(d*x+c)
^7/d/(a+I*a*tan(d*x+c))^(7/2)+64/429*I*a^3*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c
))^(5/2)+24/143*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{2\sec^6(c+dx)(390\cos(c+dx)+445\cos(3(c+dx))+7i(26\sin(c+dx)+59\sin(3(c+dx))))(i\cos(4(c+}{3003d}$$

input
```
Integrate[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^6*(390*Cos[c + d*x] + 445*Cos[3*(c + d*x)] + (7*I)*(26*Sin
[c + d*x] + 59*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*
Sqrt[a + I*a*Tan[c + d*x]])/(3003*d)
```

3.286.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^7\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{13}a\int \frac{\sec^7(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{13}a\int \frac{\sec(c+dx)^7}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{13}a\left(\frac{8}{11}a\left(\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/13)*a*Sec[c + d*x]^7)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (12*a*((((2
*I)/11)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (8*a*((((8*I)
/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/9)*a*S
ec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))))/11))/13
```

3.286.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.286. $\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.286.4 Maple [A] (verified)

Time = 8.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

method	result
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(1024i\cos(dx+c)+1024\sin(dx+c)-128i\sec(dx+c)+384\sec(dx+c)\tan(dx+c)-40i(\sec^3(dx+c))+280\tan(dx}{3003d}$

input
```
int(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3003/d*(a*(1+I*tan(d*x+c)))^(1/2)*(1024*I*cos(d*x+c)+1024*sin(d*x+c)-128
*I*sec(d*x+c)+384*sec(d*x+c)*tan(d*x+c)-40*I*sec(d*x+c)^3+280*tan(d*x+c)*s
ec(d*x+c)^3-21*I*sec(d*x+c)^5+231*tan(d*x+c)*sec(d*x+c)^5)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-429i\,e^{(6i\,dx+6i\,c)}-286i\,e^{(4i\,dx+4i\,c)}-104i\,e^{(2i\,dx+2i\,c)}-16i\right)}{3003\,(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-429*I*e^(6*I*d*x + 6
*I*c) - 286*I*e^(4*I*d*x + 4*I*c) - 104*I*e^(2*I*d*x + 2*I*c) - 16*I)/(d*e
^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c
) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x +
 2*I*c) + d)
```

3.286.6 Sympy [F]

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^7\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**7, x)
```

3.286.7 Maxima [F(-1)]

Timed out.

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.286.8 Giac [F]

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^7\,dx$$

input
```
integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^7, x)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.97

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,384\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

input

```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^7,x)
```

output

```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*128i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (exp(- c*1i
- d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*128i)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) + (exp(- c*1i - d*x*1i)*(a
 - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*384
i)/(11*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(- c*1i - d*x*1i)*(a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(e
xp(c*2i + d*x*2i) + 1)^6)
```

3.287 $\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.287.1 Optimal result 2082
3.287.2 Mathematica [A] (verified) 2082
3.287.3 Rubi [A] (verified) 2083
3.287.4 Maple [A] (verified) 2084
3.287.5 Fricas [A] (verification not implemented) 2085
3.287.6 Sympy [F] 2085
3.287.7 Maxima [F(-1)] 2085
3.287.8 Giac [F] 2086
3.287.9 Mupad [B] (verification not implemented) 2086

3.287.1 Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}}+\frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2/9*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(1/2)+64/315*I*a^3*sec(d*x+c)^5/
d/(a+I*a*tan(d*x+c))^(5/2)+16/63*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(
3/2)
```

3.287.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2\sec^4(c+dx)(36+71\cos(2(c+dx))+55i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))\sqrt{a+ia\,\mathrm{ta}}}{315d}$$

input
```
Integrate[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(2*Sec[c + d*x]^4*(36 + 71*Cos[2*(c + d*x)] + (55*I)*Sin[2*(c + d*x)])*(I*
Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(315*d)
```

3.287.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{9}a\int \frac{\sec^5(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}a\int \frac{\sec(c+dx)^5}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{9}a\left(\frac{4}{7}a\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{9}a\left(\frac{4}{7}a\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{9}a\left(\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

input

```
Int[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/9)*a*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (8*a*((((8*I
)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/7)*a*
Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))))/9
```

3.287.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.287.4 Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

method	result
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(128i\cos(dx+c)+128\sin(dx+c)-16i\sec(dx+c)+48\sec(dx+c)\tan(dx+c)-5i(\sec^3(dx+c))+35\tan(dx+c)(\sec}{315d}$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315/d*(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*cos(d*x+c)+128*sin(d*x+c)-16*I*s
ec(d*x+c)+48*sec(d*x+c)*tan(d*x+c)-5*I*sec(d*x+c)^3+35*tan(d*x+c)*sec(d*x+
c)^3)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-63i\,e^{(4i\,dx+4i\,c)}-36i\,e^{(2i\,dx+2i\,c)}-8i\right)}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-63*I*e^(4*I*d*x + 4*I*
c) - 36*I*e^(2*I*d*x + 2*I*c) - 8*I)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d
*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.287.6 Sympy [F]

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^5\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**5, x)
```

3.287.7 Maxima [F(-1)]

Timed out.

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.287.8 Giac [F]

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\,\sec\left(dx+c\right)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^5, x)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\&=\frac{32\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,36\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,63\mathrm{i}+8\mathrm{i}\right)}{315\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^5,x)
```

output
```
(32*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*36i + exp(c*4i + d*x*4i)*63i +
 8i))/(315*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.288 $\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.288.1 Optimal result 2087
3.288.2 Mathematica [A] (verified) 2087
3.288.3 Rubi [A] (verified) 2088
3.288.4 Maple [A] (verified) 2089
3.288.5 Fricas [A] (verification not implemented) 2090
3.288.6 Sympy [F] 2090
3.288.7 Maxima [B] (verification not implemented) 2090
3.288.8 Giac [F] 2091
3.288.9 Mupad [B] (verification not implemented) 2091

3.288.1 Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2/5*I*a*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+8/15*I*a^2*sec(d*x+c)^3/d/
(a+I*a*tan(d*x+c))^(3/2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2\sec(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-7i+3\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{15d}$$

input
```
Integrate[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(-2*Sec[c + d*x]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-7*I + 3*Tan[c +
 d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d)
```

3.288.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\int \frac{\sec^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \frac{\sec(c+dx)^3}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((8*I)/15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)
/5)*a*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.288.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.288.4 Maple [A] (verified)

Time = 7.95 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(8i\cos(dx+c)+8\sin(dx+c)-i\sec(dx+c)+3\sec(dx+c)\tan(dx+c))}{15d}$	62

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(a*(1+I*tan(d*x+c)))^(1/2)*(8*I*cos(d*x+c)+8*sin(d*x+c)-I*sec(d*x+c
)+3*sec(d*x+c)*tan(d*x+c))
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(2i\,dx+2i\,c)}-2i\right)}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(2*I*d*x + 2*I*c)
- 2*I)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.288.6 Sympy [F]

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**3, x)
```

3.288.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(57) = 114.

Time = 21.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.04

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{}{15\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\left(\left(\cos\left(4\,dx+4\,c\right)+2\,\cos\left(2\,dx+2\,c\right)+\right.\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
8/15*(5*I*sqrt(2)*cos(2*d*x + 2*c) - 5*sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqrt
(2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*((cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + I*sin(4*d*x + 4*c)
+ 2*I*sin(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) - sin(4*d*x + 4*c)
 - 2*sin(2*d*x + 2*c) + I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*d)
```

3.288.8 Giac [F]

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\,\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.288.9 Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{8\,e^{-c\,1i-d\,x\,1i}\,\left(e^{c\,2i+d\,x\,2i}\,5i+2i\right)\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}}{15\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^3,x)
```

output
```
(8*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*5i + 2i)*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(15*d*(exp(c*2i + d*x
*2i) + 1)^2)
```

3.289 $\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.289.1 Optimal result . . . 2092
3.289.2 Mathematica [A] (verified) . . . 2092
3.289.3 Rubi [A] (verified) . . . 2093
3.289.4 Maple [A] (verified) . . . 2094
3.289.5 Fricas [A] (verification not implemented) . . . 2094
3.289.6 Sympy [F] . . . 2094
3.289.7 Maxima [F] . . . 2095
3.289.8 Giac [F] . . . 2095
3.289.9 Mupad [B] (verification not implemented) . . . 2095

3.289.1 Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I*a*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2(i\cos(c+dx)+\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(I*Cos[c + d*x] + Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.289.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*a*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.289.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.289.4 Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(i\cos(dx+c)+\sin(dx+c))}{d}$	37

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)+sin(d*x+c))
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.289.6 Sympy [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x), x)
```

3.289.7 Maxima [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

3.289.8 Giac [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

3.289.9 Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\left(\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{d}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x),x)
```

output
```
(2*(cos(c + d*x)*1i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*
x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

3.290 $\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

3.290.1 Optimal result 2096
3.290.2 Mathematica [A] (verified) 2096
3.290.3 Rubi [A] (verified) 2097
3.290.4 Maple [B] (verified) 2098
3.290.5 Fricas [B] (verification not implemented) 2099
3.290.6 Sympy [F] 2100
3.290.7 Maxima [B] (verification not implemented) 2100
3.290.8 Giac [F] 2101
3.290.9 Mupad [F(-1)] 2101

3.290.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d} - \frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
1/2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(
1/2)/d*2^(1/2)-I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.290.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ie^{-i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{2d}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/2*I)*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh
[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c +
d*x)))
```

3.290.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.290.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

3.290.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(68) = 136.

Time = 17.33 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.39

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sin(dx+c)+\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\Bigl(\sqrt{-\frac{\cos(}{\cos(d}}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*
x+c)+(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)+I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-I*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
sin(d*x+c)+I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*cos(d*x+c))
```

3.290.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{\sqrt{2}d\sqrt{-\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{d}\right)-\sqrt{2}d\sqrt{-\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2i\,dx+2i\,c)}+d\right)}{}\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*d*sqrt(-a/d^2)*log(2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - sqrt(2)*d
*sqrt(-a/d^2)*log(-2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) - 2*sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I))/d
```

3.290.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x), x)
```

3.290.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(64) = 128$.

Time = 0.41 (sec) , antiderivative size = 774, normalized size of antiderivative = 9.33

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(I*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(
a) + (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sq
rt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(
sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x...
```

3.290.8 Giac [F]

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \cos\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.291 $\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

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3.291.1 Optimal result

Integrand size = 26, antiderivative size = 154

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{5i\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} + \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
5/16*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^
(1/2)/d*2^(1/2)+5/12*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*cos(d
*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/
2)/d
```

3.291.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{ie^{-3i(c+dx)}\left(-3+11e^{2i(c+dx)}+16e^{4i(c+dx)}+2e^{6i(c+dx)}-15e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx}}\right.\right.}{48d}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/48*I)*(-3 + 11*E^((2*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 2*E^((6
*I)*(c + d*x)) - 15*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcT
anh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I
)*(c + d*x)))
```

3.291.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow 3971$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow 3042$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow 3970$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow 219$$

$$\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

3.291. $\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

input
```
Int[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[
c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]
*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos
[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)))/6
```

3.291.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.291.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(123) = 246$.

Time = 23.32 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.56

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(15i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-15i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\Big(\sqrt{-}}{}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c)-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*sin(d*x+c)+20*I*cos(d*x+c)^2*sin(d*x+c)+15*I*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*
x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+15*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c)-4*cos(d*x+c)^3+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))+30*cos(d*x+c))
```

3.291.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(115) = 230$.

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.59

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\left(15\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{4\,d}\right)-15\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2}}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/48*(15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c)*log(5/4*(sqrt(2)*sqr
t(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*
x + 2*I*c)*log(-5/4*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) + sqrt(
2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(6*I*d*x + 6*I*c) - 16*I*e^(4
*I*d*x + 4*I*c) - 11*I*e^(2*I*d*x + 2*I*c) + 3*I))*e^(-2*I*d*x - 2*I*c)/d
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.291.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(115) = 230$.

Time = 0.48 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.07

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/192*(8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(3/4)*(I*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqr
t(a) + 12*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 4*I*sq
rt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)
*cos(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 4*sqrt(2))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 15*(2*sqrt(2)*arctan
2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(...
```

3.291.8 Giac [F]

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^3, x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input

```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output

```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.292 $\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

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3.292.1 Optimal result

Integrand size = 26, antiderivative size = 223

$$\begin{aligned}\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{63i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}d} \\ &+ \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} \\ &- \frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d} \\ &- \frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\end{aligned}$$

output
```
63/256*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*
a^(1/2)/d*2^(1/2)+21/64*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+9/40*I*a
*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-63/128*I*cos(d*x+c)*(a+I*a*tan(d*
x+c))^(1/2)/d-21/80*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d-1/5*I*cos(d*
x+c)^5*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.292.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{ie^{-5i(c+dx)}\left(-10-95e^{2i(c+dx)}+203e^{4i(c+dx)}+344e^{6i(c+dx)}+64e^{8i(c+dx)}+8e^{10i(c+dx)}-315e^{4i(c+dx)}\sqrt{1+}\right.}{1280d}$$

input
```
Integrate[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/1280*I)*(-10 - 95*E^((2*I)*(c + d*x)) + 203*E^((4*I)*(c + d*x)) + 344
*E^((6*I)*(c + d*x)) + 64*E^((8*I)*(c + d*x)) + 8*E^((10*I)*(c + d*x)) - 3
15*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((
2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((5*I)*(c + d*x)))
```

3.292.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3978, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{9}{10}a\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{10}a\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9}{10}a\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{9}{10}a\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow 3978$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow 3983$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+}}\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow 3042$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow 3971$$

3.292. $\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \; 3970$$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\,t}}{3d}\right.}{8a}\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \; 219$$

3.292. $\int\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

$$\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(}{3d}}{8a}\right.$$

$$\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

input

```
Int[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((-1/5*I)*Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d + (9*a*(((I/4)*Cos[
c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*
Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a
*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*S
qrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Ta
n[c + d*x]])/d))/(4*a)))/6))/(8*a)))/10
```

3.292.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.292.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(180) = 360$.

Time = 22.48 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.89

method	result
default	$\frac{i\sqrt{a(1+i\tan(dx+c))}\left(-288i\sin(dx+c)(\cos^4(dx+c))+32(\cos^5(dx+c))+315i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)\right.}{}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(-288*I*sin(d*x+c)*cos(d*x+c)^4+32*c
os(d*x+c)^5+315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*sin(d*x+c)-315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(
d*x+c)-420*I*sin(d*x+c)*cos(d*x+c)^2-315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-
315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)-315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+
c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+84*cos(d*
x+c)^3-315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))-630*cos(d*x+c))
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.20

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \left(315\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(4i\,dx+4i\,c)}\log\left(\frac{63\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{64\,d}\right)-315\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/1280*(315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I*d*x + 4*I*c)*log(63/64*(sqrt(2
)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(
4*I*d*x + 4*I*c)*log(-63/64*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d)
 + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(10*I*d*x + 10*I*c) -
 64*I*e^(8*I*d*x + 8*I*c) - 344*I*e^(6*I*d*x + 6*I*c) - 203*I*e^(4*I*d*x +
 4*I*c) + 95*I*e^(2*I*d*x + 2*I*c) + 10*I))*e^(-4*I*d*x - 4*I*c)/d
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.292.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2220 vs. $2(168) = 336$.

Time = 0.62 (sec) , antiderivative size = 2220, normalized size of antiderivative = 9.96

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
1/5120*(20*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c
) - 3*sqrt(2)*sin(4*d*x + 4*c) - 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin
(4*d*x + 4*c) + 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)))*sqrt(a) + 4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(8*(-I*sqrt(2)*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - I*sqrt(2)*sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 2*I*sqrt(2)*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) - I*sqrt(2))*cos(5/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)) + 5*(5*I*sqrt(2)*cos(4*d*x + 4*c) + 20*I*sqr
t(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5*sqrt(2)*sin(
4*d*x + 4*c) + 20*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - 48*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +...
```

3.292.8 Giac [F]

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^5\,dx$$

input

```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^5, x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \cos\left(c+d\,x\right)^5\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input

```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output

```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.293 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.293.1 Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d} + \frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d} - \frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d} + \frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d}$$

output

```
-16/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4/d+24/13*I*(a+I*a*tan(d*x+c))^(13/2)
/a^5/d-4/5*I*(a+I*a*tan(d*x+c))^(15/2)/a^6/d+2/17*I*(a+I*a*tan(d*x+c))^(17
/2)/a^7/d
```

3.293.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a(1+i\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}(-1767i-3641\tan(c+dx)+2717i\tan^2(c+dx)}{12155d}$$

input

```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
(2*a*(1 + I*Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c + d*x]]*(-1767*I - 3641*Tan
[c + d*x] + (2717*I)*Tan[c + d*x]^2 + 715*Tan[c + d*x]^3))/(12155*d)
```

3.293.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{9/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \; 53$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{15/2}+6a(i\tan(c+dx)a+a)^{13/2}-12a^2(i\tan(c+dx)a+a)^{11/2}+8a^3(i\tan(c+dx)a\right.}{a^7d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i(\frac{16}{11}a^3(a+ia\tan(c+dx))^{11/2}-\frac{24}{13}a^2(a+ia\tan(c+dx))^{13/2}-\frac{2}{17}(a+ia\tan(c+dx))^{17/2}+\frac{4}{5}a(a+ia\tan(c+}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(11/2))/11 - (24*a^2*(a + I*a*Tan[c
+ d*x])^(13/2))/13 + (4*a*(a + I*a*Tan[c + d*x])^(15/2))/5 - (2*(a + I*a*T
an[c + d*x])^(17/2))/17))/(a^7*d)
```

3.293.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.293.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{2a(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{2a(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^{7}}$	82

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/17*(a+I*a*tan(d*x+c))^(17/2)-2/5*a*(a+I*a*tan(d*x+c))^(15/2)+
12/13*a^2*(a+I*a*tan(d*x+c))^(13/2)-8/11*a^3*(a+I*a*tan(d*x+c))^(11/2))
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.45

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{512\sqrt{2}\left(16i\,ae^{(17i\,dx+17i\,c)}+136i\,ae^{(15i\,dx+15i\,c)}+510i\,ae^{(13i\,dx+13i\,c)}+1105i\,ae^{(11i\,dx+1}\right.}{12155\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6}\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-512/12155*sqrt(2)*(16*I*a*e^(17*I*d*x + 17*I*c) + 136*I*a*e^(15*I*d*x + 1
5*I*c) + 510*I*a*e^(13*I*d*x + 13*I*c) + 1105*I*a*e^(11*I*d*x + 11*I*c))*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*
x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70
*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*
c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2i\left(715\,(i\,a\tan\left(dx+c\right)+a)^{\frac{17}{2}}-4862\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}a+11220\,(i\,a\tan\left(dx+c\right)+a)\right.}{12155\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/12155*I*(715*(I*a*tan(d*x + c) + a)^(17/2) - 4862*(I*a*tan(d*x + c) + a)
^(15/2)*a + 11220*(I*a*tan(d*x + c) + a)^(13/2)*a^2 - 8840*(I*a*tan(d*x +
c) + a)^(11/2)*a^3)/(a^7*d)
```

3.293.8 Giac [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)^8\,dx$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^8, x)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.65

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{12155\,d}\\
&\quad-\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{12155\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3072\mathrm{i}}{12155\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\
&\quad-\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{2431\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,155136\mathrm{i}}{2431\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\\
&\quad-\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2413568\mathrm{i}}{12155\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}+\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,270336\mathrm{i}}{1105\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}\\
&\quad-\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,11776\mathrm{i}}{85\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}+\frac{a\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{17\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^8,x)
```

output

```
(a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*155136i)/(2431*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(12155*d*(exp(c*
2i + d*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*3072i)/(12155*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a
 - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512
i)/(2431*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(12155*d) - (a*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2413568i)/
(12155*d*(exp(c*2i + d*x*2i) + 1)^5) + (a*(a - (a*(exp(c*2i + d*x*2i)*1i -
 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*270336i)/(1105*d*(exp(c*2i + d*x*
2i) + 1)^6) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*11776i)/(85*d*(exp(c*2i + d*x*2i) + 1)^7) + (a*(a - (a*(ex
p(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(17*d*
(exp(c*2i + d*x*2i) + 1)^8)
```

3.293. $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

3.294 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.294.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=-\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}$$

output
```
-8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d+8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4
/d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^5/d
```

3.294.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{2ia(-i+\tan(c+dx))^4\sqrt{a+ia\tan(c+dx)}(-203+270i\tan(c+dx)+99\tan^2(c+dx))}{1287d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/1287)*a*(-I + Tan[c + d*x])^4*Sqrt[a + I*a*Tan[c + d*x]]*(-203 + (
270*I)*Tan[c + d*x] + 99*Tan[c + d*x]^2))/d
```

3.294.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int\left((i\tan(c+dx)a+a)^{11/2}-4a(i\tan(c+dx)a+a)^{9/2}+4a^2(i\tan(c+dx)a+a)^{7/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{9}a^2(a+ia\tan(c+dx))^{9/2}+\frac{2}{13}(a+ia\tan(c+dx))^{13/2}-\frac{8}{11}a(a+ia\tan(c+dx))^{11/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(9/2))/9 - (8*a*(a + I*a*Tan[c + d*x]
)^(11/2))/11 + (2*(a + I*a*Tan[c + d*x])^(13/2))/13))/(a^5*d)
```

3.294.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.294.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{4a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{4a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/13*(a+I*a*tan(d*x+c))^(13/2)+4/11*a*(a+I*a*tan(d*x+c))^(11/2
)-4/9*a^2*(a+I*a*tan(d*x+c))^(9/2))
```

3.294.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{128\,\sqrt{2}\left(8i\,ae^{(13i\,dx+13i\,c)}+52i\,ae^{(11i\,dx+11i\,c)}+143i\,ae^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1287\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-128/1287*sqrt(2)*(8*I*a*e^(13*I*d*x + 13*I*c) + 52*I*a*e^(11*I*d*x + 11*I
*c) + 143*I*a*e^(9*I*d*x + 9*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^
(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c)
 + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x +
2*I*c) + d)
```

3.294.6 Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec^6{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**6, x)
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{2i\left(99\,(i\,a\tan{(dx+c)}+a)^{\frac{13}{2}}-468\,(i\,a\tan{(dx+c)}+a)^{\frac{11}{2}}a+572\,(i\,a\tan{(dx+c)}+a)^{\frac{9}{2}}a^2\right)}{1287\,a^5d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/1287*I*(99*(I*a*tan(d*x + c) + a)^(13/2) - 468*(I*a*tan(d*x + c) + a)^(
11/2)*a + 572*(I*a*tan(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

3.294.8 Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^6, x)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.77

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = & -\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1287\,d} \\ & -\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1287\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{429\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} \\ & +\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,27136\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,58624\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} \\ & +\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,5120\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}-\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^6,x)
```

output
```
(a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*27136i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1287*d*(exp(c*2i
+ d*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*128i)/(429*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1
287*d) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*58624i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) + (a*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*5120i)/(143*d*(
exp(c*2i + d*x*2i) + 1)^5) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.294. $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

3.295 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.295.1 Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

output
```
-4/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^2/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d
```

3.295.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a(-1-i\tan(c+dx))^3(11i+7\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{63d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*(-1 - I*Tan[c + d*x])^3*(11*I + 7*Tan[c + d*x])*Sqrt[a + I*a*Tan[c +
d*x]])/(63*d)
```

3.295.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{5/2}-(i\tan(c+dx)a+a)^{7/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{7}a(a+ia\tan(c+dx))^{7/2}-\frac{2}{9}(a+ia\tan(c+dx))^{9/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(7/2))/7 - (2*(a + I*a*Tan[c + d*x])^(9
/2))/9))/(a^3*d)
```

3.295.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.295.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^3}$	44

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/9*(a+I*a*tan(d*x+c))^(9/2)-2/7*a*(a+I*a*tan(d*x+c))^(7/2))
```

3.295.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{32\sqrt{2}\left(2i\,ae^{(9i\,dx+9i\,c)}+9i\,ae^{(7i\,dx+7i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{63\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-32/63*sqrt(2)*(2*I*a*e^(9*I*d*x + 9*I*c) + 9*I*a*e^(7*I*d*x + 7*I*c))*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6
*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.295.6 Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec^4{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**4, x)
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{2i\left(7\,(i\,a\tan{(dx+c)}+a)^{\frac{9}{2}}-18\,(i\,a\tan{(dx+c)}+a)^{\frac{7}{2}}a\right)}{63\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/63*I*(7*(I*a*tan(d*x + c) + a)^(9/2) - 18*(I*a*tan(d*x + c) + a)^(7/2)*a
)/(a^3*d)
```

3.295.8 Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\int (i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 296, normalized size of antiderivative = 5.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{63\,d} - \frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{63\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,160\mathrm{i}}{21\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,608\mathrm{i}}{63\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{a\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

input

```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^4,x)
```

output

```
(a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*160i)/(21*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*d*(exp(c*2i + d*x*2
i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
 + 1))^(1/2)*64i)/(63*d) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*608i)/(63*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*
(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3
2i)/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.296 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.296.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

output
```
-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

3.296.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

3.296.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

3.296.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.296.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5ad}$	24

input

```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

3.296.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{8i\,\sqrt{2}a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(5i\,dx+5i\,c)}}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input

```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
-8/5*I*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(d*
e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.296.6 Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^2(c+dx)\,dx$$

input

```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

output

```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**2, x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i\,(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}{5\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*I*(I*a*tan(d*x + c) + a)^(5/2)/(a*d)
```

3.296.8 Giac [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^2\;dx$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.296.9 Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.28

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{4\,a\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos{(2\,c+2\,d\,x)}\;7\mathrm{i}+\cos{(4\,c+4\,d\,x)}\;4\mathrm{i}+\cos{(6\,c+6\,d\,x)}\;1\mathrm{i}-5\;\sin{(2}}{5\,d\;(15\;\cos{(2\,c+2\,d\,x)}+6\;\cos{(4\,c+4\,d\,x)}+\cos{(6\,c+6\,d}}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^2,x)
```

output
```
-(4*a*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^(1/2)*(cos(2*c + 2*d*x)*7i + cos(4*c + 4*d*x)*4i + cos(6*c + 6*d*x)*
1i - 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 4i))/(5*
d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.297 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

3.297.1 Optimal result 2141
3.297.2 Mathematica [A] (verified) 2141
3.297.3 Rubi [A] (warning: unable to verify) 2142
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3.297.8 Giac [F(-1)] 2145
3.297.9 Mupad [F(-1)] 2146

3.297.1 Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}-\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}$$

output
```
-1/4*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(
1/2)-1/2*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

3.297.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)(1-i\tan(c+dx))+2a\sqrt{a+ia\tan(c+dx)}}{4d(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]*(1
- I*Tan[c + d*x]) + 2*a*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*(I + Tan[c + d*x]
))
```

3.297.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 3968, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \ 52$$

$$-\frac{ia^3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \ 73$$

$$-\frac{ia^3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{ia^3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^3*(((I/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)
) + Sqrt[a + I*a*Tan[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x]))))/d
```

3.297.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.297.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(73) = 146$.

Time = 15.41 (sec) , antiderivative size = 576, normalized size of antiderivative = 6.19

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(i\,\text{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)\sin(dx}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(I*arctanh(
sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+I*arctanh(
sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-arctanh(sin(d*x+c)/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)^2-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+I*cos(d*x+c)^2-(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+I*cos(d*x+c)-sin(d*x+c)*cos(d
*x+c))/(I*cos(d*x+c)+I-sin(d*x+c))
```

3.297.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(68) = 136$.

Time = 0.24 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}\,d\log\left(-\frac{4(}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x
 + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^
(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sq
rt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(
2)*(-I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)))/d
```

3.297.6 Sympy [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\cos^2{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x)**2, x)
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{16\,\sqrt{i\,a\tan(dx+c)+a}a^3}{4i\,a\tan(dx+c)-4\,a}\right)}{8\,ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/8*I*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 16*sqrt(I*a*tan(d*x + c
) + a)*a^3/(4*I*a*tan(d*x + c) - 4*a))/(a*d)
```

3.297.8 Giac [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\tan(c+d\,x)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.298 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

3.298.1 Optimal result 2147
3.298.2 Mathematica [C] (verified) 2147
3.298.3 Rubi [A] (warning: unable to verify) 2148
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3.298.8 Giac [F(-1)] 2153
3.298.9 Mupad [F(-1)] 2153

3.298.1 Optimal result

Integrand size = 26, antiderivative size = 166

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{15ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

output
```
-15/64*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2
^(1/2)+15/32*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/4*I*a^4/d/(a+I*a*tan(d*x+c
))^(1/2)/(a-I*a*tan(d*x+c))^2-5/16*I*a^3/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a
*tan(d*x+c))
```

3.298.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.32

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{4d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/4)*a^2*Hypergeometric2F1[-1/2, 3, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

3.298.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3968, 52, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^5\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+}}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{}{4a(a-ia\tan(c+d}\right.}{d}$$

$$\Big\downarrow 73$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia}}\right.}{d}$$

$$\Big\downarrow 219$$

$$-\frac{ia^5\left(\frac{5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) + (
5*(1/(2*a*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*ArcT
an[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*
Tan[c + d*x]])))/(4*a)))/(8*a)))/d
```

3.298. $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

3.298.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.298.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(133) = 266$.

Time = 14.37 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.97

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(30i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)(\cos^2(dx+c))+\right.}{}$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(30*I*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+15*
I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+15*I*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+40*I*cos(d
*x+c)^2*sin(d*x+c)+30*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)
-30*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)^2-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+15*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*sin(d*x+c)-15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-24*cos(d*x+c)^3+15*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+30*co
s(d*x+c))
```

3.298.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(123) = 246.

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.73

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(i\,dx+i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-15\sqrt{\frac{1}{2}}\sqrt{}\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(15*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(
1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 15*sqrt(1/2)*sqrt
(-a^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x +
2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I*
d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-2*I*a*e^(6*I*d*x + 6*I*c) - 11
*I*a*e^(4*I*d*x + 4*I*c) - I*a*e^(2*I*d*x + 2*I*c) + 8*I*a)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(15\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(15\,(i\,a\tan(dx+c)+a)^2a^3-50\,(i\,a\tan(dx+c)+a)a^4+32\,a^5\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+4\,\sqrt{i\,a\tan(dx+c)+aa}}}{128\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/128*I*(15*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x
 + c) + a)^2*a^3 - 50*(I*a*tan(d*x + c) + a)*a^4 + 32*a^5)/((I*a*tan(d*x +
 c) + a)^(5/2) - 4*(I*a*tan(d*x + c) + a)^(3/2)*a + 4*sqrt(I*a*tan(d*x + c
) + a)*a^2))/(a*d)
```

3.298.8 Giac [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^4\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.299 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.299.1 Optimal result

Integrand size = 26, antiderivative size = 239

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{105ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}d} + \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-105/512*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d
*2^(1/2)+105/256*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)+35/128*I*a^3/d/(a+I*a*ta
n(d*x+c))^(3/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(3/2)-
3/16*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(3/2)-21/64*I*a^4/d/(
a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

3.299.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.22

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},4,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{24d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/24)*a^3*Hypergeometric2F1[-3/2, 4, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(3/2))
```

3.299.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(}\right.}{d}$$

$$\Big\downarrow \ 52$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx)}\right.}{4a}\right.}{d}$$

$$\Big\downarrow \ 61$$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\int\frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a(a-ia}\right.}{4a}\right.}{d}$$

$$\Big\downarrow \ 61$$

3.299. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+d}\right)}{8a}\right)}{4a}\right)}{d}$$

$$\Big\downarrow \; 73$$

3.299. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a}\right)}{4a}\right)}{d}$$

$$\Big\downarrow 219$$

3.299. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

$$-\frac{ia^7\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{}{4a(a-ia\tan(c+d}}{4a}}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(3/2)) +
 (3*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) + (7*(1
/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(-1/3*1/(a
*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]
])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a)))/(4*a)))/(
8*a)))/(4*a)))/d
```

3.299. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

3.299.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.299.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(193) = 386$.

Time = 14.05 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.87

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(384i\sin(dx+c)(\cos^4(dx+c))+630i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx}{(\cos(dx+c)+1)\sqrt{}}\right.\right.}{}$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1536/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(384*I*c
os(d*x+c)^4*sin(d*x+c)+630*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(si
n(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+6
30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)*sin(d*x+c)-128*cos(d*x+c)^5+315*I*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)+315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+840*I*cos(d*x+c)^2*sin(d*x+c)+6
30*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-630*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)
^2-315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*sin(d*x+c)-315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*cos(d*x+c)-504*cos(d*x+c)^3+315*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+630*cos(d*x+c))
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.30

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\left(315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(3i\,dx+3i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-315\sqrt{}\right.}{}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/1536*(315*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 315*sqrt(1
/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e
^(2*I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
 - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-8*I*a*e^(10*I*d*x
+ 10*I*c) - 58*I*a*e^(8*I*d*x + 8*I*c) - 215*I*a*e^(6*I*d*x + 6*I*c) + 43*
I*a*e^(4*I*d*x + 4*I*c) + 224*I*a*e^(2*I*d*x + 2*I*c) + 16*I*a)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/d
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(315\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4a^3-1680\,(i\,a\tan(dx+c)+a)^3a^4+2772\right.}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a+12}\right.}{3072\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/3072*I*(315*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(
d*x + c) + a)^4*a^3 - 1680*(I*a*tan(d*x + c) + a)^3*a^4 + 2772*(I*a*tan(d*
x + c) + a)^2*a^5 - 1152*(I*a*tan(d*x + c) + a)*a^6 - 256*a^7)/((I*a*tan(d
*x + c) + a)^(9/2) - 6*(I*a*tan(d*x + c) + a)^(7/2)*a + 12*(I*a*tan(d*x +
c) + a)^(5/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(3/2)*a^3))/(a*d)
```

3.299.8 Giac [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^6\,\left(a+a\,\tan(c+d\,x)\;1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.300 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.300.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

output
```
8/33*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(1/2)+2/11*I*a*sec(d*x+c)^5*(
a+I*a*tan(d*x+c))^(1/2)/d+256/1155*I*a^4*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))
^(5/2)+64/231*I*a^3*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.300.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{2a\sec^4(c+dx)(\cos(dx)-i\sin(dx))(i\cos(3c+2dx)+\sin(3c+2dx))(39+494\cos(2(c+dx)}{1155d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*Sec[c + d*x]^4*(Cos[d*x] - I*Sin[d*x])*(I*Cos[3*c + 2*d*x] + Sin[3*c
+ 2*d*x])*(39 + 494*Cos[2*(c + d*x)] + (215*I)*Sec[c + d*x]*Sin[3*(c + d*x
)] + (110*I)*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(1155*d)
```

3.300.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{11}a\int \sec^5(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{11}a\int \sec(c+dx)^5\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\int \frac{\sec^5(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\int \frac{\sec(c+dx)^5}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\left(\frac{4}{7}a\int\frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\left(\frac{4}{7}a\int\frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{11}a\left(\frac{8}{9}a\left(\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\right)+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

input
```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/11)*a*Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d + (12*a*(((((2*I
)/9)*a*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (8*a*((((8*I)/35)*
a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/7)*a*Sec[c
+ d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))))/9))/11
```

3.300.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.300.4 Maple [A] (verified)

Time = 7.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.15

method	result
default	$2a\sqrt{a(1+i\tan(dx+c))}\,(-72i\sec(dx+c)+105i(\tan^2(dx+c))(\sec^3(dx+c))+1024i\cos(dx+c)(\sin^2(dx+c))+512\sin(dx+c)-384i\cos($

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(-72*I*sec(d*x+c)+105*I*tan(d*x+c)^2
*sec(d*x+c)^3+1024*I*cos(d*x+c)*sin(d*x+c)^2+512*sin(d*x+c)-384*I*cos(d*x+
c)-35*I*sec(d*x+c)^3+192*sec(d*x+c)*tan(d*x+c)+1024*I*cos(d*x+c)^3+128*I*s
in(d*x+c)*tan(d*x+c)+140*tan(d*x+c)*sec(d*x+c)^3+120*I*tan(d*x+c)^2*sec(d*
x+c))
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{64\sqrt{2}\left(-231i\,ae^{(6i\,dx+6i\,c)}-198i\,ae^{(4i\,dx+4i\,c)}-88i\,ae^{(2i\,dx+2i\,c)}-16i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1155\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-64/1155*sqrt(2)*(-231*I*a*e^(6*I*d*x + 6*I*c) - 198*I*a*e^(4*I*d*x + 4*I*
c) - 88*I*a*e^(2*I*d*x + 2*I*c) - 16*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6
*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

3.300.6 Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec^5{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**5, x)
```

3.300.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(115) = 230$.

Time = 9.67 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.76

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
64/1155*(231*I*sqrt(2)*a*cos(6*d*x + 6*c) + 198*I*sqrt(2)*a*cos(4*d*x + 4*
c) + 88*I*sqrt(2)*a*cos(2*d*x + 2*c) - 231*sqrt(2)*a*sin(6*d*x + 6*c) - 19
8*sqrt(2)*a*sin(4*d*x + 4*c) - 88*sqrt(2)*a*sin(2*d*x + 2*c) + 16*I*sqrt(2
)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a)/(((4*cos(2*d*x + 2*c)^3 + (4*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c)^2 + 4*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + 4*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 6*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 9*cos(2*d*x + 2*c)^2 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I
*cos(2*d*x + 2*c) + I)*sin(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*c)^2 + I*sin(
2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(6*d*x + 6*c) + 6*(I*cos(2*d
*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(4*d*x +
 4*c) + 4*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*
c) + 6*cos(2*d*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + (4*I*cos(2*d*x + 2*c)^3 + (4*I*cos(2*d*x + 2*c) + I)*sin(2*d
*x + 2*c)^2 - 4*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x +
 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*
c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(6*d*x + 6*c) +
 6*(I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) ...
```

3.300.8 Giac [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^5, x)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.99

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{a\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{5\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2}-\frac{a\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,192i}{7\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3}+\frac{a\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4}-\frac{a\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{11\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^5,x)
```

output
```
(a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*64i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(- c*
1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
+ 1))^(1/2)*192i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*exp(- c*1i - d*x*1
i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*64i)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*exp(- c*1i - d*x*1i)*(a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*
d*(exp(c*2i + d*x*2i) + 1)^5)
```

3.301 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.301.1 Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}} + \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

output
```
16/35*I*a^2*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+2/7*I*a*sec(d*x+c)^3*(
a+I*a*tan(d*x+c))^(1/2)/d+64/105*I*a^3*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(
3/2)
```

3.301.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \sec^3(c+dx)(a+ia\tan(c +dx))^{3/2}\,dx = \frac{2a\sec^3(c+dx)(\cos(dx)-i\sin(dx))(28+43\cos(2(c+dx))+27i\sin(2(c+dx)))(i\cos(2c+}{105d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*Sec[c + d*x]^3*(Cos[d*x] - I*Sin[d*x])*(28 + 43*Cos[2*(c + d*x)] + (2
7*I)*Sin[2*(c + d*x)])*(I*Cos[2*c + d*x] + Sin[2*c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(105*d)
```

3.301.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{7}a\int \sec^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}a\int \sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{7}a\left(\frac{4}{5}a\int \frac{\sec^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{7}a\left(\frac{4}{5}a\int \frac{\sec(c+dx)^3}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{7}a\left(\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

input

```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/7)*a*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (8*a*((((8*I)/
15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/5)*a*Se
c[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])))/7
```

3.301.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.301.4 Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

method	result
default	$\frac{2a\sqrt{a(1+i\tan(dx+c))}\,\left(128i\cos(dx+c)\left(\sin^{2}(dx+c)\right)+128i\left(\cos^{3}(dx+c)\right)+16i\sin(dx+c)\tan(dx+c)-48i\cos(dx+c)+64\sin(dx+c)+}{105d}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*cos(d*x+c)*sin(d*x+c)^2+128*I*
cos(d*x+c)^3+16*I*sin(d*x+c)*tan(d*x+c)-48*I*cos(d*x+c)+64*sin(d*x+c)+15*I
*tan(d*x+c)^2*sec(d*x+c)-9*I*sec(d*x+c)+24*sec(d*x+c)*tan(d*x+c))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{16\sqrt{2}\left(-35i\,ae^{(4i\,dx+4i\,c)}-28i\,ae^{(2i\,dx+2i\,c)}-8i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{105\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-16/105*sqrt(2)*(-35*I*a*e^(4*I*d*x + 4*I*c) - 28*I*a*e^(2*I*d*x + 2*I*c)
- 8*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^
(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.301.6 Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(ia(\tan{(c+dx)}-i)\right)^{\frac{3}{2}}\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**3, x)
```

3.301.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(86) = 172.

Time = 0.61 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.27

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{}{105\left((2\cos{(2\,dx+2\,c)})^3+(2\cos{(2\,dx+2\,c)}+1)\sin{(2\,dx+2\,c)}^2+2i\sin{(2\,dx+2\,c)}^3+(\cdots\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
16/105*(35*I*sqrt(2)*a*cos(4*d*x + 4*c) + 28*I*sqrt(2)*a*cos(2*d*x + 2*c)
- 35*sqrt(2)*a*sin(4*d*x + 4*c) - 28*sqrt(2)*a*sin(2*d*x + 2*c) + 8*I*sqrt
(2)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*sqrt(a)/(((2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c) + 1)*sin(2*d*x
 + 2*c)^2 + 2*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c)^2 + (I
*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin
(4*d*x + 4*c) + 2*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*
d*x + 2*c) + 4*cos(2*d*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + (2*I*cos(2*d*x + 2*c)^3 + (2*I*cos(2*d*x + 2*c) + I)
*sin(2*d*x + 2*c)^2 - 2*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin
(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) + 5*I*cos(2*d
*x + 2*c)^2 - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*sin(4*d*x + 4*c) - 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(2*d*x + 2*c) + 4*I*cos(2*d*x + 2*c) + I)*sin(3/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*d)
```

3.301.8 Giac [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.301.9 Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{16\,a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,28\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,35\mathrm{i}+8\mathrm{i}\right)}{105\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^3,x)
```

output
```
(16*a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*28i + exp(c*4i + d*x*4i)*35i
 + 8i))/(105*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.301. $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

3.302 $\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.302.1 Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
8/3*I*a^2*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+2/3*I*a*sec(d*x+c)*(a+I*a*
tan(d*x+c))^(1/2)/d
```

3.302.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=-\frac{2a(\cos(c)-i\sin(c))(\cos(dx)-i\sin(dx))(-5i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*a*(Cos[c] - I*Sin[c])*(Cos[d*x] - I*Sin[d*x])*(-5*I + Tan[c + d*x])*Sq
rt[a + I*a*Tan[c + d*x]])/(3*d)
```

3.302.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((8*I)/3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a
*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.302.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.302.4 Maple [A] (verified)

Time = 6.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{2ia\sqrt{a(1+i\tan(dx+c))}\,\left(8(\sin^2(dx+c))\cos(dx+c)-4i\sin(dx+c)+8(\cos^3(dx+c))+\sin(dx+c)\tan(dx+c)-3\cos(dx+c)\right)}{3d}$	80

input

```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3*I/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(8*sin(d*x+c)^2*cos(d*x+c)-4*I*sin(d*
x+c)+8*cos(d*x+c)^3+sin(d*x+c)*tan(d*x+c)-3*cos(d*x+c))
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=-\frac{4\sqrt{2}\left(-3i\,ae^{(2i\,dx+2i\,c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input

```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
-4/3*sqrt(2)*(-3*I*a*e^(2*I*d*x + 2*I*c) - 2*I*a)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.302.6 Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x), x)
```

3.302.7 Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}\;dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.302.8 Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}\;dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.302.9 Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\,a\,\sqrt{\frac{a\,\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}+\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2\,2\mathrm{i}+\sin\left(c+d\,x\right)+\sin\left(3\right.}{3\,d\cos\left(c+d\,x\right)^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x),x)
```

output
```
(2*a*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/
2)*(sin(c + d*x) + sin(3*c + 3*d*x) + cos(c/2 + (d*x)/2)^2*8i + cos((3*c)/
2 + (3*d*x)/2)^2*2i - 5i))/(3*d*cos(c + d*x)^2)
```

3.303 $\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.303.1 Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.303.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.303.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)}dx$$
$$\Big\downarrow \text{3974}$$
$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.303.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.303.4 Maple [A] (verified)

Time = 15.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{2i\sqrt{a(1+i\tan(dx+c))}\,a\left(\left(\sin^2(dx+c)\right)\cos(dx+c)+\cos^3(dx+c)\right)}{d}$	47

input

```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*a*(sin(d*x+c)^2*cos(d*x+c)+cos(d*x+c)^3)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

input

```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output

```
sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
/d
```

3.303.6 Sympy [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\cos(c+dx)\,dx$$

input

```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output

```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x), x)
```

3.303.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(25) = 50$.

Time = 0.40 (sec) , antiderivative size = 201, normalized size of antiderivative = 6.48

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\left(i\,a^{\frac{3}{2}} - \frac{2i\,a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{i\,a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)^{\frac{3}{2}}}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} - \frac{2i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 1\right)}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*(I*a^(3/2) - 2*I*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + I*a^(3/2)
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(d*(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)*(-2*I*
sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1))
```

3.303.8 Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.303.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a\left(2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\sqrt{\frac{a\left(2\cos(c+dx)^2+\sin(2c+2dx)\,1\mathrm{i}\right)}{2\cos(c+dx)^2}}\,2\mathrm{i}}{d}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
-(a*(2*cos(c/2 + (d*x)/2)^2 - 1)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)
^2))/(2*cos(c + d*x)^2))^(1/2)*2i)/d
```

3.304 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.304.1 Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d} - \frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

output

```
1/4*I*a^(3/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))/d*2^(1/2)-1/2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x
+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

3.304.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{iae^{-i(c+dx)}\left(4+5e^{2i(c+dx)}+e^{4i(c+dx)}-3\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{12d}$$

input

```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/12*I)*a*(4 + 5*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x)) - 3*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a
*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

3.304.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3971, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{1}{2}a\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow 219$$

$$\frac{1}{2}a\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/3*I)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (a*((I*Sqrt[a]*
ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqr
t[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/2
```

3.304.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

3.304.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(97) = 194$.

Time = 11.93 (sec) , antiderivative size = 648, normalized size of antiderivative = 5.31

method	result
default	$-\frac{i(\tan(dx+c)-i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\bigg(6i\,\mathrm{arctanh}\bigg(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\bigg)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)\sin(dx}{}$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d*(tan(d*x+c)-I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(6*I*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+6*I*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+3*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+3*I*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-6*arctanh(sin
(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)^2+6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-3*I*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+10*I*
cos(d*x+c)^2-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+3*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+
3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*sin(d*x+c)*cos(d*x+c))
```

3.304.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(91) = 182$.

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.82

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{d}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\Bigg(}{}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log((sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x +
2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + I*a^2)*e^(-
I*d*x - I*c)/d) - 3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-(sqrt(2)*sqrt(1/2)*(d*
e^(2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
- I*a^2)*e^(-I*d*x - I*c)/d) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c) - 5*I*a*e
^(2*I*d*x + 2*I*c) - 4*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

3.304.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.304.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(91) = 182$.

Time = 0.83 (sec) , antiderivative size = 884, normalized size of antiderivative = 7.25

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/48*(4*(I*sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*
sqrt(a) + 12*(I*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a) + 3*(2*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) - 2*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) -
 I*sqrt(2)*a*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 +
 sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*a*log(sqrt(...
```

3.304.8 Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\int \cos\left(c+d\,x\right)^3\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2} \,d x$$

input

```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output

```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.305 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

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3.305.1 Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{7ia^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d} + \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

output
```
7/32*I*a^(3/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d*2^(1/2)+7/24*I*a^2*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-7/16*I*a*
cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-7/30*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x
+c))^(1/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d
```

3.305.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{iae^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-15+101e^{2i(c+dx)}+148e^{4i(c+dx)}+38e^{6i(c+dx)}+6e^{8i(c+dx)}-105e^{2i(c+dx)}\sqrt{1+e^{2i}}\right.}{240\sqrt{2}d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/240*I)*a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-15
 + 101*E^((2*I)*(c + d*x)) + 148*E^((4*I)*(c + d*x)) + 38*E^((6*I)*(c + d*
x)) + 6*E^((8*I)*(c + d*x)) - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c
+ d*x)))
```

3.305.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 3978, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{7}{10}a\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\int\frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7}{10}a\Bigg(\frac{5}{6}a\left(\frac{3\int\cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx}}{3d}$$
$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{7}{10}a\Bigg(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a}}{3}$$
$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}a\Bigg(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a}}{3}$$
$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3970}$$

3.305. $\int\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-i\cos^3(\right.$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow 219$$

$$\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-i\cos^3(c+\right.$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-1/3*I)
*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])
/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c +
d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]
*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)))/6))/10
```

3.305.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.305.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(155) = 310.

Time = 15.24 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.52

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(210i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)(\cos^2(dx+c))}{}$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/480/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(210*I*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-210*I*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+
168*I*cos(d*x+c)^3*sin(d*x+c)+105*I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)-105*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))*sin(d*x+c)+210*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin
(d*x+c)+210*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)^2-72*cos(d*x+c)^4-105*I*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))-210*I*sin(d*x+c)*cos(d*x+c)+105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin
(d*x+c)+105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)-105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+350*cos(d*x+c)^2)
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.43

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(\frac{7\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{8\,d}\right)-105}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/480*(105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c)*log(7/8*(sqrt(2)
*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)) + I*a^2)*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^3/d^2)*
d*e^(2*I*d*x + 2*I*c)*log(-7/8*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) +
 d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - I*a^2)*e^(-I*d*x -
I*c)/d) + sqrt(2)*(-6*I*a*e^(8*I*d*x + 8*I*c) - 38*I*a*e^(6*I*d*x + 6*I*c)
 - 148*I*a*e^(4*I*d*x + 4*I*c) - 101*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.305.7 Maxima [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.305.8 Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^5\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.306 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.306.2 Mathematica [A] (verified) 2201
3.306.3 Rubi [A] (verified) 2202
3.306.4 Maple [A] (verified) 2203
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3.306.1 Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d} - \frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d} + \frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d}$$

output
```
-16/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^4/d+8/5*I*(a+I*a*tan(d*x+c))^(15/2)/a
^5/d-12/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^6/d+2/19*I*(a+I*a*tan(d*x+c))^(19
/2)/a^7/d
```

3.306.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^6\sqrt{a+ia\tan(c+dx)}(-2429i-5291\tan(c+dx)+4095i\tan^2(c+dx)+1105\tan}{20995d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(-I + Tan[c + d*x])^6*Sqrt[a + I*a*Tan[c + d*x]]*(-2429*I - 5291*T
an[c + d*x] + (4095*I)*Tan[c + d*x]^2 + 1105*Tan[c + d*x]^3))/(20995*d)
```

3.306.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int\left(-(i\tan(c+dx)a+a)^{17/2}+6a(i\tan(c+dx)a+a)^{15/2}-12a^2(i\tan(c+dx)a+a)^{13/2}+8a^3(i\tan(c+dx)a}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{16}{13}a^3(a+ia\tan(c+dx))^{13/2}-\frac{8}{5}a^2(a+ia\tan(c+dx))^{15/2}-\frac{2}{19}(a+ia\tan(c+dx))^{19/2}+\frac{12}{17}a(a+ia\tan(c+\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(13/2))/13 - (8*a^2*(a + I*a*Tan[c +
 d*x])^(15/2))/5 + (12*a*(a + I*a*Tan[c + d*x])^(17/2))/17 - (2*(a + I*a*T
an[c + d*x])^(19/2))/19))/(a^7*d)
```

3.306.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.306.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{19}{2}}}{19}-\frac{6a(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}+\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}\right)}{d\,a^{7}}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
2*I/d/a^7*(1/19*(a+I*a*tan(d*x+c))^(19/2)-6/17*a*(a+I*a*tan(d*x+c))^(17/2)
+4/5*a^2*(a+I*a*tan(d*x+c))^(15/2)-8/13*a^3*(a+I*a*tan(d*x+c))^(13/2))
```

3.306.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{1024\sqrt{2}(16i\,a^2e^{(19i\,dx+19i\,c)}+152i\,a^2e^{(17i\,dx+17i\,c)}+646i\,a^2e^{(15i\,dx+15i\,c)}+161}{20995\,(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8i}}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1024/20995*sqrt(2)*(16*I*a^2*e^(19*I*d*x + 19*I*c) + 152*I*a^2*e^(17*I*d*
x + 17*I*c) + 646*I*a^2*e^(15*I*d*x + 15*I*c) + 1615*I*a^2*e^(13*I*d*x + 1
3*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(18*I*d*x + 18*I*c) + 9*d*e
^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*
I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6
*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i\Big(1105\,(i\,a\tan(dx+c)+a)^{\frac{19}{2}} - 7410\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}}a + 16796\,(i\,a\tan(dx+c)+}{20995\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/20995*I*(1105*(I*a*tan(d*x + c) + a)^(19/2) - 7410*(I*a*tan(d*x + c) + a
)^(17/2)*a + 16796*(I*a*tan(d*x + c) + a)^(15/2)*a^2 - 12920*(I*a*tan(d*x
+ c) + a)^(13/2)*a^3)/(a^7*d)
```

3.306.8 Giac [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^8\,dx$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^8, x)
```

3.306.9 Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 626, normalized size of antiderivative = 5.35

$$
\begin{aligned}
&\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \\
&-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16384\mathrm{i}}{20995\,d}-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{20995\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} \\
&-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6144\mathrm{i}}{20995\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{4199\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} \\
&+\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,536576\mathrm{i}}{4199\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,10484736\mathrm{i}}{20995\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} \\
&+\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,17262592\mathrm{i}}{20995\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1129472\mathrm{i}}{1615\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7} \\
&+\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,98304\mathrm{i}}{323\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{19\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^9}
\end{aligned}
$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^8,x)
```

output
```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*536576i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*2i
 + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(20995*d*(ex
p(c*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*6144i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^2) -
 (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*1024i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(20995*d) -
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*10484736i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^2*(a - (a*(exp(c*
2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*17262592i)/(2099
5*d*(exp(c*2i + d*x*2i) + 1)^6) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1129472i)/(1615*d*(exp(c*2i + d*x*2i
) + 1)^7) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*98304i)/(323*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^2*(a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1
9*d*(exp(c*2i + d*x*2i) + 1)^9)
```

3.307 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.307.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}$$

output
```
-8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3/d+8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a
^4/d-2/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^5/d
```

3.307.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2a^2(-i+\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}\left(-263+374i\tan(c+dx)+143\tan^2(c+dx)\right)}{2145d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(-I + Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c + d*x]]*(-263 + (374*I)*T
an[c + d*x] + 143*Tan[c + d*x]^2))/(2145*d)
```

3.307.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{13/2}-4a(i\tan(c+dx)a+a)^{11/2}+4a^2(i\tan(c+dx)a+a)^{9/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{11}a^2(a+ia\tan(c+dx))^{11/2}+\frac{2}{15}(a+ia\tan(c+dx))^{15/2}-\frac{8}{13}a(a+ia\tan(c+dx))^{13/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(11/2))/11 - (8*a*(a + I*a*Tan[c + d*
x])^(13/2))/13 + (2*(a + I*a*Tan[c + d*x])^(15/2))/15))/(a^5*d)
```

3.307.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.307.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}+\frac{4a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^5}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
2*I/d/a^5*(-1/15*(a+I*a*tan(d*x+c))^(15/2)+4/13*a*(a+I*a*tan(d*x+c))^(13/2
)-4/11*a^2*(a+I*a*tan(d*x+c))^(11/2))
```

3.307.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{256\sqrt{2}(8i\,a^2e^{(15i\,dx+15i\,c)}+60i\,a^2e^{(13i\,dx+13i\,c)}+195i\,a^2e^{(11i\,dx+11i\,c)})\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{2145\,(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-256/2145*sqrt(2)*(8*I*a^2*e^(15*I*d*x + 15*I*c) + 60*I*a^2*e^(13*I*d*x +
13*I*c) + 195*I*a^2*e^(11*I*d*x + 11*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x
 + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^
(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{2i\left(143\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}-660\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}a+780\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a^2\right)}{2145\,a^5d}$$

input

```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output

```
-2/2145*I*(143*(I*a*tan(d*x + c) + a)^(15/2) - 660*(I*a*tan(d*x + c) + a)^
(13/2)*a + 780*(I*a*tan(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

3.307.8 Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\sec\left(dx+c\right)^6\,dx$$

input

```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output

```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^6, x)
```

3.307.9 Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 498, normalized size of antiderivative = 5.66

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=&-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{2145\,d}-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{2145\,d\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\&-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{715\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,18176\mathrm{i}}{429\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}\\&-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,52736\mathrm{i}}{429\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,103936\mathrm{i}}{715\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}\\&-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,15616\mathrm{i}}{195\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{15\,d\,{\left({\mathrm{e}}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}\end{aligned}$$

input

```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^6,x)
```

output

```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*18176i)/(429*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(2145*d*(exp(c
*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*256i)/(715*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*
(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2
048i)/(2145*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*52736i)/(429*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^2*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1039
36i)/(715*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^2*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*15616i)/(195*d*(exp(c*2i + d
*x*2i) + 1)^6) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*256i)/(15*d*(exp(c*2i + d*x*2i) + 1)^7)
```

3.307. $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

3.308 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.308.1 Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

output
```
-4/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^2/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3
/d
```

3.308.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^4(13i+9\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{99d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(-I + Tan[c + d*x])^4*(13*I + 9*Tan[c + d*x])*Sqrt[a + I*a*Tan[c +
 d*x]])/(99*d)
```

3.308.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{7/2}-(i\tan(c+dx)a+a)^{9/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{9}a(a+ia\tan(c+dx))^{9/2}-\frac{2}{11}(a+ia\tan(c+dx))^{11/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(9/2))/9 - (2*(a + I*a*Tan[c + d*x])^(1
1/2))/11))/(a^3*d)
```

3.308.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.308.4 Maple [A] (verified)

Time = 185.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^3}$	44

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/11*(a+I*a*tan(d*x+c))^(11/2)-2/9*a*(a+I*a*tan(d*x+c))^(9/2))
```

3.308.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.93

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{64\sqrt{2}\left(2i\,a^2e^{(11i\,dx+11i\,c)}+11i\,a^2e^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{99\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-64/99*sqrt(2)*(2*I*a^2*e^(11*I*d*x + 11*I*c) + 11*I*a^2*e^(9*I*d*x + 9*I*
c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*
I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d
*e^(2*I*d*x + 2*I*c) + d)
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i\left(9\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-22\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a\right)}{99\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/99*I*(9*(I*a*tan(d*x + c) + a)^(11/2) - 22*(I*a*tan(d*x + c) + a)^(9/2)*
a)/(a^3*d)
```

3.308.8 Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

3.308.9 Mupad [B] (verification not implemented)

Time = 7.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 6.27

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= \\ &-\frac{a^2\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{99\,d}-\frac{a^2\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{99\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} \\ &+\frac{a^2\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{33\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a^2\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2944\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} \\ &+\frac{a^2\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2176\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{a^2\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^4,x)
```

output
```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*512i)/(33*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*d*(exp(c*2i + d
*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*128i)/(99*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2944i)/(99*d*(exp(c*2i + d*x*2i) + 1
)^3) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*2176i)/(99*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*
2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(
c*2i + d*x*2i) + 1)^5)
```

3.309 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.309.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

output
```
-2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

3.309.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

3.309.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

3.309.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.309.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7ad}$	24

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

3.309.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{16i\sqrt{2}a^2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(7i\,dx+7i\,c)}}{7\,(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-16/7*I*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/
(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

3.309.6 Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{5}{2}}\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x)**2, x)
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2i\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}}{7\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/7*I*(I*a*tan(d*x + c) + a)^(7/2)/(a*d)
```

3.309.8 Giac [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.309.9 Mupad [B] (verification not implemented)

Time = 7.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 8.34

$$\begin{aligned}&\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \\ &-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,16{}\mathrm{i}}{7\,d}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,48{}\mathrm{i}}{7\,d\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)} \\ &-\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,48{}\mathrm{i}}{7\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}+\frac{a^2\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,16{}\mathrm{i}}{7\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^2,x)
```

output
```
(a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*48i)/(7*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d) - (a^2*(a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(7*d*(
exp(c*2i + d*x*2i) + 1)^2) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)
/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.309. $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

3.310 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.310.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}-\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

output
```
1/2*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1
/2)-I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

3.310.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}+\frac{a^2\sqrt{a+ia\tan(c+dx)}}{d(i+\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(I*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*d) + (a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(I + Tan[c + d*x]))
```

3.310.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3042, 3968, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{\sqrt{i\tan(c+dx)a+a}}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{ia^3\left(\frac{\sqrt{a+ia\tan(c+dx)}}{a-ia\tan(c+dx)}-\frac{1}{2}\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^3\left(\frac{\sqrt{a+ia\tan(c+dx)}}{a-ia\tan(c+dx)}-\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^3\left(\frac{\sqrt{a+ia\tan(c+dx)}}{a-ia\tan(c+dx)}-\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\sqrt{a}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^3*(((-I)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*Sqrt[a])
 + Sqrt[a + I*a*Tan[c + d*x]]/(a - I*a*Tan[c + d*x])))/d
```

3.310.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.310.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(73) = 146$.

Time = 18.58 (sec) , antiderivative size = 603, normalized size of antiderivative = 6.78

method	result
default	$\frac{i(\tan(dx+c)-i)^2\sqrt{a(1+i\tan(dx+c))}\,a^2\left(\cos^2(dx+c)\right)\left(i\,\mathrm{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)\sin(d}{}$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
I/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(I*arctan
h(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+I*arctan
h(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-arctanh(sin(d*x+c)/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)^2-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-I*cos(d*x+c)^2-(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-I*cos(d*x+c)+sin(d*x+c)*cos
(d*x+c))/(-2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)-I*cos(d*x+c)+sin(d*x+c
)+I)
```

3.310.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.65

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\sqrt{-\frac{a^5}{d^2}}d\log\Bigg(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\Bigg)-\sqrt{2}\sqrt{-\frac{a^5}{d^2}}d\log\Bigg(}{}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(-a^5/d^2)*
(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d
*x - I*c)/a^2) - sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqr
t(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1)))*e^(-I*d*x - I*c)/a^2) - 2*sqrt(2)*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

3.310.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\left(\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-\frac{8\sqrt{i\,a\tan(dx+c)+a}\,a^4}{2i\,a\tan(dx+c)-2\,a}\right)}{4\,ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/4*I*(sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 8*sqrt(I*a*tan(d*x + c
) + a)*a^4/(2*I*a*tan(d*x + c) - 2*a))/(a*d)
```

3.310.8 Giac [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.311 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.311.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{3ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}$$

output

```
-3/32*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^
(1/2)-1/4*I*a^4*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2-3/16*I*a^3
*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

3.311.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{ia^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{4d}$$

input

```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/4*I)*a^2*Hypergeometric2F1[1/2, 3, 3/2, (1 + I*Tan[c + d*x])/2]*Sqrt[
a + I*a*Tan[c + d*x]])/d
```

3.311.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^5*(Sqrt[a + I*a*Tan[c + d*x]]/(4*a*(a - I*a*Tan[c + d*x])^2) + (3*
(((I/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) + Sqrt[a
 + I*a*Tan[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x]))))/(8*a)))/d
```

3.311.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.311.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(110) = 220$.

Time = 123.88 (sec) , antiderivative size = 846, normalized size of antiderivative = 6.18

method	result	size
default	Expression too large to display	846

input

```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/16/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(-3*I
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+4*I*cos(d*x+c)^2*sin(d*x+c)+6*I
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)^2*sin(d*x+c)+6*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+6*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-6*cos(d*x+c)^3
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))+6*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+6*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*
x+c)*sin(d*x+c)-6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+6*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(
d*x+c)^3-3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*
x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*I*cos(d*x+c
)*sin(d*x+c)+4*cos(d*x+c)^3+3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7*cos(d*x+c)^2+3*cos(d*x+c))/(-I*cos(...
```

3.311.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(102) = 204$.

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.92

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4\left(\right.}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*s
qrt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(
4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x
 + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2)
- sqrt(2)*(-2*I*a^2*e^(5*I*d*x + 5*I*c) - 7*I*a^2*e^(3*I*d*x + 3*I*c) - 5*
I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

3.311.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.311.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i\left(3\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^4-10\,\sqrt{i\,a\tan(dx+c)+a}a^5\right)}{(i\,a\tan(dx+c)+a)^2-4\,(i\,a\tan(dx+c)+a)a+4\,a^2}\right)}{64\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/64*I*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x +
c) + a)^(3/2)*a^4 - 10*sqrt(I*a*tan(d*x + c) + a)*a^5)/((I*a*tan(d*x + c)
+ a)^2 - 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

3.311.8 Giac [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^4\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.312 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

3.312.1 Optimal result 2235
3.312.2 Mathematica [C] (verified) 2236
3.312.3 Rubi [A] (warning: unable to verify) 2236
3.312.4 Maple [B] (verified) 2240
3.312.5 Fricas [A] (verification not implemented) 2241
3.312.6 Sympy [F(-1)] 2241
3.312.7 Maxima [A] (verification not implemented) 2242
3.312.8 Giac [F(-1)] 2242
3.312.9 Mupad [F(-1)] 2242

3.312.1 Optimal result

Integrand size = 26, antiderivative size = 210

$$\begin{aligned}\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{35ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d}\\ &\quad+\frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}\\ &\quad-\frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\\ &\quad-\frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output

```
-35/256*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*
2^(1/2)+35/128*I*a^3/d/(a+I*a*tan(d*x+c))^(1/2)-1/6*I*a^6/d/(a+I*a*tan(d*x
+c))^(1/2)/(a-I*a*tan(d*x+c))^3-7/48*I*a^5/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I
*a*tan(d*x+c))^2-35/192*I*a^4/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a*tan(d*x+c)
)
```

3.312.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.25

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{8d\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output

```
((I/8)*a^3*Hypergeometric2F1[-1/2, 4, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

3.312.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3968, 52, 52, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c}}\right.}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{12a}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{}{4a(a-ia\tan(c+dx)}\right.}{12a}\right.}{d}$$

$$\Big\downarrow \; 73$$

3.312. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan}}}{12a}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^7\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}\right)}{12a}+}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

3.312. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]) + (
7*(1/(4*a*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) + (5*(1/(2*
a*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*ArcTan[(Sqrt
[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c +
d*x]])))/(4*a)))/(8*a)))/(12*a)))/d
```

3.312.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.312.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(170) = 340$.

Time = 5.97 (sec) , antiderivative size = 933, normalized size of antiderivative = 4.44

Expression too large to display

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
1/768*I/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(42
0*I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-315*I*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)-105*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+420*arctanh(sin(d*x+c)/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)+420*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
n((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-420*cos(d*x+
c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))+210*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+210*I*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d
*x+c)*sin(d*x+c)+210*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+
448*I*cos(d*x+c)^3*sin(d*x+c)-210*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
n((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-320*cos(d*x+c)^4-105*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))-105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d...
```

3.312.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.47

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(i\,dx+i\,c)}\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-105\sqrt{\frac{1}{2}}\right.$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/768*(105*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x
 + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 105*sqrt(1/2)*
sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt
(1/2)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - sqrt(2)*(-8*I*a^2*e^(8*I*d*x + 8*I*
c) - 46*I*a^2*e^(6*I*d*x + 6*I*c) - 125*I*a^2*e^(4*I*d*x + 4*I*c) - 39*I*a
^2*e^(2*I*d*x + 2*I*c) + 48*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/d
```

3.312.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i\left(105\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(105\,(ia\tan(dx+c)+a)^3a^4-560\,(ia\tan(dx+c)+a)^2a^5+924\,(i}{(ia\tan(dx+c)+a)^{\frac{7}{2}}-6\,(ia\tan(dx+c)+a)^{\frac{5}{2}}a+12\,(ia\tan(dx+c)+a}\right.}{1536\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/1536*I*(105*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(
d*x + c) + a)^3*a^4 - 560*(I*a*tan(d*x + c) + a)^2*a^5 + 924*(I*a*tan(d*x
+ c) + a)*a^6 - 384*a^7)/((I*a*tan(d*x + c) + a)^(7/2) - 6*(I*a*tan(d*x +
c) + a)^(5/2)*a + 12*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 8*sqrt(I*a*tan(d*x
 + c) + a)*a^3))/(a*d)
```

3.312.8 Giac [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^6\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.313 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.313.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

output

```
64/105*I*a^3*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+8/21*I*a^2*sec(d*x+c)
^3*(a+I*a*tan(d*x+c))^(1/2)/d+256/315*I*a^4*sec(d*x+c)^3/d/(a+I*a*tan(d*x+
c))^(3/2)+2/9*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

3.313.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.70

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{2a^2\sec^3(c+dx)(i\cos(2c)+\sin(2c))(77+242\cos(2(c+dx))+89i\sec(c+dx)\sin(3(c+dx))}{315d(\cos(dx)+i\sin(dx))^2}$$

input

```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^3*(I*Cos[2*c] + Sin[2*c])*(77 + 242*Cos[2*(c + d*x)] +
 (89*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (54*I)*Tan[c + d*x])*Sqrt[a + I*a*
Tan[c + d*x]])/(315*d*(Cos[d*x] + I*Sin[d*x])^2)
```

3.313.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\int \sec^3(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sec(c+dx)^3(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\int \sec^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\int \sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{3}a\left(\frac{8}{7}a\left(\frac{4}{5}a\int\frac{\sec^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow 3042$$

$$\frac{4}{3}a\left(\frac{8}{7}a\left(\frac{4}{5}a\int\frac{\sec(c+dx)^3}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow 3974$$

$$\frac{4}{3}a\left(\frac{8}{7}a\left(\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\right)+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/9)*a*Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (4*a*(((((2*I
)/7)*a*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (8*a*((((8*I)/15)*a^
2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/5)*a*Sec[c +
d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])))/7))/3
```

3.313.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.313.4 Maple [A] (verified)

Time = 41.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.52

method	result
default	$\frac{2i(-\tan(dx+c)+i)^2 a^2 \sqrt{a(1+i\tan(dx+c))}\,(256i(\cos^2(dx+c))\sin(dx+c)+128i\cos(dx+c)\sin(dx+c)-256(\cos^3(dx+c))+96i\sin(dx-}{315d(4(\cos^3(dx+c))+2(\cos^2(dx+c))+4i(\cos^2(dx+c))\sin(dx+c)-}$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*I/d*(-tan(d*x+c)+I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(4*cos(d*x+c)^3
+2*cos(d*x+c)^2+4*I*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*si
n(d*x+c)-1-I*sin(d*x+c))*(256*I*cos(d*x+c)^2*sin(d*x+c)+128*I*cos(d*x+c)*s
in(d*x+c)-256*cos(d*x+c)^3+96*I*sin(d*x+c)-128*cos(d*x+c)^2-130*I*tan(d*x+
c)+32*cos(d*x+c)-35*I*tan(d*x+c)*sec(d*x+c)-226-95*sec(d*x+c)+35*sec(d*x+c
)^2)
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$
$$-\frac{32\sqrt{2}\left(-105i\,a^2e^{(6i\,dx+6i\,c)}-126i\,a^2e^{(4i\,dx+4i\,c)}-72i\,a^2e^{(2i\,dx+2i\,c)}-16i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*(-105*I*a^2*e^(6*I*d*x + 6*I*c) - 126*I*a^2*e^(4*I*d*x + 4
*I*c) - 72*I*a^2*e^(2*I*d*x + 2*I*c) - 16*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x
 + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.313.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.313.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(115) = 230.

Time = 243.69 (sec) , antiderivative size = 624, normalized size of antiderivative = 4.24

$$\int \sec^3(c+dx)(a+ia\tan(c$$

$$+dx))^{5/2}\,dx = \frac{}{315\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\left(\left(2\,\cos(2\,dx+2\,c)^3+(2\,c\right.\right.}$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
32/315*(105*I*sqrt(2)*a^2*cos(6*d*x + 6*c) + 126*I*sqrt(2)*a^2*cos(4*d*x +
 4*c) + 72*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - 105*sqrt(2)*a^2*sin(6*d*x + 6*
c) - 126*sqrt(2)*a^2*sin(4*d*x + 4*c) - 72*sqrt(2)*a^2*sin(2*d*x + 2*c) +
16*I*sqrt(2)*a^2)*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*((2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c) + 1
)*sin(2*d*x + 2*c)^2 + 2*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 5*cos(2*d*x +
2*c)^2 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*
c) + I)*sin(4*d*x + 4*c) + 2*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c)
+ I)*sin(2*d*x + 2*c) + 4*cos(2*d*x + 2*c) + 1)*cos(5/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + (2*I*cos(2*d*x + 2*c)^3 + (2*I*cos(2*d*x
+ 2*c) + I)*sin(2*d*x + 2*c)^2 - 2*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c
)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) +
5*I*cos(2*d*x + 2*c)^2 - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(2*d*x + 2*c) + 4*I*cos(2*d*x + 2*c) + I)*sin(5/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*d)
```

3.313.8 Giac [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx&=\frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\&\quad-\frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}\\&\quad-\frac{a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^3,x)
```

output
```
(a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*32i)/(3*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*exp(-
c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*96i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) + (a^2*exp(- c*1i - d*
x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*96i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(- c*1i - d*x*1i)*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)
/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.313. $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

3.314 $\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

3.314.1 Optimal result 2250
3.314.2 Mathematica [A] (verified) 2250
3.314.3 Rubi [A] (verified) 2251
3.314.4 Maple [B] (verified) 2252
3.314.5 Fricas [A] (verification not implemented) 2253
3.314.6 Sympy [F] 2253
3.314.7 Maxima [F] 2254
3.314.8 Giac [F] 2254
3.314.9 Mupad [B] (verification not implemented) 2254

3.314.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} + \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

output
```
64/15*I*a^3*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*a^2*sec(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/d+2/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)/d
```

3.314.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2a^2\sec^2(c+dx)(i\cos(c-dx)+\sin(c-dx))(20+23\cos(2(c+dx))+7i\sin(2(c+dx)))\sqrt{a}}{15d(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Sec[c + d*x]^2*(I*Cos[c - d*x] + Sin[c - d*x])*(20 + 23*Cos[2*(c +
d*x)] + (7*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d*(Cos[d*x
] + I*Sin[d*x])^2)
```

3.314.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{5}a\left(\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

input

```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/5)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (8*a*((((8*I)/
3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*Sec[c +
 d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/5
```

3.314.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.314.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(86) = 172$.

Time = 7.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.75

method	result
default	$\frac{2i(-\tan(dx+c)+i)^2 a^2 \sqrt{a(1+i\tan(dx+c))}\,(32i(\cos^2(dx+c))\sin(dx+c)-14i\cos(dx+c)\sin(dx+c)-32(\cos^3(dx+c))-3i\sin(dx+c)-}{15d(4(\cos^3(dx+c))+2(\cos^2(dx+c))+4i(\cos^2(dx+c))\sin(dx+c)-3\cos(dx+c)+2i\cos(dx+c)\sin(dx+c)-1-i}$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*I/d*(-tan(d*x+c)+I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)*(32*I*cos(d*x+c)
^2*sin(d*x+c)-14*I*cos(d*x+c)*sin(d*x+c)-32*cos(d*x+c)^3-3*I*sin(d*x+c)-46
*cos(d*x+c)^2-11*cos(d*x+c)+3)/(4*cos(d*x+c)^3+2*cos(d*x+c)^2+4*I*cos(d*x+
c)^2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-1-I*sin(d*x+c))
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{8\sqrt{2}\left(-15i\,a^2e^{(4i\,dx+4i\,c)}-20i\,a^2e^{(2i\,dx+2i\,c)}-8i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*(-15*I*a^2*e^(4*I*d*x + 4*I*c) - 20*I*a^2*e^(2*I*d*x + 2*I*c
) - 8*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)
```

3.314.6 Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{5}{2}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x), x)
```

3.314.7 Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.314.8 Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.314.9 Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{8\,a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,20\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,15\mathrm{i}+8\mathrm{i}\right)}{15\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x),x)
```

output
```
(8*a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(
c*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*20i + exp(c*4i + d*x*4i)*15
i + 8i))/(15*d*(exp(c*2i + d*x*2i) + 1)^2)
```

3.315 $\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

3.315.1 Optimal result 2255
3.315.2 Mathematica [A] (verified) 2255
3.315.3 Rubi [A] (verified) 2256
3.315.4 Maple [A] (verified) 2257
3.315.5 Fricas [A] (verification not implemented) 2257
3.315.6 Sympy [F(-1)] 2258
3.315.7 Maxima [B] (verification not implemented) 2258
3.315.8 Giac [F] 2259
3.315.9 Mupad [B] (verification not implemented) 2259

3.315.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

output
```
-8*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+2*I*a*cos(d*x+c)*(a+I*a*tan
(d*x+c))^(3/2)/d
```

3.315.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia^2(3\cos(c+dx)-i\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-2*I)*a^2*(3*Cos[c + d*x] - I*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/
d
```

3.315.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)}dx$$

$$\Big\downarrow \; 3975$$

$$4a\int \cos(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \; 3042$$

$$4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \; 3974$$

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-8*I)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c +
d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

3.315.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.315.4 Maple [A] (verified)

Time = 24.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

method	result
default	$-\frac{2(-\tan(dx+c)+i)^2\sqrt{a(1+i\tan(dx+c))}\,a^2\left(\cos^2(dx+c)\right)(3i\cos(dx+c)+\sin(dx+c))\left(2i\cos(dx+c)\sin(dx+c)-2\left(\cos^2(dx+c)\right)+1\right)}{d}$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(3*I*co
s(d*x+c)+sin(d*x+c))*(2*I*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+1)
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+2i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))/d
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.315.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(53) = 106.

Time = 0.41 (sec) , antiderivative size = 331, normalized size of antiderivative = 5.09

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2\left(-3i\,a^{\frac{5}{2}} - \frac{2\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{4\,a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{2\,a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \right.}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{4i\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{5\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{5\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right.}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*(-3*I*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*I*a^(5/2)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 9*I*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 1)^(5/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)*(4*I*sin(d*x + c)/(cos(d*x + c) +
 1) - 5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 4*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 1))
```

3.315.8 Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.315.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2\,a^2\left(\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,3\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{d}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
-(2*a^2*(cos(c + d*x)*3i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c +
 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

3.316 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.316.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

3.316.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2a^2\cos^2(c+dx)(-i\cos(c+3dx)+\sin(c+3dx))\sqrt{a+ia\tan(c+dx)}}{3d(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a^2*Cos[c + d*x]^2*((-I)*Cos[c + 3*d*x] + Sin[c + 3*d*x])*Sqrt[a + I*a*
Tan[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^2)
```

3.316.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3974}$$

$$-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

3.316.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.316.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 35.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

method	result	size
default	$-\frac{2i(-\tan(dx+c)+i)^2\sqrt{a(1+i\tan(dx+c))}\,(i\sin(dx+c)(\cos^4(dx+c))-(\cos^5(dx+c)))a^2}{3d}$	66

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)^4*sin(
d*x+c)-cos(d*x+c)^5)*a^2
```

3.316.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-2i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{6\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 2*I*a^2*e^(2*I*d*x + 2*I*c) - I*
a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.316.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(27) = 54.

Time = 0.46 (sec) , antiderivative size = 328, normalized size of antiderivative = 9.37

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2\left(i\,a^{\frac{5}{2}} - \frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{i\,a^{\frac{5}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\Bigg(\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{6i\sin(}{(\cos(dx+}}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*(I*a^(5/2) - 4*I*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*I*a^(5/
2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*I*a^(5/2)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + I*a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(
d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/
2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)^(5/2)*(-6*I*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 18*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 18*I*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 6*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 - 6*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8 - 3))
```

3.316.8 Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.316.9 Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=$$
$$-\frac{a^2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(-\sin(c+d\,x)-\sin(3\,c+3\,d\,x)+\cos(c+d\,x)\,3\mathrm{i}+\cos(3\,c+3\,d\,x)\,1\mathrm{i}}{6\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
-(a^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^(1/2)*(cos(c + d*x)*3i - sin(c + d*x) + cos(3*c + 3*d*x)*1i - sin(3*
c + 3*d*x)))/(6*d)
```

3.317 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.317.1 Optimal result

Integrand size = 26, antiderivative size = 159

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d} - \frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

output
```
1/8*I*a^(5/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))/d*2^(1/2)-1/4*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/6*I*a*cos
(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))
^(5/2)/d
```

3.317.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{ia^2e^{-i(c+dx)}\left(23+34e^{2i(c+dx)}+14e^{4i(c+dx)}+3e^{6i(c+dx)}-15\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+}}{120d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/120*I)*a^2*(23 + 34*E^((2*I)*(c + d*x)) + 14*E^((4*I)*(c + d*x)) + 3*
E^((6*I)*(c + d*x)) - 15*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

3.317.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3971, 3042, 3971, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\bigg(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\bigg)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx)}{3d}$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\bigg(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\bigg)-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx)}{3d}$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\Bigg(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\Bigg)-\frac{i\cos^3(c+dx)(a+ia\tan(c}{3d}$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\Bigg(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\Bigg)-\frac{i\cos^3(c+dx)(a+ia\tan(c+}{3d}$$

$$\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d + (a*(((-1/3*I)*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (a*((I*Sqrt[a]*ArcTanh[(Sq
rt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (
I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/2))/2
```

3.317. $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

3.317.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

3.317.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(128) = 256$.

Time = 3.30 (sec) , antiderivative size = 916, normalized size of antiderivative = 5.76

Expression too large to display

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
1/120/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(-45*
I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)+30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)
+104*I*cos(d*x+c)^3-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))-60*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^
3+60*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d
*x+c)+60*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+30*I*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)^2-30*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+30*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*
x+c)*sin(d*x+c)-30*I*cos(d*x+c)+45*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+
c)-15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*sin(d*x+c)+80*cos(d*x+c)^2*sin(d*x+c)+60*I*(-cos(d*x+c)/(cos(...
```

3.317.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(120) = 240$.

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.53

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{\left(ia^3+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log}{}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 + sqrt(2)*sqrt(1/2)*sq
rt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 - sqr
t(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-3*I*a^2*e^(6*I*d*x + 6*
I*c) - 14*I*a^2*e^(4*I*d*x + 4*I*c) - 34*I*a^2*e^(2*I*d*x + 2*I*c) - 23*I*
a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.317.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(120) = 240$.

Time = 0.47 (sec) , antiderivative size = 1076, normalized size of antiderivative = 6.77

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output

```
-1/480*(20*(I*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*sqrt(a) + 12*(5*I*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - 5*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)) + (I*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^2*sin(
2*d*x + 2*c)^2 + 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2)*cos(5/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^2*cos(2*d*x
 + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c
) + sqrt(2)*a^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sqrt(a) + 15*(2*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) - 2*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)
- I*sqrt(2)*a^2*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co...
```

3.317.8 Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\cos\left(dx+c\right)^5\,dx$$

input

```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output

```
integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^5\,{\left(a+a\,\tan(c+d\,x)\,\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.318 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

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3.318.1 Optimal result

Integrand size = 26, antiderivative size = 231

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{9ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}d} + \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d} - \frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d} - \frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

output

```
9/64*I*a^(5/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d*2^(1/2)+3/16*I*a^3*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-9/32*I*a^
2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-3/20*I*a^2*cos(d*x+c)^3*(a+I*a*tan
(d*x+c))^(1/2)/d-9/70*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d-1/7*I*co
s(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d
```

3.318.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.67

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{ia^2e^{-3i(c+dx)}\left(-35+353e^{2i(c+dx)}+544e^{4i(c+dx)}+214e^{6i(c+dx)}+68e^{8i(c+dx)}+10e^{10i(c+dx)}-315e^{2i(c+dx)}\sqrt{1}\right.}{2240d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/2240*I)*a^2*(-35 + 353*E^((2*I)*(c + d*x)) + 544*E^((4*I)*(c + d*x))
+ 214*E^((6*I)*(c + d*x)) + 68*E^((8*I)*(c + d*x)) + 10*E^((10*I)*(c + d*x
)) - 315*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(c + d*x)
))
```

3.318.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \; 3978$$

$$\frac{9}{14}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{9}{14}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3978}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9}{14}a\left(\frac{7}{10}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3978}$$

$$\frac{9}{14}a\bigg(\frac{7}{10}a\bigg(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\bigg)-\frac{i\cos^5(c+dx)(a+ia\tan(c+}{5d} \quad -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9}{14}a\bigg(\frac{7}{10}a\bigg(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\bigg)-\frac{i\cos^5(c+dx)(a+}{5} \quad -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{9}{14}a\bigg(\frac{7}{10}a\bigg(\frac{5}{6}a\bigg(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\bigg)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(}}{3d} \quad -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9}{14}a\bigg(\frac{7}{10}a\bigg(\frac{5}{6}a\bigg(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\bigg)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\bigg)-i\,\mathrm{co} \quad -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{ 3971}$$

3.318. $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+}{}\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3042$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+}{}\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 3970$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow 219$$

$$\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\,\mathrm{co}}{}\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

input

```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

3.318. $\int\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

output
```
((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d + (9*a*(((-1/5*I)
*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-1/3*I)*Cos[c +
d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[
a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sq
rt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a +
 I*a*Tan[c + d*x]])/d))/(4*a)))/6))/10))/14
```

3.318.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.318.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(188) = 376$.

Time $= 3.72$ (sec) , antiderivative size $= 945$, normalized size of antiderivative $= 4.09$

Expression too large to display

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
1/2240*I/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(
-945*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-1680*I*cos(d*x+c)^2*sin(d
*x+c)+315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*sin(d*x+c)-1260*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
n((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+1260*I*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+1260*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+1260*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-400*cos(d*x+c)^5+720*I*cos(
d*x+c)^4*sin(d*x+c)+630*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-630*
I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)*sin(d*x+c)+630*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*
sin(d*x+c)+630*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)^2-315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-315*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-c...
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.30

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{\left(315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(-\frac{9\left(-i\,a^3+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{16\,d}\right)-315\sqrt{\frac{1}{2}}\sqrt{-}\right.}{}$$

input

```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
-1/2240*(315*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*
a^3 + sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*sqrt(-a^5/
d^2)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*a^3 - sqrt(2)*sqrt(1/2)*sqrt(-a^5
/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I
*d*x - I*c)/d) - sqrt(2)*(-10*I*a^2*e^(10*I*d*x + 10*I*c) - 68*I*a^2*e^(8*
I*d*x + 8*I*c) - 214*I*a^2*e^(6*I*d*x + 6*I*c) - 544*I*a^2*e^(4*I*d*x + 4*
I*c) - 353*I*a^2*e^(2*I*d*x + 2*I*c) + 35*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.318.7 Maxima [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.318.8 Giac [F]

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\cos\left(dx+c\right)^7\,dx$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^7, x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int \cos\left(c+d\,x\right)^7\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.319 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.319.1 Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d} + \frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d} - \frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d} + \frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}$$

output
```
-16/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^4/d+24/17*I*(a+I*a*tan(d*x+c))^(17/2)
/a^5/d-12/19*I*(a+I*a*tan(d*x+c))^(19/2)/a^6/d+2/21*I*(a+I*a*tan(d*x+c))^(
21/2)/a^7/d
```

3.319.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(-i+\tan(c+dx))^7\sqrt{a+ia\tan(c+dx)}(-3243+7365i\tan(c+dx)+5865\tan^2(c+dx)}{33915d}$$

input
```
Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*(-I + Tan[c + d*x])^7*Sqrt[a + I*a*Tan[c + d*x]]*(-3243 + (7365*I)*
Tan[c + d*x] + 5865*Tan[c + d*x]^2 - (1615*I)*Tan[c + d*x]^3))/(33915*d)
```

3.319.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^8(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{13/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \; 53$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{19/2}+6a(i\tan(c+dx)a+a)^{17/2}-12a^2(i\tan(c+dx)a+a)^{15/2}+8a^3(i\tan(c+dx)a\right.}{a^7d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(\frac{16}{15}a^3(a+ia\tan(c+dx))^{15/2}-\frac{24}{17}a^2(a+ia\tan(c+dx))^{17/2}-\frac{2}{21}(a+ia\tan(c+dx))^{21/2}+\frac{12}{19}a(a+ia\tan(c+\right.}{a^7d}$$

input

```
Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(15/2))/15 - (24*a^2*(a + I*a*Tan[c
+ d*x])^(17/2))/17 + (12*a*(a + I*a*Tan[c + d*x])^(19/2))/19 - (2*(a + I*a
*Tan[c + d*x])^(21/2))/21))/(a^7*d)
```

3.319.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.319.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{21}{2}}}{21}-\frac{6a(a+ia\tan(dx+c))^{\frac{19}{2}}}{19}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}\right)}{d\,a^{7}}$$

input
```
int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
2*I/d/a^7*(1/21*(a+I*a*tan(d*x+c))^(21/2)-6/19*a*(a+I*a*tan(d*x+c))^(19/2)
+12/17*a^2*(a+I*a*tan(d*x+c))^(17/2)-8/15*a^3*(a+I*a*tan(d*x+c))^(15/2))
```

3.319.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(85) = 170$.

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{2048\sqrt{2}\left(16i\,a^3e^{(21i\,dx+21i\,c)}+168i\,a^3e^{(19i\,dx+19i\,c)}+798i\,a^3e^{(17i\,d}\right.}{33915\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de\right.}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2048/33915*sqrt(2)*(16*I*a^3*e^(21*I*d*x + 21*I*c) + 168*I*a^3*e^(19*I*d*
x + 19*I*c) + 798*I*a^3*e^(17*I*d*x + 17*I*c) + 2261*I*a^3*e^(15*I*d*x + 1
5*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(20*I*d*x + 20*I*c) + 10*d*
e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 1
4*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d
*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c
) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

3.319.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i\left(1615\,(i\,a\tan\left(dx+c\right)+a)^{\frac{21}{2}} - 10710\,(i\,a\tan\left(dx+c\right)+a)^{\frac{19}{2}}a + 23940\,(i\,a\tan\left(dx+c\right)+\right.}{33915\,a^7d}$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/33915*I*(1615*(I*a*tan(d*x + c) + a)^(21/2) - 10710*(I*a*tan(d*x + c) +
a)^(19/2)*a + 23940*(I*a*tan(d*x + c) + a)^(17/2)*a^2 - 18088*(I*a*tan(d*x
 + c) + a)^(15/2)*a^3)/(a^7*d)
```

3.319.8 Giac [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)^8\,dx$$

input
```
integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^8, x)
```

3.319.9 Mupad [B] (verification not implemented)

Time = 16.01 (sec) , antiderivative size = 690, normalized size of antiderivative = 5.90

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input
```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^8,x)
```

output

```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*247808i)/(969*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(33915*d*(ex
p(c*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(11305*d*(exp(c*2i + d*x*2i) + 1)^2) -
 (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*2048i)/(6783*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32768i)/(33915*d) -
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*1943552i)/(1615*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i
 + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*12019712i)/(4845*d
*(exp(c*2i + d*x*2i) + 1)^6) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1
i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*95516672i)/(33915*d*(exp(c*2i + d*x*2i)
 + 1)^7) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*4159488i)/(2261*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^3*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*260096i
)/(399*d*(exp(c*2i + d*x*2i) + 1)^9) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(21*d*(exp(c*2i + d*x*2i
) + 1)^10)
```

3.319. $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

3.320 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.320.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d} - \frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}$$

output
```
-8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^3/d+8/15*I*(a+I*a*tan(d*x+c))^(15/2)/a
^4/d-2/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^5/d
```

3.320.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(-i+\tan(c+dx))^6\sqrt{a+ia\tan(c+dx)}(331i+494\tan(c+dx)-195i\tan^2(c+dx))}{3315d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*(-I + Tan[c + d*x])^6*Sqrt[a + I*a*Tan[c + d*x]]*(331*I + 494*Tan[c
 + d*x] - (195*I)*Tan[c + d*x]^2))/(3315*d)
```

3.320.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int\left((i\tan(c+dx)a+a)^{15/2}-4a(i\tan(c+dx)a+a)^{13/2}+4a^2(i\tan(c+dx)a+a)^{11/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{13}a^2(a+ia\tan(c+dx))^{13/2}+\frac{2}{17}(a+ia\tan(c+dx))^{17/2}-\frac{8}{15}a(a+ia\tan(c+dx))^{15/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(13/2))/13 - (8*a*(a + I*a*Tan[c + d*
x])^(15/2))/15 + (2*(a + I*a*Tan[c + d*x])^(17/2))/17))/(a^5*d)
```

3.320.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.320.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}+\frac{4a(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}\right)}{d\,a^{5}}$$

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
2*I/d/a^5*(-1/17*(a+I*a*tan(d*x+c))^(17/2)+4/15*a*(a+I*a*tan(d*x+c))^(15/2
)-4/13*a^2*(a+I*a*tan(d*x+c))^(13/2))
```

3.320.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(64) = 128$.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.86

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{512\sqrt{2}\left(8i\,a^3e^{(17i\,dx+17i\,c)}+68i\,a^3e^{(15i\,dx+15i\,c)}+255i\,a^3e^{(13i\,dx+13i\,c)}\right)\sqrt{\frac{}{e^{(2i\,d}}}}{3315\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i}\right.}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-512/3315*sqrt(2)*(8*I*a^3*e^(17*I*d*x + 17*I*c) + 68*I*a^3*e^(15*I*d*x +
15*I*c) + 255*I*a^3*e^(13*I*d*x + 13*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x
 + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*
e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) +
 d)
```

3.320.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i\left(195\,(i\,a\tan\left(dx+c\right)+a)^{\frac{17}{2}}-884\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}a+1020\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}a^2\right)}{3315\,a^5d}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3315*I*(195*(I*a*tan(d*x + c) + a)^(17/2) - 884*(I*a*tan(d*x + c) + a)^
(15/2)*a + 1020*(I*a*tan(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

3.320.8 Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^6, x)
```

3.320.9 Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.39

$$
\begin{aligned}
\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = {} & -\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{3315\,d} \\
& -\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{3315\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1105\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} \\
& +\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,56320\mathrm{i}}{663\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,205312\mathrm{i}}{663\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} \\
& +\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,540672\mathrm{i}}{1105\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}-\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1341952\mathrm{i}}{3315\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6} \\
& +\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,44032\mathrm{i}}{255\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}-\frac{a^3\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{17\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}
\end{aligned}
$$

input

```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^6,x)
```

output

```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*56320i)/(663*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3315*d*(exp(c
*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*512i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
4096i)/(3315*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*205312i)/(663*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^3*(
a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*54
0672i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^3*(a - (a*(exp(c*2i + d*x*
2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1341952i)/(3315*d*(exp(c*
2i + d*x*2i) + 1)^6) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(
c*2i + d*x*2i) + 1))^(1/2)*44032i)/(255*d*(exp(c*2i + d*x*2i) + 1)^7) - (a
^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*512i)/(17*d*(exp(c*2i + d*x*2i) + 1)^8)
```

3.321 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.321.1 Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

output
```
-4/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^2/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a
^3/d
```

3.321.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(15-11i\tan(c+dx))(-i+\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}}{143d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*(15 - (11*I)*Tan[c + d*x])*(-I + Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c
 + d*x]])/(143*d)
```

3.321.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{9/2}-(i\tan(c+dx)a+a)^{11/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{11}a(a+ia\tan(c+dx))^{11/2}-\frac{2}{13}(a+ia\tan(c+dx))^{13/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(11/2))/11 - (2*(a + I*a*Tan[c + d*x])^
(13/2))/13))/(a^3*d)
```

3.321.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.321.4 Maple [A] (verified)

Time = 187.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{2a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{2a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^3}$	44

input

```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output

```
2*I/d/a^3*(1/13*(a+I*a*tan(d*x+c))^(13/2)-2/11*a*(a+I*a*tan(d*x+c))^(11/2)
)
```

3.321.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.14

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{128\,\sqrt{2}\left(2i\,a^3e^{(13i\,dx+13i\,c)}+13i\,a^3e^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{143\,\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input

```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-128/143*sqrt(2)*(2*I*a^3*e^(13*I*d*x + 13*I*c) + 13*I*a^3*e^(11*I*d*x + 1
1*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e
^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c)
 + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

3.321.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i\left(11\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}} - 26\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a\right)}{143\,a^3d}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/143*I*(11*(I*a*tan(d*x + c) + a)^(13/2) - 26*(I*a*tan(d*x + c) + a)^(11/
2)*a)/(a^3*d)
```

3.321.8 Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^4, x)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 434, normalized size of antiderivative = 7.36

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{143\,d}\\
&\quad-\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{143\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}+\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4480\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\
&\quad-\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,11520\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,12800\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\\
&\quad-\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6784\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}+\frac{a^3\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}\end{aligned}$$

input
```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^4,x)
```

output
```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*4480i)/(143*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(143*d*(exp(c*2i
 + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*256i)/(143*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*11520i)/(143*d*(exp(c*2i + d*x*
2i) + 1)^3) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*12800i)/(143*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6784i)/
(143*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i -
 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) +
 1)^6)
```

3.322 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.322.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

output
```
-2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

3.322.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

3.322.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int(i\tan(c+dx)a+a)^{7/2}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

3.322.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.322.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9ad}$	24

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

3.322.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{32i\sqrt{2}a^3\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(9i\,dx+9i\,c)}}{9\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-32/9*I*sqrt(2)*a^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(9*I*d*x + 9*I*c)/
(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c)
 + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.322.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}}{9\,ad}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/9*I*(I*a*tan(d*x + c) + a)^(9/2)/(a*d)
```

3.322.8 Giac [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^2, x)
```

3.322.9 Mupad [B] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 306, normalized size of antiderivative = 10.55

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d} + \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{9\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} + \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{9\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} - \frac{a^3\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

input

```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^2,x)
```

output

```
(a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*d) - (a^3*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(3*d*
(exp(c*2i + d*x*2i) + 1)^2) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i
)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)^3) -
 (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(
1/2)*32i)/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.323 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.323.2 Mathematica [C] (verified) 2303
3.323.3 Rubi [A] (warning: unable to verify) 2304
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3.323.8 Giac [F(-1)] 2308
3.323.9 Mupad [F(-1)] 2308

3.323.1 Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{3i\sqrt{2}a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d}-\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}$$

output
```
3*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
d-3*I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d-I*a^3*(a+I*a*tan(d*x+c))^(3/2)/d/(a-I
*a*tan(d*x+c))
```

3.323.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{ia\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{5/2}}{10d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/10*I)*a*Hypergeometric2F1[2, 5/2, 7/2, (1 + I*Tan[c + d*x])/2]*(a + I
*a*Tan[c + d*x])^(5/2))/d
```

3.323.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))\right)}{d}$$

$$\Big\downarrow \text{60}$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\left(2a\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))-2\sqrt{a+ia\tan(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\left(4a\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}-2\sqrt{a+ia\tan(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^3\left(\frac{(a+ia\tan(c+dx))^{3/2}}{a-ia\tan(c+dx)}-\frac{3}{2}\left(2i\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)-2\sqrt{a+ia\tan(c+dx)}\right)\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^3*((a + I*a*Tan[c + d*x])^(3/2)/(a - I*a*Tan[c + d*x]) - (3*((2*I)
*Sqrt[2]*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]] - 2*Sqrt[a + I*a*T
an[c + d*x]]))/2))/d
```

3.323.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.323.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(96) = 192.

Time = 38.36 (sec) , antiderivative size = 641, normalized size of antiderivative = 5.53

method	result
default	$-\frac{2i(\tan(dx+c)-i)^3\sqrt{a(1+i\tan(dx+c))}\,a^3\left(\cos^3(dx+c)\right)\left(3i\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)}{}$

input

```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-2*I/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(3*I*a
rctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-3*I*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+3
*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3*I*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*arctanh(
sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-I*cos(d*x+
c)^2-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-2*I*cos(
d*x+c)+sin(d*x+c)*cos(d*x+c)-I-sin(d*x+c))/(4*cos(d*x+c)^3+2*cos(d*x+c)^2+
4*I*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-1-I*sin
(d*x+c))
```

3.323.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(89) = 178.

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.03

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{3\sqrt{2}\sqrt{-\frac{a^7}{d^2}}\,d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-3\sqrt{2}\sqrt{-\frac{a^7}{d^2}}\,d\log\left(\frac{4\left(a^4e^{(i\,dx+}{}\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/2*(3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) + sqrt(-a^7/d^
2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/a^3) - 3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c)
+ sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1)))*e^(-I*d*x - I*c)/a^3) + 2*sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) +
3*I*a^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

3.323.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$
$$-\frac{i\left(3\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+4\sqrt{i\,a\tan\left(dx+c\right)+a}a^4-\frac{4\sqrt{i\,a\tan(dx+c)+a}a^5}{i\,a\tan(dx+c)-a}\right)}{2\,ad}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/2*I*(3*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*sqrt(I*a*tan(d*x +
 c) + a)*a^4 - 4*sqrt(I*a*tan(d*x + c) + a)*a^5/(I*a*tan(d*x + c) - a))/(a
*d)
```

3.323.8 Giac [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^2\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2} \,d x$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.324 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.324.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}$$

output

```
1/16*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(
1/2)-1/2*I*a^5*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2+1/8*I*a^4*(
a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

3.324.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia^2\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{3/2}}{12d}$$

input

```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/12*I)*a^2*Hypergeometric2F1[3/2, 3, 5/2, (1 + I*Tan[c + d*x])/2]*(a +
 I*a*Tan[c + d*x])^(3/2))/d
```

3.324.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3968, 51, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^5\int \frac{\sqrt{i\tan(c+dx)a+a}}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 51$$

$$-\frac{ia^5\left(\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}-\frac{1}{4}\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))\right)}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^5\left(\frac{1}{4}\left(-\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}-\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)+\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \; 73$$

$$-\frac{ia^5\left(\frac{1}{4}\left(-\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}-\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)+\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{ia^5\left(\frac{1}{4}\left(-\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}-\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)+\frac{\sqrt{a+ia\tan(c+dx)}}{2(a-ia\tan(c+dx))^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^5*(Sqrt[a + I*a*Tan[c + d*x]]/(2*(a - I*a*Tan[c + d*x])^2) + (((-1
/2*I)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - Sqrt[a +
 I*a*Tan[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x])))/4))/d
```

3.324.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.324.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(110) = 220$.

Time = 123.98 (sec) , antiderivative size = 869, normalized size of antiderivative = 6.34

method	result	size
default	Expression too large to display	869

input

```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output

```
1/8/d*(-tan(d*x+c)+I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(I*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)-3*I*cos(d*x+c)^2+2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
h(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)
*sin(d*x+c)-4*I*cos(d*x+c)^3-2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(
sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3
-2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)^2*sin(d*x+c)+I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)-2*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-2*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c)*sin(d*x+c)-2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+I*cos(d*x+c)+(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*cos(d*x+c)+4*cos(d*x+c)^2*sin(d*x+c)-2*I*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+sin(d*x+c)*cos(d*x+c))/(-2*I*cos(d*x+c)^2+2...
```

3.324.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(102) = 204$.

Time = 0.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.91

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}}{}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/16*(sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt
(1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^
4*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*
I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) + sqr
t(2)*(-2*I*a^3*e^(5*I*d*x + 5*I*c) - 3*I*a^3*e^(3*I*d*x + 3*I*c) - I*a^3*e
^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

3.324.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\left(\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left((i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^5+2\sqrt{i\,a\tan(dx+c)+a}\,a^6\right)}{(i\,a\tan(dx+c)+a)^2-4\,(i\,a\tan(dx+c)+a)a+4\,a^2}\right)}{32\,ad}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/32*I*(sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*((I*a*tan(d*x + c)
+ a)^(3/2)*a^5 + 2*sqrt(I*a*tan(d*x + c) + a)*a^6)/((I*a*tan(d*x + c) + a)
^2 - 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

3.324.8 Giac [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^4\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.325 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.325.8 Giac [F(-1)] 2321
3.325.9 Mupad [F(-1)] 2321

3.325.1 Optimal result

Integrand size = 26, antiderivative size = 181

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{5ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} - \frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))}$$

output
```
-5/128*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2
^(1/2)-1/6*I*a^6*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^3-5/48*I*a^
5*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2-5/64*I*a^4*(a+I*a*tan(d*
x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

3.325.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.29

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{8d}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/8*I)*a^3*Hypergeometric2F1[1/2, 4, 3/2, (1 + I*Tan[c + d*x])/2]*Sqrt[
a + I*a*Tan[c + d*x]])/d
```

3.325.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3968, 52, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{ia^7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^3\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\int \frac{1}{(a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{4a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow 73$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{2a}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

$$\Big\downarrow 219$$

$$-\frac{ia^7\left(\frac{5\left(\frac{3\left(\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sqrt{a+ia\tan(c+dx)}}{2a(a-ia\tan(c+dx))}\right)}{8a}+\frac{\sqrt{a+ia\tan(c+dx)}}{4a(a-ia\tan(c+dx))^2}\right)}{12a}+\frac{\sqrt{a+ia\tan(c+dx)}}{6a(a-ia\tan(c+dx))^3}\right)}{d}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^7*(Sqrt[a + I*a*Tan[c + d*x]]/(6*a*(a - I*a*Tan[c + d*x])^3) + (5*
(Sqrt[a + I*a*Tan[c + d*x]]/(4*a*(a - I*a*Tan[c + d*x])^2) + (3*(((I/2)*Ar
cTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) + Sqrt[a + I*a*Tan
[c + d*x]]/(2*a*(a - I*a*Tan[c + d*x]))))/(8*a)))/(12*a)))/d
```

3.325. $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

3.325.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.325.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(147) = 294$.

Time = 4.65 (sec) , antiderivative size = 1136, normalized size of antiderivative = 6.28

Expression too large to display

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
1/192/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(45*I
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+
35*I*cos(d*x+c)^2+60*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+
c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin(d*x
+c)-60*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4-60*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin
(d*x+c)+82*I*cos(d*x+c)^3+32*I*cos(d*x+c)^4-15*I*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*sin(d*x+c)-60*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-60*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos
(d*x+c)^2*sin(d*x+c)+60*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)+45*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)^2-15*I*cos(d*x+c)+45*arctanh(sin(d*x+c)/(cos(d*
x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)^2+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*...
```

3.325.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(136) = 272$.

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.53

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(}{}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/384*(15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)
*sqrt(1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - 15*sqrt(1/2)*sqrt(-a^7/d^2)*d*l
og(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*
d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^
3) - sqrt(2)*(-8*I*a^3*e^(7*I*d*x + 7*I*c) - 34*I*a^3*e^(5*I*d*x + 5*I*c)
- 59*I*a^3*e^(3*I*d*x + 3*I*c) - 33*I*a^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1)))/d
```

3.325.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.97

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{i\left(15\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(15\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^5-80\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^6+132\,\sqrt{i\,a\,\mathrm{ta}}\right.}{(i\,a\tan(dx+c)+a)^3-6\,(i\,a\tan(dx+c)+a)^2a+12\,(i\,a\tan(dx+c)+}\right.}{768\,ad}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/768*I*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x
 + c) + a)^(5/2)*a^5 - 80*(I*a*tan(d*x + c) + a)^(3/2)*a^6 + 132*sqrt(I*a*
tan(d*x + c) + a)*a^7)/((I*a*tan(d*x + c) + a)^3 - 6*(I*a*tan(d*x + c) + a
)^2*a + 12*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3))/(a*d)
```

3.325.8 Giac [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int \cos\left(c+d\,x\right)^6\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.326 $\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.326.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

output
```
256/35*I*a^4*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+64/35*I*a^3*sec(d*x+c)*
(a+I*a*tan(d*x+c))^(1/2)/d+24/35*I*a^2*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)
/d+2/7*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

3.326.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3\sec^2(c+dx)(i\cos(c-2dx)+\sin(c-2dx))(75+102\cos(2(c+dx))+19i\sec(c+dx)\sin}{35d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
(2*a^3*Sec[c + d*x]^2*(I*Cos[c - 2*d*x] + Sin[c - 2*d*x])*(75 + 102*Cos[2*
(c + d*x)] + (19*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (14*I)*Tan[c + d*x])*S
qrt[a + I*a*Tan[c + d*x]])/(35*d*(Cos[d*x] + I*Sin[d*x])^3)
```

3.326.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{5/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{5/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\int \sec(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a}{5}\right.$$
$$\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sec(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a}{5}\right.$$
$$\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{7}a\left(\frac{8}{5}a\left(\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3}}{5d}\right.$$
$$\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/7)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2))/d + (12*a*((((2*I)
/5)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (8*a*((((8*I)/3)*a^2*
Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*Sec[c + d*x]*S
qrt[a + I*a*Tan[c + d*x]])/d))/5))/7
```

3.326.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.326. $\int\sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.326.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(115) = 230$.

Time = 7.88 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.69

method	result
default	$\frac{2(-\tan(dx+c)+i)^3 a^3 \sqrt{a(1+i\tan(dx+c))}\,(128i(\cos^3(dx+c))\sin(dx+c)-76i(\cos^2(dx+c))\sin(dx+c)-128(\cos^4(dx+c))-22i\cos(d}{35d(8(\cos^4(dx+c))+4(\cos^3(dx+c))+8i(\cos^3(dx+c))\sin(dx+c)-8(\cos^2(dx+c))+4i(\cos^2(dx+c))\sin(dx+}$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d*(-tan(d*x+c)+I)^3*a^3*(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*cos(d*x+c)^
3*sin(d*x+c)-76*I*cos(d*x+c)^2*sin(d*x+c)-128*cos(d*x+c)^4-22*I*cos(d*x+c)
*sin(d*x+c)-204*cos(d*x+c)^3+5*I*sin(d*x+c)-54*cos(d*x+c)^2+27*cos(d*x+c)+
5)/(8*cos(d*x+c)^4+4*cos(d*x+c)^3+8*I*sin(d*x+c)*cos(d*x+c)^3-8*cos(d*x+c)
^2+4*I*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)-4*I*sin(d*x+c)*cos(d*x+c)+1-I*
sin(d*x+c))
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{16\sqrt{2}\left(-35i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-56i\,a^3e^{(2i\,dx+2i\,c)}-16i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*(-35*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*a^3*e^(4*I*d*x + 4*I*
c) - 56*I*a^3*e^(2*I*d*x + 2*I*c) - 16*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x +
2*I*c) + d)
```

3.326.6 Sympy [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.326.7 Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

3.326.8 Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.06

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d} - \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x),x)
```

output
```
(a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*16i)/d - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(d*(exp(c*
2i + d*x*2i) + 1)) + (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(5*d*(exp(c*2i + d*x*2i
) + 1)^2) - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)
*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.327 $\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.327.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

output
```
-64/3*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+16/3*I*a^2*cos(d*x+c)*(a
+I*a*tan(d*x+c))^(3/2)/d+2/3*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

3.327.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2ia^3\sec(c+dx)(12+11\cos(2(c+dx))-5i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/3)*a^3*Sec[c + d*x]*(12 + 11*Cos[2*(c + d*x)] - (5*I)*Sin[2*(c +
d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.327.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{3}a\int \cos(c+dx)(i\tan(c+dx)a+a)^{5/2}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{3}a\left(4a\int \cos(c+dx)(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}\right)+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}a\left(4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)}dx+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}\right)+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{3}a\left(\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/3)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2))/d + (8*a*(((-8*I)*
a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c + d*x]*(a
+ I*a*Tan[c + d*x])^(3/2))/d))/3
```

3.327.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.327.4 Maple [A] (verified)

Time = 29.97 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{2(-\tan(dx+c)+i)^{3}\sqrt{a(1+i\tan(dx+c))}\,a^{3}\left(10i\left(\cos^{3}(dx+c)\right)\sin(dx+c)-22\left(\cos^{4}(dx+c)\right)-\left(\cos^{2}(dx+c)\right)\right)}{3d\left(4i\left(\cos^{2}(dx+c)\right)\sin(dx+c)+4\left(\cos^{3}(dx+c)\right)-i\sin(dx+c)-3\cos(dx+c)\right)}$	122

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(-tan(d*x+c)+I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3/(4*I*cos(d*x+c)^2*
sin(d*x+c)+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c))*(10*I*cos(d*x+c)^3*si
n(d*x+c)-22*cos(d*x+c)^4-cos(d*x+c)^2)
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{4\sqrt{2}\left(3i\,a^3e^{(4i\,dx+4i\,c)}+12i\,a^3e^{(2i\,dx+2i\,c)}+8i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-4/3*sqrt(2)*(3*I*a^3*e^(4*I*d*x + 4*I*c) + 12*I*a^3*e^(2*I*d*x + 2*I*c) +
 8*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.327.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.327.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(80) = 160.

Time = 0.40 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.02

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2\left(23i\,a^{\frac{7}{2}}+\frac{20\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{88i\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{130i\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\right.}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(\frac{6i\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14i\,\sin(dx}{(\cos(dx+c}\right.}$$

---

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2*(23*I*a^(7/2) + 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*a^(7/2
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 130*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 60*a^(7
/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 88*I*a^(7/2)*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*I*a
^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x + c)/(cos(d*x +
c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*
(-18*I*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 42*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*I*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18*I*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3))
```

3.327.8 Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c), x)
```

3.327.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=$$

$$-\frac{2\,a^3\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(5\,\sin\,(c+d\,x)+5\,\sin\,(3\,c+3\,d\,x)+\cos\,(c+d\,x)\,35\mathrm{i}+\cos\,(3\,c+3\,d}{3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
-(2*a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x
) + 1))^(1/2)*(cos(c + d*x)*35i + 5*sin(c + d*x) + cos(3*c + 3*d*x)*11i +
5*sin(3*c + 3*d*x)))/(3*d*(cos(2*c + 2*d*x) + 1))
```

3.327. $\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

3.328 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.328.1 Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

output
```
8/3*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-2*I*a*cos(d*x+c)^3*(a+I*
a*tan(d*x+c))^(5/2)/d
```

3.328.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3\cos(c+dx)(i\cos(c+dx)+3\sin(c+dx))(\cos(c+4dx)+i\sin(c+4dx))\sqrt{a+ia\tan(c+}}{3d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*Cos[c + d*x]*(I*Cos[c + d*x] + 3*Sin[c + d*x])*(Cos[c + 4*d*x] + I*
Sin[c + 4*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^3
)
```

3.328.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3975}$$

$$-4a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-4a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^3}dx-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

$$\Big\downarrow \text{ 3974}$$

$$\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((8*I)/3)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((2*I)*a*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2))/d
```

3.328.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.328.4 Maple [A] (verified)

Time = 34.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

method	result	si
default	$\frac{2(\tan(dx+c)-i)^3\sqrt{a(1+i\tan(dx+c))}\,a^3\left(\cos^4(dx+c)\right)(3i\sin(dx+c)-\cos(dx+c))\left(2i\cos(dx+c)\sin(dx+c)-2\left(\cos^2(dx+c)\right)+1\right)}{3d}$	88

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^4*(3*I*si
n(d*x+c)-cos(d*x+c))*(2*I*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+1)
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{\sqrt{2}\left(-i\,a^3e^{(4i\,dx+4i\,c)}+i\,a^3e^{(2i\,dx+2i\,c)}+2i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*(-I*a^3*e^(4*I*d*x + 4*I*c) + I*a^3*e^(2*I*d*x + 2*I*c) + 2*I*
a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.328.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.328.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(57) = 114$.

Time = 0.40 (sec) , antiderivative size = 504, normalized size of antiderivative = 7.10

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{2\left(-i\,a^{\frac{7}{2}} - \frac{6\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1} + \frac{5i\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{24\,a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{10i\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{36\,a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{10i\,a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right.}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(-\frac{4i\sin(dx+c)}{\cos(dx+c)+1} + \frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{8i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \right.}$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2*(-I*a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*I*a^(7/2)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 10*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*I*a^(7/2
)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 + I*a^(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*(-2*I*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2
)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)^(7/2)*(12*I*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 24*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 42*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
- 24*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8 - 12*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 3*sin(d*x + c)^10/(c
os(d*x + c) + 1)^10 + 3))
```

3.328.8 Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^3, x)
```

3.328.9 Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{a^3\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+\cos(c+d\,x)\,3\mathrm{i}-\cos(3\,c}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
(a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(c + d*x)*3i + sin(c + d*x) - cos(3*c + 3*d*x)*1i + sin(3*c
 + 3*d*x)))/(3*d)
```

3.329 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.329.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

output
```
-2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)/d
```

3.329.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 1.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{2a^3\cos^3(c+dx)(-i\cos(2c+5dx)+\sin(2c+5dx))\sqrt{a+ia\tan(c+dx)}}{5d(\cos(dx)+i\sin(dx))^3}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*a^3*Cos[c + d*x]^3*((-I)*Cos[2*c + 5*d*x] + Sin[2*c + 5*d*x])*Sqrt[a +
I*a*Tan[c + d*x]])/(5*d*(Cos[d*x] + I*Sin[d*x])^3)
```

3.329.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \; 3974$$

$$-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

3.329.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.329.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.

Time = 4.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\frac{2(-\tan(dx+c)+i)^3 \sqrt{a(1+i\tan(dx+c))}\, a^3(-i(\cos^6(dx+c))\sin(dx+c)+\cos^7(dx+c))}{5d}$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
2/5/d*(-tan(d*x+c)+I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*(-I*cos(d*x+c)^6*si
n(d*x+c)+cos(d*x+c)^7)
```

3.329.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{2}\left(-i\,a^3e^{(6i\,dx+6i\,c)}-3i\,a^3e^{(4i\,dx+4i\,c)}-3i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{20\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/20*sqrt(2)*(-I*a^3*e^(6*I*d*x + 6*I*c) - 3*I*a^3*e^(4*I*d*x + 4*I*c) - 3
*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.329.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.329.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(27) = 54.

Time = 0.69 (sec) , antiderivative size = 454, normalized size of antiderivative = 12.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\frac{2\left(i\,a^{\frac{7}{2}}-\frac{6i\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15i\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20i\,a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\right.}{-5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10i\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\right.}$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2*(I*a^(7/2) - 6*I*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*I*a^(7
/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*I*a^(7/2)*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 15*I*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*I*
a^(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + I*a^(7/2)*sin(d*x + c)^12/
(cos(d*x + c) + 1)^12)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) +
1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-10*I*sin(d*x + c)/(
cos(d*x + c) + 1) - 20*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 50*I*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 100
*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 100*I*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 25*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 50*I*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 + 20*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 10*I*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 + 5*sin(d*x + c)^12/(cos(d*x + c) + 1)^12
 - 5))
```

3.329.8 Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\cos\left(dx+c\right)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^5, x)
```

3.329.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.20

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-2\,\sin\left(c+d\,x\right)-3\,\sin\left(3\,c+3\,d\,x\right)-\sin\left(5\,c+5\,d\,x\right)+\cos\left(c+d\,x\right)\,4\right.}{20\,d}$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
-(a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^(1/2)*(cos(c + d*x)*4i - 2*sin(c + d*x) + cos(3*c + 3*d*x)*3i + cos(
5*c + 5*d*x)*1i - 3*sin(3*c + 3*d*x) - sin(5*c + 5*d*x)))/(20*d)
```

3.330 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

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3.330.7 Maxima [B] (verification not implemented) 2349
3.330.8 Giac [F] 2350
3.330.9 Mupad [F(-1)] 2351

3.330.1 Optimal result

Integrand size = 26, antiderivative size = 196

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} - \frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d} - \frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

output
```
1/16*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(
1/2))/d*2^(1/2)-1/8*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/12*I*a^2
*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/10*I*a*cos(d*x+c)^5*(a+I*a*tan(
d*x+c))^(5/2)/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2)/d
```

3.330.2 Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.67

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{ia^3e^{-i(c+dx)}\left(176+298e^{2i(c+dx)}+188e^{4i(c+dx)}+81e^{6i(c+dx)}+15e^{8i(c+dx)}-105\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1-}\right.\right.}{1680d}$$

input
```
Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/1680*I)*a^3*(176 + 298*E^((2*I)*(c + d*x)) + 188*E^((4*I)*(c + d*x))
+ 81*E^((6*I)*(c + d*x)) + 15*E^((8*I)*(c + d*x)) - 105*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d
*x]])/(d*E^(I*(c + d*x)))
```

3.330.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3042, 3971, 3042, 3971, 3042, 3971, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \cos^3(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\left(\frac{1}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\left(\frac{1}{2}a\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a+}{5}$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\tan(c+}{5d}$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c}{3d}$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan(c}{3d}$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{3970}$$

$$\frac{1}{2}a\Bigg(\frac{1}{2}a\Bigg(\frac{1}{2}a\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia}{3d}$$
$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

$$\Big\downarrow \text{219}$$

3.330. $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

$$\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-\frac{i\cos^3(c+dx)(a+ia\tan}{3d}\right.\right.$$

$$\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/7*I)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2))/d + (a*(((-1/5*I)*C
os[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d + (a*(((-1/3*I)*Cos[c + d*x]
^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (a*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c
 + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d
*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/2))/2))/2
```

3.330.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

3.330.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(159) = 318.

Time = 3.88 (sec) , antiderivative size = 1185, normalized size of antiderivative = 6.05

Expression too large to display

input
```
int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
1/1680*I/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(-
315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)-420*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(
d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d
*x+c)-770*I*cos(d*x+c)^2+840*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(
sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3
*sin(d*x+c)-840*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4+840*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*
x+c)^3*sin(d*x+c)+105*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+1528*I*cos(d*x+c)^4-105*I*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*sin(d*x+c)-420*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+420*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)+420*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
nh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c
)^2*sin(d*x+c)-840*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+840*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2...
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.32

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{105\,\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{\left(i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{4\,d}\right)-105\,\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d}{}$$

input

```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output

```
1/1680*(105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 + sqrt(2)*sqrt(1/2)*
sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
)))*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 -
sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-15*I*a^3*e^(8*I*d*x
+ 8*I*c) - 81*I*a^3*e^(6*I*d*x + 6*I*c) - 188*I*a^3*e^(4*I*d*x + 4*I*c) -
298*I*a^3*e^(2*I*d*x + 2*I*c) - 176*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
)))/d
```

3.330.6 Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(7/2),x)
```

output

```
Timed out
```

3.330.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(149) = 298$.

Time = 0.70 (sec) , antiderivative size = 1253, normalized size of antiderivative = 6.39

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/6720*(20*(7*I*sqrt(2)*a^3*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - 7*sqrt(2)*a^3*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + 3*(I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x
 + 2*c)^2 + 2*I*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(7/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*(sqrt(2)*a^3*cos(2*d*x +
2*c)^2 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) +
 sqrt(2)*a^3)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqr
t(a) + 84*(5*I*sqrt(2)*a^3*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) - 5*sqrt(2)*a^3*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + (I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x + 2
*c)^2 + 2*I*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^3*cos(2*d*x + 2*c)^2
 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(
2)*a^3)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) +
 105*(2*sqrt(2)*a^3*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)...
```

3.330.8 Giac [F]

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\cos\left(dx+c\right)^7\,dx$$

input
```
integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^7, x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^7\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.331 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

3.331.1 Optimal result 2352
3.331.2 Mathematica [A] (verified) 2353
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3.331.1 Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= \frac{11ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}d} \\ &+ \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} \\ &- \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} \\ &- \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} \\ &- \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}\end{aligned}$$

output
```
11/128*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))
^(1/2))/d*2^(1/2)+11/96*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-11/64*
I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-11/120*I*a^3*cos(d*x+c)^3*(a+I
*a*tan(d*x+c))^(1/2)/d-11/140*I*a^2*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/
d-11/126*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d-1/9*I*cos(d*x+c)^9*(a
+I*a*tan(d*x+c))^(7/2)/d
```

3.331.2 Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{ia^3e^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-315+4303e^{2i(c+dx)}+7034e^{4i(c+dx)}+3754e^{6i(c+dx)}+1798e^{8i(c+dx)}+530e^{10i(c+}\right.}{20160\sqrt{2}d}$$

input
```
Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/20160*I)*a^3*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*
(-315 + 4303*E^((2*I)*(c + d*x)) + 7034*E^((4*I)*(c + d*x)) + 3754*E^((6*I
)*(c + d*x)) + 1798*E^((8*I)*(c + d*x)) + 530*E^((10*I)*(c + d*x)) + 70*E^
((12*I)*(c + d*x)) - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))
]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*x)))
```

3.331.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^9}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\int \cos^5(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx)}{} \right. \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\tan(}{7d} \right. \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+dx)(a+ia\,\mathrm{ta}}{5d} \right.\right. \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)-\frac{i\cos^5(c+d}{} \right.\right. \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow \text{3983}$$

3.331. $\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

$$\frac{11}{18}a\Bigg(\frac{9}{14}a\Bigg(\frac{7}{10}a\Bigg(\frac{5}{6}a\Bigg(\frac{3\int\cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\Bigg)-\frac{i\cos^3(c+dx)\sqrt{a+i}}{3d}$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow 3042$$

$$\frac{11}{18}a\Bigg(\frac{9}{14}a\Bigg(\frac{7}{10}a\Bigg(\frac{5}{6}a\Bigg(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\Bigg)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\Bigg)$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow 3971$$

$$\frac{11}{18}a\Bigg(\frac{9}{14}a\Bigg(\frac{7}{10}a\Bigg(\frac{5}{6}a\Bigg(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\Bigg)-i\cos^3$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow 3042$$

$$\frac{11}{18}a\Bigg(\frac{9}{14}a\Bigg(\frac{7}{10}a\Bigg(\frac{5}{6}a\Bigg(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\Bigg)-i\cos^3$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow 3970$$

$$\frac{11}{18}a\Bigg(\frac{9}{14}a\Bigg(\frac{7}{10}a\Bigg(\frac{5}{6}a\Bigg(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx}}$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

$$\Big\downarrow 219$$

3.331. $\int\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

$$\frac{11}{18}a\left(\frac{9}{14}a\left(\frac{7}{10}a\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)\right.\right.\right.$$

$$\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

input
```
Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/9*I)*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2))/d + (11*a*(((-1/7*I
)*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d + (9*a*(((-1/5*I)*Cos[c +
 d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-1/3*I)*Cos[c + d*x]^3*S
qrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sq
rt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan
[c + d*x]])/d))/(4*a)))/6))/10))/14))/18
```

3.331.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.331.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1210 vs. $2(219) = 438$.

Time = 4.77 (sec) , antiderivative size = 1211, normalized size of antiderivative = 4.52

Expression too large to display

input
```
int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
1/40320/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(10
395*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3465*I*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+1386
0*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)^2*sin(d*x+c)+7840*cos(d*x+c)^6-27720*cos(d*x+c)^4*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-1
3860*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-27720*cos(d*x+c)^3*arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+27720*I*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)^2-13860*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-13860*I*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+4
2504*I*cos(d*x+c)^3*sin(d*x+c)-13860*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)-3465*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*
x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-50424*cos(d*x+c)^4
+27720*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+...
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.17

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{\left(3465\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(-\frac{11\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)-3465\sqrt{\frac{1}{2}}\right.}{}$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output

```
-1/40320*(3465*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(
-I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 3465*sqrt(1/2)*sqrt(-
a^7/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(-I*a^4 - sqrt(2)*sqrt(1/2)*sqrt
(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-I*d*x - I*c)/d) - sqrt(2)*(-70*I*a^3*e^(12*I*d*x + 12*I*c) - 530*I*a^3
*e^(10*I*d*x + 10*I*c) - 1798*I*a^3*e^(8*I*d*x + 8*I*c) - 3754*I*a^3*e^(6*
I*d*x + 6*I*c) - 7034*I*a^3*e^(4*I*d*x + 4*I*c) - 4303*I*a^3*e^(2*I*d*x +
2*I*c) + 315*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c
)/d
```

3.331.6 Sympy [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**(7/2),x)
```

output

```
Timed out
```

3.331.7 Maxima [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input

```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output

```
Timed out
```

3.331.8 Giac [F]

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\cos\left(dx+c\right)^9\,dx$$

input
```
integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^9, x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^9\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.332 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

3.332.1 Optimal result 2361
3.332.2 Mathematica [A] (verified) 2362
3.332.3 Rubi [A] (verified) 2362
3.332.4 Maple [B] (verified) 2368
3.332.5 Fricas [A] (verification not implemented) 2368
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3.332.7 Maxima [F(-1)] 2369
3.332.8 Giac [F] 2370
3.332.9 Mupad [F(-1)] 2370

3.332.1 Optimal result

Integrand size = 26, antiderivative size = 342

$$\begin{aligned}\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= \frac{195ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d} \\ &+ \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} + \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d} \\ &- \frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} \\ &- \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} \\ &- \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}\end{aligned}$$

output

```
195/2048*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))/d*2^(1/2)+65/512*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+39/
448*I*a^4*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-195/1024*I*a^3*cos(d*x+c
)*(a+I*a*tan(d*x+c))^(1/2)/d-13/128*I*a^3*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^
(1/2)/d-13/168*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2)/d-65/924*I*a^2*
cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(3/2)/d-5/66*I*a*cos(d*x+c)^9*(a+I*a*tan(d
*x+c))^(5/2)/d-1/11*I*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2)/d
```

3.332.2 Mathematica [A] (verified)

Time = 6.02 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.57

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{ia^3e^{-5i(c+dx)}\left(-462-7161e^{2i(c+dx)}+47413e^{4i(c+dx)}+78800e^{6i(c+dx)}+38512e^{8i(c+dx)}+19552e^{10i(c+dx)}+\right.}{}$$

input

```
Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
((-1/473088*I)*a^3*(-462 - 7161*E^((2*I)*(c + d*x)) + 47413*E^((4*I)*(c +
d*x)) + 78800*E^((6*I)*(c + d*x)) + 38512*E^((8*I)*(c + d*x)) + 19552*E^((
10*I)*(c + d*x)) + 7184*E^((12*I)*(c + d*x)) + 1624*E^((14*I)*(c + d*x)) +
 168*E^((16*I)*(c + d*x)) - 45045*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x
]])/(d*E^((5*I)*(c + d*x)))
```

3.332.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.731$, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3978, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{7/2}}{\sec(c+dx)^{11}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\int \cos^9(c+dx)(i\tan(c+dx)a+a)^{5/2}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\int \frac{(i\tan(c+dx)a+a)^{5/2}}{\sec(c+dx)^9}dx-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\int \cos^7(c+dx)(i\tan(c+dx)a+a)^{3/2}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sec(c+dx)^7}dx-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{9d}\right)-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\int \cos^5(c+dx)\sqrt{i\tan(c+dx)a+a}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{7d}\right)-\frac{i\cos^9(c+dx)}{}\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^5}dx-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{7d}\right)-\frac{i\cos^9(c+dx)(a+ia\tan(}{9d}\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3978}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\right)-\frac{i\cos^7(c+dx)(a+ia\,\mathrm{t}}{7d}\right.\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\right)-\frac{i\cos^7(c+}{}\right.\right.$$
$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3983}$$

3.332. $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\int\cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a-}}{5d}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow\ 3042$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\right)\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow\ 3978$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int\frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)-\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow\ 3042$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\int\frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow\ 3983$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow\ 3042$$

3.332. $\int\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos}{4d\sqrt{a+}}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3971}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)}{}\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)}{}\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \text{3970}$$

3.332. $\int\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{15}{22}a\left(\frac{13}{18}a\left(\frac{11}{14}a\left(\frac{9}{10}a\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-i\right.}{8a}\right.\right.\right.\right.$$

$$\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

input
```
Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-1/11*I)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2))/d + (15*a*(((-1/9
*I)*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(5/2))/d + (13*a*(((-1/7*I)*Cos[
c + d*x]^7*(a + I*a*Tan[c + d*x])^(3/2))/d + (11*a*(((-1/5*I)*Cos[c + d*x]
^5*Sqrt[a + I*a*Tan[c + d*x]])/d + (9*a*(((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d
*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*(
(I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)
))/6))/(8*a)))/10))/14))/18))/22
```

3.332. $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

3.332.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.332.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(281) = 562$.

Time = 6.02 (sec) , antiderivative size = 1238, normalized size of antiderivative = 3.62

Expression too large to display

input
```
int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
1/473088/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(-
360360*cos(d*x+c)^3*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+37632*cos(d*
x+c)^8+101920*cos(d*x+c)^6+330330*cos(d*x+c)^2-655512*cos(d*x+c)^4+180180*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+45045*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*sin(d*x+c)+135135*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arcta
n((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-180180*arctanh(sin(d*x+c)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c
)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+360360*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-45045*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-360
360*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))-180180*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+360360*I*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin
(d*x+c)+180180*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-180180*I*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*si...
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{\Bigg(45045\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(4i\,dx+4i\,c)}\log\left(-\frac{195\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{512\,d}\right)-45045}{}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/473088*(45045*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/5
12*(-I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 45045*sqrt(1/2)*s
qrt(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/512*(-I*a^4 - sqrt(2)*sqrt(1/
2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))*e^(-I*d*x - I*c)/d) - sqrt(2)*(-168*I*a^3*e^(16*I*d*x + 16*I*c) - 1
624*I*a^3*e^(14*I*d*x + 14*I*c) - 7184*I*a^3*e^(12*I*d*x + 12*I*c) - 19552
*I*a^3*e^(10*I*d*x + 10*I*c) - 38512*I*a^3*e^(8*I*d*x + 8*I*c) - 78800*I*a
^3*e^(6*I*d*x + 6*I*c) - 47413*I*a^3*e^(4*I*d*x + 4*I*c) + 7161*I*a^3*e^(2
*I*d*x + 2*I*c) + 462*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-4*I*d*
x - 4*I*c)/d
```

3.332.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.332.7 Maxima [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.332.8 Giac [F]

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{11}\,dx$$

input
```
integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^11, x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos(c+d\,x)^{11}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{7/2}\,dx$$

input
```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.333 $\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.333.1 Optimal result 2371
3.333.2 Mathematica [A] (verified) 2371
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3.333.1 Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d} - \frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d}$$

output
```
-16/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d+8/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/
d-12/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^6/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)
/a^7/d
```

3.333.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}(-835+1421i\tan(c+dx)+945\tan^2(c+dx)-231i\tan^3(c+}{3003ad}$$

input
```
Integrate[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]*(-835 + (1421*I)*Tan[c
 + d*x] + 945*Tan[c + d*x]^2 - (231*I)*Tan[c + d*x]^3))/(3003*a*d)
```

3.333.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{5/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(-(i\tan(c+dx)a+a)^{11/2}+6a(i\tan(c+dx)a+a)^{9/2}-12a^2(i\tan(c+dx)a+a)^{7/2}+8a^3(i\tan(c+dx)a+\right.}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{16}{7}a^3(a+ia\tan(c+dx))^{7/2}-\frac{8}{3}a^2(a+ia\tan(c+dx))^{9/2}-\frac{2}{13}(a+ia\tan(c+dx))^{13/2}+\frac{12}{11}a(a+ia\tan(c+d}{a^7d}$$

input

```
Int[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(7/2))/7 - (8*a^2*(a + I*a*Tan[c + d
*x])^(9/2))/3 + (12*a*(a + I*a*Tan[c + d*x])^(11/2))/11 - (2*(a + I*a*Tan[
c + d*x])^(13/2))/13))/(a^7*d)
```

3.333.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.333.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{6a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{6a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{7}}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/13*(a+I*a*tan(d*x+c))^(13/2)-6/11*a*(a+I*a*tan(d*x+c))^(11/2)
+4/3*a^2*(a+I*a*tan(d*x+c))^(9/2)-8/7*a^3*(a+I*a*tan(d*x+c))^(7/2))
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$
$$-\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(13i\,dx+13i\,c)}+104i\,e^{(11i\,dx+11i\,c)}+286i\,e^{(9i\,dx+9i\,c)}+429i\,e^{(7i\,dx+7i\,c)}\right.}{3003\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\right.}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(13*I*d*x + 13
*I*c) + 104*I*e^(11*I*d*x + 11*I*c) + 286*I*e^(9*I*d*x + 9*I*c) + 429*I*e^
(7*I*d*x + 7*I*c))/(a*d*e^(12*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c
) + 15*a*d*e^(8*I*d*x + 8*I*c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*
I*d*x + 4*I*c) + 6*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.333.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^8\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**8/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.333.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(85) = 170.

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.54

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$
$$-\frac{2i\left(15015\sqrt{i\,a\tan\left(dx+c\right)+a}-\frac{3003\left(3\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}-10\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a+15\sqrt{i\,a\tan(dx+c)+a}a^2\right)}{a^2}+\frac{143\left(3\right.}{}\right.}{}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/15015*I*(15015*sqrt(I*a*tan(d*x + c) + a) - 3003*(3*(I*a*tan(d*x + c) +
 a)^(5/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) +
 a)*a^2)/a^2 + 143*(35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c
) + a)^(7/2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x +
 c) + a)^(3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4 - 5*(231*(I*a
*tan(d*x + c) + a)^(13/2) - 1638*(I*a*tan(d*x + c) + a)^(11/2)*a + 5005*(I
*a*tan(d*x + c) + a)^(9/2)*a^2 - 8580*(I*a*tan(d*x + c) + a)^(7/2)*a^3 + 9
009*(I*a*tan(d*x + c) + a)^(5/2)*a^4 - 6006*(I*a*tan(d*x + c) + a)^(3/2)*a
^5 + 3003*sqrt(I*a*tan(d*x + c) + a)*a^6)/a^6)/(a*d)
```

3.333.8 Giac [F]

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^8}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^8/sqrt(I*a*tan(d*x + c) + a), x)
```

3.333.9 Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.71

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=&-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{3003\,a\,d}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{3003\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\&-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{1001\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,640\mathrm{i}}{3003\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}\\&+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6784\mathrm{i}}{429\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\\&-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3456\mathrm{i}}{143\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
6784i)/(429*a*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(3003*a*d*(exp(c*2i +
 d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x
*2i) + 1))^(1/2)*256i)/(1001*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*640i)/(3003
*a*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1
i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3003*a*d) - ((a - (a*(exp(c*2i
+ d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3456i)/(143*a*d*(ex
p(c*2i + d*x*2i) + 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(
c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.333. $\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.334 $\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.334.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d} + \frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d} - \frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}$$

output
```
-8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d+8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d
-2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/d
```

3.334.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(107i+110\tan(c+dx)-35i\tan^2(c+dx))}{315ad}$$

input
```
Integrate[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(107*I + 110*Tan[c + d
*x] - (35*I)*Tan[c + d*x]^2))/(315*a*d)
```

3.334.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{7/2}-4a(i\tan(c+dx)a+a)^{5/2}+4a^2(i\tan(c+dx)a+a)^{3/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{5}a^2(a+ia\tan(c+dx))^{5/2}+\frac{2}{9}(a+ia\tan(c+dx))^{9/2}-\frac{8}{7}a(a+ia\tan(c+dx))^{7/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(5/2))/5 - (8*a*(a + I*a*Tan[c + d*x]
)^(7/2))/7 + (2*(a + I*a*Tan[c + d*x])^(9/2))/9))/(a^5*d)
```

3.334.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.334.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}+\frac{4a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}+\frac{4a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/9*(a+I*a*tan(d*x+c))^(9/2)+4/7*a*(a+I*a*tan(d*x+c))^(7/2)-4/
5*a^2*(a+I*a*tan(d*x+c))^(5/2))
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(9i\,dx+9i\,c)}+36i\,e^{(7i\,dx+7i\,c)}+63i\,e^{(5i\,dx+5i\,c)}\right)}{315\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(9*I*d*x + 9*I*c)
 + 36*I*e^(7*I*d*x + 7*I*c) + 63*I*e^(5*I*d*x + 5*I*c))/(a*d*e^(8*I*d*x +
8*I*c) + 4*a*d*e^(6*I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(
2*I*d*x + 2*I*c) + a*d)
```

3.334.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^6(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.334.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(64) = 128.

Time = 0.24 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.92

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\left(315\sqrt{i\,a\tan(dx+c)+a}-\frac{42\left(3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-10\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{i\,a\tan(dx+c)+a}a^2\right)}{a^2}+\frac{35\,(i\,a\tan(d}{}\right.}{315\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/315*I*(315*sqrt(I*a*tan(d*x + c) + a) - 42*(3*(I*a*tan(d*x + c) + a)^(5
/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^
2)/a^2 + (35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) + a)^(7/
2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c) + a)^(
3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4)/(a*d)
```

3.334.8 Giac [F]

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^6}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)
```

3.334.9 Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.48

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = {} & -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a\,d} \\ & -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{315\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{105\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} \\ & +\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,320\mathrm{i}}{63\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
320i)/(63*a*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(315*a*d*(exp(c*2i + d*x
*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
 + 1))^(1/2)*32i)/(105*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2
i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a*d) -
 ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*32i)/(9*a*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.334. $\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.335 $\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.335.1 Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=-\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}+\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

output
```
-4/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^2/d+2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d
```

3.335.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2(7-3i\tan(c+dx))(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{15ad}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*(7 - (3*I)*Tan[c + d*x])*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]
)/(15*a*d)
```

3.335.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a\sqrt{i\tan(c+dx)a+a}-(i\tan(c+dx)a+a)^{3/2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4}{3}a(a+ia\tan(c+dx))^{3/2}-\frac{2}{5}(a+ia\tan(c+dx))^{5/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*((4*a*(a + I*a*Tan[c + d*x])^(3/2))/3 - (2*(a + I*a*Tan[c + d*x])^(5
/2))/5))/(a^3*d)
```

3.335.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.335.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/5*(a+I*a*tan(d*x+c))^(5/2)-2/3*a*(a+I*a*tan(d*x+c))^(3/2))
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(5i\,dx+5i\,c)}+5i\,e^{(3i\,dx+3i\,c)}\right)}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(5*I*d*x + 5*I*c) +
 5*I*e^(3*I*d*x + 3*I*c))/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*
I*c) + a*d)
```

3.335.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\left(15\sqrt{i\,a\tan\left(dx+c\right)+a}-\frac{3\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}-10\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a+15\sqrt{i\,a\tan\left(dx+c\right)+a}a^2}{a^2}\right)}{15\,ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/15*I*(15*sqrt(I*a*tan(d*x + c) + a) - (3*(I*a*tan(d*x + c) + a)^(5/2) -
 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)/a^
2)/(a*d)
```

3.335.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

3.335.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.63

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,27\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,9\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1\mathrm{i}-5\,\mathrm{sin}\right.}{15\,a\,d\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+\right.\right.}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-(8*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2)*(cos(2*c + 2*d*x)*27i + cos(4*c + 4*d*x)*9i + cos(6*c + 6*d*x)*1
i - 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 19i))/(15
*a*d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.336 $\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.336.1 Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

output
```
-2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.336.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

3.336.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

input
```
Int[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

3.336.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.336.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{ad}$	24
default	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{ad}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a*d)
```

3.336.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{i\,a\tan(dx+c)+a}}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*I*sqrt(I*a*tan(d*x + c) + a)/(a*d)
```

3.336.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.60 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}}{ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*I*sqrt((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan
(1/2*d*x + 1/2*c)^2 - 1))/(a*d)
```

3.336.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{\frac{a\,\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,2\mathrm{i}}{a\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-(((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/2)*
2i)/(a*d)
```

3.337 $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.337.1 Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{5i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d}+\frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{5i}{8d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-5/16*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^
(1/2)+5/8*I/d/(a+I*a*tan(d*x+c))^(1/2)+5/12*I*a/d/(a+I*a*tan(d*x+c))^(3/2)
-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

3.337.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{6d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/6)*a*Hypergeometric2F1[-3/2, 2, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(a +
 I*a*Tan[c + d*x])^(3/2))
```

3.337.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3968, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+}\right.}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ia^3\left(\frac{5\left(\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (
5*(-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d
*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a)
))/(4*a)))/d
```

3.337. $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.337.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.337. $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.337.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(114) = 228$.

Time = 10.67 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.78

method	result
default	$-\frac{4i\left(\cos^3(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+4i\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-20\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-15i\cos(dx+c)\,\mathrm{ar}}{}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/d*(4*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4*I*cos(d*x+c
)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-20*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)-15*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(
d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*I*cos(d*x+c)
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-20*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)*sin(d*x+c)-15*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*I*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+15*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c))/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)
```

3.337.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(105) = 210$.

Time = 0.25 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.86

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-15i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/48*(-15*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(
2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 15*I*sqrt(1/
2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a*d*
e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2
)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(-3*I*e^(6*I*d*x + 6*I*c) + 11*I*e^(4*I*d*x + 4*I*c) + 16*I*e^
(2*I*d*x + 2*I*c) + 2*I))*e^(-3*I*d*x - 3*I*c)/(a*d)
```

3.337.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.337.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(15\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(15(ia\tan(dx+c)+a)^2a-20(ia\tan(dx+c)+a)a^2-8a^3\right)}{(ia\tan(dx+c)+a)^{\frac{5}{2}}-2(ia\tan(dx+c)+a)^{\frac{3}{2}}a}\right)}{96ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/96*I*(15*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x
+ c) + a)^2*a - 20*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3)/((I*a*tan(d*x + c)
+ a)^(5/2) - 2*(I*a*tan(d*x + c) + a)^(3/2)*a))/(a*d)
```

3.337.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(I*a*tan(d*x + c) + a), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.338 $\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.338.1 Optimal result . . . 2400
3.338.2 Mathematica [C] (verified) . . . 2401
3.338.3 Rubi [A] (warning: unable to verify) . . . 2401
3.338.4 Maple [B] (verified) . . . 2405
3.338.5 Fricas [A] (verification not implemented) . . . 2406
3.338.6 Sympy [F] . . . 2407
3.338.7 Maxima [A] (verification not implemented) . . . 2407
3.338.8 Giac [F] . . . 2407
3.338.9 Mupad [F(-1)] . . . 2408

3.338.1 Optimal result

Integrand size = 26, antiderivative size = 219

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =& -\frac{63i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d}+\frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}}\\&-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\\&-\frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\\&+\frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}}+\frac{63i}{128d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output

```
-63/256*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/
a^(1/2)+63/128*I/d/(a+I*a*tan(d*x+c))^(1/2)+63/160*I*a^2/d/(a+I*a*tan(d*x+
c))^(5/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(5/2)-9/16*I
*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+21/64*I*a/d/(a+I*a*tan(
d*x+c))^(3/2)
```

3.338.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{5}{2},3,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{20d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/20)*a^2*Hypergeometric2F1[-5/2, 3, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(5/2))
```

3.338.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{9\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{8a}+\frac{}{4a(a-i}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(}\right.}{8a}\right.}{d}$$

$$\Big\downarrow 61$$

3.338. $\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{}{2a(a-ia\tan}\right)}{8a}}{d}$$

$$\Big\downarrow \color{blue}{73}$$

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+i}\right)}{8a}}{d}$$

$$\Big\downarrow \color{blue}{219}$$

3.338. $\displaystyle\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{ia^5\left(\frac{9\left(\frac{7\left(\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/}}\right)}{8a}}{d}$$

input
```
Int[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) +
 (9*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(-1/
5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(
3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/
(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a)))/(4*a)))/(8*a)))/d
```

3.338.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.338.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(174) = 348$.

Time = 11.10 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.47

method	result
default	$-\frac{32i(\cos^{5}(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+32i(\cos^{4}(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-288\sin(dx+c)(\cos^{4}(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+84i(\cos^{3}(dx+}{}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1280/d*(32*I*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+32*I*cos(d
*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-288*sin(d*x+c)*cos(d*x+c)^4*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+84*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)-288*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
84*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-420*sin(d*x+c)*cos(d*
x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630*I*cos(d*x+c)*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)-315*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c)
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-420*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-630*I*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)-315*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))+315*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c))/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.34

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/1280*(-315*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sq
rt(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 315*I*sqr
t(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(
a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a
*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(-10*I*e^(10*I*d*x + 10*I*c) - 95*I*e^(8*I*d*x + 8*I*c) +
203*I*e^(6*I*d*x + 6*I*c) + 344*I*e^(4*I*d*x + 4*I*c) + 64*I*e^(2*I*d*x +
2*I*c) + 8*I))*e^(-5*I*d*x - 5*I*c)/(a*d)
```

3.338.6 Sympy [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^4{(c+dx)}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.338.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(315\,\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4a-1050\,(i\,a\tan(dx+c)+a)^3a^2+672\,(i\,a\tan(dx+c)+a)^2a\right.}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a+4\,(i\,a\tan(d}\right.}{2560\,ad}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2560*I*(315*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(
d*x + c) + a)^4*a - 1050*(I*a*tan(d*x + c) + a)^3*a^2 + 672*(I*a*tan(d*x +
 c) + a)^2*a^3 + 192*(I*a*tan(d*x + c) + a)*a^4 + 128*a^5)/((I*a*tan(d*x +
 c) + a)^(9/2) - 4*(I*a*tan(d*x + c) + a)^(7/2)*a + 4*(I*a*tan(d*x + c) +
a)^(5/2)*a^2))/(a*d)
```

3.338.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^4}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input

```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output

```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.339 $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.339.1 Optimal result 2409
3.339.2 Mathematica [C] (verified) 2410
3.339.3 Rubi [A] (warning: unable to verify) 2410
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3.339.8 Giac [F] 2422
3.339.9 Mupad [F(-1)] 2423

3.339.1 Optimal result

Integrand size = 26, antiderivative size = 292

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =& -\frac{429i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}d}+\frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}}\\ &-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}\\ &-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}\\ &-\frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\\ &+\frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}}\\ &+\frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}}+\frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-429/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2
)/a^(1/2)+429/1024*I/d/(a+I*a*tan(d*x+c))^(1/2)+429/896*I*a^3/d/(a+I*a*tan
(d*x+c))^(7/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(7/2)-1
3/48*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(7/2)-143/192*I*a^4/d
/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+429/1280*I*a^2/d/(a+I*a*tan(d
*x+c))^(5/2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.339.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.18

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{7}{2},4,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{56d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/56)*a^3*Hypergeometric2F1[-7/2, 4, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(7/2))
```

3.339.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^6\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}\right)}{12a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\,\mathrm{ta}}\right.}{d}$$

$$\Big\downarrow 52$$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+d}\right.}{12a}\right.}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{}{4a(a-}\right.}{12a}\right.}{d}$$

$$\Big\downarrow 61$$

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+i}}{8a}}{12a}}{d}$$

$$\Big\downarrow 61$$

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}\right.}{8a}\right.}{12a}\right.}{d}$$

$\big\downarrow$ 61

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{}{7a(a+i}\right.}{4a}\right.}{8a}\right.}{12a}\right.}{}$$

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\Big\downarrow 73$$

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{ia^7\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}\,d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7}}\right.}{4a}}{8a}}{12a}}{d}$$

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\Big\downarrow \; 219$$

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{ia^7\Bigg(\frac{13\Bigg(\frac{11\Bigg(\frac{9\left(\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{}{2a(a-i}}{8a}}{12a}}{d}$$

3.339. $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

input

```
Int[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(7/2)) +
 (13*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) + (11*
(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (9*(-1/7*1/
(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)
) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c +
 d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*
a))/(2*a))/(2*a)))/(4*a)))/(8*a)))/(12*a)))/d
```

3.339.3.1 Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.339.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(234) = 468.

Time = 9.44 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.31

method	result
default	$-\frac{-90090 i \cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-45045 i \arctan\left(\frac{\cos(dx+c)+1+i\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-33280\sin(dx+c)\left(\cos^{6}(dx+c)\right)\sqrt{-\frac{\cos(dx}{\cos(dx+}}}{}$

input
```
int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/215040/d*(-90090*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-45045*
I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))-33280*sin(d*x+c)*cos(d*x+c)^6*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+12012*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-33280*sin
(d*x+c)*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-90090*I*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-41184*sin(d*x+c)*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+4576*I*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-411
84*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4576*I*cos(d
*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-60060*sin(d*x+c)*cos(d*x+c)^2*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2560*I*cos(d*x+c)^7*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+2560*I*cos(d*x+c)^6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-6006
0*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-45045*I*cos(d*x
+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))+12012*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+45045*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*sin(d*x+c))/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(co
s(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.08

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-45045i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,d}\right.\right.}{}$$

input

```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
1/215040*(-45045*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(4
*(sqrt(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 45045
*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2)*sqrt(
1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(-280*I*e^(14*I*d*x + 14*I*c) - 2870*I*e^(12*I*d*x +
 12*I*c) - 16345*I*e^(10*I*d*x + 10*I*c) + 27029*I*e^(8*I*d*x + 8*I*c) + 4
9792*I*e^(6*I*d*x + 6*I*c) + 11072*I*e^(4*I*d*x + 4*I*c) + 2304*I*e^(2*I*d
*x + 2*I*c) + 240*I))*e^(-7*I*d*x - 7*I*c)/(a*d)
```

3.339.6 Sympy [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^6(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

input

```
integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
```

output

```
Integral(cos(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.339.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.84

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{i\left(45045\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^{6}a-240240\,(i\,a\tan(dx+c)+a)^{5}a^{2}+396396\,(i\,a\tan(\right.}{(i\,a\tan(dx+c)+a)^{\frac{13}{2}}-6\,(i\,a\tan}\right.}{430080\,ad}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/430080*I*(45045*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
 + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(45045*(I*
a*tan(d*x + c) + a)^6*a - 240240*(I*a*tan(d*x + c) + a)^5*a^2 + 396396*(I*
a*tan(d*x + c) + a)^4*a^3 - 164736*(I*a*tan(d*x + c) + a)^3*a^4 - 36608*(I
*a*tan(d*x + c) + a)^2*a^5 - 19968*(I*a*tan(d*x + c) + a)*a^6 - 15360*a^7)
/((I*a*tan(d*x + c) + a)^(13/2) - 6*(I*a*tan(d*x + c) + a)^(11/2)*a + 12*(
I*a*tan(d*x + c) + a)^(9/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(7/2)*a^3))/(a*
d)
```

3.339.8 Giac [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^6}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.340 $\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.340.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}} + \frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}} + \frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

output
```
256/6435*I*a^4*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+64/715*I*a^3*sec(d*
x+c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+8/65*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x
+c))^(5/2)+2/15*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.340.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sec^8(c+dx)(510\cos(c+dx)+731\cos(3(c+dx))+3i(90\sin(c+dx)+233\sin(3(c+dx))))(i\cos(4(c-}{6435d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^8*(510*Cos[c + d*x] + 731*Cos[3*(c + d*x)] + (3*I)*(90*Sin
[c + d*x] + 233*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])
)/(6435*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.340.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\left(\frac{4}{11}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\left(\frac{4}{11}a\int\frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{4}{5}a\left(\frac{8}{13}a\left(\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/15)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (4*a*((((
2*I)/13)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (8*a*((((8*I
)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/11)*a
*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))))/13))/5
```

3.340.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.340. $\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.340.4 Maple [A] (verified)

Time = 9.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{2048 i \sec(dx+c)}{6435}+\frac{2048 \sec(dx+c) \tan(dx+c)}{6435}+\frac{256 i \left(\sec^{3}(dx+c)\right)}{6435}+\frac{256 \tan(dx+c) \left(\sec^{3}(dx+c)\right)}{1287}+\frac{112 i \left(\sec^{5}(dx+c)\right)}{6435}+\frac{112 \tan(dx+c) \left(\sec^{5}(dx+c)\right)}{715}}{d \sqrt{a(1+i \tan(dx+c))}}$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/6435/d/(a*(1+I*tan(d*x+c)))^(1/2)*(1024*I*sec(d*x+c)+1024*sec(d*x+c)*tan
(d*x+c)+128*I*sec(d*x+c)^3+640*tan(d*x+c)*sec(d*x+c)^3+56*I*sec(d*x+c)^5+5
04*tan(d*x+c)*sec(d*x+c)^5+33*I*sec(d*x+c)^7+429*tan(d*x+c)*sec(d*x+c)^7)
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{256 \sqrt{2} \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}} \left(-715i\,e^{(6i\,dx+6i\,c)} - 390i\,e^{(4i\,dx+4i\,c)} - 120i\,e^{(2i\,dx+2i\,c)} -\right.}{6435 \left(ade^{(14i\,dx+14i\,c)} + 7\,ade^{(12i\,dx+12i\,c)} + 21\,ade^{(10i\,dx+10i\,c)} + 35\,ade^{(8i\,dx+8i\,c)} + 35\,ade^{(6i\,dx+6i\,c)} + 21\,ad\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-715*I*e^(6*I*d*x + 6
*I*c) - 390*I*e^(4*I*d*x + 4*I*c) - 120*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a*d
*e^(14*I*d*x + 14*I*c) + 7*a*d*e^(12*I*d*x + 12*I*c) + 21*a*d*e^(10*I*d*x
+ 10*I*c) + 35*a*d*e^(8*I*d*x + 8*I*c) + 35*a*d*e^(6*I*d*x + 6*I*c) + 21*a
*d*e^(4*I*d*x + 4*I*c) + 7*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.340.6 Sympy [F]

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^9\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**9/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.340.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(115) = 230.

Time = 0.43 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.14

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{2\biggl(-1241i\sqrt{a}-\frac{5194\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{6090i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2490\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14430i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33618\sqrt{a}\sin(dx+}{(\cos(dx+c)+1)}}{6435\biggl(a-\frac{8a\sin(dx+c}{(\cos(dx+c)+1}}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/6435*(-1241*I*sqrt(a) - 5194*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) +
6090*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2490*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 14430*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 33618*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13442*I*sqrt
(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18590*sqrt(a)*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 - 18590*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 13
442*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 33618*sqrt(a)*sin(d*
x + c)^11/(cos(d*x + c) + 1)^11 + 14430*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x
 + c) + 1)^12 + 2490*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 6090*
I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 5194*sqrt(a)*sin(d*x + c
)^15/(cos(d*x + c) + 1)^15 + 1241*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c)
+ 1)^16)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(
d*x + c) + 1) - 1)/((a - 8*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a*sin(d*
x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)
*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 1))
```

3.340.8 Giac [F]

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^9}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^9/sqrt(I*a*tan(d*x + c) + a), x)
```

3.340.9 Mupad [B] (verification not implemented)

Time = 10.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{9\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,768\mathrm{i}}{11\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5} + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,768\mathrm{i}}{13\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{15\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^7}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*256i)/(9*a*d*(exp(c*2i + d*x*2i) + 1)^4) - (exp(- c*1
i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*768i)/(11*a*d*(exp(c*2i + d*x*2i) + 1)^5) + (exp(- c*1i - d*x*1
i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*768i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(- c*1i - d*x*1i)*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(
15*a*d*(exp(c*2i + d*x*2i) + 1)^7)
```

3.341 $\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.341.1 Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}} + \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

output
```
64/693*I*a^3*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+16/99*I*a^2*sec(d*x+c
)^7/d/(a+I*a*tan(d*x+c))^(5/2)+2/11*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^
(3/2)
```

3.341.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sec^6(c+dx)(44+107\cos(2(c+dx))+91i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))}{693d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]^6*(44 + 107*Cos[2*(c + d*x)] + (91*I)*Sin[2*(c + d*x)])*(I
*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(693*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.341.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{11}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}a\left(\frac{4}{9}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{11}a\left(\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\right)+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

input

```
Int[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(((2*I)/11)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (8*a*((((
8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/9)
*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))))/11
```

3.341.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.341.4 Maple [A] (verified)

Time = 6.94 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

method	result	si
default	$\frac{\frac{256 i \sec \left(d x +c \right)}{693}+\frac{256 \sec \left(d x +c \right) \tan \left(d x +c \right)}{693}+\frac{32 i \left(\sec^{3}\left(d x +c \right)\right)}{693}+\frac{160 \tan \left(d x +c \right) \left(\sec^{3}\left(d x +c \right)\right)}{693}+\frac{2 i \left(\sec^{5}\left(d x +c \right)\right)}{99}+\frac{2 \tan \left(d x +c \right) \left(\sec^{5}\left(d x +c \right)\right)}{11}}{d \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}}$	99

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/693/d/(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*sec(d*x+c)+128*sec(d*x+c)*tan(d*
x+c)+16*I*sec(d*x+c)^3+80*tan(d*x+c)*sec(d*x+c)^3+7*I*sec(d*x+c)^5+63*tan(
d*x+c)*sec(d*x+c)^5)
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{64\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-99i\,e^{(4i\,dx+4i\,c)}-44i\,e^{(2i\,dx+2i\,c)}-8i\right)}{693\,\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8i\,dx+8i\,c)}+10\,ade^{(6i\,dx+6i\,c)}+10\,ade^{(4i\,dx+4i\,c)}+5\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-99*I*e^(4*I*d*x + 4*I*
c) - 44*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^
(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c
) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.341.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^7\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**7/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.341.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(86) = 172.

Time = 0.38 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.31

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\left(-151i\sqrt{a}-\frac{542\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{484i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{22\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{627i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1452\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\right.}{693\left(a-\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a\sin(dx+c)}{(\cos(dx+c)+1}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/693*(-151*I*sqrt(a) - 542*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 484
*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 22*sqrt(a)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 - 627*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
- 1452*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1452*sqrt(a)*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 + 627*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) +
 1)^8 - 22*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 484*I*sqrt(a)*sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 - 542*sqrt(a)*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 151*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*sqrt(s
in(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/((a - 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10
 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*sqrt(-2*I*sin(d*x + c)/(cos(
d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

3.341.8 Giac [F]

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^7}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^7/sqrt(I*a*tan(d*x + c) + a), x)
```

3.341.9 Mupad [B] (verification not implemented)

Time = 6.63 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{64\,{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\sqrt{a-\frac{a\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)\,1{}\mathrm{i}}{{\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1}}\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}\,44{}\mathrm{i}+{\mathrm{e}}^{c\,4{}\mathrm{i}+d\,x\,4{}\mathrm{i}}\,99{}\mathrm{i}+8{}\mathrm{i}\right)}{693\,a\,d\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^5}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(64*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*44i + exp(c*4i + d*x*4i)*99i +
 8i))/(693*a*d*(exp(c*2i + d*x*2i) + 1)^5)
```

3.341. $\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.342 $\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.342.1 Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

output
```
8/35*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)+2/7*I*a*sec(d*x+c)^5/d/
(a+I*a*tan(d*x+c))^(3/2)
```

3.342.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\sec^3(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-9i+5\tan(c+dx))}{35d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(-2*Sec[c + d*x]^3*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-9*I + 5*Tan[c
 + d*x]))/(35*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.342.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^5}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3975}$$

$$\frac{4}{7}a\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{7}a\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{3/2}}dx+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3974}$$

$$\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((8*I)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)
/7)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

3.342.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.342.4 Maple [A] (verified)

Time = 7.52 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\frac{16 i \sec \left(d x +c \right)}{35}+\frac{16 \sec \left(d x +c \right) \tan \left(d x +c \right)}{35}+\frac{2 i \left(\sec^{3}\left(d x +c \right)\right)}{35}+\frac{2 \tan \left(d x +c \right) \left(\sec^{3}\left(d x +c \right)\right)}{7}}{d \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}}$	72

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35/d/(a*(1+I*tan(d*x+c)))^(1/2)*(8*I*sec(d*x+c)+8*sec(d*x+c)*tan(d*x+c)+
I*sec(d*x+c)^3+5*tan(d*x+c)*sec(d*x+c)^3)
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(2i\,dx+2i\,c)}-2i\right)}{35\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(2*I*d*x + 2*I*c)
 - 2*I)/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*
I*d*x + 2*I*c) + a*d)
```

3.342.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^5(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**5/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.342.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(57) = 114.

Time = 0.35 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.66

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{2\left(-9i\sqrt{a}-\frac{26\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\right.}{35\left(a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/35*(-9*I*sqrt(a) - 26*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 14*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 14*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*s
qrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 26*sqrt(a)*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*sqrt(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/((a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) +
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

3.342.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/sqrt(I*a*tan(d*x + c) + a), x)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{16\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,7\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{35\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(16*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*7i + 2i)*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(35*a*d*(exp(c*2i +
d*x*2i) + 1)^3)
```

3.343 $\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.343.1 Optimal result . . . 2442
3.343.2 Mathematica [A] (verified) . . . 2442
3.343.3 Rubi [A] (verified) . . . 2443
3.343.4 Maple [A] (verified) . . . 2444
3.343.5 Fricas [A] (verification not implemented) . . . 2444
3.343.6 Sympy [F] . . . 2444
3.343.7 Maxima [B] (verification not implemented) . . . 2445
3.343.8 Giac [F] . . . 2445
3.343.9 Mupad [B] (verification not implemented) . . . 2446

3.343.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/3*I*a*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.343.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2\sec(c+dx)(i+\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sec[c + d*x]*(I + Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.343.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((2*I)/3)*a*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

3.343.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.343.4 Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\frac{2i\sec(dx+c)}{3}+\frac{2\sec(dx+c)\tan(dx+c)}{3}}{d\sqrt{a(1+i\tan(dx+c))}}$	44

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/(a*(1+I*tan(d*x+c)))^(1/2)*(I*sec(d*x+c)+sec(d*x+c)*tan(d*x+c))
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a*d*e^(2*I*d*x + 2*I*c) +
 a*d)
```

3.343.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.343.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(27) = 54$.

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.89

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{3\left(a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(a)*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/((a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) +
1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

3.343.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.80

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\;1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\;1\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\;1\mathrm{i}\right)}{3\,a\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
(2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(c + d*x)*1i + sin(c + d*x) + cos(3*c + 3*d*x)*1i + sin(3*c +
 3*d*x)))/(3*a*d*(cos(2*c + 2*d*x) + 1))
```

3.344 $\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.344.1 Optimal result 2447
3.344.2 Mathematica [A] (verified) 2447
3.344.3 Rubi [A] (verified) 2448
3.344.4 Maple [B] (verified) 2449
3.344.5 Fricas [B] (verification not implemented) 2449
3.344.6 Sympy [F] 2450
3.344.7 Maxima [F] 2450
3.344.8 Giac [F] 2450
3.344.9 Mupad [F(-1)] 2451

3.344.1 Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2)
/d/a^(1/2)
```

3.344.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ie^{i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.344.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3970}$$

$$\frac{2i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

input

```
Int[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(I*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[a]*d)
```

3.344.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

3.344.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(41) = 82$.

Time = 7.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.35

method	result	size
default	$\frac{(i\cos(dx+c)+i-\sin(dx+c))\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)}{d(\cos(dx+c)+1)\sqrt{a(1+i\tan(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$	122

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(I*cos(d*x+c)+I-sin(d*x+c))*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(co
s(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(a*(1+I*tan
(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

3.344.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(39) = 78$.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.87

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)\\ &\quad+\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)\end{aligned}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) +
1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d)
```

3.344.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec{(c+dx)}}{\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.344.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.344.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec{(dx+c)}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input

```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output

```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.345 $\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.345.1 Optimal result 2452
3.345.2 Mathematica [A] (verified) 2452
3.345.3 Rubi [A] (verified) 2453
3.345.4 Maple [B] (verified) 2455
3.345.5 Fricas [B] (verification not implemented) 2456
3.345.6 Sympy [F] 2456
3.345.7 Maxima [B] (verification not implemented) 2457
3.345.8 Giac [F] 2457
3.345.9 Mupad [F(-1)] 2458

3.345.1 Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}$$

output
```
3/8*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d*2
^(1/2)/a^(1/2)+1/2*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+c
)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.345.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sec(c+dx)\left(3i\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-i(1+\cos(2(c+dx))+3i\sin(2(c+dx)))\right)}{8d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(Sec[c + d*x]*((3*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*
I)*(c + d*x))]] - I*(1 + Cos[2*(c + d*x)] + (3*I)*Sin[2*(c + d*x)])))/(8*d
*Sqrt[a + I*a*Tan[c + d*x]])
```

3.345.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow 219$$

$$\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

input

```
Int[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTa
nh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*
d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)
```

3.345.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.345.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(97) = 194$.

Time = 9.74 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2i\left(\cos^{2}(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-3i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+2i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-6\sqrt{-\frac{c}{\cos}}}{8d\sqrt{-\frac{\cos}{\cos(}}}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*(2*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*cos(d*x+c)
*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))+2*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-6*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-3*I*arctan(1/2*(I*sin(
d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3*
arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*sin(d*x+c)-6*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(
1/2)
```

3.345.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(91) = 182$.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.01

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{\left(-3i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(2i\,dx+2i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)+3i\sqrt{\frac{1}{2}}ad}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/8*(-3*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt
(2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 3*I*sqrt(1/2)*a*d*sqrt(1/
(a*d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x
+ 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(
-I*d*x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*
d*x + 4*I*c) - I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

3.345.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.345.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(91) = 182$.

Time = 0.44 (sec) , antiderivative size = 837, normalized size of antiderivative = 6.86

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/32*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqr
t(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*
cos(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 2*sqrt(2))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(2*sqrt(2)*arctan2(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) ...
```

3.345.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.346 $\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.346.1 Optimal result 2459
3.346.2 Mathematica [A] (verified) 2460
3.346.3 Rubi [A] (verified) 2460
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3.346.7 Maxima [B] (verification not implemented) 2465
3.346.8 Giac [F] 2466
3.346.9 Mupad [F(-1)] 2467

3.346.1 Optimal result

Integrand size = 26, antiderivative size = 193

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{35i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} + \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}$$

output
```
35/128*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/
d*2^(1/2)/a^(1/2)+35/96*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+1/4*I*cos(
d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-35/64*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^
(1/2)/a/d-7/24*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.346.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.61

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sec(c+dx)\left(-41i+105i\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-43i\cos(2(c+dx))-2i\cos(4(c+dx))\right.}{384d\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(Sec[c + d*x]*(-41*I + (105*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[
1 + E^((2*I)*(c + d*x))]] - (43*I)*Cos[2*(c + d*x)] - (2*I)*Cos[4*(c + d*x
)] + 133*Sin[2*(c + d*x)] + 14*Sin[4*(c + d*x)]))/(384*d*Sqrt[a + I*a*Tan[
c + d*x]])
```

3.346.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sec(c+dx)^3\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{ 3983}$$
$$\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3970}$$

3.346. $\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+d}}{3d}\right.}{8a}$$

$$\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}$$

$$\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[
c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*S
qrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])
/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt
[a + I*a*Tan[c + d*x]])/d))/(4*a)))/6))/(8*a)
```

3.346.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.346. $\int\frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.346.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(156) = 312$.

Time = 11.70 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.51

method	result
default	$\frac{-16i\left(\cos^{4}(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+112\sin(dx+c)\left(\cos^{3}(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-16i\left(\cos^{3}(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+112\sin(dx+c)}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d*(-16*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+112*sin(d*x
+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-16*I*cos(d*x+c)^3*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+112*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)-70*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2
10*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-70*I*cos(d*x+c
)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+105*I*cos(d*x+c)*arctan(1/2*(I*sin(d*
x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+210*
sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*arctan(1/2*(I*sin(d*x+c)
-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+
c)+105*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)))/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1
)/(a*(1+I*tan(d*x+c)))^(1/2)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-105i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)+105i\sqrt{\frac{1}{2}}a}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/384*(-105*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32
*(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 105*I*sqrt(1/2)*a*d
*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32*(sqrt(2)*sqrt(1/2)*(-I*d*e
^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)
) - I)*e^(-I*d*x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8
*I*e^(8*I*d*x + 8*I*c) - 88*I*e^(6*I*d*x + 6*I*c) - 41*I*e^(4*I*d*x + 4*I*
c) + 45*I*e^(2*I*d*x + 2*I*c) + 6*I))*e^(-4*I*d*x - 4*I*c)/(a*d)
```

3.346.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.346.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(146) = 292$.

Time = 0.52 (sec) , antiderivative size = 1938, normalized size of antiderivative = 10.04

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output

```
-1/1536*(4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c
) - 3*sqrt(2)*sin(4*d*x + 4*c) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin
(4*d*x + 4*c) - 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)))*sqrt(a) + 12*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((-I*sqrt(2)*cos(4*
d*x + 4*c) - 12*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - sqrt(2)*sin(4*d*x + 4*c) - 12*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 24*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)) + (sqrt(2)*cos(4*d*x + 4*c) + 12*sqrt(2)*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - I*sqrt(2)*sin(4*d*x + 4*c) - 12
*I*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 24*sqrt(
2))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sqrt(a) + 1...
```

3.346.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input

```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output

```
integrate(cos(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.347 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.347.1 Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d} + \frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d} - \frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d} + \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}$$

output
```
-16/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d+24/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5
/d-4/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^6/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a
^7/d
```

3.347.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(-533i-755\tan(c+dx)+455i\tan^2(c+dx)+105\tan^3(c+d}{1155a^2d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(-533*I - 755*Tan[c +
 d*x] + (455*I)*Tan[c + d*x]^2 + 105*Tan[c + d*x]^3))/(1155*a^2*d)
```

3.347.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int (-(i\tan(c+dx)a+a)^{9/2}+6a(i\tan(c+dx)a+a)^{7/2}-12a^2(i\tan(c+dx)a+a)^{5/2}+8a^3(i\tan(c+dx)a+}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{16}{5}a^3(a+ia\tan(c+dx))^{5/2}-\frac{24}{7}a^2(a+ia\tan(c+dx))^{7/2}-\frac{2}{11}(a+ia\tan(c+dx))^{11/2}+\frac{4}{3}a(a+ia\tan(c+d}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(5/2))/5 - (24*a^2*(a + I*a*Tan[c +
d*x])^(7/2))/7 + (4*a*(a + I*a*Tan[c + d*x])^(9/2))/3 - (2*(a + I*a*Tan[c
+ d*x])^(11/2))/11))/(a^7*d)
```

3.347.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.347.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^7}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/11*(a+I*a*tan(d*x+c))^(11/2)-2/3*a*(a+I*a*tan(d*x+c))^(9/2)+1
2/7*a^2*(a+I*a*tan(d*x+c))^(7/2)-8/5*a^3*(a+I*a*tan(d*x+c))^(5/2))
```

3.347.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(11i\,dx+11i\,c)}+88i\,e^{(9i\,dx+9i\,c)}+198i\,e^{(7i\,dx+7i\,c)}+231i\,e^{(5i\,dx+5i\,c)}\right)}{1155\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-64/1155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(11*I*d*x + 11*
I*c) + 88*I*e^(9*I*d*x + 9*I*c) + 198*I*e^(7*I*d*x + 7*I*c) + 231*I*e^(5*I
*d*x + 5*I*c))/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*e^(8*I*d*x + 8*I*c)
+ 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x + 4*I*c) + 5*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

3.347.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^8(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\left(105\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}-770\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a+1980\,(i\,a\tan\right.}{1155\,a^7d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output

```
2/1155*I*(105*(I*a*tan(d*x + c) + a)^(11/2) - 770*(I*a*tan(d*x + c) + a)^(
9/2)*a + 1980*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 1848*(I*a*tan(d*x + c) +
a)^(5/2)*a^3)/(a^7*d)
```

3.347.8 Giac [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^8}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output

```
integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.347.9 Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.16

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1155\,a^2\,d}\\ &\quad-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1155\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{385\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\ &\quad-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{33\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}\end{aligned}$$

input

```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
256i)/(33*a^2*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1155*a^2*d*(exp(c*2i
+ d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*128i)/(385*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(231
*a^2*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)
*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1155*a^2*d) - ((a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*a^2*d*
(exp(c*2i + d*x*2i) + 1)^5)
```

3.347. $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.348 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.348.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d} + \frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d} - \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}$$

output
```
-8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d+8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d
-2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5/d
```

3.348.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(-71+54i\tan(c+dx)+15\tan^2(c+dx))}{105a^2d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(-71 + (54*I)*Tan[c + d
*x] + 15*Tan[c + d*x]^2))/(105*a^2*d)
```

3.348.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{5/2}-4a(i\tan(c+dx)a+a)^{3/2}+4a^2\sqrt{i\tan(c+dx)a+a}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{8}{3}a^2(a+ia\tan(c+dx))^{3/2}+\frac{2}{7}(a+ia\tan(c+dx))^{7/2}-\frac{8}{5}a(a+ia\tan(c+dx))^{5/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*((8*a^2*(a + I*a*Tan[c + d*x])^(3/2))/3 - (8*a*(a + I*a*Tan[c + d*x]
)^(5/2))/5 + (2*(a + I*a*Tan[c + d*x])^(7/2))/7))/(a^5*d)
```

3.348.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.348.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{4a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{4a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^5}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/7*(a+I*a*tan(d*x+c))^(7/2)+4/5*a*(a+I*a*tan(d*x+c))^(5/2)-4/
3*a^2*(a+I*a*tan(d*x+c))^(3/2))
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(7i\,dx+7i\,c)}+28i\,e^{(5i\,dx+5i\,c)}+35i\,e^{(3i\,dx+3i\,c)}\right)}{105\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-16/105*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(7*I*d*x + 7*I*c)
 + 28*I*e^(5*I*d*x + 5*I*c) + 35*I*e^(3*I*d*x + 3*I*c))/(a^2*d*e^(6*I*d*x
+ 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2
*d)
```

3.348.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^6\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2i\left(15\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}-84\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a+140\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^2\right)}{105\,a^5d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/105*I*(15*(I*a*tan(d*x + c) + a)^(7/2) - 84*(I*a*tan(d*x + c) + a)^(5/2
)*a + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

3.348.8 Giac [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^6}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.75

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{105\,a^2\,d} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{105\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16i}{35\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} + \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16i}{7\,a^2\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
16i)/(7*a^2*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(105*a^2*d*(exp(c*2i + d*
x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*16i)/(35*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(105*a^2*
d)
```

3.349 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.349.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=-\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}+\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

output
```
-4*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d
```

3.349.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=-\frac{2(5i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3a^2d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*(5*I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*a^2*d)
```

3.349.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \; 53$$

$$-\frac{i\int \left(\frac{2a}{\sqrt{i\tan(c+dx)a+a}}-\sqrt{i\tan(c+dx)a+a}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{i\left(4a\sqrt{a+ia\tan(c+dx)}-\frac{2}{3}(a+ia\tan(c+dx))^{3/2}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*(4*a*Sqrt[a + I*a*Tan[c + d*x]] - (2*(a + I*a*Tan[c + d*x])^(3/2))/3
))/(a^3*d)
```

3.349.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.349.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^3}$	44

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(1/3*(a+I*a*tan(d*x+c))^(3/2)-2*a*(a+I*a*tan(d*x+c))^(1/2))
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(3i\,dx+3i\,c)}+3i\,e^{(i\,dx+i\,c)}\right)}{3\,(a^2de^{(2i\,dx+2i\,c)}+a^2d)}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(3*I*d*x + 3*I*c) +
3*I*e^(I*d*x + I*c))/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.349.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^4{(c+dx)}}{(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\left((i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}-6\sqrt{i\,a\tan{(dx+c)}+a}\,a\right)}{3\,a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/3*I*((I*a*tan(d*x + c) + a)^(3/2) - 6*sqrt(I*a*tan(d*x + c) + a)*a)/(a^3
*d)
```

3.349.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2\left(\cos\left(2\,c+2\,d\,x\right)\,5\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}}{3\,a^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
-(2*(cos(2*c + 2*d*x)*5i + sin(2*c + 2*d*x) + 5i)*((a*(cos(2*c + 2*d*x) +
sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(3*a^2*d*(cos(2*c
 + 2*d*x) + 1))
```

3.350 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.350.1 Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.350.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.350.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3968$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \ 17$$

$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.350.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.350.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2i}{ad\sqrt{a+ia\tan(dx+c)}}$	24
default	$\frac{2i}{ad\sqrt{a+ia\tan(dx+c)}}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.350.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(-
I*d*x - I*c)/(a^2*d)
```

3.350.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^2{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{\sqrt{i\,a\tan{(dx+c)}+a}ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*I/(sqrt(I*a*tan(d*x + c) + a)*a*d)
```

3.350.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^2}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.350.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\cos\left(c+d\,x\right)^2\,2\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{a\,\left(2\,\cos\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\,\cos\left(c+d\,x\right)^2}}}{a^2\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((sin(2*c + 2*d*x) + cos(c + d*x)^2*2i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c
 + d*x)^2))/(2*cos(c + d*x)^2))^(1/2))/(a^2*d)
```

3.351 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.351.8 Giac [F] 2495
3.351.9 Mupad [F(-1)] 2495

3.351.1 Optimal result

Integrand size = 26, antiderivative size = 175

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{7i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}\\ &\quad+\frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\\ &\quad+\frac{7i}{24d(a+ia\tan(c+dx))^{3/2}}+\frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-7/32*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^
(1/2)+7/16*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+7/20*I*a/d/(a+I*a*tan(d*x+c))^(5
/2)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+7/24*I/d/(a+I*
a*tan(d*x+c))^(3/2)
```

3.351.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.29

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{10d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/10)*a*Hypergeometric2F1[-5/2, 2, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(a
+ I*a*Tan[c + d*x])^(5/2))
```

3.351.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3968, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{7\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\,\mathrm{t}}}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{}{2a(a-ia\,\mathrm{ta}}}{d}$$

$$\Big\downarrow 73$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a}}{d}$$

$$\Big\downarrow 219$$

$$-\frac{ia^3\left(\frac{7\left(\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+d}}{d}$$

3.351. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (
7*(-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d
*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)
) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a)))/(4*a)))/d
```

3.351.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.351.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(137) = 274$.

Time = 10.01 (sec) , antiderivative size = 595, normalized size of antiderivative = 3.40

method	result
default	$-72i(\cos^3(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+168\sin(dx+c)(\cos^2(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-72i(\cos^2(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+168\sqrt{-\frac{\cos(d}{\cos(dx}}$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/480/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+
c)))^(1/2)/(1+I*tan(d*x+c))/a*(-72*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+168*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-
72*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+168*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+210*I*cos(d*x+c)*arctan(1/2*(cos(d
*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3
50*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-210*arctan(1/2*(cos(d*x
+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin
(d*x+c)-210*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+105*I*arctan(1/2
*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))+350*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*tan(d*x+c)*arctan(1/2*(
cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))-210*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*I*sec(d*x+c)*arc
tan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)))
```

3.351.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(126) = 252$.

Time = 0.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-105i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e^{(2i\,dx}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/480*(-105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(4*
(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(8*I*d*x + 8*I*c) + 101*I*e^(6
*I*d*x + 6*I*c) + 148*I*e^(4*I*d*x + 4*I*c) + 38*I*e^(2*I*d*x + 2*I*c) + 6
*I))*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

3.351.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^2{(c+dx)}}{\left(ia\left(\tan{(c+dx)}-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(105\,(i\,a\tan(dx+c)+a)^3-140\,(i\,a\tan(dx+c)+a)^2a-\right.}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-2\,(i\,a\tan(dx}\right.}{960\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/960*I*(105*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(
sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(105*(I*a*tan(d
*x + c) + a)^3 - 140*(I*a*tan(d*x + c) + a)^2*a - 56*(I*a*tan(d*x + c) + a
)*a^2 - 48*a^3)/((I*a*tan(d*x + c) + a)^(7/2) - 2*(I*a*tan(d*x + c) + a)^(
5/2)*a))/(a*d)
```

3.351.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.352 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.352.8 Giac [F] . . . 2505
3.352.9 Mupad [F(-1)] . . . 2505

3.352.1 Optimal result

Integrand size = 26, antiderivative size = 248

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = {} & -\frac{99i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} \\ & + \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\ & - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}} \\ & + \frac{33i}{128d(a+ia\tan(c+dx))^{3/2}} + \frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-99/512*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*
2^(1/2)+99/256*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+99/224*I*a^2/d/(a+I*a*tan(d*
x+c))^(7/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(7/2)-11/1
6*I*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+99/320*I*a/d/(a+I*a*
tan(d*x+c))^(5/2)+33/128*I/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.352.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia^2 \operatorname{Hypergeometric2F1}\left(-\frac{7}{2},3,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{28d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/28)*a^2*Hypergeometric2F1[-7/2, 3, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(7/2))
```

3.352.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c}\right.}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{8a}+\frac{}{4a(a-}\right.}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \text{61}$$

3.352. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{}{2a}}{8a}}{d}$$

$$\Big\downarrow 61$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{}{7a(a+ia\tan}}{4a}}{8a}}{d}$$

3.352. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \; 73$$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \; 219$$

3.352. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$-\frac{ia^5\left(\frac{11\left(\frac{9\left(\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{}{2a(a-ia\tan}\right.}{8a}}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) +
 (11*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (9*(-1
/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^
(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Ta
n[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]])
)/(2*a))/(2*a))/(2*a)))/(4*a)))/(8*a)))/d
```

3.352. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.352.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.352. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.352.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(197) = 394$.

Time = 8.62 (sec) , antiderivative size = 729, normalized size of antiderivative = 2.94

method	result
default	$-\frac{-11550 i \cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-3465 i \arctan\left(\frac{\cos(dx+c)+1+i\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-3520\sin(dx+c)\left(\cos^4(dx+c)\right)\sqrt{-\frac{\cos(dx+c}{\cos(dx+c)}}}{}$

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/17920/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d
*x+c)))^(1/2)/(1+I*tan(d*x+c))/a*(-11550*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-3465*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-3520*sin(d*x+c)*cos(d*x+c)^4*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)+3465*I*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*s
in(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-3520*sin(d*x
+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2376*I*cos(d*x+c)^2*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-5544*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)-11550*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+960*I*co
s(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-5544*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+960*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)-6930*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(co
s(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6930*sin(d*x+c)*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+6930*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+2376*I*cos(d*x+c)
^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6930*tan(d*x+c)*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+3465*tan(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.27

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-3465i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e^{(2i}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/17920*(-3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log
(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
 + 3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sq
rt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqr
t(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-70*I*e^(12*I*d*x + 12*I*c) - 805*
I*e^(10*I*d*x + 10*I*c) + 2833*I*e^(8*I*d*x + 8*I*c) + 4584*I*e^(6*I*d*x +
 6*I*c) + 1304*I*e^(4*I*d*x + 4*I*c) + 328*I*e^(2*I*d*x + 2*I*c) + 40*I))*
e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

3.352.6 Sympy [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^4\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{3465\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}} + \frac{4\left(3465\,(i\,a\tan(dx+c)+a)^5-11550\,(i\,a\tan(dx+c)+a)}{(i\,a\tan(dx+c}\right.}{3584}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/35840*I*(3465*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(3465*(I*a*t
an(d*x + c) + a)^5 - 11550*(I*a*tan(d*x + c) + a)^4*a + 7392*(I*a*tan(d*x
+ c) + a)^3*a^2 + 2112*(I*a*tan(d*x + c) + a)^2*a^3 + 1408*(I*a*tan(d*x +
c) + a)*a^4 + 1280*a^5)/((I*a*tan(d*x + c) + a)^(11/2) - 4*(I*a*tan(d*x +
c) + a)^(9/2)*a + 4*(I*a*tan(d*x + c) + a)^(7/2)*a^2))/(a*d)
```

3.352.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.353 $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.353.1 Optimal result . . . 2506
3.353.2 Mathematica [C] (verified) . . . 2507
3.353.3 Rubi [A] (warning: unable to verify) . . . 2507
3.353.4 Maple [B] (verified) . . . 2519
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3.353.6 Sympy [F] . . . 2521
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3.353.8 Giac [F] . . . 2522
3.353.9 Mupad [F(-1)] . . . 2522

3.353.1 Optimal result

Integrand size = 26, antiderivative size = 321

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{715i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}a^{3/2}d}\\
&+\frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}}\\
&-\frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\\
&-\frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\\
&+\frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}}+\frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}}\\
&+\frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}}+\frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-715/4096*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/
d*2^(1/2)+715/2048*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+715/1152*I*a^3/d/(a+I*a*
tan(d*x+c))^(9/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(9/2
)-5/16*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(9/2)-65/64*I*a^4/d
/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+715/1792*I*a^2/d/(a+I*a*tan(d
*x+c))^(7/2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(5/2)+715/3072*I/d/(a+I*a*ta
n(d*x+c))^(3/2)
```

3.353.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.17

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{9}{2},4,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{72d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Integrate[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/72)*a^3*Hypergeometric2F1[-9/2, 4, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(9/2))
```

3.353.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3968, 52, 52, 52, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^6(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{1}{(a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^7\left(\frac{5\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{6a(a-ia\tan(c+dx))^3(a+ia\,\mathrm{ta}}\right.}{d}$$

$$\Big\downarrow \; 52$$

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+}\right.}{4a}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{}{4a(a-}\right.}{4a}\right.}{d}$$

$$\Big\downarrow \; 61$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+i}\right.}{8a}\right.}{4a}\right.}{d}$$

$$\Big\downarrow \textbf{61}$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}}{8a}\right.}{4a}\right.}{d}$$

$\downarrow$ 61

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$-\frac{ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}\,d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\right.}{4a}\right.}{8a}\right.}{4a}\right.}{}$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.353.

$$\Big\downarrow 61$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}\,d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{}{7a(a+i}}{2a}\right.}{4a}\right.}{8a}\right.}{4a}\right.$$

3.353. $\displaystyle\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\downarrow 73$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7}}}{2a}\right.}{4a}\right.}{8a}\right.}{4a}\right.$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\Big\downarrow \ 219$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$ia^7\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{}{9a(a+ia\tan}\right.}{4a}\right.}{8a}\right.}{4a}\right.$$

3.353. $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

input
```
Int[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*a^7*(1/(6*a*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(9/2)) +
 (5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) + (13*(
1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (11*(-1/9*1/
(a*(a + I*a*Tan[c + d*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)
) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c +
 d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/
2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2*a)))/(4*a)
))/(8*a)))/(4*a)))/d
```

3.353.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.353.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(257) = 514.

Time = 9.31 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.53

method	result	size
default	Expression too large to display	811

input
```
int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/258048/d*(90090*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+28672*si
n(d*x+c)*cos(d*x+c)^8*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+45045*I*arctan(1/
2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))+28672*sin(d*x+c)*cos(d*x+c)^7*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-12
012*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+33280*sin(d*x+c)*cos
(d*x+c)^6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+28672*I*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)^8+33280*sin(d*x+c)*cos(d*x+c)^5*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)+90090*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+41184*sin(d*x
+c)*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-4576*I*cos(d*x+c)^5*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+41184*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)-4576*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)^4+60060*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2867
2*I*cos(d*x+c)^9*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+60060*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-2560*I*cos(d*x+c)^7*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)-2560*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
^6-45045*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+45045*I*arctan(1/2*(cos(d*x+c)+1+I*sin
(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-120
12*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(a*(1+I*tan(d*x+c)))
^(1/2)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.05

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e^{(2}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/258048*(-45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*l
og(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)) + 45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*
(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-168*I*e^(16*I*d*x + 16*I*c) -
1974*I*e^(14*I*d*x + 14*I*c) - 13209*I*e^(12*I*d*x + 12*I*c) + 33301*I*e^(
10*I*d*x + 10*I*c) + 57632*I*e^(8*I*d*x + 8*I*c) + 17344*I*e^(6*I*d*x + 6*
I*c) + 5440*I*e^(4*I*d*x + 4*I*c) + 1136*I*e^(2*I*d*x + 2*I*c) + 112*I))*e
^(-9*I*d*x - 9*I*c)/(a^2*d)
```

3.353.6 Sympy [F]

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^6(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.81

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{45045\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}} + \frac{4\left(45045\,(i\,a\tan(dx+c)+a)^7-240240\,(i\,a\tan(dx+c)}{}\right.}{}$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/516096*I*(45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(45045*(I*
a*tan(d*x + c) + a)^7 - 240240*(I*a*tan(d*x + c) + a)^6*a + 396396*(I*a*ta
n(d*x + c) + a)^5*a^2 - 164736*(I*a*tan(d*x + c) + a)^4*a^3 - 36608*(I*a*t
an(d*x + c) + a)^3*a^4 - 19968*(I*a*tan(d*x + c) + a)^2*a^5 - 15360*(I*a*t
an(d*x + c) + a)*a^6 - 14336*a^7)/((I*a*tan(d*x + c) + a)^(15/2) - 6*(I*a*
tan(d*x + c) + a)^(13/2)*a + 12*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 8*(I*a
*tan(d*x + c) + a)^(9/2)*a^3))/(a*d)
```

3.353.8 Giac [F]

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^6}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.354 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.354.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}} + \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}} + \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

output
```
256/12155*I*a^4*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+64/1105*I*a^3*se
c(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+8/85*I*a^2*sec(d*x+c)^11/d/(a+I*a*t
an(d*x+c))^(7/2)+2/17*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(5/2)
```

3.354.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^9(c+dx)(i\cos(4(c+dx))+\sin(4(c+dx)))(475i-2242i\cos(2(c+dx}{12155ad(-i+\tan(c+dx))\sqrt{a+}}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^9*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(475*I - (2242*I
)*Cos[2*(c + d*x)] + 1089*Sec[c + d*x]*Sin[3*(c + d*x)] + 374*Tan[c + d*x]
))/(12155*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.354.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{17}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{17}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{17}a\left(\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow 3974$$

$$\frac{12}{17}a\left(\frac{8}{15}a\left(\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/17)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (12*a*((
((2*I)/15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (8*a*((((
8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I)
/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))))/15))/17
```

3.354.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.354.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x)
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{512\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1105i\,e^{(6i\,dx+6i\,c)}-510i\,e^{(4i\,dx+4i\,c)}-136\right.}{12155\left(a^2de^{(16i\,dx+16i\,c)}+8\,a^2de^{(14i\,dx+14i\,c)}+28\,a^2de^{(12i\,dx+12i\,c)}+56\,a^2de^{(10i\,dx+10i\,c)}+70\,a^2de^{(8i\,dx+8i\,c)}+\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-512/12155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1105*I*e^(6*I*d*x +
 6*I*c) - 510*I*e^(4*I*d*x + 4*I*c) - 136*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a
^2*d*e^(16*I*d*x + 16*I*c) + 8*a^2*d*e^(14*I*d*x + 14*I*c) + 28*a^2*d*e^(1
2*I*d*x + 12*I*c) + 56*a^2*d*e^(10*I*d*x + 10*I*c) + 70*a^2*d*e^(8*I*d*x +
 8*I*c) + 56*a^2*d*e^(6*I*d*x + 6*I*c) + 28*a^2*d*e^(4*I*d*x + 4*I*c) + 8*
a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.354.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.354.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(115) = 230$.

Time = 0.48 (sec) , antiderivative size = 764, normalized size of antiderivative = 5.20

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/12155*(-1767*I*sqrt(a) - 6854*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) +
 2088*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 16438*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - 5661*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 56984*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 13328*I*sqr
t(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 129336*sqrt(a)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 7514*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 -
 156468*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 156468*sqrt(a)*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 - 7514*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x
 + c) + 1)^12 - 129336*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 133
28*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 56984*sqrt(a)*sin(d*x
 + c)^15/(cos(d*x + c) + 1)^15 + 5661*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x +
 c) + 1)^16 - 16438*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2088*I
*sqrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 6854*sqrt(a)*sin(d*x + c)
^19/(cos(d*x + c) + 1)^19 + 1767*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) +
 1)^20)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)^(3/2)/((a^2 - 10*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 45*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 120*a^2*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^2*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12/(cos(d*x + c
) + 1)^12 - 120*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a^2*sin(...
```

3.354.8 Giac [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{11}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.354.9 Mupad [B] (verification not implemented)

Time = 11.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{11\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} \\
&\quad - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1536\mathrm{i}}{13\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6} + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{5\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7} \\
&\quad - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{17\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*512i)/(11*a^2*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(-
c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*1536i)/(13*a^2*d*(exp(c*2i + d*x*2i) + 1)^6) + (exp(- c*1i -
 d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*512i)/(5*a^2*d*(exp(c*2i + d*x*2i) + 1)^7) - (exp(- c*1i - d*x*1i)
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
512i)/(17*a^2*d*(exp(c*2i + d*x*2i) + 1)^8)
```

3.355 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.355.1 Optimal result 2529
3.355.2 Mathematica [A] (verified) 2529
3.355.3 Rubi [A] (verified) 2530
3.355.4 Maple [F(-1)] 2531
3.355.5 Fricas [A] (verification not implemented) 2532
3.355.6 Sympy [F] 2532
3.355.7 Maxima [B] (verification not implemented) 2532
3.355.8 Giac [F] 2533
3.355.9 Mupad [B] (verification not implemented) 2534

3.355.1 Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}} + \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

output
```
64/1287*I*a^3*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+16/143*I*a^2*sec(d*x
+c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+2/13*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c)
)^(5/2)
```

3.355.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^8(c+dx)(52+151\cos(2(c+dx))+135i\sin(2(c+dx)))(\cos(3(c+dx}{1287ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^8*(52 + 151*Cos[2*(c + d*x)] + (135*I)*Sin[2*(c + d*x)])*(
Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(1287*a*d*(-I + Tan[c + d*x])*Sqrt
[a + I*a*Tan[c + d*x]])
```

3.355.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{13}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{13}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{13}a\left(\frac{4}{11}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{13}a\left(\frac{4}{11}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{13}a\left(\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\right)+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

3.355. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$

output
```
(((2*I)/13)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (8*a*((((
8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/11
)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))))/13
```

3.355.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.355.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^9(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-143i\,e^{(4i\,dx+4i\,c)}-52i\,e^{(2i\,dx+2i\,c)}-8i\right)}{1287\left(a^2de^{(12i\,dx+12i\,c)}+6\,a^2de^{(10i\,dx+10i\,c)}+15\,a^2de^{(8i\,dx+8i\,c)}+20\,a^2de^{(6i\,dx+6i\,c)}+15\,a^2de^{(4i\,dx+4i\,c)}+6\,a^2d\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-128/1287*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-143*I*e^(4*I*d*x + 4
*I*c) - 52*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a
^2*d*e^(10*I*d*x + 10*I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*
I*d*x + 6*I*c) + 15*a^2*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c
) + a^2*d)
```

3.355.6 Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^9(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**9/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.355.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(86) = 172.

Time = 0.43 (sec) , antiderivative size = 626, normalized size of antiderivative = 5.69

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(-203i\sqrt{a}-\frac{678\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1802\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{26i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3614\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\right.}{1287\left(a^2-\frac{8a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/1287*(-203*I*sqrt(a) - 678*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*
I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1802*sqrt(a)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 26*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 3614*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 858*I*sqrt(a)*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 - 6578*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 6578*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 858*I*sqrt(a)*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10 - 3614*sqrt(a)*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + 26*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 180
2*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2*I*sqrt(a)*sin(d*x + c)
^14/(cos(d*x + c) + 1)^14 - 678*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)
^15 + 203*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(
cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/(
(a^2 - 8*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a^2*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 + 28*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^2*sin(d*x +
c)^14/(cos(d*x + c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d
*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)^(3/2))
```

3.355.8 Giac [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^9}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.355.9 Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,52\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,143\mathrm{i}+8\mathrm{i}\right)}{1287\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

input

```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output

```
(128*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*52i + exp(c*4i + d*x*4i)*143i
 + 8i))/(1287*a^2*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.356 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.356.1 Optimal result 2535
3.356.2 Mathematica [A] (verified) 2535
3.356.3 Rubi [A] (verified) 2536
3.356.4 Maple [F(-1)] 2537
3.356.5 Fricas [A] (verification not implemented) 2537
3.356.6 Sympy [F] 2538
3.356.7 Maxima [B] (verification not implemented) 2538
3.356.8 Giac [F] 2539
3.356.9 Mupad [B] (verification not implemented) 2539

3.356.1 Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

output
```
8/63*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+2/9*I*a*sec(d*x+c)^7/d/
(a+I*a*tan(d*x+c))^(5/2)
```

3.356.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^5(c+dx)(i\cos(2(c+dx))+\sin(2(c+dx)))(-11i+7\tan(c+dx))}{63ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^5*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-11*I + 7*Tan[c
 + d*x]))/(63*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.356.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{9}a\int \frac{\sec^7(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{9}a\int \frac{\sec(c+dx)^7}{(i\tan(c+dx)a+a)^{5/2}}dx+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)
/9)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.356.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.356.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{7}\left(dx+c\right)}{\left(a+ia\tan\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x)
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(2i\,dx+2i\,c)}-2i\right)}{63\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+4i\,c)}+4\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-32/63*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(2*I*d*x + 2*I*c)
 - 2*I)/(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d
*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.356.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^7(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.356.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(57) = 114.

Time = 0.37 (sec) , antiderivative size = 488, normalized size of antiderivative = 6.68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(-11i\sqrt{a}-\frac{30\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{12i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{86\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{108\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\right.}{63\left(a^2-\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/63*(-11*I*sqrt(a) - 30*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 12*I*s
qrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 86*sqrt(a)*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 9*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 108*
sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 108*sqrt(a)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 - 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 86
*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 12*I*sqrt(a)*sin(d*x + c)^1
0/(cos(d*x + c) + 1)^10 - 30*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
 + 11*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(
d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2
 - 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)
^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

3.356.8 Giac [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^7}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{32\,e^{-c\,1i-d\,x\,1i}\,\left(e^{c\,2i+d\,x\,2i}\,9i+2i\right)\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}}{63\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4}$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
(32*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*9i + 2i)*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(63*a^2*d*(exp(c*2i
+ d*x*2i) + 1)^4)
```

3.357 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.357.1 Optimal result 2540
3.357.2 Mathematica [A] (verified) 2540
3.357.3 Rubi [A] (verified) 2541
3.357.4 Maple [F(-1)] 2542
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3.357.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

output
```
2/5*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)
```

3.357.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\sec^3(c+dx)(1-i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sec[c + d*x]^3*(1 - I*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x])*Sqrt[a
+ I*a*Tan[c + d*x]])
```

3.357.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/5)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.357.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.357.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{5}\left(dx+c\right)}{\left(a+ia\tan\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x)
```

3.357.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{8i\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{5\,\left(a^{2}de^{(4i\,dx+4i\,c)}+2\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
8/5*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^2*d*e^(4*I*d*x + 4*I*c)
 + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.357.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^{5}\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.357.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(27) = 54.

Time = 0.34 (sec) , antiderivative size = 350, normalized size of antiderivative = 10.00

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{6\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\right.}{5\left(a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(-\frac{2i\sin(dx}{\cos(dx+c)}\right.}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 6*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*I*sqrt(a)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)/(c
os(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((
a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

3.357.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^5}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.357.9 Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.97

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(\cos(d\,x)-\sin(d\,x)\ 1\mathrm{i})\ (\cos(c)-\sin(c)\ 1\mathrm{i})\ \sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\ 1\mathrm{i}}{\cos(2\,c+2\,d\,x)+1}}}{5\,a^2\,d\ (4\ \cos(2\,c+}$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*((a*(cos(2*c + 2*d*x) + sin
(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(2*cos(2*c + 2*d*x) +
 cos(4*c + 4*d*x) - sin(2*c + 2*d*x)*2i - sin(4*c + 4*d*x)*1i + 1)*4i)/(5*
a^2*d*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3))
```

3.358 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.358.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/
2)/a^(3/2)/d-2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{8e^{3i(c+dx)}\left(-1+\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(8*E^((3*I)*(c + d*x))*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]]))/(a*d*(1 + E^((2*I)*(c + d*x)))^2*(-I + Tan[c + d
*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.358.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 3972, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3972}$$

$$\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3970}$$

$$\frac{4i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{ad}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((2*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*
x]])
```

3.358.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3972
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Simp[2*(d^2/a)   Int[(d*Sec[e + f*x])^
(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

3.358.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(71) = 142$.

Time = 8.94 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.59

method	result
default	$-\frac{2(-\csc(dx+c)+\cot(dx+c)+i)^{3}\left(-\sqrt{2}\,\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1}}\right)\sqrt{(\csc^{2}(dx+c))(1-\cos(dx+c))^{2}-1}+i(\csc(dx+}{d\left(-\frac{a\left(2i(\csc(dx+c)-\cot(dx+c))-\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1\right)}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1}\right)^{\frac{3}{2}}\left((\csc^{2}(dx+c))(1-\cos(dx+c))^{2}-1\right)^{2}}$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-csc(d*x+c)+cot(d*x+c)+I)^3*(-2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(
d*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*
(1-cos(d*x+c))^2-1)^(1/2)+I*(csc(d*x+c)-cot(d*x+c))-1)/(-a*(2*I*(csc(d*x+c
)-cot(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c)
)^2-1))^(3/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^2
```

3.358.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(67) = 134$.

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}a^2d\sqrt{\frac{1}{a^3d^2}}\log\left(-\frac{8\left(\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{ad}\right)}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((I*a*d*e^(2*I*d*x + 2*I*c) + I
*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x -
 I*c)/(a*d)) + I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((-I*a*d*e^(2*I*d*
x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) -
I)*e^(-I*d*x - I*c)/(a*d)) - 2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)/(a^2*d)
```

3.358.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.358.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(67) = 134$.

Time = 0.45 (sec) , antiderivative size = 813, normalized size of antiderivative = 9.45

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output

```
-1/2*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sq
rt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(
sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)...
```

3.358.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.359 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.359.1 Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
1/4*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(
3/2)/d*2^(1/2)+1/2*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.359.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{\left(2+\frac{2e^{2i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)\sec(c+dx)}{4ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((2 + (2*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/Sqrt[
1 + E^((2*I)*(c + d*x))])*Sec[c + d*x])/(4*a*d*(-I + Tan[c + d*x])*Sqrt[a
+ I*a*Tan[c + d*x]])
```

3.359.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])
])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3
/2))
```

3.359.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.359.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(68) = 136$.

Time = 8.67 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.78

method	result
default	$-\frac{i\left(2i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+i\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+2i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-2\,\mathrm{arct}\right.}{4d(\mathrm{ta}}$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/d/(tan(d*x+c)-I)/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)/a/(cos(d*x+c)+1)*(2*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(
d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+I*arctan(1/2*
(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))+2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*arctan(1/2*(I*sin(d*x+c)-cos
(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-I
*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+2*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)-tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

3.359.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(64) = 128$.

Time $= 0.26$ (sec) , antiderivative size $= 246$, normalized size of antiderivative $= 2.83$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}+i\right.}{ad}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*
sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt(1/(a^3*d^2)) + I)*e^(-I*d*x - I*c)/(a*d)) - I*sqrt(1/2)*a^2*d*s
qrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*sqrt(1/2)*(-I*a*d*e^(2*I
*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2))
 + I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
(I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

3.359.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.359.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.359.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.360 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.360.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{15i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} \\ &\quad+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}+\frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} \\ &\quad-\frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}\end{aligned}$$

output
```
15/64*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a
^(3/2)/d*2^(1/2)+5/16*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)-15/32*I*co
s(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/4*I*cos(d*x+c)/d/(a+I*a*tan(d*x+
c))^(3/2)
```

3.360.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sec(c+dx)\Bigg(\frac{30e^{2i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}-2(-9+6\cos(2(c+dx))+10}{64ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
(Sec[c + d*x]*((30*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(-9 + 6*Cos[2*(c + d*x)] + (10*I)*S
in[2*(c + d*x)])))/(64*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.360.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3983, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{3970}$$

$$\frac{5\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{5\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I/4)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(((I/2)*Cos[c +
 d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Se
c[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c
+ d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)))/(8*a)
```

3.360. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.360.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.360. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.360.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(126) = 252$.

Time = 9.95 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.43

method	result
default	$-\frac{24i\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-30i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+24i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-40\sqrt{}}{}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+
c)))^(1/2)/(1+I*tan(d*x+c))/a*(24*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)-30*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+24*I*cos(d*x+c)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-40*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*
x+c)-15*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))-30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+30*arcta
n(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*sin(d*x+c)-40*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+15*
I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))-30*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+15*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

3.360.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(118) = 236$.

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.72

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\Bigg(-15i\,\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(4i\,dx+4i\,c)}\log\Bigg(-\frac{15\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+}}\right.}{16\,ad}}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/64*(-15*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/
16*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 15*I*sq
rt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/16*(sqrt(2)*sq
rt(1/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(-8*I*e^(6*I*d*x + 6*I*c) + I*e^(4*I*d*x + 4*I*c) +
11*I*e^(2*I*d*x + 2*I*c) + 2*I))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

3.360.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.360.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1821 vs. 2(118) = 236.

Time = 0.55 (sec) , antiderivative size = 1821, normalized size of antiderivative = 11.60

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/256*(36*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c)
- sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)) + (sqrt(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2
*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 4*(cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*((7*I*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*sin(4*d*x +
 4*c) + 8*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) - (7*sqrt(2)*cos(4*d*x + 4*c) - 7*I*sqrt(2)*sin(4*d*x + 4*c) + 8*sqrt(2)
)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 15*(2*
sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x ...
```

3.360.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.361 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.361.1 Optimal result

Integrand size = 26, antiderivative size = 233

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{105i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}$$

output
```
105/512*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))
/a^(3/2)/d*2^(1/2)+35/128*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)+3/16*I
*cos(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(1/2)-105/256*I*cos(d*x+c)*(a+I*a*tan
(d*x+c))^(1/2)/a^2/d-7/32*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/
6*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.361.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.62

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sec(c+dx)\left(\frac{630e^{2i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(158\cos(2(c+dx)) + 8\cos(}{1536ad(-i+\tan(c+dx))\sqrt{a}}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(Sec[c + d*x]*((630*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x
))]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(158*Cos[2*(c + d*x)] + 8*Cos[4*(c
 + d*x)] + (3*I)*(55*I + 86*Sin[2*(c + d*x)] + 8*Sin[4*(c + d*x)]))))/(153
6*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.361.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+}}\right.}{4a}$$

$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.361. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+}}\right.}{4a}$$

$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3970$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.}{4a}$$

$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i}{4d\sqrt{}}\right.}{4a}$$

$$\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

3.361. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

output
```
((I/6)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (3*(((I/4)*Cos[c
 + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*S
qrt[a + I*a*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sq
rt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan
[c + d*x]])/d))/(4*a)))/6))/(8*a)))/(4*a)
```

3.361.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.361. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.361.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(190) = 380$.

Time = 10.02 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.89

method	result
default	$-\frac{128i\left(\cos^{4}(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+128i\left(\cos^{3}(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-384\sin(dx+c)\left(\cos^{3}(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+504i(\cos^{2}(}{}$

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1536/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(1+I*tan(d*x+c
))/(a*(1+I*tan(d*x+c)))^(1/2)/a*(128*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)+128*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-384*sin
(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+504*I*cos(d*x+c)^2
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-384*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+504*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)-630*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-840*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)*sin(d*x+c)-630*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-315*I*arct
an(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))-840*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+630*arctan(1
/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*sin(d*x+c)-630*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+315
*I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))+315*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(
d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.25

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{105\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c}}}}{128\,ad}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/1536*(-315*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-
105/128*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 31
5*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-105/128*(sq
rt(2)*sqrt(1/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(-16*I*e^(10*I*d*x + 10*I*c) - 224*I*e^(8*I*
d*x + 8*I*c) - 43*I*e^(6*I*d*x + 6*I*c) + 215*I*e^(4*I*d*x + 4*I*c) + 58*I
*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

3.361.6 Sympy [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.361.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2632 vs. $2(178) = 356$.

Time = 0.53 (sec) , antiderivative size = 2632, normalized size of antiderivative = 11.30

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/6144*(8*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(
6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-4*I*sqrt(2)*cos(6*d*x + 6*c
) - 45*I*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 4*
sqrt(2)*sin(6*d*x + 6*c) - 45*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c))) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x
+ 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c))) + 1)) + (4*sqrt(2)*cos(6*d*x + 6*c) + 45*sqrt(2)*cos(2/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c))) - 4*I*sqrt(2)*sin(6*d*x + 6*c) - 45*I*s
qrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 8*sqrt(2))*s
in(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos
(6*d*x + 6*c))) + 1)^(1/4)*(((-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*
d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (-I*s
qrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*(-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2
)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) -
 I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2...
```

3.361.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.362 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.362.1 Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=-\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d}+\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d}-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d}$$

output
```
-32/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d+64/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6
/d-16/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^7/d+16/11*I*(a+I*a*tan(d*x+c))^(11/2)
/a^8/d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^9/d
```

3.362.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(9683i+16700\tan(c+dx)-142}{15015a^3d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(9683*I + 16700*Tan[c
+ d*x] - (14210*I)*Tan[c + d*x]^2 - 6300*Tan[c + d*x]^3 + (1155*I)*Tan[c +
 d*x]^4))/(15015*a^3*d)
```

3.362.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4(i\tan(c+dx)a+a)^{3/2}d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{ 53}$$

$$-\frac{i\int \left((i\tan(c+dx)a+a)^{11/2}-8a(i\tan(c+dx)a+a)^{9/2}+24a^2(i\tan(c+dx)a+a)^{7/2}-32a^3(i\tan(c+dx)a+\right.}{a^9d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{32}{5}a^4(a+ia\tan(c+dx))^{5/2}-\frac{64}{7}a^3(a+ia\tan(c+dx))^{7/2}+\frac{16}{3}a^2(a+ia\tan(c+dx))^{9/2}+\frac{2}{13}(a+ia\tan(c+\right.}{a^9d}$$

input

```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output

```
((-I)*((32*a^4*(a + I*a*Tan[c + d*x])^(5/2))/5 - (64*a^3*(a + I*a*Tan[c +
d*x])^(7/2))/7 + (16*a^2*(a + I*a*Tan[c + d*x])^(9/2))/3 - (16*a*(a + I*a*
Tan[c + d*x])^(11/2))/11 + (2*(a + I*a*Tan[c + d*x])^(13/2))/13))/(a^9*d)
```

3.362.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.362.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{8a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{16a^4(a+ia\tan(dx+c))}{5}\right.}{d\,a^9}$
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{8a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{16a^4(a+ia\tan(dx+c))}{5}\right.}{d\,a^9}$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^9*(-1/13*(a+I*a*tan(d*x+c))^(13/2)+8/11*a*(a+I*a*tan(d*x+c))^(11/2
)-8/3*a^2*(a+I*a*tan(d*x+c))^(9/2)+32/7*a^3*(a+I*a*tan(d*x+c))^(7/2)-16/5*
a^4*(a+I*a*tan(d*x+c))^(5/2))
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{128\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(128i\,e^{(13i\,dx+13i\,c)}+832i\,e^{(11i\,dx+11i\,c)}+2288i\,e^{(9i\,dx+9i\,c)}+3432i\,e^{(7i\,dx+7i\,c)}+300\right.}{15015\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a^3\right.}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-128/15015*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(13*I*d*x +
13*I*c) + 832*I*e^(11*I*d*x + 11*I*c) + 2288*I*e^(9*I*d*x + 9*I*c) + 3432*
I*e^(7*I*d*x + 7*I*c) + 3003*I*e^(5*I*d*x + 5*I*c))/(a^3*d*e^(12*I*d*x + 1
2*I*c) + 6*a^3*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20
*a^3*d*e^(6*I*d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I
*d*x + 2*I*c) + a^3*d)
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{2i\left(1155\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}-10920\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a+40040\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a^2-686\right.}{15015\,a^9d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/15015*I*(1155*(I*a*tan(d*x + c) + a)^(13/2) - 10920*(I*a*tan(d*x + c) +
 a)^(11/2)*a + 40040*(I*a*tan(d*x + c) + a)^(9/2)*a^2 - 68640*(I*a*tan(d*x
 + c) + a)^(7/2)*a^3 + 48048*(I*a*tan(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

3.362.8 Giac [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{10}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.97

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=&-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16384\mathrm{i}}{15015\,a^3\,d}\\&-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{15015\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{5005\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\&-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{3003\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{429\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\\&+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1792\mathrm{i}}{143\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
1792i)/(143*a^3*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(15015*a^3*d*(exp(c
*2i + d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*2048i)/(5005*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a
 - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*102
4i)/(3003*a^3*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(429*a^3*d*(exp(c*2i +
 d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*16384i)/(15015*a^3*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*a^3*d*(exp(c*2i + d*x
*2i) + 1)^6)
```

3.362. $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.363 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.363.1 Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=-\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}-\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}$$

output
```
-16/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d+24/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5
/d-12/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^
7/d
```

3.363.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2(1+i\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(-319i-321\tan(c+dx)+165i}{315a^3d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*(1 + I*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(-319*I - 321*Tan[c + d
*x] + (165*I)*Tan[c + d*x]^2 + 35*Tan[c + d*x]^3))/(315*a^3*d)
```

3.363.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^3\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \Big(-(i\tan(c+dx)a+a)^{7/2}+6a(i\tan(c+dx)a+a)^{5/2}-12a^2(i\tan(c+dx)a+a)^{3/2}+8a^3\sqrt{i\tan(c+dx)a}}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{16}{3}a^3(a+ia\tan(c+dx))^{3/2}-\frac{24}{5}a^2(a+ia\tan(c+dx))^{5/2}-\frac{2}{9}(a+ia\tan(c+dx))^{9/2}+\frac{12}{7}a(a+ia\tan(c+dx}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((16*a^3*(a + I*a*Tan[c + d*x])^(3/2))/3 - (24*a^2*(a + I*a*Tan[c +
d*x])^(5/2))/5 + (12*a*(a + I*a*Tan[c + d*x])^(7/2))/7 - (2*(a + I*a*Tan[c
 + d*x])^(9/2))/9))/(a^7*d)
```

3.363.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.363.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{6a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{6a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^{7}}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/9*(a+I*a*tan(d*x+c))^(9/2)-6/7*a*(a+I*a*tan(d*x+c))^(7/2)+12/
5*a^2*(a+I*a*tan(d*x+c))^(5/2)-8/3*a^3*(a+I*a*tan(d*x+c))^(3/2))
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{32\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(9i\,dx+9i\,c)}+72i\,e^{(7i\,dx+7i\,c)}+126i\,e^{(5i\,dx+5i\,c)}+105i\,e^{(3i\,dx+3i\,c)}\right)}{315\left(a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(9*I*d*x + 9*I*c
) + 72*I*e^(7*I*d*x + 7*I*c) + 126*I*e^(5*I*d*x + 5*I*c) + 105*I*e^(3*I*d*
x + 3*I*c))/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a
^3*d*e^(4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.363.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^8\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\,\Big(35\,(i\,a\tan\,(dx+c)+a)^{\frac{9}{2}}-270\,(i\,a\tan\,(dx+c)+a)^{\frac{7}{2}}a+756\,(i\,a\tan\,(a}{315\,a^7d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/315*I*(35*(I*a*tan(d*x + c) + a)^(9/2) - 270*(I*a*tan(d*x + c) + a)^(7/2
)*a + 756*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 840*(I*a*tan(d*x + c) + a)^(3
/2)*a^3)/(a^7*d)
```

3.363.8 Giac [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^8}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.363.9 Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.62

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{315\,a^3\,d} \\ &\quad -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{105\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} \\ &\quad -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{63\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
32i)/(9*a^3*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a^3*d*(exp(c*2i + d
*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*64i)/(105*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp
(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*a^3*
d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/
(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(315*a^3*d)
```

3.364 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.364.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}$$

output

```
-8*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d-2
/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d
```

3.364.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{a+ia\tan(c+dx)}(-43+14i\tan(c+dx)+3\tan^2(c+dx))}{15a^3d}$$

input

```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output

```
(((2*I)/15)*Sqrt[a + I*a*Tan[c + d*x]]*(-43 + (14*I)*Tan[c + d*x] + 3*Tan[
c + d*x]^2))/(a^3*d)
```

3.364.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{4a^2}{\sqrt{i\tan(c+dx)a+a}}-4\sqrt{i\tan(c+dx)a+a}a+(i\tan(c+dx)a+a)^{3/2}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(8a^2\sqrt{a+ia\tan(c+dx)}+\frac{2}{5}(a+ia\tan(c+dx))^{5/2}-\frac{8}{3}a(a+ia\tan(c+dx))^{3/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*(8*a^2*Sqrt[a + I*a*Tan[c + d*x]] - (8*a*(a + I*a*Tan[c + d*x])^(3/2
))/3 + (2*(a + I*a*Tan[c + d*x])^(5/2))/5))/(a^5*d)
```

3.364.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.364.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^{2}\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{5}}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^{2}\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{5}}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/5*(a+I*a*tan(d*x+c))^(5/2)+4/3*a*(a+I*a*tan(d*x+c))^(3/2)-4*
a^2*(a+I*a*tan(d*x+c))^(1/2))
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(5i\,dx+5i\,c)}+20i\,e^{(3i\,dx+3i\,c)}+15i\,e^{(i\,dx+i\,c)}\right)}{15\left(a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(5*I*d*x + 5*I*c) +
 20*I*e^(3*I*d*x + 3*I*c) + 15*I*e^(I*d*x + I*c))/(a^3*d*e^(4*I*d*x + 4*I*
c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.364.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^6(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2i\left(3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-20\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+60\,\sqrt{i\,a\tan(dx+c)+a}a^2\right)}{15\,a^5d}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/15*I*(3*(I*a*tan(d*x + c) + a)^(5/2) - 20*(I*a*tan(d*x + c) + a)^(3/2)*
a + 60*sqrt(I*a*tan(d*x + c) + a)*a^2)/(a^5*d)
```

3.364.8 Giac [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^6}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.364.9 Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.80

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{4\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,321\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,132\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,23\mathrm{i}+35\right.}{15\,a^3\,d\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\right.\right.}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
-(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2)*(cos(2*c + 2*d*x)*321i + cos(4*c + 4*d*x)*132i + cos(6*c + 6*d*x
)*23i + 35*sin(2*c + 2*d*x) + 28*sin(4*c + 4*d*x) + 7*sin(6*c + 6*d*x) + 2
12i))/(15*a^3*d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*
x) + 10))
```

3.365 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.365.1 Optimal result . . . 2590
3.365.2 Mathematica [A] (verified) . . . 2590
3.365.3 Rubi [A] (verified) . . . 2591
3.365.4 Maple [A] (verified) . . . 2592
3.365.5 Fricas [A] (verification not implemented) . . . 2593
3.365.6 Sympy [F] . . . 2593
3.365.7 Maxima [A] (verification not implemented) . . . 2593
3.365.8 Giac [F] . . . 2594
3.365.9 Mupad [B] (verification not implemented) . . . 2594

3.365.1 Optimal result

Integrand size = 26, antiderivative size = 55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

output
```
4*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+2*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

3.365.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{6i-2\tan(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(6*I - 2*Tan[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.365.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^{3/2}}-\frac{1}{\sqrt{i\tan(c+dx)a+a}}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{4a}{\sqrt{a+ia\tan(c+dx)}}-2\sqrt{a+ia\tan(c+dx)}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*((-4*a)/Sqrt[a + I*a*Tan[c + d*x]] - 2*Sqrt[a + I*a*Tan[c + d*x]]))/
(a^3*d)
```

3.365.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.365.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{2a}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{3}}$	42
default	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{2a}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{3}}$	42

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*((a+I*a*tan(d*x+c))^(1/2)+2*a/(a+I*a*tan(d*x+c))^(1/2))
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{(-i\,dx-i\,c)}}{a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(2*I*d*x + 2*I*c) - I
)*e^(-I*d*x - I*c)/(a^3*d)
```

3.365.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\left(\frac{\sqrt{i\,a\tan(dx+c)+a}}{a^2}+\frac{2}{\sqrt{i\,a\tan(dx+c)+a}\,a}\right)}{ad}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*I*(sqrt(I*a*tan(d*x + c) + a)/a^2 + 2/(sqrt(I*a*tan(d*x + c) + a)*a))/(a
*d)
```

3.365.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output

```
integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.365.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+2\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}{a^3\,d}$$

input

```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output

```
(2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 2i)*((a*(cos(2*c + 2*d*x) + s
in(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(a^3*d)
```

3.366 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.366.1 Optimal result . . . 2595
3.366.2 Mathematica [A] (verified) . . . 2595
3.366.3 Rubi [A] (verified) . . . 2596
3.366.4 Maple [A] (verified) . . . 2597
3.366.5 Fricas [B] (verification not implemented) . . . 2597
3.366.6 Sympy [F] . . . 2598
3.366.7 Maxima [A] (verification not implemented) . . . 2598
3.366.8 Giac [F] . . . 2598
3.366.9 Mupad [B] (verification not implemented) . . . 2599

3.366.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.366.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

3.366.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

3.366.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.366.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24
default	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.366.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^3d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 2*I
*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

3.366.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/3*I/((I*a*tan(d*x + c) + a)^(3/2)*a*d)
```

3.366.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.366.9 Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\mathrm{i}}{3\,a\,d\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
2i/(3*a*d*(a + a*tan(c + d*x)*1i)^(3/2))
```

3.367 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.367.1 Optimal result 2600
3.367.2 Mathematica [C] (verified) 2601
3.367.3 Rubi [A] (warning: unable to verify) 2601
3.367.4 Maple [B] (verified) 2605
3.367.5 Fricas [B] (verification not implemented) 2606
3.367.6 Sympy [F] 2606
3.367.7 Maxima [A] (verification not implemented) 2607
3.367.8 Giac [F] 2607
3.367.9 Mupad [F(-1)] 2607

3.367.1 Optimal result

Integrand size = 26, antiderivative size = 204

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{9i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}+\frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} + \frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}}+\frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-9/64*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^
(1/2)+9/32*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+9/28*I*a/d/(a+I*a*tan(d*x+c))^
(7/2)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+9/40*I/d/(a+
I*a*tan(d*x+c))^(5/2)+3/16*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.367.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.25

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{7}{2},2,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{14d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/14)*a*Hypergeometric2F1[-7/2, 2, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(a
+ I*a*Tan[c + d*x])^(7/2))
```

3.367.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3968, 52, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{9\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\right)}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\,t}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{}{2a(}\right.}{d}$$

$$\Big\downarrow \; 61$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+}\right.}{4a}\right.}{d}$$

3.367. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \textcolor{blue}{73}$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\right.}{d}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$-\frac{ia^3\left(\frac{9\left(\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}\right)}{4a}+\frac{\ }{2a(a-ia\tan}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (
9*(-1/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d
*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[
a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d
*x]]))/(2*a))/(2*a))/(2*a)))/(4*a)))/d
```

3.367. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.367.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.367. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.367.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 779 vs. 2(160) = 320.

Time = 10.47 (sec) , antiderivative size = 780, normalized size of antiderivative = 3.82

method	result
default	$2184i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+720\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+630i\arctan\left(\frac{\cos(dx+c)+1+i\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+720$

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2240/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x
+c)))^(1/2)/(1+I*tan(d*x+c))^2/a^2*(2184*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+720*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+630*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))+720*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)*sin(d*x+c)-400*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2184*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1260*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x
+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-1680*si
n(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630*I*sec(d*x+c)^2*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)-945*I*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*
x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-630*tan(d*x+c)*ar
ctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))-1680*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630*I*sec
(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1260*I*cos(d*x+c)*arctan(1/2*(c
os(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))+315*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d
*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-400*I*cos(d*x+c)^3*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-315*I*sec(d*x+c)^2*arctan(1/2*(cos(d*x+c)+1+I*sin
(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

3.367.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(147) = 294$.

Time = 0.25 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.50

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{}{e^{(2i\,dx}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/2240*(-315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(4
*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
 315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(
2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-35*I*e^(10*I*d*x + 10*I*c) + 353*I*e
^(8*I*d*x + 8*I*c) + 544*I*e^(6*I*d*x + 6*I*c) + 214*I*e^(4*I*d*x + 4*I*c)
 + 68*I*e^(2*I*d*x + 2*I*c) + 10*I))*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

3.367.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.367.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\left(\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4-420\,(i\,a\tan(dx+c)+a)^3a-168\,(i\,a\tan(dx+c)+a)^2a^2-144\,(i\,a\tan(dx+c\right.}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a-2\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2}\right.}{4480\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/4480*I*(4*(315*(I*a*tan(d*x + c) + a)^4 - 420*(I*a*tan(d*x + c) + a)^3*a
 - 168*(I*a*tan(d*x + c) + a)^2*a^2 - 144*(I*a*tan(d*x + c) + a)*a^3 - 160
*a^4)/((I*a*tan(d*x + c) + a)^(9/2)*a - 2*(I*a*tan(d*x + c) + a)^(7/2)*a^2
) + 315*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(
2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2))/(a*d)
```

3.367.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^2}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.368 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.368.1 Optimal result 2608
3.368.2 Mathematica [C] (verified) 2609
3.368.3 Rubi [A] (warning: unable to verify) 2609
3.368.4 Maple [B] (verified) 2617
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3.368.7 Maxima [A] (verification not implemented) 2619
3.368.8 Giac [F] 2619
3.368.9 Mupad [F(-1)] 2619

3.368.1 Optimal result

Integrand size = 26, antiderivative size = 277

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{143i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d}\\ &\quad+\frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\\ &\quad-\frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\\ &\quad+\frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}}+\frac{143i}{640d(a+ia\tan(c+dx))^{5/2}}\\ &\quad+\frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}}+\frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output

```
-143/1024*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/
d*2^(1/2)+143/512*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+143/288*I*a^2/d/(a+I*a*
tan(d*x+c))^(9/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(9/2
)-13/16*I*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+143/448*I*a/d/
(a+I*a*tan(d*x+c))^(7/2)+143/640*I/d/(a+I*a*tan(d*x+c))^(5/2)+143/768*I/a/
d/(a+I*a*tan(d*x+c))^(3/2)
```

3.368.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{9}{2},3,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{36d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/36)*a^2*Hypergeometric2F1[-9/2, 3, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(9/2))
```

3.368.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan}\right.}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{8a}+\frac{}{4a(a-}\right.}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\,\mathrm{ta}}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \text{61}$$

3.368. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\right.}{8a}\right.}{d}$$

$$\downarrow 61$$

3.368. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{}{9a}\right.}{4a}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \textcolor{blue}{61}$$

3.368. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{}{7a(a+ia\,\mathrm{ta}}}{2a}\right.}{4a}\right.}{8a}\right.}{d}$$

$$\Big\downarrow 73$$

3.368. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}\right.}{4a}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \text{219}$$

3.368. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$-\frac{ia^5\left(\frac{13\left(\frac{11\left(\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan}\right.}{4a}\right.}{8a}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) +
 (13*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (11*(-
1/9*1/(a*(a + I*a*Tan[c + d*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c + d*x])
^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*T
an[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]
*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2*a)))
/(4*a)))/(8*a)))/d
```

3.368. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.368.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.368. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.368.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(220) = 440$.

Time = 9.16 (sec) , antiderivative size = 914, normalized size of antiderivative = 3.30

method	result	size
default	Expression too large to display	914

input

```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/322560/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(1+I*tan(d*x+
c))^2/(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(312312*I*cos(d*x+c)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+90090*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+58240*sin(d*x+c)*cos(d*x+c)^4*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-57200*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+58240*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)+312312*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+102960*sin(d*x+c)*cos(d
*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-22400*I*cos(d*x+c)^5*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-90090*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)+102960*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-135
135*I*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))-22400*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-180180*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-240240*sin(d*x+c)*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)-90090*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)+180180*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*
x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-90090*tan(d*x+c)*arctan(1/2*(c
os(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))-240240*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-57200*I*cos(d*x+c)
^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-45045*I*sec(d*x+c)^2*arctan(1/2*(...
```

3.368.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.18

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/322560*(-45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*l
og(4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)) + 45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*
(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-630*I*e^(14*I*d*x + 14*I*c) -
8505*I*e^(12*I*d*x + 12*I*c) + 42709*I*e^(10*I*d*x + 10*I*c) + 69392*I*e^(
8*I*d*x + 8*I*c) + 26752*I*e^(6*I*d*x + 6*I*c) + 10144*I*e^(4*I*d*x + 4*I*
c) + 2480*I*e^(2*I*d*x + 2*I*c) + 280*I))*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

3.368.6 Sympy [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos^4\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^6-150150\,(i\,a\tan(dx+c)+a)^5a+96096\,(i\,a\tan(dx+c)+a)^4a^2+27456\,(i\right.}{(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a-4\,(i\,a\tan(dx+c)+}\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/645120*I*(4*(45045*(I*a*tan(d*x + c) + a)^6 - 150150*(I*a*tan(d*x + c) +
 a)^5*a + 96096*(I*a*tan(d*x + c) + a)^4*a^2 + 27456*(I*a*tan(d*x + c) + a
)^3*a^3 + 18304*(I*a*tan(d*x + c) + a)^2*a^4 + 16640*(I*a*tan(d*x + c) + a
)*a^5 + 17920*a^6)/((I*a*tan(d*x + c) + a)^(13/2)*a - 4*(I*a*tan(d*x + c)
+ a)^(11/2)*a^2 + 4*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 45045*sqrt(2)*log(
-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*
a*tan(d*x + c) + a)))/a^(3/2))/(a*d)
```

3.368.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^4}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^4}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.369 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.369.1 Optimal result 2620
3.369.2 Mathematica [A] (verified) 2620
3.369.3 Rubi [A] (verified) 2621
3.369.4 Maple [F(-1)] 2623
3.369.5 Fricas [A] (verification not implemented) 2623
3.369.6 Sympy [F(-1)] 2624
3.369.7 Maxima [B] (verification not implemented) 2624
3.369.8 Giac [F] 2625
3.369.9 Mupad [B] (verification not implemented) 2626

3.369.1 Optimal result

Integrand size = 26, antiderivative size = 147

$$\begin{aligned}\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}} \\ &+ \frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}} \\ &+ \frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}\end{aligned}$$

output
```
256/20995*I*a^4*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+64/1615*I*a^3*se
c(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+24/323*I*a^2*sec(d*x+c)^13/d/(a+I*
a*tan(d*x+c))^(9/2)+2/19*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(7/2)
```

3.369.2 Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^{12}(c+dx)(798\cos(c+dx)+1631\cos(3(c+dx))+13i(38\sin(c+dx)+}{20995a^2d(-i+\tan(c+dx))^2\sqrt{a}}$$

input
```
Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^12*(798*Cos[c + d*x] + 1631*Cos[3*(c + d*x)] + (13*I)*(38*Si
n[c + d*x] + 123*Sin[3*(c + d*x)]))*((-2*I)*Cos[4*(c + d*x)] - 2*Sin[4*(c
+ d*x)]))/(20995*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.369.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{13}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{19}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{19}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\left(\frac{4}{15}a\int\frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\left(\frac{4}{15}a\int\frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{19}a\left(\frac{8}{17}a\left(\frac{8ia^2\sec^{13}(c+dx)}{195d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/19)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (12*a*((
((2*I)/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (8*a*((((
8*I)/195)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((2*I)
/15)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2))))/17))/19
```

3.369.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.369. $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.369.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x)
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.35

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{1024\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1615i\,e^{(6i\,dx+6i\,c)}-646i\,e^{(4i\,d}\right.}{20995\left(a^3de^{(18i\,dx+18i\,c)}+9\,a^3de^{(16i\,dx+16i\,c)}+36\,a^3de^{(14i\,dx+14i\,c)}+84\,a^3de^{(12i\,dx+12i\,c)}+126\,a^3de^{(10i\,dx+10i\,c)}\right.}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1024/20995*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1615*I*e^(6*I*d*x
+ 6*I*c) - 646*I*e^(4*I*d*x + 4*I*c) - 152*I*e^(2*I*d*x + 2*I*c) - 16*I)/(
a^3*d*e^(18*I*d*x + 18*I*c) + 9*a^3*d*e^(16*I*d*x + 16*I*c) + 36*a^3*d*e^(
14*I*d*x + 14*I*c) + 84*a^3*d*e^(12*I*d*x + 12*I*c) + 126*a^3*d*e^(10*I*d*
x + 10*I*c) + 126*a^3*d*e^(8*I*d*x + 8*I*c) + 84*a^3*d*e^(6*I*d*x + 6*I*c)
 + 36*a^3*d*e^(4*I*d*x + 4*I*c) + 9*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.369.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(115) = 230$.

Time = 1.05 (sec) , antiderivative size = 902, normalized size of antiderivative = 6.14

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/20995*(-2429*I*sqrt(a) - 8850*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
 5122*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 45190*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 - 12924*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 152478*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 40470*I*s
qrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 397594*sqrt(a)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 50065*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 - 722228*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 19380*I*sqrt(a)*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10 - 936700*sqrt(a)*sin(d*x + c)^11/(cos
(d*x + c) + 1)^11 - 936700*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 +
 19380*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 722228*sqrt(a)*si
n(d*x + c)^15/(cos(d*x + c) + 1)^15 + 50065*I*sqrt(a)*sin(d*x + c)^16/(cos
(d*x + c) + 1)^16 - 397594*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 +
 40470*I*sqrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 152478*sqrt(a)*si
n(d*x + c)^19/(cos(d*x + c) + 1)^19 + 12924*I*sqrt(a)*sin(d*x + c)^20/(cos
(d*x + c) + 1)^20 - 45190*sqrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 +
5122*I*sqrt(a)*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 - 8850*sqrt(a)*sin(d*
x + c)^23/(cos(d*x + c) + 1)^23 + 2429*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x
+ c) + 1)^24)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(c
os(d*x + c) + 1) - 1)^(5/2)/((a^3 - 12*a^3*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 66*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 220*a^3*sin(d*x + c...
```

3.369.8 Giac [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{13}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{13\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

$$-\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{5\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^7} + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3072\mathrm{i}}{17\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^8}$$

$$-\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{19\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^9}$$

input
```
int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
(exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*1024i)/(13*a^3*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(-
 c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*1024i)/(5*a^3*d*(exp(c*2i + d*x*2i) + 1)^7) + (exp(- c*1i -
 d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*3072i)/(17*a^3*d*(exp(c*2i + d*x*2i) + 1)^8) - (exp(- c*1i - d*x*1
i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*1024i)/(19*a^3*d*(exp(c*2i + d*x*2i) + 1)^9)
```

3.370 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.370.1 Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}} + \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

output
```
64/2145*I*a^3*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+16/195*I*a^2*sec(d
*x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+2/15*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*
x+c))^(7/2)
```

3.370.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^{10}(c+dx)(60+203\cos(2(c+dx))+187i\sin(2(c+dx)))(-2i\cos(3(c+}{2145a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(Sec[c + d*x]^10*(60 + 203*Cos[2*(c + d*x)] + (187*I)*Sin[2*(c + d*x)])*((
-2*I)*Cos[3*(c + d*x)] - 2*Sin[3*(c + d*x)]))/(2145*a^2*d*(-I + Tan[c + d*
x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.370.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{15}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{15}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{15}a\left(\frac{4}{13}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{15}a\left(\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\right)+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

input

```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

3.370. $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$

output
```
(((2*I)/15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (8*a*(((
(8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I
)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))))/15
```

3.370.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.370.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x)
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.44

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(4i\,dx+4i\,c)}-60i\,e^{(2i\,dx+2i\,c)}-8i\right)}{2145\left(a^3de^{(14i\,dx+14i\,c)}+7\,a^3de^{(12i\,dx+12i\,c)}+21\,a^3de^{(10i\,dx+10i\,c)}+35\,a^3de^{(8i\,dx+8i\,c)}+35\,a^3de^{(6i\,dx+6i\,c)}+21\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-256/2145*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-195*I*e^(4*I*d*x + 4
*I*c) - 60*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^3*d*e^(14*I*d*x + 14*I*c) + 7*a
^3*d*e^(12*I*d*x + 12*I*c) + 21*a^3*d*e^(10*I*d*x + 10*I*c) + 35*a^3*d*e^(
8*I*d*x + 8*I*c) + 35*a^3*d*e^(6*I*d*x + 6*I*c) + 21*a^3*d*e^(4*I*d*x + 4*
I*c) + 7*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.370.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.370.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(86) = 172.

Time = 0.48 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.95

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output

```
-2/2145*(-263*I*sqrt(a) - 830*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 76
0*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4270*sqrt(a)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 - 1085*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 - 11576*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2000*I*sqrt(a)*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 - 23000*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 2470*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 33540*
sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 33540*sqrt(a)*sin(d*x + c)^1
1/(cos(d*x + c) + 1)^11 + 2470*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1
)^12 - 23000*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2000*I*sqrt(a
)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 11576*sqrt(a)*sin(d*x + c)^15/(c
os(d*x + c) + 1)^15 + 1085*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 - 4270*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 760*I*sqrt(a)*sin(
d*x + c)^18/(cos(d*x + c) + 1)^18 - 830*sqrt(a)*sin(d*x + c)^19/(cos(d*x +
 c) + 1)^19 + 263*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
^(5/2)/((a^3 - 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^3*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 120*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 210*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^3*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + 210*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120
*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a^3*sin(d*x + c)^16/(co...
```

3.370.8 Giac [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{11}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output

```
integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{256\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,60\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,195\mathrm{i}+8\mathrm{i}\right)}{2145\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
(256*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*60i + exp(c*4i + d*x*4i)*195i
 + 8i))/(2145*a^3*d*(exp(c*2i + d*x*2i) + 1)^7)
```

3.371 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.371.1 Optimal result 2633
3.371.2 Mathematica [A] (verified) 2633
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3.371.1 Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

output
```
8/99*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+2/11*I*a*sec(d*x+c)^9/d
/(a+I*a*tan(d*x+c))^(7/2)
```

3.371.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2\sec^7(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-13i+9\tan(c+dx))}{99a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*Sec[c + d*x]^7*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-13*I + 9*Tan[c
 + d*x]))/(99*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.371.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4}{11}a\int \frac{\sec^9(c+dx)}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{11}a\int \frac{\sec(c+dx)^9}{(i\tan(c+dx)a+a)^{7/2}}dx+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)
/11)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

3.371.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.371.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^9(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x)
```

3.371.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(2i\,dx+2i\,c)}-2i\right)}{99\left(a^3de^{(10i\,dx+10i\,c)}+5\,a^3de^{(8i\,dx+8i\,c)}+10\,a^3de^{(6i\,dx+6i\,c)}+10\,a^3de^{(4i\,dx+4i\,c)}+5\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-64/99*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(2*I*d*x + 2*I*c
) - 2*I)/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a
^3*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d
*x + 2*I*c) + a^3*d)
```

3.371.6 Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^9\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**9/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.371.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(57) = 114$.

Time = 0.43 (sec) , antiderivative size = 626, normalized size of antiderivative = 8.58

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\,\Big(-13i\sqrt{a}-\frac{34\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{46i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{174\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{54i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{394\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{22i\sqrt{a}\sin}{(\cos(dx+}}{99\,\Big(a^3-\frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/99*(-13*I*sqrt(a) - 34*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 46*I*s
qrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 174*sqrt(a)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 - 54*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 39
4*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 22*I*sqrt(a)*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - 550*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 550*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 22*I*sqrt(a)*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 - 394*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 + 54*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 174*sqrt(a)*s
in(d*x + c)^13/(cos(d*x + c) + 1)^13 + 46*I*sqrt(a)*sin(d*x + c)^14/(cos(d
*x + c) + 1)^14 - 34*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 13*I*
sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 8*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 2
8*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(d
*x + c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2
))
```

3.371.8 Giac [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^9}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{64\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,11\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{99\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

input

```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output

```
(64*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*11i + 2i)*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(99*a^3*d*(exp(c*2i
 + d*x*2i) + 1)^5)
```

3.371. $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.372 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.372.1 Optimal result 2639
3.372.2 Mathematica [A] (verified) 2639
3.372.3 Rubi [A] (verified) 2640
3.372.4 Maple [F(-1)] 2641
3.372.5 Fricas [B] (verification not implemented) 2641
3.372.6 Sympy [F] 2641
3.372.7 Maxima [B] (verification not implemented) 2642
3.372.8 Giac [F] 2642
3.372.9 Mupad [B] (verification not implemented) 2643

3.372.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

output
```
2/7*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)
```

3.372.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sec^5(c+dx)(i+\tan(c+dx))}{7a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*Sec[c + d*x]^5*(I + Tan[c + d*x]))/(7*a^2*d*(-I + Tan[c + d*x])^2*Sqrt
[a + I*a*Tan[c + d*x]])
```

3.372.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/7)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

3.372.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.372.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^7(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x)
```

3.372.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(27) = 54$.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{16i\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{7\,(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(2i\,dx+2i\,c)}+a^3d)}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
16/7*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^3*d*e^(6*I*d*x + 6*I*c
) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.372.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^7(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.372.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(27) = 54$.

Time = 0.38 (sec) , antiderivative size = 488, normalized size of antiderivative = 13.94

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{20\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right.}{7\left(a^3-\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(d}{(\cos(dx+c}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/7*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*sqrt(a)*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 5*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*sqrt(a)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*sqrt(a)*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 + 5*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10*sqrt(a)
*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4*I*sqrt(a)*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 - 2*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + I*sqrt(
a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 6*a^3*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 20*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 - 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin
(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

3.372.8 Giac [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^7}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.372.9 Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\mathrm{e}^{-c\,4\mathrm{i}-d\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}}\,2\mathrm{i}}{7\,a^3\,d\cos\left(c+d\,x\right)^3}$$

input

```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output

```
(exp(- c*4i - d*x*4i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(7*
a^3*d*cos(c + d*x)^3)
```

3.373 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.373.2 Mathematica [A] (verified) 2644
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3.373.1 Optimal result

Integrand size = 26, antiderivative size = 123

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
4*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/
2)/a^(5/2)/d-4*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*sec(d*x+c
)^3/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.373.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sec(c+dx)\left(7i-6i\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)+\tan(c+dx)\right)}{3a^2d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*Sec[c + d*x]*(7*I - (6*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]] + Tan[c + d*x]))/(3*a^2*d*Sqrt[a + I*a*Tan[c + d*
x]])
```

3.373.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3972, 3042, 3972, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3972}$$

$$\frac{2\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3972}$$

$$\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{2\left(\frac{4i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{ad}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow 219$$

$$\frac{2\left(\frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((2*I
)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]]))
)/a
```

3.373.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3972
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Simp[2*(d^2/a)   Int[(d*Sec[e + f*x])^
(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

3.373.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(102) = 204$.

Time = 9.84 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.18

method	result
default	$\frac{2(-\csc(dx+c)+\cot(dx+c)+i)^5\left(6\sqrt{2}\,\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1\right)^{\frac{3}{2}}-7i(\csc^3(d}{3d\left(-\frac{a\left(2i(\csc(dx+c)-\cot(dx+c))-\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1\right)}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}\right)^{\frac{5}{2}}\left(\left(\csc^2\right.\right.}$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(-csc(d*x+c)+cot(d*x+c)+I)^5*(6*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-co
t(d*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^
2*(1-cos(d*x+c))^2-1)^(3/2)-7*I*csc(d*x+c)^3*(1-cos(d*x+c))^3+9*I*(csc(d*x
+c)-cot(d*x+c))+9*csc(d*x+c)^2*(1-cos(d*x+c))^2-7)/(-a*(2*I*(csc(d*x+c)-co
t(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-
1))^(5/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^4
```

3.373.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(96) = 192$.

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.20

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(3\sqrt{2}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(-\frac{16\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{a^2d}\right)+3\sqrt{}\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(1/(a^5*d^2))*
log(-16*((I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*sqrt(2)*(-I*
a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(1/(a^5*d^2))*log(-16*((-I*a^2*d*
e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a
^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(3*I*e^(2*I*d*x + 2*I*c) + 4*I))/(a^3*d*e^(2*I*d*x + 2*I*c) +
a^3*d)
```

3.373.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.373.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1074 vs. 2(96) = 192.

Time = 0.48 (sec) , antiderivative size = 1074, normalized size of antiderivative = 8.73

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output

```
1/3*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((-3*I*sqrt(2)*cos(2*d*x + 2*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - 4*I*s
qrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*sqrt
(2)*cos(2*d*x + 2*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2))*sin(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - 3*(2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*(sqrt
(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - (I
*sqrt(2)*cos(2*d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*I*sqrt(2)*c
os(2*d*x + 2*c) + I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^...
```

3.373.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output

```
integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.374 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.374.1 Optimal result 2651
3.374.2 Mathematica [A] (verified) 2651
3.374.3 Rubi [A] (verified) 2652
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3.374.5 Fricas [B] (verification not implemented) 2655
3.374.6 Sympy [F] 2655
3.374.7 Maxima [B] (verification not implemented) 2656
3.374.8 Giac [F] 2656
3.374.9 Mupad [F(-1)] 2657

3.374.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=-\frac{i\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}+\frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
-1/2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(5/2)/d*2^(1/2)+I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.374.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.73

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{ie^{-\frac{1}{2}i(2c+dx)}\Big(-1-e^{2i(c+dx)}+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\mathrm{arctanh}\Big(\sqrt{1+e^{2i(c+dx)}}\Big)}{2a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/2)*(-1 - E^((2*I)*(c + d*x)) + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^3*(Cos[c +
 (d*x)/2] + I*Sin[c + (d*x)/2]))/(a^2*d*E^((I/2)*(2*c + d*x))*(-I + Tan[c
+ d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.374.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.47, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 3982, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{3983}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{3970}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}$$

input

```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output

```
((2*I)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*(((I/2)*ArcTa
nh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*
a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/a
```

3.374.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3982

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

3.374. $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.374.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(71) = 142$.

Time = 9.18 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.60

method	result
default	$\frac{\Bigg(\sqrt{2}\,\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)+2i\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\sqrt{2}\,(\csc(dx+c)-\cot(dx+c))-(\csc^2}{2d\Big((\csc^2(dx+c}$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c
)^2*(1-cos(d*x+c))^2-1)^(1/2))+2*I*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1
)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*2^(1/2)*(csc(d*x+c)-cot
(d*x+c))-csc(d*x+c)^2*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(cs
c(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-2*(csc(d*x+
c)^2*(1-cos(d*x+c))^2-1)^(1/2)+2*I*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2)
*(csc(d*x+c)-cot(d*x+c)))*(-csc(d*x+c)+cot(d*x+c)+I)^3/(csc(d*x+c)^2*(1-co
s(d*x+c))^2-1)^(5/2)/(-a*(2*I*(csc(d*x+c)-cot(d*x+c))-csc(d*x+c)^2*(1-cos(
d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(5/2)
```

3.374.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(67) = 134$.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.85

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(i\sqrt{2}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{2\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e}{a^2d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/4*(I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log(2*((I*a^2*d
*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(
a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) - I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2
))*e^(2*I*d*x + 2*I*c)*log(2*((-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^
2*d)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c
) - I))*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

3.374.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.374.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(67) = 134$.

Time = 0.70 (sec) , antiderivative size = 827, normalized size of antiderivative = 9.62

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output

```
-1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(2*d*x + 2*c
) - I*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
- 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*...
```

3.374.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.375 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.375.1 Optimal result 2658
3.375.2 Mathematica [A] (verified) 2658
3.375.3 Rubi [A] (verified) 2659
3.375.4 Maple [B] (verified) 2661
3.375.5 Fricas [B] (verification not implemented) 2661
3.375.6 Sympy [F] 2662
3.375.7 Maxima [F] 2662
3.375.8 Giac [F] 2663
3.375.9 Mupad [F(-1)] 2663

3.375.1 Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{3i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/32*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(5/2)/d*2^(1/2)+1/4*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(5/2)+3/16*I*sec(d*x
+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.375.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\sec^3(c+dx)\Big(7+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)+7\cos(2(c+dx))+3i\sin(2(c+dx))}{32a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/32*I)*Sec[c + d*x]^3*(7 + 3*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] + 7*Cos[2*(c + d*x)] + (3*
I)*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*
x]])
```

3.375.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3983, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3970}$$

$$\frac{3\left(\frac{i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow 219$$

$$\frac{3\left(\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/4)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (3*(((I/2)*ArcTanh
[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^
(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/(8*a)
```

3.375.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.375. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.375.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(97) = 194$.

Time = 9.23 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.97

method	result
default	$-\frac{i\Bigg(12i\arctan\Bigg(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\Bigg)\sin(dx+c)+6i\tan(dx+c)\arctan\Bigg(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\Bigg)+6i\tan(dx+c)}{}$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*I/d/(tan(d*x+c)-I)^2/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)/a^2/(cos(d*x+c)+1)*(12*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c
)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+6*I*tan
(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))+6*I*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
12*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))-3*I*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(I*sin(
d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*
I*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*arctan(1/2*(I
*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))+14*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-9*sec(d*x+c)*arctan(1/2*(I*sin(d*
x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+14*s
ec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*sec(d*x+c)^2*arctan(1/2*(I*
sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
))
```

3.375.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(91) = 182$.

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.19

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\Bigg(-3i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(4i\,dx+4i\,c)}\log\Bigg(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right.}{8\,a^2d}}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(-3*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*
(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*I*
sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*(sqrt(2)*sq
rt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(2)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(4*I*d*x + 4*I*c) + 7*I*e^(2*I*d*x + 2*
I*c) + 2*I))*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

3.375.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.375.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec{(dx+c)}}{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.375.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.376 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.376.1 Optimal result . . . 2664
3.376.2 Mathematica [A] (verified) . . . 2664
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3.376.4 Maple [B] (verified) . . . 2669
3.376.5 Fricas [A] (verification not implemented) . . . 2670
3.376.6 Sympy [F] . . . 2670
3.376.7 Maxima [B] (verification not implemented) . . . 2671
3.376.8 Giac [F] . . . 2671
3.376.9 Mupad [F(-1)] . . . 2672

3.376.1 Optimal result

Integrand size = 24, antiderivative size = 192

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{35i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d}$$

output
```
35/256*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/
a^(5/2)/d*2^(1/2)+35/192*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-35/12
8*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+1/6*I*cos(d*x+c)/d/(a+I*a*ta
n(d*x+c))^(5/2)+7/48*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.376.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.74

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec^3(c+dx)\left(-125-105e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-\right.}{768a^2d(-i+\tan(c+}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/768)*Sec[c + d*x]^3*(-125 - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*
(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 85*Cos[2*(c + d*x)] +
 40*Cos[4*(c + d*x)] + (7*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/
(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.376.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{5\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3970}$$

3.376. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow 219$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/6)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(((I/4)*Cos[c +
 d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (5*(((I/2)*Cos[c + d*x])/(d*Sqrt
[a + I*a*Tan[c + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(S
qrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a
+ I*a*Tan[c + d*x]])/d))/(4*a)))/(8*a)))/(12*a)
```

3.376. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.376.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.376. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.376.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(155) = 310$.

Time = 10.36 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.80

method	result
default	$-\frac{320i\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+320i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-420i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-4}{}$

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/768/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x
+c)))^(1/2)/(1+I*tan(d*x+c))^2/a^2*(320*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+320*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-420*I*
cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))-448*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)*sin(d*x+c)-490*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-210*I*arctan(1/2*(I
*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))-448*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+420*arctan(1/2*(I*sin
(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*s
in(d*x+c)-490*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+315*I*sec(d*
x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))+210*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+210
*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+105*I*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-co
s(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+210*tan(d*x
+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*tan(d*x+c)*sec(d*x+c
)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)))
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.51

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-105i\,\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i}}}\right.}{64\,a^2d}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/768*(-105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-3
5/64*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) +
 105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-35/64*(s
qrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(
2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-48*I*e^(8*I*d*x + 8*I*c) + 39*I*e^(
6*I*d*x + 6*I*c) + 125*I*e^(4*I*d*x + 4*I*c) + 46*I*e^(2*I*d*x + 2*I*c) +
8*I))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

3.376.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.376.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(145) = 290$.

Time = 0.51 (sec) , antiderivative size = 2297, normalized size of antiderivative = 11.96

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/3072*(544*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(
1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(6*d*x + 6*c)
 - sqrt(2)*sin(6*d*x + 6*c))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
))) + 1)) + (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(3/
2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*ar
ctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1/3*ar
ctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x +
 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*
x + 6*c))) + 1)^(1/4)*(29*((I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x
 + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (I*sqrt(
2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x
+ 6*c), cos(6*d*x + 6*c)))^2 + 2*(I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin
(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + I*sq
rt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2(sin(1/3
*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x +
 6*c), cos(6*d*x + 6*c))) + 1)) + (19*I*sqrt(2)*cos(6*d*x + 6*c) + 19*sqrt
(2)*sin(6*d*x + 6*c) - 16*I*sqrt(2))*cos(1/2*arctan2(sin(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(...
```

3.376.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.377 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.377.1 Optimal result 2673
3.377.2 Mathematica [A] (verified) 2674
3.377.3 Rubi [A] (verified) 2674
3.377.4 Maple [B] (verified) 2680
3.377.5 Fricas [A] (verification not implemented) 2681
3.377.6 Sympy [F] 2682
3.377.7 Maxima [B] (verification not implemented) 2682
3.377.8 Giac [F] 2683
3.377.9 Mupad [F(-1)] 2684

3.377.1 Optimal result

Integrand size = 26, antiderivative size = 270

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{1155i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} \\ &+ \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} \\ &+ \frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d} - \frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}\end{aligned}$$

output
```
1155/8192*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)+385/2048*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+
33/256*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1155/4096*I*cos(d*x+c
)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d-77/512*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^
(1/2)/a^3/d+1/8*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I*cos(d*x+
c)^3/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.377.2 Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.61

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec^3(c+dx)\left(-3325-3465e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right.\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/24576)*Sec[c + d*x]^3*(-3325 - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2
*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 1605*Cos[2*(c + d
*x)] + 1800*Cos[4*(c + d*x)] + 80*Cos[6*(c + d*x)] + (1111*I)*Sin[2*(c + d
*x)] + (2552*I)*Sin[4*(c + d*x)] + (176*I)*Sin[6*(c + d*x)]))/(a^2*d*(-I +
 Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.377.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{11\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{11\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{3\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{3\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{3\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{3\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

3.377. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3971}$$

3.377. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx}}\right.}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx}}\right.}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3970}$$

3.377. $\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow 219$$

3.377. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{\frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}\right)}{}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{}{4d\sqrt{}}\right.}{4a}\right.}{16a}$$

$$\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/8)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (11*(((I/6)*Cos[
c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (3*(((I/4)*Cos[c + d*x]^3)/
(d*Sqrt[a + I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a
*Tan[c + d*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x
]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*
Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d))/(4*a)))/6))/(8*a)))/(4*a)))/(16*a)
```

3.377.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.377.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(221) = 442$.

Time $= 9.11$ (sec) , antiderivative size $= 864$, normalized size of antiderivative $= 3.20$

method	result	size
default	Expression too large to display	864

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/24576/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*
x+c)))^(1/2)/(1+I*tan(d*x+c))^2/a^2*(-10560*I*cos(d*x+c)*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)+5632*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)-10560*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5632*sin(d
*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+13860*I*cos(d*x+c)*a
rctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+14784*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x
+c)+16170*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2560*I*cos(d*x+c)^4*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)+14784*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)-13860*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+6930*I*arctan(1/2*(I*sin(d*x+c)-cos
(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+16170*I*sec(
d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-6930*tan(d*x+c)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-6930*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2560*I*cos(d*x+c)^3*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-10395*I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c
)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-6930*ta
n(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3465*tan(d*x+c)*sec
(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))-3465*I*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-co...
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.15

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-3465i\,\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(8i\,dx+8i\,c)}\log\left(-\frac{1155\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{}{e^{(2i\,dx}}}\right.}{2048\,a^2d}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
1/24576*(-3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log
(-1155/2048*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^
2*d)) + 3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log(-
1155/2048*(sqrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2
*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(12*I*d*x + 12*
I*c) - 2176*I*e^(10*I*d*x + 10*I*c) + 247*I*e^(8*I*d*x + 8*I*c) + 3325*I*e
^(6*I*d*x + 6*I*c) + 1358*I*e^(4*I*d*x + 4*I*c) + 376*I*e^(2*I*d*x + 2*I*c
) + 48*I))*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

3.377.6 Sympy [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos^3\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input

```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

output

```
Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.377.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3789 vs. 2(207) = 414.

Time = 0.60 (sec) , antiderivative size = 3789, normalized size of antiderivative = 14.03

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\text{Too large to display}$$

input

```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/98304*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(15*((-I*sqrt(2)*cos(8*d*x +
8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*s
in(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos(
8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c),
 cos(8*d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c
))*cos(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), c
os(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (55*I*sqrt(2)*
cos(8*d*x + 8*c) + 960*I*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c))) - 1296*I*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c))) + 55*sqrt(2)*sin(8*d*x + 8*c) + 960*sqrt(2)*sin(3/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))) - 1296*sqrt(2)*sin(1/2*arctan2(sin(8*d*x +
8*c), cos(8*d*x + 8*c))) + 128*I*sqrt(2))*cos(3/2*arctan2(sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c))) + 1)) + 15*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d
*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(
2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*
x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2...
```

3.377.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.378 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.378.1 Optimal result 2685
3.378.2 Mathematica [A] (verified) 2685
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3.378.1 Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d} -\frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8d}-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9d}$$

output
```
-32/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d+64/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6
/d-48/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d+16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a
^8/d-2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^9/d
```

3.378.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(5419-6396i\tan(c+dx)-4530}{3465a^4d}$$

input
```
Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(5419 - (6396*I)*Tan[c +
 d*x] - 4530*Tan[c + d*x]^2 + (1820*I)*Tan[c + d*x]^3 + 315*Tan[c + d*x]^4
))/(3465*a^4*d)
```

3.378.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{10}}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^4\sqrt{i\tan(c+dx)a+a}d(ia\tan(c+dx))}{a^9d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \Big((i\tan(c+dx)a+a)^{9/2}-8a(i\tan(c+dx)a+a)^{7/2}+24a^2(i\tan(c+dx)a+a)^{5/2}-32a^3(i\tan(c+dx)a+}{a^9d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(\frac{32}{3}a^4(a+ia\tan(c+dx))^{3/2}-\frac{64}{5}a^3(a+ia\tan(c+dx))^{5/2}+\frac{48}{7}a^2(a+ia\tan(c+dx))^{7/2}+\frac{2}{11}(a+ia\tan(c+}{a^9d}$$

input

```
Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
((-I)*((32*a^4*(a + I*a*Tan[c + d*x])^(3/2))/3 - (64*a^3*(a + I*a*Tan[c +
d*x])^(5/2))/5 + (48*a^2*(a + I*a*Tan[c + d*x])^(7/2))/7 - (16*a*(a + I*a*
Tan[c + d*x])^(9/2))/9 + (2*(a + I*a*Tan[c + d*x])^(11/2))/11))/(a^9*d)
```

3.378.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.378.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{8a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{24a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{16a^4(a+ia\tan(dx+c))}{3}\right.}{d\,a^9}$
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{8a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{24a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{16a^4(a+ia\tan(dx+c))}{3}\right.}{d\,a^9}$

input
```
int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^9*(-1/11*(a+I*a*tan(d*x+c))^(11/2)+8/9*a*(a+I*a*tan(d*x+c))^(9/2)-
24/7*a^2*(a+I*a*tan(d*x+c))^(7/2)+32/5*a^3*(a+I*a*tan(d*x+c))^(5/2)-16/3*a
^4*(a+I*a*tan(d*x+c))^(3/2))
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\bigl(128i\,e^{(11i\,dx+11i\,c)}+704i\,e^{(9i\,dx+9i\,c)}+1584i\,e^{(7i\,dx+7i\,c)}+1848i\,e^{(5i\,dx+5i\,c)}+1155i\,e^{(3i\,d}}{3465\,\bigl(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-64/3465*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(11*I*d*x + 11
*I*c) + 704*I*e^(9*I*d*x + 9*I*c) + 1584*I*e^(7*I*d*x + 7*I*c) + 1848*I*e^
(5*I*d*x + 5*I*c) + 1155*I*e^(3*I*d*x + 3*I*c))/(a^4*d*e^(10*I*d*x + 10*I*
c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*d*e^(6*I*d*x + 6*I*c) + 10*a^4*d
*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

3.378.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\,\Bigl(315\,(i\,a\tan{(dx+c)}+a)^{\frac{11}{2}}-3080\,(i\,a\tan{(dx+c)}+a)^{\frac{9}{2}}a+11880\,(i\,a\tan{(dx+c)}+a)^{\frac{7}{2}}a^2-22176}{3465\,a^9d}$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3465*I*(315*(I*a*tan(d*x + c) + a)^(11/2) - 3080*(I*a*tan(d*x + c) + a)
^(9/2)*a + 11880*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 22176*(I*a*tan(d*x + c
) + a)^(5/2)*a^3 + 18480*(I*a*tan(d*x + c) + a)^(3/2)*a^4)/(a^9*d)
```

3.378.8 Giac [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{10}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.378.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.53

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx&=-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{3465\,a^4\,d}\\&\quad-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{3465\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1155\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\&\quad-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{99\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
64i)/(11*a^4*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2i)*
1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(3465*a^4*d*(exp(c*2i
+ d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*1024i)/(1155*a^4*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(
693*a^4*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i -
1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*a^4*d*(exp(c*2i + d*x*2i)
 + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*8192i)/(3465*a^4*d)
```

3.378. $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.379 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.379.2 Mathematica [A] (verified) 2691
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3.379.1 Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=-\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d}-\frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d}$$

output
```
-16*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d+8*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d-12
/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d
```

3.379.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2\sqrt{a+ia\tan(c+dx)}(-177i-71\tan(c+dx)+27i\tan^2(c+dx)+5\tan^3(}{35a^4d}$$

input
```
Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(2*Sqrt[a + I*a*Tan[c + d*x]]*(-177*I - 71*Tan[c + d*x] + (27*I)*Tan[c + d
*x]^2 + 5*Tan[c + d*x]^3))/(35*a^4*d)
```

3.379.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^8}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^3}{\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{a^7d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{8a^3}{\sqrt{i\tan(c+dx)a+a}}-12\sqrt{i\tan(c+dx)a+a}a^2+6(i\tan(c+dx)a+a)^{3/2}a-(i\tan(c+dx)a+a)^{5/2}\right)d(ia\tan}{a^7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(16a^3\sqrt{a+ia\tan(c+dx)}-8a^2(a+ia\tan(c+dx))^{3/2}-\frac{2}{7}(a+ia\tan(c+dx))^{7/2}+\frac{12}{5}a(a+ia\tan(c+dx))^5\right.}{a^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*(16*a^3*Sqrt[a + I*a*Tan[c + d*x]] - 8*a^2*(a + I*a*Tan[c + d*x])^(3
/2) + (12*a*(a + I*a*Tan[c + d*x])^(5/2))/5 - (2*(a + I*a*Tan[c + d*x])^(7
/2))/7))/(a^7*d)
```

3.379.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.379.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}-8a^3\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}-8a^3\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^7}$	82

input
```
int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^7*(1/7*(a+I*a*tan(d*x+c))^(7/2)-6/5*a*(a+I*a*tan(d*x+c))^(5/2)+4*a
^2*(a+I*a*tan(d*x+c))^(3/2)-8*a^3*(a+I*a*tan(d*x+c))^(1/2))
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(7i\,dx+7i\,c)}+56i\,e^{(5i\,dx+5i\,c)}+70i\,e^{(3i\,dx+3i\,c)}+35i\,e^{(i\,dx+i\,c)}\right)}{35\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(7*I*d*x + 7*I*c)
 + 56*I*e^(5*I*d*x + 5*I*c) + 70*I*e^(3*I*d*x + 3*I*c) + 35*I*e^(I*d*x + I
*c))/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) + 3*a^4*d*e^
(2*I*d*x + 2*I*c) + a^4*d)
```

3.379.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\left(5\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}-42\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a+140\left(i\,a\tan\left(dx\right.\right.}{35\,a^7d}$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 42*(I*a*tan(d*x + c) + a)^(5/2)*a
 + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 280*sqrt(I*a*tan(d*x + c) + a)*a
^3)/(a^7*d)
```

3.379.8 Giac [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^8}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.379.9 Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.14

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{35\,a^4\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{35\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{35\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
- ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*256i)/(35*a^4*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*128i)/(35*a^4*d*(exp(c*2i + d*x*2i) + 1)) - ((a - (a*
(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*96i)/(35*
a^4*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*
1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*a^4*d*(exp(c*2i + d*x*2i) + 1)
^3)
```

3.380 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.380.1 Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}$$

output
```
8*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+8*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-2/3*
I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d
```

3.380.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2i(23+10i\tan(c+dx)+\tan^2(c+dx))}{3a^3d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/3)*(23 + (10*I)*Tan[c + d*x] + Tan[c + d*x]^2))/(a^3*d*Sqrt[a + I*
a*Tan[c + d*x]])
```

3.380.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{(a-ia\tan(c+dx))^2}{(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{4a^2}{(i\tan(c+dx)a+a)^{3/2}}-\frac{4a}{\sqrt{i\tan(c+dx)a+a}}+\sqrt{i\tan(c+dx)a+a}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{8a^2}{\sqrt{a+ia\tan(c+dx)}}-8a\sqrt{a+ia\tan(c+dx)}+\frac{2}{3}(a+ia\tan(c+dx))^{3/2}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((-8*a^2)/Sqrt[a + I*a*Tan[c + d*x]] - 8*a*Sqrt[a + I*a*Tan[c + d*x]
] + (2*(a + I*a*Tan[c + d*x])^(3/2))/3))/(a^5*d)
```

3.380.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.380.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a\sqrt{a+ia\tan(dx+c)}+\frac{4a^{2}}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{5}}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a\sqrt{a+ia\tan(dx+c)}+\frac{4a^{2}}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^{5}}$	63

input
```
int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^5*(-1/3*(a+I*a*tan(d*x+c))^(3/2)+4*a*(a+I*a*tan(d*x+c))^(1/2)+4*a^
2/(a+I*a*tan(d*x+c))^(1/2))
```

3.380.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(4i\,dx+4i\,c)}-12i\,e^{(2i\,dx+2i\,c)}-3i\right)}{3\left(a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right)}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(4*I*d*x + 4*I*c) -
 12*I*e^(2*I*d*x + 2*I*c) - 3*I)/(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d
*x + I*c))
```

3.380.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^6(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\left(\frac{12}{\sqrt{i\,a\tan(dx+c)+a}a^2}-\frac{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}-12\sqrt{i\,a\tan(dx+c)+a}a}{a^4}\right)}{3\,ad}$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/3*I*(12/(sqrt(I*a*tan(d*x + c) + a)*a^2) - ((I*a*tan(d*x + c) + a)^(3/2)
 - 12*sqrt(I*a*tan(d*x + c) + a)*a)/a^4)/(a*d)
```

3.380.8 Giac [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{6}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.380.9 Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,23{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,3{}\mathrm{i}+\right.}{3\,a^4\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(2*c + 2*d*x)*23i + cos(4*c + 4*d*x)*3i + 7*sin(2*c + 2*d*x)
+ 3*sin(4*c + 4*d*x) + 20i))/(3*a^4*d*(cos(2*c + 2*d*x) + 1))
```

3.381 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.381.4 Maple [A] (verified) 2703
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3.381.7 Maxima [A] (verification not implemented) 2704
3.381.8 Giac [F] 2705
3.381.9 Mupad [F(-1)] 2705

3.381.1 Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

output

```
-2*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+4/3*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.381.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\left(-\frac{4a}{3(a+ia\tan(c+dx))^{3/2}}+\frac{2}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^3d}$$

input

```
Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
((-I)*((-4*a)/(3*(a + I*a*Tan[c + d*x])^(3/2)) + 2/Sqrt[a + I*a*Tan[c + d*
x]]))/(a^3*d)
```

3.381.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{a-ia\tan(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(\frac{2a}{(i\tan(c+dx)a+a)^{5/2}}-\frac{1}{(i\tan(c+dx)a+a)^{3/2}}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2}{\sqrt{a+ia\tan(c+dx)}}-\frac{4a}{3(a+ia\tan(c+dx))^{3/2}}\right)}{a^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*((-4*a)/(3*(a + I*a*Tan[c + d*x])^(3/2)) + 2/Sqrt[a + I*a*Tan[c + d*
x]]))/(a^3*d)
```

3.381. $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$

3.381.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.381.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2i\left(-\frac{1}{\sqrt{a+ia\tan(dx+c)}}+\frac{2a}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d\,a^{3}}$	44
default	$\frac{2i\left(-\frac{1}{\sqrt{a+ia\tan(dx+c)}}+\frac{2a}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d\,a^{3}}$	44

input
```
int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a^3*(-1/(a+I*a*tan(d*x+c))^(1/2)+2/3*a/(a+I*a*tan(d*x+c))^(3/2))
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(4i\,dx+4i\,c)}-i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^4d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x + 4*I*c) -
I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

3.381.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\left(3i\,a\tan\left(dx+c\right)+a\right)}{3\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a^3d}$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3*I*(3*I*a*tan(d*x + c) + a)/((I*a*tan(d*x + c) + a)^(3/2)*a^3*d)
```

3.381.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.382 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.382.2 Mathematica [A] (verified) . . . 2706
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3.382.7 Maxima [A] (verification not implemented) . . . 2709
3.382.8 Giac [F] . . . 2709
3.382.9 Mupad [B] (verification not implemented) . . . 2710

3.382.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

output
```
2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

3.382.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.382.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int \frac{1}{(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.382.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.382.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24
default	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24

input
```
int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

3.382.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(6i\,dx+6i\,c)}+3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{20\,a^4d}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/20*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(6*I*d*x + 6*I*c) + 3*
I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/
(a^4*d)
```

3.382.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec^2{(c+dx)}}{(ia\,(\tan{(c+dx)}-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

3.382.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2i}{5\,(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}ad}$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
2/5*I/((I*a*tan(d*x + c) + a)^(5/2)*a*d)
```

3.382.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec{(dx+c)}^2}{(i\,a\tan{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.382.9 Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2\mathrm{i}}{5\,a\,d\,{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
2i/(5*a*d*(a + a*tan(c + d*x)*1i)^(5/2))
```

3.383 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.383.1 Optimal result 2711
3.383.2 Mathematica [C] (verified) 2712
3.383.3 Rubi [A] (warning: unable to verify) 2712
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3.383.5 Fricas [A] (verification not implemented) 2717
3.383.6 Sympy [F(-1)] 2718
3.383.7 Maxima [A] (verification not implemented) 2718
3.383.8 Giac [F] 2718
3.383.9 Mupad [F(-1)] 2719

3.383.1 Optimal result

Integrand size = 26, antiderivative size = 233

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{11i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}a^{7/2}d}\\ &+\frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\\ &+\frac{11i}{56d(a+ia\tan(c+dx))^{7/2}}+\frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}}\\ &+\frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-11/128*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(7/2)/d*
2^(1/2)+11/64*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+11/36*I*a/d/(a+I*a*tan(d*x+
c))^(9/2)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+11/56*I/
d/(a+I*a*tan(d*x+c))^(7/2)+11/80*I/a/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I/a^
2/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.383.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{9}{2},2,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{18d(a+ia\tan(c+dx))^{9/2}}$$

input

```
Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
((I/18)*a*Hypergeometric2F1[-9/2, 2, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(a
+ I*a*Tan[c + d*x])^(9/2))
```

3.383.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3968, 52, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^3\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^3\left(\frac{11\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\right)}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia}}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}\right)}{4a}+\frac{}{2a}}{d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{}{9a(a+i}}{4a}}{d}$$

3.383. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \text{61}$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c-}}{2a}\right.}{4a}\right.}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{}{9}\right.}{4a}\right.}{d}$$

$$\Big\downarrow \text{219}$$

3.383. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$-\frac{ia^3\left(\frac{11\left(\frac{\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+}\right.}{4a}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*a^3*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (
11*(-1/9*1/(a*(a + I*a*Tan[c + d*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c +
d*x])^(7/2)) + (-1/5*1/(a*(a + I*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a +
I*a*Tan[c + d*x])^(3/2)) + ((I*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sq
rt[2]*a^(3/2)) - 1/(a*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2
*a)))/(4*a)))/d
```

3.383.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

3.383. $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.383.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(183) = 366$.

Time = 10.36 (sec) , antiderivative size = 977, normalized size of antiderivative = 4.19

method	result	size
default	Expression too large to display	977

input
```
int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40320/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(1+I*tan(d*x+
c))^3/(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(-50424*I*cos(d*x+c)*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-13860*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-12320*sin(d*x+c)*cos(d*x+c)^2*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+7840*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-3465*I*sec(d*x+c)^3*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-12320*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-50424*I*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+25410*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+42504*sin
(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+27720*arctan(1/2*(cos(d*x+c)+1+
I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c
)+27720*I*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+25410*I*sec(d*x+c)*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)+42504*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+13860
*tan(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))-27720*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*
sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7840*I*cos(
d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-6930*tan(d*x+c)*sec(d*x+c)*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-13860*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(co
s(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1...
```

3.383.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.36

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-3465i\,\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{}{e^{(2i}}}\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/40320*(-3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log
(4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
 + 3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sq
rt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqr
t(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-315*I*e^(12*I*d*x + 12*I*c) + 430
3*I*e^(10*I*d*x + 10*I*c) + 7034*I*e^(8*I*d*x + 8*I*c) + 3754*I*e^(6*I*d*x
 + 6*I*c) + 1798*I*e^(4*I*d*x + 4*I*c) + 530*I*e^(2*I*d*x + 2*I*c) + 70*I)
)*e^(-9*I*d*x - 9*I*c)/(a^4*d)
```

3.383.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\left(\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^5-4620\,(i\,a\tan(dx+c)+a)^4a-1848\,(i\,a\tan(dx+c)+a)^3a^2-1584\,(i\,a\tan(\right.}{(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^2-2\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a}\right.}{80640}$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/80640*I*(4*(3465*(I*a*tan(d*x + c) + a)^5 - 4620*(I*a*tan(d*x + c) + a)^
4*a - 1848*(I*a*tan(d*x + c) + a)^3*a^2 - 1584*(I*a*tan(d*x + c) + a)^2*a^
3 - 1760*(I*a*tan(d*x + c) + a)*a^4 - 2240*a^5)/((I*a*tan(d*x + c) + a)^(1
1/2)*a^2 - 2*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 3465*sqrt(2)*log(-(sqrt(2
)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*
x + c) + a)))/a^(5/2))/(a*d)
```

3.383.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos(dx+c)^2}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{{\cos\left(c+d\,x\right)}^2}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}} \,d x$$

input
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.384 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.384.1 Optimal result 2720
3.384.2 Mathematica [C] (verified) 2721
3.384.3 Rubi [A] (warning: unable to verify) 2721
3.384.4 Maple [B] (verified) 2730
3.384.5 Fricas [A] (verification not implemented) 2731
3.384.6 Sympy [F(-1)] 2731
3.384.7 Maxima [A] (verification not implemented) 2732
3.384.8 Giac [F] 2732
3.384.9 Mupad [F(-1)] 2732

3.384.1 Optimal result

Integrand size = 26, antiderivative size = 306

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{195i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}a^{7/2}d} \\ &+ \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} \\ &- \frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}} + \frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}} \\ &+ \frac{195i}{896d(a+ia\tan(c+dx))^{7/2}} + \frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-195/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(7/2)/
d*2^(1/2)+195/1024*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+195/352*I*a^2/d/(a+I*a
*tan(d*x+c))^(11/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(1
1/2)-15/16*I*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(11/2)+65/192*I*a
/d/(a+I*a*tan(d*x+c))^(9/2)+195/896*I/d/(a+I*a*tan(d*x+c))^(7/2)+39/256*I/
a/d/(a+I*a*tan(d*x+c))^(5/2)+65/512*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.384.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.17

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{11}{2},3,-\frac{9}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{44d(a+ia\tan(c+dx))^{11/2}}$$

input
```
Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/44)*a^2*Hypergeometric2F1[-11/2, 3, -9/2, (1 + I*Tan[c + d*x])/2])/(d*
(a + I*a*Tan[c + d*x])^(11/2))
```

3.384.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3968, 52, 52, 61, 61, 61, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{1}{(a-ia\tan(c+dx))^3(i\tan(c+dx)a+a)^{13/2}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{15\int \frac{1}{(a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{13/2}}d(ia\tan(c+dx))}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}\right)}{d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{13/2}}d(ia\tan(c+dx))}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}\right)}{8a}+\frac{1}{4a(a-ia\tan(c+dx))^2(a+ia\,\mathrm{ta}}\right.}{d}$$

$$\Big\downarrow \color{blue}{61}$$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{11/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{11a(a+ia\tan(c+dx))^{11/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}\right)}{8a}+\frac{}{4}\right.}{d}$$

$$\Big\downarrow \color{blue}{61}$$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{9/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}}{2a}-\frac{1}{11a(a+ia\tan(c+dx))^{11/2}}\right)}{4a}+\frac{1}{2a(a-ia\tan(c+dx))(a+ia}\right.}{8a}\right.}{d}$$

$$\Big\downarrow \color{blue}{61}$$

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$-\frac{i a^5 \left(\frac{15 \left(\frac{13 \left(\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{7/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}}{2a}-\frac{1}{11a(a+ia\tan(c+dx))^{11/2}}\right)}{4a}+\right.}{8a}\right.}{d}$$

$$\Big\downarrow 61$$

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{5/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan(c+dx))^{9/2}}}{2a}-\frac{}{11}\right.}{4a}\right.}{8a}\right.}{d}$$

$$\Big\downarrow 61$$

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{3/2}}d(ia\tan(c+dx))}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{}{9a}}{2a}\right.}{4a}\right.}{8a}\right.}{d}$$

$$\Big\downarrow 61$$

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\frac{\frac{\int \frac{1}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-}{2a}}{2a}\right)}{4a}}{8a}\right)}{}$$

$$\Big\downarrow 73$$

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\frac{\int \frac{1}{a^2\tan^2(c+dx)+2a}\,d\sqrt{i\tan(c+dx)a+a}}{a}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}}{2a}\right.}{4a}}{8a}\right.}{}$$

$\downarrow$ 219

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$-\frac{ia^5\left(\frac{15\left(\frac{13\left(\frac{\frac{\frac{\frac{\frac{i\arctan\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}a^{3/2}}-\frac{1}{a\sqrt{a+ia\tan(c+dx)}}}{2a}-\frac{1}{3a(a+ia\tan(c+dx))^{3/2}}}{2a}-\frac{1}{5a(a+ia\tan(c+dx))^{5/2}}}{2a}-\frac{1}{7a(a+ia\tan(c+dx))^{7/2}}}{2a}-\frac{1}{9a(a+ia\tan}}{2a}}{4a}}{8a}}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

output
```
((-I)*a^5*(1/(4*a*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(11/2))
+ (15*(1/(2*a*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(11/2)) + (13*
(-1/11*1/(a*(a + I*a*Tan[c + d*x])^(11/2)) + (-1/9*1/(a*(a + I*a*Tan[c + d
*x])^(9/2)) + (-1/7*1/(a*(a + I*a*Tan[c + d*x])^(7/2)) + (-1/5*1/(a*(a + I
*a*Tan[c + d*x])^(5/2)) + (-1/3*1/(a*(a + I*a*Tan[c + d*x])^(3/2)) + ((I*A
rcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[2]])/(Sqrt[2]*a^(3/2)) - 1/(a*Sqrt[a + I
*a*Tan[c + d*x]]))/(2*a))/(2*a))/(2*a))/(2*a))/(2*a)))/(4*a)))/(8*a)))/d
```

3.384.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.384.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1110 vs. $2(243) = 486$.

Time = 9.97 (sec) , antiderivative size = 1111, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	1111

input
```
int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/473088/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d
*x+c)))^(1/2)/(1+I*tan(d*x+c))^3/a^3*(-360360*arctan(1/2*(cos(d*x+c)+1+I*s
in(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+1
80180*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*
x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+655512*I*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+180180*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+
1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+90090*tan(d*x+c)*sec(d*x+c)*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)-180180*tan(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*s
in(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+45045*I*sec(
d*x+c)^3*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))-37632*I*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)-37632*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-101920*I*co
s(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-101920*I*cos(d*x+c)^2*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)+360360*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+
I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+655512*I*
cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-360360*I*sec(d*x+c)*arctan(1
/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))-330330*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-135135*I*se
c(d*x+c)^2*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))-552552*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1...
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.10

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(11i\,dx+11i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\right.\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/473088*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)
*log(4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)*log
(-4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-462*I*e^(16*I*d*x + 16*I*c
) - 7161*I*e^(14*I*d*x + 14*I*c) + 47413*I*e^(12*I*d*x + 12*I*c) + 78800*I
*e^(10*I*d*x + 10*I*c) + 38512*I*e^(8*I*d*x + 8*I*c) + 19552*I*e^(6*I*d*x
+ 6*I*c) + 7184*I*e^(4*I*d*x + 4*I*c) + 1624*I*e^(2*I*d*x + 2*I*c) + 168*I
))*e^(-11*I*d*x - 11*I*c)/(a^4*d)
```

3.384.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.384.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^7-150150\,(i\,a\tan(dx+c)+a)^6a+96096\,(i\,a\tan(dx+c)+a)^5a^2+27456\,(i}{(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a^2-4\,(}\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/946176*I*(4*(45045*(I*a*tan(d*x + c) + a)^7 - 150150*(I*a*tan(d*x + c) +
 a)^6*a + 96096*(I*a*tan(d*x + c) + a)^5*a^2 + 27456*(I*a*tan(d*x + c) + a
)^4*a^3 + 18304*(I*a*tan(d*x + c) + a)^3*a^4 + 16640*(I*a*tan(d*x + c) + a
)^2*a^5 + 17920*(I*a*tan(d*x + c) + a)*a^6 + 21504*a^7)/((I*a*tan(d*x + c)
 + a)^(15/2)*a^2 - 4*(I*a*tan(d*x + c) + a)^(13/2)*a^3 + 4*(I*a*tan(d*x +
c) + a)^(11/2)*a^4) + 45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d
*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2))/(a*
d)
```

3.384.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.384. $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.385 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.385.1 Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

output
```
64/3315*I*a^3*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+16/255*I*a^2*sec(d
*x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+2/17*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d
*x+c))^(9/2)
```

3.385.2 Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=-\frac{2\sec^{12}(c+dx)(68+263\cos(2(c+dx))+247i\sin(2(c+dx)))(\cos(3(c+dx))-i\sin(3(c+dx)))}{3315a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(-2*Sec[c + d*x]^12*(68 + 263*Cos[2*(c + d*x)] + (247*I)*Sin[2*(c + d*x)])
*(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(3315*a^3*d*(-I + Tan[c + d*x])^
3*Sqrt[a + I*a*Tan[c + d*x]])
```

3.385.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^{13}}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{17}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{17}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3975}$$

$$\frac{8}{17}a\left(\frac{4}{15}a\int \frac{\sec^{13}(c+dx)}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{17}a\left(\frac{4}{15}a\int \frac{\sec(c+dx)^{13}}{(i\tan(c+dx)a+a)^{11/2}}dx+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3974}$$

$$\frac{8}{17}a\left(\frac{8ia^2\sec^{13}(c+dx)}{195d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{15d(a+ia\tan(c+dx))^{11/2}}\right)+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (8*a*(((
(8*I)/195)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((2*I
)/15)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2))))/17
```

3.385.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.385.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(dx+c)}{(a+ia\tan(dx+c))^{\frac{7}{2}}}dx$$

input
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x)
```

3.385.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(86) = 172$.

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.57

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{512\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}(-255i\,e^{(4i\,dx+4i\,c)} - 68i\,e^{(2i\,dx+2i\,c}}{3315\,(a^4de^{(16i\,dx+16i\,c)} + 8\,a^4de^{(14i\,dx+14i\,c)} + 28\,a^4de^{(12i\,dx+12i\,c)} + 56\,a^4de^{(10i\,dx+10i\,c)} + 70\,a^4de^{(8i\,dx+8i\,c)} + 56}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-512/3315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-255*I*e^(4*I*d*x + 4
*I*c) - 68*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^4*d*e^(16*I*d*x + 16*I*c) + 8*a
^4*d*e^(14*I*d*x + 14*I*c) + 28*a^4*d*e^(12*I*d*x + 12*I*c) + 56*a^4*d*e^(
10*I*d*x + 10*I*c) + 70*a^4*d*e^(8*I*d*x + 8*I*c) + 56*a^4*d*e^(6*I*d*x +
6*I*c) + 28*a^4*d*e^(4*I*d*x + 4*I*c) + 8*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*
d)
```

3.385.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.385.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(86) = 172$.

Time = 0.57 (sec) , antiderivative size = 902, normalized size of antiderivative = 8.20

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/3315*(-331*I*sqrt(a) - 998*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 18
38*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 7522*sqrt(a)*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 - 4836*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 27882*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8954*I*sqrt(a)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 68926*sqrt(a)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 - 12631*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 1250
52*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10540*I*sqrt(a)*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 - 168980*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c
) + 1)^11 - 168980*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 10540*I
*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 125052*sqrt(a)*sin(d*x +
c)^15/(cos(d*x + c) + 1)^15 + 12631*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c
) + 1)^16 - 68926*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 8954*I*s
qrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 27882*sqrt(a)*sin(d*x + c)^
19/(cos(d*x + c) + 1)^19 + 4836*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) +
1)^20 - 7522*sqrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 1838*I*sqrt(a
)*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 - 998*sqrt(a)*sin(d*x + c)^23/(cos
(d*x + c) + 1)^23 + 331*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(
sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1
) - 1)^(7/2)/((a^4 - 12*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^4*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 220*a^4*sin(d*x + c)^6/(cos(d*x + ...
```

3.385.8 Giac [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{13}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.385.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{512\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,68\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,255\mathrm{i}+8\mathrm{i}\right)}{3315\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8}$$

input
```
int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(512*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*68i + exp(c*4i + d*x*4i)*255i
 + 8i))/(3315*a^4*d*(exp(c*2i + d*x*2i) + 1)^8)
```

3.386 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.386.1 Optimal result . . . 2740
3.386.2 Mathematica [A] (verified) . . . 2740
3.386.3 Rubi [A] (verified) . . . 2741
3.386.4 Maple [F(-1)] . . . 2742
3.386.5 Fricas [B] (verification not implemented) . . . 2742
3.386.6 Sympy [F(-1)] . . . 2743
3.386.7 Maxima [B] (verification not implemented) . . . 2743
3.386.8 Giac [F] . . . 2744
3.386.9 Mupad [B] (verification not implemented) . . . 2745

3.386.1 Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

output
```
8/143*I*a^2*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+2/13*I*a*sec(d*x+c)^
11/d/(a+I*a*tan(d*x+c))^(9/2)
```

3.386.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\sec^9(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-15i+11\tan(c+dx))}{143a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/143)*Sec[c + d*x]^9*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-15*
I + 11*Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*
x]])
```

3.386.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^{11}}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3975}$$

$$\frac{4}{13}a\int \frac{\sec^{11}(c+dx)}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{13}a\int \frac{\sec(c+dx)^{11}}{(i\tan(c+dx)a+a)^{9/2}}dx+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3974}$$

$$\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2
*I)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

3.386.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.386.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(dx+c)}{(a+ia\tan(dx+c))^{\frac{7}{2}}}dx$$

input
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x)
```

3.386.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(57) = 114.

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(2i\,dx+2i\,c)}-2i\right)}{143\left(a^4de^{(12i\,dx+12i\,c)}+6\,a^4de^{(10i\,dx+10i\,c)}+15\,a^4de^{(8i\,dx+8i\,c)}+20\,a^4de^{(6i\,dx+6i\,c)}+15\,a^4de^{(4i\,dx+4i\,c)}+6\,a^4de\right.}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-128/143*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(2*I*d*x + 2*I
*c) - 2*I)/(a^4*d*e^(12*I*d*x + 12*I*c) + 6*a^4*d*e^(10*I*d*x + 10*I*c) +
15*a^4*d*e^(8*I*d*x + 8*I*c) + 20*a^4*d*e^(6*I*d*x + 6*I*c) + 15*a^4*d*e^(
4*I*d*x + 4*I*c) + 6*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

3.386.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.386.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(57) = 114$.

Time = 0.47 (sec) , antiderivative size = 764, normalized size of antiderivative = 10.47

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/143*(-15*I*sqrt(a) - 38*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*
sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 278*sqrt(a)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 213*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
920*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 272*I*sqrt(a)*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 1848*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 182*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2548*sqrt(a)*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 - 2548*sqrt(a)*sin(d*x + c)^11/(cos(d*x +
c) + 1)^11 + 182*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1848*sq
rt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 272*I*sqrt(a)*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14 - 920*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^1
5 + 213*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 278*sqrt(a)*sin(
d*x + c)^17/(cos(d*x + c) + 1)^17 + 88*I*sqrt(a)*sin(d*x + c)^18/(cos(d*x
+ c) + 1)^18 - 38*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 + 15*I*sqr
t(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*x + c) +
1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 10*a^4*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^4*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 120*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^4*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 - 252*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
210*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^4*sin(d*x + c)^14/(c
os(d*x + c) + 1)^14 + 45*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10...
```

3.386.8 Giac [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{11}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.386.9 Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,13\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{143\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

input
```
int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(128*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*13i + 2i)*(a - (a*(exp(c*2i
+ d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(143*a^4*d*(exp(c*
2i + d*x*2i) + 1)^6)
```

3.387 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.387.1 Optimal result . . . 2746
3.387.2 Mathematica [A] (verified) . . . 2746
3.387.3 Rubi [A] (verified) . . . 2747
3.387.4 Maple [F(-1)] . . . 2748
3.387.5 Fricas [B] (verification not implemented) . . . 2748
3.387.6 Sympy [F(-1)] . . . 2748
3.387.7 Maxima [B] (verification not implemented) . . . 2749
3.387.8 Giac [F] . . . 2750
3.387.9 Mupad [B] (verification not implemented) . . . 2750

3.387.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

output
```
2/9*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)
```

3.387.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{2i\sec^7(c+dx)(i+\tan(c+dx))}{9a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/9)*Sec[c + d*x]^7*(I + Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3
*Sqrt[a + I*a*Tan[c + d*x]])
```

3.387.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^9}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

input
```
Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(((2*I)/9)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

3.387.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.387.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{9}\left(dx+c\right)}{\left(a+ia\tan\left(dx+c\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x)
```

3.387.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{32i\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{9\left(a^4de^{(8i\,dx+8i\,c)}+4\,a^4de^{(6i\,dx+6i\,c)}+6\,a^4de^{(4i\,dx+4i\,c)}+4\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
32/9*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^4*d*e^(8*I*d*x + 8*I*c
) + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*I*d*x + 4*I*c) + 4*a^4*d*e^
(2*I*d*x + 2*I*c) + a^4*d)
```

3.387.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.387.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(27) = 54$.

Time $= 0.72$ (sec) , antiderivative size $= 626$, normalized size of antiderivative $= 17.89$

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{6i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{42\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right.}{9\left(a^4-\frac{8a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-56a^4\right.}$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/9*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*I*sqrt(a)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 14*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*sqrt(a
)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt(a)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 - 70*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 70*sqrt(
a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 14*I*sqrt(a)*sin(d*x + c)^10/(cos
(d*x + c) + 1)^10 - 42*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 14*
I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 14*sqrt(a)*sin(d*x + c)^
13/(cos(d*x + c) + 1)^13 + 6*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^
14 - 2*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + I*sqrt(a)*sin(d*x +
 c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 8*a^4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 28*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*a^4*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^4*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 56*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^4*sin(d*x + c
)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 +
 a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x
+ c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))
```

3.387.8 Giac [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^9}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.387.9 Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}}\,2\mathrm{i}}{9\,a^4\,d\cos\left(c+d\,x\right)^4}$$

input
```
int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
(exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(9*
a^4*d*cos(c + d*x)^4)
```

3.388 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.388.1 Optimal result 2751
3.388.2 Mathematica [A] (verified) 2751
3.388.3 Rubi [A] (verified) 2752
3.388.4 Maple [B] (warning: unable to verify) 2754
3.388.5 Fricas [B] (verification not implemented) 2755
3.388.6 Sympy [F] 2756
3.388.7 Maxima [B] (verification not implemented) 2756
3.388.8 Giac [F] 2757
3.388.9 Mupad [F(-1)] 2758

3.388.1 Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

output
```
8*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/
2)/a^(7/2)/d-8*I*sec(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-2/5*I*sec(d*x+c
)^5/a/d/(a+I*a*tan(d*x+c))^(5/2)-4/3*I*sec(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c
))^(3/2)
```

3.388.2 Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{128e^{7i(c+dx)}\left(-23-35e^{2i(c+dx)}-15e^{4i(c+dx)}+15\left(1+e^{2i(c+dx)}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{15a^3d\left(1+e^{2i(c+dx)}\right)^6(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(-128*E^((7*I)*(c + d*x))*(-23 - 35*E^((2*I)*(c + d*x)) - 15*E^((4*I)*(c +
 d*x)) + 15*(1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[Sqrt[1 + E^((2*I)*(c +
 d*x))]]))/(15*a^3*d*(1 + E^((2*I)*(c + d*x)))^6*(-I + Tan[c + d*x])^3*Sqr
t[a + I*a*Tan[c + d*x]])
```

3.388.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 3972, 3042, 3972, 3042, 3972, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3972}$$

$$\frac{2\int \frac{\sec^5(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sec(c+dx)^5}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3972}$$

$$\frac{2\left(\frac{2\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3972}$$

$$\frac{2\left(\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{2\left(\frac{2\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{a}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3970$$

$$\frac{2\left(\frac{2\left(\frac{4i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{ad}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\left(\frac{2\left(\frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}\right)}{a}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}\right)}{a}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

input

```
Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
(((-2*I)/5)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + (2*((((-2
*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((2*I)*Sqr
t[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])
/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])))/a))
/a
```

3.388. $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.388.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3972
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Simp[2*(d^2/a)   Int[(d*Sec[e + f*x])^
(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

3.388.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(133) = 266$.

Time = 10.79 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2(-\csc(dx+c)+\cot(dx+c)+i)^7 \left(-60\sqrt{2}\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1\right)^{\frac{5}{2}}+73i(}{15d\Big(-\frac{a\big(2i(\csc(dx+c)-\cot(dx+c)}{\big(\csc^2(dx+}}$

input
```
int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*(-csc(d*x+c)+cot(d*x+c)+I)^7*(-60*2^(1/2)*arctan(1/2*(I*(csc(d*x+c
)-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x
+c)^2*(1-cos(d*x+c))^2-1)^(5/2)+73*I*csc(d*x+c)^5*(1-cos(d*x+c))^5-190*I*c
sc(d*x+c)^3*(1-cos(d*x+c))^3-105*csc(d*x+c)^4*(1-cos(d*x+c))^4+105*I*(csc(
d*x+c)-cot(d*x+c))+190*csc(d*x+c)^2*(1-cos(d*x+c))^2-73)/(-a*(2*I*(csc(d*x
+c)-cot(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+
c))^2-1))^(7/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^6
```

3.388.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(125) = 250$.

Time = 0.26 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.04

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{4\left(15\sqrt{2}\left(i\,a^4de^{(4i\,dx+4i\,c)}+2i\,a^4de^{(2i\,dx+2i\,c)}+i\,a^4d\right)\sqrt{\frac{1}{a^7d^2}}\log\left(-\frac{32\left(\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}\right.}{a^3d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-4/15*(15*sqrt(2)*(I*a^4*d*e^(4*I*d*x + 4*I*c) + 2*I*a^4*d*e^(2*I*d*x + 2*
I*c) + I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((I*a^3*d*e^(2*I*d*x + 2*I*c) +
I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*
x - I*c)/(a^3*d)) + 15*sqrt(2)*(-I*a^4*d*e^(4*I*d*x + 4*I*c) - 2*I*a^4*d*e
^(2*I*d*x + 2*I*c) - I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((-I*a^3*d*e^(2*I*
d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)
) - I)*e^(-I*d*x - I*c)/(a^3*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*(15*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 23*I))/(a^4*d*
e^(4*I*d*x + 4*I*c) + 2*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

3.388.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^7{(c+dx)}}{\left(ia\left(\tan{(c+dx)}-i\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

3.388.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1164 vs. 2(125) = 250.

Time = 0.72 (sec) , antiderivative size = 1164, normalized size of antiderivative = 7.28

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-2/15*(15*(2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*
sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2
 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - 1) + (-I*sqrt(2)*cos(2*d*x + 2*c)^2 - I*sqrt(2)*sin(2*d*x
+ 2*c)^2 - 2*I*sqrt(2)*cos(2*d*x + 2*c) - I*sqrt(2))*log(sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + (I*sqrt(2)*cos(2*d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)
^2 + 2*I*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d...
```

3.388.8 Giac [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^7}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^7\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}} \,d x$$

input
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.389 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.389.1 Optimal result . . . 2759
3.389.2 Mathematica [A] (verified) . . . 2759
3.389.3 Rubi [A] (verified) . . . 2760
3.389.4 Maple [B] (warning: unable to verify) . . . 2762
3.389.5 Fricas [B] (verification not implemented) . . . 2763
3.389.6 Sympy [F] . . . 2764
3.389.7 Maxima [F(-1)] . . . 2764
3.389.8 Giac [F] . . . 2764
3.389.9 Mupad [F(-1)] . . . 2765

3.389.1 Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{3i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
-3*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1
/2)/a^(7/2)/d-2*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+6*I*sec(d*x+c)
/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.389.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{16e^{5i(c+dx)}\Big(-1-3e^{2i(c+dx)}+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\Big(\sqrt{1+e^{2i(c+dx)}}}{a^3d\left(1+e^{2i(c+dx)}\right)^4(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(16*E^((5*I)*(c + d*x))*(-1 - 3*E^((2*I)*(c + d*x)) + 3*E^((2*I)*(c + d*x)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(
a^3*d*(1 + E^((2*I)*(c + d*x)))^4*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c
 + d*x]])
```

3.389.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3982, 3042, 3982, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{6\int \frac{\sec^3(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{\sec(c+dx)^3}{(i\tan(c+dx)a+a)^{5/2}}dx}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3982}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3970$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{6\left(\frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{2\left(\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{a}\right)}{a}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-2*I)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + (6*(((2*I)*Se
c[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*(((I/2)*ArcTanh[(Sqrt[
a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^(3/2)*d
) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/a))/a
```

3.389. $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.389.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.389.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(102) = 204$.

Time = 10.22 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.51

method	result
default	$\left(3\sqrt{2}\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\sqrt{(\csc^2(dx+c))(1-\cos(dx+c))^2-1}+6i\sqrt{2}\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2}}\right.\right.$

input
```
int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(3*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c
)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2)+6*I
*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1
-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2)*(csc(d*x+
c)-cot(d*x+c))-3*csc(d*x+c)^2*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c)
)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*(1-cos
(d*x+c))^2-1)^(1/2)*(1-cos(d*x+c))^2+4+4*I*csc(d*x+c)^3*(1-cos(d*x+c))^3)*
(-csc(d*x+c)+cot(d*x+c)+I)^5/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^4/(-a*(2*I*
(csc(d*x+c)-cot(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-
cos(d*x+c))^2-1))^(7/2)
```

3.389.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(96) = 192$.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-3i\sqrt{2}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(2i\,dx+2i\,c)}\log\left(-\frac{12\left(\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d}}\right.}{a^3d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/2*(-3*I*sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((I*
a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + 3*I*sqrt(2)*a^4*d*sqrt(1/(
a^7*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a
^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x -
 I*c)/(a^3*d)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(2*I*
d*x + 2*I*c) - I))*e^(-2*I*d*x - 2*I*c)/(a^4*d)
```

3.389.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec^5{\left(c+dx \right)}}{\left(ia\left(\tan{\left(c+dx \right)}-i\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

3.389.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.389.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{5}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output

```
int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.390 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.390.1 Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
-1/16*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a
^(7/2)/d*2^(1/2)+1/2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)-1/8*I*sec(d
*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.390.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\sec^3(c+dx)\left(-3+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-3\cos\right.}{16a^3d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx}}$$

input
```
Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/16)*Sec[c + d*x]^3*(-3 + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*
x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 3*Cos[2*(c + d*x)] + I*Sin[2
*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.390.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 3982, 3042, 3983, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3982}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{3a}$$

$$\Big\downarrow \text{3983}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \text{3983}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \textbf{3970}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{i\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\left(\frac{3\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{3a}$$

input

```
Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output

```
(((2*I)/3)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) - (2*(((I/4)*S
ec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (3*(((I/2)*ArcTanh[(Sqrt[a
]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^(3/2)*d)
 + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/(8*a)))/(3*a)
```

3.390. $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.390.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.390.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(100) = 200$.

Time $= 10.65$ (sec) , antiderivative size $= 795$, normalized size of antiderivative $= 6.36$

method	result
default	$\frac{8i\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sin(dx+c)+4i\tan(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-4i\tan(dx+c)\sqrt{-}}{}$

input
```
int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/d/(tan(d*x+c)-I)^3/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)/a^3/(cos(d*x+c)+1)*(8*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+4*I*tan(d*x
+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+8*co
s(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c
)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*I*tan
(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4*arctan(1/2*(I*sin(
d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*tan(d*x+c)*sec(d*x+c)^2*arctan(1/2*(I
*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))-8*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)-3*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)+sec(d*x+c)^3*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2*sec(d*x+c)^3*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))
```

3.390.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(94) = 188.

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.14

$$\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{\left(-i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{}\right.}{4\,a^3d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/16*(-I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(s
qrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + I*sqrt
(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(sqrt(2)*sqrt(1
/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) +
 2*I))*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

3.390.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

3.390.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(94) = 188$.

Time = 0.46 (sec) , antiderivative size = 977, normalized size of antiderivative = 7.82

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/64*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(4*d*x + 4*
c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*
d*x + 4*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sq
rt(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*...
```

3.390.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

input

```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output

```
int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.391 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.391.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{5i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}$$

output
```
5/128*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a
^(7/2)/d*2^(1/2)+1/6*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(7/2)+5/48*I*sec(d*
x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+5/64*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c
))^(3/2)
```

3.391.2 Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.76

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sec^3(c+dx)\left(52+\frac{30e^{4i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+82\cos(2(c+dx))+50i\sin(2(c+dx))\right)}{384a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
-1/384*(Sec[c + d*x]^3*(52 + (30*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((
2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 82*Cos[2*(c + d*x)] + (
50*I)*Sin[2*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c +
 d*x]])
```

3.391.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{5\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(\frac{3\int \frac{\sec(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \textcolor{blue}{3983}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \textcolor{blue}{3970}$$

$$\frac{5\left(\frac{3\left(\frac{i\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{2ad}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{5\left(\frac{3\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\right)}{8a}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}\right)}{12a}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/6)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (5*(((I/4)*Sec[c +
 d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (3*(((I/2)*ArcTanh[(Sqrt[a]*Sec[
c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I
/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2))))/(8*a)))/(12*a)
```

3.391. $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.391.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.391.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(126) = 252$.

Time = 9.87 (sec) , antiderivative size = 798, normalized size of antiderivative = 5.08

method	result
default	$120i \arctan\left(\frac{i \sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right) \sin(dx+c)+60i \tan(dx+c) \arctan\left(\frac{i \sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+100i \tan(dx+c)$

input
```
int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/d/(-tan(d*x+c)+I)^3/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)/a^3/(cos(d*x+c)+1)*(120*I*sin(d*x+c)*arctan(1/2*(I*sin(d*x+c
)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+60*I*ta
n(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))+100*I*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+120*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-60*I*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(I
*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))+100*I*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+60*arcta
n(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))+164*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-15*I*tan(d*x+c)*sec(d*x+
c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))-120*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+164*sec(d*x+c)*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)-45*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d
*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*sec(d*x+c)^
2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+15*sec(d*x+c)^3*arctan(1/2*(I*sin(d*x
+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*se
c(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

3.391.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(118) = 236$.

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.77

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-15i\,\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}\right.}{32\,a^3d}\right.\right.}{}$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/384*(-15*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/
32*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + 1
5*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/32*(sqrt(
2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(33*I*e^(6*I*d*x + 6*I*c) + 59*I*e^(4*I*d
*x + 4*I*c) + 34*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^4*d
)
```

3.391.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec{(c+dx)}}{(ia\,(\tan{(c+dx)}-i))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

3.391.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec{(dx+c)}}{(i\,a\tan{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.391.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

output
```
int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.392 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

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3.392.1 Optimal result

Integrand size = 24, antiderivative size = 227

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= \frac{315i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}a^{7/2}d} \\ &+ \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}\end{aligned}$$

output

```
315/4096*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(7/2)/d*2^(1/2)+105/1024*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-3
15/2048*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d+1/8*I*cos(d*x+c)/d/(a+
I*a*tan(d*x+c))^(7/2)+3/32*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+21/25
6*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.392.2 Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$
$$-\frac{\sec^3(c+dx)\Big(420+\frac{630e^{4i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+826\cos(2(c+dx))-224\cos(4(c+dx))+474i\sin(}{4096a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
-1/4096*(Sec[c + d*x]^3*(420 + (630*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E
^((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 826*Cos[2*(c + d*x)]
 - 224*Cos[4*(c + d*x)] + (474*I)*Sin[2*(c + d*x)] - (288*I)*Sin[4*(c + d*
x)]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

3.392.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^{5/2}}dx}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{7\int \frac{\cos(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{7\int \frac{1}{\sec(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

3.392. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3971}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}\right)}{16a}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3970}$$

3.392. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{ia\int \frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\,\mathrm{co}}{6d(a+ia\,\mathrm{t}}\right)}{16a}$$

$$\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow 219$$

3.392. $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)}{8a}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}\right)}{12a}+\frac{i\cos(c+}{6d(a+ia\tan(}}{16a}$$

$$\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((I/8)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (9*(((I/6)*Cos[c +
 d*x])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (7*(((I/4)*Cos[c + d*x])/(d*(a +
 I*a*Tan[c + d*x])^(3/2)) + (5*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c
 + d*x]]) + (3*((I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a
+ I*a*Tan[c + d*x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c +
d*x]])/d))/(4*a)))/(8*a)))/(12*a)))/(16*a)
```

3.392.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.392.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(184) = 368$.

Time = 12.80 (sec) , antiderivative size = 923, normalized size of antiderivative = 4.07

method	result	size
default	Expression too large to display	923

input
```
int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4096/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*
x+c)))^(1/2)/(1+I*tan(d*x+c))^3/a^3*(1792*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+1792*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-252
0*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))-2304*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos
(d*x+c)*sin(d*x+c)+630*I*sec(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3
15*I*sec(d*x+c)^3*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2304*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+2520*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3444*I*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+630*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2100*tan(
d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1260*tan(d*x+c)*arctan(1/2*(I*si
n(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-
3444*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1260*I*arctan(1/2*(I*
sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)+2100*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1260*tan(d
*x+c)*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))+2520*I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c
)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+945*I*s
ec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos...
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(8i\,dx+8i\,c)}\log\left(-\frac{315\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2}}}\right.}{1024\,a^3d}\right.\right.}{}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/4096*(-315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-
315/1024*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d
)) + 315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-315/
1024*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d))
+ sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(10*I*d*x + 10*I*c)
+ 197*I*e^(8*I*d*x + 8*I*c) + 535*I*e^(6*I*d*x + 6*I*c) + 298*I*e^(4*I*d*x
 + 4*I*c) + 104*I*e^(2*I*d*x + 2*I*c) + 16*I))*e^(-8*I*d*x - 8*I*c)/(a^4*d
)
```

3.392.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.392.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2779 vs. $2(172) = 344$.

Time $= 0.49$ (sec) , antiderivative size $= 2779$, normalized size of antiderivative $= 12.24$

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
-1/16384*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(325*((-I*sqrt(2)*cos(8*d*x +
 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*
sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos
(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*
c))*cos(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))),
cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 643*(-I*sqrt(
2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(3/2*arctan2(sin(1/4*ar
ctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*
c), cos(8*d*x + 8*c))) + 1)) + 325*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*
sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 +
 (sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sq
rt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
)) + sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(7/2*arctan
2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(si
n(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 643*(sqrt(2)*cos(8*d*x + 8*c...
```

3.392.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

input
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.393 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

3.393.1 Optimal result . . . 2792
3.393.2 Mathematica [A] (verified) . . . 2793
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3.393.9 Mupad [F(-1)] . . . 2805

3.393.1 Optimal result

Integrand size = 26, antiderivative size = 307

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= \frac{3003i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}a^{7/2}d} \\ &+ \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d} \\ &- \frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}\end{aligned}$$

output

```
3003/32768*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))/a^(7/2)/d*2^(1/2)+1001/8192*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2
)+429/5120*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-3003/16384*I*cos(
d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-1001/10240*I*cos(d*x+c)^3*(a+I*a*tan
(d*x+c))^(1/2)/a^4/d+1/10*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(7/2)+13/160
*I*cos(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+143/1920*I*cos(d*x+c)^3/a^2/d
/(a+I*a*tan(d*x+c))^(3/2)
```

3.393.2 Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.57

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{\Big(42140+20048e^{-2i(c+dx)}+71190e^{2i(c+dx)}+5856e^{-4i(c+dx)}-48640e^{4i(c+dx)}+768e^{-6i(c+dx)}-2560e^{6i(c+d}}{491520a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
-1/491520*((42140 + 20048/E^((2*I)*(c + d*x)) + 71190*E^((2*I)*(c + d*x))
+ 5856/E^((4*I)*(c + d*x)) - 48640*E^((4*I)*(c + d*x)) + 768/E^((6*I)*(c +
 d*x)) - 2560*E^((6*I)*(c + d*x)) + (90090*E^((4*I)*(c + d*x))*ArcTanh[Sqr
t[1 + E^((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))])*Sec[c + d*x]^3
)/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

3.393.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.654$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3971, 3042, 3970, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+ia\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{13\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^{5/2}}dx}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^{5/2}}dx}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{13\left(\frac{11\int \frac{\cos^3(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13\left(\frac{11\int \frac{1}{\sec(c+dx)^3(i\tan(c+dx)a+a)^{3/2}}dx}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{13\left(\frac{11\left(\frac{3\int \frac{\cos^3(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13\left(\frac{11\left(\frac{3\int \frac{1}{\sec(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int \cos^3(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

3.393. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)^3}dx}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}\right)}{20a}+\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow 3978$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{\cos(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/}}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow 3042$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\int \frac{1}{\sec(c+dx)\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}+\frac{i\cos^3(c-}{8d(a+ia\tan(}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

3.393. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\Big\downarrow \ 3983$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\cos(c+dx)\sqrt{i\tan(c+dx)a+a}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx)}\right.}{16a}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sec(c+dx)}dx}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}\right)}{16a}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \ 3971$$

3.393. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dz}{4d\sqrt{a+ia\tan(c}}}{4a}}{16a}}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\frac{i\cos^3(c+dz}{4d\sqrt{a+ia\tan(c}}}{4a}}{16a}}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

3.393. $\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$\downarrow$ 3970

3.393. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{ia\int\frac{1}{2-\frac{a\sec^2(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sec(c+dx)}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right.}{8a}\right.}{4a}\right.}{16a}\right.}{20a}$$

3.393. $\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$ $\quad\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$

$$\Big\downarrow 219$$

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5}{6}a\left(\frac{3\left(\frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}\right)-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\right)}{8a}+\right.}{4a}\right.}{16a}\right.}{20a}$$

$$\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

input
```
Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

3.393. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

output

```
((I/10)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (13*(((I/8)*Cos
[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (11*(((I/6)*Cos[c + d*x]^3
)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (3*(((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (7*(((-1/3*I)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d
*x]])/d + (5*a*(((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*(
(I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d))/(4*a)
))/6))/(8*a)))/(4*a)))/(16*a)))/(20*a)
```

3.393.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3970

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Simp[-2*(a/(b*f))   Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/
Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 3971

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Simp[a/(2*d^2)   Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
 f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
 EqQ[m/2 + n, 0] && GtQ[n, 0]
```

rule 3978

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.393. $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.393.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1056 vs. 2(252) = 504.

Time = 11.08 (sec) , antiderivative size = 1057, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	1057

input
```
int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/491520/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d
*x+c)))^(1/2)/(1+I*tan(d*x+c))^3/a^3*(-256256*I*cos(d*x+c)*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)-256256*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+106496*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+36036
0*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))+106496*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)-90090*I*sec(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+45045*I*sec(d*x+c)^3*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+329472*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)*sin(d*x+c)+492492*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)-90090*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-360360*arctan(1/
2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*sin(d*x+c)+329472*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-573
44*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+492492*I*sec(d*x+c)*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-180180*tan(d*x+c)*arctan(1/2*(I*sin(d*x+
c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-300300
*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+180180*I*arctan(1/2*(I*sin(
d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-57
344*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+180180*tan(d*x+c)*se
c(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*...
```

3.393.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\Bigg(-45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(10i\,dx+10i\,c)}\log\Bigg(-\frac{3003\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{}\right.}{8192\,a^3}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/491520*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c)
*log(-3003/8192*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)
/(a^3*d)) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c
)*log(-3003/8192*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*
c)/(a^3*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1280*I*e^(14*I*d
*x + 14*I*c) - 25600*I*e^(12*I*d*x + 12*I*c) + 11275*I*e^(10*I*d*x + 10*I*
c) + 56665*I*e^(8*I*d*x + 8*I*c) + 31094*I*e^(6*I*d*x + 6*I*c) + 12952*I*e
^(4*I*d*x + 4*I*c) + 3312*I*e^(2*I*d*x + 2*I*c) + 384*I))*e^(-10*I*d*x - 1
0*I*c)/(a^4*d)
```

3.393.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.393.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5821 vs. $2(236) = 472$.

Time = 0.62 (sec) , antiderivative size = 5821, normalized size of antiderivative = 18.96

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output

```
-1/1966080*(40*(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2
 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*
arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(3/4)*((79*(-I*sqrt(
2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 79*(-I*sqrt(2)*cos(10*d*x + 10*c) -
 sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c)))^2 + 837*(-I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10
*d*x + 10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
 + 10*c))) - I*sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 1
0*c))) + 158*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*
cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 837*(sqrt(2)*co
s(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2)*sin(1/5
*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*cos(1/5*ar
ctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*sin(4/5*arctan2(
sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 79*I*sqrt(2)*cos(10*d*x + 10*c)
 - 79*sqrt(2)*sin(10*d*x + 10*c))*cos(7/2*arctan2(sin(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(
10*d*x + 10*c))) + 1)) + (-49*I*sqrt(2)*cos(10*d*x + 10*c) - 1155*I*sqrt(2
)*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*I*sqr...
```

3.393.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

input

```
integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.394 $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

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3.394.1 Optimal result

Integrand size = 30, antiderivative size = 524

$$
\begin{aligned}
\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{ia^{3/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{ia^{3/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{ia^{3/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

output
```
I*a*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*e^(3/2)*
arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(
1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/
2)+1/2*I*a^(3/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)
/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)+1/4*I*a^(3/2)*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(
1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d
*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(
1/2)-1/4*I*a^(3/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))
^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^
(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

3.394.2 Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.71

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e\sqrt{e\sec(c+dx)}(\cos(c)-i\sin(c))\left(\operatorname{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right.}}\right.\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Sqrt[e*Sec[c + d*x]]*(Cos[c] - I*Sin[c])*(ArcTanh[(Sqrt[1 + I*Cos[c] -
Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan
[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] -
 Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] + Sin[c]]*(Sqrt[-1 -
I*Cos[c] - Sin[c]]*(I*Cos[d*x] + Sin[d*x])*Sqrt[I - Tan[(d*x)/2]] - ArcTan
h[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c]
 - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*Cos[c] + Sin[c
]]*Sqrt[I + Tan[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[-1 - I*Cos
[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

3.394.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.79, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3980}$$

$$\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{2ia^2e^3\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(}{\sqrt{e\sec(c+}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

3.394. $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\,\mathrm{ta}}{e}}}{2\sqrt{2}\sqrt{a}e}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 1103$$

$$\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

3.394. $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

input

```
Int[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*e
^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sq
rt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt
[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*S
qrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I
*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x
])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a -
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*
x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan
[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.394.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.394. $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

3.394.4 Maple [A] (verified)

Time = 11.39 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.62

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\left(2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)-i\,\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)-i\,\mathrm{arcta}}{}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c))^(1/2)*(2*I*(1/(cos(d*x+
c)+1))^(1/2)*cos(d*x+c)-I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c
)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-I*arctanh(1/2*(cos(d*x+c)-sin(d*
x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+2*I*(1/(cos(d*
x+c)+1))^(1/2)+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+arctanh(1/2*(cos(d*x+
c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-arcta
nh(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c))*(I*cos(d*x+c)+I+sin(d*x+c))*e/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1)
)^(1/2)
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.80

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{4i\,e\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{i\,ae^3}{d^2}}d\log\left(\frac{2\left(\left(ee^{(2i\,dx+2i\,c)}+e\right.\right.}{}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*(4*I*e*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(I*a*e^3/d^2)*d*log(2*((e*e^(2*I*d*x +
2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c) + I*sqrt(I*a*e^3/d^2)*d)/e) - sqrt(I*a*e^3/d
^2)*d*log(2*((e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(I*a*e^
3/d^2)*d)/e) + sqrt(-I*a*e^3/d^2)*d*log(2*((e*e^(2*I*d*x + 2*I*c) + e)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*
d*x + 1/2*I*c) + I*sqrt(-I*a*e^3/d^2)*d)/e) - sqrt(-I*a*e^3/d^2)*d*log(2*(
(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(-I*a*e^3/d^2)*d)/e)
)/d
```

3.394.6 Sympy [F]

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\sec(c+dx))^{\frac{3}{2}}\sqrt{ia(\tan(c+dx)-i)}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.394.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1870 vs. 2(396) = 792.

Time = 0.48 (sec) , antiderivative size = 1870, normalized size of antiderivative = 3.57

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output

```
-8*(2*(sqrt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)
*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*
(sqrt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*ar
ctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt
(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*arctan2
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*e*
cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*arctan2(sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*e*co
s(2*d*x + 2*c) + sqrt(2)*e*sin(2*d*x + 2*c) - I*sqrt(2)*e)*arctan2(sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) - 2*(I*sqrt(2)*e*cos(2*d*x + 2*c) - sqrt(2)*e*sin(2*d*x + 2*c) +
I*sqrt(2)*e)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(...
```

3.394.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\int (e\sec(dx+c))^{\frac{3}{2}}\sqrt{i\,a\tan(dx+c)+a}\,dx$$

input

```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output

```
integrate((e*sec(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.395 $\int \sqrt{e \sec (c+d x)} \sqrt{a+i a \tan (c+d x)} \, dx$

3.395.1 Optimal result 2817
3.395.2 Mathematica [A] (verified) 2818
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3.395.1 Optimal result

Integrand size = 30, antiderivative size = 323

$$
\begin{aligned}
\int \sqrt{e \sec (c+d x)} \sqrt{a+i a \tan (c+d x)} \, dx
&= \frac{i \sqrt{2} \sqrt{a} \sqrt{e} \arctan \left(1-\frac{\sqrt{2} \sqrt{e} \sqrt{a+i a \tan (c+d x)}}{\sqrt{a} \sqrt{e \sec (c+d x)}}\right)}{d} \\
&\quad -\frac{i \sqrt{2} \sqrt{a} \sqrt{e} \arctan \left(1+\frac{\sqrt{2} \sqrt{e} \sqrt{a+i a \tan (c+d x)}}{\sqrt{a} \sqrt{e \sec (c+d x)}}\right)}{d} \\
&\quad -\frac{i \sqrt{a} \sqrt{e} \log \left(a-\frac{\sqrt{2} \sqrt{a} \sqrt{e} \sqrt{a+i a \tan (c+d x)}}{\sqrt{e \sec (c+d x)}}+\cos (c+d x)(a+i a \tan (c+d x))\right)}{\sqrt{2} d} \\
&\quad +\frac{i \sqrt{a} \sqrt{e} \log \left(a+\frac{\sqrt{2} \sqrt{a} \sqrt{e} \sqrt{a+i a \tan (c+d x)}}{\sqrt{e \sec (c+d x)}}+\cos (c+d x)(a+i a \tan (c+d x))\right)}{\sqrt{2} d}
\end{aligned}
$$

output

```
-1/2*I*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c)
)^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)*e^(1/2)/d*2^(1/2)+1/2*I*ln(
a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+co
s(d*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)*e^(1/2)/d*2^(1/2)+I*arctan(1-2^(1/2)*
e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)*a^(
1/2)*e^(1/2)/d-I*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)
/(e*sec(d*x+c))^(1/2))*2^(1/2)*a^(1/2)*e^(1/2)/d
```

3.395.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.86

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{2e\left(\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}-\operatorname{arctanh}\left(\right.\right.}{d\sqrt{e\sec(c+dx)}\sqrt{1+}}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(-2*e*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[
-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[
c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqr
t[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])
]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*Sqrt[I + Tan[(
d*x)/2]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[1 + Cos[
2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

3.395.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3976}$$

$$-\frac{4iae^2\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{826}$$

$$-\frac{4iae^2\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{4iae^2\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\sqrt{i}}{2e}}{2e}\right.}{d}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{4iae^2\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a}{a^2+}}{}\right.}{d}$$

$$\Big\downarrow \text{217}$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d}$$

$$\Big\downarrow \text{25}$$

3.395. $\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\,t}}{\sqrt{}}}{2\sqrt{2}\sqrt{a}e}\right.}{d}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{4iae^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-4*I)*a*e^2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (S
qrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]
/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*
Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e
]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a
*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d
```

3.395. $\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

3.395.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.395. $\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.395.4 Maple [A] (verified)

Time = 11.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.49

method	result
default	$\frac{(-1+i)\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(i\,\mathrm{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)}{d(-i\cos(dx+c)+\sin(dx+c)-i)\sqrt{\frac{1}{\cos(dx+c)+1}}}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1+I)/d*(e*sec(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(I*arctanh(1/2*(-
cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-arctanh(
1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))*co
s(d*x+c)/(-I*cos(d*x+c)+sin(d*x+c)-I)/(1/(cos(d*x+c)+1))^(1/2)
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00

$$
\begin{aligned}
&\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. + d\sqrt{\frac{4i\,ae}{d^2}}\right) \\
&\quad - \frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. - d\sqrt{\frac{4i\,ae}{d^2}}\right) \\
&\quad - \frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. + d\sqrt{-\frac{4i\,ae}{d^2}}\right) \\
&\quad + \frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right. \\
&\qquad \left. - d\sqrt{-\frac{4i\,ae}{d^2}}\right)
\end{aligned}
$$

input

```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output

```
1/2*sqrt(4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) +
d*sqrt(4*I*a*e/d^2)) - 1/2*sqrt(4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(
1/2*I*d*x + 1/2*I*c) - d*sqrt(4*I*a*e/d^2)) - 1/2*sqrt(-4*I*a*e/d^2)*log(2
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2
*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) + d*sqrt(-4*I*a*e/d^2)) + 1/2
*sqrt(-4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) - d*
sqrt(-4*I*a*e/d^2))
```

3.395.6 Sympy [F]

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{e\sec\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,dx$$

input

```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output

```
Integral(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.395.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(239) = 478$.

Time = 0.48 (sec) , antiderivative size = 1400, normalized size of antiderivative = 4.33

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=\text{Too large to display}$$

input

```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output

```
1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqr
t(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2
*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 1) + I*sqrt(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*...
```

3.395.8 Giac [F]

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{e\sec\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}\,dx$$

input

```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output

```
integrate(sqrt(e*sec(d*x + c))*sqrt(I*a*tan(d*x + c) + a), x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input

```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output

```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.396 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx$

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3.396.1 Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

output
```
-2*I*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

3.396.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3969}$$

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

3.396.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.396.4 Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2i\sqrt{a(1+i\tan(dx+c))}}{d\sqrt{e\sec(dx+c)}}$	32
risch	$-\frac{2i\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}}{\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	59

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)
```

3.396.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(28) = 56$.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{de}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-I*
e^(2*I*d*x + 2*I*c) - I)*e^(1/2*I*d*x + 1/2*I*c)/(d*e)
```

3.396.6 Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\sqrt{ia\,(\tan{(c+dx)}-i)}}{\sqrt{e\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*sec(c + d*x)), x)
```

3.396.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{a}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{d\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-2*I*sqrt(a)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)/(d*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1))
```

3.396.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\sqrt{i\,a\tan{(dx+c)}+a}}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

3.396.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2), x)
```

3.397 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$

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3.397.9 Mupad [B] (verification not implemented) . . . 2836

3.397.1 Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx=\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

output
```
4/3*I*a*(e*sec(d*x+c))^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*(a+I*a*t
an(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(3/2)
```

3.397.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx=\frac{2(i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(2*(I + 2*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(e*Sec[c + d*x])^
(3/2))
```

3.397.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))
```

3.397.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.397.4 Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{2(i\cos(dx+c)+2\sin(dx+c))\sqrt{a(1+i\tan(dx+c))}}{3d\sqrt{e\sec(dx+c)}\,e}$	52
risch	$-\frac{i\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3e\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	80

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c)
)^(1/2)/e
```

3.397.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,de^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
I*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-1/2*I*d*x - 1/2
*I*c)/(d*e^2)
```

3.397.6 Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(3/2), x)
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}(-i\,\cos(\frac{3}{2}\,dx+\frac{3}{2}\,c)+3i\,\cos(\frac{1}{2}\,dx+\frac{1}{2}\,c)+\sin(\frac{3}{2}\,dx+\frac{3}{2}\,c)+3\,\sin(\frac{1}{2}\,dx}{3\,de^{\frac{3}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
1/3*sqrt(a)*(-I*cos(3/2*d*x + 3/2*c) + 3*I*cos(1/2*d*x + 1/2*c) + sin(3/2*
d*x + 3/2*c) + 3*sin(1/2*d*x + 1/2*c))/(d*e^(3/2))
```

3.397.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

3.397.9 Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1i)}{\cos(2\,c+2\,d\,x)+1}}\,(\cos\left(2\,c+2\,d\,x\right)\,1i+2\,\sin(2\,c+2}{3\,d\,e^2}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(3/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/
(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 2*sin(2*c + 2*d*x) +
1i))/(3*d*e^2)
```

3.398 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$

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3.398.1 Optimal result

Integrand size = 30, antiderivative size = 122

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

output
```
8/15*I*a/d/e^2/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-2/5*I*(a+I*a*
tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(5/2)-16/15*I*(a+I*a*tan(d*x+c))^(1/2)/
d/e^2/(e*sec(d*x+c))^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{i(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

output
```
((I/15)*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan
[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

3.398.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

input

```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

output

```
(((-2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*
(((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3
)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)
```

3.398.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.398.4 Maple [A] (verified)

Time = 10.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2\left(i\left(\cos^{2}(dx+c)\right)+4\sin(dx+c)\cos(dx+c)-8i\right)\sqrt{a(1+i\tan(dx+c))}}{15d\sqrt{e\sec(dx+c)}\,e^{2}}$	62
risch	$-\frac{i\sqrt{\frac{a\,{\mathrm e}^{2i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,(30-2\cos(2dx+2c)+8i\sin(2dx+2c))}{30e^{2}\sqrt{\frac{e\,{\mathrm e}^{i(dx+c)}}{{\mathrm e}^{2i(dx+c)}+1}}\,d}$	87

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-8*I)*(a*(1+I*tan(d*x+c)))^(
1/2)/(e*sec(d*x+c))^(1/2)/e^2
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(6i\,dx+6i\,c)} - 33i\,e^{(4i\,dx+4i\,c)} - 25i\,e^{(2i\,dx+2i\,c)} + \right.}{30\,de^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-3*I*e^(6*I*d*x + 6*I*c) - 33*I*e^(4*I*d*x + 4*I*c) - 25*I*e^(2*I*d*x + 2*
I*c) + 5*I)*e^(-3/2*I*d*x - 3/2*I*c)/(d*e^3)
```

3.398.6 Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(5/2), x)
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(5i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/30*sqrt(a)*(5*I*cos(3/2*d*x + 3/2*c) - 3*I*cos(5/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 5*sin(3/2*d*x + 3/2*c) + 3*sin(5/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))))/(d*e^(5/2))
```

3.398.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

3.398.9 Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(4\,\sin\left(c+d\,x\right)+4\,\sin\left(3\,c+3\,d\,x\right)\right.}{30\,d\,e^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(5/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/
(cos(2*c + 2*d*x) + 1))^(1/2)*(4*sin(c + d*x) - cos(c + d*x)*29i + cos(3*c
 + 3*d*x)*1i + 4*sin(3*c + 3*d*x)))/(30*d*e^3)
```

3.398. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$

3.399 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$

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3.399.1 Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}$$

output
```
12/35*I*a/d/e^2/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/35*I*a*(e
*sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*(a+I*a*tan(d*x+c))
^(1/2)/d/(e*sec(d*x+c))^(7/2)-16/35*I*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*se
c(d*x+c))^(3/2)
```

3.399.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+6\sin(3(c+dx)))\sqrt{a+}}{70de^3\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Sin[c + d*x] + 6*Sin[3*(c
+ d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d*e^3*Sqrt[e*Sec[c + d*x]])
```

3.399.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3978, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6a\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) + (6*a*
(((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((
4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*
I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(7*
e^2)
```

3.399.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.399. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.399.4 Maple [A] (verified)

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result	size
default	$-\frac{2i\sqrt{a(1+i\tan(dx+c))}\,(6i(\cos^2(dx+c))\sin(dx+c)-(\cos^3(dx+c))+16i\sin(dx+c)-8\cos(dx+c))}{35d\sqrt{e\sec(dx+c)}\,e^3}$	80

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(6*I*cos(d*x+c)^2*sin(d*x+c)-cos(d*x+
c)^3+16*I*sin(d*x+c)-8*cos(d*x+c))/(e*sec(d*x+c))^(1/2)/e^3
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(8i\,dx+8i\,c)}-40i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+\right.}{140\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="fric
as")
```

output
```
1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-5*I*e^(8*I*d*x + 8*I*c) - 40*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4
*I*c) + 112*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-5/2*I*d*x - 5/2*I*c)/(d*e^4)
```

3.399.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx=\frac{\sqrt{a}\big(7i\,\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,\cos\left(\frac{7}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
1/140*sqrt(a)*(7*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(7/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 105*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))) + 7*sin(5/2*d*x + 5/2*c) + 5*sin(7/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*sin(3/5*arctan2(sin(5/2*d*x + 5
/2*c), cos(5/2*d*x + 5/2*c))) + 105*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))))/(d*e^(7/2))
```

3.399.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

3.399.9 Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,(\cos\left(2\,c+2\,d\,x\right)\,36\mathrm{i}+\cos\left(4\,c+4\,d\right.}{140\,d\,e^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(7/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/
(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*36i + cos(4*c + 4*d*x)*1i
+ 76*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + 35i))/(140*d*e^4)
```

3.400 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

3.400.1 Optimal result 2849
3.400.2 Mathematica [A] (verified) 2850
3.400.3 Rubi [A] (verified) 2851
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3.400.5 Fricas [A] (verification not implemented) 2857
3.400.6 Sympy [F(-1)] 2858
3.400.7 Maxima [B] (verification not implemented) 2858
3.400.8 Giac [F] 2859
3.400.9 Mupad [F(-1)] 2860

3.400.1 Optimal result

Integrand size = 30, antiderivative size = 453

$$\begin{aligned}\int (e\sec(c+dx))^{5/2}(a&\\ +ia\tan(c+dx))^{3/2}\,dx &= \frac{7ia^{3/2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}\\ &- \frac{7ia^{3/2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}\\ &- \frac{7ia^{3/2}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d}\\ &+ \frac{7ia^{3/2}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d}\\ &+ \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d}\\ &+ \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d}\end{aligned}$$

output
```
7/16*I*a^(3/2)*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a
^(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)-7/16*I*a^(3/2)*e^(5/2)*arctan(1+2^(
1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1
/2)-7/32*I*a^(3/2)*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))
^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)+7/32*
I*a^(3/2)*e^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e
*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)+7/12*I*a^2*(e*
sec(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*I*a*(e*sec(d*x+c))^(5/2)*
(a+I*a*tan(d*x+c))^(1/2)/d-7/8*I*a*e^2*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x
+c))^(1/2)/d
```

3.400.2 Mathematica [A] (verified)

Time = 4.19 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.83

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{a(e\sec(c+dx))^{5/2}\left(2i\sqrt{1+\cos(2c)+i\sin(2c)}(-9+7\cos(2c+2dx)+14i\sin(2c+2dx))\sqrt{i-\tan\left(\frac{dx}{2}\right.}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
-1/96*(a*(e*Sec[c + d*x])^(5/2)*((2*I)*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*(-9
 + 7*Cos[2*c + 2*d*x] + (14*I)*Sin[2*c + 2*d*x])*Sqrt[I - Tan[(d*x)/2]] +
84*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 -
 I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[-1 - I*Co
s[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] - 84*Arc
Tanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos
[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[1 - I*Cos[c] +
Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*
Tan[c + d*x]])/(d*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

3.400.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.03, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3979, 3042, 3979, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \, 3042$$

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}dx$$

$$\Big\downarrow \, 3979$$

$$\frac{7}{6}a\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \, 3042$$

$$\frac{7}{6}a\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \, 3979$$

$$\frac{7}{6}a\left(\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \, 3042$$

$$\frac{7}{6}a\left(\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \, 3982$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+}{2d\sqrt{a+ia\,\mathrm{ta}}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+}{2d\sqrt{a+ia\,\mathrm{ta}}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\int\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{}{2}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{826}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a}}{}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+}{e}}}{2e}}{2e}\right.}{d}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow \text{1082}$$

3.400. $\int(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow 217$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a-}}{\sqrt{e\sec(c+dx)}}}{2e}\right.}{d}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow 1479$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\,t}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow 25$$

3.400. $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow 27$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

$$\Big\downarrow 1103$$

$$\frac{7}{6}a\left(\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

3.400. $\int(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

input

```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
((I/3)*a*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (7*a*(((I/
2)*a*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*a*(((-2*I
)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*S
qrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2
]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
+ I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c +
d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
 + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c +
 d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)))/4))/6
```

3.400.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

3.400. $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

3.400.4 Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.16

method	result
default	$\frac{(-\frac{1}{48}+\frac{i}{48}) \sec(dx+c)(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,a\,e^{2}\Big(-8\sqrt{\frac{1}{\cos(dx+c)+1}}+8i\sqrt{\frac{1}{\cos(dx+c)+1}}+21\sin(dx+c)(}{}$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/48+1/48*I)/d*sec(d*x+c)*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*(e*
sec(d*x+c))^(1/2)*a*e^2*(-8*(1/(cos(d*x+c)+1))^(1/2)+8*I*(1/(cos(d*x+c)+1)
)^(1/2)+21*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)-7*I*(1/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)^2+14*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1
/2)-8*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+21*cos(d*x+c)^3*arctanh(1/2*(-
cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+14*I*(1/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-21*I*(1/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)^3+21*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+7*(1/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)^2+21*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+
22*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-22*(1/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)-8*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+21*I*cos(d*x+c)^3*arctanh(1/2*
(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))/(-2*I*
cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)-I*cos(d*x+c)+sin(d*x+c)+I)/(1/(cos(d*
x+c)+1))^(1/2)
```

3.400.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.42

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\left(-21i\,ae^{2}e^{(5i\,dx+5i\,c)}+18i\,ae^{2}e^{(3i\,dx+3i\,c)}+7i\,ae^{2}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+}}}{}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output

```
1/12*((-21*I*a*e^2*e^(5*I*d*x + 5*I*c) + 18*I*a*e^2*e^(3*I*d*x + 3*I*c) +
7*I*a*e^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 6*sqrt(49/64*I*a^3*e^5/d^2)
*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e
^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*sqrt(49/64*I*a^3*e^5/d^2
)*d)/(a*e^2)) - 6*sqrt(49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e
^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*
d*x + 1/2*I*c) - 8*sqrt(49/64*I*a^3*e^5/d^2)*d)/(a*e^2)) - 6*sqrt(-49/64*I
*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/
7*(7*(a*e^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*sqrt(-49/64
*I*a^3*e^5/d^2)*d)/(a*e^2)) + 6*sqrt(-49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x +
 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e^(2*I*d*x + 2*I*
c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c) - 8*sqrt(-49/64*I*a^3*e^5/d^2)*d)/(a*e^2)))/
(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input

```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output

```
Timed out
```

3.400.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(331) = 662$.

Time = 0.62 (sec) , antiderivative size = 3005, normalized size of antiderivative = 6.63

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-192*(336*a*e^2*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 28
8*a*e^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 112*a*e^2*c
os(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 336*I*a*e^2*sin(11/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 288*I*a*e^2*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 112*I*a*e^2*sin(3/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*s
qrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(
2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(
2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) +
3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sq
rt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sq
rt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c
) + 3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) +
I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*
I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arcta...
```

3.400.8 Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.400.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.401 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

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3.401.1 Optimal result

Integrand size = 30, antiderivative size = 571

$$\begin{aligned}
\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx &= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5ia^{5/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{5ia^{5/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{5ia^{5/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5ia^{5/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}
\end{aligned}$$

output
```
5/4*I*a^2*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*a^(5/2)*e^
(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x
+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c
))^(1/2)+5/8*I*a^(5/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))
^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c
))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/16*I*a^(5/2)*e^(3/2)*ln(a-2^(1/2)*a^(1
/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*
a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*
x+c))^(1/2)-5/16*I*a^(5/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan
(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x
+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a*(e
*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.401.2 Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.66

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\cos^3(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)-i\sin(dx))\Bigg(2\sec^2(c+dx)(i\cos(c)+\sin(c)}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^3*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] - I*Sin[d*x])*(2*Sec[c +
d*x]^2*(I*Cos[c] + Sin[c]) + 5*Sec[c + d*x]*(I*Cos[2*c + d*x] + Sin[2*c +
d*x]) + (5*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(
Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] -
 Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]
]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)
/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c]
 - I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[
-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(3
/2))/(4*d)
```

3.401.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.80, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3979, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{5}{4}a\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3980}$$

$$\frac{5}{4}a\left(\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{4}a\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow 3976$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow 826$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow 1476$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+dx}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow 1082$$

3.401. $\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan}}{\sqrt{a}\sqrt{e\sec(c+}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow 217$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-}}{\sqrt{}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow 1479$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan}}{\sqrt{e\sec(}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\mathrm{co}\right.}}{2\sqrt{}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow 25$$

3.401. $\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(}{\sqrt{e\sec(c+d}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+\right.}}{2\sqrt{2}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+dx)(}}{2\sqrt{2}\sqrt{a}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{t}}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}\right.$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

3.401. $\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

input

```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
((I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*((I*a
*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*e^3*(
(-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e
*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[
a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*T
an[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*
Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/4
```

3.401.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.401. $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

3.401.4 Maple [A] (verified)

Time = 10.54 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.74

method	result
default	$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-\tan(dx+c)+i\right)\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\left(5i\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-5i\sqrt{\frac{}{\cos(d}}\right.}{}$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/16+1/16*I)/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c))^
(1/2)*(5*I*cos(d*x+c)^2*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+
1)/(1/(cos(d*x+c)+1))^(1/2))-5*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+5*I
*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-7*I*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)-2*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)^2*arcta
nh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))
-5*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-5*sin(d*x+c)*cos(d*x+c)*(1/(cos(d
*x+c)+1))^(1/2)-2*I*(1/(cos(d*x+c)+1))^(1/2)-7*(1/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*(1/(cos(d*x+c)+1))^(1/2))
*(2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)+I*cos(d*x+c)+sin(d*x+c)-I)*a*e/
(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
```

3.401.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.94

$$\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx=\frac{\left(9i\,aee^{(2i\,dx+2i\,c)}+5i\,ae\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{25i\,a^3e^3}{16\,d^2}}\left(de^{(2i\,dx+\right.}}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*((9*I*a*e*e^(2*I*d*x + 2*I*c) + 5*I*a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(25/1
6*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e*e^(2*I*d*x +
2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*I*sqrt(25/16*I*a^3*e^3/d^2)*d)/(a*e))
- sqrt(25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e*e^
(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*I*sqrt(25/16*I*a^3*e^3/d^2)
*d)/(a*e)) + sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/
5*(5*(a*e*e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*I*sqrt(-25/16*I
*a^3*e^3/d^2)*d)/(a*e)) - sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c
) + d)*log(2/5*(5*(a*e*e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*I*
sqrt(-25/16*I*a^3*e^3/d^2)*d)/(a*e)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.401.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.401.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2367 vs. $2(427) = 854$.

Time = 0.53 (sec) , antiderivative size = 2367, normalized size of antiderivative = 4.15

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output

```
32*(144*a*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 80*a*e*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 144*I*a*e*sin(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 80*I*a*e*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*s
qrt(2)*a*e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)
*a*e*sin(2*d*x + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2
)*a*e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*
sin(2*d*x + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*
e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(
2*d*x + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*e*cos
(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(2*d*x
 + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 10*(-I*sqrt(2)*a*e*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a*e...
```

3.401.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \int (e\sec\left(dx+c\right))^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

input

```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output

```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.401.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.402 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$

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3.402.1 Optimal result

Integrand size = 30, antiderivative size = 364

$$\begin{aligned}\int \sqrt{e\sec(c+dx)}(a&+ia\tan(c+dx))^{3/2}\,dx = \frac{3ia^{3/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}\\ &-\frac{3ia^{3/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}\\ &-\frac{3ia^{3/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d}\\ &+\frac{3ia^{3/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d}\\ &+\frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}\end{aligned}$$

output
```
3/2*I*a^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-3/2*I*a^(3/2)*arctan(1+2^(1/2)*e^(1/
2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2
)-3/4*I*a^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*s
ec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+3/4*I*a^
(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))
^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+I*a*(e*sec(d*x+c))
^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.402.2 Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.93

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ae\left(i\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}-3\text{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan(}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan(}}\right.\right.}{}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(a*e*(I*Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]
] - 3*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-
1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c
]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + 3*ArcTanh[(Sqrt[1
+ I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]
*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] +
 Sin[c]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec
[c + d*x]]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

3.402.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3979, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}$$

$$\Big\downarrow \text{826}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\,t}}{\sqrt{}}}{2e}}{2e}\right.}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\int\frac{a-}{a^2+c}\right.}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}$$

$$\Big\downarrow \text{1479}$$

3.402. $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-$$

$$\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-$$

$$\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}c}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-$$

$$\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\,\mathrm{ta}}}{\sqrt{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d}$$

$$\Big\downarrow \; 1103$$

$$\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-$$

$$\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{}{2}}{}\right.}{d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

3.402. $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$

output
```
((-6*I)*a^2*e^2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]]
)/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 +
(Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]]
)]/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[
e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*
a*Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt
[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I
*a*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d + (I*a*Sqrt[e*Sec
[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.402.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

3.402.4 Maple [A] (verified)

Time = 10.48 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.99

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,a\cos(dx+c)\left(3i\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-3i\,\mathrm{arcta}\right.}{}$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c))^(1/2)*a*c
os(d*x+c)*(3*I*cos(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+
c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/
(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*I*(1/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)-3*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/
(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+
1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*I*(1/(cos(d*x+c)+
1))^(1/2)+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2))/(2*I*cos(d*x+c)^2+I*cos(d
*x+c)-2*sin(d*x+c)*cos(d*x+c)-I-sin(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.15

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{4i\,a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}+\sqrt{\frac{9i\,a^3e}{d^2}}d\log\left(\frac{2\left(3\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{}{e^{(2i}}}}{3\,a}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(4*I*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*
d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(9*I*a^3*e/d^2)*d)/a) - sqrt(9*I
*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - s
qrt(9*I*a^3*e/d^2)*d)/a) - sqrt(-9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x
 + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*
c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(-9*I*a^3*e/d^2)*d)/a) + sqrt(-9*I*
a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - sq
rt(-9*I*a^3*e/d^2)*d)/a))/d
```

3.402.6 Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx=\int \sqrt{e\sec{(c+dx)}}(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.402.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1871 vs. 2(268) = 536.

Time = 0.49 (sec) , antiderivative size = 1871, normalized size of antiderivative = 5.14

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-8*(6*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)
*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*
(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*ar
ctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt
(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*a*
cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*a*co
s(2*d*x + 2*c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*a)*arctan2(sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) + 6*(I*sqrt(2)*a*cos(2*d*x + 2*c) - sqrt(2)*a*sin(2*d*x + 2*c) +
I*sqrt(2)*a)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(...
```

3.402.8 Giac [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx=\int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.402.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx=\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.403 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

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3.403.2 Mathematica [A] (verified) . . . 2884
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3.403.5 Fricas [A] (verification not implemented) . . . 2890
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3.403.8 Giac [F] . . . 2892
3.403.9 Mupad [F(-1)] . . . 2893

3.403.1 Optimal result

Integrand size $=30$, antiderivative size $=520$

$$\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad+\frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}
\end{aligned}$$

output

```
-1/2*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*se
c(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2
)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a^(5/2)*ln(a+2^(
1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x
+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1
/2)/(a+I*a*tan(d*x+c))^(1/2)+I*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan
(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/e^(1/2)/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*a^(5/2)*arctan(1+2^(1/
2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+
c)*2^(1/2)/d/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4*I
*a*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

3.403.2 Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2ae(\cos(dx)-i\sin(dx))\left(\operatorname{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i}\right.}{}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

output

```
(2*a*e*(Cos[d*x] - I*Sin[d*x])*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[
I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*
Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*((-I)*Cos[2*c] -
Sin[2*c])*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] + Sin[c]]*(2*Sqrt[-1
 - I*Cos[c] - Sin[c]]*(Cos[c] - I*Sin[c])*Sqrt[I - Tan[(d*x)/2]] + ArcTanh
[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c]
- Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*(I*Cos[2*c]
 + Sin[2*c])*Sqrt[I + Tan[(d*x)/2]]))*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan
[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1
 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

3.403.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.79, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3977, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3980}$$

$$-\frac{a^2\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a^2\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3976}$$

$$-\frac{4ia^3e\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow\ 1476$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-i}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow\ 1082$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow\ 217$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan}}{\sqrt{e\sec(c}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow\ 1479$$

3.403. $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+d}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+i}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,t}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \ 1103$$

$$-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

3.403. $\int\frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-4*I)*a^3*e*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (S
qrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]
/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*
Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e
]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a
*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[
a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4*I)*a*Sqrt[a + I*a*
Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

3.403.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.403. $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

3.403.4 Maple [A] (warning: unable to verify)

Time = 10.45 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\left(i\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-\cot(dx+c)+\csc(dx+c)-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\right)\sqrt{(\csc^2(dx+c))(1-\cos(dx+c))^2+1}+i\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-\cot(dx+c)+\csc(dx+c)+}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+}}\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(I*2^(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+
c)^2*(1-cos(d*x+c))^2+1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)+I*
2^(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-co
s(d*x+c))^2+1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)-2^(1/2)*arct
anh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+
1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)+2^(1/2)*arctanh(1/2*(-co
t(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*(c
sc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)-8*I*(csc(d*x+c)-cot(d*x+c))-8)*(csc(
d*x+c)^2*(1-cos(d*x+c))^2-1)*(-a*(2*I*(csc(d*x+c)-cot(d*x+c))-csc(d*x+c)^2
*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(3/2)/(-csc(d*x+c)
+cot(d*x+c)+I)^3/(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-co
s(d*x+c))^2-1))^(1/2)
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx =$$

$$-\frac{de\sqrt{\frac{4i\,a^3}{d^2e}}\log\left(\frac{2\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}+i\,de\sqrt{\frac{4i\,a^3}{d^2e}}}{a}\right)-de\sqrt{\frac{4i\,a^3}{d^2e}}\log\left(\frac{2\left(ae^{(2i\,dx+2i\,c)}+\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*(d*e*sqrt(4*I*a^3/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
+ 1/2*I*c) + I*d*e*sqrt(4*I*a^3/(d^2*e)))/a) - d*e*sqrt(4*I*a^3/(d^2*e))*l
og((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*d*e*sqrt(4*I*a^3/(
d^2*e)))/a) + d*e*sqrt(-4*I*a^3/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1
/2*I*d*x + 1/2*I*c) + I*d*e*sqrt(-4*I*a^3/(d^2*e)))/a) - d*e*sqrt(-4*I*a^3
/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*d*e*sqr
t(-4*I*a^3/(d^2*e)))/a) + 8*(I*a*e^(2*I*d*x + 2*I*c) + I*a)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))/(d*e)
```

3.403.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{\sqrt{e\sec{(c+dx)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)/sqrt(e*sec(c + d*x)), x)
```

3.403.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1462 vs. 2(396) = 792.

Time = 0.50 (sec) , antiderivative size = 1462, normalized size of antiderivative = 2.81

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/4*(2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*sqrt(2)*a*arctan2(sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
 + 2*sqrt(2)*a*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + I*sqrt(2)*a*log(2*sqrt(2)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4...
```

3.403.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)/sqrt(e*sec(d*x + c)), x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2), x)
```

3.404 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx$

3.404.1 Optimal result 2894
3.404.2 Mathematica [A] (verified) 2894
3.404.3 Rubi [A] (verified) 2895
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3.404.1 Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx=-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

output
```
-2/3*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(3/2)
```

3.404.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx=-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

3.404.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3969}$$

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

3.404.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.404.4 Maple [A] (verified)

Time = 9.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{2a\sqrt{a(1+i\tan(dx+c))}\,(-i\cos(dx+c)+\sin(dx+c))}{3d\sqrt{e\sec(dx+c)}\,e}$	51
risch	$-\frac{2ia\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{i(dx+c)}}{3e\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

input

```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/d*a*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e*(-I*cos(d*x+c)+s
in(d*x+c))
```

3.404.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{2\left(-i\,ae^{(3i\,dx+3i\,c)}-i\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{3\,de^2}$$

input

```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="fric
as")
```

output

```
2/3*(-I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*
e^2)
```

3.404.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)/(e*sec(c + d*x))**(3/2), x)
```

3.404.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28)=56$.

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx=-\frac{2i\,a^{\frac{3}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{3\,de^{\frac{3}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-2/3*I*a^(3/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 1)^(3/2)/(d*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)^(3/2))
```

3.404.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(3/2), x)
```

3.404.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2), x)
```

3.405 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx$

3.405.1 Optimal result 2899
3.405.2 Mathematica [A] (verified) 2899
3.405.3 Rubi [A] (verified) 2900
3.405.4 Maple [A] (verified) 2901
3.405.5 Fricas [A] (verification not implemented) 2902
3.405.6 Sympy [F] 2902
3.405.7 Maxima [A] (verification not implemented) 2902
3.405.8 Giac [F] 2903
3.405.9 Mupad [B] (verification not implemented) 2903

3.405.1 Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

output
```
-4/5*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(1/2)-2/5*I*(a+I*a*
tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(5/2)
```

3.405.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2a(\cos(dx)-i\sin(dx))(\cos(c+2dx)+i\sin(c+2dx))(3i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{5de(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(-2*a*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*(3*I + 2
*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*e*(e*Sec[c + d*x])^(3/2))
```

3.405.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3969}$$

$$-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]]) - (
((2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))
```

3.405.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.405.4 Maple [A] (verified)

Time = 9.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{2\cos(dx+c)(\tan(dx+c)-i)a\sqrt{a(1+i\tan(dx+c))}\,(2i\sin(dx+c)-3\cos(dx+c))}{5d\sqrt{e\sec(dx+c)}\,e^2}$	68
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+5)}{5e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	74

input

```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/5/d*cos(d*x+c)*(tan(d*x+c)-I)*a*(a*(1+I*tan(d*x+c)))^(1/2)*(2*I*sin(d*x
+c)-3*cos(d*x+c))/(e*sec(d*x+c))^(1/2)/e^2
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(-i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}-5i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{(\frac{1}{2}i\,dx+\frac{1}{2}i}}{5\,de^3}$$

input

```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="fric
as")
```

output

```
1/5*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) - 5*I*a)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
+ 1/2*I*c)/(d*e^3)
```

3.405.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input

```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(5/2),x)
```

output

```
Integral((I*a*(tan(c + d*x) - I))**(3/2)/(e*sec(c + d*x))**(5/2), x)
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(-i\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\sin(\right.}{5\,de^{\frac{5}{2}}}$$

input

```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output

```
1/5*(-I*a*cos(5/2*d*x + 5/2*c) - 5*I*a*cos(1/2*d*x + 1/2*c) + a*sin(5/2*d*
x + 5/2*c) + 5*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(5/2))
```

3.405.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(5/2), x)
```

3.405.9 Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx=$$
$$-\frac{a\,\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,11\mathrm{i}+\cos\left(3\right.\right.}{10\,d\,e^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(5/2),x)
```

output
```
-(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*11i - sin(c + d*x) + cos(3*
c + 3*d*x)*1i - sin(3*c + 3*d*x)))/(10*d*e^3)
```

3.406 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx$

3.406.1 Optimal result 2904
3.406.2 Mathematica [A] (verified) 2904
3.406.3 Rubi [A] (verified) 2905
3.406.4 Maple [A] (verified) 2906
3.406.5 Fricas [A] (verification not implemented) 2907
3.406.6 Sympy [F(-1)] 2907
3.406.7 Maxima [A] (verification not implemented) 2907
3.406.8 Giac [F] 2908
3.406.9 Mupad [B] (verification not implemented) 2908

3.406.1 Optimal result

Integrand size = 30, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}} - \frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

output
```
16/21*I*a^2*(e*sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-8/21*I*a*(
a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a*tan(d*x+c)
)^(3/2)/d/(e*sec(d*x+c))^(7/2)
```

3.406.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a(\cos(dx)-i\sin(dx))(-7i+9i\cos(2(c+dx))+12\sin(2(c+dx)))(\cos(c+}{21de^3\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(a*(Cos[d*x] - I*Sin[d*x])*(-7*I + (9*I)*Cos[2*(c + d*x)] + 12*Sin[2*(c +
d*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(21
*d*e^3*Sqrt[e*Sec[c + d*x]])
```

3.406.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4a\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

---

output
```
(((-2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2)) + (4*
a*((((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(7*e^
2)
```

3.406.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.406.4 Maple [A] (verified)

Time = 9.70 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2 \cos \left(d x +c \right) \left(\tan \left(d x +c \right)-i\right) a \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}\, \left(-12 i \cos \left(d x +c \right) \sin \left(d x +c \right)+9 \left(\cos^{2}\left(d x +c \right)\right)-8\right)}{21 d \sqrt{e \sec \left(d x +c \right)}\, e^{3}}$	77
risch	$-\frac{i a \sqrt{\frac{a \,{\mathrm e}^{2 i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, \left(3 \,{\mathrm e}^{3 i \left(d x +c \right)}-7 \cos \left(d x +c \right)+35 i \sin \left(d x +c \right)\right)}{42 e^{3} \sqrt{\frac{e \,{\mathrm e}^{i \left(d x +c \right)}}{{\mathrm e}^{2 i \left(d x +c \right)}+1}}\, d}$	92

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/d*cos(d*x+c)*(tan(d*x+c)-I)*a*(a*(1+I*tan(d*x+c)))^(1/2)*(-12*I*sin(
d*x+c)*cos(d*x+c)+9*cos(d*x+c)^2-8)/(e*sec(d*x+c))^(1/2)/e^3
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,ae^{(6i\,dx+6i\,c)}-17i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}+21i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{}}{42\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="fric
as")
```

output
```
1/42*(-3*I*a*e^(6*I*d*x + 6*I*c) - 17*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2
*I*d*x + 2*I*c) + 21*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4)
```

3.406.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,a\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-14i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+21i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{42\,de^{\frac{7}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
1/42*(-3*I*a*cos(7/2*d*x + 7/2*c) - 14*I*a*cos(3/2*d*x + 3/2*c) + 21*I*a*c
os(1/2*d*x + 1/2*c) + 3*a*sin(7/2*d*x + 7/2*c) + 14*a*sin(3/2*d*x + 3/2*c)
 + 21*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(7/2))
```

3.406.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(7/2), x)
```

3.406.9 Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a\,\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,4\mathrm{i}-\cos(4\,c+}{84\,d\,e^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(7/2),x)
```

output
```
(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*4i - cos(4*c + 4*d*x)*3i
 + 38*sin(2*c + 2*d*x) + 3*sin(4*c + 4*d*x) + 7i))/(84*d*e^4)
```

3.407 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx$

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3.407.1 Optimal result

Integrand size = 30, antiderivative size = 167

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

output
```
16/45*I*a^2/d/e^4/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/15*I*a*(
a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(5/2)-32/45*I*a*(a+I*a*tan(d*
x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-2/9*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e
*sec(d*x+c))^(9/2)
```

3.407.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{a(\cos(dx)-i\sin(dx))(-81i\cos(c+dx)+5i\cos(3(c+dx))-54\sin(c+dx}{90de^4\sqrt{e\,\mathrm{se}}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(a*(Cos[d*x] - I*Sin[d*x])*((-81*I)*Cos[c + d*x] + (5*I)*Cos[3*(c + d*x)]
- 54*Sin[c + d*x] + 10*Sin[3*(c + d*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x
])*Sqrt[a + I*a*Tan[c + d*x]])/(90*d*e^4*Sqrt[e*Sec[c + d*x]])
```

3.407.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3978, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3978}$$

$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3978}$$

$$\frac{2a\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3983}$$

$$\frac{2a\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{2a\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{3e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

output
```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(9/2)) + (2*
a*((((-2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4
*a*(((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I
)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(3
*e^2)
```

3.407.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.407. $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx$

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.407.4 Maple [A] (verified)

Time = 9.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57

method	result	si
default	$\frac{2\cos(dx+c)(\tan(dx+c)-i)a\sqrt{a(1+i\tan(dx+c))}\,(10i(\cos^2(dx+c))\sin(dx+c)-5(\cos^3(dx+c))-16i\sin(dx+c)+24\cos(dx+c))}{45d\sqrt{e\sec(dx+c)}\,e^4}$	95
risch	$-\frac{ia\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(5\,\mathrm{e}^{4i(dx+c)}+135+12\cos(2dx+2c)+42i\sin(2dx+2c))}{180e^4\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	99

input
```
int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d*cos(d*x+c)*(tan(d*x+c)-I)*a*(a*(1+I*tan(d*x+c)))^(1/2)*(10*I*cos(d*
x+c)^2*sin(d*x+c)-5*cos(d*x+c)^3-16*I*sin(d*x+c)+24*cos(d*x+c))/(e*sec(d*x
+c))^(1/2)/e^4
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{(-5i\,ae^{(8i\,dx+8i\,c)}-32i\,ae^{(6i\,dx+6i\,c)}-162i\,ae^{(4i\,dx+4i\,c)}-120i\,ae^{(2i\,dx+2i\,c)}+1}{180\,de^5}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="fric
as")
```

output
```
1/180*(-5*I*a*e^(8*I*d*x + 8*I*c) - 32*I*a*e^(6*I*d*x + 6*I*c) - 162*I*a*e
^(4*I*d*x + 4*I*c) - 120*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I
*c)/(d*e^5)
```

3.407.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{(-5i\,a\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+15i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-27i\,a\cos(\frac{5}{3}\,\arctan(\sin(\frac{3}{2}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxi
ma")
```

output
```
1/180*(-5*I*a*cos(9/2*d*x + 9/2*c) + 15*I*a*cos(3/2*d*x + 3/2*c) - 27*I*a*
cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 135*I*a*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*a*sin(9/2*d*
x + 9/2*c) + 15*a*sin(3/2*d*x + 3/2*c) + 27*a*sin(5/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 135*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(9/2))
```

3.407.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}{(e\sec\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(9/2), x)
```

3.407.9 Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx =$$
$$-\frac{a\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-42\sin\left(c+d\,x\right)-47\sin\left(3\,c+3\,d\,x\right)-5\sin\left(5\,c+5\,d\,x\right)+\right.}{360\,d\,e^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(9/2),x)
```

output
```
-(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*282i - 42*sin(c + d*x) + co
s(3*c + 3*d*x)*17i + cos(5*c + 5*d*x)*5i - 47*sin(3*c + 3*d*x) - 5*sin(5*c
 + 5*d*x)))/(360*d*e^5)
```

3.408 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

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3.408.1 Optimal result

Integrand size = 30, antiderivative size = 612

$$\begin{aligned}
\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx &= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{15ia^{7/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{15ia^{7/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{15ia^{7/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{15ia^{7/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d}
\end{aligned}$$

output
```
15/8*I*a^3*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-15/16*I*a^(7/2)
*e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(
d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*
x+c))^(1/2)+15/16*I*a^(7/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*
x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+15/32*I*a^(7/2)*e^(3/2)*ln(a-2^(1/2
)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)
*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*
tan(d*x+c))^(1/2)-15/32*I*a^(7/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-
I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*
sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/4
*I*a^2*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/3*I*a*(e*sec(d*x+
c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2)/d
```

3.408.2 Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.63

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\cos^4(c+dx)(e\sec(c+dx))^{3/2}\left(\frac{1}{6}\sec^3(c+dx)(63+79\cos(2(c+dx))+34i\sin(2(c\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^4*(e*Sec[c + d*x])^(3/2)*((Sec[c + d*x]^3*(63 + 79*Cos[2*(c
+ d*x)] + (34*I)*Sin[2*(c + d*x)])*(I*Cos[3*c + d*x] + Sin[3*c + d*x]))/6
+ (15*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[
-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[
c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqr
t[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])
]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[3*c] - I*
Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 +
I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/
(8*d*(Cos[d*x] + I*Sin[d*x])^2)
```

3.408.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.82, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3979, 3042, 3979, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\Bigg(\frac{5}{4}a\Bigg(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\Bigg)+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}\Bigg)+$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3980}$$

$$\frac{3}{2}a\Bigg(\frac{5}{4}a\Bigg(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\Bigg)+\frac{ia\sqrt{a+ia\tan(c+}}{}$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\Bigg(\frac{5}{4}a\Bigg(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\Bigg)+\frac{ia\sqrt{a+ia\tan(c+}}{}$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{3}{2}a\Bigg(\frac{5}{4}a\Bigg(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\Bigg)+\frac{ia\sqrt{a}}{}$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{826}$$

$$\frac{3}{2}a\Bigg(\frac{5}{4}a\Bigg(\frac{2ia^2e^3\sec(c+dx)\Bigg(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{1476}$$

3.408. $\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\mathrm{co}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 1082$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a}}{\sqrt{a}\sqrt{}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 217$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 1479$$

3.408. $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}}{}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e}\sqrt{e\sec(c+dx)}}\right.}}{}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow 25$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-}}{\sqrt{e\,s}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\right.}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow 27$$

3.408. $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia}}{\sqrt{e\sec(}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{2}a\left(\frac{5}{4}a\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\mathrm{co}\right.}{2\sqrt{2}\sqrt{a}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}\right.\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/3)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2))/d + (3*a*(((
I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (5*a*((I*a*(
e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*e^3*((-
(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*S
ec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]
*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan
[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Ta
n[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]/(
2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c + d
*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/4))/2
```

3.408. $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

3.408.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.408.4 Maple [A] (verified)

Time = 9.46 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.91

method	result
default	$\left(-\frac{1}{96}-\frac{i}{96}\right)\left(-\tan(dx+c)+i\right)^{2}\sqrt{a\left(1+i\tan(dx+c)\right)}\,\sqrt{e\sec(dx+c)}\,\Bigg(-45\sin(dx+c)\left(\cos^{2}(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)+1}}+34\sin(dx+c)\cos(dx$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/96-1/96*I)/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c
))^(1/2)*(-45*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+34*sin(d*x+
c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+45*I*cos(d*x+c)^3*arctanh(1/2*(cos(
d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+8*I*(1/(cos(
d*x+c)+1))^(1/2)-79*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+45*cos(d*x+c)^
3*arctanh(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^
(1/2))-45*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-79*(1/(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)^2-26*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+8*sin(d*x+c)*(1/(cos
(d*x+c)+1))^(1/2)-8*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-34*I*(1/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-45*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)^3+45*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-26*I*(1/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)+8*(1/(cos(d*x+c)+1))^(1/2))*(4*I*cos(d*x+c)^2*si
n(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-4*cos(d*x+c)^3-I*sin(d*x+c)-2*cos(d*x+c
)^2+3*cos(d*x+c)+1)*a^2*e/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.04

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\left(113i\,a^2ee^{(4i\,dx+4i\,c)}+126i\,a^2ee^{(2i\,dx+2i\,c)}+45i\,a^2e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{(\frac{1}{2}i\,dx}}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/12*((113*I*a^2*e*e^(4*I*d*x + 4*I*c) + 126*I*a^2*e*e^(2*I*d*x + 2*I*c) +
 45*I*a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(1/2*I*d*x + 1/2*I*c) + 6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*
x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x +
 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*I*sqrt(225/64*I*a^5*e^3/d^2)*d)/(a^
2*e)) - 6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c) - 8*I*sqrt(225/64*I*a^5*e^3/d^2)*d)/(a^2*e)) + 6*sqrt(-225/64*I*
a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/1
5*(15*(a^2*e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*I*sqrt(-22
5/64*I*a^5*e^3/d^2)*d)/(a^2*e)) - 6*sqrt(-225/64*I*a^5*e^3/d^2)*(d*e^(4*I*
d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x
 + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 8*I*sqrt(-225/64*I*a^5*e^3/d^2)*d)/
(a^2*e)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.408.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.408.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(458) = 916$.

Time = 0.65 (sec) , antiderivative size = 3005, normalized size of antiderivative = 4.91

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
192*(1808*a^2*e*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 201
6*a^2*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 720*a^2*e*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1808*I*a^2*e*sin(9/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2016*I*a^2*e*sin(5/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 720*I*a^2*e*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 3*
sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*sqrt
(2)*a^2*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*sqrt
(2)*a^2*e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) +
 3*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*s
qrt(2)*a^2*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*s
qrt(2)*a^2*e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*
c) + 3*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) +
 I*sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3
*I*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arct...
```

3.408.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx=\int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2} \,d x$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.409 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

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3.409.1 Optimal result

Integrand size = 30, antiderivative size = 411

$$\begin{aligned}
\int \sqrt{e \sec(c+dx)}(a &+ia\tan(c+dx))^{5/2}\,dx = \frac{21ia^{5/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} \\
&-\frac{21ia^{5/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} \\
&-\frac{21ia^{5/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} \\
&+\frac{21ia^{5/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} \\
&+\frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} \\
&+\frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}
\end{aligned}$$

output
```
21/8*I*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(
e*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-21/8*I*a^(5/2)*arctan(1+2^(1/2)*e^(
1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1
/2)-21/16*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/
(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+21/1
6*I*a^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+7/4*I*a^2*(e
*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/2*I*a*(e*sec(d*x+c))^(1/2)
*(a+I*a*tan(d*x+c))^(3/2)/d
```

3.409.2 Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.94

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{a^2\sqrt{e\sec(c+dx)}(\cos(2dx)+i\sin(2dx))\sqrt{a+ia\tan(c+dx)}\left(21\text{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)c}{}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
-1/4*(a^2*Sqrt[e*Sec[c + d*x]]*(Cos[2*d*x] + I*Sin[2*d*x])*Sqrt[a + I*a*Ta
n[c + d*x]]*(21*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]
])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqr
t[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2
]] - 21*ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt
[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*
Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqr
t[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*(-9*I + 2*Tan[c + d*x]
)))/(d*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*(Cos[d*x] + I*Sin[d*x])^2*Sqrt[I -
Tan[(d*x)/2]])
```

3.409.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3979, 3042, 3979, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}dx$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3979}$$

$$\frac{7}{4}a\left(\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}a\left(\frac{3}{2}a\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx+\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{3976}$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{826}$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan}{a^2+\cos^2(c+dx)(i\tan}}{}\right.}{d}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+}{2e}\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{}{-\frac{\cos(c+dx}{}}}{}}{2e}\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \text{217}$$

3.409. $\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\int\right.}{d}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 1479$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 25$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\int\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

3.409. $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

$$\Big\downarrow 1103$$

$$\frac{7}{4}a\left(\frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}-\frac{6ia^2e^2\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I/2)*a*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/d + (7*a*(((-6
*I)*a^2*e^2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(S
qrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqr
t[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(
Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d + (I*a*Sqrt[e*Sec[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d))/4
```

3.409. $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

3.409.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.409. $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

3.409.4 Maple [A] (verified)

Time = 10.24 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.09

method	result
default	$\left(\frac{1}{8}-\frac{i}{8}\right)(\tan(dx+c)-i)^2\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a^2\cos(dx+c)\left(11i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)\sin(dx+c)-11i\sqrt{\frac{1}{\cos(dx+c)+}}\right.$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/8-1/8*I)/d*(tan(d*x+c)-I)^2*(e*sec(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(
1/2)*a^2*cos(d*x+c)*(11*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)-1
1*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-21*I*arctanh(1/2*(cos(d*x+c)+sin
(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+11*sin(d*
x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+11*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-9*I*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)-21*cos(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(
d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+
9*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+2*I*(1/(cos(d*x+c)+1))^(1/2)-2*(1/(c
os(d*x+c)+1))^(1/2))/(1/(cos(d*x+c)+1))^(1/2)/(4*cos(d*x+c)^3+2*cos(d*x+c)
^2+4*I*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-1-I*
sin(d*x+c))
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.28

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\left(11i\,a^2e^{(3i\,dx+3i\,c)}+7i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{441i\,a^5e}{16\,d^2}}\left(de^{(2i\,dx+2}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output

```
1/2*((11*I*a^2*e^(3*I*d*x + 3*I*c) + 7*I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c) + sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21*
(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*sqrt(441/16*I*a^5*e/
d^2)*d)/a^2) - sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/
21*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*sqrt(441/16*I
*a^5*e/d^2)*d)/a^2) - sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d
)*log(2/21*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*sqrt(
-441/16*I*a^5*e/d^2)*d)/a^2) + sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2
*I*c) + d)*log(2/21*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
- 4*sqrt(-441/16*I*a^5*e/d^2)*d)/a^2))/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.409.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input

```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.409.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2431 vs. $2(299) = 598$.

Time = 0.87 (sec) , antiderivative size = 2431, normalized size of antiderivative = 5.91

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input

```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
32*(176*a^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*a^2
*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*I*a^2*sin(7/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*I*a^2*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2
*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(
2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt
(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^
2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*
a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*si
n(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*c
os(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d
*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 42*(-I*sqrt(2)*a^2*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a...
```

3.409.8 Giac [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx=\int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.410 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

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3.410.1 Optimal result

Integrand size = 30, antiderivative size = 563

$$\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{5ia^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5ia^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5ia^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{5ia^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}
\end{aligned}$$

output
```
5/2*I*a^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/
(a+I*a*tan(d*x+c))^(1/2)-5/2*I*a^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan
(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/4*I*a^(7/2)*ln(a-2^(1/
2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c
)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2
)/(a+I*a*tan(d*x+c))^(1/2)+5/4*I*a^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I
*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*s
ec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1
/2)-10*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)+I*a*(a+I*a*ta
n(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(1/2)
```

3.410.2 Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{e^2(a+ia\tan(c+dx))^{5/2}\left(\frac{5\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{d}{2}\right.}}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(e^2*(a + I*a*Tan[c + d*x])^(5/2)*((5*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]
*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/
2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*(Cos[3*c] - I*Sin[3*c])*Sqrt[I + Tan[(d*
x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]]) - (5*ArcTanh
[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c]
+ Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 + I*Cos[c] - Sin[c]]*(Cos[3*c] -
 I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I
- Tan[(d*x)/2]]) - (Cos[2*c] - I*Sin[2*c])*(9*I + Tan[c + d*x])))/(d*(e*Se
c[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

3.410.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.81, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3979, 3042, 3977, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{5}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sqrt{e\sec(c+dx)}}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{2}a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{\sqrt{e\sec(c+dx)}}dx+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3977}$$

$$\frac{5}{2}a\left(-\frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{2}a\left(-\frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{e^2}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3980}$$

$$\frac{5}{2}a\left(-\frac{a^2\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{2}a\left(-\frac{a^2\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3976}$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\right)+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+}}{}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 1082}$$

3.410. $\int\frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}dx$

$$\frac{5}{2}a\Bigg(-\frac{4ia^3e\sec(c+dx)\Bigg(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\Big(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan}}{\sqrt{a}\sqrt{e\sec(c}}}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 217$$

$$\frac{5}{2}a\Bigg(-\frac{4ia^3e\sec(c+dx)\Bigg(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a}}{\sqrt{}}}{2e}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 1479$$

$$\frac{5}{2}a\Bigg(-\frac{4ia^3e\sec(c+dx)\Bigg(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia}}{\sqrt{e\sec}}}{\sqrt{e}\Big(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+c}}{2}}{}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{}}$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 25$$

3.410. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan}}{\sqrt{e\sec(c+}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c\right.}}{2\sqrt{2}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c-}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+dx}}{2\sqrt{2}\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 1103$$

$$\frac{5}{2}a\left(-\frac{4ia^3e\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+d\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+d}}\right.$$

$$\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

3.410. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

output
```
(I*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sqrt[e*Sec[c + d*x]]) + (5*a*(((-4*I
)*a^3*e*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[
a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]
*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt
[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[
a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
+ d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
 + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4*I)*a*Sqrt[a + I*a*Tan[c
+ d*x]])/(d*Sqrt[e*Sec[c + d*x]])))/2
```

3.410.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.410. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.410.4 Maple [A] (verified)

Time = 10.55 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.17

method	result
default	$-\frac{(\tan(dx+c)-i)^2\sqrt{a(1+i\tan(dx+c))}\,a^2\cos(dx+c)\Bigg(-5i\,\operatorname{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))+5}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)*(-5*I*ar
ctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/
2))*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+5*I*(1/(cos(d*x+c)+1))^(1/2)*arc
tanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
)*cos(d*x+c)^2-5*cos(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(
d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)^
2*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^
(1/2))*(1/(cos(d*x+c)+1))^(1/2)-5*I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)
/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)+5*I*(1/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(co
s(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-16*I*cos(d*x+c)^2-5*cos(d
*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)
+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+
sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^
(1/2)+16*sin(d*x+c)*cos(d*x+c)-18*I*cos(d*x+c)-2*sin(d*x+c)-2*I)/(e*sec(d*
x+c))^(1/2)/(4*cos(d*x+c)^3+2*cos(d*x+c)^2+4*I*cos(d*x+c)^2*sin(d*x+c)-3*c
os(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-1-I*sin(d*x+c))
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{\frac{25i\,a^5}{d^2e}}de\log\left(\frac{2\left(5\,\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}+i\,\sqrt{\frac{25i\,a^5}{d^2e}}de\right)}{5\,a^2}\right)-\sqrt{\frac{25i\,a^5}{d^2e}}de\log\Bigg(\frac{2\Big(5\,(}{}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*(sqrt(25*I*a^5/(d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1
/2*I*d*x + 1/2*I*c) + I*sqrt(25*I*a^5/(d^2*e))*d*e)/a^2) - sqrt(25*I*a^5/(
d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) -
 I*sqrt(25*I*a^5/(d^2*e))*d*e)/a^2) + sqrt(-25*I*a^5/(d^2*e))*d*e*log(2/5*
(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + I*sqrt(-25*I*a^5/(d
^2*e))*d*e)/a^2) - sqrt(-25*I*a^5/(d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x
+ 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(-25*I*a^5/(d^2*e))*d*e)/a^2) +
4*(4*I*a^2*e^(2*I*d*x + 2*I*c) + 5*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e)
```

3.410.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.410.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2013 vs. $2(427) = 854$.

Time = 1.02 (sec) , antiderivative size = 2013, normalized size of antiderivative = 3.58

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
8*(10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqr
t(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
) + 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sq
rt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) +
sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) + 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) +
 sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1) - 10*(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c
) - I*sqrt(2)*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 10*(I*sqrt(2)*a^2*cos(2*d*x +
 2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*arctan2(-sqrt(2)*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*...
```

3.410.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}{\sqrt{e\sec{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)/sqrt(e*sec(d*x + c)), x)
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{5/2}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input

```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2),x)
```

output

```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2), x)
```

3.411 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$

3.411.1 Optimal result 2953
3.411.2 Mathematica [A] (verified) 2954
3.411.3 Rubi [A] (verified) 2954
3.411.4 Maple [A] (warning: unable to verify) 2959
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3.411.8 Giac [F] 2961
3.411.9 Mupad [F(-1)] 2962

3.411.1 Optimal result

Integrand size = 30, antiderivative size = 362

$$\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx &= -\frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} \\
&+ \frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} \\
&+ \frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}} \\
&- \frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}} \\
&- \frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}
\end{aligned}$$

output
```
1/2*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/e^(3/2)*2^(1/2)-1/2*I*a^(5
/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(
1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/e^(3/2)*2^(1/2)-I*a^(5/2)*arctan(1-2
^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1
/2)/d/e^(3/2)+I*a^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/
a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d/e^(3/2)-4/3*I*a*(a+I*a*tan(d*x+c))
^(3/2)/d/(e*sec(d*x+c))^(3/2)
```

3.411.2 Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{e\left(-\frac{4}{3}i\cos(dx)(\cos(c)-i\sin(c))+\frac{4}{3}(\cos(c)-i\sin(c))\sin(dx)+\frac{2\left(\text{arctan}\right.}{}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

output
```
(e*(((-4*I)/3)*Cos[d*x]*(Cos[c] - I*Sin[c]) + (4*(Cos[c] - I*Sin[c])*Sin[d
*x])/3 + (2*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/
(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c]
- Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c
]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x
)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c
] - I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*S
qrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^
(5/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

3.411.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3977, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3977}$$

$$-\frac{a^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$-\frac{a^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{e^2}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3976$$

$$\frac{4ia^3\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 826$$

$$\frac{4ia^3\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 1476$$

$$\frac{4ia^3\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d}{2e}}{2e}\right.}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 1082$$

$$\frac{4ia^3\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 217$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

3.411. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}dx$

$$\Big\downarrow \; 1479$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d\sqrt{}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c}}{\sqrt{e\sec}}}{2\sqrt{2}\sqrt{a}e}}{}\right.}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 1103$$

$$\frac{4ia^3\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\log}{}}{2e}\right.}{d}$$

$$\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

3.411. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$

input

```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

output

```
((4*I)*a^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (((4*I)/3)*a*(a + I*
a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

3.411.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3977
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Simp[b^2*((m + 2*n - 2)/(d^2*m))   Int[(d*Sec[e + f*x])
^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x]
&& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
 (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &
& LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)]))
&& IntegerQ[2*m]
```

3.411. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$

3.411.4 Maple [A] (warning: unable to verify)

Time = 10.55 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.30

method	result
default	$\frac{\left(\frac{1}{6}+\frac{i}{6}\right)\left(\cos^{2}(dx+c)\right)\left(\tan(dx+c)-i\right)^{2}\sqrt{a(1+i\tan(dx+c))}\,a^{2}\Bigg(-4i\sin(dx+c)-4i\cos(dx+c)-3\sqrt{\frac{1}{\cos(dx+c)+1}}\ \operatorname{arctanh}\Bigg(\frac{\cos(dx+c)}{2(\cos(dx+c)+}}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6+1/6*I)/d*cos(d*x+c)^2*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2
*(-4*I*sin(d*x+c)-4*I*cos(d*x+c)-3*(1/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*(c
os(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+3*I*(1/(c
os(d*x+c)+1))^(1/2)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/
(1/(cos(d*x+c)+1))^(1/2))+4*sin(d*x+c)-4*cos(d*x+c)+3*sin(d*x+c)*arctanh(1
/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/
(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(
cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)+3*I*arcta
nh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))
*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c
)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c))*(2*I*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+1)/e/(e*sec(d*x+c))^(1/
2)
```

3.411.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.40

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx =$$

$$-\ 3\,de^{2}\sqrt{\frac{4i\,a^{5}}{d^{2}e^{3}}}\log\left(\frac{de^{2}\sqrt{\frac{4i\,a^{5}}{d^{2}e^{3}}}+2\left(a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}}{a^{2}}\right)-3\,de^{2}\sqrt{\frac{4i\,a^{5}}{d^{2}e^{3}}}\log\Bigg(-\frac{de^{2}\sqrt{}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
-1/6*(3*d*e^2*sqrt(4*I*a^5/(d^2*e^3))*log((d*e^2*sqrt(4*I*a^5/(d^2*e^3)) +
 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/a^2) - 3*d*e^2*sqrt(
4*I*a^5/(d^2*e^3))*log(-(d*e^2*sqrt(4*I*a^5/(d^2*e^3)) - 2*(a^2*e^(2*I*d*x
 + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/a^2) - 3*d*e^2*sqrt(-4*I*a^5/(d^2*e^3)
)*log((d*e^2*sqrt(-4*I*a^5/(d^2*e^3)) + 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2
*I*d*x + 1/2*I*c))/a^2) + 3*d*e^2*sqrt(-4*I*a^5/(d^2*e^3))*log(-(d*e^2*sqr
t(-4*I*a^5/(d^2*e^3)) - 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
))/a^2) + 8*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c))/(d*e^2)
```

3.411.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.411.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1492 vs. $2(268) = 536$.

Time $= 0.50$ (sec) , antiderivative size $= 1492$, normalized size of antiderivative $= 4.12$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output

```
-1/12*(-6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*sqrt(2)*a^2*arctan2(sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) + 6*sqrt(2)*a^2*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*I*sqrt(2)*a^2*
log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2...
```

3.411.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}{(e\sec\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

input

```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac
")
```

output

```
integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(3/2), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2), x)
```

3.412 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx$

3.412.1 Optimal result 2963
3.412.2 Mathematica [A] (verified) 2963
3.412.3 Rubi [A] (verified) 2964
3.412.4 Maple [A] (verified) 2965
3.412.5 Fricas [B] (verification not implemented) 2965
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3.412.7 Maxima [B] (verification not implemented) 2966
3.412.8 Giac [F] 2966
3.412.9 Mupad [B] (verification not implemented) 2967

3.412.1 Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

output
```
-2/5*I*(a+I*a*tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(5/2)
```

3.412.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

3.412.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3969}$$

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

3.412.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.412.4 Maple [A] (verified)

Time = 9.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{2i\left(\cos^{2}\left(dx+c\right)\right)\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,a^{2}\left(\tan\left(dx+c\right)-i\right)^{2}}{5d\sqrt{e\sec\left(dx+c\right)}\,e^{2}}$	57
risch	$-\frac{2ia^{2}\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{2i(dx+c)}}{5e^{2}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	74

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*I/d*cos(d*x+c)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(tan(d*x+c)-I)^2/(e*se
c(d*x+c))^(1/2)/e^2
```

3.412.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{2\left(-i\,a^{2}e^{(4i\,dx+4i\,c)}-i\,a^{2}e^{(2i\,dx+2i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\,de^{3}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
2/5*(-I*a^2*e^(4*I*d*x + 4*I*c) - I*a^2*e^(2*I*d*x + 2*I*c))*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c)/(d*e^3)
```

3.412.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.412.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i\,a^{\frac{5}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{5\,de^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
-2/5*I*a^(5/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 1)^(5/2)/(d*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1)^(5/2))
```

3.412.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(5/2), x)
```

3.412.9 Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.74

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{a^2\,\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}+\cos\left(3\,c\right.\right.}{5\,d\,e^3}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(5/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*1i - sin(c + d*x) + cos(3
*c + 3*d*x)*1i - sin(3*c + 3*d*x)))/(5*d*e^3)
```

3.413 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx$

3.413.1 Optimal result 2968
3.413.2 Mathematica [A] (verified) 2968
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3.413.1 Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

output
```
-4/21*I*a*(a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a
*tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(7/2)
```

3.413.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2a^2(\cos(2(c+2dx))+i\sin(2(c+2dx)))(5i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(-2*a^2*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*(5*I + 2*Tan[c + d*x])
*Sqrt[a + I*a*Tan[c + d*x]])/(21*d*e^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] +
I*Sin[d*x])^2)
```

3.413.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3978$$

$$\frac{2a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 3969$$

$$-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

output
```
(((-4*I)/21)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(3/2)
) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(7/2))
```

3.413.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.413.4 Maple [A] (verified)

Time = 9.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2(\tan(dx+c)-i)^{2} a^{2} \sqrt{a(1+i\tan(dx+c))}\,(5i(\cos^{3}(dx+c))+2(\cos^{2}(dx+c))\sin(dx+c))}{21d\sqrt{e\sec(dx+c)}\,e^{3}}$	76
risch	$-\frac{ia^{2}\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(3\,\mathrm{e}^{3i(dx+c)}+7\,\mathrm{e}^{i(dx+c)})}{21e^{3}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	88

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d*(tan(d*x+c)-I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2
)/e^3*(5*I*cos(d*x+c)^3+2*cos(d*x+c)^2*sin(d*x+c))
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,a^2e^{(5i\,dx+5i\,c)}-10i\,a^2e^{(3i\,dx+3i\,c)}-7i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2}}}}{21\,de^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="fric
as")
```

output
```
1/21*(-3*I*a^2*e^(5*I*d*x + 5*I*c) - 10*I*a^2*e^(3*I*d*x + 3*I*c) - 7*I*a^
2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^4)
```

3.413.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-7i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,a^2\cos\left(\frac{7}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\right.\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
1/21*(-7*I*a^2*cos(3/2*d*x + 3/2*c) - 3*I*a^2*cos(7/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 7*a^2*sin(3/2*d*x + 3/2*c) + 3*a^2*sin(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(7/
2))
```

3.413.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}{(e\sec\left(dx+c\right))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(7/2), x)
```

3.413.9 Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx=$$
$$-\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,10\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,3\mathrm{i}-10\,\sin\left(2\,c+2\,d\right.\right.}{42\,d\,e^4}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(7/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*10i + cos(4*c + 4*d*x
)*3i - 10*sin(2*c + 2*d*x) - 3*sin(4*c + 4*d*x) + 7i))/(42*d*e^4)
```

3.414 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx$

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3.414.1 Optimal result

Integrand size = 30, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

output

```
-16/45*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-8/45*I*a*
(a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(5/2)-2/9*I*(a+I*a*tan(d*x+c
))^(5/2)/d/(e*sec(d*x+c))^(9/2)
```

3.414.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{a^2(9+25\cos(2(c+dx))-20i\sin(2(c+dx)))(-i\cos(2(c+2dx))+\sin(2(c}{45de^4\sqrt{e\sec(c+dx)}(\cos(dx)+i\sin(dx))^2}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

output

```
(a^2*(9 + 25*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((-I)*Cos[2*(c +
2*d*x)] + Sin[2*(c + 2*d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(45*d*e^4*Sqrt[e
*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

3.414.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{5/2}}dx}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\right)}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\left(\frac{2a\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{5e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\right)}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4a\left(-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\right)}{9e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

3.414. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}dx$

output
```
(((-2*I)/9)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(9/2)) + (4*
a*((((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
- (((2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))))/(9
*e^2)
```

3.414.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.414.4 Maple [A] (verified)

Time = 9.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2(\tan(dx+c)-i)^{2}a^{2}\sqrt{a(1+i\tan(dx+c))}\,(25i(\cos^{4}(dx+c))+20(\cos^{3}(dx+c))\sin(dx+c)-8i(\cos^{2}(dx+c)))}{45d\sqrt{e\sec(dx+c)}\,e^{4}}$	87
risch	$-\frac{ia^{2}\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(5\,\mathrm{e}^{4i(dx+c)}+18\,\mathrm{e}^{2i(dx+c)}+45)}{90e^{4}\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	89

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/d*(tan(d*x+c)-I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2
)/e^4*(25*I*cos(d*x+c)^4+20*cos(d*x+c)^3*sin(d*x+c)-8*I*cos(d*x+c)^2)
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a^2e^{(6i\,dx+6i\,c)} - 23i\,a^2e^{(4i\,dx+4i\,c)} - 63i\,a^2e^{(2i\,dx+2i\,c)} - 45i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}}{90\,de^5}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="fric
as")
```

output
```
1/90*(-5*I*a^2*e^(6*I*d*x + 6*I*c) - 23*I*a^2*e^(4*I*d*x + 4*I*c) - 63*I*a
^2*e^(2*I*d*x + 2*I*c) - 45*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^5)
```

3.414.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(9/2),x)
```

output
```
Timed out
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a^2\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right) - 18i\,a^2\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right) - 45i\,a^2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) +}{90\,de}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxi
ma")
```

output
```
1/90*(-5*I*a^2*cos(9/2*d*x + 9/2*c) - 18*I*a^2*cos(5/2*d*x + 5/2*c) - 45*I
*a^2*cos(1/2*d*x + 1/2*c) + 5*a^2*sin(9/2*d*x + 9/2*c) + 18*a^2*sin(5/2*d*
x + 5/2*c) + 45*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(9/2))
```

3.414.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}{(e\sec\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(9/2), x)
```

3.414.9 Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx=$$
$$-\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-18\,\sin\left(c+d\,x\right)-23\,\sin\left(3\,c+3\,d\,x\right)-5\,\sin\left(5\,c+5\,d\,x\right)-\right.}{180\,d\,e^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(9/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*108i - 18*sin(c + d*x) +
cos(3*c + 3*d*x)*23i + cos(5*c + 5*d*x)*5i - 23*sin(3*c + 3*d*x) - 5*sin(5
*c + 5*d*x)))/(180*d*e^5)
```

3.415 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx$

3.415.1 Optimal result . . . 2978
3.415.2 Mathematica [A] (verified) . . . 2978
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3.415.1 Optimal result

Integrand size = 30, antiderivative size = 169

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx=\frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}}-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}}-\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

output

```
32/77*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^6/(a+I*a*tan(d*x+c))^(1/2)-16/77*I*a^
2*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(3/2)-12/77*I*a*(a+I*a*tan
(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(7/2)-2/11*I*(a+I*a*tan(d*x+c))^(5/2)/
d/(e*sec(d*x+c))^(11/2)
```

3.415.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx=\frac{a^2(-55i\cos(c+dx)+35i\cos(3(c+dx))-22\sin(c+dx)+42\sin(3(c+dx}{154de^5\sqrt{e\sec(c+dx)}(\cos(dx}$$

input

```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(a^2*((-55*I)*Cos[c + d*x] + (35*I)*Cos[3*(c + d*x)] - 22*Sin[c + d*x] + 4
2*Sin[3*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*Sqrt[a + I
*a*Tan[c + d*x]])/(154*d*e^5*Sqrt[e*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^
2)
```

3.415.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3978, 3042, 3978, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\int \frac{(i\tan(c+dx)a+a)^{3/2}}{(e\sec(c+dx))^{7/2}}dx}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\left(\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4a\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6a\left(\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6a\left(\frac{4a\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6a\left(\frac{4a\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\right)}{11e^2}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

output
```
(((-2*I)/11)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(11/2)) + (
6*a*((((-2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2))
 + (4*a*((((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x
]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/
(7*e^2)))/(11*e^2)
```

3.415.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.415. $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx$

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.415.4 Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{i a^{2} \sqrt{\frac{a\,{\mathrm e}^{2 i (d x+c)}}{{\mathrm e}^{2 i (d x+c)}+1}}\, \left(7\,{\mathrm e}^{5 i (d x+c)}+33\,{\mathrm e}^{3 i (d x+c)}+154 i \sin(d x+c)\right)}{308 e^{5} \sqrt{\frac{e\,{\mathrm e}^{i (d x+c)}}{{\mathrm e}^{2 i (d x+c)}+1}}\, d}$
default	$\frac{2 i (\tan(d x+c)-i)^{2} a^{2} \sqrt{a (1+i \tan(d x+c))}\, (42 i \sin(d x+c) (\cos^{4}(d x+c))-35 (\cos^{5}(d x+c))-16 i (\cos^{2}(d x+c)) \sin(d x+c)+40 (\cos^{3}(d x+}{77 d \sqrt{e \sec(d x+c)}\, e^{5}}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/308*I*a^2/e^5/(e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)*(a*exp(2*I*
(d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)/d*(7*exp(5*I*(d*x+c))+33*exp(3*I*(d*x
+c))+154*I*sin(d*x+c))
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx=\frac{\left(-7i\,a^{2}e^{(8i\,dx+8i\,c)}-40i\,a^{2}e^{(6i\,dx+6i\,c)}-110i\,a^{2}e^{(4i\,dx+4i\,c)}+77i\,a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i}}}}{308\,de^{6}}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="fri
cas")
```

output
```
1/308*(-7*I*a^2*e^(8*I*d*x + 8*I*c) - 40*I*a^2*e^(6*I*d*x + 6*I*c) - 110*I
*a^2*e^(4*I*d*x + 4*I*c) + 77*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^6)
```

3.415.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(11/2),x)
```

output
```
Timed out
```

3.415.7 Maxima [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\left(-7i\,a^2\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)-33i\,a^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-77i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="max
ima")
```

output
```
1/308*(-7*I*a^2*cos(11/2*d*x + 11/2*c) - 33*I*a^2*cos(7/2*d*x + 7/2*c) - 7
7*I*a^2*cos(3/2*d*x + 3/2*c) + 77*I*a^2*cos(1/2*d*x + 1/2*c) + 7*a^2*sin(1
1/2*d*x + 11/2*c) + 33*a^2*sin(7/2*d*x + 7/2*c) + 77*a^2*sin(3/2*d*x + 3/2
*c) + 77*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(11/2))
```

3.415.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="gia
c")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(11/2), x)
```

3.415.9 Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx =$$
$$-\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-187\,\sin\left(2\,c+2\,d\,x\right)-40\,\sin\left(4\,c+4\,d\,x\right)-7\,\sin\left(6\,c+6\right.\right.}{616\,d\,e^6}$$

input
```
int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(11/2),x)
```

output
```
-(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*33i + cos(4*c + 4*d*x
)*40i + cos(6*c + 6*d*x)*7i - 187*sin(2*c + 2*d*x) - 40*sin(4*c + 4*d*x) -
 7*sin(6*c + 6*d*x)))/(616*d*e^6)
```

3.416 $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.416.1 Optimal result 2984
3.416.2 Mathematica [A] (verified) 2985
3.416.3 Rubi [A] (verified) 2985
3.416.4 Maple [B] (warning: unable to verify) 2990
3.416.5 Fricas [A] (verification not implemented) 2990
3.416.6 Sympy [F(-1)] 2991
3.416.7 Maxima [B] (verification not implemented) 2991
3.416.8 Giac [F] 2992
3.416.9 Mupad [F(-1)] 2993

3.416.1 Optimal result

Integrand size = 30, antiderivative size = 369

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} \\ &- \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} \\ &- \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} \\ &+ \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} \\ &- \frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}\end{aligned}$$

output
```
1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/
2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2
)-1/4*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*s
ec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)/a^(1/2)+1/4*I*e^
(5/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))
^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)/a^(1/2)-I*e^2*(e*sec(d*x+c
))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.416.2 Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.95

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^3\left(\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}-i\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos(c}}{\sqrt{-1-i\cos(}}\right.\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e^3*(Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]
- I*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1
- I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]
*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + I*ArcTanh[(Sqrt[1 +
I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*S
qrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + S
in[c]]*Sqrt[I + Tan[(d*x)/2]])*(-I + Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]
]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I*a*Tan[
c + d*x]])
```

3.416.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \ 3982$$

$$\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow 3042$$

$$\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow 3976$$

$$-\frac{2ie^4\int\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow 826$$

$$-\frac{2ie^4\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow 1476$$

$$-\frac{2ie^4\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\mathrm{t}}}{\sqrt{}}}{2e}}{2e}\right.}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow 1082$$

$$-\frac{2ie^4\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\mathrm{c}}{a^2+\mathrm{c}}}{}\right.}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

$$\Big\downarrow 217$$

$$-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

3.416. $\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$

$$\Big\downarrow \text{ 1479}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d}{2\sqrt{2}\sqrt{a}e}}{}\right.}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}$$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-}{2}\right.}{d}}{\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}$$

3.416. $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

input
```
Int[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(S
qrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqr
t[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(
Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

3.416.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

3.416. $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.416.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(285) = 570$.

Time = 15.95 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.07

method	result	size
default	Expression too large to display	765

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^
2-1))^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-csc(d*x+c)+cot(d*x+c)+I)*(
I*csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^
2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2+I*csc(d*x+c)^2*arcta
nh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+cs
c(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-co
s(d*x+c))^2+csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(c
sc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-I*arctanh(
1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(
1/2))*2^(1/2)-I*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)
^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)-4*I*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1
)^(1/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c
)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))-2^(1/2)*arctanh(1/2*
(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
)-4*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))/(-a*(2*I*(csc(d*x+c)-cot(d*x+
c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1
/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(5/2)
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.25

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-4i\,e^{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)} + \sqrt{\frac{i\,e^{5}}{ad^{2}}}ad\log\left(\frac{2\left(\left(e^{2}e^{(2i\,dx+2i\,c)}+e^{2}\right.\right.}{}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*(-4*I*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2
*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*
x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(I*e^5/(a*d^2))*a*d)/e^2) -
 sqrt(I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c) - sqrt(I*e^5/(a*d^2))*a*d)/e^2) - sqrt(-I*e^5/(a*d^2))*a*d*log(2*(
(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(-I*e^5/(a*d^2))*a
*d)/e^2) + sqrt(-I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2*I*d*x + 2*I*c) + e^2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/
2*I*d*x + 1/2*I*c) - sqrt(-I*e^5/(a*d^2))*a*d)/e^2))/(a*d)
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.416.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2258 vs. 2(273) = 546.

Time = 0.78 (sec) , antiderivative size = 2258, normalized size of antiderivative = 6.12

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-8*(16*e^2*cos(3/2*d*x + 3/2*c) + 16*I*e^2*sin(3/2*d*x + 3/2*c) + 2*(sqrt(
2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sq
rt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + s
qrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) + 2*(sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*e^2*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2)*a
rctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 1) + 2*(sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(-I*sqrt(2)*e^2*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2*sin(4/3*arct...
```

3.416.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.417 $\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.417.1 Optimal result

Integrand size = 30, antiderivative size = 483

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{i\sqrt{a}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{i\sqrt{a}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output

```
1/2*I*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d*2^(1/2)
/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*e^(3/2)*ln(a+2^(1
/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+
c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)-I*e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(
d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)*a^(1/2)/d/(
a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+I*e^(3/2)*arctan(1+2^(1/2
)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c
)*2^(1/2)*a^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

3.417.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.63

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2e\sqrt{e\sec(c+dx)}\Bigg(\operatorname{arctanh}\Bigg(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\Bigg)\sqrt{-1-i\cos(c)-}}{d\sqrt{}}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*e*Sqrt[e*Sec[c + d*x]]*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - T
an[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[
-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*
Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqr
t[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin
[c]])*(Cos[d*x] + I*Sin[d*x])*Sqrt[I + Tan[(d*x)/2]])/(d*Sqrt[-1 - I*Cos[c
] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I
*a*Tan[c + d*x]])
```

3.417.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.77, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3980$$

$$\frac{e\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{e\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 3976$$

$$\frac{4iae^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 826$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 1476$$

$$4iae^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}}}{2e}}{2e}\right.$$

$$d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}$$

$$\Big\downarrow \ 1082$$

$$4iae^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.$$

$$d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}$$

$$\Big\downarrow \ 217$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c}}{\sqrt{e\sec(c+d}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 1479$$

3.417. $\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,t}}$$

$$\Big\downarrow \; 25$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-i}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}$$

$$\Big\downarrow \; 27$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\,\mathrm{tan}}{e}}}{2\sqrt{2}\sqrt{a}e}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c-}}$$

$$\Big\downarrow \; 1103$$

$$\frac{4iae^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\,\mathrm{ta}\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

3.417. $\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

output
```
((4*I)*a*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(
Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sq
rt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*
Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a
 - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.417.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

---

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.417.4 Maple [A] (verified)

Time = 14.69 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\,\mathrm{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)\sqrt{e\sec(dx+c)}\,e(\cos(dx+c)+1+i\sin}{d(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{a(1+i\tan(dx+c))}}$

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2-1/2*I)/d*(I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/
(cos(d*x+c)+1))^(1/2))+arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1
)/(1/(cos(d*x+c)+1))^(1/2)))*(e*sec(d*x+c))^(1/2)*e*(cos(d*x+c)+1+I*sin(d*
x+c))/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.80

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{1}{2}\sqrt{\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\,a}{e}\right.$$

$$-\frac{1}{2}\sqrt{\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-i\,ad\sqrt{\frac{4i\,e^3}{ad^2}}}{e}\right)$$

$$+\frac{1}{2}\sqrt{-\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\,ad\sqrt{-\frac{4i\,e^3}{ad^2}}}{e}\right)$$

$$-\frac{1}{2}\sqrt{-\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-i\,ad\sqrt{-\frac{4i\,e^3}{ad^2}}}{e}\right)$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/2*sqrt(4*I*e^3/(a*d^2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c) + I*a*d*sqrt(4*I*e^3/(a*d^2)))/e) - 1/2*sqrt(4*I*e^3/(a*d^2))*log((2*
(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*a*d*sqrt(4*I*e^3/(a*d^2)
))/e) + 1/2*sqrt(-4*I*e^3/(a*d^2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c) + I*a*d*sqrt(-4*I*e^3/(a*d^2)))/e) - 1/2*sqrt(-4*I*e^3/(a*d^
2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*a*d*sqrt(-4*I
*e^3/(a*d^2)))/e)
```

3.417.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.417.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.50

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/4*(2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(
1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*c
os(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e
*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) +
 1) - 2*sqrt(2)*e*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqr
t(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*e*arctan2(-sqrt(
2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + co
s(d*x + c) + 1) + I*sqrt(2)*e*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2
*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 +
 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e*log(-2*sqrt(2)*sin(d*x + c
)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c)
 + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - sqrt(2)*e*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(...
```

3.417.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.418 $\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.418.1 Optimal result 3003
3.418.2 Mathematica [A] (verified) 3003
3.418.3 Rubi [A] (verified) 3004
3.418.4 Maple [A] (verified) 3005
3.418.5 Fricas [B] (verification not implemented) 3005
3.418.6 Sympy [F] 3006
3.418.7 Maxima [B] (verification not implemented) 3006
3.418.8 Giac [F] 3006
3.418.9 Mupad [B] (verification not implemented) 3007

3.418.1 Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.418.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.418.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3969}$$

$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.418.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.418.4 Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}}{d\sqrt{a(1+i\tan(dx+c))}}$	32
risch	$\frac{2i\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}{\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	59

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

3.418.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ad}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(I*e
^(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c)/(a*d)
```

3.418.6 Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\sec\left(c+dx\right)}}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.418.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
2*I*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1))
```

3.418.8 Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(e*sec(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

3.418.9 Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,2\mathrm{i}}{d\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}}}$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*2i)/(d*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2
))
```

3.419 $\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.419.1 Optimal result . . . 3008
3.419.2 Mathematica [A] (verified) . . . 3008
3.419.3 Rubi [A] (verified) . . . 3009
3.419.4 Maple [A] (verified) . . . 3010
3.419.5 Fricas [A] (verification not implemented) . . . 3011
3.419.6 Sympy [F] . . . 3011
3.419.7 Maxima [A] (verification not implemented) . . . 3011
3.419.8 Giac [F] . . . 3012
3.419.9 Mupad [B] (verification not implemented) . . . 3012

3.419.1 Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

output
```
2/3*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(a+I*a*tan(d*x
+c))^(1/2)/a/d/(e*sec(d*x+c))^(1/2)
```

3.419.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-2i+4\tan(c+dx)}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(-2*I + 4*Tan[c + d*x])/(3*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])
```

3.419.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)
*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])
```

3.419.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.419.4 Maple [A] (verified)

Time = 10.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2(i-2\tan(dx+c))}{3d\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}$	42
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}-1\right)}{3\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	85

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3/d/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I-2*tan(d*x+c))
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ade}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
3*I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2
*I*c)/(a*d*e)
```

3.419.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sec\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,\cos\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sin\bigl(\frac{1}{3}\,\mathrm{ar}}{3\,\sqrt{a}d\sqrt{e}}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/3*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d*sqrt(e))
```

3.419.8 Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.419.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\,\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(-2\,\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1{}\mathrm{i}\right)}{3\,d\,e\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-(2*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*1i - 2*sin(c + d*x)))/(3*d*e*((a*
(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2
))
```

3.420 $\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.420.1 Optimal result 3013
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3.420.1 Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}$$

output

```
2/5*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*sec(d*x+c
))^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(a+I*a*tan(d*x+c))^(1/2)/a/
d/(e*sec(d*x+c))^(3/2)
```

3.420.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=-\frac{i\sec^2(c+dx)(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output

```
((-1/15*I)*Sec[c + d*x]^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]
))/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

3.420.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

input

```
Int[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output

```
((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4
*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I
)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)
```

3.420.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.420.4 Maple [A] (verified)

Time = 10.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{2(i\cos(dx+c)-4\sin(dx+c)-8i\sec(dx+c))}{15d\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e}$	61

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/15/d/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/e*(I*cos(d*x+c)-4*
sin(d*x+c)-8*I*sec(d*x+c))
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(6i\,dx+6i\,c)}+25i\,e^{(4i\,dx+4i\,c)}+\right.}{30\,ade^2}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-5*I*e^(6*I*d*x + 6*I*c) + 25*I*e^(4*I*d*x + 4*I*c) + 33*I*e^(2*I*d*x + 2*
I*c) + 3*I)*e^(-5/2*I*d*x - 5/2*I*c)/(a*d*e^2)
```

3.420.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/30*(3*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c),
 cos(5/2*d*x + 5/2*c))) + 30*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 3*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d*e^(3/2))
```

3.420.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.420.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(4\sin\left(2\,c+2\,d\,x\right)-\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+15\mathrm{i}\right)}{15\,d\,e^2\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(4*sin(2*c + 2*d*x) - cos(2*c + 2*d*x)*1i + 15i))/
(15*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d
*x) + 1))^(1/2))
```

3.420. $\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.421 $\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.421.1 Optimal result . . . 3019
3.421.2 Mathematica [A] (verified) . . . 3019
3.421.3 Rubi [A] (verified) . . . 3020
3.421.4 Maple [A] (verified) . . . 3022
3.421.5 Fricas [A] (verification not implemented) . . . 3023
3.421.6 Sympy [F] . . . 3023
3.421.7 Maxima [A] (verification not implemented) . . . 3023
3.421.8 Giac [F] . . . 3024
3.421.9 Mupad [B] (verification not implemented) . . . 3024

3.421.1 Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}+\frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}}-\frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}}$$

output
```
2/7*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+16/35*I/d/e^2/(e*sec
(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(a+I*a*tan(d*x+c))^(1/2)/a
/d/(e*sec(d*x+c))^(5/2)-32/35*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*
x+c))^(1/2)
```

3.421.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.48

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx=-\frac{i(17+\cos(2(c+dx))+3i\sec(c+dx)\sin(3(c+dx))+35i\tan(c+dx))}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((-1/35*I)*(17 + Cos[2*(c + d*x)] + (3*I)*Sec[c + d*x]*Sin[3*(c + d*x)] +
(35*I)*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])
```

3.421.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3983, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (6*((((-
2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2
*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sq
rt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(7*a)
```

3.421. $\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.421.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.421.4 Maple [A] (verified)

Time = 10.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

method	result	size
default	$-\frac{2\left(i\left(\cos^{2}\left(dx+c\right)\right)-6\sin\left(dx+c\right)\cos\left(dx+c\right)+8i-16\tan\left(dx+c\right)\right)}{35d\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,\sqrt{e\sec\left(dx+c\right)}\,e^{2}}$	70

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/35/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e^2*(I*cos(d*x+c)^
2-6*sin(d*x+c)*cos(d*x+c)+8*I-16*tan(d*x+c))
```

3.421.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(8i\,dx+8i\,c)} - 112i\,e^{(6i\,dx+6i\,c)}\right.}{140\,ade^3}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-7*I*e^(8*I*d*x + 8*I*c) - 112*I*e^(6*I*d*x + 6*I*c) - 70*I*e^(4*I*d*x +
4*I*c) + 40*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a*d*e^3
)
```

3.421.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(5/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

3.421.7 Maxima [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{5i\,\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right) - 7i\,\cos\left(\frac{5}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\right.\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/140*(5*I*cos(7/2*d*x + 7/2*c) - 7*I*cos(5/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c))) + 35*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))) - 105*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d
*x + 7/2*c))) + 5*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*
d*x + 7/2*c))))/(sqrt(a)*d*e^(5/2))
```

3.421.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}\sqrt{ia\tan(dx+c)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.421.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(-\sin\left(c+d\,x\right)-\frac{3\,\sin(3\,c+3\,d\,x)}{35}+\frac{\cos(c+d\,x)\,1\mathrm{i}}{2}+\frac{\cos(3\,c+3\,d\,x)\,1\mathrm{i}}{70}\right)}{d\,e^3\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
-((e/cos(c + d*x))^(1/2)*((cos(c + d*x)*1i)/2 - sin(c + d*x) + (cos(3*c +
3*d*x)*1i)/70 - (3*sin(3*c + 3*d*x))/35))/(d*e^3*((a*(cos(2*c + 2*d*x) + s
in(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.422 $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.422.1 Optimal result 3025
3.422.2 Mathematica [A] (verified) 3025
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3.422.9 Mupad [B] (verification not implemented) 3031

3.422.1 Optimal result

Integrand size = 30, antiderivative size = 206

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\\ &+ \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}}\\ &- \frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}\end{aligned}$$

output
```
2/9*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+32/105*I/d/e^2/(e*se
c(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+256/315*I*(e*sec(d*x+c))^(1/2)/d/
e^4/(a+I*a*tan(d*x+c))^(1/2)-16/63*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*sec(d
*x+c))^(7/2)-128/315*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*x+c))^(3/
2)
```

3.422.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.42

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e\sec(c+dx)}(945i-84i\cos(2(c+dx))-5i\cos(4(c+dx)}{1260de^4\sqrt{a+ia\tan(c}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(Sqrt[e*Sec[c + d*x]]*(945*I - (84*I)*Cos[2*(c + d*x)] - (5*I)*Cos[4*(c +
d*x)] + 336*Sin[2*(c + d*x)] + 40*Sin[4*(c + d*x)]))/(1260*d*e^4*Sqrt[a +
I*a*Tan[c + d*x]])
```

3.422.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3978, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{8\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\left(\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6a\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3978}$$

$$\frac{8\left(\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

3.422. $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\frac{8\left(\frac{6a\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3969$$

$$\frac{8\left(\frac{6a\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)}{9a}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}$$

input

```
Int[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output

```
((2*I)/9)/(d*(e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (8*((((-
2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) + (6*a*(((2
*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)
/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3
)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(7*e^2)
))/(9*a)
```

3.422. $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.422.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.422.4 Maple [A] (verified)

Time = 10.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{-\frac{2i\left(\cos^{3}(dx+c)\right)}{63}+\frac{16\left(\cos^{2}(dx+c)\right)\sin(dx+c)}{63}-\frac{32i\cos(dx+c)}{315}+\frac{128\sin(dx+c)}{315}+\frac{256i\sec(dx+c)}{315}}{d\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e^{3}}$	88

input
```
int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2/315/d/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/e^3*(-5*I*cos(d*x+
c)^3+40*cos(d*x+c)^2*sin(d*x+c)-16*I*cos(d*x+c)+64*sin(d*x+c)+128*I*sec(d*
x+c))
```

3.422.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-45i\,e^{(10i\,dx+10i\,c)} - 465i\,e^{(8i\,dx+8}\right.$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/2520*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*(-45*I*e^(10*I*d*x + 10*I*c) - 465*I*e^(8*I*d*x + 8*I*c) + 1470*I*e^(6*I*
d*x + 6*I*c) + 2142*I*e^(4*I*d*x + 4*I*c) + 287*I*e^(2*I*d*x + 2*I*c) + 35
*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a*d*e^4)
```

3.422.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = 35i\,\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right) - 45i\,\cos\left(\frac{7}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\mathrm{co}\right.\right.$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
1/2520*(35*I*cos(9/2*d*x + 9/2*c) - 45*I*cos(7/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) + 252*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) - 420*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) + 1890*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d
*x + 9/2*c))) + 35*sin(9/2*d*x + 9/2*c) + 45*sin(7/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c))))/(sqrt(a)*d*e^(7/2))
```

3.422.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\sec(dx+c))^{\frac{7}{2}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.422.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(336\,\sin\left(2\,c+2\,d\,x\right)-\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}-\cos\left(\right.\right.}{1260\,d\,e^4\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\mathrm{s}\right.}{\cos\left(2\,c+2\,d\right.}}}$$

input
```
int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(336*sin(2*c + 2*d*x) - cos(4*c + 4*d*x)*5i - cos(
2*c + 2*d*x)*84i + 40*sin(4*c + 4*d*x) + 945i))/(1260*d*e^4*((a*(cos(2*c +
 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.423 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.423.1 Optimal result 3032
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3.423.9 Mupad [F(-1)] 3044

3.423.1 Optimal result

Integrand size = 30, antiderivative size = 529

$$
\begin{aligned}
\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3ie^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3ie^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

output
```
-I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)-3/2*I*e^(7/2)*arc
tan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2
))*sec(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)
)^(1/2)+3/2*I*e^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^
(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c)
)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/4*I*e^(7/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2
)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+
c)))*sec(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+
c))^(1/2)-3/4*I*e^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1
/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/
2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

3.423.2 Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.64

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{e(e\sec(c+dx))^{5/2}\left(-i\cos(c+dx)+\sin(c+dx)+\frac{3\cos(c+dx)(\cos(c)+i\sin(c))\left(\right.}{}\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(e*(e*Sec[c + d*x])^(5/2)*((-I)*Cos[c + d*x] + Sin[c + d*x] + (3*Cos[c + d
*x]*(Cos[c] + I*Sin[c])*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan
[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1
 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Co
s[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[
I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c
]])*(Cos[d*x] + I*Sin[d*x])^2*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c]
- Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I
*a*Tan[c + d*x])^(3/2))
```

3.423.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.88, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3981, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3979}$$

$$\frac{3e^2\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3e^2\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3980}$$

$$\frac{3e^2\left(\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3e^2\left(\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec}{d\sqrt{a+ia}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

3.423. $\int\frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 217$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+da}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 1479$$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}\right.}{a}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 25$$

3.423. $\int\frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$3e^2\left(\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-i}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(}}}{a^2}\right.$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow 27$$

$$3e^2\left(\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}}{a^2}\right.$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow 1103$$

3.423. $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{a^2}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-4*I)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*
e^2*((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*
a^2*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[
a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]
*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt
[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[
a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
+ d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
 + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/a^2
```

3.423.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.423. $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.423. $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.423.4 Maple [A] (verified)

Time = 16.85 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.35

method	result
default	$\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{e\sec(dx+c)}\,e^{3}\left(6i\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)+2i\tan(dx+c)\sec(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}-3i\sec(dx$

input
```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/4-1/4*I)/d*(e*sec(d*x+c))^(1/2)*e^3/(-tan(d*x+c)+I)/a/(a*(1+I*tan(d*x+c
)))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(6*I*cos(d*x+c)*arctanh(
1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+2*I
*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-3*I*sec(d*x+c)*arctanh(1/2
*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*ta
n(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)+2*I*sec(d*x
+c)*(1/(cos(d*x+c)+1))^(1/2)-6*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+
c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-6*arctanh(1/2*(-cos(d*x+c)+
sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*I*tan(
d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c
)+1))^(1/2))-6*I*sin(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*
x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*
(1/(cos(d*x+c)+1))^(1/2)+3*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*
x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*tan(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin
(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*arctanh(1/2*(-cos(d*
x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*tan(d*x+c)*s
ec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3
*sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(
d*x+c)+1))^(1/2)))
```

3.423.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.91

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-4i\,e^3\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\sqrt{\frac{9i\,e^7}{a^3d^2}}a^2d\log\left(-\frac{2\left(i\sqrt{\frac{9i\,e^7}{a^3d^2}}a^2\right.}{}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output

```
1/2*(-4*I*e^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) - sqrt(9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(
I*sqrt(9*I*e^7/(a^3*d^2))*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c))/e^3) + sqrt(9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(-I*sqrt(9*I*e^
7/(a^3*d^2))*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/
e^3) - sqrt(-9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(I*sqrt(-9*I*e^7/(a^3*d^2))
*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^3) + sqrt(
-9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(-I*sqrt(-9*I*e^7/(a^3*d^2))*a^2*d - 3*
(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^3))/(a^2*d)
```

3.423.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input

```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output

```
Timed out
```

3.423.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1817 vs. $2(401) = 802$.

Time = 0.51 (sec) , antiderivative size = 1817, normalized size of antiderivative = 3.43

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output

```
-8*(6*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/
2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) +
 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*arctan2(sqrt(2)*cos
(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*a
rctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1
) + 3*I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
 3*I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*
I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*I*s
qrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 16*e^3*c
os(1/2*d*x + 1/2*c) + 16*I*e^3*sin(1/2*d*x + 1/2*c) - 6*(-I*sqrt(2)*e^3*co
s(2*d*x + 2*c) + sqrt(2)*e^3*sin(2*d*x + 2*c) - I*sqrt(2)*e^3)*arctan2(sqr
t(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x + 1/2*c) + c
os(d*x + c) + 1) - 6*(I*sqrt(2)*e^3*cos(2*d*x + 2*c) - sqrt(2)*e^3*sin(2*d
*x + 2*c) + I*sqrt(2)*e^3)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x
 + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 3*(2*sqrt(2)*e^
3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c...
```

3.423.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{(e\sec\left(dx+c\right))^{\frac{7}{2}}}{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

input

```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output

```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan\left(c+d\,x\right)\ 1\mathrm{i})^{3/2}}\,dx$$

input

```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output

```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.424 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.424.1 Optimal result . . . 3045
3.424.2 Mathematica [A] (verified) . . . 3046
3.424.3 Rubi [A] (verified) . . . 3046
3.424.4 Maple [B] (verified) . . . 3051
3.424.5 Fricas [A] (verification not implemented) . . . 3052
3.424.6 Sympy [F(-1)] . . . 3052
3.424.7 Maxima [B] (verification not implemented) . . . 3053
3.424.8 Giac [F] . . . 3054
3.424.9 Mupad [F(-1)] . . . 3054

3.424.1 Optimal result

Integrand size = 30, antiderivative size = 365

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx =& -\frac{i\sqrt{2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}\\ &+\frac{i\sqrt{2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}\\ &+\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d}\\ &-\frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d}\\ &+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output

```
1/2*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(3/2)/d*2^(1/2)-1/2*I*e^(5
/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(
1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(3/2)/d*2^(1/2)-I*e^(5/2)*arctan(1-2
^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1
/2)/a^(3/2)/d+I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/
a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+4*I*e^2*(e*sec(d*x+c))^(1/
2)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.424.2 Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.93

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{e(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2 \left(\cos(dx)(4i\cos(c)-4\sin(c))+4(\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(e*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*(Cos[d*x]*((4*I)*Cos[c
] - 4*Sin[c]) + 4*(Cos[c] + I*Sin[c])*Sin[d*x] + (2*(ArcTanh[(Sqrt[1 - I*C
os[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt
[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[
c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-
1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]
]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I + Tan[(d*x)
/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I
*a*Tan[c + d*x])^(3/2))
```

3.424.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3981, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}-\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}-\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3976}$$

$$\frac{4ie^4\int\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{4ie^4\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{ad}+$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{4ie^4\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(}{\sqrt{e\,se}}}{2e}}{2e}}{ad}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{4ie^4\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos}{a^2+\cos^2}}{}}{ad}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{217}$$

3.424. $\int\frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{ad}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 1479$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{ad}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d\sqrt{}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{ad}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c}}{\sqrt{e\sec}}}{2\sqrt{2}\sqrt{a}e}}{}\right.}{ad}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 1103$$

3.424. $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\frac{4ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-}{2e}\right.}{ad}$$

$$\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((4*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/(a*d) + ((4*I)*e^2*Sqrt[
e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.424.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.424. $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.424. $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.424.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1089 vs. 2(285) = 570.

Time = 16.18 (sec) , antiderivative size = 1090, normalized size of antiderivative = 2.99

method	result	size
default	Expression too large to display	1090

input

```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/d*e^2*(e*sec(d*x+c))^(1/2)/(tan(d*x+c)-I)/a/(a*(1+I*tan(d*x+c)))^(1/2)
/(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(8*(1/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)+2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+
c)+1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-
1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*arctanh(1/2*(cos(d*x+c)+sin(
d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)
*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(
1/2))+8*(1/(cos(d*x+c)+1))^(1/2)+2*I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+si
n(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+arctanh(1/2*(-cos(d*x
+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-tan(d*x+c)*arct
anh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
)-arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^
(1/2))-tan(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/
(cos(d*x+c)+1))^(1/2))-I*sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)
/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+I*arctanh(1/2*(-cos(d*x+c)+sin(d
*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+I*tan(d*x+c)*arctanh(1/2
*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-I*tan
(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c
)+1))^(1/2))+2*I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/
(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+s...
```

3.424.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.48

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{\left(a^2d\sqrt{\frac{4i\,e^5}{a^3d^2}}e^{(i\,dx+i\,c)}\log\left(\frac{a^2d\sqrt{\frac{4i\,e^5}{a^3d^2}}+2\left(e^2e^{(2i\,dx+2i\,c)}+e^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{e^2}\right)-a^2d\sqrt{\frac{4i\,e^5}{a^3d^2}}e^{(i\,dx}\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
-1/2*(a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(4*I*e^
5/(a^3*d^2)) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) -
 a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(4*I*e^5/(a
^3*d^2)) - 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) - a^2
*d*sqrt(-4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(-4*I*e^5/(a^3*
d^2)) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) + a^2*d*
sqrt(-4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(-4*I*e^5/(a^3*d^
2)) - 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) + 8*(-I*e^
2*e^(2*I*d*x + 2*I*c) - I*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e
^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(a^2*d)
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.424.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(273) = 546$.

Time = 0.47 (sec) , antiderivative size = 778, normalized size of antiderivative = 2.13

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output

```
-1/4*(2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*si
n(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/
2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqr
t(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqr
t(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x +
1/2*c) + 1) + 2*sqrt(2)*e^2*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x
 + c), sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - 2*sqrt(2)*e^2*ar
ctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x +
 1/2*c) + cos(d*x + c) + 1) + I*sqrt(2)*e^2*log(2*sqrt(2)*sin(d*x + c)*sin
(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + co
s(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e^2*log(-2*sqr
t(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) -
 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) + sqr
t(2)*e^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^
2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*...
```

3.424.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.425 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.425.1 Optimal result 3055
3.425.2 Mathematica [A] (verified) 3055
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3.425.9 Mupad [F(-1)] 3059

3.425.1 Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/3*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.425.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

3.425.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3969}$$

$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

3.425.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.425.4 Maple [A] (verified)

Time = 11.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{2i\sec(dx+c)e\sqrt{e\sec(dx+c)}}{3d(1+i\tan(dx+c))a\sqrt{a(1+i\tan(dx+c))}}$	55
risch	$\frac{2ie\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-i(dx+c)}}{3a\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d*sec(d*x+c)*e*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))/a/(a*(1+I*tan(d
*x+c)))^(1/2)
```

3.425.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\left(i\,ee^{(2i\,dx+2i\,c)}+i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
2/3*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/(a^2*d)
```

3.425.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.425.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\,e^{\frac{3}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
2/3*I*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(a^(3/2)*d*
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1)^(3/2))
```

3.425.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{3}{2}}}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.426 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

3.426.1 Optimal result 3060
3.426.2 Mathematica [A] (verified) 3060
3.426.3 Rubi [A] (verified) 3061
3.426.4 Maple [A] (verified) 3062
3.426.5 Fricas [A] (verification not implemented) 3063
3.426.6 Sympy [F] 3063
3.426.7 Maxima [A] (verification not implemented) 3063
3.426.8 Giac [F] 3064
3.426.9 Mupad [B] (verification not implemented) 3064

3.426.1 Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}+\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
4/5*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)+2/5*I*(e*sec(d*x+c
))^(1/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\sqrt{e\sec(c+dx)}(3+2i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[e*Sec[c + d*x]]*(3 + (2*I)*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x
])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.426.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(((2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I
)/5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.426.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.426.4 Maple [A] (verified)

Time = 14.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}\,(2i\tan(dx+c)+3)}{5d(1+i\tan(dx+c))a\sqrt{a(1+i\tan(dx+c))}}$	59

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*I/d*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))/a/(a*(1+I*tan(d*x+c)))^(1/2)
*(2*I*tan(d*x+c)+3)
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^2d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(5
*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5/2*I*d*x - 5/2*
I*c)/(a^2*d)
```

3.426.6 Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{e\sec\left(c+dx\right)}}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(i\,\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5i\,\cos\left(\frac{1}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right.\right.}{5\,a}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
1/5*sqrt(e)*(I*cos(5/2*d*x + 5/2*c) + 5*I*cos(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) + sin(5/2*d*x + 5/2*c) + 5*sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(a^(3/2)*d)
```

3.426.8 Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.426.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5\mathrm{i}\right)}{5\,a\,d\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 5i))/(5*
a*d*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2))
```

3.427 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$

3.427.1 Optimal result 3065
3.427.2 Mathematica [A] (verified) 3065
3.427.3 Rubi [A] (verified) 3066
3.427.4 Maple [A] (verified) 3067
3.427.5 Fricas [A] (verification not implemented) 3068
3.427.6 Sympy [F] 3068
3.427.7 Maxima [A] (verification not implemented) 3068
3.427.8 Giac [F] 3069
3.427.9 Mupad [B] (verification not implemented) 3069

3.427.1 Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{8i}{21ad\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}}$$

output

```
8/21*I/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-16/21*I*(a+I*a*ta
n(d*x+c))^(1/2)/a^2/d/(e*sec(d*x+c))^(1/2)+2/7*I/d/(e*sec(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(3/2)
```

3.427.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)(-7+9\cos(2(c+dx))+12i\sin(2(c+dx)))}{21ad\sqrt{e\sec(c+dx)}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output

```
-1/21*(Sec[c + d*x]^2*(-7 + 9*Cos[2*(c + d*x)] + (12*I)*Sin[2*(c + d*x)]))
/(a*d*Sqrt[e*Sec[c + d*x]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.427.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

input

```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
((2*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + (4*(((2*
I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqr
t[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(7*a)
```

3.427.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.427.4 Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{2\left(9 i-12 \tan \left(d x+c\right)-8 i \left(\sec^{2}\left(d x+c\right)\right)\right)}{21 d \left(1+i \tan \left(d x+c\right)\right) \sqrt{e \sec \left(d x+c\right)}\, \sqrt{a \left(1+i \tan \left(d x+c\right)\right)}\, a}$	69

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/21/d/(1+I*tan(d*x+c))/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/a
*(9*I-12*tan(d*x+c)-8*I*sec(d*x+c)^2)
```

3.427.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-21i\,e^{(6i\,dx+6i\,c)}-7i\,e^{(4i\,dx+4i\,c)}-\right.}{42\,a^2de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fr
icas")
```

output
```
1/42*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-21*I*e^(6*I*d*x + 6*I*c) - 7*I*e^(4*I*d*x + 4*I*c) + 17*I*e^(2*I*d*x + 2*
I*c) + 3*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a^2*d*e)
```

3.427.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{e\sec{(c+dx)}}\,(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

3.427.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{3i\,\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+14i\,\cos\left(\frac{3}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="ma
xima")
```

output
```
1/42*(3*I*cos(7/2*d*x + 7/2*c) + 14*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c))) - 21*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(
7/2*d*x + 7/2*c))) + 3*sin(7/2*d*x + 7/2*c) + 14*sin(3/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 21*sin(1/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c))))/(a^(3/2)*d*sqrt(e))
```

3.427.8 Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,(35\,\sin\,(c+d\,x)+3\,\sin\,(3\,c+3\,d\,x)-\cos\,(c+d}{42\,a\,d\,e\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(35*sin(c + d*x) - cos(c + d*x)*7i + cos(3*c + 3*d
*x)*3i + 3*sin(3*c + 3*d*x)))/(42*a*d*e*((a*(cos(2*c + 2*d*x) + sin(2*c +
2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.428 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

3.428.1 Optimal result 3070
3.428.2 Mathematica [A] (verified) 3070
3.428.3 Rubi [A] (verified) 3071
3.428.4 Maple [A] (verified) 3073
3.428.5 Fricas [A] (verification not implemented) 3074
3.428.6 Sympy [F] 3074
3.428.7 Maxima [A] (verification not implemented) 3074
3.428.8 Giac [F] 3075
3.428.9 Mupad [B] (verification not implemented) 3075

3.428.1 Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}$$

output
```
4/15*I/a/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/45*I*(e*sec(d*
x+c))^(1/2)/a/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-16/45*I*(a+I*a*tan(d*x+c))^(1
/2)/a^2/d/(e*sec(d*x+c))^(3/2)+2/9*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x
+c))^(3/2)
```

3.428.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)(-81\cos(c+dx)+5\cos(3(c+dx))-54i\sin(c+dx)+10i\sin(3(c+dx)))}{90ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
-1/90*(Sec[c + d*x]^3*(-81*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - (54*I)*Sin[
c + d*x] + (10*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan
[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.428.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3983, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3978$$

$$\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3969$$

$$\frac{2\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
((2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((
2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I
)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/
3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(3*a)
```

3.428. $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

3.428.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.428.4 Maple [A] (verified)

Time = 9.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2(5i\cos(dx+c)-10\sin(dx+c)-24i\sec(dx+c)+16\sec(dx+c)\tan(dx+c))}{45d(1+i\tan(dx+c))\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,ae}$	91

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/45/d/(1+I*tan(d*x+c))/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/a
/e*(5*I*cos(d*x+c)-10*sin(d*x+c)-24*I*sec(d*x+c)+16*sec(d*x+c)*tan(d*x+c))
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,(-15i\,e^{(8i\,dx+8i\,c)}+120i\,e^{(6i\,dx+}}{180\,a^{2}}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fr
icas")
```

output
```
1/180*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-15*I*e^(8*I*d*x + 8*I*c) + 120*I*e^(6*I*d*x + 6*I*c) + 162*I*e^(4*I*d*x
+ 4*I*c) + 32*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^2*d
*e^2)
```

3.428.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\,(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{5i\,\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+27i\,\cos\left(\frac{5}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),c\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="ma
xima")
```

output
```
1/180*(5*I*cos(9/2*d*x + 9/2*c) + 27*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) - 15*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))) + 135*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*
d*x + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 27*sin(5/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) + 15*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 135*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/
2*d*x + 9/2*c))))/(a^(3/2)*d*e^(3/2))
```

3.428.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

3.428.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,12{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5{}\mathrm{i}+42\right.}{180\,a\,d\,e^2\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+}{\cos(2\,c+2}}}$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*12i + cos(4*c + 4*d*x)*5i + 42*s
in(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 135i))/(180*a*d*e^2*((a*(cos(2*c +
2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.429 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

3.429.1 Optimal result 3076
3.429.2 Mathematica [A] (verified) 3076
3.429.3 Rubi [A] (verified) 3077
3.429.4 Maple [A] (verified) 3080
3.429.5 Fricas [A] (verification not implemented) 3081
3.429.6 Sympy [F(-1)] 3081
3.429.7 Maxima [A] (verification not implemented) 3081
3.429.8 Giac [F] 3082
3.429.9 Mupad [B] (verification not implemented) 3082

3.429.1 Optimal result

Integrand size = 30, antiderivative size = 209

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\\ &+ \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{128i}{385ade^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &- \frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}} - \frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}}\end{aligned}$$

output
```
16/77*I/a/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+128/385*I/a/d/e^
2/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-96/385*I*(a+I*a*tan(d*x+c)
)^(1/2)/a^2/d/(e*sec(d*x+c))^(5/2)-256/385*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/
d/e^2/(e*sec(d*x+c))^(1/2)+2/11*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)
)^(3/2)
```

3.429.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.48

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{(e\sec(c+dx))^{3/2}(-385+660\cos(2(c+dx))+21\cos(4(c+dx))+880i\sin(2(c+dx))+56i\sin(4(c+d}{1540ade^4(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output

```
-1/1540*((e*Sec[c + d*x])^(3/2)*(-385 + 660*Cos[2*(c + d*x)] + 21*Cos[4*(c
 + d*x)] + (880*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(a*d*e^4*(
-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

3.429.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3978$$

$$\frac{8\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{8\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3983$$

$$\frac{8\left(\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

3.429. $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

$$\frac{8\left(\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 3969$$

$$\frac{8\left(\frac{6\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

output
```
((2*I)/11)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)) + (8*((
(2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (6*((((-2
*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2*
I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqr
t[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(7*a)))/(1
1*a)
```

3.429.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.429. $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.429.4 Maple [A] (verified)

Time = 9.83 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{2\left(21 i \left(\cos^{2}\left(d x+c\right)\right)-56 \sin\left(d x+c\right) \cos\left(d x+c\right)+144 i-192 \tan\left(d x+c\right)-128 i \left(\sec^{2}\left(d x+c\right)\right)\right)}{385 d \sqrt{a \left(1+i \tan\left(d x+c\right)\right)}\, \sqrt{e \sec\left(d x+c\right)}\, \left(1+i \tan\left(d x+c\right)\right) a \,e^{2}}$	97

input
```
int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/385/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))/
a/e^2*(21*I*cos(d*x+c)^2-56*sin(d*x+c)*cos(d*x+c)+144*I-192*tan(d*x+c)-128
*I*sec(d*x+c)^2)
```

3.429.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(10i\,dx+10i\,c)} - 1617i\,e^{(8i}\right)}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fr
icas")
```

output
```
1/3080*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*(-77*I*e^(10*I*d*x + 10*I*c) - 1617*I*e^(8*I*d*x + 8*I*c) - 770*I*e^(6*I*
d*x + 6*I*c) + 990*I*e^(4*I*d*x + 4*I*c) + 255*I*e^(2*I*d*x + 2*I*c) + 35*
I)*e^(-11/2*I*d*x - 11/2*I*c)/(a^2*d*e^3)
```

3.429.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.429.7 Maxima [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{35i\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+220i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}\,dx+\right.\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="ma
xima")
```

output
```
1/3080*(35*I*cos(11/2*d*x + 11/2*c) + 220*I*cos(7/11*arctan2(sin(11/2*d*x
+ 11/2*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(5/11*arctan2(sin(11/2*d*x +
 11/2*c), cos(11/2*d*x + 11/2*c))) + 770*I*cos(3/11*arctan2(sin(11/2*d*x +
 11/2*c), cos(11/2*d*x + 11/2*c))) - 1540*I*cos(1/11*arctan2(sin(11/2*d*x
+ 11/2*c), cos(11/2*d*x + 11/2*c))) + 35*sin(11/2*d*x + 11/2*c) + 220*sin(
7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(5/1
1*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 770*sin(3/11*
arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1540*sin(1/11*a
rctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(3/2)*d*e^(5/2
))
```

3.429.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

3.429.9 Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(2310\,\sin\left(c+d\,x\right)+297\,\sin\left(3\,c+3\,d\,x\right)+35\right.}{3080\,a\,d}$$

input
```
int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(2310*sin(c + d*x) - cos(c + d*x)*770i + cos(3*c +
 3*d*x)*143i + cos(5*c + 5*d*x)*35i + 297*sin(3*c + 3*d*x) + 35*sin(5*c +
5*d*x)))/(3080*a*d*e^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2))
```

3.430 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.430.1 Optimal result 3083
3.430.2 Mathematica [A] (verified) 3084
3.430.3 Rubi [A] (verified) 3084
3.430.4 Maple [B] (verified) 3090
3.430.5 Fricas [A] (verification not implemented) 3091
3.430.6 Sympy [F(-1)] 3092
3.430.7 Maxima [B] (verification not implemented) 3092
3.430.8 Giac [F] 3093
3.430.9 Mupad [F(-1)] 3094

3.430.1 Optimal result

Integrand size = 30, antiderivative size = 411

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = &-\frac{5ie^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}\\
&+\frac{5ie^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}\\
&+\frac{5ie^{9/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d}\\
&-\frac{5ie^{9/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d}\\
&+\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}+\frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d}\end{aligned}$$

output
```
-5/2*I*e^(9/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(
e*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+5/2*I*e^(9/2)*arctan(1+2^(1/2)*e^(1
/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/
2)+5/4*I*e^(9/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*
sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(5/2)/d*2^(1/2)-5/4*I*e
^(9/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c)
)^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(5/2)/d*2^(1/2)+5*I*e^4*(e*sec(d*
x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+4*I*e^2*(e*sec(d*x+c))^(5/2)/a/
d/(a+I*a*tan(d*x+c))^(3/2)
```

3.430.2 Mathematica [A] (verified)

Time = 5.50 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.90

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e^2(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^3\left(\cos(dx)(8i\cos(2c)-8\sin(2c))\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(e^2*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*(Cos[d*x]*((8*I)*Cos
[2*c] - 8*Sin[2*c]) + Sec[c + d*x]*(I*Cos[3*c] - Sin[3*c]) + 8*(Cos[2*c] +
 I*Sin[2*c])*Sin[d*x] + (5*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I -
Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt
[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I
*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sq
rt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Si
n[c]])*(Cos[3*c] + I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c]
+ I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.430.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3981, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3982}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(\frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{3976}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\int\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e\left(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2\right)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)}{a^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\int\frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+}}{ad}\right.}{a^2}$$

$$\Big\downarrow \text{1476}$$

3.430. $\int\frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}dx$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-5e^2\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}}{2e}}{2e}\right.}{d}\right.}{a^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-5e^2\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{}{a^2}}{}\right.}{d}\right.}{a^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{i}{}\right.}{a^2}$$

$$\Big\downarrow \; 1479$$

3.430. $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.}{}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.}{a}$$

$$\Big\downarrow \textcolor{blue}{27}$$

3.430. $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\sqrt{}}{2\sqrt{2}\sqrt{a}e}\right.}{d}\right.}{a^2}$$

$$\Big\downarrow 1103$$

$$\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{5e^2\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.}{a^2}$$

input
```
Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) - (5
*e^2*(((-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x
]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1
+ (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x
]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a +
I*a*Tan[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a +
 I*a*Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e
*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)))/a^2
```

3.430. $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.430.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.430. $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3982

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

3.430.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1611 vs. 2(321) = 642.

Time = 15.10 (sec) , antiderivative size = 1612, normalized size of antiderivative = 3.92

method	result	size
default	Expression too large to display	1612

input

```
int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/d*(e*sec(d*x+c))^(1/2)*e^4/(tan(d*x+c)-I)^2/a^2/(a*(1+I*tan(d*x+c))
)^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(-20*arctanh(1/2*(cos(d*x+
c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+80*(1
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+10*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1
)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+20*arctanh(1/2*(-cos(d*x+c)+sin
(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-20*sin(d*x+
c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1)
)^(1/2))-10*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d
*x+c)+1))^(1/2))-4*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-4*I*ta
n(d*x+c)*sec(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)-5*I*tan(d*x+c)*sec(d*x+c)*a
rctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1
/2))+5*I*tan(d*x+c)*sec(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(
d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-10*tan(d*x+c)*arctanh(1/2*(-cos(d*x+c)
+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+15*sec(d*x+c)*arct
anh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))
+10*I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)
+1))^(1/2))+10*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(
cos(d*x+c)+1))^(1/2))-5*sec(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1
)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+5*sec(d*x+c)^2*arctanh(1/2*(cos
(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-44*sec(d...
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.32

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(\sqrt{\frac{25i\,e^9}{a^5d^2}}a^3de^{(i\,dx+i\,c)}\log\left(\frac{2\left(\sqrt{\frac{25i\,e^9}{a^5d^2}}a^3d+5\left(e^4e^{(2i\,dx+2i\,c)}+e^4\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)}{5\,e^4}\right)-\sqrt{\frac{25i\,e^9}{a^5d^2}}a^3}{}$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output

```
-1/2*(sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(2/5*(sqrt(25*I*e^
9/(a^5*d^2))*a^3*d + 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/
e^4) - sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(-2/5*(sqrt(25*I*
e^9/(a^5*d^2))*a^3*d - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
)/e^4) - sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(2/5*(sqrt(-25
*I*e^9/(a^5*d^2))*a^3*d + 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))/e^4) + sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(-2/5*(sqrt
(-25*I*e^9/(a^5*d^2))*a^3*d - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c))/e^4) + 4*(-5*I*e^4*e^(2*I*d*x + 2*I*c) - 4*I*e^4)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*
c))*e^(-I*d*x - I*c)/(a^3*d)
```

3.430.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.430.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2449 vs. $2(307) = 614$.

Time = 0.92 (sec) , antiderivative size = 2449, normalized size of antiderivative = 5.96

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
-(64*e^4*cos(1/2*d*x + 1/2*c)^2 + 64*e^4*sin(1/2*d*x + 1/2*c)^2 + 16*e^4 -
 10*(-I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) - I*sqrt(2)*e^4*cos(1/2*d*x + 1/2
*c) - sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) + sqrt(2)*e^4*sin(1/2*d*x + 1/2*c))
*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x
+ 1/2*c) + cos(d*x + c) + 1) - 10*(I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) + I*
sqrt(2)*e^4*cos(1/2*d*x + 1/2*c) + sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) - sqrt
(2)*e^4*sin(1/2*d*x + 1/2*c))*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(
d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - (10*sqrt(2)*
e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*
sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1
/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt
(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 5*I*sqr
t(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2
)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*I*sqrt(2)*e
^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*e^4*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos...
```

3.430.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.430.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.431 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.431.1 Optimal result . . . 3095
3.431.2 Mathematica [A] (warning: unable to verify) . . . 3096
3.431.3 Rubi [A] (verified) . . . 3097
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3.431.8 Giac [F] . . . 3105
3.431.9 Mupad [F(-1)] . . . 3105

3.431.1 Optimal result

Integrand size = 30, antiderivative size = 527

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} \\ &+ \frac{i\sqrt{2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{i\sqrt{2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-1/2*I*e^(7/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*se
c(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/a^(3/2)/d*2^(1/2
)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*e^(7/2)*ln(a+2^(
1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x
+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/a^(3/2)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1
/2)/(a+I*a*tan(d*x+c))^(1/2)+I*e^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan
(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/a^(3/2)/d/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*e^(7/2)*arctan(1+2^(1/
2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+
c)*2^(1/2)/a^(3/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+4/3
*I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.431.2 Mathematica [A] (warning: unable to verify)

Time = 4.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.68

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^3\left(\frac{4}{3}i\cos(2dx)(\cos(c)+i\sin(c))+\right.}{}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*(((4*I)/3)*Cos[2*d*x]*
(Cos[c] + I*Sin[c]) + (4*(Cos[c] + I*Sin[c])*Sin[2*d*x])/3 - (2*(ArcTanh[(
Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] +
Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*C
os[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/
2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos
[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I
 + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c
]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.431.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.80, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3981, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3981}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3980}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^3\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{a^2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{e^3\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{a^2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 1476$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}}}{2e}}{2e}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 1082$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int\frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 217$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+dx}}}{2e}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 1479$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan}}$$

3.431. $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \ 25$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}$$

$$\Big\downarrow \ 27$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}$$

$$\Big\downarrow \ 1103$$

$$\frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4ie^5\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input

```
Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

3.431. $\int\frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

output

```
(((4*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
- ((4*I)*e^5*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(
Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sq
rt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*
Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(a*d*Sqrt
[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.431.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.431. $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

3.431. $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.431.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(421) = 842$.

Time = 15.14 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.75

method	result	size
default	Expression too large to display	920

input

```
int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
(1/6+1/6*I)/d*e^3*(e*sec(d*x+c))^(1/2)/(tan(d*x+c)-I)^2/a^2/(a*(1+I*tan(d*
x+c)))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(-12*I*cos(d*x+c)*arc
tanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
)+3*I*sec(d*x+c)^2*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1
/(cos(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)
+6*I*tan(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(
cos(d*x+c)+1))^(1/2))+12*I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x
+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3*I*tan(d*x+c)*sec(d*x+c)*arct
anh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
)+12*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)+12*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/
(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*(1/(cos(d*x+c)+1))
^(1/2)-4*I*(1/(cos(d*x+c)+1))^(1/2)-4*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)+9*I*sec(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(
cos(d*x+c)+1))^(1/2))+6*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)
+1)/(1/(cos(d*x+c)+1))^(1/2))+6*tan(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x
+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-4*(1/(cos(d*x+c)+1))^(1/2)
+4*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-6*I*arctanh(1/2*(cos(d*x+c)+sin(d*x
+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-9*sec(d*x+c)*arctanh(1/2*(
-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*ta...
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.03

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(3\,a^3d\sqrt{\frac{4i\,e^7}{a^5d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{i\,a^3d\sqrt{\frac{4i\,e^7}{a^5d^2}}+2\left(e^3e^{(2i\,dx+2i\,c)}+e^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}}{e^3}\right)-3\,a^3d\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
-1/6*(3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a^3*d*sqr
t(4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
)/e^3) - 3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((-I*a^3*d
*sqrt(4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c))/e^3) + 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a
^3*d*sqrt(-4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c))/e^3) - 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log
((-I*a^3*d*sqrt(-4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I
*d*x + 1/2*I*c))/e^3) + 8*(-I*e^3*e^(2*I*d*x + 2*I*c) - I*e^3)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/
2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

3.431.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.431.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1466 vs. $2(401) = 802$.

Time = 0.67 (sec) , antiderivative size = 1466, normalized size of antiderivative = 2.78

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output

```
1/12*(6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3
*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 1) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 6*sqrt(2)*e^3*arctan2(sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*sqrt(2)*e^3*arctan2(-sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*I*sqrt(2)*e^3*log(2*sqrt(2)*sin(2/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(si...
```

3.431.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.432 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.432.1 Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

output
```
2/5*I*(e*sec(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)
```

3.432.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.432.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3969}$$

$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

3.432.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.432.4 Maple [A] (verified)

Time = 11.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{2i(\sec^2(dx+c))e^2\sqrt{e\sec(dx+c)}}{5d(1+i\tan(dx+c))^2a^2\sqrt{a(1+i\tan(dx+c))}}$	59

input
```
int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*I/d*sec(d*x+c)^2*e^2*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))^2/a^2/(a*(1
+I*tan(d*x+c)))^(1/2)
```

3.432.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\left(i\,e^2e^{(2i\,dx+2i\,c)} + i\,e^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
2/5*(I*e^2*e^(2*I*d*x + 2*I*c) + I*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a^3*d)
```

3.432.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.432.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\,e^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
2/5*I*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)/(a^(5/2)*d*
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1)^(5/2))
```

3.432.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.432. $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

3.433 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.433.1 Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}+\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}$$

output
```
2/7*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(5/2)+4/21*I*(e*sec(d*x+c)
)^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.433.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2(e\sec(c+dx))^{3/2}(-5i+2\tan(c+dx))}{21a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(2*(e*Sec[c + d*x])^(3/2)*(-5*I + 2*Tan[c + d*x]))/(21*a^2*d*(-I + Tan[c +
 d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.433.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3983}$$
$$\frac{2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^{3/2}}dx}{7a}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^{3/2}}dx}{7a}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$
$$\Big\downarrow \text{3969}$$
$$\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/7)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((4
*I)/21)*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

3.433.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.433.4 Maple [A] (verified)

Time = 14.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}\,e(2i\tan(dx+c)\sec(dx+c)+5\sec(dx+c))}{21d(1+i\tan(dx+c))^{2}a^{2}\sqrt{a(1+i\tan(dx+c))}}$	73

input
```
int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*I/d*(e*sec(d*x+c))^(1/2)*e/(1+I*tan(d*x+c))^2/a^2/(a*(1+I*tan(d*x+c))
)^(1/2)*(2*I*sec(d*x+c)*tan(d*x+c)+5*sec(d*x+c))
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(7i\,ee^{(4i\,dx+4i\,c)}+10i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{7}{2}i\,dx-\right.}}{21\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/21*(7*I*e*e^(4*I*d*x + 4*I*c) + 10*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*
d*x - 7/2*I*c)/(a^3*d)
```

3.433.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.433.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(3i\,e\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+7i\,e\cos\left(\frac{3}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right.\right.\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/21*(3*I*e*cos(7/2*d*x + 7/2*c) + 7*I*e*cos(3/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c))) + 3*e*sin(7/2*d*x + 7/2*c) + 7*e*sin(3/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(a^(5/2)*d)
```

3.433.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.433.9 Mupad [B] (verification not implemented)

Time = 4.77 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e\sqrt{\frac{e}{\cos(c+d\,x)}}\,(7\,\sin(c+d\,x)+3\,\sin(3\,c+3\,d\,x)+\cos(c+d\,x)\,7\mathrm{i}+\cos(3\,c}{21\,a^2\,d\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(e*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*7i + 7*sin(c + d*x) + cos(3*c + 3*
d*x)*3i + 3*sin(3*c + 3*d*x)))/(21*a^2*d*((a*(cos(2*c + 2*d*x) + sin(2*c +
 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.434 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia \tan(c+dx))^{5/2}} \, dx$

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3.434.1 Optimal result

Integrand size = 30, antiderivative size = 121

$$\begin{aligned}\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia \tan(c+dx))^{5/2}} \, dx &= \frac{2i\sqrt{e \sec(c+dx)}}{9d(a+ia \tan(c+dx))^{5/2}} \\ &\quad + \frac{8i\sqrt{e \sec(c+dx)}}{45ad(a+ia \tan(c+dx))^{3/2}} + \frac{16i\sqrt{e \sec(c+dx)}}{45a^2d\sqrt{a+ia \tan(c+dx)}}\end{aligned}$$

output

```
16/45*I*(e*sec(d*x+c))^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+2/9*I*(e*sec(d
*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+8/45*I*(e*sec(d*x+c))^(1/2)/a/d/(a
+I*a*tan(d*x+c))^(3/2)
```

3.434.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\begin{aligned}&\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia \tan(c+dx))^{5/2}} \, dx = \\ &-\frac{i \sec^2(c+dx)\sqrt{e \sec(c+dx)}(9+25\cos(2(c+dx))+20i\sin(2(c+dx)))}{45a^2d(-i+\tan(c+dx))^2\sqrt{a+ia \tan(c+dx)}}\end{aligned}$$

input

```
Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/45*I)*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(9 + 25*Cos[2*(c + d*x)] +
(20*I)*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c
+ d*x]])
```

3.434.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^{3/2}}dx}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^{3/2}}dx}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}\right)}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{5a}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}\right)}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{4\left(\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}\right)}{9a}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(((2*I)/9)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (4*(((
(2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I)/
5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])))/(9*a)
```

3.434.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.434.4 Maple [A] (verified)

Time = 13.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}\,(20i\tan(dx+c)+25-8(\sec^2(dx+c)))}{45d(1+i\tan(dx+c))^2\sqrt{a(1+i\tan(dx+c))}\,a^2}$	69

input
```
int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*I/d*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))^2/(a*(1+I*tan(d*x+c)))^(1/2
)/a^2*(20*I*tan(d*x+c)+25-8*sec(d*x+c)^2)
```

3.434.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\big(45i\,e^{(6i\,dx+6i\,c)}+63i\,e^{(4i\,dx+4i\,c)}+23i\,e^{(2i\,dx+2i\,c)}}{90\,a^3d}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/90*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
45*I*e^(6*I*d*x + 6*I*c) + 63*I*e^(4*I*d*x + 4*I*c) + 23*I*e^(2*I*d*x + 2*
I*c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^3*d)
```

3.434.6 Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{e\sec{(c+dx)}}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.434.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \sqrt{e}\big(5i\,\cos\big(\tfrac{9}{2}\,dx+\tfrac{9}{2}\,c\big)+18i\,\cos\big(\tfrac{5}{9}\,\arctan\big(\sin\big(\tfrac{9}{2}\,dx+\tfrac{9}{2}\,c\big),\cos\big(\tfrac{9}{2}\,dx+\tfrac{9}{2}$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
1/90*sqrt(e)*(5*I*cos(9/2*d*x + 9/2*c) + 18*I*cos(5/9*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 18*sin(5/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*sin(1/9*arctan2(sin(9/2*d
*x + 9/2*c), cos(9/2*d*x + 9/2*c))))/(a^(5/2)*d)
```

3.434.8 Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.434.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,18\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}+18\,\sin\left(2\,c+2\,d\,x\right)+\right.}{90\,a^2\,d\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

input
```
int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*18i + cos(4*c + 4*d*x)*5i + 18*s
in(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 45i))/(90*a^2*d*((a*(cos(2*c + 2*d*
x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.435 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.435.1 Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77a^2d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}}$$

output
```
16/77*I/a^2/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-32/77*I*(a+I*a
*tan(d*x+c))^(1/2)/a^3/d/(e*sec(d*x+c))^(1/2)+2/11*I/d/(e*sec(d*x+c))^(1/2
)/(a+I*a*tan(d*x+c))^(5/2)+12/77*I/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x
+c))^(3/2)
```

3.435.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec^3(c+dx)(-55\cos(c+dx)+35\cos(3(c+dx))-22i\sin}{154a^2d\sqrt{e\sec(c+dx)}(-i+\tan(c+dx))^2\sqrt{a}}$$

input
```
Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

output
```
((I/154)*Sec[c + d*x]^3*(-55*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - (22*I)*S
in[c + d*x] + (42*I)*Sin[3*(c + d*x)]))/(a^2*d*Sqrt[e*Sec[c + d*x]]*(-I +
Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.435.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{6\left(\frac{4\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{7a}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}\right)}{11a}+\frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

output
```
((2*I)/11)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + (6*(((2
*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + (4*(((2*I)/
3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a
 + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])))/(7*a)))/(11*a)
```

3.435. $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$

3.435.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.435.4 Maple [A] (verified)

Time = 10.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{-\frac{10i}{11}+\frac{12\tan(dx+c)}{11}+\frac{80i\left(\sec^2(dx+c)\right)}{77}-\frac{32\left(\sec^2(dx+c)\right)\tan(dx+c)}{77}}{d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,(1+i\tan(dx+c))^2a^2}$	85

input
```
int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
2/77/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))^2/
a^2*(-35*I+42*tan(d*x+c)+40*I*sec(d*x+c)^2-16*sec(d*x+c)^2*tan(d*x+c))
```

3.435.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}(-77i\,e^{(8i\,dx+8i\,c)}+110i\,e^{(4i\,dx+4i\,c}}{308\,a^3de}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fr
icas")
```

output
```
1/308*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-77*I*e^(8*I*d*x + 8*I*c) + 110*I*e^(4*I*d*x + 4*I*c) + 40*I*e^(2*I*d*x +
 2*I*c) + 7*I)*e^(-11/2*I*d*x - 11/2*I*c)/(a^3*d*e)
```

3.435.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{e\sec\left(c+dx\right)}\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(5/2)), x)
```

3.435.7 Maxima [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{7i\,\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+33i\,\cos\left(\frac{7}{11}\,\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="ma
xima")
```

output
```
1/308*(7*I*cos(11/2*d*x + 11/2*c) + 33*I*cos(7/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c))) + 77*I*cos(3/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(1/11*arctan2(sin(11/2*d*x + 11/
2*c), cos(11/2*d*x + 11/2*c))) + 7*sin(11/2*d*x + 11/2*c) + 33*sin(7/11*ar
ctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(3/11*arcta
n2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(1/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(5/2)*d*sqrt(e))
```

3.435.8 Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(5/2)), x)
```

3.435.9 Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,(154\,\sin\left(c+d\,x\right)+33\,\sin\left(3\,c+3\,d\,x\right)+7\,\sin\left(5\right.}{308\,a^2\,d\,e\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x}{\cos(2}}}$$

input
```
int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(154*sin(c + d*x) + cos(3*c + 3*d*x)*33i + cos(5*c
 + 5*d*x)*7i + 33*sin(3*c + 3*d*x) + 7*sin(5*c + 5*d*x)))/(308*a^2*d*e*((a
*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/
2))
```

3.436 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.436.1 Optimal result

Integrand size = 30, antiderivative size = 206

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\\ &+ \frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\\ &+ \frac{32i}{195a^2d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\\ &+ \frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}}\end{aligned}$$

output
```
32/195*I/a^2/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+256/585*I*(e*
sec(d*x+c))^(1/2)/a^2/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-128/585*I*(a+I*a*tan(
d*x+c))^(1/2)/a^3/d/(e*sec(d*x+c))^(3/2)+2/13*I/d/(e*sec(d*x+c))^(3/2)/(a+
I*a*tan(d*x+c))^(5/2)+16/117*I/a/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))
^(3/2)
```

3.436.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.52

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^4(c+dx)(-351i-1300i\cos(2(c+dx))+75i\cos(4(c+}{2340a^2d(e\sec(c+dx))^{3/2}(-i+\tan(c}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

output
```
(Sec[c + d*x]^4*(-351*I - (1300*I)*Cos[2*(c + d*x)] + (75*I)*Cos[4*(c + d*
x)] + 1040*Sin[2*(c + d*x)] - 120*Sin[4*(c + d*x)]))/(2340*a^2*d*(e*Sec[c
+ d*x])^(3/2)*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

3.436.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3983, 3042, 3983, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3983$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{8\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^{3/2}}dx}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3983$$

$$\frac{8\left(\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3983}$$

$$\frac{8\left(\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{2\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3978}$$

3.436. $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$

$$\frac{8\left(\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{8\left(\frac{2\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3969$$

$$\frac{8\left(\frac{2\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{3a}+\frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\right)}{13a}+\frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

3.436. $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$

output
```
((2*I)/13)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)) + (8*((
(2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + (2*(((2
*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)
/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3
)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(3*a)))
/(13*a)
```

3.436.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

3.436.4 Maple [A] (verified)

Time = 8.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{2(75i\cos(dx+c)-120\sin(dx+c)-400i\sec(dx+c)+320\sec(dx+c)\tan(dx+c)+128i(\sec^{3}(dx+c)))}{585d\sqrt{a(1+i\tan(dx+c))}\,(1+i\tan(dx+c))^{2}\sqrt{e\sec(dx+c)}\,a^{2}e}$	102

input
```
int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/585/d/(a*(1+I*tan(d*x+c)))^(1/2)/(1+I*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2
)/a^2/e*(75*I*cos(d*x+c)-120*sin(d*x+c)-400*I*sec(d*x+c)+320*sec(d*x+c)*ta
n(d*x+c)+128*I*sec(d*x+c)^3)
```

3.436.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(10i\,dx+10i\,c)}+2145i\,e^{(8}\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fr
icas")
```

output
```
1/4680*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*(-195*I*e^(10*I*d*x + 10*I*c) + 2145*I*e^(8*I*d*x + 8*I*c) + 3042*I*e^(6*
I*d*x + 6*I*c) + 962*I*e^(4*I*d*x + 4*I*c) + 305*I*e^(2*I*d*x + 2*I*c) + 4
5*I)*e^(-13/2*I*d*x - 13/2*I*c)/(a^3*d*e^2)
```

3.436.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.436.7 Maxima [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{45i\cos\left(\frac{13}{2}\,dx+\frac{13}{2}\,c\right)+260i\cos\left(\frac{9}{13}\arctan\left(\sin\left(\frac{13}{2}\,dx+\right.\right.\right.}{}$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="ma
xima")
```

output
```
1/4680*(45*I*cos(13/2*d*x + 13/2*c) + 260*I*cos(9/13*arctan2(sin(13/2*d*x
+ 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*I*cos(5/13*arctan2(sin(13/2*d*x
+ 13/2*c), cos(13/2*d*x + 13/2*c))) - 195*I*cos(3/13*arctan2(sin(13/2*d*x
+ 13/2*c), cos(13/2*d*x + 13/2*c))) + 2340*I*cos(1/13*arctan2(sin(13/2*d*x
 + 13/2*c), cos(13/2*d*x + 13/2*c))) + 45*sin(13/2*d*x + 13/2*c) + 260*sin
(9/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*sin(5
/13*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 195*sin(3/1
3*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 2340*sin(1/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))))/(a^(5/2)*d*e^(3
/2))
```

3.436.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(5/2)), x)
```

3.436.9 Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.66

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,507\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,260\mathrm{i}+\right.}{46}$$

input
```
int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

output
```
((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*507i + cos(4*c + 4*d*x)*260i + c
os(6*c + 6*d*x)*45i + 897*sin(2*c + 2*d*x) + 260*sin(4*c + 4*d*x) + 45*sin
(6*c + 6*d*x) + 2340i))/(4680*a^2*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c +
2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.437 $\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.437.1 Optimal result 3135
3.437.2 Mathematica [A] (verified) 3135
3.437.3 Rubi [A] (verified) 3136
3.437.4 Maple [F] 3138
3.437.5 Fricas [F] 3138
3.437.6 Sympy [F(-1)] 3139
3.437.7 Maxima [F] 3139
3.437.8 Giac [F] 3139
3.437.9 Mupad [F(-1)] 3140

3.437.1 Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i2^{2/3}a\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{7/3}}{7d(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/7*I*2^(2/3)*a*hypergeom([1/3, 7/6],[13/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d
*x+c))^(7/3)*(1+I*tan(d*x+c))^(1/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.437.2 Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\sqrt[3]{2}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{4/3}\left(4+\left(1+e^{2i(c+dx)}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)\right)}{5d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((-3*I)/5)*2^(1/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(
c + d*x))))^(4/3)*(4 + (1 + E^((2*I)*(c + d*x)))^(5/6)*Hypergeometric2F1[2
/3, 5/6, 5/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.437.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{7/3}\int (a-ia\tan(c+dx))^{7/6}(i\tan(c+dx)a+a)^{2/3}dx}{(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{7/3}\int (a-ia\tan(c+dx))^{7/6}(i\tan(c+dx)a+a)^{2/3}dx}{(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{7/3}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)a+a}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\int \frac{\sqrt[3]{2}\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)+1}}d\tan(c+dx)}{\sqrt[3]{2}d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)+1}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{3i2^{2/3}a\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{7d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((3*I)/7)*2^(2/3)*a*Hypergeometric2F1[1/3, 7/6, 13/6, (1 - I*Tan[c + d*x]
)/2]*(e*Sec[c + d*x])^(7/3)*(1 + I*Tan[c + d*x])^(1/3))/(d*(a + I*a*Tan[c
+ d*x])^(3/2))
```

3.437.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.437.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.437.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/5*(-6*I*2^(5/6)*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2
*I*c) + 1))^(1/3)*e^(4/3*I*d*x + 4/3*I*c) + 5*a*d*integral(-2/5*I*2^(5/6)*
e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*
e^(1/3*I*d*x + 1/3*I*c)/(a*d), x))/(a*d)
```

3.437.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(7/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.437.7 Maxima [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.437.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.438 $\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.438.1 Optimal result 3141
3.438.2 Mathematica [A] (verified) 3141
3.438.3 Rubi [A] (verified) 3142
3.438.4 Maple [F] 3144
3.438.5 Fricas [F] 3144
3.438.6 Sympy [F(-1)] 3145
3.438.7 Maxima [F] 3145
3.438.8 Giac [F] 3145
3.438.9 Mupad [F(-1)] 3146

3.438.1 Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{3i\sqrt[3]{2}a\,\text{Hypergeometric2F1}\big(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\big)\,(e\sec(c+dx))^{5/3}(1}{5d(a+ia\tan(c+dx))^{3/2}}$$

output
```
3/5*I*2^(1/3)*a*hypergeom([2/3, 5/6],[11/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d
*x+c))^(5/3)*(1+I*tan(d*x+c))^(2/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

3.438.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{3i2^{2/3}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\Big(-2+\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\big(\frac{1}{6},\frac{1}{3},}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((3*I)*2^(2/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d
*x))))^(2/3)*(-2 + (1 + E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[1/6,
1/3, 4/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.438.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{5/3}\int (a-ia\tan(c+dx))^{5/6}\sqrt[3]{i\tan(c+dx)a+a}dx}{(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{5/3}\int (a-ia\tan(c+dx))^{5/6}\sqrt[3]{i\tan(c+dx)a+a}dx}{(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{5/3}\int \frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{2/3}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\int \frac{2^{2/3}}{(i\tan(c+dx)+1)^{2/3}\sqrt[6]{a-ia\tan(c+dx)}}d\tan(c+dx)}{2^{2/3}d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\int \frac{1}{(i\tan(c+dx)+1)^{2/3}\sqrt[6]{a-ia\tan(c+dx)}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{3i\sqrt[3]{2}a(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{5d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(((3*I)/5)*2^(1/3)*a*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - I*Tan[c + d*x]
)/2]*(e*Sec[c + d*x])^(5/3)*(1 + I*Tan[c + d*x])^(2/3))/(d*(a + I*a*Tan[c
+ d*x])^(3/2))
```

3.438.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.438. $\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.438.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.438.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-(6*2^(1/6)*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(2/3*I*d*x + 2/3*I*c) - a*d*inte
gral(2^(1/6)*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(-4/3*I*d*x - 4/3*I*c)/(a*d), x
))/(a*d)
```

3.438.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.438.7 Maxima [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.438.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.439 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.439.1 Optimal result

Integrand size = 30, antiderivative size = 85

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2/3}\sqrt[6]{1+i}}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

output
```
3/4*I*hypergeom([1/3, 7/6],[4/3],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(2/3
)*(1+I*tan(d*x+c))^(1/6)*2^(5/6)/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.439.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{3i\sqrt[6]{2}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)}{d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

input
```
Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((3*I)*2^(1/6)*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 +
E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c
+ d*x))])/(d*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

3.439.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(c+dx)}}{\sqrt[6]{i\tan(c+dx)a+a}}dx}{\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(c+dx)}}{\sqrt[6]{i\tan(c+dx)a+a}}dx}{\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(i\tan(c+dx)a+a)^{7/6}}d\tan(c+dx)}{d\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\int \frac{2\sqrt[6]{2}}{(i\tan(c+dx)+1)^{7/6}(a-ia\tan(c+dx))^{2/3}}d\tan(c+dx)}{2\sqrt[6]{2}d\sqrt[3]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\int \frac{1}{(i\tan(c+dx)+1)^{7/6}(a-ia\tan(c+dx))^{2/3}}d\tan(c+dx)}{d\sqrt[3]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{3i\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

3.439. $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}dx$

input

```
Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(((3*I)/2)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - I*Tan[c + d*x])/2]*(e*Sec
[c + d*x])^(2/3)*(1 + I*Tan[c + d*x])^(1/6))/(2^(1/6)*d*Sqrt[a + I*a*Tan[c
 + d*x]])
```

3.439.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.439. $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 4006

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.439.4 Maple [F]

$$\int \frac{(e\sec (dx+c))^{\frac{2}{3}}}{\sqrt{a+ia\tan (dx+c)}}dx$$

input

```
int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output

```
int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.439.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\sec (dx+c))^{\frac{2}{3}}}{\sqrt{i\,a\tan (dx+c)+a}}\,dx$$

input

```
integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output

```
-(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1)
)^(2/3)*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*e^(2/3*I*d*
x + 2/3*I*c) - (a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2*I*d*x + 2*I*c) + a*d*
e^(I*d*x + I*c))*integral(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^
(2*I*d*x + 2*I*c) + 1))^(2/3)*(I*e^(4*I*d*x + 4*I*c) + 7*I*e^(3*I*d*x + 3*
I*c) + 5*I*e^(2*I*d*x + 2*I*c) + 7*I*e^(I*d*x + I*c) + 4*I)*e^(2/3*I*d*x +
 2/3*I*c)/(a*d*e^(4*I*d*x + 4*I*c) - 3*a*d*e^(3*I*d*x + 3*I*c) + 3*a*d*e^(
2*I*d*x + 2*I*c) - a*d*e^(I*d*x + I*c)), x))/(a*d*e^(3*I*d*x + 3*I*c) - 2*
a*d*e^(2*I*d*x + 2*I*c) + a*d*e^(I*d*x + I*c))
```

3.439.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\sec{(c+dx)})^{\frac{2}{3}}}{\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(2/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(2/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.439.7 Maxima [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\sec{(dx+c)})^{\frac{2}{3}}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.439.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\sec{(dx+c)})^{\frac{2}{3}}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.440 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.440.1 Optimal result

Integrand size = 30, antiderivative size = 83

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[3]{e\sec(c+dx)}\sqrt[3]{1+i\tan(c+dx)}}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

output
```
3/2*I*hypergeom([1/6, 4/3],[7/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(1/3
)*(1+I*tan(d*x+c))^(1/3)*2^(2/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

3.440.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3\left(8i-\frac{2ie^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)}{\sqrt[6]{1+e^{2i(c+dx)}}}\right)\sqrt[3]{e\sec(c+dx)}}{16d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(3*(8*I - ((2*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 5/6, 5/3, -E^(
(2*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(1/6))*(e*Sec[c + d*x])^(1/3)
)/(16*d*Sqrt[a + I*a*Tan[c + d*x]])
```

3.440.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{\sqrt[3]{e\sec(c+dx)}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)a+a}}dx}{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[3]{e\sec(c+dx)}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{i\tan(c+dx)a+a}}dx}{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2\sqrt[3]{e\sec(c+dx)}\int \frac{1}{(a-ia\tan(c+dx))^{5/6}(i\tan(c+dx)a+a)^{4/3}}d\tan(c+dx)}{d\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{a\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\int \frac{2\sqrt[3]{2}}{(i\tan(c+dx)+1)^{4/3}(a-ia\tan(c+dx))^{5/6}}d\tan(c+dx)}{2\sqrt[3]{2}d\sqrt[6]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\int \frac{1}{(i\tan(c+dx)+1)^{4/3}(a-ia\tan(c+dx))^{5/6}}d\tan(c+dx)}{d\sqrt[6]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{3i\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

3.440. $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$

input
```
Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((3*I)*Hypergeometric2F1[1/6, 4/3, 7/6, (1 - I*Tan[c + d*x])/2]*(e*Sec[c +
 d*x])^(1/3)*(1 + I*Tan[c + d*x])^(1/3))/(2^(1/3)*d*Sqrt[a + I*a*Tan[c + d
*x]])
```

3.440.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.440. $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.440.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.440.5 Fricas [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/4*(4*a*d*e^(I*d*x + I*c)*integral(-1/4*I*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(1/3*I*d*x + 1/3*I*c)/(a*
d), x) - 3*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*
c) + 1))^(1/3)*(-I*e^(2*I*d*x + 2*I*c) - I)*e^(1/3*I*d*x + 1/3*I*c))*e^(-I
*d*x - I*c)/(a*d)
```

3.440.6 Sympy [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\sqrt[3]{e\sec\left(c+dx\right)}}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**(1/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.440.7 Maxima [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((e*sec(d*x + c))^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.440.8 Giac [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*sec(d*x + c))^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{1/3}}{\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.441 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.441.1 Optimal result

Integrand size $= 30$, antiderivative size $= 83$

$$\begin{aligned}&\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx\\&=-\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{2/3}}{2^{2/3}d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-3/2*I*hypergeom([-1/6, 5/3],[5/6],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^
(2/3)*2^(1/3)/d/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2)
```

3.441.2 Mathematica [A] (verified)

Time $= 1.35$ (sec) , antiderivative size $= 95$, normalized size of antiderivative $= 1.14$

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{12i-\frac{30ie^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{4}{3},-e^{2i(c+dx)}\right)}{\left(1+e^{2i(c+dx)}\right)^{5/6}}}{16d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(12*I - ((30*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/6, 1/3, 4/3, -E^((
2*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(5/6))/(16*d*(e*Sec[c + d*x])^
(1/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

3.441.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\int \frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{2/3}}dx}{\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\int \frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{2/3}}dx}{\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\int \frac{1}{(a-ia\tan(c+dx))^{7/6}(i\tan(c+dx)a+a)^{5/3}}d\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a(1+i\tan(c+dx))^{2/3}\sqrt[6]{a-ia\tan(c+dx)}\int \frac{2\ 2^{2/3}}{(i\tan(c+dx)+1)^{5/3}(a-ia\tan(c+dx))^{7/6}}d\tan(c+dx)}{2\ 2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(1+i\tan(c+dx))^{2/3}\sqrt[6]{a-ia\tan(c+dx)}\int \frac{1}{(i\tan(c+dx)+1)^{5/3}(a-ia\tan(c+dx))^{7/6}}d\tan(c+dx)}{d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{3i(1+i\tan(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

3.441. $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}dx$

input
```
Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((-3*I)*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - I*Tan[c + d*x])/2]*(1 + I*T
an[c + d*x])^(2/3))/(2^(2/3)*d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c +
 d*x]])
```

3.441.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.441. $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.441.4 Maple [F]

$$\int \frac{1}{\left(e \sec \left(d x +c \right)\right)^{\frac{1}{3}} \sqrt{a +i a \tan \left(d x +c \right)}}d x$$

input
```
int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.441.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\left(e \sec \left(d x +c \right)\right)^{\frac{1}{3}} \sqrt{i\, a \tan \left(d x +c \right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
-1/8*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c)
+ 1))^(2/3)*(4*I*e^(6*I*d*x + 6*I*c) + 9*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*
I*d*x + 2*I*c) + I)*e^(2/3*I*d*x + 2/3*I*c) - 8*(a*d*e*e^(4*I*d*x + 4*I*c)
 - a*d*e*e^(2*I*d*x + 2*I*c))*integral(-15/16*2^(1/6)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(3*I*e^(4*I*d*x + 4*I*c)
 + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(2/3*I*d*x + 2/3*I*c)/(a*d*e*e^(6*I*d*x
+ 6*I*c) - 2*a*d*e*e^(4*I*d*x + 4*I*c) + a*d*e*e^(2*I*d*x + 2*I*c)), x))/(
a*d*e*e^(4*I*d*x + 4*I*c) - a*d*e*e^(2*I*d*x + 2*I*c))
```

3.441.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\sqrt[3]{e\sec\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(1/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

3.441.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/((e*sec(d*x + c))^(1/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.441.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(1/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{1/3}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.442 $\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.442.1 Optimal result

Integrand size = 30, antiderivative size = 88

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

output
```
-3/16*I*hypergeom([-2/3, 13/6],[1/3],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+
c))^(1/2)*(1+I*tan(d*x+c))^(1/6)*2^(5/6)/a/d/(e*sec(d*x+c))^(4/3)
```

3.442.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\sec^2(c+dx)\Bigl(3+3\cos(2(c+dx))-55\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)+11i}{112d(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(((-3*I)/112)*Sec[c + d*x]^2*(3 + 3*Cos[2*(c + d*x)] - 55*(1 + E^((2*I)*(c
 + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c + d*x))] +
(11*I)*Sin[2*(c + d*x)]))/(d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d
*x]])
```

3.442.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 3986, 3042, 4006, 80, 27, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(i\tan(c+dx)a+a)^{7/6}}dx}{(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(i\tan(c+dx)a+a)^{7/6}}dx}{(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\int \frac{1}{(a-ia\tan(c+dx))^{5/3}(i\tan(c+dx)a+a)^{13/6}}d\tan(c+dx)}{d(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \text{80}$$

$$\frac{\sqrt[6]{1+i\tan(c+dx)}(a-ia\tan(c+dx))^{2/3}\sqrt{a+ia\tan(c+dx)}\int \frac{4\sqrt[6]{2}}{(i\tan(c+dx)+1)^{13/6}(a-ia\tan(c+dx))^{5/3}}d\tan(c+dx)}{4\sqrt[6]{2}d(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[6]{1+i\tan(c+dx)}(a-ia\tan(c+dx))^{2/3}\sqrt{a+ia\tan(c+dx)}\int\frac{1}{(i\tan(c+dx)+1)^{13/6}(a-ia\tan(c+dx))^{5/3}}d\tan(c+dx)}{d(e\sec(c+dx))^{4/3}}$$

$$\Big\downarrow 79$$

$$-\frac{3i\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

input
```
Int[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(((-3*I)/8)*Hypergeometric2F1[-2/3, 13/6, 1/3, (1 - I*Tan[c + d*x])/2]*(1
+ I*Tan[c + d*x])^(1/6)*Sqrt[a + I*a*Tan[c + d*x]])/(2^(1/6)*a*d*(e*Sec[c
+ d*x])^(4/3))
```

3.442.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.442.4 Maple [F]

$$\int \frac{1}{(e\sec (dx+c))^{\frac{4}{3}}\sqrt{a+ia\tan (dx+c)}}dx$$

input
```
int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.442.5 Fricas [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\sec (dx+c))^{\frac{4}{3}}\sqrt{i\,a\tan (dx+c)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
-1/112*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c
) + 1))^(2/3)*(7*I*e^(8*I*d*x + 8*I*c) - 14*I*e^(7*I*d*x + 7*I*c) - 38*I*e
^(6*I*d*x + 6*I*c) - 20*I*e^(5*I*d*x + 5*I*c) - 101*I*e^(4*I*d*x + 4*I*c)
+ 2*I*e^(3*I*d*x + 3*I*c) - 60*I*e^(2*I*d*x + 2*I*c) + 8*I*e^(I*d*x + I*c)
 - 4*I)*e^(2/3*I*d*x + 2/3*I*c) - 112*(a*d*e^2*e^(5*I*d*x + 5*I*c) - 2*a*d
*e^2*e^(4*I*d*x + 4*I*c) + a*d*e^2*e^(3*I*d*x + 3*I*c))*integral(-55/112*2
^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/
3)*(-I*e^(4*I*d*x + 4*I*c) - 7*I*e^(3*I*d*x + 3*I*c) - 5*I*e^(2*I*d*x + 2*
I*c) - 7*I*e^(I*d*x + I*c) - 4*I)*e^(2/3*I*d*x + 2/3*I*c)/(a*d*e^2*e^(4*I*
d*x + 4*I*c) - 3*a*d*e^2*e^(3*I*d*x + 3*I*c) + 3*a*d*e^2*e^(2*I*d*x + 2*I*
c) - a*d*e^2*e^(I*d*x + I*c)), x))/(a*d*e^2*e^(5*I*d*x + 5*I*c) - 2*a*d*e^
2*e^(4*I*d*x + 4*I*c) + a*d*e^2*e^(3*I*d*x + 3*I*c))
```

3.442.6 Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{4}{3}}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))**(4/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*sec(c + d*x))**(4/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

3.442.7 Maxima [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(1/((e*sec(d*x + c))^(4/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.442.8 Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*sec(d*x + c))^(4/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{4/3}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.443 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

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3.443.1 Optimal result

Integrand size $=30$, antiderivative size $=437$

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx &= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} \\
&- \frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\
&+ \frac{5i\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{12\ 2^{2/3}\sqrt{3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\
&- \frac{5i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\
&- \frac{5i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\
&+ \frac{5i(d\sec(e+fx))^{2/3}}{24f\sqrt[3]{a+ia\tan(e+fx)}\,(a^2+ia^2\tan(e+fx))}
\end{aligned}
$$

output
```
1/4*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(7/3)-5/144*x*(d*sec(f*x+e
))^(2/3)*2^(1/3)/a^(5/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
-5/144*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f/(a-I*a*tan(
f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-5/48*I*ln(2^(1/3)*a^(1/3)-(a-I*a*ta
n(f*x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f/(a-I*a*tan(f*x+e))
^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+5/72*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*
tan(f*x+e))^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f
*3^(1/2)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+5/24*I*(d*sec(f
*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)/(a^2+I*a^2*tan(f*x+e))
```

3.443.2 Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.55

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{e^{-2i(e+fx)}\left(9i+33ie^{2i(e+fx)}+24ie^{4i(e+fx)}-10e^{4i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}fx-\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

output
```
((9*I + (33*I)*E^((2*I)*(e + f*x)) + (24*I)*E^((4*I)*(e + f*x)) - 10*E^((4
*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*x - (10*I)*Sqrt[3]*E^((4*
I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*
(e + f*x)))^(1/3))/Sqrt[3]] - (15*I)*E^((4*I)*(e + f*x))*(1 + E^((2*I)*(e
+ f*x)))^(1/3)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)])*Sec[e + f*x]^2*(d
*Sec[e + f*x])^(2/3))/(144*E^((2*I)*(e + f*x))*f*(a + I*a*Tan[e + f*x])^(7
/3))
```

3.443.3 Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.57, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^2}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{(i\tan(e+fx)a+a)^2}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4005}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \cos^4(e+fx)(a-ia\tan(e+fx))^{7/3}dx}{a^4\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{(a-ia\tan(e+fx))^{7/3}}{\sec(e+fx)^4}dx}{a^4\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)^3}d(-ia\tan(e+fx))}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{52}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)^2}d(-ia\tan(e+fx))}{12a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{4a(a+ia\tan(e+fx))^2}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{52}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)}d(-ia\tan(e+fx))}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right)}{12a}+\frac{\sqrt[3]{a-ia\tan(e+}}{4a(a+ia\tan(e+fx))}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{69}$$

3.443. $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{\frac{3\int\frac{1}{ia\tan(e+fx)+\sqrt[3]{2}\sqrt[3]{a}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int\frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3a}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(}$$

$$\Big\downarrow \textcolor{blue}{16}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{\frac{3\int\frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\mathrm{lo}}{}}{3a}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{1082}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{-\frac{3\int\frac{1}{a^2\tan^2(e+fx)-3}d\left(1-i2^{2/3}a^{2/3}\tan(e+fx)\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-ia}}{2a(a+ia}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{217}$$

3.443. $\int\frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{5\left(\frac{-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(e+fx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right.}{12a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2/3)*((a - I*a*Tan[e + f*x])^(1/3)/(4*a*(a + I*a*Ta
n[e + f*x])^2) + (5*((((-I)*Sqrt[3]*ArcTanh[(a*Tan[e + f*x])/Sqrt[3]])/(2^
(2/3)*a^(2/3)) - (3*Log[2^(1/3)*a^(1/3) + I*a*Tan[e + f*x]])/(2*2^(2/3)*a^
(2/3)) + Log[a + I*a*Tan[e + f*x]]/(2*2^(2/3)*a^(2/3)))/(3*a) + (a - I*a*T
an[e + f*x])^(1/3)/(2*a*(a + I*a*Tan[e + f*x]))))/(12*a)))/(f*(a - I*a*Tan
[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

3.443.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

3.443. $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

3.443. $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

3.443.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{7}{3}}}dx$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.21

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{\left(48\,a^3 f\left(\frac{125i\,d^2}{186624\,a^7f^3}\right)^{\frac{1}{3}} e^{(6i\,fx+6i\,e)}\log\left(-\frac{2}{5}\left(72i\,a^3 f\left(\frac{125i\,d^2}{186624\,a^7f^3}\right)^{\frac{1}{3}} e^{(2i\,fx+2i\,e)}\right.\right.\right.}{}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="fric
as")
```

output
```
1/48*(48*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(
-2/5*(72*I*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) -
5*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e
)) + 2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) +
 1))^(2/3)*(8*I*e^(6*I*f*x + 6*I*e) + 19*I*e^(4*I*f*x + 4*I*e) + 14*I*e^(2
*I*f*x + 2*I*e) + 3*I)*e^(2*I*f*x + 2*I*e) - 24*(-I*sqrt(3)*a^3*f + a^3*f)
*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(2/5*(5*2^(1/3)
*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(
e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 36*(sqrt(3)*a^3*f + I*a^3*f
)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*
I*e)) - 24*(I*sqrt(3)*a^3*f + a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^
(6*I*f*x + 6*I*e)*log(2/5*(5*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(
d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x +
2*I*e) - 36*(sqrt(3)*a^3*f - I*a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e
^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

3.443.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(7/3),x)
```

output
```
Timed out
```

3.443.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3902 vs. $2(324) = 648$.

Time $= 0.55$ (sec) , antiderivative size $= 3902$, normalized size of antiderivative $= 8.93$

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="maxi
ma")
```

output
```
1/288*(48*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/
2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(5/6)*((I*2^(1/3)*cos(4*f*x + 4*e) +
2^(1/3)*sin(4*f*x + 4*e))*cos(5/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e)
, cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))
 + 1)) - (2^(1/3)*cos(4*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(5/3*a
rctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)))*d^(2/3) + 30*(cos(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*
e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
 4*e))) + 1)^(1/3)*((-I*2^(1/3)*cos(4*f*x + 4*e) - 2^(1/3)*sin(4*f*x + 4*e
))*cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), c
os(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + (2^(1/3)*cos(4
*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(2/3*arctan2(sin(1/2*arctan2(
sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), co
s(4*f*x + 4*e))) + 1)))*d^(2/3) + 5*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt
(3)*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arct
an2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x +
 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(...
```

3.443.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx=\int \frac{(d\sec\left(fx+e\right))^{\frac{2}{3}}}{(i\,a\tan\left(fx+e\right)+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(7/3), x)
```

3.443.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx=\int \frac{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{2/3}}{{\left(a+a\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{7/3}}\,dx$$

input
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3),x)
```

output
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3), x)
```

3.444 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

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3.444.1 Optimal result

Integrand size = 30, antiderivative size = 378

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx &= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}}\\ &- \frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &+ \frac{i\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}\sqrt{3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &- \frac{i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &- \frac{i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\end{aligned}$$

output
```
1/2*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(4/3)-1/12*x*(d*sec(f*x+e)
)^(2/3)*2^(1/3)/a^(2/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-
1/12*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f/(a-I*a*tan(f*
x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-1/4*I*ln(2^(1/3)*a^(1/3)-(a-I*a*tan(f
*x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f/(a-I*a*tan(f*x+e))^(1
/3)/(a+I*a*tan(f*x+e))^(1/3)+1/6*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*tan(
f*x+e))^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f*3^(
1/2)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

3.444.2 Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.58

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{e^{-i(e+fx)}\Big(3i+3ie^{2i(e+fx)}-2e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}fx-2i\sqrt{3}e^{2i(e+fx)}\sqrt[3]{1}}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

output
```
((3*I + (3*I)*E^((2*I)*(e + f*x)) - 2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e
 + f*x)))^(1/3)*f*x - (2*I)*Sqrt[3]*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e +
 f*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*x)))^(1/3))/Sqrt[3]] - (3
*I)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Log[1 - (1 + E^((2
*I)*(e + f*x)))^(1/3)])*(d*Sec[e + f*x])^(5/3))/(12*d*E^(I*(e + f*x))*f*(a
 + I*a*Tan[e + f*x])^(4/3))
```

3.444.3 Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.53, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{i\tan(e+fx)a+a}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{i\tan(e+fx)a+a}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{4005}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \cos^2(e+fx)(a-ia\tan(e+fx))^{4/3}dx}{a^2\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \frac{(a-ia\tan(e+fx))^{4/3}}{\sec(e+fx)^2}dx}{a^2\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)^2}d(-ia\tan(e+fx))}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{52}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{\int \frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)}d(-ia\tan(e+fx))}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{69}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{\frac{3\int \frac{1}{ia\tan(e+fx)+\sqrt[3]{2}\sqrt[3]{a}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int \frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-}}{2\sqrt[3]{2}\sqrt[3]{a}}}{3a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{\frac{3\int \frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a}{2}}{3a}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{1082}$$

3.444. $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{-\frac{3\int\frac{1}{a^2\tan^2(e+fx)-3}d\left(1-i2^{2/3}a^{2/3}\tan(e+fx)\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-ia\,t}}{2a(a+ia\,t}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow 217$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(e+fx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}}{3a}+\frac{\sqrt[3]{a-ia\tan(e+fx)}}{2a(a+ia\tan(e+fx))}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2/3)*((((-I)*Sqrt[3]*ArcTanh[(a*Tan[e + f*x])/Sqrt[
3]])/(2^(2/3)*a^(2/3)) - (3*Log[2^(1/3)*a^(1/3) + I*a*Tan[e + f*x]])/(2*2^
(2/3)*a^(2/3)) + Log[a + I*a*Tan[e + f*x]]/(2*2^(2/3)*a^(2/3)))/(3*a) + (a
 - I*a*Tan[e + f*x])^(1/3)/(2*a*(a + I*a*Tan[e + f*x]))))/(f*(a - I*a*Tan[
e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

3.444.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

3.444. $\int\frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

3.444. $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

3.444.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.36

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{\left(4\,a^2 f\left(\frac{i\,d^2}{108\,a^4 f^3}\right)^{\frac{1}{3}} e^{(4i\,fx+4i\,e)}\log\left(-2\left(6i\,a^2 f\left(\frac{i\,d^2}{108\,a^4 f^3}\right)^{\frac{1}{3}} e^{(2i\,fx+2i\,e)} - 2^{\frac{1}{3}}\left(\right.\right.\right.\right.}{}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="fric
as")
```

output
```
1/4*(4*a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log(-2*(6*I
*a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 2^(1/3)*(a/(e^(
2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f
*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 2^(1/3)*(a/(
e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(I*e^(
4*I*f*x + 4*I*e) + 2*I*e^(2*I*f*x + 2*I*e) + I)*e^(2*I*f*x + 2*I*e) - 2*(-
I*sqrt(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)
*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e
) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 3*(sqrt(3)*a
^2*f + I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I
*f*x - 2*I*e)) - 2*(I*sqrt(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)
*e^(4*I*f*x + 4*I*e)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d
/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2
*I*e) - 3*(sqrt(3)*a^2*f - I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f
*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

3.444.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{2}{3}}}{(ia(\tan{(e+fx)}-i))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(4/3),x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(4/3), x)
```

3.444.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1906 vs. 2(279) = 558.

Time = 0.44 (sec) , antiderivative size = 1906, normalized size of antiderivative = 5.04

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="maxi
ma")
```

output
```
-1/24*(6*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/3)*((-I*2^(1/3)*cos(2*f*x + 2*e) - 2^(1/3)*sin(2*f*x + 2*e))*cos(2/3*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*cos(2*f*x + 2*
e) - I*2^(1/3)*sin(2*f*x + 2*e))*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)))*d^(2/3) - (-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/
3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt
(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3)))
 - 2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/
3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)
*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(1/3*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(2...
```

3.444.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx=\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(4/3), x)
```

3.444.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{4/3}}\,dx$$

input
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3),x)
```

output
```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3), x)
```

3.445 $\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$

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3.445.1 Optimal result

Integrand size = 30, antiderivative size = 340

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = &-\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &+\frac{i\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &-\frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\ &-\frac{3i\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\end{aligned}$$

output
```
-1/4*a^(1/3)*x*(d*sec(f*x+e))^(2/3)*2^(1/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*
a*tan(f*x+e))^(1/3)-1/4*I*a^(1/3)*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1
/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-3/4*I*a^(1/3)*ln(2
^(1/3)*a^(1/3)-(a-I*a*tan(f*x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/f/(a
-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+1/2*I*a^(1/3)*arctan(1/3*(
a^(1/3)+2^(2/3)*(a-I*a*tan(f*x+e))^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^
(2/3)*3^(1/2)*2^(1/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

3.445.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.47

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = -\frac{\left(\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{2/3}\sqrt[3]{1+e^{2i(e+fx)}}\left(2fx+2i\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+e^{2i(e+fx)}}}{\sqrt{3}}\right)+3i\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right)\right)}{2\,2^{2/3}\sqrt[3]{\frac{ae^{2i(e+fx)}}{1+e^{2i(e+fx)}}}\,f}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

output
```
-1/2*(((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(2/3)*(1 + E^((2*I)*
(e + f*x)))^(1/3)*(2*f*x + (2*I)*Sqrt[3]*ArcTan[(1 + 2*(1 + E^((2*I)*(e +
f*x)))^(1/3))/Sqrt[3]] + (3*I)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)]))/
(2^(2/3)*((a*E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1/3)*f)
```

3.445.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.45, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3973, 3042, 3962, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3973}$$

$$\frac{(d\sec(e+fx))^{2/3}\int \sqrt[3]{a-ia\tan(e+fx)}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\sec(e+fx))^{2/3}\int\sqrt[3]{a-ia\tan(e+fx)}dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{3962}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\int\frac{1}{(a-ia\tan(e+fx))^{2/3}(i\tan(e+fx)a+a)}d(-ia\tan(e+fx))}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{69}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{3\int\frac{1}{ia\tan(e+fx)+\sqrt[3]{2}\sqrt[3]{a}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int\frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(\frac{3\int\frac{1}{-a^2\tan^2(e+fx)-i\sqrt[3]{2}a^{4/3}\tan(e+fx)+2^{2/3}a^{2/3}}d\sqrt[3]{a-ia\tan(e+fx)}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}-\right.}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(-\frac{3\int\frac{1}{a^2\tan^2(e+fx)-3}d\left(1-i2^{2/3}a^{2/3}\tan(e+fx)\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{ia(d\sec(e+fx))^{2/3}\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(e+fx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}+ia\tan(e+fx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a+ia\tan(e+fx))}{2\ 2^{2/3}a^{2/3}}\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

input

```
Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

output

```
(I*a*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[e + f*x])/Sqrt[3]])/(2^(2/3)*a^(2/3)) -
 (3*Log[2^(1/3)*a^(1/3) + I*a*Tan[e + f*x]])/(2*2^(2/3)*a^(2/3)) + Log[a +
 I*a*Tan[e + f*x]]/(2*2^(2/3)*a^(2/3)))*(d*Sec[e + f*x])^(2/3))/(f*(a - I*
a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

3.445. $\int\frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}dx$

3.445.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 3973
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
   Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
&& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

3.445. $\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$

3.445.4 Maple [F]

$$\int \frac{(d\sec (fx+e))^{\frac{2}{3}}}{(a+ia\tan (fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.08

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx=\frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}(e\right.\right.$$
$$+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(e^{(2i\,fx+2i\,e)}+1\right)e^{(2i\,fx+2i\,e)}-\right.\right.$$
$$+\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(e^{(2i\,fx+2i\,e)}+1\right)e^{(2i\,fx+2i\,e)}-2i\,af\left(\frac{i\,d^2}{4\,af^3}\right)\right.\right.$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="fric
as")
```

output
```
1/2*(I*sqrt(3) - 1)*(1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*
x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2
*I*e) + 1)*e^(2*I*f*x + 2*I*e) + (sqrt(3)*a*f + I*a*f)*(1/4*I*d^2/(a*f^3))
^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 1/2*(-I*sqrt(3) - 1)*(
1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3
)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x
 + 2*I*e) - (sqrt(3)*a*f - I*a*f)*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I*f*x + 2
*I*e))*e^(-2*I*f*x - 2*I*e)) + (1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a
/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(
2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 2*I*a*f*(1/4*I*d^2/(a*f^3))^(1
/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e))
```

3.445.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{2}{3}}}{\sqrt[3]{ia\,(\tan{(e+fx)}-i)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(1/3),x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(1/3), x)
```

3.445.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(251) = 502$.

Time $= 0.47$ (sec) , antiderivative size $= 1753$, normalized size of antiderivative $= 5.16$

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="maxi
ma")
```

output

```
1/8*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*sqrt(3)*2^(1/3)*
arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2) - 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*...
```

3.445.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx=\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input

```
integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="giac
")
```

output

```
integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(1/3), x)
```

3.445.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{1/3}}\,dx$$

input

```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3),x)
```

output

```
int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3), x)
```

3.446 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx$

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3.446.1 Optimal result

Integrand size = 30, antiderivative size = 37

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

output
```
3*I*a*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)
```

3.446.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3d^2(i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{f(d\sec(e+fx))^{4/3}}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

output
```
(3*d^2*(I + Tan[e + f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(f*(d*Sec[e + f*x]
)^(4/3))
```

3.446.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \ 3974$$

$$\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

output
```
((3*I)*a*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
```

3.446.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.446.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{2}{3}} (a+ia\tan (fx+e))^{\frac{2}{3}}\,dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = -\frac{3\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(-i\,e^{(2i\,fx+2i\,e)}-i\right)}{f}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="fric
as")
```

output
```
-3*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1
))^(2/3)*(-I*e^(2*I*f*x + 2*I*e) - I)/f
```

3.446.6 Sympy [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \int (d\sec (e+fx))^{\frac{2}{3}} (ia(\tan (e+fx)-i))^{\frac{2}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(2/3),x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)*(I*a*(tan(e + f*x) - I))**(2/3), x)
```

3.446.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(29) = 58$.

Time = 0.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = -\frac{3\Bigl(-i\cdot 2^{\frac{1}{3}}\cos\bigl(\frac{1}{3}\arctan(\sin(2\,fx+2\,e),\cos(2\,fx+2\,e)+1)\bigr)-2^{\frac{1}{3}}\sin\bigl(\frac{1}{3}\arctan(\sin(2\,fx+2\,e),\cos(2}{\left(\cos(2\,fx+2\,e)^2+\sin(2\,fx+2\,e)^2+2\,\cos(2\,fx+2\,e)+1\right)^{\frac{1}{6}}f}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="maxi
ma")
```

output
```
-3*(-I*2^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
2^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*
d^(2/3)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/6)*f)
```

3.446.8 Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\,(i\,a\tan(fx+e)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(2/3), x)
```

3.446.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \frac{3\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\left(\cos\left(2\,e+2\,f\,x\right)\,1\mathrm{i}+\sin\left(2\,e+2\,f\,x\right)+1\mathrm{i}\right)\left(\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+}{\cos(2\,e+2\,f\,x)+1}\right.}{2\,f}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(2/3),x)
```

output
```
(3*(d/cos(e + f*x))^(2/3)*(cos(2*e + 2*f*x)*1i + sin(2*e + 2*f*x) + 1i)*((
a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(2
/3))/(2*f)
```

3.447 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx$

3.447.1 Optimal result 3203
3.447.2 Mathematica [A] (verified) 3203
3.447.3 Rubi [A] (verified) 3204
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3.447.1 Optimal result

Integrand size = 30, antiderivative size = 81

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx=\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}$$

output
```
9/2*I*a^2*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+3/4*I*a*(d*sec(f
*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f
```

3.447.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx=-\frac{3ad(\cos(e)-i\sin(e))(\cos(fx)-i\sin(fx))(-7i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{4f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

output
```
(-3*a*d*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(-7*I + Tan[e + f*x])*
(a + I*a*Tan[e + f*x])^(2/3))/(4*f*(d*Sec[e + f*x])^(1/3))
```

3.447.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

$$\Big\downarrow \text{3974}$$

$$\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

output
```
(((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) +
(((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f
```

3.447.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.447.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{2}{3}} (a+ia\tan (fx+e))^{\frac{5}{3}}\,dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

3.447.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = -\frac{3 \cdot 2^{\frac{1}{3}} \left(-4i\,ae^{\left(2i\,fx+2i\,e\right)} - 3i\,a\right) \left(\frac{a}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{2}{3}}}{2\,f}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="fric
as")
```

output
```
-3/2*2^(1/3)*(-4*I*a*e^(2*I*f*x + 2*I*e) - 3*I*a)*(a/(e^(2*I*f*x + 2*I*e)
+ 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)/f
```

3.447.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(5/3),x)
```

output
```
Timed out
```

3.447.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(61) = 122.

Time = 0.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{3\Big(\Big(-i\cdot 2^{\frac{1}{3}}a\cos\big(\tfrac{4}{3}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\big)-2^{\frac{1}{3}}a\sin\big(\tfrac{4}{3}\ \mathrm{arc}}{}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="maxi
ma")
```

output
```
3/2*((-I*2^(1/3)*a*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
) - 2^(1/3)*a*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sq
rt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(2/
3)*d^(2/3) + 4*((I*2^(1/3)*a*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a*sin(2*f*x +
2*e)^2 + 2*I*2^(1/3)*a*cos(2*f*x + 2*e) + I*2^(1/3)*a)*cos(1/3*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*a*cos(2*f*x + 2*e)^2 + 2^
(1/3)*a*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a*cos(2*f*x + 2*e) + 2^(1/3)*a)*sin
(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^(2/3))/((
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(7/6)*f)
```

3.447.8 Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\,(ia\tan(fx+e)+a)^{\frac{5}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(5/3), x)
```

3.447.9 Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int (d\sec(e+fx))^{2/3}(a +ia\tan(e+fx))^{5/3}\,dx = \frac{3\,a\left(\frac{d}{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}\right)^{2/3}\left(\cos\left(e+f\,x\right)^2\,6\mathrm{i}+3\,\sin\left(2\,e+2\,f\,x\right)+1\mathrm{i}\right)\,\left(\frac{a\left(2\cos(e+f\,x)^2+\right.}{2\cos(e}\right.}{4\,f}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(5/3),x)
```

output
```
(3*a*(d/(2*cos(e/2 + (f*x)/2)^2 - 1))^(2/3)*(3*sin(2*e + 2*f*x) + cos(e +
f*x)^2*6i + 1i)*((a*(sin(2*e + 2*f*x)*1i + 2*cos(e + f*x)^2))/(2*cos(e + f
*x)^2))^(2/3))/(4*f)
```

3.448 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$

3.448.1 Optimal result 3208
3.448.2 Mathematica [A] (verified) 3208
3.448.3 Rubi [A] (verified) 3209
3.448.4 Maple [F] 3210
3.448.5 Fricas [A] (verification not implemented) 3211
3.448.6 Sympy [F(-1)] 3211
3.448.7 Maxima [B] (verification not implemented) 3211
3.448.8 Giac [F] 3212
3.448.9 Mupad [B] (verification not implemented) 3212

3.448.1 Optimal result

Integrand size = 30, antiderivative size = 122

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f}$$

output
```
54/7*I*a^3*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+9/7*I*a^2*(d*se
c(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+3/7*I*a*(d*sec(f*x+e))^(2/3)*(a
+I*a*tan(f*x+e))^(5/3)/f
```

3.448.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{3a^2(d\sec(e+fx))^{5/3}(i\cos(e-fx)+\sin(e-fx))(21+23\cos(2(e+fx))+5i\sin(}{14df(\cos(fx)+i\sin(fx))^2}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

output
```
(3*a^2*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - f*x] + Sin[e - f*x])*(21 + 23*Cos
[2*(e + f*x)] + (5*I)*Sin[2*(e + f*x)])*(a + I*a*Tan[e + f*x])^(2/3))/(14*
d*f*(Cos[f*x] + I*Sin[f*x])^2)
```

3.448.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3975}$$

$$\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

$$\Big\downarrow \text{3974}$$

$$\frac{12}{7}a\left(\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

output
```
(((3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f + (12*
a*((((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
 + (((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f))/7
```

3.448.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.448.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}} (a+ia\tan(fx+e))^{\frac{8}{3}}\,dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx =$$

$$-\frac{6\cdot 2^{\frac{1}{3}}\left(-14i\,a^2e^{(4i\,fx+4i\,e)}-21i\,a^2e^{(2i\,fx+2i\,e)}-9i\,a^2\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{(2i\,fx+2i\,e)}}{7\left(fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="fric
as")
```

output
```
-6/7*2^(1/3)*(-14*I*a^2*e^(4*I*f*x + 4*I*e) - 21*I*a^2*e^(2*I*f*x + 2*I*e)
 - 9*I*a^2)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) +
1))^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*
e))
```

3.448.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(8/3),x)
```

output
```
Timed out
```

3.448.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(92) = 184.

Time = 0.89 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.30

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{6\Bigg(7\Big(-i\cdot 2^{\frac{1}{3}}a^2\cos\left(\frac{4}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-2^{\frac{1}{3}}a^2\sin\Big(\frac{4}{3}}{}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="maxi
ma")
```

output
```
6/7*(7*(-I*2^(1/3)*a^2*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - 2^(1/3)*a^2*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)))*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*a^(2/3)*d^(2/3) + 2*(I*2^(1/3)*a^2*cos(7/3*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)) + 2^(1/3)*a^2*sin(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)) + 7*(I*2^(1/3)*a^2*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^2*sin(
2*f*x + 2*e)^2 + 2*I*2^(1/3)*a^2*cos(2*f*x + 2*e) + I*2^(1/3)*a^2)*cos(1/3
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(2^(1/3)*a^2*cos(2*f
*x + 2*e)^2 + 2^(1/3)*a^2*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^2*cos(2*f*x + 2
*e) + 2^(1/3)*a^2)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
))*a^(2/3)*d^(2/3))/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(7/6)*f)
```

3.448.8 Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx=\int (d\sec(fx+e))^{\frac{2}{3}}\,(ia\tan(fx+e)+a)^{\frac{8}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(8/3), x)
```

3.448.9 Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx=\frac{3\,a^2\,{\left(\frac{d}{\cos(e+f\,x)}\right)}^{2/3}\,{\left(\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}\right)}^{2/3}\,\left(\cos\left(2\,e+2\,f\,x\right)\,44\mathrm{i}+\cos\left(4\right.\right.}{14\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+\right.}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(8/3),x)
```

output

```
(3*a^2*(d/cos(e + f*x))^(2/3)*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i
+ 1))/(cos(2*e + 2*f*x) + 1))^(2/3)*(cos(2*e + 2*f*x)*44i + cos(4*e + 4*f*
x)*9i + 16*sin(2*e + 2*f*x) + 9*sin(4*e + 4*f*x) + 35i))/(14*f*(cos(2*e +
2*f*x) + 1))
```

3.448. $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$

3.449 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx$

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3.449.1 Optimal result

Integrand size = 30, antiderivative size = 163

$$\begin{aligned}\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx &= \frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}}\\ &+ \frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f}\\ &+ \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f}\\ &+ \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f}\end{aligned}$$

output
```
486/35*I*a^4*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+81/35*I*a^3*(
d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+27/35*I*a^2*(d*sec(f*x+e))^
(2/3)*(a+I*a*tan(f*x+e))^(5/3)/f+3/10*I*a*(d*sec(f*x+e))^(2/3)*(a+I*a*tan(
f*x+e))^(8/3)/f
```

3.449.2 Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \frac{3a^3(d\sec(e+fx))^{5/3}(i\cos(e-2fx)+\sin(e-2fx))(364+442\cos(2(e+fx))+}{140df(\cos(fx)+i}$$

input

```
Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

output

```
(3*a^3*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - 2*f*x] + Sin[e - 2*f*x])*(364 + 4
42*Cos[2*(e + f*x)] + (59*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (45*I)*Tan[e
+ f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(140*d*f*(Cos[f*x] + I*Sin[f*x])^3)
```

3.449.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3975, 3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{11/3}(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{11/3}(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{9}{5}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{8/3}dx+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{5}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{8/3}dx+\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \text{3975}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}\right)+$$
$$\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{5/3}dx+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}\right)+$$
$$\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \ \color{blue}{3975}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+$$
$$\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \ \color{blue}{3042}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\left(\frac{3}{2}a\int (d\sec(e+fx))^{2/3}(i\tan(e+fx)a+a)^{2/3}dx+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+$$
$$\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

$$\Big\downarrow \ \color{blue}{3974}$$

$$\frac{9}{5}a\left(\frac{12}{7}a\left(\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}\right)+\frac{3ia(a+ia\tan(e+fx))^{5/3}}{7f}$$
$$\frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

output
```
(((3*I)/10)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3))/f + (9*
a*((((3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f + (
12*a*((((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/
3)) + (((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f)
)/7))/5
```

3.449. $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx$

3.449.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.449.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}}(a+ia\tan(fx+e))^{\frac{11}{3}}\,dx$$

input
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

output
```
int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx =$$

$$-\frac{6\cdot 2^{\frac{1}{3}}\left(-140i\,a^3e^{(6i\,fx+6i\,e)}-315i\,a^3e^{(4i\,fx+4i\,e)}-270i\,a^3e^{(2i\,fx+2i\,e)}-81i\,a^3\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)}{35\left(fe^{(6i\,fx+6i\,e)}+2\,fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="fri
cas")
```

output
```
-6/35*2^(1/3)*(-140*I*a^3*e^(6*I*f*x + 6*I*e) - 315*I*a^3*e^(4*I*f*x + 4*I
*e) - 270*I*a^3*e^(2*I*f*x + 2*I*e) - 81*I*a^3)*(a/(e^(2*I*f*x + 2*I*e) +
1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(6*
I*f*x + 6*I*e) + 2*f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

3.449.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(11/3),x)
```

output
```
Timed out
```

3.449.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(123) = 246$.

Time $= 0.40$ (sec) , antiderivative size $= 983$, normalized size of antiderivative $= 6.03$

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="max
ima")
```

output
```
6/35*(7*(-2*I*2^(1/3)*a^3*cos(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)) - 2*2^(1/3)*a^3*sin(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)) + 15*(-I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 - I*2^(1/3)*a^3*sin(2*f*
x + 2*e)^2 - 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) - I*2^(1/3)*a^3)*cos(4/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 15*(2^(1/3)*a^3*cos(2*f*x
+ 2*e)^2 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e)
 + 2^(1/3)*a^3)*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*
sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(
2/3)*d^(2/3) + 20*(3*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^3*sin
(2*f*x + 2*e)^2 + 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) + I*2^(1/3)*a^3)*cos(7/
3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(I*2^(1/3)*a^3*cos(
2*f*x + 2*e)^4 + I*2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*I*2^(1/3)*a^3*cos(2*
f*x + 2*e)^3 + 6*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*I*2^(1/3)*a^3*cos(2*
f*x + 2*e) + I*2^(1/3)*a^3 + 2*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2*I*2^(
1/3)*a^3*cos(2*f*x + 2*e) + I*2^(1/3)*a^3)*sin(2*f*x + 2*e)^2)*cos(1/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 3*(2^(1/3)*a^3*cos(2*f*x +
 2*e)^2 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e)
+ 2^(1/3)*a^3)*sin(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
7*(2^(1/3)*a^3*cos(2*f*x + 2*e)^4 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*2^(
1/3)*a^3*cos(2*f*x + 2*e)^3 + 6*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*2^(1...
```

3.449.8 Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx=\int (d\sec(fx+e))^{\frac{2}{3}}\,(i\,a\tan(fx+e)+a)^{\frac{11}{3}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="gia
c")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(11/3), x)
```

3.449.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.86

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \frac{\left(-\frac{d}{2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}-1}\right)^{2/3}\left(2\sin\left(2e+2fx\right)^{2}+\sin\left(4e+4fx\right)\,1\mathrm{i}-1\right)\left(\frac{a^{3}\left(a-\frac{a\,\sin(}{2\sin\left(\frac{e}{2}\right.}\right.}{}\right.}{}$$

input
```
int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(11/3),x)
```

output
```
((-d/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*e + 4*f*x)*1i + 2*sin(2*e
+ 2*f*x)^2 - 1)*((a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1
))^(2/3)*243i)/(35*f) + (a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2
)^2 - 1))^(2/3)*(sin(2*e + 2*f*x)*1i - 2*sin(e + f*x)^2 + 1)*162i)/(7*f) +
 (a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*
e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*27i)/f + (a^3*(a - (a*sin(e + f*
x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(6*e + 6*f*x)*1i - 2*sin(3*
e + 3*f*x)^2 + 1)*12i)/f))/(4*(sin(e + f*x)^2 - 1))
```

3.450 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx$

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3.450.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \frac{i2^{5+\frac{m}{2}}a^5\,\text{Hypergeometric2F1}\left(-4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))}{dm}$$

output
```
I*2^(5+1/2*m)*a^5*hypergeom([1/2*m, -4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

3.450.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 210 vs. 2(86) = 172.

Time = 6.42 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.44

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \frac{a^5(e\sec(c+dx))^m\Bigg(\frac{i(16(8+6m+m^2)-12m(4+m)\sec^2(c+dx)+m(2+m)\sec^4(c+dx))}{8+6m+m^2}+5\cot(c+dx)\,\text{Hypergeometric2F1}}{}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(a^5*(e*Sec[c + d*x])^m*((I*(16*(8 + 6*m + m^2) - 12*m*(4 + m)*Sec[c + d*x
]^2 + m*(2 + m)*Sec[c + d*x]^4))/(8 + 6*m + m^2) + 5*Cot[c + d*x]*Hypergeo
metric2F1[-3/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2] + 10
*Cot[c + d*x]*Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt
[-Tan[c + d*x]^2] + Cot[c + d*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Se
c[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

3.450.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^5(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^5(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+10}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+10}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^6 2^{\frac{m}{2}+4}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m+8}{2}}(a-ia\tan(}{d}$$

$$\Big\downarrow 79$$

$$\frac{ia^5 2^{\frac{m}{2}+5}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2}(-m-8),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

output
```
(I*2^(5 + m/2)*a^5*Hypergeometric2F1[(-8 - m)/2, m/2, (2 + m)/2, (1 - I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

3.450.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.450.4 Maple [F]

$$\int (e\sec (dx+c))^{m} (a+ia\tan (dx+c))^{5} dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

3.450.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx=\int (i\,a\tan (dx+c)+a)^5(e\sec (dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

output
```
integral(32*a^5*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(10*I*
d*x + 10*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

3.450.6 Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int \left(-i(e\sec(c+dx))^m\right)\,dx + \int 5(e\sec(c+dx))^m\tan(c+dx)\,dx + \int \left(-10(e\sec(c+dx))^m\tan^3(c+dx)\right)\,dx + \int (e\sec(c+dx))^m\tan^5(c+dx)\,dx + \int 10i(e\sec(c+dx))^m\tan^2(c+dx)\,dx + \int \left(-5i(e\sec(c+dx))^m\tan^4(c+dx)\right)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**5,x)
```

output
```
I*a**5*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral(5*(e*sec(c + d*x))*
*m*tan(c + d*x), x) + Integral(-10*(e*sec(c + d*x))**m*tan(c + d*x)**3, x)
 + Integral((e*sec(c + d*x))**m*tan(c + d*x)**5, x) + Integral(10*I*(e*sec
(c + d*x))**m*tan(c + d*x)**2, x) + Integral(-5*I*(e*sec(c + d*x))**m*tan(
c + d*x)**4, x))
```

3.450.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan(dx+c)+a)^5(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

3.450.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan\left(dx+c\right)+a)^5(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^5\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5, x)
```

3.451 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx$

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3.451.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{i2^{3+\frac{m}{2}}a^3\,\text{Hypergeometric2F1}\left(-2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))}{dm}$$

output
```
I*2^(3+1/2*m)*a^3*hypergeom([1/2*m, -2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

3.451.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{ia^3(e\sec(c+dx))^m\Big(-3i(2+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\tan(c+dx)-i(2+}{dm(2+}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((-I)*a^3*(e*Sec[c + d*x])^m*((-3*I)*(2 + m)*Hypergeometric2F1[-1/2, m/2,
(2 + m)/2, Sec[c + d*x]^2]*Tan[c + d*x] - I*(2 + m)*Hypergeometric2F1[1/2,
 m/2, (2 + m)/2, Sec[c + d*x]^2]*Tan[c + d*x] + (-8 - 4*m + m*Sec[c + d*x]
^2)*Sqrt[-Tan[c + d*x]^2]))/(d*m*(2 + m)*Sqrt[-Tan[c + d*x]^2])
```

3.451.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+6}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+6}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^4 2^{\frac{m}{2}+2}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m+4}{2}}(a-ia\tan(c}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia^3 2^{\frac{m}{2}+3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \,\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-4),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(I*2^(3 + m/2)*a^3*Hypergeometric2F1[(-4 - m)/2, m/2, (2 + m)/2, (1 - I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

3.451.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.451.4 Maple [F]

$$\int (e\sec (dx+c))^{m} (a+ia\tan (dx+c))^{3}dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

3.451.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx=\int (i\,a\tan (dx+c)+a)^{3}(e\sec (dx+c))^{m}\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(8*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(6*I*d*
x + 6*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2
*I*c) + 1), x)
```

3.451.6 Sympy [F]

$$\begin{aligned}\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx=-ia^{3}\bigg(\int i(e\sec (c+dx))^{m}\,dx\\+\int (-3(e\sec (c+dx))^{m}\tan (c+dx))\,dx\\+\int (e\sec (c+dx))^{m}\tan^{3}(c+dx)\,dx\\+\int (-3i(e\sec (c+dx))^{m}\tan^{2}(c+dx))\,dx\bigg)\end{aligned}$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*a**3*(Integral(I*(e*sec(c + d*x))**m, x) + Integral(-3*(e*sec(c + d*x))
**m*tan(c + d*x), x) + Integral((e*sec(c + d*x))**m*tan(c + d*x)**3, x) +
Integral(-3*I*(e*sec(c + d*x))**m*tan(c + d*x)**2, x))
```

3.451.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int (i\,a\tan(dx+c)+a)^3(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

3.451.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int (i\,a\tan(dx+c)+a)^3(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\,1\mathrm{i})^3\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3, x)
```

3.452 $\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^2\,dx$

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3.452.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^2\,dx = \frac{i2^{2+\frac{m}{2}}a^2\,\text{Hypergeometric2F1}\left(-1-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan (c+dx))\right)(e\sec (c+dx))^m(1+i\tan (c+dx))}{dm}$$

output
```
I*2^(2+1/2*m)*a^2*hypergeom([1/2*m, -1-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

3.452.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

$$\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^2\,dx = \frac{a^2(e\sec (c+dx))^m\Big(2i+\cot (c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec ^2(c+dx)\right)\sqrt{-\tan ^2(c+dx)}+}{dm}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(a^2*(e*Sec[c + d*x])^m*(2*I + Cot[c + d*x]*Hypergeometric2F1[-1/2, m/2, (
2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2] + Cot[c + d*x]*Hypergeomet
ric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

3.452.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+4}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+4}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx)}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+4}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+4}{2},\frac{m+6}{2},\right.}{d(m+4)}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (4 + m)/2, (6 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (4 + m)
/2))/(d*(4 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

3.452.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.452.4 Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^2\,dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

3.452.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(4*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(4*I*d*
x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

3.452.6 Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int (-(e\sec(c+dx))^m)\,dx + \int (e\sec(c+dx))^m\tan^2(c+dx)\,dx + \int (-2i(e\sec(c+dx))^m\tan(c+dx))\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*ta
n(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**m*tan(c + d*x), x))
```

3.452.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

3.452.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\;1\mathrm{i})^2\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

3.453 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$

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3.453.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \frac{i2^{1+\frac{m}{2}}a\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

output
```
I*2^(1+1/2*m)*a*hypergeom([1/2*m, -1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

3.453.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \frac{a(e\sec(c+dx))^m\left(i+\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}\right)}{dm}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(e*Sec[c + d*x])^m*(I + Cot[c + d*x]*Hypergeometric2F1[1/2, m/2, (2 + m
)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

3.453.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+2}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+2}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx)}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+2}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+2}{2},\frac{m+4}{2},\right.}{d(m+2)}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (2 + m)/2, (4 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (2 + m)
/2))/(d*(2 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

3.453.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.453. $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$

3.453.4 Maple [F]

$$\int (e\sec (dx+c))^{m} (a+ia\tan (dx+c))\,dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

3.453.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx=\int (i\,a\tan (dx+c)+a)(e\sec (dx+c))^{m}\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(2*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(2*I*d*x
+ 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

3.453.6 Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx=ia\left(\int (-i(e\sec (c+dx))^{m})\,dx\right.$$
$$\left.+\int (e\sec (c+dx))^{m}\tan (c+dx)\,dx\right)$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*ta
n(c + d*x), x))
```

3.453.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

3.453.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m (a+a\tan(c+d\,x)\;1\mathrm{i})\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

3.454 $\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

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3.454.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \frac{i2^{-1+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/}}{adm}$$

output
```
I*2^(-1+1/2*m)*hypergeom([1/2*m, 2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/a/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

3.454.2 Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.77

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i2^{-1+m}e^{-i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-1+m,\frac{m}{2},-e^{2i(c+dx)}\right.}{d(-2+m)(a+ia\tan(c+dx))}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(-1 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*
I)*(c + d*x)))^m*Hypergeometric2F1[(-2 + m)/2, -1 + m, m/2, -E^((2*I)*(c +
 d*x))]*Sec[c + d*x]^(1 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x]))/(
d*E^(I*(c + 2*d*x))*(-2 + m)*(a + I*a*Tan[c + d*x]))
```

3.454.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-2}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-2}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m-4}{2}}(a-ia\tan(c+}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m}{2}-1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{4-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output
```
(I*2^(-1 + m/2)*Hypergeometric2F1[(4 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m)/(a*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

3.454.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.454.4 Maple [F]

$$\int \frac{(e\sec (dx+c))^{m}}{a+ia\tan (dx+c)}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

3.454.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec (dx+c))^{m}}{i\,a\tan (dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(2*I*d*x
 + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

3.454.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\sec (c+dx))^{m}}{\tan (c+dx)-i}\,dx}{a}$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*sec(c + d*x))**m/(tan(c + d*x) - I), x)/a
```

3.454.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.454.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

3.455 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

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3.455.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \frac{i2^{-2+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(3-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m}}{a^2dm}$$

output
```
I*2^(-2+1/2*m)*hypergeom([1/2*m, 3-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/a^2/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

3.455.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-2+m}e^{-2i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-4+m),-2+m,\frac{1}{2}(-2+m),\right.}{d(-4+m)(a+ia\tan(c+dx))^2}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(-2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*
I)*(c + d*x)))^m*Hypergeometric2F1[(-4 + m)/2, -2 + m, (-2 + m)/2, -E^((2*
I)*(c + d*x))]*Sec[c + d*x]^(2 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d
*x])^2)/(d*E^((2*I)*(c + 2*d*x))*(-4 + m)*(a + I*a*Tan[c + d*x])^2)
```

3.455.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{3042, 3986, 3042, 4006, 80, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-4}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-4}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m-6}{2}}(a-ia\tan(c+}{ad}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{6-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(I*2^(-2 + m/2)*Hypergeometric2F1[(6 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m)/(a^2*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

3.455.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.455. $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

3.455.4 Maple [F]

$$\int \frac{(e \sec (dx+c))^{m}}{(a+ia \tan (dx+c))^{2}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

3.455.5 Fricas [F]

$$\int \frac{(e \sec(c+dx))^m}{(a+ia \tan(c+dx))^2} \, dx = \int \frac{(e \sec (dx+c))^{m}}{(i \, a \tan (dx+c)+a)^{2}} \, dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/4*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(4*I*d*x
 + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

3.455.6 Sympy [F]

$$\int \frac{(e \sec(c+dx))^m}{(a+ia \tan(c+dx))^2} \, dx = -\frac{\int \frac{(e \sec (c+dx))^{m}}{\tan^{2} (c+dx)-2i \tan (c+dx)-1}\, dx}{a^{2}}$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*sec(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

3.455.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.455.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^m}{(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

3.456 $\int \frac{(e \sec(c+dx))^m}{(a+ia \tan(c+dx))^3} \, dx$

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3.456.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e \sec(c+dx))^m}{(a+ia \tan(c+dx))^3} \, dx = \frac{i 2^{-3+\frac{m}{2}} \text{Hypergeometric2F1}\left(4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m}}{a^3 d m}$$

output
```
I*2^(-3+1/2*m)*hypergeom([1/2*m, 4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m/a^3/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

3.456.2 Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.78

$$\int \frac{(e \sec(c+dx))^m}{(a+ia \tan(c+dx))^3} \, dx = \frac{2^{-3+m} e^{-3i(c+2dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m \left(1+e^{2i(c+dx)}\right)^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-6+m),-3+m,\frac{1}{2}(-4+m),-e\right.}{a^3 d(-6+m)(-i+\tan(c+dx))^3}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(2^(-3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c
 + d*x)))^m*Hypergeometric2F1[(-6 + m)/2, -3 + m, (-4 + m)/2, -E^((2*I)*(c
 + d*x))]*Sec[c + d*x]^(3 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x])^
3)/(a^3*d*E^((3*I)*(c + 2*d*x))*(-6 + m)*(-I + Tan[c + d*x])^3)
```

3.456.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow 3042$$

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow 3986$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-6}{2}}dx$$

$$\Big\downarrow 3042$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-6}{2}}dx$$

$$\Big\downarrow 4006$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int(a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow 80$$

$$\frac{2^{\frac{m}{2}-4}(1+i\tan(c+dx))^{-m/2}(a-ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{\frac{m-8}{2}}(a-ia\tan(c+}{a^2d}$$

$$\Big\downarrow 79$$

$$\frac{i2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{8-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^3dm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(I*2^(-3 + m/2)*Hypergeometric2F1[(8 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m)/(a^3*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

3.456.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.456.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^3}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

3.456.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
integral(1/8*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(6*I*d*x
 + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-6*I*d*x
 - 6*I*c)/a^3, x)
```

3.456.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^m}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*Integral((e*sec(c + d*x))**m/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*
tan(c + d*x) + I), x)/a**3
```

3.456.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.456.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^3, x)
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3, x)
```

3.457 $\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^{7/2}\,dx$

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3.457.1 Optimal result

Integrand size = 28, antiderivative size = 109

$$\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^{7/2}\,dx=\frac{i2^{\frac{7+m}{2}}a^3\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan (c+dx))\right)(e\sec (c+dx))^m(1+}{dm}$$

```
output
I*2^(7/2+1/2*m)*a^3*hypergeom([1/2*m, -5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(
d*x+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1
/2*m)/d/m
```

3.457.2 Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^{7/2}\,dx = -\frac{i2^{\frac{7}{2}+m}e^{3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{7}{2}+m,\frac{7+m}{2},\frac{9+m}{2},-e^{2i(c+dx)}\right.}{d(7+m)(\cos (dx)+i\sin (dx))^{7/2}}$$

```
input
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
((-I)*2^(7/2 + m)*E^((3*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hy
pergeometric2F1[7/2 + m, (7 + m)/2, (9 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c
 + d*x]^(-7/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2))/(d*(7
+ m)*(Cos[d*x] + I*Sin[d*x])^(7/2))
```

3.457.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{7/2}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{7/2}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+7}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+7}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^4 2^{\frac{m+5}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia^3 2^{\frac{m+5}{2}+1}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\,\big(\tfrac{1}{2}(-m-5}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

output
```
(I*2^(1 + (5 + m)/2)*a^3*Hypergeometric2F1[(-5 - m)/2, m/2, (2 + m)/2, (1
- I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*
(a + I*a*Tan[c + d*x])^(-1/2*m + (1 + m)/2))/(d*m)
```

3.457.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.457.4 Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^{\frac{7}{2}}\,dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

3.457.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
integral(8*sqrt(2)*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(e^(6*I*d*x + 6*I*c)
+ 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

3.457.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.457.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

3.457.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.458 $\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^{5/2}\,dx$

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3.458.1 Optimal result

Integrand size = 28, antiderivative size = 109

$$\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^{5/2}\,dx=\frac{i2^{\frac{5+m}{2}}a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan (c+dx))\right)(e\sec (c+dx))^m(1+}{dm}$$

output

```
I*2^(5/2+1/2*m)*a^2*hypergeom([1/2*m, -3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(
d*x+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1
/2*m)/d/m
```

3.458.2 Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int (e\sec (c+dx))^m(a+ia\tan (c+dx))^{5/2}\,dx=-\frac{i2^{\frac{5}{2}+m}e^{2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{5}{2}+m,\frac{5+m}{2},\frac{7+m}{2},-e^{2i(c+dx)}}{d(5+m)(\cos (dx)+i\sin (dx))^{5/2}}$$

input

```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*2^(5/2 + m)*E^((2*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hy
pergeometric2F1[5/2 + m, (5 + m)/2, (7 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c
 + d*x]^(-5/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2))/(d*(5
+ m)*(Cos[d*x] + I*Sin[d*x])^(5/2))
```

3.458.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+5}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+5}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^3 2^{\frac{m+3}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int (\frac{1}{2}i\tan(}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia^2 2^{\frac{m+3}{2}+1}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m \text{Hypergeometric2F1}\,\big(\tfrac{1}{2}(-m-3}{dm}$$

input
```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(I*2^(1 + (3 + m)/2)*a^2*Hypergeometric2F1[(-3 - m)/2, m/2, (2 + m)/2, (1
- I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*
(a + I*a*Tan[c + d*x])^(-1/2*m + (1 + m)/2))/(d*m)
```

3.458.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.458.4 Maple [F]

$$\int \left(e \sec \left(d x +c \right)\right)^{m} \left(a +i a \tan \left(d x +c \right)\right)^{\frac{5}{2}}d x$$

input

```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

output

```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

3.458.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \int \left(i\, a \tan \left(dx + c\right) + a\right)^{\frac{5}{2}} \left(e \sec \left(dx + c\right)\right)^{m}\,dx$$

input

```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output

```
integral(4*sqrt(2)*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(e^(4*I*d*x + 4*I*c)
+ 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

3.458.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input

```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(5/2),x)
```

output

```
Timed out
```

3.458.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx=\int (ia\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

3.458.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx=\int (ia\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

3.458.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.459 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx$

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3.459.1 Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=\frac{i2^{\frac{3+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+}{dm}$$

output
```
I*2^(3/2+1/2*m)*a*hypergeom([1/2*m, -1/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1/2
*m)/d/m
```

3.459.2 Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=-\frac{i2^{\frac{3}{2}+m}e^{i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{3}{2}+m,\frac{3+m}{2},\frac{5+m}{2},-e^{2i(c+dx)}\right)}{d(3+m)(\cos(dx)+i\sin(dx))^{3/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*2^(3/2 + m)*E^(I*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hyperg
eometric2F1[3/2 + m, (3 + m)/2, (5 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d
*x]^(-3/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2))/(d*(3 + m)
*(Cos[d*x] + I*Sin[d*x])^(3/2))
```

3.459.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \ 3986$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+3}{2}}dx$$

$$\Big\downarrow \ 3042$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+3}{2}}dx$$

$$\Big\downarrow \ 4006$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \ 80$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx)}{d}$$

$$\Big\downarrow \ 79$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+3}{2}-\frac{m}{2}}(e\sec(c+dx))^{m}\,\text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+3}{2},\frac{m+5}{2},\right.}{d(m+3)}$$

input

```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (3 + m)/2, (5 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (3 + m)
/2))/(d*(3 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

3.459.3.1 Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.459.4 Maple [F]

$$\int (e\sec (dx+c))^{m} (a+ia\tan (dx+c))^{\frac{3}{2}}dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

3.459.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=\int (i\,a\tan (dx+c)+a)^{\frac{3}{2}}(e\sec (dx+c))^{m}\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(2*sqrt(2)*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(e^(2*I*d*x + 2*I*c) +
1), x)
```

3.459.6 Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=\int (e\sec (c+dx))^{m} (ia(\tan (c+dx)-i))^{\frac{3}{2}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.459.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

3.459.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}(e\sec(dx+c))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.460 $\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$

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3.460.1 Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i2^{\frac{1+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

output
```
I*2^(1/2+1/2*m)*a*hypergeom([1/2*m, -1/2*m+1/2],[1+1/2*m],1/2-1/2*I*tan(d*
x+c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2*m+1/2)/d/m/(a+I*a*tan(d*x+c)
)^(1/2)
```

3.460.2 Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i2^{\frac{1}{2}+m}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1+m}{2},\frac{3+m}{2},-e^{2i(c+dx)}\right)\sec^{-}}{d(1+m)\sqrt{\cos(dx)+i\sin(dx)}}$$

input
```
Integrate[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(1/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hypergeometric2F1[1/2 +
m, (1 + m)/2, (3 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - m)*(e*
Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]])/(d*(1 + m)*Sqrt[Cos[d*x] + I*S
in[d*x]])
```

3.460.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^m dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+1}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m+1}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int(a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int\left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx)}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m+1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m+1}{2},\frac{m+3}{2},\right.}{d(m+1)}$$

input

```
Int[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, (1 + m)/2, (3 + m)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(-1/2*m + (1 + m)
/2))/(d*(1 + m)*(1 - I*Tan[c + d*x])^(m/2))
```

3.460.3.1 Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.460.4 Maple [F]

$$\int (e\sec (dx+c))^{m} \sqrt{a+ia\tan (dx+c)}dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

3.460.5 Fricas [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan (dx+c)+a}(e\sec (dx+c))^{m}\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

3.460.6 Sympy [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\sec (c+dx))^{m} \sqrt{ia (\tan (c+dx)-i)}\,dx$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.460.7 Maxima [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan\left(dx+c\right)+a}(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

3.460.8 Giac [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan\left(dx+c\right)+a}(e\sec\left(dx+c\right))^m\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.461 $\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.461.1 Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i2^{\frac{1}{2}(-1+m)}\,\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

output

```
I*2^(-1/2+1/2*m)*hypergeom([1/2*m, 3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2*m+1/2)/d/m/(a+I*a*tan(d*x+c))^
(1/2)
```

3.461.2 Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.64

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i2^{-\frac{1}{2}+m}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),-\frac{1}{2}+m,\frac{1+m}{2},-e^{2i(c+dx}\right.}{d\sqrt{e^{idx}}(-1+m)\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(-1/2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + m)*
(1 + E^((2*I)*(c + d*x)))^(-1/2 + m)*Hypergeometric2F1[(-1 + m)/2, -1/2 +
m, (1 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - m)*(e*Sec[c + d*x]
)^m*Sqrt[Cos[d*x] + I*Sin[d*x]])/(d*Sqrt[E^(I*d*x)]*(-1 + m)*Sqrt[a + I*a*
Tan[c + d*x]])
```

3.461.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-1}{2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-1}{2}}dx$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{a2^{\frac{m-3}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int \left(\frac{1}{2}i\tan(c+\right.}{d}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{i2^{\frac{m-3}{2}+1}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(\right.}{dm}$$

input

```
Int[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(I*2^(1 + (-3 + m)/2)*Hypergeometric2F1[(3 - m)/2, m/2, (2 + m)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2)*(a +
I*a*Tan[c + d*x])^((-1 + m)/2 - m/2))/(d*m)
```

3.461.3.1 Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.461.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.461.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c)/
a, x)
```

3.461.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(c+dx))^m}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*sec(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.461.7 Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\sec(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

3.461.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\sec(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\sqrt{a+a\tan(c+d\,x)\ 1\mathrm{i}}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.462 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$

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3.462.1 Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{i2^{\frac{1}{2}(-3+m)}\,\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c}{adm\sqrt{a+ia\tan(c+dx)}}$$

output
```
I*2^(-3/2+1/2*m)*hypergeom([1/2*m, 5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2*m+1/2)/a/d/m/(a+I*a*tan(d*x+c)
)^(1/2)
```

3.462.2 Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.63

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx=$$
$$-\frac{i2^{-\frac{3}{2}+m}e^{-2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{1}{2}(-1+m),-e^{2i(c}\right.}{d(-3+m)(a+ia\tan(c+dx))^{3/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*2^(-3/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F1[1, 1 - m/2, (
-1 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - m)*(e*Sec[c + d*x])^m
*(Cos[d*x] + I*Sin[d*x])^(3/2))/(d*E^((2*I)*(c + 2*d*x))*(-3 + m)*(a + I*a
*Tan[c + d*x])^(3/2))
```

3.462.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int(a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-3}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int(a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-3}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int(a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m-5}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int\left(\frac{1}{2}i\tan(c+d\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m-5}{2}+1}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^{m}\,\text{Hypergeometric2F1}\bigl(\frac{5-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(}{adm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(I*2^(1 + (-5 + m)/2)*Hypergeometric2F1[(5 - m)/2, m/2, (2 + m)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2)*(a +
I*a*Tan[c + d*x])^((-1 + m)/2 - m/2))/(a*d*m)
```

3.462.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.462.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^{m}}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

3.462.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{(e\sec(dx+c))^{m}}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(1/4*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c
) + 1)*e^(-3*I*d*x - 3*I*c)/a^2, x)
```

3.462.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{(e\sec(c+dx))^{m}}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

3.462.7 Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{(e\sec\left(dx+c\right))^m}{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.462.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{(e\sec\left(dx+c\right))^m}{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.463 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx$

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3.463.1 Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i2^{\frac{1}{2}(-5+m)}\,\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c +}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

output
```
I*2^(-5/2+1/2*m)*hypergeom([1/2*m, 7/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2*m+1/2)/a^2/d/m/(a+I*a*tan(d*x+
c))^(1/2)
```

3.463.2 Mathematica [A] (verified)

Time = 5.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.63

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{i2^{-\frac{5}{2}+m}e^{-3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^4\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{1}{2}(-3+m),-e^{2i(c-}\right.}{d(-5+m)(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((-I)*2^(-5/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x
))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 1 - m/2, (
-3 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(5/2 - m)*(e*Sec[c + d*x])^m
*(Cos[d*x] + I*Sin[d*x])^(5/2))/(d*E^((3*I)*(c + 2*d*x))*(-5 + m)*(a + I*a
*Tan[c + d*x])^(5/2))
```

3.463.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-5}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m-5}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int(a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{\frac{m-7}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m\int(\frac{1}{2}i\tan(c+dx}{ad}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{\frac{m-7}{2}+1}(1+i\tan(c+dx))^{\frac{1-m}{2}}(a+ia\tan(c+dx))^{\frac{m-1}{2}-\frac{m}{2}}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(\right.}{a^2dm}$$

input
```
Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(I*2^(1 + (-7 + m)/2)*Hypergeometric2F1[(7 - m)/2, m/2, (2 + m)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2)*(a +
I*a*Tan[c + d*x])^((-1 + m)/2 - m/2))/(a^2*d*m)
```

3.463.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.463.4 Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

input
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

3.463.5 Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(1/8*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c
) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c)/a^3, x)
```

3.463.6 Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(c+dx))^m}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

3.463.7 Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.463.8 Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.464 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

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3.464.1 Optimal result

Integrand size = 26, antiderivative size = 105

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{\frac{m}{2}+n}\,\text{Hypergeometric2F1}\left(\frac{m}{2},1-\frac{m}{2}-n,\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))}{dm}$$

output
```
I*2^(1/2*m+n)*hypergeom([1/2*m, 1-1/2*m-n],[1+1/2*m],1/2-1/2*I*tan(d*x+c))
*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2*m-n)*(a+I*a*tan(d*x+c))^n/d/m
```

3.464.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.57

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i2^{m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+n}\left(1+e^{2i(c+dx)}\right)^{m+n}\text{Hypergeometric2F1}\left(\frac{m}{2}+n,m+n,1+\frac{m}{2}+n,-e^{2i(c}\right.}{d(m+2n)}$$

input
```
Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
(m + n)*(1 + E^((2*I)*(c + d*x)))^(m + n)*Hypergeometric2F1[m/2 + n, m + n
, 1 + m/2 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-m - n)*(e*Sec[c + d*x]
)^m*(a + I*a*Tan[c + d*x])^n)/(d*(m + 2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

3.464.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^mdx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m}{2}+n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{m/2}(i\tan(c+dx)a+a)^{\frac{m}{2}+n}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int (a-ia\tan(c+dx))^{\frac{m-2}{2}}(i\tan(c+dx)a+}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{\frac{m}{2}-1}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\int \left(\frac{1}{2}-\frac{1}{2}i\tan(c+dx)\right)^{\frac{m-2}{2}}(i\tan(c+dx)}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{m/2}(1-i\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^n(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{2-m}{2},\frac{m}{2}+n,\frac{m}{2}+n+}{d(m+2n)}$$

input

```
Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(m/2)*Hypergeometric2F1[(2 - m)/2, m/2 + n, 1 + m/2 + n, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n)/(d*(m + 2*n)*(
1 - I*Tan[c + d*x])^(m/2))
```

3.464.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.464.4 Maple [F]

$$\int (e \sec (dx+c))^{m} (a+ia \tan (dx+c))^{n} dx$$

input
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

3.464.5 Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e \sec (dx+c))^{m} (i\,a \tan (dx+c)+a)^{n} dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(I*d*n*x + I*
c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)),
x)
```

3.464.6 Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e \sec (c+dx))^{m} \left(ia(\tan (c+dx)-i)\right)^{n}\, dx$$

input
```
integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

3.464.7 Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^m(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

3.464.8 Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^m(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

3.465 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

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3.465.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{4i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)} + \frac{4i(a+ia\tan(c+dx))^{4+n}}{a^4d(4+n)} - \frac{i(a+ia\tan(c+dx))^{5+n}}{a^5d(5+n)}$$

output

```
-4*I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d/(3+n)+4*I*(a+I*a*tan(d*x+c))^(4+n)/a^4
/d/(4+n)-I*(a+I*a*tan(d*x+c))^(5+n)/a^5/d/(5+n)
```

3.465.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^{3+n}\left(\frac{4a^2}{3+n}-\frac{4a(a+ia\tan(c+dx))}{4+n}+\frac{(a+ia\tan(c+dx))^2}{5+n}\right)}{a^5d}$$

input

```
Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^(3 + n)*((4*a^2)/(3 + n) - (4*a*(a + I*a*Tan[
c + d*x]))/(4 + n) + (a + I*a*Tan[c + d*x])^2/(5 + n)))/(a^5*d)
```

3.465.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))^2(i\tan(c+dx)a+a)^{n+2}d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(4a^2(i\tan(c+dx)a+a)^{n+2}-4a(i\tan(c+dx)a+a)^{n+3}+(i\tan(c+dx)a+a)^{n+4}\right)d(ia\tan(c+dx))}{a^5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{4a^2(a+ia\tan(c+dx))^{n+3}}{n+3}-\frac{4a(a+ia\tan(c+dx))^{n+4}}{n+4}+\frac{(a+ia\tan(c+dx))^{n+5}}{n+5}\right)}{a^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*((4*a^2*(a + I*a*Tan[c + d*x])^(3 + n))/(3 + n) - (4*a*(a + I*a*Tan[
c + d*x])^(4 + n))/(4 + n) + (a + I*a*Tan[c + d*x])^(5 + n)/(5 + n)))/(a^5
*d)
```

3.465.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.465.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(91) = 182$.

Time = 1.83 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.80

method	result
derivativedivides	$\frac{(\tan^5(dx+c))\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(5+n)}+\frac{(n^2+15n+60)\tan(dx+c)\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(3+n)(4+n)(5+n)}-\frac{in(\tan^4(dx+c))\mathrm{e}^{n\ln(a+ia\tan}}{(nd+4d)(5+n)}$
default	$\frac{(\tan^5(dx+c))\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(5+n)}+\frac{(n^2+15n+60)\tan(dx+c)\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(3+n)(4+n)(5+n)}-\frac{in(\tan^4(dx+c))\mathrm{e}^{n\ln(a+ia\tan}}{(nd+4d)(5+n)}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/d/(5+n)*tan(d*x+c)^5*exp(n*ln(a+I*a*tan(d*x+c)))+(n^2+15*n+60)/d/(3+n)/(
4+n)/(5+n)*tan(d*x+c)*exp(n*ln(a+I*a*tan(d*x+c)))-I*n/(d*n+4*d)/(5+n)*tan(
d*x+c)^4*exp(n*ln(a+I*a*tan(d*x+c)))+2*(n^2+11*n+20)/d/(3+n)/(4+n)/(5+n)*t
an(d*x+c)^3*exp(n*ln(a+I*a*tan(d*x+c)))-I*(n^2+11*n+32)/d/(3+n)/(4+n)/(5+n
)*exp(n*ln(a+I*a*tan(d*x+c)))-2*I*n*(n+7)/d/(3+n)/(4+n)/(5+n)*tan(d*x+c)^2
*exp(n*ln(a+I*a*tan(d*x+c)))
```

3.465.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(85) = 170.

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.55

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{32\,(2\,(i\,n+5i)e^{(8i\,dx+}}{dn^3+12\,dn^2+47\,dn+(dn^3+12\,dn^2+47\,dn+60\,d)e^{(10i\,dx+10i\,c)}+5\,(dn^3+12\,dn^2+47\,dn+60\,d)e^{(8}}$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
-32*(2*(I*n + 5*I)*e^(8*I*d*x + 8*I*c) + (I*n^2 + 9*I*n + 20*I)*e^(6*I*d*x
 + 6*I*c) + 2*I*e^(10*I*d*x + 10*I*c))*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*
x + 2*I*c) + 1))^n/(d*n^3 + 12*d*n^2 + 47*d*n + (d*n^3 + 12*d*n^2 + 47*d*n
 + 60*d)*e^(10*I*d*x + 10*I*c) + 5*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(8
*I*d*x + 8*I*c) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(6*I*d*x + 6*I*c
) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(4*I*d*x + 4*I*c) + 5*(d*n^3 +
 12*d*n^2 + 47*d*n + 60*d)*e^(2*I*d*x + 2*I*c) + 60*d)
```

3.465.6 Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\sec^6(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**6, x)
```

3.465.7 Maxima [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)
```

3.465.8 Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)
```

3.465.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx =\frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,{\left(a+\frac{a\,\sin\left(c+d\,x\right)\,1\mathrm{i}}{\cos\left(c+d\,x\right)}\right)}^n\,\left(\frac{64\,\mathrm{e}^{c\,10\mathrm{i}+d\,x\,10\mathrm{i}}}{d\,\left(n^3\,1\mathrm{i}+n^2\,12\mathrm{i}+n\,47\mathrm{i}+60\mathrm{i}\right)}+\frac{\mathrm{e}^{c\,6\mathrm{i}+d\,x\,6\mathrm{i}}\,\left(32\,n^2+288\,n+640\right)}{d\,\left(n^3\,1\mathrm{i}+n^2\,12\mathrm{i}+n\,47\mathrm{i}+60\mathrm{i}\right)}+\frac{\mathrm{e}^{c\,8\mathrm{i}+d\,x\,8\mathrm{i}}\,\left(64\,n+320\right)}{d\,\left(n^3\,1\mathrm{i}+n^2\,12\mathrm{i}+n\,47\mathrm{i}+60\mathrm{i}\right)}\right)}{32\,{\cos\left(c+d\,x\right)}^5}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^6,x)
```

output
```
(exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*((64*exp(c*
10i + d*x*10i))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*6i + d*x*6i)
*(288*n + 32*n^2 + 640))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*8i
+ d*x*8i)*(64*n + 320))/(d*(n*47i + n^2*12i + n^3*1i + 60i))))/(32*cos(c +
 d*x)^5)
```

3.466 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

3.466.1 Optimal result 3302
3.466.2 Mathematica [A] (verified) 3302
3.466.3 Rubi [A] (verified) 3303
3.466.4 Maple [B] (verified) 3304
3.466.5 Fricas [B] (verification not implemented) 3305
3.466.6 Sympy [F] 3305
3.466.7 Maxima [F] 3305
3.466.8 Giac [F] 3306
3.466.9 Mupad [B] (verification not implemented) 3306

3.466.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{2i(a+ia\tan(c+dx))^{2+n}}{a^2d(2+n)}+\frac{i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}$$

output
```
-2*I*(a+I*a*tan(d*x+c))^(2+n)/a^2/d/(2+n)+I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d
/(3+n)
```

3.466.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{i\left(\frac{2a(a+ia\tan(c+dx))^{2+n}}{2+n}-\frac{(a+ia\tan(c+dx))^{3+n}}{3+n}\right)}{a^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^(2 + n))/(2 + n) - (a + I*a*Tan[c + d*x
])^(3 + n)/(3 + n)))/(a^3*d)
```

3.466.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3968, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{n+1}d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{53}$$

$$-\frac{i\int \left(2a(i\tan(c+dx)a+a)^{n+1}-(i\tan(c+dx)a+a)^{n+2}\right)d(ia\tan(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{2a(a+ia\tan(c+dx))^{n+2}}{n+2}-\frac{(a+ia\tan(c+dx))^{n+3}}{n+3}\right)}{a^3d}$$

input

```
Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output

```
((-I)*((2*a*(a + I*a*Tan[c + d*x])^(2 + n))/(2 + n) - (a + I*a*Tan[c + d*x
])^(3 + n)/(3 + n)))/(a^3*d)
```

3.466.3.1 Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.466.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(61) = 122$.

Time = 1.01 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.31

method	result
derivativedivides	$\frac{(\tan^3(dx+c))\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(3+n)}+\frac{(n+6)\tan(dx+c)\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{i(4+n)\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-$ $in(\mathrm{ta}$
default	$\frac{(\tan^3(dx+c))\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(3+n)}+\frac{(n+6)\tan(dx+c)\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{i(4+n)\mathrm{e}^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-$ $in(\mathrm{ta}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/d/(3+n)*tan(d*x+c)^3*exp(n*ln(a+I*a*tan(d*x+c)))+(n+6)/d/(2+n)/(3+n)*tan
(d*x+c)*exp(n*ln(a+I*a*tan(d*x+c)))-I*(4+n)/d/(2+n)/(3+n)*exp(n*ln(a+I*a*t
an(d*x+c)))-I*n/d/(2+n)/(3+n)*tan(d*x+c)^2*exp(n*ln(a+I*a*tan(d*x+c)))
```

3.466.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(57) = 114$.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.18

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{8\left((i\,n+3i)e^{(4i\,dx+4i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n}{dn^2+5\,dn+(dn^2+5\,dn+6\,d)e^{(6i\,dx+6i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(4i\,dx+4i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(2i\,dx}}$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
-8*((I*n + 3*I)*e^(4*I*d*x + 4*I*c) + I*e^(6*I*d*x + 6*I*c))*(2*a*e^(2*I*d
*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(d*n^2 + 5*d*n + (d*n^2 + 5*d*n +
 6*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^2 + 5*d*n + 6*d)*e^(4*I*d*x + 4*I*c) +
3*(d*n^2 + 5*d*n + 6*d)*e^(2*I*d*x + 2*I*c) + 6*d)
```

3.466.6 Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**4, x)
```

3.466.7 Maxima [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

3.466.8 Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

3.466.9 Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.32

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{4\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n\left(n\,3\mathrm{i}+\cos\left(2\,c+2\,d\,x\right)\,15\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1\mathrm{i}\right.}{d\,\left(n^2+5\,n+6\right.}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^4,x)
```

output
```
-(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^n*(n*3i + cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos(6*c + 6*d*x
)*1i - 9*sin(2*c + 2*d*x) - 6*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + n*cos(
2*c + 2*d*x)*4i + n*cos(4*c + 4*d*x)*1i - 2*n*sin(2*c + 2*d*x) - n*sin(4*c
 + 4*d*x) + 10i))/(d*(5*n + n^2 + 6)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*
d*x) + cos(6*c + 6*d*x) + 10))
```

3.467 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

3.467.1 Optimal result . . . 3307
3.467.2 Mathematica [A] (verified) . . . 3307
3.467.3 Rubi [A] (verified) . . . 3308
3.467.4 Maple [A] (verified) . . . 3309
3.467.5 Fricas [B] (verification not implemented) . . . 3309
3.467.6 Sympy [F] . . . 3310
3.467.7 Maxima [A] (verification not implemented) . . . 3310
3.467.8 Giac [B] (verification not implemented) . . . 3310
3.467.9 Mupad [B] (verification not implemented) . . . 3311

3.467.1 Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

output
```
-I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

3.467.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

input
```
Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

3.467.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{i\int (i\tan(c+dx)a+a)^n d(ia\tan(c+dx))}{ad}$$

$$\Big\downarrow \text{17}$$

$$-\frac{i(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

input
```
Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

3.467.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.467.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
default	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
risch	$-\frac{2ia^{n}(\mathrm{e}^{2i(dx+c)}+1)^{-n}(\mathrm{e}^{i(dx+c)})^{2n}2^{n}\mathrm{e}^{\frac{i(-\operatorname{csgn}(\frac{i\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1})^{3}\pi n+\operatorname{csgn}(\frac{i\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1})^{2}\pi\ \operatorname{csgn}(\frac{i}{\mathrm{e}^{2i(dx+c)}+1})n+\operatorname{csgn}(}{}}}{}$

input
```
int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

3.467.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{2i\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}e^{(2i\,dx+2i\,c)}}{dn+(dn+d)e^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
-2*I*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(2*I*d*x + 2*
I*c)/(d*n + (d*n + d)*e^(2*I*d*x + 2*I*c) + d)
```

3.467.6 Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan{(c+dx)}-i))^n\sec^2{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**2, x)
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\,(i\,a\tan{(dx+c)}+a)^{n+1}}{ad(n+1)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
-I*(I*a*tan(d*x + c) + a)^(n + 1)/(a*d*(n + 1))
```

3.467.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(28) = 56.

Time = 0.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)^{n+1}}{ad(n+1)}$$

input
```
integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
-I*((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2*d
*x + 1/2*c)^2 - 1))^(n + 1)/(a*d*(n + 1))
```

3.467.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{2\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1\mathrm{i}\right)\,{\left(\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}\right)}^n}{d\,\left(n+1\right)\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}-\sin\left(2\,c+2\,d\,x\right)+1\mathrm{i}\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^2,x)
```

output
```
(2*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*((a*(cos(2*c + 2*
d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(n + 1)*(co
s(2*c + 2*d*x)*1i - sin(2*c + 2*d*x) + 1i))
```

3.468 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

3.468.1 Optimal result 3312
3.468.2 Mathematica [A] (verified) 3312
3.468.3 Rubi [A] (verified) 3313
3.468.4 Maple [F] 3314
3.468.5 Fricas [F] 3314
3.468.6 Sympy [F] 3314
3.468.7 Maxima [F] 3315
3.468.8 Giac [F] 3315
3.468.9 Mupad [F(-1)] 3315

3.468.1 Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(2,-1+n,n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(1-n)}$$

output
```
1/4*I*a*hypergeom([2, -1+n],[n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(
-1+n)/d/(1-n)
```

3.468.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia\,\text{Hypergeometric2F1}\left(2,-1+n,n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(-1+n)}$$

input
```
Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/4*I)*a*Hypergeometric2F1[2, -1 + n, n, (1 + I*Tan[c + d*x])/2]*(a + I
*a*Tan[c + d*x])^(-1 + n))/(d*(-1 + n))
```

3.468.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \; 3968$$

$$-\frac{ia^3\int \frac{(i\tan(c+dx)a+a)^{n-2}}{(a-ia\tan(c+dx))^2}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 78$$

$$\frac{ia(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(2,n-1,n,\frac{i\tan(c+dx)a+a}{2a}\right)}{4d(1-n)}$$

input
```
Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/4)*a*Hypergeometric2F1[2, -1 + n, n, (a + I*a*Tan[c + d*x])/(2*a)]*(a
+ I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n))
```

3.468.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.468.4 Maple [F]

$$\int \left(\cos^2\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^n dx$$

input
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

3.468.5 Fricas [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \left(i\,a\tan\left(dx+c\right)+a\right)^n\cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/4*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(4*I
*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c), x)
```

3.468.6 Sympy [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \left(ia(\tan\left(c+dx\right)-i)\right)^n\cos^2\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**2, x)
```

3.468.7 Maxima [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

3.468.8 Giac [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \cos(c+d\,x)^2\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n, x)
```

3.469 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

3.469.1 Optimal result 3316
3.469.2 Mathematica [A] (verified) 3316
3.469.3 Rubi [A] (verified) 3317
3.469.4 Maple [F] 3318
3.469.5 Fricas [F] 3318
3.469.6 Sympy [F] 3318
3.469.7 Maxima [F] 3319
3.469.8 Giac [F] 3319
3.469.9 Mupad [F(-1)] 3319

3.469.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(2-n)}$$

output
```
1/8*I*a^2*hypergeom([3, -2+n],[-1+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+
c))^(-2+n)/d/(2-n)
```

3.469.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia^2\,\text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(-2+n)}$$

input
```
Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/8*I)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (1 + I*Tan[c + d*x])/2]
*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(-2 + n))
```

3.469.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^5\int \frac{(i\tan(c+dx)a+a)^{n-3}}{(a-ia\tan(c+dx))^3}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{ia^2(a+ia\tan(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(3,n-2,n-1,\frac{i\tan(c+dx)a+a}{2a}\right)}{8d(2-n)}$$

input
```
Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/8)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (a + I*a*Tan[c + d*x])/(2*
a)]*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(2 - n))
```

3.469.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.469.4 Maple [F]

$$\int \left(\cos^4(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

input
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

3.469.5 Fricas [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/16*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(8*
I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*
d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c), x)
```

3.469.6 Sympy [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**4, x)
```

3.469.7 Maxima [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

3.469.8 Giac [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \cos(c+d\,x)^4\,(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

input
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n, x)
```

3.470 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

3.470.1 Optimal result 3320
3.470.2 Mathematica [A] (verified) 3320
3.470.3 Rubi [A] (verified) 3321
3.470.4 Maple [F] 3322
3.470.5 Fricas [F] 3322
3.470.6 Sympy [F(-1)] 3322
3.470.7 Maxima [F] 3323
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3.470.9 Mupad [F(-1)] 3323

3.470.1 Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(4,-3+n,-2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(3-n)}$$

output
```
1/16*I*a^3*hypergeom([4, -3+n],[-2+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x
+c))^(-3+n)/d/(3-n)
```

3.470.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia^3\,\text{Hypergeometric2F1}\left(4,-3+n,-2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(-3+n)}$$

input
```
Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/16*I)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (1 + I*Tan[c + d*x])/2
]*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(-3 + n))
```

3.470.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{3968}$$

$$-\frac{ia^7\int \frac{(i\tan(c+dx)a+a)^{n-4}}{(a-ia\tan(c+dx))^4}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{ia^3(a+ia\tan(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(4,n-3,n-2,\frac{i\tan(c+dx)a+a}{2a}\right)}{16d(3-n)}$$

input
```
Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/16)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (a + I*a*Tan[c + d*x])/(2
*a)]*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(3 - n))
```

3.470.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

3.470.4 Maple [F]

$$\int \left(\cos^{6}\left(dx+c\right)\right) \left(a+ia\tan\left(dx+c\right)\right)^{n} dx$$

input
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

3.470.5 Fricas [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \left(i\,a\tan\left(dx+c\right)+a\right)^{n}\cos\left(dx+c\right)^{6}\,dx$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/64*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(12
*I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e
^(6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*e
^(-6*I*d*x - 6*I*c), x)
```

3.470.6 Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Timed out
```

3.470.7 Maxima [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

3.470.8 Giac [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^6\,dx$$

input
```
integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^6\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n, x)
```

3.471 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

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3.471.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{i2^{\frac{5}{2}+n}a^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2}-n,\frac{7}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^5(c+dx)(1+i\tan(c+dx))^{-\frac{1}{2}-n}(a+}{5d}$$

output
```
1/5*I*2^(5/2+n)*a^2*hypergeom([5/2, -3/2-n],[7/2],1/2-1/2*I*tan(d*x+c))*se
c(d*x+c)^5*(1+I*tan(d*x+c))^(-1/2-n)*(a+I*a*tan(d*x+c))^(-2+n)/d
```

3.471.2 Mathematica [A] (verified)

Time = 14.72 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i2^{5+n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\text{Hypergeometric2F1}\left(-\frac{3}{2},1,\frac{7}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)}{d\left(1+e^{2i(c+dx)}\right)^4(5+2n)}$$

input
```
Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(5 + n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x))))^n*Hypergeometric2F1[-3/2, 1, 7/2 + n, -E^((2*I)*(c + d
*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^4*(5 + 2*n)*S
ec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.471.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sec^5(c+dx)\int (a-ia\tan(c+dx))^{5/2}(i\tan(c+dx)a+a)^{n+\frac{5}{2}}dx}{(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^5(c+dx)\int (a-ia\tan(c+dx))^{5/2}(i\tan(c+dx)a+a)^{n+\frac{5}{2}}dx}{(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sec^5(c+dx)\int (a-ia\tan(c+dx))^{3/2}(i\tan(c+dx)a+a)^{n+\frac{3}{2}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^3 2^{n+\frac{3}{2}}\sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n+\frac{3}{2}}(a-ia\tan(c+dx}{d(a-ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia^2 2^{n+\frac{5}{2}}\sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-n-\frac{3}{2},\frac{7}{2},\frac{1}{2}(1-i\,\mathrm{ta}\right.}{5d}$$

input
```
Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/5)*2^(5/2 + n)*a^2*Hypergeometric2F1[5/2, -3/2 - n, 7/2, (1 - I*Tan[c
+ d*x])/2]*Sec[c + d*x]^5*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c +
 d*x])^(-2 + n))/d
```

3.471.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.471.4 Maple [F]

$$\int \left(\sec^5(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

input
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

3.471.5 Fricas [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(5*I*d
*x + 5*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x
 + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

3.471.6 Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\sec^5(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**5, x)
```

3.471.7 Maxima [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

3.471.8 Giac [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos\left(c+d\,x\right)^5}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5,x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5, x)
```

3.472 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

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3.472.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{3}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-n,\frac{5}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^3(c+dx)(1+i\tan(c+dx))^{-\frac{1}{2}-n}(a+}{3d}$$

output
```
1/3*I*2^(3/2+n)*a*hypergeom([3/2, -1/2-n],[5/2],1/2-1/2*I*tan(d*x+c))*sec(
d*x+c)^3*(1+I*tan(d*x+c))^(-1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

3.472.2 Mathematica [A] (verified)

Time = 13.91 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{3+n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{5}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)}{d\left(1+e^{2i(c+dx)}\right)^2(3+2n)}$$

input
```
Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(3 + n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^
((2*I)*(c + d*x))))^n*Hypergeometric2F1[-1/2, 1, 5/2 + n, -E^((2*I)*(c + d
*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^2*(3 + 2*n)*S
ec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.472.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^3(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \ 3986$$

$$\frac{\sec^3(c+dx)\int(a-ia\tan(c+dx))^{3/2}(i\tan(c+dx)a+a)^{n+\frac{3}{2}}dx}{(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sec^3(c+dx)\int(a-ia\tan(c+dx))^{3/2}(i\tan(c+dx)a+a)^{n+\frac{3}{2}}dx}{(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4006$$

$$\frac{a^2\sec^3(c+dx)\int\sqrt{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{2}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 80$$

$$\frac{a^2 2^{n+\frac{1}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\int\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n+\frac{1}{2}}\sqrt{a-ia\tan(c+}{d(a-ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 79$$

$$\frac{ia2^{n+\frac{3}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{2},-n-\frac{1}{2},\frac{5}{2},\frac{1}{2}(1-i\right.}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/3)*2^(3/2 + n)*a*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (1 - I*Tan[c +
d*x])/2]*Sec[c + d*x]^3*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d
*x])^(-1 + n))/d
```

3.472.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.472.4 Maple [F]

$$\int \left(\sec^3(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

input
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

3.472.5 Fricas [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^3\;dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(3*I*d*
x + 3*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2
*I*c) + 1), x)
```

3.472.6 Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**3, x)
```

3.472.7 Maxima [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

3.472.8 Giac [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3,x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3, x)
```

3.473 $\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$

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3.473.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{i2^{\frac{1}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\,t}{d}$$

output
```
I*2^(1/2+n)*a*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*I*tan(d*x+c))*sec(d*x+c
)*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

3.473.2 Mathematica [A] (verified)

Time = 9.96 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i2^{1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1+n}\left(1+e^{2i(c+dx)}\right)^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2}+n,1+n,\frac{3}{2}+n,-e^{2i(c+dx)}\right)\sec}{d(1+2n)}$$

input
```
Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
(1 + n)*(1 + E^((2*I)*(c + d*x)))^(1 + n)*Hypergeometric2F1[1/2 + n, 1 + n
, 3/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.473.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sec(c+dx)\int \sqrt{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{2}}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(c+dx)\int \sqrt{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{2}}dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\sec(c+dx)\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}d\tan(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{n-\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}d\tan(c+dx)}{d\sqrt{a-ia\tan(c+dx)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia2^{n+\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c+\right.}{d}$$

input
```
Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(1/2 + n)*a*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - I*Tan[c + d*x])
/2]*Sec[c + d*x]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^(-1
 + n))/d
```

3.473.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.473.4 Maple [F]

$$\int \sec \left(dx+c\right)\left(a+ia\tan \left(dx+c\right)\right)^{n}dx$$

input
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

3.473.5 Fricas [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \left(i\,a\tan\left(dx+c\right)+a\right)^{n}\sec\left(dx+c\right)\,dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*x
+ I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

3.473.6 Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (ia(\tan\left(c+dx\right)-i))^n\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x), x)
```

3.473.7 Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)\;dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

3.473.8 Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)\;dx$$

input
```
integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

3.473.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^n}{\cos(c+d\,x)}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x), x)
```

3.474 $\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$

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3.474.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{-\frac{1}{2}+n}\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+}{d}$$

output
```
-I*2^(-1/2+n)*cos(d*x+c)*hypergeom([-1/2, 3/2-n],[1/2],1/2-1/2*I*tan(d*x+c
))*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^n/d
```

3.474.2 Mathematica [A] (verified)

Time = 13.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.72

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{-1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1+n}\left(1+e^{2i(c+dx)}\right)^{-1+n}\text{Hypergeometric2F1}\left(-1+n,-\frac{1}{2}+n,\frac{1}{2}+n,-e^{2i(c+}\right.}{d(-1+2n)}$$

input
```
Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(-1 + n)*(1 + E^((2*I)*(c + d*x)))^(-1 + n)*Hypergeometric2F1[-1 + n, -1/
2 + n, 1/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(-1 + 2
*n)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.474.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3986}$$

$$\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{2}}}{\sqrt{a-ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{n-\frac{3}{2}}\cos(c+dx)\sqrt{a-ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}d\tan(c+}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{1}{2}}\cos(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i\tan(c\right.}{d}$$

input
```
Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[-1/2, 3/2 - n, 1/2, (1 -
 I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^
n)/d
```

3.474.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.474.4 Maple [F]

$$\int \cos \left(dx+c\right) \left(a+ia\tan \left(dx+c\right)\right)^{n}dx$$

input
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

3.474.5 Fricas [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \left(i\,a\tan \left(dx+c\right)+a\right)^{n}\cos \left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(2*I
*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c), x)
```

3.474.6 Sympy [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \left(ia(\tan \left(c+dx\right)-i)\right)^{n}\cos \left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x), x)
```

3.474.7 Maxima [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

3.474.8 Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)\,dx$$

input
```
integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos\left(c+d\,x\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.475 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

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3.475.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{3}{2}+n}\cos^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(}{3ad}$$

output
```
-1/3*I*2^(-3/2+n)*cos(d*x+c)^3*hypergeom([-3/2, 5/2-n],[-1/2],1/2-1/2*I*ta
n(d*x+c))*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d
```

3.475.2 Mathematica [A] (verified)

Time = 14.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-3+n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^4\text{Hypergeometric2F1}\left(1,\frac{5}{2},-\frac{1}{2}+n,-e^{2i(c+dx)}\right)\sec^{-}}{d(-3+2n)}$$

input
```
Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-3 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^n*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 5/2, -1/2 + n, -E^((2*
I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(-3 + 2*n)
*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.475.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \; 3986$$

$$\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{2}}}{(a-ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 4006$$

$$\frac{a^2\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 80$$

$$\frac{2^{n-\frac{5}{2}}\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}d\,\mathrm{tan}}{d}$$

$$\Big\downarrow \; 79$$

$$-\frac{i2^{n-\frac{3}{2}}\cos^3(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\,(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\,\mathrm{ta}}{3ad}$$

input
```
Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/3*I)*2^(-3/2 + n)*Cos[c + d*x]^3*Hypergeometric2F1[-3/2, 5/2 - n, -1/
2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c +
 d*x])^(1 + n))/(a*d)
```

3.475.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.475. $\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

3.475.4 Maple [F]

$$\int \left(\cos^{3}\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^{n}dx$$

input
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

3.475.5 Fricas [F]

$$\int \cos^{3}(c+dx)(a+ia\tan(c+dx))^{n}\,dx=\int \left(i\,a\tan\left(dx+c\right)+a\right)^{n}\cos\left(dx+c\right)^{3}\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(6*I
*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-3*I
*d*x - 3*I*c), x)
```

3.475.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{3}(c+dx)(a+ia\tan(c+dx))^{n}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Timed out
```

3.475.7 Maxima [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

3.475.8 Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^3\,(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

input
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n, x)
```

3.476 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

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3.476.1 Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{5}{2}+n}\cos^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(}{5a^2d}$$

output
```
-1/5*I*2^(-5/2+n)*cos(d*x+c)^5*hypergeom([-5/2, 7/2-n],[-3/2],1/2-1/2*I*ta
n(d*x+c))*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(2+n)/a^2/d
```

3.476.2 Mathematica [A] (warning: unable to verify)

Time = 15.68 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-5+n}e^{-5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^6\text{Hypergeometric2F1}\left(1,\frac{7}{2},-\frac{3}{2}+n,-e^{2i(c+dx)}\right)\sec}{d(-5+2n)}$$

input
```
Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-5 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^n*(1 + E^((2*I)*(c + d*x)))^6*Hypergeometric2F1[1, 7/2, -3/2 + n, -E^((2*
I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((5*I)*(c + d*x))*(-5 + 2*n)
*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.476.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3986}$$

$$\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{2}}}{(a-ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{7}{2}}}{(a-ia\tan(c+dx))^{7/2}}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{n-\frac{7}{2}}\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{7}{2}}}{(a-ia\tan(c+dx))^{7/2}}d\tan}{ad}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{5}{2}}\cos^5(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\text{Hypergeometric2F1}\,(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\tan}{5a^2d}$$

input
```
Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

3.476. $\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

output

```
((-1/5*I)*2^(-5/2 + n)*Cos[c + d*x]^5*Hypergeometric2F1[-5/2, 7/2 - n, -3/
2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c +
 d*x])^(2 + n))/(a^2*d)
```

3.476.3.1 Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.476.4 Maple [F]

$$\int \left(\cos^{5}\left(dx+c\right)\right)\left(a+ia\tan\left(dx+c\right)\right)^{n}dx$$

input
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

3.476.5 Fricas [F]

$$\int \cos^{5}(c+dx)(a+ia\tan(c+dx))^{n}\,dx=\int \left(i\,a\tan\left(dx+c\right)+a\right)^{n}\cos\left(dx+c\right)^{5}\,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(1/32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(10
*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6*I*c) + 10*e^(
4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c), x)
```

3.476.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{5}(c+dx)(a+ia\tan(c+dx))^{n}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Timed out
```

3.476.7 Maxima [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

3.476.8 Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx=\int \cos\left(c+d\,x\right)^5\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

input
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n, x)
```

3.477 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$

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3.477.1 Optimal result

Integrand size = 28, antiderivative size = 96

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx=\frac{i2^{\frac{9}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{5}{4},-\frac{1}{4}-n,\frac{9}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/2}(1-}{5d}$$

output
```
1/5*I*2^(9/4+n)*a*hypergeom([5/4, -1/4-n],[9/4],1/2-1/2*I*tan(d*x+c))*(e*s
ec(d*x+c))^(5/2)*(1+I*tan(d*x+c))^(-1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

3.477.2 Mathematica [A] (verified)

Time = 10.90 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.89

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx=$$

$$-\frac{i2^{\frac{7}{2}+n}e^{2i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(\frac{5}{4}+n,\frac{5}{2}+n,\frac{9}{4}+n,-e^{2i(c+d}\right.}{d(5+4n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(7/2 + n)*E^((2*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x))))^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hyperg
eometric2F1[5/4 + n, 5/2 + n, 9/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^
(-5/2 - n)*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n)/(d*(5 + 4*n)*(
Cos[d*x] + I*Sin[d*x])^n)
```

3.477.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(e\sec(c+dx))^{5/2}\int (a-ia\tan(c+dx))^{5/4}(i\tan(c+dx)a+a)^{n+\frac{5}{4}}dx}{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e\sec(c+dx))^{5/2}\int (a-ia\tan(c+dx))^{5/4}(i\tan(c+dx)a+a)^{n+\frac{5}{4}}dx}{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(e\sec(c+dx))^{5/2}\int \sqrt[4]{a-ia\tan(c+dx)}(i\tan(c+dx)a+a)^{n+\frac{1}{4}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{n+\frac{1}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n+\frac{1}{4}}\sqrt[4]{a-ia\tan(}}{d(a-ia\tan(c+dx))^{5/4}}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia2^{n+\frac{9}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},-n-\frac{1}{4},\frac{9}{4},\frac{1}{2}(1-\right.}{5d}$$

input
```
Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/5)*2^(9/4 + n)*a*Hypergeometric2F1[5/4, -1/4 - n, 9/4, (1 - I*Tan[c +
d*x])/2]*(e*Sec[c + d*x])^(5/2)*(1 + I*Tan[c + d*x])^(-1/4 - n)*(a + I*a*T
an[c + d*x])^(-1 + n))/d
```

3.477.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.477.4 Maple [F]

$$\int (e\sec (dx+c))^{\frac{5}{2}} (a+ia\tan (dx+c))^n\, dx$$

input
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

3.477.5 Fricas [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\, dx = \int (e\sec (dx+c))^{\frac{5}{2}} (i\,a\tan (dx+c)+a)^n\, dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(4*sqrt(2)*e^2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))
^n*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(5/2*I*d*x + 5/2*I*c)/(e^(4*I*d*x +
 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

3.477.6 Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\, dx = \text{Timed out}$$

input
```
integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Timed out
```

3.477.7 Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.477.8 Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.477.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.478 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$

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3.478.1 Optimal result

Integrand size = 28, antiderivative size = 96

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx=\frac{i2^{\frac{7}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3/2}(1+}{3d}$$

output
```
1/3*I*2^(7/4+n)*a*hypergeom([3/4, 1/4-n],[7/4],1/2-1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(3/2)*(1+I*tan(d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

3.478.2 Mathematica [A] (verified)

Time = 11.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.77

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx=-\frac{i2^{\frac{5}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{3}{2}+n}\text{Hypergeometric2F1}\left(\frac{3}{4}+n,\frac{3}{2}+n,\frac{7}{4}+n,-e^{2i(c+dx)}\right)\sec^{-}}{d(3+4n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(5/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
)^(3/2 + n)*(1 + E^((2*I)*(c + d*x)))^(3/2 + n)*Hypergeometric2F1[3/4 + n,
 3/2 + n, 7/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3/2 - n)*(e*Sec[c
+ d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n)/(d*(3 + 4*n)*(Cos[d*x] + I*Sin[d*x
])^n)
```

3.478.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{(e\sec(c+dx))^{3/2}\int (a-ia\tan(c+dx))^{3/4}(i\tan(c+dx)a+a)^{n+\frac{3}{4}}dx}{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e\sec(c+dx))^{3/2}\int (a-ia\tan(c+dx))^{3/4}(i\tan(c+dx)a+a)^{n+\frac{3}{4}}dx}{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2(e\sec(c+dx))^{3/2}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{a^2 2^{n-\frac{1}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}d\tan(c+dx)}{d(a-ia\tan(c+dx))^{3/4}}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{ia2^{n+\frac{7}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\,(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\,t}{3d}$$

input
```
Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/3)*2^(7/4 + n)*a*Hypergeometric2F1[3/4, 1/4 - n, 7/4, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan
[c + d*x])^(-1 + n))/d
```

3.478.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.478.4 Maple [F]

$$\int \left(e \sec \left(dx+c\right)\right)^{\frac{3}{2}} \left(a+ia\tan \left(dx+c\right)\right)^{n}dx$$

input
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

3.478.5 Fricas [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int \left(e \sec \left(dx+c\right)\right)^{\frac{3}{2}} \left(i\,a\tan \left(dx+c\right)+a\right)^{n}dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(2*sqrt(2)*e*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)/(e^(2*I*d*x + 2
*I*c) + 1), x)
```

3.478.6 Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int \left(e \sec \left(c+dx\right)\right)^{\frac{3}{2}} \left(ia(\tan \left(c+dx\right)-i)\right)^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.478.7 Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.478.8 Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.478.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx=\int \left(\frac{e}{\cos{(c+d\,x)}}\right)^{3/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.479 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx$

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3.479.1 Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{5}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+}{d}$$

output
```
I*2^(5/4+n)*a*hypergeom([1/4, 3/4-n],[5/4],1/2-1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(1/2)*(1+I*tan(d*x+c))^(3/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

3.479.2 Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{3}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(\frac{1}{4}+n,\frac{1}{2}+n,\frac{5}{4}+n,-e^{2i(c+dx)}\right)\text{se}}{d(1+4n)}$$

input
```
Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(3/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
)^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[1/4 + n,
 1/2 + n, 5/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - n)*Sqrt[e*Se
c[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 4*n)*(Cos[d*x] + I*Sin[d*x])
^n)
```

3.479.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e \sec (c+d x)} (a+i a \tan (c+d x))^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{e \sec (c+d x)} (a+i a \tan (c+d x))^n dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{\sqrt{e \sec (c+d x)} \int \sqrt[4]{a-i a \tan (c+d x)} (i \tan (c+d x) a+a)^{n+\frac{1}{4}} dx}{\sqrt[4]{a-i a \tan (c+d x)} \sqrt[4]{a+i a \tan (c+d x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{e \sec (c+d x)} \int \sqrt[4]{a-i a \tan (c+d x)} (i \tan (c+d x) a+a)^{n+\frac{1}{4}} dx}{\sqrt[4]{a-i a \tan (c+d x)} \sqrt[4]{a+i a \tan (c+d x)}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2 \sqrt{e \sec (c+d x)} \int \frac{(i \tan (c+d x) a+a)^{n-\frac{3}{4}}}{(a-i a \tan (c+d x))^{3/4}} d \tan (c+d x)}{d \sqrt[4]{a-i a \tan (c+d x)} \sqrt[4]{a+i a \tan (c+d x)}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{n-\frac{3}{4}} \sqrt{e \sec (c+d x)} (1+i \tan (c+d x))^{\frac{3}{4}-n} (a+i a \tan (c+d x))^{n-1} \int \frac{\left(\frac{1}{2} i \tan (c+d x)+\frac{1}{2}\right)^{n-\frac{3}{4}}}{(a-i a \tan (c+d x))^{3/4}} d \tan (c+d x)}{d \sqrt[4]{a-i a \tan (c+d x)}}$$

$$\Big\downarrow \text{79}$$

$$\frac{ia2^{n+\frac{5}{4}}\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\,(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan}{d}$$

input
```
Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(5/4 + n)*a*Hypergeometric2F1[1/4, 3/4 - n, 5/4, (1 - I*Tan[c + d*x])
/2]*Sqrt[e*Sec[c + d*x]]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d
*x])^(-1 + n))/d
```

3.479.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.479.4 Maple [F]

$$\int \sqrt{e \sec \left(dx+c\right)}\, \left(a+ia \tan \left(dx+c\right)\right)^{n} dx$$

input
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

3.479.5 Fricas [F]

$$\int \sqrt{e \sec(c+dx)}(a+ia \tan(c+dx))^n\, dx = \int \sqrt{e \sec \left(dx+c\right)}\left(i\, a \tan \left(dx+c\right)+a\right)^{n} dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c), x)
```

3.479.6 Sympy [F]

$$\int \sqrt{e \sec(c+dx)}(a+ia \tan(c+dx))^n\, dx = \int \sqrt{e \sec \left(c+dx\right)}\left(ia\left(\tan \left(c+dx\right)-i\right)\right)^{n}\, dx$$

input
```
integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**n, x)
```

3.479.7 Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

3.479.8 Giac [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

3.479.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^n\,dx$$

input
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.480 $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx$

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3.480.1 Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{i2^{\frac{3}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+d}{d\sqrt{e\sec(c+dx)}}$$

output
```
-I*2^(3/4+n)*hypergeom([-1/4, 5/4-n],[3/4],1/2-1/2*I*tan(d*x+c))*(1+I*tan(
d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^n/d/(e*sec(d*x+c))^(1/2)
```

3.480.2 Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.57

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{i2^{\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+n}\text{Hypergeometric2F1}\left(-\frac{1}{2}+n,-\frac{1}{4}+n,\frac{3}{4}+n,-e^{2i(c+dx)}\right)\sec}{d(-1+4n)\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-I)*2^(1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + n)*(
1 + E^((2*I)*(c + d*x)))^(-1/2 + n)*Hypergeometric2F1[-1/2 + n, -1/4 + n,
3/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x
])^n)/(d*(-1 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

3.480.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{ 3986}$$

$$\frac{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}dx}{\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{1}{4}}}{\sqrt[4]{a-ia\tan(c+dx)}}dx}{\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}d\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{a2^{n-\frac{5}{4}}\sqrt[4]{a-ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}d\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{n+\frac{3}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{d\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

output
```
((-I)*2^(3/4 + n)*Hypergeometric2F1[-1/4, 5/4 - n, 3/4, (1 - I*Tan[c + d*x
])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[e*S
ec[c + d*x]])
```

3.480.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.480. $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.480.4 Maple [F]

$$\int \frac{(a + ia\tan(dx + c))^n}{\sqrt{e\sec(dx + c)}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

3.480.5 Fricas [F]

$$\int \frac{(a + ia\tan(c + dx))^n}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{(i\,a\tan(dx + c) + a)^n}{\sqrt{e\sec(dx + c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x
 - 1/2*I*c)/e, x)
```

3.480.6 Sympy [F]

$$\int \frac{(a + ia\tan(c + dx))^n}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{(ia(\tan(c + dx) - i))^n}{\sqrt{e\sec(c + dx)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/sqrt(e*sec(c + d*x)), x)
```

3.480.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

3.480.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

3.480.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(a+a\tan(c+d\,x)\,\,1{}\mathrm{i})^n}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2), x)
```

3.481 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx$

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3.481.1 Optimal result

Integrand size = 28, antiderivative size = 93

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i2^{\frac{1}{4}+n}\operatorname{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

output
```
-1/3*I*2^(1/4+n)*hypergeom([-3/4, 7/4-n],[1/4],1/2-1/2*I*tan(d*x+c))*(1+I*
tan(d*x+c))^(3/4-n)*(a+I*a*tan(d*x+c))^n/d/(e*sec(d*x+c))^(3/2)
```

3.481.2 Mathematica [A] (verified)

Time = 12.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i2^{-\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{3}{2}+n}\operatorname{Hypergeometric2F1}\left(-\frac{3}{2}+n,-\frac{3}{4}+n,\frac{1}{4}+n,-e^{2i(c+dx)}\right)\sec^{\frac{3}{2}}}{d(-3+4n)(e\sec(c+dx))^{3/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

output

```
((-I)*2^(-1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-3/2 + n)*
(1 + E^((2*I)*(c + d*x)))^(-3/2 + n)*Hypergeometric2F1[-3/2 + n, -3/4 + n,
 1/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - n)*(a + I*a*Tan[c + d*
x])^n)/(d*(-3 + 4*n)*(e*Sec[c + d*x])^(3/2))
```

3.481.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{4}}}{(a-ia\tan(c+dx))^{3/4}}dx}{(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{3}{4}}}{(a-ia\tan(c+dx))^{3/4}}dx}{(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{7}{4}}}{(a-ia\tan(c+dx))^{7/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{n-\frac{7}{4}}(a-ia\tan(c+dx))^{3/4}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{7}{4}}}{(a-ia\tan(c+dx))^{7/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n+\frac{1}{4}}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{3d(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

output
```
((-1/3*I)*2^(1/4 + n)*Hypergeometric2F1[-3/4, 7/4 - n, 1/4, (1 - I*Tan[c +
 d*x])/2]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(e*S
ec[c + d*x])^(3/2))
```

3.481.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.481. $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx$

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.481.4 Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{3}{2}}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

3.481.5 Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral(1/4*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2
*I*c) + 1)*e^(-3/2*I*d*x - 3/2*I*c)/e^2, x)
```

3.481.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(3/2), x)
```

3.481.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{(e\sec\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

3.481.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{(e\sec\left(dx+c\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

3.481.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2), x)
```

3.482 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx$

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3.482.1 Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{i2^{-\frac{1}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+}{5ad(e\sec(c+dx))^{5/2}}$$

output
```
-1/5*I*2^(-1/4+n)*hypergeom([-5/4, 9/4-n],[-1/4],1/2-1/2*I*tan(d*x+c))*(1+
I*tan(d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(e*sec(d*x+c))^(5/2)
```

3.482.2 Mathematica [A] (verified)

Time = 12.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.60

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{i2^{-\frac{3}{2}+n}e^{-3i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(-\frac{5}{2}+n,-\frac{5}{4}+n,-\frac{1}{4}+n,-e^{2i(c+}\right.}{de^2(-5+4n)\sqrt{e\sec(c+dx)}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

output
```
((-I)*2^(-3/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(
1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[-5/2 + n, -5/4 + n, -
1/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x
])^n)/(d*e^2*E^((3*I)*(c + d*x))*(-5 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

3.482.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3986}$$

$$\frac{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}dx}{(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{5}{4}}}{(a-ia\tan(c+dx))^{5/4}}dx}{(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\int \frac{(i\tan(c+dx)a+a)^{n-\frac{9}{4}}}{(a-ia\tan(c+dx))^{9/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{n-\frac{9}{4}}(a-ia\tan(c+dx))^{5/4}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\int \frac{\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{n-\frac{9}{4}}}{(a-ia\tan(c+dx))^{9/4}}d\tan(c+dx)}{d(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{n-\frac{1}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{5ad(e\sec(c+dx))^{5/2}}$$

input

```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

output

```
((-1/5*I)*2^(-1/4 + n)*Hypergeometric2F1[-5/4, 9/4 - n, -1/4, (1 - I*Tan[c
 + d*x])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))
/(a*d*(e*Sec[c + d*x])^(5/2))
```

3.482.3.1 Defintions of rubi rules used

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.482.4 Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{5}{2}}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

3.482.5 Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(1/8*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4
*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5/2*I*d*x - 5/2*I*c)/e^3, x)
```

3.482.6 Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(5/2), x)
```

3.482.7 Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

3.482.8 Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2), x)
```

3.483 $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$

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3.483.1 Optimal result

Integrand size = 30, antiderivative size = 269

$$\begin{aligned}\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} \\ &\quad+\frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)} \\ &\quad-\frac{12i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(2-n)(4-n)n} \\ &\quad+\frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{3+n}}{a^3d(4-n)n\,(4-n^2)} \\ &\quad-\frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{4+n}}{a^4dn\,(64-20n^2+n^4)}\end{aligned}$$

output
```
I*(e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n/d/(4-n)+4*I*(e*sec(d*x+c))^(-
4-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-6*n+8)-12*I*(e*sec(d*x+c))^(-4-n)*(
a+I*a*tan(d*x+c))^(2+n)/a^2/d/(2-n)/(4-n)/n+24*I*(e*sec(d*x+c))^(-4-n)*(a+
I*a*tan(d*x+c))^(3+n)/a^3/d/(4-n)/n/(-n^2+4)-24*I*(e*sec(d*x+c))^(-4-n)*(a
+I*a*tan(d*x+c))^(4+n)/a^4/d/n/(n^4-20*n^2+64)
```

3.483.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.61

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i(e\sec(c+dx))^{-n}\left(192-60n^2+3n^4+4n^2(-16+n^2)\cos(2(c+dx))+n^2(-4+n^2)\cos(4(c+dx))+\right.}{8de^4(-4+n)(-}$$

input
```
Integrate[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/8*I)*(192 - 60*n^2 + 3*n^4 + 4*n^2*(-16 + n^2)*Cos[2*(c + d*x)] + n^2
*(-4 + n^2)*Cos[4*(c + d*x)] + (128*I)*n*Sin[2*(c + d*x)] - (8*I)*n^3*Sin[
2*(c + d*x)] + (16*I)*n*Sin[4*(c + d*x)] - (4*I)*n^3*Sin[4*(c + d*x)])*(a
+ I*a*Tan[c + d*x])^n)/(d*e^4*(-4 + n)*(-2 + n)*n*(2 + n)*(4 + n)*(e*Sec[c
 + d*x])^n)
```

3.483.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3985, 3042, 3985, 3042, 3985, 3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}dx$$

$$\Big\downarrow \text{3985}$$

$$\frac{4\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+1}dx}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+1}dx}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{4\left(\frac{3\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+2}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+2}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{4\left(\frac{3\left(-\frac{2\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+3}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{3\left(-\frac{2\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+3}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}\right)}{a(4-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{4\left(\frac{3\left(-\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+4}dx}{a(n+2)}-\frac{i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{d(n+2)}\right)}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+i(\right.}{a(4-n)}$$

$$\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3042}$$

3.483. $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{n}\,dx$

$$\frac{4\left(\frac{3\left(-\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-4}(i\tan(c+dx)a+a)^{n+4}dx}{a(n+2)}-\frac{i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{d(n+2)}\right)}{an}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}\right)}{a(2-n)}+i(\right.}{a(4-n)}$$

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}+$$

$$\frac{4\left(\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{d(2-n)}+\frac{3\left(-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{dn}-\frac{2\left(\frac{i(a+ia\tan(c+dx))^{n+4}(e\sec(c+dx))^{-n-4}}{ad(n+2)(n+4)}-i(a+ia\right.}{an}\right.}{a(2-n)}\right.}{a(4-n)}$$

input
```
Int[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(4 - n)) + (4*((
I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(d*(2 - n)) +
(3*(((-I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(d*n)
- (2*(((-I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(3 + n))/(d*(
2 + n)) + (I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(4 + n))/(a*
d*(2 + n)*(4 + n))))/(a*n)))/(a*(2 - n))))/(a*(4 - n))
```

3.483.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.483. $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

3.483.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.43 (sec) , antiderivative size = 4331, normalized size of antiderivative = 16.10

method	result	size
risch	Expression too large to display	4331

input
```
int((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I/(n-4)/d*a^n/(e^n)/e^4*exp(I*(d*x+c))^n*exp(-1/2*I*(-csgn(I*a*exp(2
*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c)
)+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csg
n(I*a)*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-csgn(I/
(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*
(d*x+c))+1))^2*Pi*n-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n
*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x
+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn
(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n-
2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I
*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*
(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(
2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))*n-n
*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+
c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))^2+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*
Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d
*x+c))+1))*n+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))^2-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+8...
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.25

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{(-i\,n^4 - 4i\,n^3 + 4i\,n^2 + (-i\,n^4 + 4i\,n^3 + 4i\,n^2 - 16i\,n)e^{(8i\,dx+8i\,c)} - 4\,(i\,n^4 - 2i\,n^3 - 16i\,n^2 + 32i\,n)e^{(6i\,dx}}{dn^5 - 20\,dn^3 + 64\,dn + (dn^5 - 20\,dn^3 + 64\,dn)e^{(8i\,dx+8i\,c)} + 4\,(}$$

input
```
integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
(-I*n^4 - 4*I*n^3 + 4*I*n^2 + (-I*n^4 + 4*I*n^3 + 4*I*n^2 - 16*I*n)*e^(8*I
*d*x + 8*I*c) - 4*(I*n^4 - 2*I*n^3 - 16*I*n^2 + 32*I*n)*e^(6*I*d*x + 6*I*c
) - 6*(I*n^4 - 20*I*n^2 + 64*I)*e^(4*I*d*x + 4*I*c) - 4*(I*n^4 + 2*I*n^3 -
 16*I*n^2 - 32*I*n)*e^(2*I*d*x + 2*I*c) + 16*I*n)*(2*e*e^(I*d*x + I*c)/(e^
(2*I*d*x + 2*I*c) + 1))^(-n - 4)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x +
 I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^5 - 20*d*n^3 + 64*d*n
+ (d*n^5 - 20*d*n^3 + 64*d*n)*e^(8*I*d*x + 8*I*c) + 4*(d*n^5 - 20*d*n^3 +
64*d*n)*e^(6*I*d*x + 6*I*c) + 6*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(4*I*d*x + 4
*I*c) + 4*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(2*I*d*x + 2*I*c))
```

3.483.6 Sympy [F]

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec{(c+dx)})^{-n-4}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

input
```
integrate((e*sec(d*x+c))**(-4-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-n - 4)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.62

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^nn^4+4i\,a^nn^3+4i\,a^nn^2-16i\,a^nn)\cos\left((dx+c)(n+4)\right)-4\left(i\,a^nn^4-2i\,a^nn^3-16i\,a^nn^2+32i\,a^nn\right)}{}$$

input
```
integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/16*((-I*a^n*n^4 + 4*I*a^n*n^3 + 4*I*a^n*n^2 - 16*I*a^n*n)*cos((d*x + c)*
(n + 4)) - 4*(I*a^n*n^4 - 2*I*a^n*n^3 - 16*I*a^n*n^2 + 32*I*a^n*n)*cos((d*
x + c)*(n + 2)) - 4*(I*a^n*n^4 + 2*I*a^n*n^3 - 16*I*a^n*n^2 - 32*I*a^n*n)*
cos((d*x + c)*(n - 2)) + (-I*a^n*n^4 - 4*I*a^n*n^3 + 4*I*a^n*n^2 + 16*I*a^
n*n)*cos((d*x + c)*(n - 4)) - 6*(I*a^n*n^4 - 20*I*a^n*n^2 + 64*I*a^n)*cos(
(d*x + c)*n) + (a^n*n^4 - 4*a^n*n^3 - 4*a^n*n^2 + 16*a^n*n)*sin((d*x + c)*
(n + 4)) + 4*(a^n*n^4 - 2*a^n*n^3 - 16*a^n*n^2 + 32*a^n*n)*sin((d*x + c)*(
n + 2)) + 4*(a^n*n^4 + 2*a^n*n^3 - 16*a^n*n^2 - 32*a^n*n)*sin((d*x + c)*(n
 - 2)) + (a^n*n^4 + 4*a^n*n^3 - 4*a^n*n^2 - 16*a^n*n)*sin((d*x + c)*(n - 4
)) + 6*(a^n*n^4 - 20*a^n*n^2 + 64*a^n)*sin((d*x + c)*n))/((e^(n + 4)*n^5 -
 20*e^(n + 4)*n^3 + 64*e^(n + 4)*n)*d)
```

3.483.8 Giac [F]

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec(dx+c))^{-n-4}\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 4)*(I*a*tan(d*x + c) + a)^n, x)
```

3.483.9 Mupad [B] (verification not implemented)

Time = 11.48 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.90

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{\left(2\sin\left(2\,c+2\,d\,x\right)^2+\sin\left(4\,c+4\,d\,x\right)\,1\mathrm{i}-1\right)\left(\frac{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\,\left(-n^3-4\,n^2+4\,n+16\right)}{d\,\left(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i}\right)}+\frac{4\,\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n}{}\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 4),x)
```

output
```
((sin(4*c + 4*d*x)*1i + 2*sin(2*c + 2*d*x)^2 - 1)*(((a - (a*sin(c + d*x)*1
i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(4*n - 4*n^2 - n^3 + 16))/(d*(n^4*1i -
n^2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1)
)^n*(sin(2*c + 2*d*x)*1i - 2*sin(c + d*x)^2 + 1)*(16*n - 2*n^2 - n^3 + 32)
)/(d*(n^4*1i - n^2*20i + 64i)) + ((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d
*x)/2)^2 - 1))^n*(sin(8*c + 8*d*x)*1i - 2*sin(4*c + 4*d*x)^2 + 1)*(4*n + 4
*n^2 - n^3 - 16))/(d*(n^4*1i - n^2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1
i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d*
x)^2 + 1)*(16*n + 2*n^2 - n^3 - 32))/(d*(n^4*1i - n^2*20i + 64i)) - ((a -
(a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4*d*x)*1i -
 2*sin(2*c + 2*d*x)^2 + 1)*(6*n^4 - 120*n^2 + 384))/(d*n*(n^4*1i - n^2*20i
 + 64i))))/(16*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 4)*(sin(c + d*x)^2 -
 1)^2)
```

3.484 $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$

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3.484.1 Optimal result

Integrand size = 30, antiderivative size = 205

$$\begin{aligned}\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} \\ &\quad+\frac{3i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}}{ad\left(3-4n+n^2\right)} \\ &\quad-\frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(3-n)\left(1-n^2\right)} \\ &\quad+\frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{3+n}}{a^3d\left(9-10n^2+n^4\right)}\end{aligned}$$

output
```
I*(e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n/d/(3-n)+3*I*(e*sec(d*x+c))^(-
3-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-4*n+3)-6*I*(e*sec(d*x+c))^(-3-n)*(a
+I*a*tan(d*x+c))^(2+n)/a^2/d/(3-n)/(-n^2+1)+6*I*(e*sec(d*x+c))^(-3-n)*(a+I
*a*tan(d*x+c))^(3+n)/a^3/d/(n^4-10*n^2+9)
```

3.484.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{(e\sec(c+dx))^{-n}\left(-3in(-9+n^2)\cos(c+dx)-in(-1+n^2)\cos(3(c+dx))-6(-5+n^2+(-1+n^2)\text{co}\right.}{4de^3(-3+n)(-1+n)(1+n)(3+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(((-3*I)*n*(-9 + n^2)*Cos[c + d*x] - I*n*(-1 + n^2)*Cos[3*(c + d*x)] - 6*(
-5 + n^2 + (-1 + n^2)*Cos[2*(c + d*x)])*Sin[c + d*x])*(a + I*a*Tan[c + d*x
])^n)/(4*d*e^3*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(e*Sec[c + d*x])^n)
```

3.484.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3985, 3042, 3985, 3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}dx$$

$$\Big\downarrow \text{3985}$$

$$\frac{3\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+1}dx}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+1}dx}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{3\left(\frac{2\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+2}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+2}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{3\left(\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+3}dx}{a(n+1)}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{d(n+1)}\right)}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-3}(i\tan(c+dx)a+a)^{n+3}dx}{a(n+1)}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{d(n+1)}\right)}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}\right)}{a(3-n)}+\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i(a+ia\tan(c+dx))^{n}(e\sec(c+dx))^{-n-3}}{d(3-n)}+\frac{3\left(\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{d(1-n)}+\frac{2\left(\frac{i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-3}}{ad(n+1)(n+3)}-\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{d(n+1)}\right)}{a(1-n)}\right)}{a(3-n)}$$

input
```
Int[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

3.484. $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$

output
```
(I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(3 - n)) + (3*((
I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(d*(1 - n)) +
(2*(((-I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(d*(1
+ n)) + (I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(3 + n))/(a*d*
(1 + n)*(3 + n))))/(a*(1 - n))))/(a*(3 - n))
```

3.484.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

3.484.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.88 (sec) , antiderivative size = 4982, normalized size of antiderivative = 24.30

method	result	size
risch	Expression too large to display	4982

input
```
int((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I/(-3+n)/d/(e^n)*exp(I*(d*x+c))^n/e^3*a^n*exp(-1/2*I*(6*c-3*Pi*csgn(I
*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*a*exp(2*I*(d*x+c))/(exp(2
*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*ex
p(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+6*d
*x-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x
+c)))^3*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n+Pi*csg
n(I*exp(2*I*(d*x+c)))^3*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*
Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x
+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*
(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n+3*Pi*csgn
(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I/(exp(2*I
*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I*ex
p(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/
(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+3*Pi*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*
I*(d*x+c))+1))^2-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi
*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi
*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*csgn(I/...
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{\left(-i\,n^3 - 3i\,n^2 + \left(-i\,n^3 + 3i\,n^2 + i\,n - 3i\right)e^{(6i\,dx+6i\,c)} - 3\left(i\,n^3 - i\,n^2 - 9i\,n + 9i\right)e^{(4i\,dx+4i\,c)} - 3\left(i\,n^3 + i\,n\right.\right.}{dn^4 - 10\,dn^2 + \left(dn^4 - 10\,dn^2 + 9\,d\right)e^{(6i\,dx+6i\,c)} + 3\left(dn^4 - 10\,dn^2 + \right.}$$

input
```
integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
(-I*n^3 - 3*I*n^2 + (-I*n^3 + 3*I*n^2 + I*n - 3*I)*e^(6*I*d*x + 6*I*c) - 3
*(I*n^3 - I*n^2 - 9*I*n + 9*I)*e^(4*I*d*x + 4*I*c) - 3*(I*n^3 + I*n^2 - 9*
I*n - 9*I)*e^(2*I*d*x + 2*I*c) + I*n + 3*I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d
*x + 2*I*c) + 1))^(-n - 3)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/
(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^4 - 10*d*n^2 + (d*n^4 - 10*d
*n^2 + 9*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(4*I*d*x +
4*I*c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(2*I*d*x + 2*I*c) + 9*d)
```

3.484.6 Sympy [F]

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{-n-3}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

input
```
integrate((e*sec(d*x+c))**(-3-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.69

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{(-i\,a^n n^3+3i\,a^n n^2+i\,a^n n-3i\,a^n)\cos\left((dx+c)(n+3)\right)-3\left(i\,a^n n^3-i\,a^n n^2-9i\,a^n n+9i\,a^n\right)\cos\left((dx+\right.}{}$$

input
```
integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/8*((-I*a^n*n^3 + 3*I*a^n*n^2 + I*a^n*n - 3*I*a^n)*cos((d*x + c)*(n + 3))
 - 3*(I*a^n*n^3 - I*a^n*n^2 - 9*I*a^n*n + 9*I*a^n)*cos((d*x + c)*(n + 1))
- 3*(I*a^n*n^3 + I*a^n*n^2 - 9*I*a^n*n - 9*I*a^n)*cos((d*x + c)*(n - 1)) +
 (-I*a^n*n^3 - 3*I*a^n*n^2 + I*a^n*n + 3*I*a^n)*cos((d*x + c)*(n - 3)) + (
a^n*n^3 - 3*a^n*n^2 - a^n*n + 3*a^n)*sin((d*x + c)*(n + 3)) + 3*(a^n*n^3 -
 a^n*n^2 - 9*a^n*n + 9*a^n)*sin((d*x + c)*(n + 1)) + 3*(a^n*n^3 + a^n*n^2
- 9*a^n*n - 9*a^n)*sin((d*x + c)*(n - 1)) + (a^n*n^3 + 3*a^n*n^2 - a^n*n -
 3*a^n)*sin((d*x + c)*(n - 3)))/((e^(n + 3)*n^4 - 10*e^(n + 3)*n^2 + 9*e^(
n + 3))*d)
```

3.484.8 Giac [F]

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-n-3}\,(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.484.9 Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.07

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+\sin\left(3\,c+3\,d\,x\right)\,1\mathrm{i}-1\right)\,\left(\frac{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\,(-n^3-3\,n^2+n+3)}{d\,(n^4\,1\mathrm{i}-n^2\,10\mathrm{i}+9\mathrm{i})}\right.}{}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 3),x)
```

output
```
-((2*sin(c/2 + (d*x)/2)^2 - 1)*(sin(3*c + 3*d*x)*1i + 2*sin((3*c)/2 + (3*d
*x)/2)^2 - 1)*(((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(
n - 3*n^2 - n^3 + 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*sin(c + d*x)*
1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d
*x)^2 + 1)*(n + 3*n^2 - n^3 - 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*s
in(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(2*c + 2*d*x)*1i - 2*s
in(c + d*x)^2 + 1)*(27*n - 3*n^2 - 3*n^3 + 27))/(d*(n^4*1i - n^2*10i + 9i)
) + ((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4
*d*x)*1i - 2*sin(2*c + 2*d*x)^2 + 1)*(27*n + 3*n^2 - 3*n^3 - 27))/(d*(n^4*
1i - n^2*10i + 9i))))/(8*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 3)*(sin(c
+ d*x)^2 - 1)^2)
```

3.484. $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$

3.485 $\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$

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3.485.1 Optimal result

Integrand size = 30, antiderivative size = 148

$$\begin{aligned}\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)}\\ &\quad-\frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{1+n}}{ad(2-n)n}\\ &\quad+\frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{2+n}}{a^2dn\,(4-n^2)}\end{aligned}$$

output

```
I*(e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n/d/(2-n)-2*I*(e*sec(d*x+c))^(-
2-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(2-n)/n+2*I*(e*sec(d*x+c))^(-2-n)*(a+I*a
*tan(d*x+c))^(2+n)/a^2/d/n/(-n^2+4)
```

3.485.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}\left(-4+n^2+n^2\cos(2(c+dx))-2in\sin(2(c+dx))\right)(a+ia\tan(c+dx))^n}{2de^2(-2+n)n(2+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/2*I)*(-4 + n^2 + n^2*Cos[2*(c + d*x)] - (2*I)*n*Sin[2*(c + d*x)])*(a
+ I*a*Tan[c + d*x])^n)/(d*e^2*(-2 + n)*n*(2 + n)*(e*Sec[c + d*x])^n)
```

3.485.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3985, 3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}dx$$

$$\Big\downarrow \text{3985}$$

$$\frac{2\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+1}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+1}dx}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3985}$$

$$\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+2}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\frac{\int (e\sec(c+dx))^{-n-2}(i\tan(c+dx)a+a)^{n+2}dx}{an}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{dn}\right)}{a(2-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}+\frac{2\left(\frac{i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-2}}{adn(n+2)}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{dn}\right)}{a(2-n)}$$

input
```
Int[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(2 - n)) + (2*((
(-I)*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(d*n) + (I*
(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a*d*n*(2 + n)))
)/(a*(2 - n))
```

3.485.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

3.485.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.42 (sec) , antiderivative size = 2581, normalized size of antiderivative = 17.44

method	result	size
risch	Expression too large to display	2581

input
```
int((e*sec(d*x+c))^(-n-2)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/(n-2)/d*exp(I*(d*x+c))^n/(e^n)/e^2*a^n*exp(-1/2*I*(csgn(I*a*exp(2*I
*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*
(d*x+c))+1))^2*csgn(I*a)*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c))
)*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n+csgn(I/(exp(2*I*(
d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1
))*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n-csg
n(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n
-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c
))+1))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*
(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))*n-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I
*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2
*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I/(exp(2*I*(d*x+c))
+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi
*csgn(I*exp(2*I*(d*x+c)))^3*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I
*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+4*d...
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{\left(-i\,n^2+(-i\,n^2+2i\,n)e^{(4i\,dx+4i\,c)}-2\,(i\,n^2-4i)e^{(2i\,dx+2i\,c)}-2i\,n\right)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-2}e^{\left(i\,dnx+i\,cn+n\log\right.}}{dn^3-4\,dn+(dn^3-4\,dn)e^{(4i\,dx+4i\,c)}+2\,(dn^3-4\,dn)e^{(2i\,dx+2i\,c)}}$$

input
```
integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
(-I*n^2 + (-I*n^2 + 2*I*n)*e^(4*I*d*x + 4*I*c) - 2*(I*n^2 - 4*I)*e^(2*I*d*
x + 2*I*c) - 2*I*n)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n -
2)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1
)) + n*log(a/e))/(d*n^3 - 4*d*n + (d*n^3 - 4*d*n)*e^(4*I*d*x + 4*I*c) + 2*
(d*n^3 - 4*d*n)*e^(2*I*d*x + 2*I*c))
```

3.485.6 Sympy [F]

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{-n-2}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

input
```
integrate((e*sec(d*x+c))**(-2-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{(-i\,a^n n^2+2i\,a^n n)\cos{((dx+c)(n+2))}+(-i\,a^n n^2-2i\,a^n n)\cos{((dx+c)(n-2))}-2\,(i\,a^n n^2-4i\,a^n)\,c}{4}$$

input
```
integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/4*((-I*a^n*n^2 + 2*I*a^n*n)*cos((d*x + c)*(n + 2)) + (-I*a^n*n^2 - 2*I*a
^n*n)*cos((d*x + c)*(n - 2)) - 2*(I*a^n*n^2 - 4*I*a^n)*cos((d*x + c)*n) +
(a^n*n^2 - 2*a^n*n)*sin((d*x + c)*(n + 2)) + (a^n*n^2 + 2*a^n*n)*sin((d*x
+ c)*(n - 2)) + 2*(a^n*n^2 - 4*a^n)*sin((d*x + c)*n))/((e^(n + 2)*n^3 - 4*
e^(n + 2)*n)*d)
```

3.485.8 Giac [F]

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n-2}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.485.9 Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.53

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = \frac{(\cos(2\,c+2\,d\,x)-\sin(2\,c+2\,d\,x)\ 1\mathrm{i})\left(\frac{\left(a+\frac{a\ \sin(c+d\,x)\ 1\mathrm{i}}{\cos(c+d\,x)}\right)^n(n+2)}{d\,(n^2\ 1\mathrm{i}-4\mathrm{i})}+\frac{(\cos(4\,c+4\,d\,x)+\sin(4\,c+4\,d\,x)\ 1\mathrm{i})\left(a+\frac{a\ \sin(c+d\,x)\ 1\mathrm{i}}{\cos(c+d\,x)}\right)^n(}{d\,(n^2\ 1\mathrm{i}-4\mathrm{i})}\right.}{4\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)\left(\frac{e}{\cos(c+d\,x)}\right)^{n+2}}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 2),x)
```

output
```
((cos(2*c + 2*d*x) - sin(2*c + 2*d*x)*1i)*(((a + (a*sin(c + d*x)*1i)/cos(c
 + d*x))^n*(n + 2))/(d*(n^2*1i - 4i)) + ((cos(4*c + 4*d*x) + sin(4*c + 4*d
*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n - 2))/(d*(n^2*1i - 4i)
) + ((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(2*n^2 - 8)*(a + (a*sin(c +
d*x)*1i)/cos(c + d*x))^n)/(d*n*(n^2*1i - 4i))))/(4*(cos(2*c + 2*d*x)/2 + 1
/2)*(e/cos(c + d*x))^(n + 2))
```

3.486 $\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$

3.486.1 Optimal result 3406
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3.486.1 Optimal result

Integrand size = 30, antiderivative size = 94

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)} - \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(1-n^2)}$$

output
```
I*(e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n/d/(1-n)-I*(e*sec(d*x+c))^(-1-
n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(-n^2+1)
```

3.486.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.62

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-1-n}(n-i\tan(c+dx))(a+ia\tan(c+dx))^n}{d(-1+n)(1+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*(e*Sec[c + d*x])^(-1 - n)*(n - I*Tan[c + d*x])*(a + I*a*Tan[c + d*x]
)^n)/(d*(-1 + n)*(1 + n))
```

3.486.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3985, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}dx$$

$$\Big\downarrow \text{3985}$$

$$\frac{\int (e\sec(c+dx))^{-n-1}(i\tan(c+dx)a+a)^{n+1}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{-n-1}(i\tan(c+dx)a+a)^{n+1}dx}{a(1-n)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}$$

$$\Big\downarrow \text{3969}$$

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-1}}{ad(1-n)(n+1)}$$

input
```
Int[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n)) - (I*(e
*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 - n)*(1 +
n))
```

3.486.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3985
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

3.486.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.10 (sec) , antiderivative size = 2484, normalized size of antiderivative = 26.43

method	result	size
risch	Expression too large to display	2484

input
```
int((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/(1+n)/d*e^(-n)*a^n/e*exp(I*(d*x+c))^n*exp(1/2*I*(2*c+Pi*csgn(I*e*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^2*csgn(I*a)*Pi*n+2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(
d*x+c)))*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+2*d*x+n*
Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))
^3*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-Pi*csgn(I*e
xp(2*I*(d*x+c)))^3*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*cs
gn(I/(exp(2*I*(d*x+c))+1))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))
^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n-Pi*csgn(I*e)*c
sgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I/(exp(2*I*(d*x+c))
+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*exp(I*(d*x+c)
))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))^3+Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x
+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(I*(d*
x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^
2+n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*Pi*csgn(I*exp(I*(
d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn
(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I/(exp(2*I*(d*x+c))...
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \frac{\left((-i\,n+i)e^{(2i\,dx+2i\,c)}-i\,n-i\right)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-1}e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)\right)}}{dn^2+(dn^2-d)e^{(2i\,dx+2i\,c)}-d}$$

input
```
integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

output
```
((-I*n + I)*e^(2*I*d*x + 2*I*c) - I*n - I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*
x + 2*I*c) + 1))^(-n - 1)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(
e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^2 + (d*n^2 - d)*e^(2*I*d*x +
2*I*c) - d)
```

3.486.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(71) = 142.

Time = 0.51 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.62

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \begin{cases} x(e\sec{(c)})^{-n-1}\,(ia\tan{(c)}+a)^n & \text{for } d=0 \\ \frac{dx\tan{(c+dx)}}{2ad\tan{(c+dx)}-2iad} - \frac{idx}{2ad\tan{(c+dx)}-2iad} + \frac{1}{2ad\tan{(c+dx)}-2iad} & \text{for } n=-1 \\ \frac{\frac{ax\tan^2{(c+dx)}}{2\sec^2{(c+dx)}} + \frac{ax}{2\sec^2{(c+dx)}} + \frac{a\tan{(c+dx)}}{2d\sec^2{(c+dx)}} - \frac{ia}{2d\sec^2{(c+dx)}}}{e^2} & \text{for } n=1 \\ -\frac{in(e\sec{(c+dx)})^{-n-1}(ia\tan{(c+dx)}+a)^n}{dn^2-d} - \frac{(e\sec{(c+dx)})^{-n-1}(ia\tan{(c+dx)}+a)^n\tan{(c+dx)}}{dn^2-d} & \text{otherwise} \end{cases}$$

input
```
integrate((e*sec(d*x+c))**(-1-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Piecewise((x*(e*sec(c))**(-n - 1)*(I*a*tan(c) + a)**n, Eq(d, 0)), (d*x*tan
(c + d*x)/(2*a*d*tan(c + d*x) - 2*I*a*d) - I*d*x/(2*a*d*tan(c + d*x) - 2*I
*a*d) + 1/(2*a*d*tan(c + d*x) - 2*I*a*d), Eq(n, -1)), ((a*x*tan(c + d*x)**
2/(2*sec(c + d*x)**2) + a*x/(2*sec(c + d*x)**2) + a*tan(c + d*x)/(2*d*sec(
c + d*x)**2) - I*a/(2*d*sec(c + d*x)**2))/e**2, Eq(n, 1)), (-I*n*(e*sec(c
+ d*x))**(-n - 1)*(I*a*tan(c + d*x) + a)**n/(d*n**2 - d) - (e*sec(c + d*x)
)**(-n - 1)*(I*a*tan(c + d*x) + a)**n*tan(c + d*x)/(d*n**2 - d), True))
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^n n + i\,a^n)\cos{((dx+c)(n+1))} + (-i\,a^n n - i\,a^n)\cos{((dx+c)(n-1))} + (a^n n - a^n)\sin{((dx+c)(n}}{2\,(e^{n+1}n^2 - e^{n+1})d}$$

input
```
integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

output
```
1/2*((-I*a^n*n + I*a^n)*cos((d*x + c)*(n + 1)) + (-I*a^n*n - I*a^n)*cos((d
*x + c)*(n - 1)) + (a^n*n - a^n)*sin((d*x + c)*(n + 1)) + (a^n*n + a^n)*si
n((d*x + c)*(n - 1)))/((e^(n + 1)*n^2 - e^(n + 1))*d)
```

3.486.8 Giac [F]

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-n-1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.486.9 Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n\,(\sin{(c+d\,x)}+\sin{(3\,c+3\,d\,x)}+n\,\cos{(c+d\,x)}\,3\mathrm{i}+n\,\cos{(3\,c+3\,d\,x}}{2\,d\,e\,\left(\cos{(2\,c+2\,d\,x)}+1\right)\,\left(n^2-1\right)\,{\left(\frac{e}{\cos(c+d\,x)}\right)}^n}$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 1),x)
```

output
```
-(((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^n*(sin(c + d*x) + sin(3*c + 3*d*x) + n*cos(c + d*x)*3i + n*cos(3*c + 3*d
*x)*1i))/(2*d*e*(cos(2*c + 2*d*x) + 1)*(n^2 - 1)*(e/cos(c + d*x))^n)
```

3.487 $\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx$

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3.487.2 Mathematica [A] (verified) . . . 3412
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3.487.4 Maple [C] (warning: unable to verify) . . . 3414
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3.487.9 Mupad [F(-1)] . . . 3416

3.487.1 Optimal result

Integrand size = 28, antiderivative size = 37

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx=-\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

output
```
-I*(a+I*a*tan(d*x+c))^n/d/n/((e*sec(d*x+c))^n)
```

3.487.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx=-\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

3.487.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}dx$$

$$\Big\downarrow \ 3969$$

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}}{dn}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
```

output
```
((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

3.487.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

3.487.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.52 (sec) , antiderivative size = 842, normalized size of antiderivative = 22.76

method	result	size
risch	Expression too large to display	842

input
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x,method=_RETURNVERBOSE)
```

output
```
-I*exp(I*(d*x+c))^(2*n)*a^n/(exp(I*(d*x+c))^n)/(e^n)/n/d*exp(1/2*I*n*Pi*(-
csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+csgn(I*a*exp(2*I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-csgn(I/(exp(2*I*
(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+
1))*csgn(I*a)-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3+csgn(I/(exp(
2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*exp(2*I*(d*x+c)))+csgn(I/(exp(2
*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))-csgn(I/(ex
p(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*ex
p(2*I*(d*x+c)))+csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))^2-csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+csgn(I*e
)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp
(2*I*(d*x+c))+1))+csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*exp
(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-csgn(I/(exp(2*I
*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+csgn(I/(exp(2*
I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))-csgn(I*exp(2*I*(d*x+c)))^3+2*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp
(I*(d*x+c)))-csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2))
```

3.487.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(33) = 66.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)\right)}}{dn\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n}$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="fricas")
```

output
```
-I*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1
)) + n*log(a/e))/(d*n*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n)
```

3.487.6 Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0 \wedge n=0 \\ x(e\sec(c))^{-n}\left(ia\tan(c)+a\right)^n & \text{for } d=0 \\ x & \text{for } n=0 \\ -\frac{i(e\sec(c+dx))^{-n}(ia\tan(c+dx)+a)^n}{dn} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**n),x)
```

output
```
Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(I*a*tan(c) + a)**n/(e*sec(c))**n,
Eq(d, 0)), (x, Eq(n, 0)), (-I*(I*a*tan(c + d*x) + a)**n/(d*n*(e*sec(c + d*
x))**n), True))
```

3.487.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(33) = 66.

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.32

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,a^n e^{\left(n\log\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)-n\log\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)\right)}}{de^n n}$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="maxima")
```

output
```
-I*a^n*e^(n*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 1) - n*log(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/(
d*e^n*n)
```

3.487.8 Giac [F]

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^n}{(e\sec{(dx+c)})^n}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^n, x)
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^n}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n,x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n, x)
```

3.488 $\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$

3.488.1 Optimal result 3417
3.488.2 Mathematica [A] (verified) 3417
3.488.3 Rubi [A] (verified) 3418
3.488.4 Maple [F] 3420
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3.488.7 Maxima [F] 3421
3.488.8 Giac [F] 3421
3.488.9 Mupad [F(-1)] 3422

3.488.1 Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{1-n}(1+i\tan(c+dx))^{\frac{1}{2}(-1-}}{d(1-n)}$$

output
```
I*2^(1/2+1/2*n)*hypergeom([1/2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],1/2-1/2*I*tan
(d*x+c))*(e*sec(d*x+c))^(1-n)*(1+I*tan(d*x+c))^(-1/2-1/2*n)*(a+I*a*tan(d*x
+c))^n/d/(1-n)
```

3.488.2 Mathematica [A] (verified)

Time = 5.84 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{e(\text{Hypergeometric2F1}(1,n,1+n,i\cos(c+dx)-\sin(c+dx))-\text{Hypergeometric2F1}(1,n,1+n,-i\,\text{co}}{dn}$$

input
```
Integrate[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
-((e*(Hypergeometric2F1[1, n, 1 + n, I*Cos[c + d*x] - Sin[c + d*x]] - Hype
rgeometric2F1[1, n, 1 + n, (-I)*Cos[c + d*x] + Sin[c + d*x]])*(a + I*a*Tan
[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n))
```

3.488.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int (a-ia\tan(c+dx))^{\frac{1-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+1}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int (a-ia\tan(c+dx))^{\frac{1-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+1}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(-n-1)}(i\tan(c+dx)a}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{-\frac{n}{2}-\frac{1}{2}}(1-i\tan(c+dx))^{\frac{n+1}{2}}(a-ia\tan(c+dx))^{\frac{1}{2}(-n-1)+\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}}(e\sec(c+dx))^{1-n}\int \left(\frac{1}{2}-\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{1}{2}-\frac{n}{2}}(1-i\tan(c+dx))^{\frac{n+1}{2}}(a-ia\tan(c+dx))^{\frac{1}{2}(-n-1)+\frac{n-1}{2}}(a+ia\tan(c+dx))^{\frac{n-1}{2}+\frac{n+1}{2}}(e\sec(c+dx))^{1-n}\,\mathrm{H}}{d(n+1)}$$

input
```
Int[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1/2 - n/2)*a*Hypergeometric2F1[(1 + n)/2, (1 + n)/2, (3 + n)/2, (
1 + I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(1 - n)*(1 - I*Tan[c + d*x])^((1 +
 n)/2)*(a - I*a*Tan[c + d*x])^((-1 - n)/2 + (-1 + n)/2)*(a + I*a*Tan[c + d
*x])^((-1 + n)/2 + (1 + n)/2))/(d*(1 + n))
```

3.488.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.488.4 Maple [F]

$$\int (e\sec (dx+c))^{1-n} (a+ia\tan (dx+c))^n\, dx$$

input
```
int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

3.488.5 Fricas [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\, dx = \int (e\sec (dx+c))^{-n+1} (i\,a\tan (dx+c)+a)^n\, dx$$

input
```
integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 1)*e^(I*d*n
*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

3.488.6 Sympy [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\, dx = \int (e\sec (c+dx))^{1-n} (ia(\tan (c+dx)-i))^n\, dx$$

input
```
integrate((e*sec(d*x+c))**(1-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(1 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.488.7 Maxima [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+1}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
-2*(a^n*e*cos(c*n + (d*n + d)*x + c) + I*a^n*e*sin(c*n + (d*n + d)*x + c)
- 2*(I*a^n*d*e^(n + 1)*n - I*a^n*d*e^(n + 1) + (I*a^n*d*e^(n + 1)*n - I*a^
n*d*e^(n + 1))*cos(2*d*x + 2*c) - (a^n*d*e^(n + 1)*n - a^n*d*e^(n + 1))*si
n(2*d*x + 2*c))*integrate(((cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*cos
(c*n + (d*n + d)*x + c) + (sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(c*n
+ (d*n + d)*x + c))/((e^n*n - e^n)*cos(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*co
s(2*d*x + 2*c)^2 + (e^n*n - e^n)*sin(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(e^n*n - e^n)*sin(2*d*x + 2*c)^2 + e^n*n
 + 2*(e^n*n + 2*(e^n*n - e^n)*cos(2*d*x + 2*c) - e^n)*cos(4*d*x + 4*c) + 4
*(e^n*n - e^n)*cos(2*d*x + 2*c) - e^n), x) + 2*(a^n*d*e^(n + 1)*n - a^n*d*
e^(n + 1) + (a^n*d*e^(n + 1)*n - a^n*d*e^(n + 1))*cos(2*d*x + 2*c) - (-I*a
^n*d*e^(n + 1)*n + I*a^n*d*e^(n + 1))*sin(2*d*x + 2*c))*integrate(-((sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(c*n + (d*n + d)*x + c) - (cos(4*d*x
+ 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(c*n + (d*n + d)*x + c))/((e^n*n - e^n
)*cos(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*cos(2*d*x + 2*c)^2 + (e^n*n - e^n)*
sin(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*(e^n*n - e^n)*sin(2*d*x + 2*c)^2 + e^n*n + 2*(e^n*n + 2*(e^n*n - e^n)*cos
(2*d*x + 2*c) - e^n)*cos(4*d*x + 4*c) + 4*(e^n*n - e^n)*cos(2*d*x + 2*c) -
 e^n), x))/(-I*d*e^n*n + I*d*e^n + (-I*d*e^n*n + I*d*e^n)*cos(2*d*x + 2*c)
 + (d*e^n*n - d*e^n)*sin(2*d*x + 2*c))
```

3.488.8 Giac [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+1}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{1-n}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n \,d x$$

input
```
int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.489 $\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$

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3.489.1 Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{1+\frac{n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{2-n}{2},-\frac{n}{2},\frac{4-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2-n}(1+i\tan(c+dx))^{-n/}}{d(2-n)}$$

output
```
I*2^(1+1/2*n)*a*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*tan(d*x+c)
)*(e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(2-n)/((1+I*tan(d*x+c))
^(1/2*n))
```

3.489.2 Mathematica [A] (verified)

Time = 14.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{4e^2\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},-\cos(2(c+dx))+i\sin(2(c+dx))\right)(e\sec(c+dx))^{-n}(\cos(2c)-}{d(-2+n)(-1-i\tan(dx))}$$

input
```
Integrate[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(4*e^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, -Cos[2*(c + d*x)] + I*Sin[2*
(c + d*x)]]*(Cos[2*c] - I*Sin[2*c])*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^
n)/(d*(-2 + n)*(e*Sec[c + d*x])^n*(-1 - I*Tan[d*x]))
```

3.489.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{2-n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{2-n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{n-2}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int (a-ia\tan(c+dx))^{\frac{2-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+2}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{n-2}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int (a-ia\tan(c+dx))^{\frac{2-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+2}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{n-2}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int (a-ia\tan(c+dx))^{-n/2}(i\tan(c+dx)a+a}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{-n/2}(1-i\tan(c+dx))^{n/2}(a-ia\tan(c+dx))^{\frac{n-2}{2}-\frac{n}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}}(e\sec(c+dx))^{2-n}\int \left(\frac{1}{2}-\frac{1}{2}i\tan(\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{1-\frac{n}{2}}(1-i\tan(c+dx))^{n/2}(a-ia\tan(c+dx))^{\frac{n-2}{2}-\frac{n}{2}}(a+ia\tan(c+dx))^{\frac{n-2}{2}+\frac{n+2}{2}}(e\sec(c+dx))^{2-n}\,\text{Hyperge}}{d(n+2)}$$

input

```
Int[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1 - n/2)*a*Hypergeometric2F1[n/2, (2 + n)/2, (4 + n)/2, (1 + I*Ta
n[c + d*x])/2]*(e*Sec[c + d*x])^(2 - n)*(1 - I*Tan[c + d*x])^(n/2)*(a - I*
a*Tan[c + d*x])^((-2 + n)/2 - n/2)*(a + I*a*Tan[c + d*x])^((-2 + n)/2 + (2
 + n)/2))/(d*(2 + n))
```

3.489.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.489.4 Maple [F]

$$\int (e\sec(dx+c))^{2-n}(a+ia\tan(dx+c))^{n}\,dx$$

input
```
int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

3.489.5 Fricas [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^{n}\,dx=\int (e\sec(dx+c))^{-n+2}(i\,a\tan(dx+c)+a)^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
1/2*((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 2)*(I*e^(2*I*d*
x + 2*I*c) + I)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
 + 2*I*c) + 1)) + n*log(a/e)) + 2*d*e^(2*I*d*x + 2*I*c)*integral(1/2*(n*e^
(2*I*d*x + 2*I*c) + n)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n
 + 2)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
 + 2*I*c) + 1)) + n*log(a/e) - 2*I*c), x))*e^(-2*I*d*x - 2*I*c)/d
```

3.489.6 Sympy [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^{n}\,dx=\int (e\sec(c+dx))^{2-n}\,(ia(\tan(c+dx)-i))^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))**(2-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(2 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.489.7 Maxima [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+2}\left(i\,a\tan(dx+c)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
4*(4*a^n*e^2*cos(d*n*x + c*n) + 4*I*a^n*e^2*sin(d*n*x + c*n) - (a^n*e^2*n
- 4*a^n*e^2)*cos(c*n + (d*n + 2*d)*x + 2*c) - 4*(I*a^n*d*e^(n + 2)*n^3 - 6
*I*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n + (I*a^n*d*e^(n + 2)*n^3 -
6*I*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n)*cos(4*d*x + 4*c) + 2*(I*a
^n*d*e^(n + 2)*n^3 - 6*I*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n)*cos(
2*d*x + 2*c) - (a^n*d*e^(n + 2)*n^3 - 6*a^n*d*e^(n + 2)*n^2 + 8*a^n*d*e^(n
 + 2)*n)*sin(4*d*x + 4*c) - 2*(a^n*d*e^(n + 2)*n^3 - 6*a^n*d*e^(n + 2)*n^2
 + 8*a^n*d*e^(n + 2)*n)*sin(2*d*x + 2*c))*integrate(((cos(6*d*x + 6*c) + 3
*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(d*n*x + c*n) + (sin(6*d*x
+ 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*sin(d*n*x + c*n))/(e^n*n
^2 + (e^n*n^2 - 6*e^n*n + 8*e^n)*cos(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^n*n
 + 8*e^n)*cos(4*d*x + 4*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2
*c)^2 + (e^n*n^2 - 6*e^n*n + 8*e^n)*sin(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^
n*n + 8*e^n)*sin(4*d*x + 4*c)^2 + 18*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(4*d*x
 + 4*c)*sin(2*d*x + 2*c) + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(2*d*x + 2*c)^
2 - 6*e^n*n + 2*(e^n*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(4*d
*x + 4*c) + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 8*e^n)*cos(6*
d*x + 6*c) + 6*(e^n*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*
x + 2*c) + 8*e^n)*cos(4*d*x + 4*c) + 6*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d
*x + 2*c) + 6*((e^n*n^2 - 6*e^n*n + 8*e^n)*sin(4*d*x + 4*c) + (e^n*n^2 ...
```

3.489.8 Giac [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+2}\left(i\,a\tan(dx+c)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{2-n}\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

input
```
int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.490 $\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$

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3.490.1 Optimal result

Integrand size = 30, antiderivative size = 121

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{i2^{\frac{3+n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-n),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3-n}(1+i\tan(c+d}{d(3-n)}$$

output
```
I*2^(3/2+1/2*n)*a*hypergeom([3/2-1/2*n, -1/2-1/2*n],[5/2-1/2*n],1/2-1/2*I*
tan(d*x+c))*(e*sec(d*x+c))^(3-n)*(1+I*tan(d*x+c))^(-1/2-1/2*n)*(a+I*a*tan(
d*x+c))^(-1+n)/d/(3-n)
```

3.490.2 Mathematica [A] (verified)

Time = 17.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{8e^3\,\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},-\cos(2(c+dx))+i\sin(2(c+dx))\right)\sec(dx)(e\sec(c+dx))^{-n}(i+\text{ta}}{d(-3+n)(\cos(c)+i\sin(c))^3(-i+\tan(dx))^2}$$

input
```
Integrate[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(8*e^3*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, -Cos[2*(c + d*x)] + I*Si
n[2*(c + d*x)]]*Sec[d*x]*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)/(d*(-3 +
 n)*(e*Sec[c + d*x])^n*(Cos[c] + I*Sin[c])^3*(-I + Tan[d*x])^2)
```

3.490.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int (a-ia\tan(c+dx))^{\frac{3-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+3}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int (a-ia\tan(c+dx))^{\frac{3-n}{2}}(i\tan(c+dx)a+a)^{\frac{n+3}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int (a-ia\tan(c+dx))^{\frac{1-n}{2}}(i\tan(c+dx)a+a)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{\frac{1}{2}-\frac{n}{2}}(1-i\tan(c+dx))^{\frac{n-1}{2}}(a-ia\tan(c+dx))^{\frac{1-n}{2}+\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}}(e\sec(c+dx))^{3-n}\int \left(\frac{1}{2}-\frac{1}{2}i\,\text{ta}\right.}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{3}{2}-\frac{n}{2}}(1-i\tan(c+dx))^{\frac{n-1}{2}}(a-ia\tan(c+dx))^{\frac{1-n}{2}+\frac{n-3}{2}}(a+ia\tan(c+dx))^{\frac{n-3}{2}+\frac{n+3}{2}}(e\sec(c+dx))^{3-n}\,\text{Hyper}}{d(n+3)}$$

input
```
Int[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(3/2 - n/2)*a*Hypergeometric2F1[(-1 + n)/2, (3 + n)/2, (5 + n)/2,
(1 + I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3 - n)*(1 - I*Tan[c + d*x])^((-1
 + n)/2)*(a - I*a*Tan[c + d*x])^((1 - n)/2 + (-3 + n)/2)*(a + I*a*Tan[c +
d*x])^((-3 + n)/2 + (3 + n)/2))/(d*(3 + n))
```

3.490.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.490.4 Maple [F]

$$\int (e\sec (dx+c))^{3-n} (a+ia\tan (dx+c))^{n}dx$$

input
```
int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

3.490.5 Fricas [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec (dx+c))^{-n+3} (i\,a\tan (dx+c)+a)^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
1/8*(((-I*n - I)*e^(4*I*d*x + 4*I*c) - 2*I*n*e^(2*I*d*x + 2*I*c) - I*n + I
)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 3)*e^(I*d*n*x + I*
c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)) +
 8*d*e^(2*I*d*x + 2*I*c)*integral(-1/8*(n^2 + (n^2 - 1)*e^(4*I*d*x + 4*I*c
) + 2*(n^2 - 1)*e^(2*I*d*x + 2*I*c) - 1)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
+ 2*I*c) + 1))^(-n + 3)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x
+ I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e) - 2*I*c), x))*e^(-2*I*d*x -
 2*I*c)/d
```

3.490.6 Sympy [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec (c+dx))^{3-n} (ia(\tan (c+dx)-i))^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3-n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(3 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.490.7 Maxima [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(dx+c)})^{-n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
8*(6*a^n*e^3*cos(c*n + (d*n + d)*x + c) + 6*I*a^n*e^3*sin(c*n + (d*n + d)*
x + c) - (a^n*e^3*n - 5*a^n*e^3)*cos(c*n + (d*n + 3*d)*x + 3*c) - 6*((I*a^
n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n + 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I
*a^n*d*e^(n + 3))*cos(c*n) + ((I*a^n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n + 3)
*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n) - (a^n*d*e^(
n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n +
 3))*sin(c*n))*cos(6*d*x + 6*c) + 3*((I*a^n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^
(n + 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n) - (a^
n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d
*e^(n + 3))*sin(c*n))*cos(4*d*x + 4*c) + 3*((I*a^n*d*e^(n + 3)*n^3 - 7*I*a
^n*d*e^(n + 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n
) - (a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 1
5*a^n*d*e^(n + 3))*sin(c*n))*cos(2*d*x + 2*c) - (a^n*d*e^(n + 3)*n^3 - 7*a
^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n + 3))*sin(c*n) -
((a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a
^n*d*e^(n + 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n
^2 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*sin(6*d*x + 6
*c) - 3*((a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*
n + 15*a^n*d*e^(n + 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(
n + 3)*n^2 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*si...
```

3.490.8 Giac [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(dx+c)})^{-n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(\frac{e}{\cos(c+d\,x)}\right)^{3-n}(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.491 $\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$

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3.491.1 Optimal result

Integrand size = 30, antiderivative size = 156

$$\begin{aligned}\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx &= \frac{8ia^3(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-3+n}}{d(5-n)\left(12-7n+n^2\right)} \\ &\quad+\frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(20-9n+n^2\right)} \\ &\quad+\frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)}\end{aligned}$$

output
```
8*I*a^3*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-3+n)/d/(-n^3+12*n^2-47
*n+60)+4*I*a^2*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-9*n
+20)+I*a*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(5-n)
```

3.491.2 Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{e^6\sec^5(c+dx)(e\sec(c+dx))^{-2n}\left(-2(-5+n)+\left(22-9n+n^2\right)\cos(2(c+dx))+i\left(18-9n+n^2\right)\sin(}{d(-5+n)(-4+n)(-3+n)}$$

input

```
Integrate[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output

```
-((e^6*Sec[c + d*x]^5*(-2*(-5 + n) + (22 - 9*n + n^2)*Cos[2*(c + d*x)] + I
*(18 - 9*n + n^2)*Sin[2*(c + d*x)])*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]
)*(a + I*a*Tan[c + d*x])^n)/(d*(-5 + n)*(-4 + n)*(-3 + n)*(e*Sec[c + d*x])
^(2*n)))
```

3.491.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{6-2n}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{6-2n}dx$$

$$\Big\downarrow \text{ 3975}$$

$$\frac{4a\int (e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-1}dx}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4a\int (e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-1}dx}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \text{ 3975}$$

$$\frac{4a\left(\frac{2a\int (e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-2}dx}{4-n}+\frac{ia(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d(4-n)}\right)}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4a\left(\frac{2a\int(e\sec(c+dx))^{6-2n}(i\tan(c+dx)a+a)^{n-2}dx}{4-n}+\frac{ia(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d(4-n)}\right)}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

$$\Big\downarrow \text{3974}$$

$$\frac{4a\left(\frac{2ia^2(a+ia\tan(c+dx))^{n-3}(e\sec(c+dx))^{6-2n}}{d(3-n)(4-n)}+\frac{ia(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d(4-n)}\right)}{5-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

input
```
Int[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*a*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(5 - n
)) + (4*a*(((2*I)*a^2*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-
3 + n))/(d*(3 - n)*(4 - n)) + (I*a*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan
[c + d*x])^(-2 + n))/(d*(4 - n))))/(5 - n)
```

3.491.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.491.4 Maple [F]

$$\int (e\sec (dx+c))^{6-2n} (a+ia\tan (dx+c))^{n} dx$$

input
```
int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.491.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{n}\,dx$$
$$= \frac{((-i\,n^{2}+9i\,n-20i)e^{(6i\,dx+6i\,c)}+(-i\,n^{2}+11i\,n-30i)e^{(4i\,dx+4i\,c)}-2\,(-i\,n+6i)e^{(2i\,dx+2i\,c)}-2i)\left(\frac{2\,ee^{(i\,dx}}{e^{(2i\,dx+2}}\right.}{2\,(dn^{3}-12\,dn^{2}+47\,dn-60\,d)}$$

input
```
integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
1/2*((-I*n^2 + 9*I*n - 20*I)*e^(6*I*d*x + 6*I*c) + (-I*n^2 + 11*I*n - 30*I
)*e^(4*I*d*x + 4*I*c) - 2*(-I*n + 6*I)*e^(2*I*d*x + 2*I*c) - 2*I)*(2*e*e^(
I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 6)*e^(I*d*n*x + I*c*n - 6*
I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)
- 6*I*c)/(d*n^3 - 12*d*n^2 + 47*d*n - 60*d)
```

3.491.6 Sympy [F]

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{n}\,dx = \int (e\sec (c+dx))^{6-2n} (ia(\tan (c+dx)-i))^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))**(6-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(6 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.491.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(140) = 280$.

Time = 1.37 (sec) , antiderivative size = 1067, normalized size of antiderivative = 6.84

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \text{Too large to display}$$

input

```
integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output

```
-32*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/2*n)*a^n*e^6*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2
*I*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2
*n)*a^n*e^6*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^n*
e^6*n^2 - 9*a^n*e^6*n + 20*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(4*d*x + n*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1) + 4*c) - 2*(a^n*e^6*n - 5*a^n*e^6)*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(2*d*x + n
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c) - (-I*a^n*e^6*n^2
+ 9*I*a^n*e^6*n - 20*I*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(4*d*x + n*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1) + 4*c) - 2*(I*a^n*e^6*n - 5*I*a^n*e^6)*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^3
 + 12*I*e^(2*n)*n^2 - 47*I*e^(2*n)*n + 60*I*e^(2*n))*2^n*cos(10*d*x + 10*c
) - 5*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2
^n*cos(8*d*x + 8*c) - 10*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*
n - 60*I*e^(2*n))*2^n*cos(6*d*x + 6*c) - 10*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*
n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2^n*cos(4*d*x + 4*c) - 5*(I*e^(2*n)*n
^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2^n*cos(2*d*x + ...
```

3.491.8 Giac [F]

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+6}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 6)*(I*a*tan(d*x + c) + a)^n, x)
```

3.491.9 Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.04

$$\begin{aligned}\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx &= (\cos(6c+6dx) \\ &\quad - \sin(6c+6dx)\ 1\mathrm{i})\left(\frac{e}{\cos(c+dx)}\right)^{6-2n}\left(\frac{\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}\right. \\ &\quad - \frac{(2n-12)\ (\cos(2c+2dx)+\sin(2c+2dx)\ 1\mathrm{i})\ \left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} \\ &\quad + \frac{(\cos(6c+6dx)+\sin(6c+6dx)\ 1\mathrm{i})\ \left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-9n+20)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} \\ &\quad \left.+ \frac{(\cos(4c+4dx)+\sin(4c+4dx)\ 1\mathrm{i})\ \left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-11n+30)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}\right)\end{aligned}$$

input
```
int((e/cos(c + d*x))^(6 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
(cos(6*c + 6*d*x) - sin(6*c + 6*d*x)*1i)*(e/cos(c + d*x))^(6 - 2*n)*((a +
(a*sin(c + d*x)*1i)/cos(c + d*x))^n/(d*(n*47i - n^2*12i + n^3*1i - 60i)) -
 ((2*n - 12)*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(a + (a*sin(c + d*x)
*1i)/cos(c + d*x))^n)/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(6*c +
 6*d*x) + sin(6*c + 6*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n
^2 - 9*n + 20))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(4*c + 4*d*x
) + sin(4*c + 4*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n^2 - 1
1*n + 30))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)))
```

3.491. $\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$

3.492 $\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx$

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3.492.1 Optimal result

Integrand size = 30, antiderivative size = 97

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{\frac{5}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}(-3+2n),\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{5-2n}(1-i\tan(c+d}{5d}$$

output
```
-1/5*I*2^(5/2-n)*hypergeom([5/2, -3/2+n],[7/2],1/2+1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(5-2*n)*(1-I*tan(d*x+c))^(-5/2+n)*(a+I*a*tan(d*x+c))^n/d
```

3.492.2 Mathematica [A] (verified)

Time = 15.72 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{5-n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{5}{2},5-n,\frac{7}{2},-e^{2i(c+dx)}\right)\sec^{-5}}{5d}$$

input
```
Integrate[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/5*I)*2^(5 - n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[5/
2, 5 - n, 7/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5 + n)*(e*Sec[c + d*x]
)^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.492.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(5-2n)}(i\tan(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(5-2n)}(i\tan(c+dx)a+a)^{5/2}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(3-2n)}(i\tan(c}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^3 2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-5)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)}(e\sec(c+dx))^{5-2n}\int (}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia^2 2^{\frac{5}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-5)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-5)+\frac{5}{2}}(e\sec(c+dx))^{5-}}{5d}$$

input
```
Int[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/5*I)*2^(5/2 - n)*a^2*Hypergeometric2F1[5/2, (-3 + 2*n)/2, 7/2, (1 + I
*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(5 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 +
n)*(a - I*a*Tan[c + d*x])^(1/2 - n + (-5 + 2*n)/2)*(a + I*a*Tan[c + d*x])^
(5/2 + (-5 + 2*n)/2))/d
```

3.492.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.492.4 Maple [F]

$$\int (e \sec (dx + c))^{5-2n} (a + ia \tan (dx + c))^{n} \, dx$$

input
```
int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.492.5 Fricas [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n \, dx \\ = \int (e \sec (dx + c))^{-2\,n+5} (i\,a \tan (dx + c) + a)^{n} \, dx$$

input
```
integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 5)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

3.492.6 Sympy [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n \, dx = \int (e \sec (c + dx))^{5-2n} (ia(\tan (c + dx) - i))^{n} \, dx$$

input
```
integrate((e*sec(d*x+c))**(5-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(5 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.492.7 Maxima [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+5}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

3.492.8 Giac [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+5}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5-2n}(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.493 $\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$

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3.493.1 Optimal result

Integrand size = 30, antiderivative size = 98

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{2ia^2(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-2+n}}{d\,(6-5n+n^2)} + \frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)}$$

output
```
2*I*a^2*(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-5*n+6)+I*a
*(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(3-n)
```

3.493.2 Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{e^4\sec^2(c+dx)(e\sec(c+dx))^{-2n}(\cos(2(c+dx))-i\sin(2(c+dx)))(a+ia\tan(c+dx))^n(-i(-4+n)+}{d(-3+n)(-2+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(e^4*Sec[c + d*x]^2*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(a + I*a*Tan[c
 + d*x])^n*((-I)*(-4 + n) + (-2 + n)*Tan[c + d*x]))/(d*(-3 + n)*(-2 + n)*(
e*Sec[c + d*x])^(2*n))
```

3.493.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{4-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{4-2n}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{2a\int (e\sec(c+dx))^{4-2n}(i\tan(c+dx)a+a)^{n-1}dx}{3-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\int (e\sec(c+dx))^{4-2n}(i\tan(c+dx)a+a)^{n-1}dx}{3-n}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

$$\Big\downarrow \text{3974}$$

$$\frac{2ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{4-2n}}{d(2-n)(3-n)}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

input
```
Int[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((2*I)*a^2*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*
(2 - n)*(3 - n)) + (I*a*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^
(-1 + n))/(d*(3 - n))
```

3.493.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.493.4 Maple [F]

$$\int (e\sec(dx+c))^{4-2n}(a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.493.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{((-in+3i)e^{(4i\,dx+4i\,c)}+(-in+4i)e^{(2i\,dx+2i\,c)}+i)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+4}e^{\left(i\,dnx+i\,cn-4i\,dx+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)-\right.}}{2\,(dn^2-5\,dn+6\,d)}$$

input
```
integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
1/2*((-I*n + 3*I)*e^(4*I*d*x + 4*I*c) + (-I*n + 4*I)*e^(2*I*d*x + 2*I*c) +
 I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 4)*e^(I*d*n*x
+ I*c*n - 4*I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) +
 n*log(a/e) - 4*I*c)/(d*n^2 - 5*d*n + 6*d)
```

3.493.6 Sympy [F]

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(c+dx)})^{4-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(4-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(4 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.493.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(88) = 176$.

Time = 1.08 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.07

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{8\Big(\big(\cos{(2\,dx+2\,c)}^2+\sin{(2\,dx+2\,c)}^2+2\,\cos{(2\,dx+2\,c)}+1\big)^{\frac{1}{2}\,n}a^ne^4\cos{(n\arctan{(\sin{(2\,dx+2\,c)}\,,\cos}}{}$$

input
```
integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
8*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2
*n)*a^n*e^4*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + I*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*a^
n*e^4*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (a^n*e^4*n
- 3*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/2*n)*cos(2*d*x + n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
) + 2*c) + (-I*a^n*e^4*n + 3*I*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^2 + 5*I*e^(2*n)*n - 6
*I*e^(2*n))*2^n*cos(6*d*x + 6*c) - 3*(I*e^(2*n)*n^2 - 5*I*e^(2*n)*n + 6*I*
e^(2*n))*2^n*cos(4*d*x + 4*c) - 3*(I*e^(2*n)*n^2 - 5*I*e^(2*n)*n + 6*I*e^(
2*n))*2^n*cos(2*d*x + 2*c) + (e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*s
in(6*d*x + 6*c) + 3*(e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(4*d*x
+ 4*c) + 3*(e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(2*d*x + 2*c) +
(-I*e^(2*n)*n^2 + 5*I*e^(2*n)*n - 6*I*e^(2*n))*2^n)*d)
```

3.493.8 Giac [F]

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(e\sec\left(dx+c\right)\right)^{-2\,n+4}\left(i\,a\tan\left(dx+c\right)+a\right)^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 4)*(I*a*tan(d*x + c) + a)^n, x)
```

3.493.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{4\,e^4\,\left(\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}\right)^n\,\left(4\,\sin\left(2\,c+2\,d\,x\right)+\cos\left(2\,c+2\,d\,x\right)\,4\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,1\mathrm{i}-n\,1\mathrm{i}+\right.}{d\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+\right.}$$

input
```
int((e/cos(c + d*x))^(4 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
(4*e^4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
 + 1))^n*(cos(2*c + 2*d*x)*4i - n*1i + cos(4*c + 4*d*x)*1i + 4*sin(2*c + 2
*d*x) + sin(4*c + 4*d*x) - n*cos(2*c + 2*d*x)*1i - n*sin(2*c + 2*d*x) + 3i
))/(d*(e/cos(c + d*x))^(2*n)*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3)*(
n^2 - 5*n + 6))
```

3.493. $\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$

3.494 $\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$

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3.494.1 Optimal result

Integrand size = 30, antiderivative size = 97

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{3}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1+2n),\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{3-2n}(1-i\tan(c+d}{3d}$$

output
```
-1/3*I*2^(3/2-n)*hypergeom([3/2, -1/2+n],[5/2],1/2+1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(3-2*n)*(1-I*tan(d*x+c))^(-3/2+n)*(a+I*a*tan(d*x+c))^n/d
```

3.494.2 Mathematica [A] (verified)

Time = 14.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{3-n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},3-n,\frac{5}{2},-e^{2i(c+dx)}\right)\sec^{-3}}{3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/3*I)*2^(3 - n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[3/
2, 3 - n, 5/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3 + n)*(e*Sec[c + d*x]
)^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.494.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.44, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(3-2n)}(i\tan(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(3-2n)}(i\tan(c+dx)a+a)^{3/2}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(1-2n)}\sqrt{i\tan(}}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-3)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)}(e\sec(c+dx))^{3-2n}\int (}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-3)+\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-3)+\frac{3}{2}}(e\sec(c+dx))^{3-2}}{3d}$$

input

```
Int[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/3*I)*2^(3/2 - n)*a*Hypergeometric2F1[3/2, (-1 + 2*n)/2, 5/2, (1 + I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^(3 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 + n)
*(a - I*a*Tan[c + d*x])^(1/2 - n + (-3 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(3
/2 + (-3 + 2*n)/2))/d
```

3.494.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.494.4 Maple [F]

$$\int (e\sec (dx+c))^{3-2n} (a+ia\tan (dx+c))^{n} dx$$

input
```
int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.494.5 Fricas [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec (dx+c))^{-2\,n+3} (i\,a\tan (dx+c)+a)^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 3)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

3.494.6 Sympy [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec (c+dx))^{3-2n} (ia(\tan (c+dx)-i))^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))**(3-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(3 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.494.7 Maxima [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+3}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.494.8 Giac [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+3}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.494.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3-2\,n}(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

input
```
int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.495 $\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$

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3.495.1 Optimal result

Integrand size = 30, antiderivative size = 46

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx=\frac{ia(e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^{-1+n}}{d(1-n)}$$

output
```
I*a*(e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(1-n)
```

3.495.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=-\frac{e^2(e\sec(c+dx))^{-2n}(i+\sec(c)\sec(c+dx)\sin(dx)+\tan(c))(a+ia\tan(c+dx))^n}{d(-1+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
-((e^2*(I + Sec[c]*Sec[c + d*x]*Sin[d*x] + Tan[c])*(a + I*a*Tan[c + d*x])^
n)/(d*(-1 + n)*(e*Sec[c + d*x])^(2*n)))
```

3.495.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{2-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{2-2n}dx$$

$$\Big\downarrow \text{3974}$$

$$\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-2n}}{d(1-n)}$$

input
```
Int[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*a*(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n
))
```

3.495.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.495.4 Maple [F]

$$\int (e \sec (dx + c))^{2-2n} (a + ia \tan (dx + c))^{n} \, dx$$

input
```
int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.495.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(40) = 80$.

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.37

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+2}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(i\,dnx+i\,cn-2i\,dx+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)-2i\,c\right)}}{2\,(dn-d)}$$

input
```
integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
1/2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 2)*(-I*e^(2*I*
d*x + 2*I*c) - I)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x + I*c)
/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e) - 2*I*c)/(d*n - d)
```

3.495.6 Sympy [F]

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec (c+dx))^{2-2n} (ia(\tan (c+dx)-i))^{n} \, dx$$

input
```
integrate((e*sec(d*x+c))**(2-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(2 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.495.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(40) = 80$.

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 4.72

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{\left(-i\,a^n e^2 - \frac{2\,a^n e^2 \sin(dx+c)}{\cos(dx+c)+1} + \frac{i\,a^n e^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right) e^{\left(n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)+n\log\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c}{(\cos(dx+c)+}\right.\right.}}{\left(e^{2\,n}(n-1) - \frac{e^{2\,n}(n-1)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

input
```
integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
(-I*a^n*e^2 - 2*a^n*e^2*sin(d*x + c)/(cos(d*x + c) + 1) + I*a^n*e^2*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*e^(n*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1) + n*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1) + n*log(-2*I*sin(d*x + c)/
(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1) - 2*n*log(-s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/((e^(2*n)*(n - 1) - e^(2*n)*(n -
1)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*d)
```

3.495.8 Giac [F]

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec(dx+c))^{-2\,n+2}\,(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.495.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=-\frac{e^2\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1{}\mathrm{i}\right)\,{\left(\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}\right)}^n}{d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{2\,n}\,\left(n-1\right)}$$

input
```
int((e/cos(c + d*x))^(2 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
-(e^2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i)*((a*(cos(2*c + 2*d*x)
+ sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(cos(2*c + 2*d*x
) + 1)*(e/cos(c + d*x))^(2*n)*(n - 1))
```

3.496 $\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$

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3.496.1 Optimal result

Integrand size = 30, antiderivative size = 95

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{\frac{1}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+2n),\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{1-2n}(1-i\tan(c+dx)}{d}$$

output
```
-I*2^(1/2-n)*hypergeom([1/2, 1/2+n],[3/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*x
+c))^(1-2*n)*(1-I*tan(d*x+c))^(-1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

3.496.2 Mathematica [A] (verified)

Time = 10.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.62

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{1-n}e\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1-n}\left(1+e^{2i(c+dx)}\right)^{1-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-e^{2i(c+dx)}\right)\sec^n(c+d}{d}$$

input
```
Integrate[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1 - n)*e*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
)^(1 - n)*(1 + E^((2*I)*(c + d*x)))^(1 - n)*Hypergeometric2F1[1/2, 1 - n,
3/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x])^n)/(d*(e*
Sec[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

3.496.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.45, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}dx$$

$$\Big\downarrow \; 3986$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(1-2n)}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \; 3042$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}\int (a-ia\tan(c+dx))^{\frac{1}{2}(1-2n)}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \; 4006$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-1)}}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx}{d}$$

$$\Big\downarrow \; 80$$

$$\frac{a^2 2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)}(e\sec(c+dx))^{1-2n}\int}{d}$$

$$\Big\downarrow \; 79$$

$$-\frac{ia2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n-1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n-1)+\frac{1}{2}}(e\sec(c+dx))^{1-2}}{d}$$

input
```
Int[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(1/2 - n)*a*Hypergeometric2F1[1/2, (1 + 2*n)/2, 3/2, (1 + I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^(1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a -
I*a*Tan[c + d*x])^(-1/2 - n + (-1 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(1/2 +
(-1 + 2*n)/2))/d
```

3.496.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.496. $\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$

3.496.4 Maple [F]

$$\int \left(e\sec\left(dx+c\right)\right)^{1-2n}\left(a+ia\tan\left(dx+c\right)\right)^{n}dx$$

input
```
int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.496.5 Fricas [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^{n}\,dx = \int \left(e\sec\left(dx+c\right)\right)^{-2\,n+1}\left(i\,a\tan\left(dx+c\right)+a\right)^{n}dx$$

input
```
integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 1)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

3.496.6 Sympy [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^{n}\,dx = \int \left(e\sec\left(c+dx\right)\right)^{1-2n}\left(ia(\tan\left(c+dx\right)-i)\right)^{n}\,dx$$

input
```
integrate((e*sec(d*x+c))**(1-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(1 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.496.7 Maxima [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(e\sec\left(dx+c\right)\right)^{-2\,n+1}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.496.8 Giac [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(e\sec\left(dx+c\right)\right)^{-2\,n+1}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{1-2\,n}\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^n\,dx$$

input
```
int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.497 $\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$

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3.497.1 Optimal result

Integrand size = 28, antiderivative size = 65

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n}{2dn}$$

output
```
-1/2*I*hypergeom([1, -n],[1-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/
d/n/((e*sec(d*x+c))^(2*n))
```

3.497.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(65) = 130.

Time = 2.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,1+e^{2i(c+dx)}\right)\sec^n(c+}{d(1+n)}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

output
```
(I*2^(-1 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1,
 1 + n, 2 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*
x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(1 + n)*(e*Sec[c +
 d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

3.497.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3042, 3973, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}dx$$

$$\Big\downarrow \text{3973}$$

$$(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\int (a-ia\tan(c+dx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\int (a-ia\tan(c+dx))^{-n}dx$$

$$\Big\downarrow \text{3962}$$

$$\frac{ia(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\int \frac{(a-ia\tan(c+dx))^{-n-1}}{i\tan(c+dx)a+a}d(-ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\,\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{a-ia\tan(c+dx)}{2a}\right)}{2dn}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

output
```
((-1/2*I)*Hypergeometric2F1[1, -n, 1 - n, (a - I*a*Tan[c + d*x])/(2*a)]*(a
 + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^(2*n))
```

3.497.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 3973
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
   Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
&& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

3.497.4 Maple [F]

$$\int (a+ia\tan(dx+c))^{n} (e\sec(dx+c))^{-2n} \, dx$$

input
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

output
```
int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

3.497.5 Fricas [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^n}{(e\sec{(dx+c)})^{2\,n}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="fricas
")
```

output
```
integral(e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*
c) + 1)) + n*log(a/e))/(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(2*
n), x)
```

3.497.6 Sympy [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(c+dx)})^{-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**(2*n)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(2*n), x)
```

3.497.7 Maxima [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^n}{(e\sec{(dx+c)})^{2\,n}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="maxima
")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

3.497.8 Giac [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^n}{(e\sec\left(dx+c\right))^{2\,n}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n), x)
```

3.498 $\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$

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3.498.1 Optimal result

Integrand size = 30, antiderivative size = 95

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-\frac{1}{2}-n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(3+2n),\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-1-2n}(1-i\tan(c+d}{d}$$

output
```
I*2^(-1/2-n)*hypergeom([-1/2, 3/2+n],[1/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(-1-2*n)*(1-I*tan(d*x+c))^(1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

3.498.2 Mathematica [A] (verified)

Time = 14.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1-n}\left(1+e^{2i(c+dx)}\right)^{-1-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-n,\frac{1}{2},-e^{2i(c+dx)}\right)\sec^{1+n}}{d}$$

input
```
Integrate[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(-1 - n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-
1 - n)*(1 + E^((2*I)*(c + d*x)))^(-1 - n)*Hypergeometric2F1[-1/2, -1 - n,
1/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1 + n)*(e*Sec[c + d*x])^(-1 - 2*n
)*(a + I*a*Tan[c + d*x])^n)/(d*(Cos[d*x] + I*Sin[d*x])^n)
```

3.498.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-1)}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-1)}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-3)}}{(i\tan(c+dx)a+a)^{3/2}}d\tan(c+d}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)}(e\sec(c+dx))^{-2n-1}}{d}$$

$$\Big\downarrow \text{79}$$

$$\frac{i2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+1)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+1)-\frac{1}{2}}(e\sec(c+dx))^{-2n-}}{d}$$

input
```
Int[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

3.498. $\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$

output
```
(I*2^(-1/2 - n)*Hypergeometric2F1[-1/2, (3 + 2*n)/2, 1/2, (1 + I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(-1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a - I*
a*Tan[c + d*x])^(-1/2 - n + (1 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(-1/2 + (1
 + 2*n)/2))/d
```

3.498.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.498.4 Maple [F]

$$\int (e\sec(dx+c))^{-1-2n}(a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.498.5 Fricas [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n-1}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="frica
s")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 1)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

3.498.6 Sympy [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(c+dx))^{-2n-1}(ia(\tan(c+dx)-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(-1-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-2*n - 1)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.498.7 Maxima [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-1}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxim
a")
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.498.8 Giac [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-1}(ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac"
)
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx=\int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+1}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1), x)
```

3.499 $\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$

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3.499.1 Optimal result

Integrand size = 30, antiderivative size = 74

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,\text{Hypergeometric2F1}\left(2,-1-n,-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2(1+n)}(a+ia\tan(c+dx))^{1}}{4ad(1+n)}$$

output
```
-1/4*I*hypergeom([2, -1-n],[-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(
1+n)/a/d/(1+n)/((e*sec(d*x+c))^(2+2*n))
```

3.499.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.

Time = 14.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-3-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(2,3+n,4+n,1+e^{2i(c+dx)}\right)\sec^n(c-}{de^2(3+n)}$$

input
```
Integrate[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-3 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2
F1[2, 3 + n, 4 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c
 + d*x])^n)/(d*e^2*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(3 + n)*(
e*Sec[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

3.499.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-2}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \frac{(a-ia\tan(c+dx))^{-n-1}}{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \frac{(a-ia\tan(c+dx))^{-n-1}}{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{4005}$$

$$\frac{(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \cos^2(c+dx)(a-ia\tan(c+dx))^{-n}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \frac{(a-ia\tan(c+dx))^{-n}}{\sec(c+dx)^2}dx}{a^2}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(a-ia\tan(c+dx))^{n+1}(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\int \frac{(a-ia\tan(c+dx))^{-n-2}}{(i\tan(c+dx)a+a)^2}d(-ia\tan(c+dx))}{d}$$

$$\Big\downarrow 78$$

$$-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\,\text{Hypergeometric2F1}\left(2,-n-1,-n,\frac{a-ia\tan(c+dx)}{2a}\right)}{4ad(n+1)}$$

input
```
Int[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-1/4*I)*Hypergeometric2F1[2, -1 - n, -n, (a - I*a*Tan[c + d*x])/(2*a)]*(
a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n)*(e*Sec[c + d*x])^(2*(1 + n)))
```

3.499.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

3.499. $\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

3.499.4 Maple [F]

$$\int (e\sec(dx+c))^{-2-2n} (a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.499.5 Fricas [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n-2} (ia\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="frica
s")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 2)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

3.499.6 Sympy [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{-2n-2}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(-2-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-2*n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.499.7 Maxima [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n-2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxim
a")
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.499.8 Giac [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n-2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac"
)
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2), x)
```

3.500 $\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$

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3.500.1 Optimal result

Integrand size = 30, antiderivative size = 97

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-\frac{3}{2}-n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(5+2n),-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-3-2n}(1-i\tan(c+}{3d}$$

output
```
1/3*I*2^(-3/2-n)*hypergeom([-3/2, 5/2+n],[-1/2],1/2+1/2*I*tan(d*x+c))*(e*s
ec(d*x+c))^(-3-2*n)*(1-I*tan(d*x+c))^(3/2+n)*(a+I*a*tan(d*x+c))^n/d
```

3.500.2 Mathematica [A] (verified)

Time = 14.99 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-3-n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-3-n,-\frac{1}{2},-e^{2i(c+dx)}\right)}{3d}$$

input
```
Integrate[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((I/3)*2^(-3 - n)*(E^(I*d*x))^n*Hypergeometric2F1[-3/2, -3 - n, -1/2, -E^(
(2*I)*(c + d*x))]*Sec[c + d*x]^(3 + n)*(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*
a*Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.500.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}dx$$

$$\Big\downarrow \text{ 3986}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-3)}}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(a-ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-3)}}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}\int \frac{(a-ia\tan(c+dx))^{\frac{1}{2}(-2n-5)}}{(i\tan(c+dx)a+a)^{5/2}}d\tan(c+d}{d}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{2^{-n-\frac{5}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+3)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)}(e\sec(c+dx))^{-2n-3}\int}{d}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{i2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a-ia\tan(c+dx))^{-n+\frac{1}{2}(2n+3)-\frac{1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(2n+3)-\frac{3}{2}}(e\sec(c+dx))^{-2n-}}{3ad}$$

input
```
Int[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

3.500. $\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$

output
```
((I/3)*2^(-3/2 - n)*Hypergeometric2F1[-3/2, (5 + 2*n)/2, -1/2, (1 + I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^(-3 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a
 - I*a*Tan[c + d*x])^(-1/2 - n + (3 + 2*n)/2)*(a + I*a*Tan[c + d*x])^(-3/2
 + (3 + 2*n)/2))/(a*d)
```

3.500.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.500.4 Maple [F]

$$\int (e\sec(dx+c))^{-3-2n}(a+ia\tan(dx+c))^n\,dx$$

input
```
int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

3.500.5 Fricas [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n-3}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="frica
s")
```

output
```
integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 3)*e^(I*d
*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*lo
g(a/e)), x)
```

3.500.6 Sympy [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(c+dx))^{-2n-3}(ia(\tan(c+dx)-i))^n\,dx$$

input
```
integrate((e*sec(d*x+c))**(-3-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*sec(c + d*x))**(-2*n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

3.500.7 Maxima [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-3}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxim
a")
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.500.8 Giac [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-3}(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac"
)
```

output
```
integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+3}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3), x)
```

3.501 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$

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3.501.8 Giac [F] 3493
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3.501.1 Optimal result

Integrand size = 32, antiderivative size = 66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(3,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{8a^2fn}$$

output
```
1/8*I*hypergeom([3, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a^
2/f/n/((a+I*a*tan(f*x+e))^n)
```

3.501.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 13.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = -\frac{i2^{-3+n}e^{2ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)^{3}\text{Hypergeometric2F1}\left(3,3-n,4-n,1+e^{2i(e+fx)}\right)\sec}{f(-3+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

output
```
((-I)*2^(-3 + n)*E^((2*I)*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n
*(1 + E^((2*I)*(e + f*x)))^3*Hypergeometric2F1[3, 3 - n, 4 - n, 1 + E^((2*
I)*(e + f*x))]*Sec[e + f*x]^(2 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*S
in[f*x])^(2 + n)*(a + I*a*Tan[e + f*x])^(-2 - n))/((E^(I*f*x))^n*f*(-3 + n
))
```

3.501.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{-n-2}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{-n-2}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{(i\tan(e+fx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{(i\tan(e+fx)a+a)^2}dx$$

$$\Big\downarrow \text{4005}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \cos^4(e+fx)(a-ia\tan(e+fx))^{n+2}dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n+2}}{\sec(e+fx)^4}dx}{a^4}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n-1}}{(i\tan(e+fx)a+a)^3}d(-ia\tan(e+fx))}{f}$$

$$\Big\downarrow 78$$

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(3,n,n+1,\frac{a-ia\tan(e+fx)}{2a}\right)}{8a^2fn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

output
```
((I/8)*Hypergeometric2F1[3, n, 1 + n, (a - I*a*Tan[e + f*x])/(2*a)]*(d*Sec
[e + f*x])^(2*n))/(a^2*f*n*(a + I*a*Tan[e + f*x])^n)
```

3.501.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

3.501.4 Maple [F]

$$\int (d\sec (fx+e))^{2n} (a+ia\tan (fx+e))^{-n-2}\,dx$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-2),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-2),x)
```

3.501.5 Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$$
$$=\int (d\sec (fx+e))^{2\,n} (i\,a\tan (fx+e)+a)^{-n-2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="fri
cas")
```

output
```
integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n +
 (-I*f*n - 2*I*f)*x - (n + 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e)
 + 1)) - (n + 2)*log(a/d) - 2*I*e), x)
```

3.501.6 Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$$
$$=\int (d\sec (e+fx))^{2n} (ia(\tan (e+fx)-i))^{-n-2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-2-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(-n - 2), x)
```

3.501.7 Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.501.8 Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int (d\sec(fx+e))^{2n}(i\,a\tan(fx+e)+a)^{-n-2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="gia
c")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 2), x)
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{n+2}}\,dx$$

input
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2),x)
```

output
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2), x)
```

3.502 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$

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3.502.5 Fricas [F] 3497
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3.502.7 Maxima [F(-2)] 3498
3.502.8 Giac [F] 3498
3.502.9 Mupad [F(-1)] 3498

3.502.1 Optimal result

Integrand size = 32, antiderivative size = 66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(2,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{4afn}$$

output
```
1/4*I*hypergeom([2, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a/
f/n/((a+I*a*tan(f*x+e))^n)
```

3.502.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 12.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \frac{i2^{-2+n}e^{ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)^{2}\text{Hypergeometric2F1}\left(2,2-n,3-n,1+e^{2i(e+fx)}\right)\sec^{1-n}}{f(-2+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

output
```
(I*2^(-2 + n)*E^(I*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E
^((2*I)*(e + f*x)))^2*Hypergeometric2F1[2, 2 - n, 3 - n, 1 + E^((2*I)*(e +
 f*x))]*Sec[e + f*x]^(1 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x]
)^(1 + n)*(a + I*a*Tan[e + f*x])^(-1 - n))/((E^(I*f*x))^n*f*(-2 + n))
```

3.502.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3042, 3986, 3042, 4005, 3042, 3968, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{-n-1}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{-n-1}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{i\tan(e+fx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^n}{i\tan(e+fx)a+a}dx$$

$$\Big\downarrow \text{4005}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \cos^2(e+fx)(a-ia\tan(e+fx))^{n+1}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n+1}}{\sec(e+fx)^2}dx}{a^2}$$

$$\Big\downarrow \text{3968}$$

$$\frac{ia(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n-1}}{(i\tan(e+fx)a+a)^2}d(-ia\tan(e+fx))}{f}$$

$$\Big\downarrow 78$$

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(2,n,n+1,\frac{a-ia\tan(e+fx)}{2a}\right)}{4afn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

output
```
((I/4)*Hypergeometric2F1[2, n, 1 + n, (a - I*a*Tan[e + f*x])/(2*a)]*(d*Sec
[e + f*x])^(2*n))/(a*f*n*(a + I*a*Tan[e + f*x])^n)
```

3.502.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3968
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(a^(m - 2)*b*f)   Subst[Int[(a - x)^(m/2 - 1)*(a + x
)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
 EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 4005
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Sec[e + f*x]^(2*m)*(c +
 d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[
m, 0] || GtQ[m, n]))
```

3.502.4 Maple [F]

$$\int (d\sec(fx+e))^{2n} (a+ia\tan(fx+e))^{-1-n}\,dx$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

3.502.5 Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \int (d\sec(fx+e))^{2n} (ia\tan(fx+e)+a)^{-n-1}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="fri
cas")
```

output
```
integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n +
 (-I*f*n - I*f)*x - (n + 1)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) +
 1)) - (n + 1)*log(a/d) - I*e), x)
```

3.502.6 Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \int (d\sec(e+fx))^{2n} (ia(\tan(e+fx)-i))^{-n-1}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-1-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(-n - 1), x)
```

3.502.7 Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.502.8 Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$$
$$= \int (d\sec(fx+e))^{2n}(ia\tan(fx+e)+a)^{-n-1}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="gia
c")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 1), x)
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan(e+f\,x)\;1\mathrm{i})^{n+1}}\,dx$$

input
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1),x)
```

output
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1), x)
```

3.503 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx$

3.503.1 Optimal result 3499
3.503.2 Mathematica [B] (verified) 3499
3.503.3 Rubi [A] (verified) 3500
3.503.4 Maple [F] 3501
3.503.5 Fricas [F] 3502
3.503.6 Sympy [F] 3502
3.503.7 Maxima [F] 3502
3.503.8 Giac [F] 3503
3.503.9 Mupad [F(-1)] 3503

3.503.1 Optimal result

Integrand size = 30, antiderivative size = 63

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{2fn}$$

output
```
1/2*I*hypergeom([1, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/f/
n/((a+I*a*tan(f*x+e))^n)
```

3.503.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 150 vs. 2(63) = 126.

Time = 2.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.38

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = -\frac{i2^{-1+n}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)\text{Hypergeometric2F1}\left(1,1-n,2-n,1+e^{2i(e+fx)}\right)\sec^{-n}(e}{f(-1+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

output
```
((-I)*2^(-1 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^((2*
I)*(e + f*x)))*Hypergeometric2F1[1, 1 - n, 2 - n, 1 + E^((2*I)*(e + f*x))]
*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^n)/((E^(I*f*x))^n*f*(-1 +
n)*Sec[e + f*x]^n*(a + I*a*Tan[e + f*x])^n)
```

3.503.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3973, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{3973}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int (a-ia\tan(e+fx))^{n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int (a-ia\tan(e+fx))^{n}dx$$

$$\Big\downarrow \text{3962}$$

$$\frac{ia(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\int \frac{(a-ia\tan(e+fx))^{n-1}}{i\tan(e+fx)a+a}d(-ia\tan(e+fx))}{f}$$

$$\Big\downarrow \text{78}$$

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{a-ia\tan(e+fx)}{2a}\right)}{2fn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

output
```
((I/2)*Hypergeometric2F1[1, n, 1 + n, (a - I*a*Tan[e + f*x])/(2*a)]*(d*Sec
[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

3.503.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 3973
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
   Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
&& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

3.503.4 Maple [F]

$$\int (a(i\tan(fx+e)+1))^{-n} (d\sec(fx+e))^{2n}\, dx$$

input
```
int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

output
```
int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

3.503.5 Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int \frac{(d\sec(fx+e))^{2n}}{(ia\tan(fx+e)+a)^{n}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="fricas
")
```

output
```
integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*f*n*x
 - I*e*n - n*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - n*log(a/
d)), x)
```

3.503.6 Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int (d\sec(e+fx))^{2n}\,(ia(\tan(e+fx)-i))^{-n}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)/((a+I*a*tan(f*x+e))**n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)/(I*a*(tan(e + f*x) - I))**n, x)
```

3.503.7 Maxima [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int \frac{(d\sec(fx+e))^{2n}}{(ia\tan(fx+e)+a)^{n}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="maxima
")
```

output
```
integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

3.503.8 Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int \frac{(d\sec(fx+e))^{2n}}{(i\,a\tan(fx+e)+a)^{n}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{n}}\,dx$$

input
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n,x)
```

output
```
int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n, x)
```

3.504 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$

3.504.1 Optimal result 3504
3.504.2 Mathematica [A] (verified) 3504
3.504.3 Rubi [A] (verified) 3505
3.504.4 Maple [C] (warning: unable to verify) 3506
3.504.5 Fricas [B] (verification not implemented) 3507
3.504.6 Sympy [F] 3507
3.504.7 Maxima [B] (verification not implemented) 3508
3.504.8 Giac [F] 3508
3.504.9 Mupad [B] (verification not implemented) 3509

3.504.1 Optimal result

Integrand size = 32, antiderivative size = 40

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

output
```
I*a*(d*sec(f*x+e))^(2*n)/f/n/((a+I*a*tan(f*x+e))^n)
```

3.504.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

3.504.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \ 3974$$

$$\frac{ia(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

3.504.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

3.504.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.25 (sec) , antiderivative size = 1261, normalized size of antiderivative = 31.52

method	result	size
risch	Expression too large to display	1261

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x,method=_RETURNVERBOSE)
```

output
```
-I/((exp(2*I*(f*x+e))+1)^n)/f/n*2^n*(d^n)^2*a/(a^n)*exp(-1/2*I*Pi*(-n*csgn
(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/(exp(2*I*(f*x+e))+1)*ex
p(2*I*(f*x+e)))*csgn(I*a)+2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*
csgn(I*d)*csgn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))+2*n*csgn(I/(exp(2*
I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+
e))+1))-n*csgn(I*exp(I*(f*x+e)))^2*csgn(I*exp(2*I*(f*x+e)))+2*n*csgn(I*exp
(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+2*n*csgn(I*d*exp(I*(f*x+e))/(exp(2*I*(
f*x+e))+1))^3-n*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3-n*csgn(I/(
exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1)*
exp(2*I*(f*x+e)))+n*csgn(I*exp(2*I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1)*e
xp(2*I*(f*x+e)))^2+csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/
(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a)+csgn(I/(exp(2*I*(f*x+e))+
1)*exp(2*I*(f*x+e)))^3-2*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+
e))/(exp(2*I*(f*x+e))+1))^2+csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f
*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))-csgn(I*exp(2*I*(f*x+
e)))*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2-csgn(I*a/(exp(2*I*(f*
x+e))+1)*exp(2*I*(f*x+e)))^2*csgn(I*a)+n*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp
(2*I*(f*x+e)))^2*csgn(I*a)+n*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))
*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2+csgn(I*exp(2*I*(f*x+e))
)^3+csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3+n*csgn(I/(exp(2*I...
```

3.504.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$=\frac{\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2\,n}\left(i\,e^{(2i\,fx+2i\,e)}+i\right)e^{\left(-i\,en+(-i\,fn+i\,f)x-2i\,fx-(n-1)\log\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)-(n-1)\log\left(\frac{a}{d}\right)-i\,e\right)}}{2\,fn}$$

input

```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="fric
as")
```

output

```
1/2*(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*(I*e^(2*I*f*x +
2*I*e) + I)*e^(-I*e*n + (-I*f*n + I*f)*x - 2*I*f*x - (n - 1)*log(2*d*e^(I*
f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - (n - 1)*log(a/d) - I*e)/(f*n)
```

3.504.6 Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$=\int (d\sec(e+fx))^{2n}\,(ia(\tan(e+fx)-i))^{1-n}\,dx$$

input

```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(1-n),x)
```

output

```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(1 - n), x)
```

3.504.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(36) = 72$.

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.42

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$= \frac{i\,a^{-n+1}d^{2\,n}e^{\left(-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)-n\log\left(-\frac{2i\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)+2\,n\log\left(-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)\right.}}{fn}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="maxi
ma")
```

output
```
I*a^(-n + 1)*d^(2*n)*e^(-n*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1) - n*log(-2*I*sin(f*x + e)/(cos(f*x
+ e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1) + 2*n*log(-sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 1))/(f*n)
```

3.504.8 Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$= \int (d\sec(fx+e))^{2\,n}(i\,a\tan(fx+e)+a)^{-n+1}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 1), x)
```

3.504.9 Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx = \frac{a\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}\,1\mathrm{i}}{f\,n\,\left(\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{2\,\cos(e+f\,x)^2}\right)^n}$$

input
```
int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(1 - n),x)
```

output
```
(a*(d/cos(e + f*x))^(2*n)*1i)/(f*n*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x
)*1i + 1))/(2*cos(e + f*x)^2))^n)
```

3.505 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$

3.505.1 Optimal result 3510
3.505.2 Mathematica [A] (verified) 3510
3.505.3 Rubi [A] (verified) 3511
3.505.4 Maple [F] 3512
3.505.5 Fricas [A] (verification not implemented) 3512
3.505.6 Sympy [F] 3513
3.505.7 Maxima [B] (verification not implemented) 3513
3.505.8 Giac [F] 3514
3.505.9 Mupad [B] (verification not implemented) 3514

3.505.1 Optimal result

Integrand size = 32, antiderivative size = 92

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)} + \frac{2ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn(1+n)}$$

output
```
I*a*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(1+n)+2*I*a^2*(d*sec(f
*x+e))^(2*n)/f/n/(1+n)/((a+I*a*tan(f*x+e))^n)
```

3.505.2 Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = -\frac{a^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}(-i(2+n)+n\tan(e+fx))}{fn(1+n)}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

output
```
-((a^2*(d*Sec[e + f*x])^(2*n)*((-I)*(2 + n) + n*Tan[e + f*x]))/(f*n*(1 + n
)*(a + I*a*Tan[e + f*x])^n))
```

3.505.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \; 3975$$

$$\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

$$\Big\downarrow \; 3974$$

$$\frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 + n)) +
((2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x])^n)
```

3.505.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.505.4 Maple [F]

$$\int (d\sec(fx+e))^{2n}(a+ia\tan(fx+e))^{2-n}\,dx$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

3.505.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.51

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$

$$= \frac{\left((i\,n+i)e^{(4i\,fx+4i\,e)}+(i\,n+2i)e^{(2i\,fx+2i\,e)}+i\right)\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2\,n}e^{\left(-i\,en+(-i\,fn+2i\,f)x-4i\,fx-(n-2)\log\left(\frac{2\,de^{(i\,fx+i}}{e^{(2i\,fx+2i\,e)}}\right.\right.}}{2\,(fn^2+fn)}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="fric
as")
```

output
```
1/2*((I*n + I)*e^(4*I*f*x + 4*I*e) + (I*n + 2*I)*e^(2*I*f*x + 2*I*e) + I)*
(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n + (-I*f*n
+ 2*I*f)*x - 4*I*f*x - (n - 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e
) + 1)) - (n - 2)*log(a/d) - 2*I*e)/(f*n^2 + f*n)
```

3.505.6 Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$=\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{2-n}\,dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(2-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(2 - n), x)
```

3.505.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(84) = 168$.

Time = 0.44 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.30

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$=\frac{2^{n+1}a^2d^{2\,n}\cos{(n\arctan{(\sin{(2\,fx+2\,e)},\cos{(2\,fx+2\,e)}+1)})}-i\cdot 2^{n+1}a^2d^{2\,n}\sin{(n\arctan{(\sin{(2\,fx+2}}}{(-i\,a^nn^2-i\,a^nn+(-}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="maxi
ma")
```

output
```
(2^(n + 1)*a^2*d^(2*n)*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) - I*2^(n + 1)*a^2*d^(2*n)*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) + 2*(a^2*d^(2*n)*n + a^2*d^(2*n))*2^n*cos(-2*f*x + n*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + 2*(-I*a^2*d^(2*n)*n - I*a^2
*d^(2*n))*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1) - 2*e))/((-I*a^n*n^2 - I*a^n*n + (-I*a^n*n^2 - I*a^n*n)*cos(2*f*x + 2*e
) + (a^n*n^2 + a^n*n)*sin(2*f*x + 2*e))*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*f)
```

3.505.8 Giac [F]

$$\begin{aligned}\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx \\ = \int (d\sec(fx+e))^{2n}\,(ia\tan(fx+e)+a)^{-n+2}\,dx\end{aligned}$$

input

```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="giac
")
```

output

```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 2), x)
```

3.505.9 Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.83

$$\begin{aligned}&\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx \\ &= -\mathrm{e}^{-e\,4\mathrm{i}-f\,x\,4\mathrm{i}}\left(\frac{d}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}\right)^{2\,n}\left(\frac{\left(a-\frac{a\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}\right)^{2-n}}{2\,f\,n\,\left(n\,1\mathrm{i}+1\mathrm{i}\right)}\right. \\ &\quad+\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\left(a-\frac{a\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}\right)^{2-n}\,\left(n+2\right)}{2\,f\,n\,\left(n\,1\mathrm{i}+1\mathrm{i}\right)} \\ &\quad\left.+\frac{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\left(a-\frac{a\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}\right)^{2-n}\,\left(n+1\right)}{2\,f\,n\,\left(n\,1\mathrm{i}+1\mathrm{i}\right)}\right)\end{aligned}$$

input

```
int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(2 - n),x)
```

output

```
-exp(- e*4i - f*x*4i)*(d/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^
(2*n)*((a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^
(2 - n)/(2*f*n*(n*1i + 1i)) + (exp(e*2i + f*x*2i)*(a - (a*(exp(e*2i + f*x*
2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(2 - n)*(n + 2))/(2*f*n*(n*1i +
 1i)) + (exp(e*4i + f*x*4i)*(a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(
e*2i + f*x*2i) + 1))^(2 - n)*(n + 1))/(2*f*n*(n*1i + 1i)))
```

3.506 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$

3.506.1 Optimal result 3515
3.506.2 Mathematica [A] (verified) 3515
3.506.3 Rubi [A] (verified) 3516
3.506.4 Maple [F] 3518
3.506.5 Fricas [A] (verification not implemented) 3518
3.506.6 Sympy [F] 3518
3.506.7 Maxima [F(-2)] 3519
3.506.8 Giac [F] 3519
3.506.9 Mupad [B] (verification not implemented) 3520

3.506.1 Optimal result

Integrand size = 32, antiderivative size = 148

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f\left(2+3n+n^2\right)} + \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)} + \frac{8ia^3(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn\left(2+3n+n^2\right)}$$

output
```
4*I*a^2*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(n^2+3*n+2)+I*a*(d
*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n)/f/(2+n)+8*I*a^3*(d*sec(f*x+e))
^(2*n)/f/n/(n^2+3*n+2)/((a+I*a*tan(f*x+e))^n)
```

3.506.2 Mathematica [A] (verified)

Time = 3.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \frac{ia^3\sec^2(e+fx)(d\sec(e+fx))^{2n}(\cos(3fx)+i\sin(3fx))\left(2(2+n)+\left(4+3n+n^2\right)\cos(2(e+fx))+in(\right.}{fn(1+n)(2+n)(\cos(fx)+i\sin(fx))^3}$$

input
```
Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

output
```
(I*a^3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(2*n)*(Cos[3*f*x] + I*Sin[3*f*x])*(
2*(2 + n) + (4 + 3*n + n^2)*Cos[2*(e + f*x)] + I*n*(3 + n)*Sin[2*(e + f*x)
]))/(f*n*(1 + n)*(2 + n)*(Cos[f*x] + I*Sin[f*x])^3*(a + I*a*Tan[e + f*x])^
n)
```

3.506.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3975, 3042, 3975, 3042, 3974}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{3-n}(d\sec(e+fx))^{2n}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{3-n}(d\sec(e+fx))^{2n}dx$$

$$\Big\downarrow \text{3975}$$

$$\frac{4a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{2-n}dx}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{2-n}dx}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

$$\Big\downarrow \text{3975}$$

$$\frac{4a\left(\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}\right)}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4a\left(\frac{2a\int (d\sec(e+fx))^{2n}(i\tan(e+fx)a+a)^{1-n}dx}{n+1}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}\right)}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

$$\Big\downarrow \ 3974$$

$$\frac{4a\left(\frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}\right)}{n+2}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}$$

input
```
Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

output
```
(I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n))/(f*(2 + n)) +
(4*a*((I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 +
n)) + ((2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x
])^n)))/(2 + n)
```

3.506.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3974
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

rule 3975
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Inte
gerQ[n]
```

3.506.4 Maple [F]

$$\int (d\sec (fx+e))^{2n} (a+ia\tan (fx+e))^{3-n}\, dx$$

input
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

output
```
int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

3.506.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.16

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\, dx$$

$$= \frac{\left((i\,n^2+3i\,n+2i)e^{(6i\,fx+6i\,e)}+(i\,n^2+5i\,n+6i)e^{(4i\,fx+4i\,e)}-2\,(-i\,n-3i)e^{(2i\,fx+2i\,e)}+2i\right)\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2}}{2\,(fn^3+3\,fn^2+2\,fn)}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="fric
as")
```

output
```
1/2*((I*n^2 + 3*I*n + 2*I)*e^(6*I*f*x + 6*I*e) + (I*n^2 + 5*I*n + 6*I)*e^(
4*I*f*x + 4*I*e) - 2*(-I*n - 3*I)*e^(2*I*f*x + 2*I*e) + 2*I)*(2*d*e^(I*f*x
 + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n + (-I*f*n + 3*I*f)*x -
6*I*f*x - (n - 3)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - (n
- 3)*log(a/d) - 3*I*e)/(f*n^3 + 3*f*n^2 + 2*f*n)
```

3.506.6 Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\, dx$$
$$= \int (d\sec (e+fx))^{2n} (ia(\tan (e+fx)-i))^{3-n}\, dx$$

input
```
integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(3-n),x)
```

output
```
Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(3 - n), x)
```

3.506.7 Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.506.8 Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$=\int (d\sec(fx+e))^{2n}(i\,a\tan(fx+e)+a)^{-n+3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="giac
")
```

output
```
integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 3), x)
```

3.506.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.17

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$= -(\cos(6\,e+6\,f\,x)-\sin(6\,e+6\,f\,x)\ 1\mathrm{i})\ \left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}\left(\frac{\left(a+\frac{a\,\sin(e+f\,x)\,1\mathrm{i}}{\cos(e+f\,x)}\right)^{3-n}}{f\,n\ (n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}\right.$$
$$+\frac{(\cos(4\,e+4\,f\,x)+\sin(4\,e+4\,f\,x)\ 1\mathrm{i})\ \left(a+\frac{a\,\sin(e+f\,x)\,1\mathrm{i}}{\cos(e+f\,x)}\right)^{3-n}(n^2+5\,n+6)}{2\,f\,n\ (n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}$$
$$+\frac{(\cos(6\,e+6\,f\,x)+\sin(6\,e+6\,f\,x)\ 1\mathrm{i})\ \left(a+\frac{a\,\sin(e+f\,x)\,1\mathrm{i}}{\cos(e+f\,x)}\right)^{3-n}(n^2+3\,n+2)}{2\,f\,n\ (n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}$$
$$\left.+\frac{(2\,n+6)\ (\cos(2\,e+2\,f\,x)+\sin(2\,e+2\,f\,x)\ 1\mathrm{i})\ \left(a+\frac{a\,\sin(e+f\,x)\,1\mathrm{i}}{\cos(e+f\,x)}\right)^{3-n}}{2\,f\,n\ (n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}\right)$$

input
```
int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(3 - n),x)
```

output
```
-(cos(6*e + 6*f*x) - sin(6*e + 6*f*x)*1i)*(d/cos(e + f*x))^(2*n)*((a + (a*
sin(e + f*x)*1i)/cos(e + f*x))^(3 - n)/(f*n*(n*3i + n^2*1i + 2i)) + ((cos(
4*e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))
^(3 - n)*(5*n + n^2 + 6))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((cos(6*e + 6*f*x
) + sin(6*e + 6*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n)*(3
*n + n^2 + 2))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((2*n + 6)*(cos(2*e + 2*f*x)
 + sin(2*e + 2*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n))/(2
*f*n*(n*3i + n^2*1i + 2i)))
```

3.507 $\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$

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3.507.1 Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

output
```
1/6*b*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^
5/d
```

3.507.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^6(c+dx)}{6d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
 + d*x]^5/5))/d
```

3.507.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^6(c+dx)dx+\frac{b\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^6dx+\frac{b\sec^6(c+dx)}{6d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\sec^6(c+dx)}{6d}-\frac{a\int\left(\tan^4(c+dx)+2\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sec^6(c+dx)}{6d}-\frac{a\left(-\frac{1}{5}\tan^5(c+dx)-\frac{2}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^6)/(6*d) - (a*(-Tan[c + d*x] - (2*Tan[c + d*x]^3)/3 - Tan[
c + d*x]^5/5))/d
```

3.507.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.507.4 Maple [A] (verified)

Time = 14.75 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{\frac{b\left(\tan^{6}(dx+c)\right)}{6}+\frac{a\left(\tan^{5}(dx+c)\right)}{5}+\frac{\left(\tan^{4}(dx+c)\right)b}{2}+\frac{2a\left(\tan^{3}(dx+c)\right)}{3}+\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	69
default	$\frac{\frac{b\left(\tan^{6}(dx+c)\right)}{6}+\frac{a\left(\tan^{5}(dx+c)\right)}{5}+\frac{\left(\tan^{4}(dx+c)\right)b}{2}+\frac{2a\left(\tan^{3}(dx+c)\right)}{3}+\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	69
risch	$\frac{\frac{32ia\,e^{6i(dx+c)}}{3}+\frac{32b\,e^{6i(dx+c)}}{3}+16ia\,e^{4i(dx+c)}+\frac{32ia\,e^{2i(dx+c)}}{5}+\frac{16ia}{15}}{d\left(e^{2i(dx+c)}+1\right)^{6}}$	75

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/6*b*tan(d*x+c)^6+1/5*a*tan(d*x+c)^5+1/2*tan(d*x+c)^4*b+2/3*a*tan(d*
x+c)^3+1/2*b*tan(d*x+c)^2+a*tan(d*x+c))
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\left(8\,a\cos\left(dx+c\right)^5+4\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^6}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d
*x + c) + 5*b)/(d*cos(d*x + c)^6)
```

3.507.6 Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^5\left(c+dx\right)}{5}+\frac{2\tan^3\left(c+dx\right)}{3}+\tan\left(c+dx\right)\right)+\frac{b\sec^6\left(c+dx\right)}{6}}{d} & \text{for } d\neq 0 \\ x(a+b\tan\left(c\right))\sec^6\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + b
*sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**6, True))
```

3.507.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a\,\mathrm{ta}}{30\,d}$$

input

```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output

```
1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a
*tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

3.507.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a\,\mathrm{ta}}{30\,d}$$

input

```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output

```
1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a
*tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

3.507.9 Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{\frac{b\tan(c+d\,x)^6}{6}+\frac{a\tan(c+d\,x)^5}{5}+\frac{b\tan(c+d\,x)^4}{2}+\frac{2\,a\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

input

```
int((a + b*tan(c + d*x))/cos(c + d*x)^6,x)
```

output

```
(a*tan(c + d*x) + (2*a*tan(c + d*x)^3)/3 + (a*tan(c + d*x)^5)/5 + (b*tan(c
 + d*x)^2)/2 + (b*tan(c + d*x)^4)/2 + (b*tan(c + d*x)^6)/6)/d
```

3.508 $\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$

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3.508.8 Giac [B] (verification not implemented) 3530
3.508.9 Mupad [B] (verification not implemented) 3531

3.508.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+1/5*b*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x+
c)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

3.508.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c
+ d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.508.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^5(c+dx)dx+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \; 4257$$

$$a\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)+\frac{b\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^5)/(5*d) + a*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(Ar
cTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)
```

3.508.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.508.4 Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^{5}}}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^{5}}}{d}$
risch	$\frac{-15ia\,\mathrm{e}^{9i(dx+c)}-70ia\,\mathrm{e}^{7i(dx+c)}+128b\,\mathrm{e}^{5i(dx+c)}+70ia\,\mathrm{e}^{3i(dx+c)}+15ia\,\mathrm{e}^{i(dx+c)}}{20d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{3a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8d}+\frac{3a\ln\left(\mathrm{e}^{i(dx+c}\right.}{8d}$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+t
an(d*x+c)))+1/5*b/cos(d*x+c)^5)
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \sec^{5}(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{15\,a\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+10\,\left(3\,a\cos\left(dx+c\right)^{3}\right.}{80\,d\cos\left(dx+c\right)^{5}}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x +
c) + 16*b)/(d*cos(d*x + c)^5)
```

3.508.6 Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sec^5(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sec(c + d*x)**5, x)
```

3.508.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{16\,b}{\cos(dx+c)^5}}{80\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*
b/cos(d*x + c)^5)/d
```

3.508.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.91

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-1\right.}{}}{40\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)
^8 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

3.508.9 Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.36

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

$$-\frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^5,x)
```

output
```
(3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((2*b)/5 + (5*a*tan(c/2 + (d*x)/2)
)/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 - (5*a*tan(c
/2 + (d*x)/2)^9)/4 + 4*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/
(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/
2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.509 $\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$

3.509.1 Optimal result 3532
3.509.2 Mathematica [A] (verified) 3532
3.509.3 Rubi [A] (verified) 3533
3.509.4 Maple [A] (verified) 3534
3.509.5 Fricas [A] (verification not implemented) 3535
3.509.6 Sympy [A] (verification not implemented) 3535
3.509.7 Maxima [A] (verification not implemented) 3535
3.509.8 Giac [A] (verification not implemented) 3536
3.509.9 Mupad [B] (verification not implemented) 3536

3.509.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
1/4*b*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

3.509.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^4(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.509.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3967, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))dx$$

$$\Big\downarrow \; 3967$$

$$a\int \sec^4(c+dx)dx+\frac{b\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \; 3042$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx+\frac{b\sec^4(c+dx)}{4d}$$

$$\Big\downarrow \; 4254$$

$$\frac{b\sec^4(c+dx)}{4d}-\frac{a\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\sec^4(c+dx)}{4d}-\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d}$$

input

```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output

```
(b*Sec[c + d*x]^4)/(4*d) - (a*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/d
```

3.509.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.509.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^{4}(dx+c)\right)b}{4}+\frac{a\left(\tan^{3}(dx+c)\right)}{3}+\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	47
default	$\frac{\frac{\left(\tan^{4}(dx+c)\right)b}{4}+\frac{a\left(\tan^{3}(dx+c)\right)}{3}+\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	47
risch	$\frac{4ia\,e^{4i(dx+c)}+4b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}+\frac{4ia}{3}}{d\left(e^{2i(dx+c)}+1\right)^{4}}$	62

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*tan(d*x+c)^4*b+1/3*a*tan(d*x+c)^3+1/2*b*tan(d*x+c)^2+a*tan(d*x+c)
)
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{4\left(2\,a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^4}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d
*x + c)^4)
```

3.509.6 Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{b\sec^4(c+dx)}{4}}{d} & \text{for } d\neq 0 \\ x(a+b\tan(c))\sec^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + b*sec(c + d*x)**4/4)/d,
 Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**4, True))
```

3.509.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*
tan(d*x + c))/d
```

3.509.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*
tan(d*x + c))/d
```

3.509.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{\frac{b\tan(c+d\,x)^4}{4}+\frac{a\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^4,x)
```

output
```
(a*tan(c + d*x) + (a*tan(c + d*x)^3)/3 + (b*tan(c + d*x)^2)/2 + (b*tan(c +
 d*x)^4)/4)/d
```

3.510 $\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$

3.510.1 Optimal result 3537
3.510.2 Mathematica [A] (verified) 3537
3.510.3 Rubi [A] (verified) 3538
3.510.4 Maple [A] (verified) 3539
3.510.5 Fricas [A] (verification not implemented) 3540
3.510.6 Sympy [F] 3540
3.510.7 Maxima [A] (verification not implemented) 3540
3.510.8 Giac [B] (verification not implemented) 3541
3.510.9 Mupad [B] (verification not implemented) 3541

3.510.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x+
c)/d
```

3.510.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*
x]*Tan[c + d*x])/(2*d)
```

3.510.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3967, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))dx$$

$$\Big\downarrow \; 3967$$

$$a\int \sec^3(c+dx)dx+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \; 3042$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \; 4255$$

$$a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \; 3042$$

$$a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \; 4257$$

$$a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{b\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^3)/(3*d) + a*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*
Tan[c + d*x])/(2*d))
```

3.510.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.510.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$	50
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$	50
risch	$\frac{-3ia\,\mathrm{e}^{5i(dx+c)}+8b\,\mathrm{e}^{3i(dx+c)}+3ia\,\mathrm{e}^{i(dx+c)}}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$	97

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/3*b/cos
(d*x+c)^3)
```

3.510.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+6\,a\cos\left(dx+c\right)\sin\left(d\right.}{12\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

3.510.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

3.510.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= -\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{4\,b}{\cos(dx+c)^3}}{12\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

3.510.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(46) = 92$.

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,a\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 -
3*a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.510.9 Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.02

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d - ((2*b)/3 + a*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.511 $\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$

3.511.1 Optimal result 3542
3.511.2 Mathematica [A] (verified) 3542
3.511.3 Rubi [A] (verified) 3543
3.511.4 Maple [A] (verified) 3544
3.511.5 Fricas [A] (verification not implemented) 3545
3.511.6 Sympy [A] (verification not implemented) 3545
3.511.7 Maxima [A] (verification not implemented) 3545
3.511.8 Giac [A] (verification not implemented) 3546
3.511.9 Mupad [B] (verification not implemented) 3546

3.511.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

output
```
1/2*b*sec(d*x+c)^2/d+a*tan(d*x+c)/d
```

3.511.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

3.511.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3967, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec^2(c+dx)dx+\frac{b\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{b\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{b\sec^2(c+dx)}{2d}-\frac{a\int 1d(-\tan(c+dx))}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

3.511.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.511.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	25
default	$\frac{\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	25
risch	$\frac{2ia\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}+2ia}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}$	48

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b*tan(d*x+c)^2+a*tan(d*x+c))
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

3.511.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\tan\left(c+dx\right)+\frac{b\tan^2\left(c+dx\right)}{2}}{d} & \text{for}\ d\neq 0 \\ x(a+b\tan\left(c\right))\sec^2\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**2/2)/d, Ne(d, 0)), (x*(a + b*
tan(c))*sec(c)**2, True))
```

3.511.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\left(b\tan\left(dx+c\right)+a\right)^2}{2\,bd}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(b*tan(d*x + c) + a)^2/(b*d)
```

3.511.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\tan\left(dx+c\right)^2+2\,a\tan\left(dx+c\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

3.511.9 Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\tan(c+d\,x)\;\left(2\,a+b\tan(c+d\,x)\right)}{2\,d}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(tan(c + d*x)*(2*a + b*tan(c + d*x)))/(2*d)
```

3.512 $\int \sec(c+dx)(a+b\tan(c+dx))\,dx$

3.512.1 Optimal result 3547
3.512.2 Mathematica [A] (verified) 3547
3.512.3 Rubi [A] (verified) 3548
3.512.4 Maple [A] (verified) 3549
3.512.5 Fricas [B] (verification not implemented) 3549
3.512.6 Sympy [A] (verification not implemented) 3550
3.512.7 Maxima [A] (verification not implemented) 3550
3.512.8 Giac [B] (verification not implemented) 3550
3.512.9 Mupad [B] (verification not implemented) 3551

3.512.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+b*sec(d*x+c)/d
```

3.512.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

3.512.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3967, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sec(c+dx)dx+\frac{b\sec(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\sec(c+dx)}{d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

3.512.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.512.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
default	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2b\,\mathrm{e}^{i(dx+c)}}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{a\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{a\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$	67

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

3.512.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)+2\,b}{2\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*b)/(d*cos(d*x + c))
```

3.512.6 Sympy [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+b\sec(c+dx)}{d} & \text{for } d\neq 0 \\ x(a+b\tan(c))\sec(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*sec(c + d*x))/d, Ne(d,
0)), (x*(a + b*tan(c))*sec(c), True))
```

3.512.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\log(\sec(dx+c)+\tan(dx+c))+\frac{b}{\cos(dx+c)}}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sec(d*x + c) + tan(d*x + c)) + b/cos(d*x + c))/d
```

3.512.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.512.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,b}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.513 $\int \cos(c+dx)(a+b\tan(c+dx))\,dx$

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3.513.2 Mathematica [A] (verified) . . . 3552
3.513.3 Rubi [A] (verified) . . . 3553
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3.513.7 Maxima [A] (verification not implemented) . . . 3555
3.513.8 Giac [B] (verification not implemented) . . . 3555
3.513.9 Mupad [B] (verification not implemented) . . . 3556

3.513.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
-b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

3.513.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
-((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
 (b*Sin[c]*Sin[d*x])/d
```

3.513.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3967, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos(c+dx)dx-\frac{b\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\cos(c+dx)}{d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

output
```
-((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

3.513.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.513.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
default	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b*cos(d*x+c)+a*sin(d*x+c))
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos{(dx+c)}-a\sin{(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/d
```

3.513.6 Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan{(c+dx)})\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x), x)
```

3.513.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos{(dx+c)}-a\sin{(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/d
```

3.513.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(24) = 48.

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.38

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx =$$

$$-\frac{b\tan\left(\frac{1}{2}\,dx\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+2\,a\tan\left(\frac{1}{2}\,dx\right)^{2}\tan\left(\frac{1}{2}\,c\right)+2\,a\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx\right)^{2}-4\,b\tan}{d\tan\left(\frac{1}{2}\,dx\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+d\tan\left(\frac{1}{2}\,dx\right)^{2}+d\tan}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-(b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*tan(
1/2*d*x)*tan(1/2*c)^2 - b*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b
*tan(1/2*c)^2 - 2*a*tan(1/2*d*x) - 2*a*tan(1/2*c) + b)/(d*tan(1/2*d*x)^2*t
an(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

3.513.9 Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x)),x)
```

output
```
-(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

3.514 $\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$

3.514.1 Optimal result 3557
3.514.2 Mathematica [A] (verified) 3557
3.514.3 Rubi [A] (verified) 3558
3.514.4 Maple [A] (verified) 3559
3.514.5 Fricas [A] (verification not implemented) 3560
3.514.6 Sympy [F] 3560
3.514.7 Maxima [A] (verification not implemented) 3560
3.514.8 Giac [B] (verification not implemented) 3561
3.514.9 Mupad [B] (verification not implemented) 3561

3.514.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a*x-1/2*b*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

3.514.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{a(c+dx)}{2d}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d
)
```

3.514.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3967, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^2(c+dx)dx-\frac{b\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{b\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3115}$$

$$a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{b\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{b\cos^2(c+dx)}{2d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

output
```
-1/2*(b*Cos[c + d*x]^2)/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

3.514.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.514.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{ax}{2}-\frac{b\cos(2dx+2c)}{4d}+\frac{a\sin(2dx+2c)}{4d}$	36
derivativedivides	$\frac{-\frac{b\left(\cos^{2}(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41
default	$\frac{-\frac{b\left(\cos^{2}(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x-1/4*b/d*cos(2*d*x+2*c)+1/4*a/d*sin(2*d*x+2*c)
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{adx-b\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

3.514.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^2\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

3.514.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{(dx+c)a+\frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((d*x + c)*a + (a*tan(d*x + c) - b)/(tan(d*x + c)^2 + 1))/d
```

3.514.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(37) = 74$.

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.40

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{2\,adx\tan{(dx)}^2\tan{(c)}^2 + 2\,adx\tan{(dx)}^2 + 2\,adx\tan{(c)}^2 - b\tan{(dx)}^2\tan{(c)}^2 - 2\,a\tan{(dx)}^2\tan{(c)} - 2}{4\left(d\tan{(dx)}^2\tan{(c)}^2 + d\right.}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 -
 b*tan(d*x)^2*tan(c)^2 - 2*a*tan(d*x)^2*tan(c) - 2*a*tan(d*x)*tan(c)^2 + 2
*a*d*x + b*tan(d*x)^2 + 4*b*tan(d*x)*tan(c) + b*tan(c)^2 + 2*a*tan(d*x) +
2*a*tan(c) - b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.514.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\,x}{2} - \frac{\cos{(c+d\,x)}^2\left(\frac{b}{2} - \frac{a\tan(c+d\,x)}{2}\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

output
```
(a*x)/2 - (cos(c + d*x)^2*(b/2 - (a*tan(c + d*x))/2))/d
```

3.515 $\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$

3.515.1 Optimal result . . . 3562
3.515.2 Mathematica [A] (verified) . . . 3562
3.515.3 Rubi [A] (verified) . . . 3563
3.515.4 Maple [A] (verified) . . . 3564
3.515.5 Fricas [A] (verification not implemented) . . . 3565
3.515.6 Sympy [F] . . . 3565
3.515.7 Maxima [A] (verification not implemented) . . . 3565
3.515.8 Giac [B] (verification not implemented) . . . 3566
3.515.9 Mupad [B] (verification not implemented) . . . 3566

3.515.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
-1/3*b*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.515.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

3.515.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3967, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \cos^3(c+dx)dx-\frac{b\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d}-\frac{b\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d - (a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/d
```

3.515.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.515.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{b\left(\cos^{3}(dx+c)\right)}{3}+\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
default	$\frac{-\frac{b\left(\cos^{3}(dx+c)\right)}{3}+\frac{a\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
risch	$-\frac{b\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)}{12d}+\frac{a\sin(3dx+3c)}{12d}$	56

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*b*cos(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos(dx+c)^3 - \left(a\cos(dx+c)^2 + 2\,a\right)\sin(dx+c)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

3.515.6 Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

3.515.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos(dx+c)^3 + \left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

3.515.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11886 vs. $2(40) = 80$.

Time = 2.42 (sec) , antiderivative size = 11886, normalized size of antiderivative = 270.14

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output

```
1/96*(3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^6*tan(1/2*c)^6 - 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t
an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)
^6*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x...
```

3.515.9 Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d} - \frac{b\cos\left(c+d\,x\right)^3}{3\,d} + \frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x)),x)
```

output
```
(2*a*sin(c + d*x))/(3*d) - (b*cos(c + d*x)^3)/(3*d) + (a*cos(c + d*x)^2*si
n(c + d*x))/(3*d)
```

3.515. $\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$

3.516 $\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$

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3.516.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3ax}{8} - \frac{b\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
3/8*a*x-1/4*b*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c
)^3*sin(d*x+c)/d
```

3.516.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a(c+dx)}{8d} - \frac{b\cos^4(c+dx)}{4d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4
*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

3.516.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3967, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\tan(c+dx)}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3967}$$

$$a\int \cos^4(c+dx)dx-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{ 3115}$$

$$a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{ 3115}$$

$$a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)-\frac{b\cos^4(c+dx)}{4d}$$

$$\Big\downarrow \text{ 24}$$

$$a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)-\frac{b\cos^4(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

output
```
-1/4*(b*Cos[c + d*x]^4)/d + a*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x
/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.516.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.516.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{b\left(\cos^{4}(dx+c)\right)}{4}+a\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	52
default	$\frac{-\frac{b\left(\cos^{4}(dx+c)\right)}{4}+a\left(\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	52
risch	$\frac{3ax}{8}-\frac{b\cos(4dx+4c)}{32d}+\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	66

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*b*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/8*d*x+3/8*c))
```

3.516.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-\left(2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c
))*sin(d*x + c))/d
```

3.516.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

output
```
Integral((a + b*tan(c + d*x))*cos(c + d*x)**4, x)
```

3.516.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\left(dx+c\right)a+\frac{3\,a\tan\left(dx+c\right)^3+5\,a\tan\left(dx+c\right)-2\,b}{\tan\left(dx+c\right)^4+2\,\tan\left(dx+c\right)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*b)/(tan(d*
x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

3.516.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(57) = 114$.

Time = 0.52 (sec) , antiderivative size = 426, normalized size of antiderivative = 6.55

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{12\,adx\tan{(dx)}^4\tan{(c)}^4 + 24\,adx\tan{(dx)}^4\tan{(c)}^2 + 24\,adx\tan{(dx)}^2\tan{(c)}^4 - 5\,b\tan{(dx)}^4\tan{(c)}^4 -}{}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*d
*x*tan(d*x)^2*tan(c)^4 - 5*b*tan(d*x)^4*tan(c)^4 - 20*a*tan(d*x)^4*tan(c)^
3 - 20*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x*tan(d*x)^2*t
an(c)^2 + 6*b*tan(d*x)^4*tan(c)^2 + 32*b*tan(d*x)^3*tan(c)^3 + 12*a*d*x*ta
n(c)^4 + 6*b*tan(d*x)^2*tan(c)^4 - 12*a*tan(d*x)^4*tan(c) + 24*a*tan(d*x)^
3*tan(c)^2 + 24*a*tan(d*x)^2*tan(c)^3 - 12*a*tan(d*x)*tan(c)^4 + 24*a*d*x*
tan(d*x)^2 + 3*b*tan(d*x)^4 + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x)^2*tan(c)^2
 + 3*b*tan(c)^4 + 12*a*tan(d*x)^3 - 24*a*tan(d*x)^2*tan(c) - 24*a*tan(d*x)
*tan(c)^2 + 12*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2 + 32*b*tan(d*x)*tan(
c) + 6*b*tan(c)^2 + 20*a*tan(d*x) + 20*a*tan(c) - 5*b)/(d*tan(d*x)^4*tan(c
)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4
*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

3.516.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,x}{8} + \frac{{\cos\left(c+d\,x\right)}^4\,\left(\frac{3\,a\,{\mathrm{tan}\left(c+d\,x\right)}^3}{8}+\frac{5\,a\,\mathrm{tan}\left(c+d\,x\right)}{8}-\frac{b}{4}\right)}{d}$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

output
```
(3*a*x)/8 + (cos(c + d*x)^4*((5*a*tan(c + d*x))/8 - b/4 + (3*a*tan(c + d*x
)^3)/8))/d
```

3.517 $\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$

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3.517.9 Mupad [B] (verification not implemented) 3578

3.517.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\begin{aligned}\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{ab\sec^8(c+dx)}{4d}+\frac{a^2\tan(c+dx)}{d}\\ &\quad+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}\\ &\quad+\frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+\frac{b^2\tan^9(c+dx)}{9d}\end{aligned}$$

output
```
1/4*a*b*sec(d*x+c)^8/d+a^2*tan(d*x+c)/d+1/3*(3*a^2+b^2)*tan(d*x+c)^3/d+3/5
*(a^2+b^2)*tan(d*x+c)^5/d+1/7*(a^2+3*b^2)*tan(d*x+c)^7/d+1/9*b^2*tan(d*x+c
)^9/d
```

3.517.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \tan(c+dx)\left(1260a^2+1260ab\tan(c+dx)+420(3a^2+b^2)\tan^2(c+dx)+1890ab\tan^3(c+dx)+756(a^2\right.$$

input
```
Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

output
```
(Tan[c + d*x]*(1260*a^2 + 1260*a*b*Tan[c + d*x] + 420*(3*a^2 + b^2)*Tan[c
+ d*x]^2 + 1890*a*b*Tan[c + d*x]^3 + 756*(a^2 + b^2)*Tan[c + d*x]^4 + 1260
*a*b*Tan[c + d*x]^5 + 180*(a^2 + 3*b^2)*Tan[c + d*x]^6 + 315*a*b*Tan[c + d
*x]^7 + 140*b^2*Tan[c + d*x]^8))/(1260*d)
```

3.517.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow 3042$$

$$\int \sec(c+dx)^8(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow 3987$$

$$\frac{\int \frac{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^3}{b^6}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow 27$$

$$\frac{\int (a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^3d(b\tan(c+dx))}{b^7d}$$

$$\Big\downarrow 475$$

$$\frac{\int \left(b^8\tan^8(c+dx)+b^6\left(a^2+3b^2\right)\tan^6(c+dx)+3b^6\left(a^2+b^2\right)\tan^4(c+dx)+b^6\left(3a^2+b^2\right)\tan^2(c+dx)+a^2b^6\right)d}{b^7d}$$

$$\Big\downarrow 2009$$

$$\frac{a^2b^7\tan(c+dx)+\frac{1}{7}b^7\left(a^2+3b^2\right)\tan^7(c+dx)+\frac{3}{5}b^7\left(a^2+b^2\right)\tan^5(c+dx)+\frac{1}{3}b^7\left(3a^2+b^2\right)\tan^3(c+dx)+\frac{1}{4}a(b}{b^7d}$$

input
```
Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*b^7*Tan[c + d*x] + (b^7*(3*a^2 + b^2)*Tan[c + d*x]^3)/3 + (3*b^7*(a^2
 + b^2)*Tan[c + d*x]^5)/5 + (b^7*(a^2 + 3*b^2)*Tan[c + d*x]^7)/7 + (b^9*Ta
n[c + d*x]^9)/9 + (a*(b^2 + b^2*Tan[c + d*x]^2)^4)/4)/(b^7*d)
```

3.517.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.517.4 Maple [A] (verified)

Time = 113.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{ab}{4\cos(dx+c)^8}+b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}-\right.}{d}$
default	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{ab}{4\cos(dx+c)^8}+b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}-\right.}{d}$
risch	$\frac{32i\left(-630iab\,\mathrm{e}^{10i(dx+c)}+315a^2\mathrm{e}^{10i(dx+c)}-315b^2\mathrm{e}^{10i(dx+c)}-630iab\,\mathrm{e}^{8i(dx+c)}+819a^2\mathrm{e}^{8i(dx+c)}+189b^2\mathrm{e}^{8i(dx+c)}+756a^2\right.}{315d\left(\mathrm{e}^{2i(dx+c)}+1\right)^9}$

input
```
int(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*ta
n(d*x+c)+1/4*a*b/cos(d*x+c)^8+b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(
d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/
cos(d*x+c)^3))
```

3.517.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{315\,ab\cos(dx+c)+4\left(16\left(9\,a^2-b^2\right)\cos(dx+c)^8+8\left(9\,a^2-b^2\right)\cos(dx+c)^6+6\left(9\,a^2-b^2\right)\cos(dx+c}{1260\,d\cos(dx+c)^9}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/1260*(315*a*b*cos(d*x + c) + 4*(16*(9*a^2 - b^2)*cos(d*x + c)^8 + 8*(9*a
^2 - b^2)*cos(d*x + c)^6 + 6*(9*a^2 - b^2)*cos(d*x + c)^4 + 5*(9*a^2 - b^2
)*cos(d*x + c)^2 + 35*b^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

3.517.6 Sympy [F]

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx=\int\left(a+b\tan\left(c+dx\right)\right)^2\sec^8\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**8, x)
```

3.517.7 Maxima [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{140\,b^2\tan{(dx+c)}^9 + 315\,ab\tan{(dx+c)}^8 + 1260\,ab\tan{(dx+c)}^6 + 180\,(a^2+3\,b^2)\tan{(dx+c)}^7 + 1890}{}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 1260*a*b*tan(d*x
 + c)^6 + 180*(a^2 + 3*b^2)*tan(d*x + c)^7 + 1890*a*b*tan(d*x + c)^4 + 756
*(a^2 + b^2)*tan(d*x + c)^5 + 1260*a*b*tan(d*x + c)^2 + 420*(3*a^2 + b^2)*
tan(d*x + c)^3 + 1260*a^2*tan(d*x + c))/d
```

3.517.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{140\,b^2\tan{(dx+c)}^9 + 315\,ab\tan{(dx+c)}^8 + 180\,a^2\tan{(dx+c)}^7 + 540\,b^2\tan{(dx+c)}^7 + 1260\,ab\tan{(dx}{}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 180*a^2*tan(d*x
+ c)^7 + 540*b^2*tan(d*x + c)^7 + 1260*a*b*tan(d*x + c)^6 + 756*a^2*tan(d*
x + c)^5 + 756*b^2*tan(d*x + c)^5 + 1890*a*b*tan(d*x + c)^4 + 1260*a^2*tan
(d*x + c)^3 + 420*b^2*tan(d*x + c)^3 + 1260*a*b*tan(d*x + c)^2 + 1260*a^2*
tan(d*x + c))/d
```

3.517.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{\tan(c+d\,x)^3\left(a^2+\frac{b^2}{3}\right)+a^2\tan(c+d\,x)+\tan(c+d\,x)^5\left(\frac{3\,a^2}{5}+\frac{3\,b^2}{5}\right)+\tan(c+d\,x)^7\left(\frac{a^2}{7}+\frac{3\,b^2}{7}\right)+ b^2\tan}{d}$$

input

```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^8,x)
```

output

```
(tan(c + d*x)^3*(a^2 + b^2/3) + a^2*tan(c + d*x) + tan(c + d*x)^5*((3*a^2)
/5 + (3*b^2)/5) + tan(c + d*x)^7*(a^2/7 + (3*b^2)/7) + (b^2*tan(c + d*x)^9
)/9 + a*b*tan(c + d*x)^2 + (3*a*b*tan(c + d*x)^4)/2 + a*b*tan(c + d*x)^6 +
 (a*b*tan(c + d*x)^8)/4)/d
```

3.518 $\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$

3.518.1 Optimal result 3579
3.518.2 Mathematica [A] (verified) 3579
3.518.3 Rubi [A] (verified) 3580
3.518.4 Maple [A] (verified) 3581
3.518.5 Fricas [A] (verification not implemented) 3582
3.518.6 Sympy [F] 3582
3.518.7 Maxima [A] (verification not implemented) 3583
3.518.8 Giac [A] (verification not implemented) 3583
3.518.9 Mupad [B] (verification not implemented) 3584

3.518.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{ab\sec^6(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d} +\frac{(2a^2+b^2)\tan^3(c+dx)}{3d} +\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}$$

output
```
1/3*a*b*sec(d*x+c)^6/d+a^2*tan(d*x+c)/d+1/3*(2*a^2+b^2)*tan(d*x+c)^3/d+1/5
*(a^2+2*b^2)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

3.518.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{\tan(c+dx)\left(105a^2+105ab\tan(c+dx)+35(2a^2+b^2)\tan^2(c+dx)+105ab\tan^3(c+dx)+21(a^2+2b^2)\right.}{105d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

output
```
(Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d
*x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*
Tan[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

3.518.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{475}$$

$$\frac{\int \left(b^6\tan^6(c+dx)+b^4\left(a^2+2b^2\right)\tan^4(c+dx)+b^4\left(2a^2+b^2\right)\tan^2(c+dx)+a^2b^4\right)d(b\tan(c+dx))+\frac{1}{3}a(b^2\tan^2}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2b^5\tan(c+dx)+\frac{1}{5}b^5\left(a^2+2b^2\right)\tan^5(c+dx)+\frac{1}{3}b^5\left(2a^2+b^2\right)\tan^3(c+dx)+\frac{1}{3}a\left(b^2\tan^2(c+dx)+b^2\right)^3+\frac{1}{7}b^7\,\mathrm{ta}}{b^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*b^5*Tan[c + d*x] + (b^5*(2*a^2 + b^2)*Tan[c + d*x]^3)/3 + (b^5*(a^2 +
 2*b^2)*Tan[c + d*x]^5)/5 + (b^7*Tan[c + d*x]^7)/7 + (a*(b^2 + b^2*Tan[c +
 d*x]^2)^3)/3)/(b^5*d)
```

3.518.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.518.4 Maple [A] (verified)

Time = 28.99 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^{6}}+b^{2}\left(\frac{\sin^{3}(dx+c)}{7\cos(dx+c)^{7}}+\frac{4\left(\sin^{3}(dx+c)\right)}{35\cos(dx+c)^{5}}+\frac{8\left(\sin^{3}(dx+c)\right)}{105\cos(dx+c)^{3}}\right.}{d}$
default	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^{6}}+b^{2}\left(\frac{\sin^{3}(dx+c)}{7\cos(dx+c)^{7}}+\frac{4\left(\sin^{3}(dx+c)\right)}{35\cos(dx+c)^{5}}+\frac{8\left(\sin^{3}(dx+c)\right)}{105\cos(dx+c)^{3}}\right.}{d}$
risch	$\frac{16i\left(-140iab\,\mathrm{e}^{8i(dx+c)}+70a^{2}\mathrm{e}^{8i(dx+c)}-70b^{2}\mathrm{e}^{8i(dx+c)}-140iab\,\mathrm{e}^{6i(dx+c)}+175a^{2}\mathrm{e}^{6i(dx+c)}+35b^{2}\mathrm{e}^{6i(dx+c)}+147a^{2}\mathrm{e}^{4i(dx+\right.}{105d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{7}}$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*b/co
s(d*x+c)^6+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)
^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{35\,ab\cos\left(dx+c\right)+\left(8\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^6+4\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^4+3\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^2+}{105\,d\cos\left(dx+c\right)^7}$$

input

```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output

```
1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 -
b^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c
))/(d*cos(d*x + c)^7)
```

3.518.6 Sympy [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx=\int \left(a+b\tan\left(c+dx\right)\right)^2\sec^6\left(c+dx\right)dx$$

input

```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

output

```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**6, x)
```

3.518.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+105\,ab\tan\left(dx+c\right)^4+21\left(a^2+2\,b^2\right)\tan\left(dx+c\right)^5+105\,ab\tan}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 105*a*b*tan(d*x + c
)^4 + 21*(a^2 + 2*b^2)*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^2 + 35*(2*a^2
 + b^2)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

3.518.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5+42\,b^2\tan\left(dx+c\right)^5+105\,ab\tan\left(dx+c\right)^4}{105\,d}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)
^5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^
3 + 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))
/d
```

3.518.9 Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x)+\tan(c+d\,x)^3\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\tan(c+d\,x)^5\left(\frac{a^2}{5}+\frac{2\,b^2}{5}\right)+\frac{b^2\tan(c+d\,x)^7}{7}+a\,b\tan(c+d\,x)^2+}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^6,x)
```

output
```
(a^2*tan(c + d*x) + tan(c + d*x)^3*((2*a^2)/3 + b^2/3) + tan(c + d*x)^5*(a
^2/5 + (2*b^2)/5) + (b^2*tan(c + d*x)^7)/7 + a*b*tan(c + d*x)^2 + a*b*tan(
c + d*x)^4 + (a*b*tan(c + d*x)^6)/3)/d
```

3.519 $\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$

3.519.1 Optimal result . . . 3585
3.519.2 Mathematica [A] (verified) . . . 3585
3.519.3 Rubi [A] (verified) . . . 3586
3.519.4 Maple [A] (verified) . . . 3587
3.519.5 Fricas [A] (verification not implemented) . . . 3588
3.519.6 Sympy [F] . . . 3588
3.519.7 Maxima [A] (verification not implemented) . . . 3588
3.519.8 Giac [A] (verification not implemented) . . . 3589
3.519.9 Mupad [B] (verification not implemented) . . . 3589

3.519.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{(a^2+b^2)\,(a+b\tan(c+dx))^3}{3b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}+\frac{(a+b\tan(c+dx))^5}{5b^3d}$$

output
```
1/3*(a^2+b^2)*(a+b*tan(d*x+c))^3/b^3/d-1/2*a*(a+b*tan(d*x+c))^4/b^3/d+1/5*
(a+b*tan(d*x+c))^5/b^3/d
```

3.519.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{(a+b\tan(c+dx))^3\,(a^2+10b^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx))}{30b^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
 d*x]^2))/(30*b^3*d)
```

3.519.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \color{blue}{3042}$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \color{blue}{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}{b^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\int (a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \color{blue}{476}$$

$$\frac{\int \left((a+b\tan(c+dx))^4-2a(a+b\tan(c+dx))^3+\left(a^2+b^2\right)(a+b\tan(c+dx))^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \color{blue}{2009}$$

$$\frac{\frac{1}{3}\left(a^2+b^2\right)(a+b\tan(c+dx))^3+\frac{1}{5}(a+b\tan(c+dx))^5-\frac{1}{2}a(a+b\tan(c+dx))^4}{b^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
(((a^2 + b^2)*(a + b*Tan[c + d*x])^3)/3 - (a*(a + b*Tan[c + d*x])^4)/2 + (
a + b*Tan[c + d*x])^5/5)/(b^3*d)
```

3.519.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.519.4 Maple [A] (verified)

Time = 7.98 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
risch	$\frac{4i\left(-30iab\,e^{6i(dx+c)}+15a^2e^{6i(dx+c)}-15b^2e^{6i(dx+c)}-30iab\,e^{4i(dx+c)}+35a^2e^{4i(dx+c)}+5b^2e^{4i(dx+c)}+25a^2e^{2i(dx+c)}-5b\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/2*a*b/cos(d*x+c)^4+b^2*(1/5
*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3))
```

3.519.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{15\,ab\cos\left(dx+c\right)+2\left(2\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^4+\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^2+3\,b^2\right)\sin\left(dx+c\right)}{30\,d\cos\left(dx+c\right)^5}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b
^2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.519.6 Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\sec^4\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**4, x)
```

3.519.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+30\,ab\tan\left(dx+c\right)^2+10\left(a^2+b^2\right)\tan\left(dx+c\right)^3+30\,a^2\tan\left(dx+\right.}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^2
 + 10*(a^2 + b^2)*tan(d*x + c)^3 + 30*a^2*tan(d*x + c))/d
```

3.519.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+10\,a^2\tan\left(dx+c\right)^3+10\,b^2\tan\left(dx+c\right)^3+30\,ab\tan\left(dx+c\right)^2+}{30\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
 + 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

3.519.9 Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{a^2\tan(c+d\,x)+\tan(c+d\,x)^3\left(\frac{a^2}{3}+\frac{b^2}{3}\right)+\frac{b^2\tan(c+d\,x)^5}{5}+a\,b\tan(c+d\,x)^2+\frac{a\,b\tan(c+d\,x)^4}{2}}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
(a^2*tan(c + d*x) + tan(c + d*x)^3*(a^2/3 + b^2/3) + (b^2*tan(c + d*x)^5)/
5 + a*b*tan(c + d*x)^2 + (a*b*tan(c + d*x)^4)/2)/d
```

3.520 $\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$

3.520.1 Optimal result 3590
3.520.2 Mathematica [B] (verified) 3590
3.520.3 Rubi [A] (verified) 3591
3.520.4 Maple [B] (verified) 3592
3.520.5 Fricas [B] (verification not implemented) 3592
3.520.6 Sympy [F] 3593
3.520.7 Maxima [A] (verification not implemented) 3593
3.520.8 Giac [B] (verification not implemented) 3593
3.520.9 Mupad [B] (verification not implemented) 3594

3.520.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{(a+b\tan(c+dx))^3}{3bd}$$

output
```
1/3*(a+b*tan(d*x+c))^3/b/d
```

3.520.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

3.520.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int (a+b\tan(c+dx))^2d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+b\tan(c+dx))^3}{3bd}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a + b*Tan[c + d*x])^3/(3*b*d)
```

3.520.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.520.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(20) = 40$.

Time = 2.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	48
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	48
risch	$-\frac{2i\left(6iab\,\mathrm{e}^{4i(dx+c)}-3a^2\mathrm{e}^{4i(dx+c)}+3b^2\mathrm{e}^{4i(dx+c)}+6iab\,\mathrm{e}^{2i(dx+c)}-6a^2\mathrm{e}^{2i(dx+c)}-3a^2+b^2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	99

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3+a*b/cos(d*x+c)^2+a^2*tan(d*x+c))
```

3.520.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{3\,ab\cos(dx+c)+\left((3\,a^2-b^2)\cos(dx+c)^2+b^2\right)\sin(dx+c)}{3\,d\cos(dx+c)^3}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c
))/(d*cos(d*x + c)^3)
```

3.520.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

3.520.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(b\tan(dx+c)+a)^3}{3\,bd}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3*(b*tan(d*x + c) + a)^3/(b*d)
```

3.520.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan(dx+c)^3+3\,ab\tan(dx+c)^2+3\,a^2\tan(dx+c)}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

3.520.9 Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x)+a\,b\tan(c+d\,x)^2+\frac{b^2\tan(c+d\,x)^3}{3}}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
(a^2*tan(c + d*x) + (b^2*tan(c + d*x)^3)/3 + a*b*tan(c + d*x)^2)/d
```

3.521 $\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$

3.521.1 Optimal result 3595
3.521.2 Mathematica [A] (verified) 3595
3.521.3 Rubi [A] (verified) 3596
3.521.4 Maple [A] (verified) 3597
3.521.5 Fricas [A] (verification not implemented) 3598
3.521.6 Sympy [F] 3598
3.521.7 Maxima [A] (verification not implemented) 3598
3.521.8 Giac [B] (verification not implemented) 3599
3.521.9 Mupad [B] (verification not implemented) 3599

3.521.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{2}\left(a^2+b^2\right)x-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

output
```
1/2*(a^2+b^2)*x-1/2*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d
```

3.521.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\left(a^2+b^2\right)(c+dx)-2ab\cos(2(c+dx))+\left(a^2-b^2\right)\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
 d*x)])/(4*d)
```

3.521.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int \frac{b^4(a+b\tan(c+dx))^2}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b^3\int \frac{(a+b\tan(c+dx))^2}{(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 487}$$

$$\frac{b^3\left(\frac{(a^2+b^2)\int \frac{1}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{b^3\left(\frac{(a^2+b^2)\arctan(\tan(c+dx))}{2b^3}-\frac{(a+b\tan(c+dx))(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^3*(((a^2 + b^2)*ArcTan[Tan[c + d*x]])/(2*b^3) - ((a + b*Tan[c + d*x])*(
b^2 - a*b*Tan[c + d*x]))/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2))))/d
```

3.521.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.521.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

method	result	size
risch	$\frac{a^2 x}{2}+\frac{b^2 x}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$	64
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70
default	$\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x+1/2*b^2*x-1/2*a*b/d*cos(2*d*x+2*c)+1/4/d*sin(2*d*x+2*c)*a^2-1/4/
d*sin(2*d*x+2*c)*b^2
```

3.521.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\,ab\cos\left(dx+c\right)^2-(a^2+b^2)dx-(a^2-b^2)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*si
n(d*x + c))/d
```

3.521.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan\left(c+dx\right))^2\cos^2\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

3.521.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+b^2)(dx+c)-\frac{2\,ab-(a^2-b^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*((a^2 + b^2)*(d*x + c) - (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x +
 c)^2 + 1))/d
```

3.521.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(46) = 92$.

Time = 0.50 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.00

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2dx\tan{(dx)}^2\tan{(c)}^2 + b^2dx\tan{(dx)}^2\tan{(c)}^2 + a^2dx\tan{(dx)}^2 + b^2dx\tan{(dx)}^2 + a^2dx\tan{(c)}^2 + b^2dx}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(a^2*d*x*tan(d*x)^2*tan(c)^2 + b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*t
an(d*x)^2 + b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(c)^2 + b^2*d*x*tan(c)^2 - a*b
*tan(d*x)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(c) + b^2*tan(d*x)^2*tan(c) - a^2
*tan(d*x)*tan(c)^2 + b^2*tan(d*x)*tan(c)^2 + a^2*d*x + b^2*d*x + a*b*tan(d
*x)^2 + 4*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2 + a^2*tan(d*x) - b^2*tan(d*x)
 + a^2*tan(c) - b^2*tan(c) - a*b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 +
d*tan(c)^2 + d)
```

3.521.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = x\left(\frac{a^2}{2}+\frac{b^2}{2}\right) - \frac{\cos{(c+d\,x)}^2\left(a\,b-\tan(c+d\,x)\,\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)}{d}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

output
```
x*(a^2/2 + b^2/2) - (cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)))/
d
```

3.522 $\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$

3.522.1 Optimal result 3600
3.522.2 Mathematica [B] (verified) 3600
3.522.3 Rubi [A] (verified) 3601
3.522.4 Maple [A] (verified) 3603
3.522.5 Fricas [A] (verification not implemented) 3604
3.522.6 Sympy [F] 3604
3.522.7 Maxima [A] (verification not implemented) 3604
3.522.8 Giac [B] (verification not implemented) 3605
3.522.9 Mupad [B] (verification not implemented) 3605

3.522.1 Optimal result

Integrand size = 21, antiderivative size = 88

$$\begin{aligned}\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{1}{8}\left(3a^2+b^2\right)x - \frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} \\ &\quad - \frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+dx)\right)}{8d}\end{aligned}$$

output
```
1/8*(3*a^2+b^2)*x-1/4*cos(d*x+c)^4*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d-1/8
*cos(d*x+c)^2*(2*a*b-(3*a^2+b^2)*tan(d*x+c))/d
```

3.522.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.

Time = 3.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.24

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{\left(3a^2+b^2\right)\left(2ab\left(2a^2+b^2\right)-2ab\left(a^2+b^2\right)\cos(2(c+dx))+\frac{b\left(a^2+b^2\right)^2\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{\sqrt{-b^2}}-\frac{b\left(a^2+b^2\right)^2\log\left(\sqrt{-b^2}\right.}{\sqrt{-b^2}}\right.}{16\left(a^2+\right.}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output
```
((3*a^2 + b^2)*(2*a*b*(2*a^2 + b^2) - 2*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] +
 (b*(a^2 + b^2)^2*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/Sqrt[-b^2] - (b*(a^2 +
 b^2)^2*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/Sqrt[-b^2] + (a^4 - b^4)*Sin[2*(
c + d*x)]) + 4*(a^2 + b^2)*Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[
c + d*x])^3)/(16*(a^2 + b^2)^2*d)
```

3.522.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3987, 27, 495, 454, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \; 3987$$

$$\frac{\int \frac{b^6(a+b\tan(c+dx))^2}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \; 27$$

$$\frac{b^5\int \frac{(a+b\tan(c+dx))^2}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 495$$

$$\frac{b^5\left(\frac{\int \frac{3a^2+2b\tan(c+dx)a+b^2}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{4b^2\left(b^2\tan^2(c+dx)+b^2\right)^2}\right)}{d}$$

$$\Big\downarrow \; 454$$

$$\frac{b^5\left(\frac{\frac{1}{2}\left(\frac{3a^2}{b^2}+1\right)\int \frac{1}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))-\frac{2ab^2-b\left(3a^2+b^2\right)\tan(c+dx)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{4b^2\left(b^2\tan^2(c+dx)+b^2\right)^2}\right)}{d}$$

$$\Big\downarrow 216$$

$$\frac{b^5\left(\frac{\frac{\left(\frac{3a^2}{b^2}+1\right)\arctan(\tan(c+dx))}{2b}-\frac{2ab^2-b\left(3a^2+b^2\right)\tan(c+dx)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{4b^2\left(b^2\tan^2(c+dx)+b^2\right)^2}\right)}{d}$$

input

```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

output

```
(b^5*(-1/4*((a + b*Tan[c + d*x])*(b^2 - a*b*Tan[c + d*x]))/(b^2*(b^2 + b^2
*Tan[c + d*x]^2)^2) + (((1 + (3*a^2)/b^2)*ArcTan[Tan[c + d*x]])/(2*b) - (2
*a*b^2 - b*(3*a^2 + b^2)*Tan[c + d*x])/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2)))
/(4*b^2)))/d
```

3.522.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454

```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.522.4 Maple [A] (verified)

Time = 8.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{b^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\Bigg(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+}{d}$
default	$\frac{b^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\Bigg(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+}{d}$
risch	$\frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos(4dx+4c)}{16d}+\frac{\sin(4dx+4c)a^2}{32d}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(2dx+2c)}{4d}+\frac{\sin(2dx+2c)a^2}{4d}$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1
/8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)+3/8*d*x+3/8*c))
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{4\,ab\cos\left(dx+c\right)^4-(3\,a^2+b^2)dx-\left(2\,(a^2-b^2)\cos\left(dx+c\right)^3+(3\,a^2+b^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x +
c)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.522.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**4, x)
```

3.522.7 Maxima [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(3\,a^2+b^2)(dx+c)+\frac{(3\,a^2+b^2)\tan(dx+c)^3-4\,ab+(5\,a^2-b^2)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*((3*a^2 + b^2)*(d*x + c) + ((3*a^2 + b^2)*tan(d*x + c)^3 - 4*a*b + (5*
a^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

3.522.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2286 vs. $2(83) = 166$.

Time = 2.60 (sec) , antiderivative size = 2286, normalized size of antiderivative = 25.98

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output

```
1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^2*d
*x*tan(d*x)^4*tan(c)^4 + 8*b^2*d*x*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*sgn(-2*t
an(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4
*tan(c)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 6*
pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*b^2*arctan((ta
n(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*b^2*arctan
(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 48*a^2*
d*x*tan(d*x)^4*tan(c)^2 + 16*b^2*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*b^2*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)
^4*tan(c)^2 + 48*a^2*d*x*tan(d*x)^2*tan(c)^4 + 16*b^2*d*x*tan(d*x)^2*tan(c
)^4 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
 - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 20*a*b*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*s
gn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)
^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*ta
n(c))*tan(d*x)^2*tan(c)^2 + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*ta
n(c) - 1))*tan(d*x)^4*tan(c)^2 - 12*b^2*arctan(-(tan(d*x) - tan(c))/(ta...
```

3.522.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= x\left(\frac{3\,a^2}{8}+\frac{b^2}{8}\right)+\frac{\left(\frac{3\,a^2}{8}+\frac{b^2}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{5\,a^2}{8}-\frac{b^2}{8}\right)\,\tan(c+d\,x)-\frac{a\,b}{2}}{d\,\left({\tan\left(c+d\,x\right)}^4+2\,{\tan\left(c+d\,x\right)}^2+1\right)}$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

output
```
x*((3*a^2)/8 + b^2/8) + (tan(c + d*x)*((5*a^2)/8 - b^2/8) - (a*b)/2 + tan(
c + d*x)^3*((3*a^2)/8 + b^2/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)
)
```

3.522. $\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$

3.523 $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$

3.523.1 Optimal result . . . 3607
3.523.2 Mathematica [A] (verified) . . . 3608
3.523.3 Rubi [A] (verified) . . . 3608
3.523.4 Maple [A] (verified) . . . 3612
3.523.5 Fricas [A] (verification not implemented) . . . 3612
3.523.6 Sympy [F] . . . 3613
3.523.7 Maxima [A] (verification not implemented) . . . 3613
3.523.8 Giac [B] (verification not implemented) . . . 3613
3.523.9 Mupad [B] (verification not implemented) . . . 3614

3.523.1 Optimal result

Integrand size = 21, antiderivative size = 163

$$\begin{aligned}\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{5(8a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{128d}+\frac{9ab\sec^7(c+dx)}{56d}\\ &+\frac{5(8a^2-b^2)\sec(c+dx)\tan(c+dx)}{128d}\\ &+\frac{5(8a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{192d}\\ &+\frac{(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)}{48d}\\ &+\frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}\end{aligned}$$

output

```
5/128*(8*a^2-b^2)*arctanh(sin(d*x+c))/d+9/56*a*b*sec(d*x+c)^7/d+5/128*(8*a
^2-b^2)*sec(d*x+c)*tan(d*x+c)/d+5/192*(8*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/
d+1/48*(8*a^2-b^2)*sec(d*x+c)^5*tan(d*x+c)/d+1/8*b*sec(d*x+c)^7*(a+b*tan(d
*x+c))/d
```

3.523.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

$$\begin{aligned}\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx=&\frac{5a^2\text{arctanh}(\sin(c+dx))}{16d}-\frac{5b^2\text{arctanh}(\sin(c+dx))}{128d}\\&+\frac{2ab\sec^7(c+dx)}{7d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{16d}\\&-\frac{5b^2\sec(c+dx)\tan(c+dx)}{128d}\\&+\frac{5a^2\sec^3(c+dx)\tan(c+dx)}{24d}\\&-\frac{5b^2\sec^3(c+dx)\tan(c+dx)}{192d}\\&+\frac{a^2\sec^5(c+dx)\tan(c+dx)}{6d}\\&-\frac{b^2\sec^5(c+dx)\tan(c+dx)}{48d}\\&+\frac{b^2\sec^7(c+dx)\tan(c+dx)}{8d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

output
```
(5*a^2*ArcTanh[Sin[c + d*x]])/(16*d) - (5*b^2*ArcTanh[Sin[c + d*x]])/(128*
d) + (2*a*b*Sec[c + d*x]^7)/(7*d) + (5*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*
d) - (5*b^2*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^2*Sec[c + d*x]^3*Tan
[c + d*x])/(24*d) - (5*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + (a^2*Sec
[c + d*x]^5*Tan[c + d*x])/(6*d) - (b^2*Sec[c + d*x]^5*Tan[c + d*x])/(48*d)
 + (b^2*Sec[c + d*x]^7*Tan[c + d*x])/(8*d)
```

3.523.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3991, 27, 3042, 3086, 15, 4159, 298, 215, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^7(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^7(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sec^7(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \sec^7(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sec^7(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^7\left(a^2+b^2\tan(c+dx)^2\right)dx+2ab\int \sec(c+dx)^7\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sec(c+dx)^7\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\int \sec^6(c+dx)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \sec(c+dx)^7\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{(1-\sin^2(c+dx))^5}d\sin(c+dx)}{d}+\frac{2ab\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{8}\left(8a^2-b^2\right)\int \frac{1}{(1-\sin^2(c+dx))^4}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{8(1-\sin^2(c+dx))^4}}{d}+\frac{2ab\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{8}\left(8a^2-b^2\right)\left(\frac{5}{6}\int \frac{1}{(1-\sin^2(c+dx))^3}d\sin(c+dx)+\frac{\sin(c+dx)}{6(1-\sin^2(c+dx))^3}\right)+\frac{b^2\sin(c+dx)}{8(1-\sin^2(c+dx))^4}}{d}+\frac{2ab\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \text{215}$$

3.523. $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$

$$\frac{\frac{1}{8}(8a^2-b^2)\left(\frac{5}{6}\left(\frac{3}{4}\int\frac{1}{(1-\sin^2(c+dx))^2}d\sin(c+dx)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{\sin(c+dx)}{6(1-\sin^2(c+dx))^3}\right)+\frac{b^2\sin(c+dx)}{8(1-\sin^2(c+dx))^4}}{d}+\frac{2ab\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{1}{8}(8a^2-b^2)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{\sin(c+dx)}{6(1-\sin^2(c+dx))^3}\right)+\frac{b^2}{8(1-}}{d}$$

$$\frac{2ab\sec^7(c+dx)}{7d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{8}(8a^2-b^2)\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{\sin(c+dx)}{6(1-\sin^2(c+dx))^3}\right)+\frac{b^2\sin(c+}{8(1-\sin^2(c+}}{d}$$

$$\frac{2ab\sec^7(c+dx)}{7d}$$

input
```
Int[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*a*b*Sec[c + d*x]^7)/(7*d) + ((b^2*Sin[c + d*x])/(8*(1 - Sin[c + d*x]^2)
^4) + ((8*a^2 - b^2)*(Sin[c + d*x]/(6*(1 - Sin[c + d*x]^2)^3) + (5*(Sin[c
+ d*x]/(4*(1 - Sin[c + d*x]^2)^2) + (3*(ArcTanh[Sin[c + d*x]]/2 + Sin[c +
d*x]/(2*(1 - Sin[c + d*x]^2))))/4))/6))/8)/d
```

3.523.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.523. $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$

3.523.4 Maple [A] (verified)

Time = 67.51 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{7\cos(dx+c)^7}+b^2}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{7\cos(dx+c)^7}+b^2}{d}$
risch	$-\, i e^{i(dx+c)}(840a^2 e^{14i(dx+c)}-105b^2 e^{14i(dx+c)}+6440a^2 e^{12i(dx+c)}-805b^2 e^{12i(dx+c)}+21448a^2 e^{10i(dx+c)}-2681b^2$

input

```
int(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(a^2*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c
)+5/16*ln(sec(d*x+c)+tan(d*x+c)))+2/7*a*b/cos(d*x+c)^7+b^2*(1/8*sin(d*x+c)
^3/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*x+c
)^4+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+t
an(d*x+c))))
```

3.523.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= 105\left(8a^2-b^2\right)\cos(dx+c)^8\log(\sin(dx+c)+1)-105\left(8a^2-b^2\right)\cos(dx+c)^8\log(-\sin(dx+c)+1)+$$

input

```
integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output

```
1/5376*(105*(8*a^2 - b^2)*cos(d*x + c)^8*log(sin(d*x + c) + 1) - 105*(8*a^
2 - b^2)*cos(d*x + c)^8*log(-sin(d*x + c) + 1) + 1536*a*b*cos(d*x + c) + 1
4*(15*(8*a^2 - b^2)*cos(d*x + c)^6 + 10*(8*a^2 - b^2)*cos(d*x + c)^4 + 8*(
8*a^2 - b^2)*cos(d*x + c)^2 + 48*b^2)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

3.523.6 Sympy [F]

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\sec^7{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**7, x)
```

3.523.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.35

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{7\,b^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1} - 15\,\log{(\sin{(dx+c)}+1)} + 15\,\log{(\sin{(dx+c)}-}\right.}{}$$

input
```
integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/5376*(7*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^
3 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4
 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
- 1)) - 56*a^2*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(
d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 1536*a*b/cos(d*x + c)^7)/d
```

3.523.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(151) = 302$.

Time = 0.59 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.68

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{105\,(8\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 105\,(8\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(1848\,a^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2688*(105*(8*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(8*a^2
- b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(1848*a^2*tan(1/2*d*x + 1/2*
c)^15 + 105*b^2*tan(1/2*d*x + 1/2*c)^15 - 5376*a*b*tan(1/2*d*x + 1/2*c)^14
 - 3416*a^2*tan(1/2*d*x + 1/2*c)^13 + 2779*b^2*tan(1/2*d*x + 1/2*c)^13 + 5
376*a*b*tan(1/2*d*x + 1/2*c)^12 + 6328*a^2*tan(1/2*d*x + 1/2*c)^11 + 6265*
b^2*tan(1/2*d*x + 1/2*c)^11 - 26880*a*b*tan(1/2*d*x + 1/2*c)^10 - 4760*a^2
*tan(1/2*d*x + 1/2*c)^9 + 12355*b^2*tan(1/2*d*x + 1/2*c)^9 + 26880*a*b*tan
(1/2*d*x + 1/2*c)^8 - 4760*a^2*tan(1/2*d*x + 1/2*c)^7 + 12355*b^2*tan(1/2*
d*x + 1/2*c)^7 - 16128*a*b*tan(1/2*d*x + 1/2*c)^6 + 6328*a^2*tan(1/2*d*x +
 1/2*c)^5 + 6265*b^2*tan(1/2*d*x + 1/2*c)^5 + 16128*a*b*tan(1/2*d*x + 1/2*
c)^4 - 3416*a^2*tan(1/2*d*x + 1/2*c)^3 + 2779*b^2*tan(1/2*d*x + 1/2*c)^3 -
 768*a*b*tan(1/2*d*x + 1/2*c)^2 + 1848*a^2*tan(1/2*d*x + 1/2*c) + 105*b^2*
tan(1/2*d*x + 1/2*c) + 768*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

3.523.9 Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.65

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{5\,a^2}{8}-\frac{5\,b^2}{64}\right)}{d} + \frac{\left(\frac{11\,a^2}{8}+\frac{5\,b^2}{64}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\left(\frac{397\,b^2}{192}-\frac{61\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^7,x)
```

output

```
(atanh(tan(c/2 + (d*x)/2))*((5*a^2)/8 - (5*b^2)/64))/d + ((4*a*b)/7 + tan(
c/2 + (d*x)/2)^15*((11*a^2)/8 + (5*b^2)/64) - tan(c/2 + (d*x)/2)^3*((61*a^
2)/24 - (397*b^2)/192) - tan(c/2 + (d*x)/2)^13*((61*a^2)/24 - (397*b^2)/19
2) + tan(c/2 + (d*x)/2)^5*((113*a^2)/24 + (895*b^2)/192) + tan(c/2 + (d*x)
/2)^11*((113*a^2)/24 + (895*b^2)/192) - tan(c/2 + (d*x)/2)^7*((85*a^2)/24
- (1765*b^2)/192) - tan(c/2 + (d*x)/2)^9*((85*a^2)/24 - (1765*b^2)/192) +
tan(c/2 + (d*x)/2)*((11*a^2)/8 + (5*b^2)/64) - (4*a*b*tan(c/2 + (d*x)/2)^2
)/7 + 12*a*b*tan(c/2 + (d*x)/2)^4 - 12*a*b*tan(c/2 + (d*x)/2)^6 + 20*a*b*t
an(c/2 + (d*x)/2)^8 - 20*a*b*tan(c/2 + (d*x)/2)^10 + 4*a*b*tan(c/2 + (d*x)
/2)^12 - 4*a*b*tan(c/2 + (d*x)/2)^14)/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(
c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*
tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14
 + tan(c/2 + (d*x)/2)^16 + 1))
```

3.523. $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$

3.524 $\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$

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3.524.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\begin{aligned}\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{(6a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{7ab\sec^5(c+dx)}{30d}\\&\quad+\frac{(6a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d}\\&\quad+\frac{(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{24d}\\&\quad+\frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}\end{aligned}$$

output
```
1/16*(6*a^2-b^2)*arctanh(sin(d*x+c))/d+7/30*a*b*sec(d*x+c)^5/d+1/16*(6*a^2
-b^2)*sec(d*x+c)*tan(d*x+c)/d+1/24*(6*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1
/6*b*sec(d*x+c)^5*(a+b*tan(d*x+c))/d
```

3.524.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.28

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{8d} - \frac{b^2\text{arctanh}(\sin(c+dx))}{16d} + \frac{2ab\sec^5(c+dx)}{5d} + \frac{3a^2\sec(c+dx)\tan(c+dx)}{8d} - \frac{b^2\sec(c+dx)\tan(c+dx)}{16d} + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{b^2\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}$$

input
```
Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
 (2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) -
(b^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Ta
n[c + d*x])/(6*d)
```

3.524.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3991, 27, 3042, 3086, 15, 4159, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sec^5(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \sec^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sec^5(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5\left(a^2+b^2\tan(c+dx)^2\right)dx+2ab\int \sec(c+dx)^5\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sec(c+dx)^5\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\int \sec^4(c+dx)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \sec(c+dx)^5\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^4}d\sin(c+dx)}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{6}\left(6a^2-b^2\right)\int \frac{1}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{6\left(1-\sin^2(c+dx)\right)^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}\left(6a^2-b^2\right)\left(\frac{3}{4}\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)+\frac{\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)+\frac{b^2\sin(c+dx)}{6\left(1-\sin^2(c+dx)\right)^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{6}\left(6a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)+\frac{b^2\sin(c+dx)}{6\left(1-\sin^2(c+dx)\right)^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

$$\Big\downarrow \text{219}$$

3.524. $\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$

$$\frac{\frac{1}{6}(6a^2-b^2)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{6(1-\sin^2(c+dx))^3}}{d}+\frac{2ab\sec^5(c+dx)}{5d}$$

input

```
Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output

```
(2*a*b*Sec[c + d*x]^5)/(5*d) + ((b^2*Sin[c + d*x])/(6*(1 - Sin[c + d*x]^2)
^3) + ((6*a^2 - b^2)*(Sin[c + d*x]/(4*(1 - Sin[c + d*x]^2)^2) + (3*(ArcTan
h[Sin[c + d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c + d*x]^2))))/4))/6)/d
```

3.524.3.1 Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.524.4 Maple [A] (verified)

Time = 15.70 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a^{2}\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^{5}}+b^{2}\left(\frac{\sin^{3}(dx+c)}{6\cos(dx+c)^{6}}+\frac{\sin^{3}(dx}{8\cos(dx}\right.}{d}$
default	$\frac{a^{2}\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^{5}}+b^{2}\left(\frac{\sin^{3}(dx+c)}{6\cos(dx+c)^{6}}+\frac{\sin^{3}(dx}{8\cos(dx}\right.}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(90a^{2}e^{10i(dx+c)}-15b^{2}e^{10i(dx+c)}+510a^{2}e^{8i(dx+c)}-85b^{2}e^{8i(dx+c)}+420a^{2}e^{6i(dx+c)}+570b^{2}e^{6i(dx+c)}+1536ia\right.}{120d\left(e^{2i(dx+\right.}}$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))+2/5*a*b/cos(d*x+c)^5+b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*
sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1
/16*ln(sec(d*x+c)+tan(d*x+c))))
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{15\,(6\,a^2-b^2)\cos{(dx+c)}^6\log{(\sin{(dx+c)}+1)}-15\,(6\,a^2-b^2)\cos{(dx+c)}^6\log{(-\sin{(dx+c)}+1)}+1}{480\,d\cos{(dx}}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
 b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3
*(6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

3.524.6 Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx=\int (a+b\tan{(c+dx)})^2\sec^5{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**5, x)
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.37

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{5\,b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log{(\sin{(dx+c)}+1)}+3\,\log{(\sin{(dx+c)}-1)}\right)-30\,a^2\Bigg(\frac{2\,(}{\mathrm{si}}}{480\,d}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d
*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d
```

3.524.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(121) = 242$.

Time = 0.54 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.62

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^1
1 + 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210
*a^2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan
(1/2*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^
5 + 390*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*
a^2*tan(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1
/2*d*x + 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/
2*c) + 96*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.524.9 Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.50

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{\left(\frac{5\,a^2}{4}+\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{47\,b^2}{24}-\frac{7\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\mathrm{ta}\right.}$$

$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{4}-\frac{b^2}{8}\right)}{d}$$

input

```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^5,x)
```

output

```
((4*a*b)/5 + tan(c/2 + (d*x)/2)^5*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2
)^7*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)^11*((5*a^2)/4 + b^2/8) - tan
(c/2 + (d*x)/2)^3*((7*a^2)/4 - (47*b^2)/24) - tan(c/2 + (d*x)/2)^9*((7*a^2
)/4 - (47*b^2)/24) + tan(c/2 + (d*x)/2)*((5*a^2)/4 + b^2/8) - (4*a*b*tan(c
/2 + (d*x)/2)^2)/5 + 8*a*b*tan(c/2 + (d*x)/2)^4 - 8*a*b*tan(c/2 + (d*x)/2)
^6 + 4*a*b*tan(c/2 + (d*x)/2)^8 - 4*a*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(
c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*t
an(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)
) + (atanh(tan(c/2 + (d*x)/2))*((3*a^2)/4 - b^2/8))/d
```

3.525 $\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$

3.525.1 Optimal result 3624
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3.525.1 Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{(4a^2-b^2)\,\text{arctanh}(\sin(c+dx))}{8d}+\frac{5ab\sec^3(c+dx)}{12d}+\frac{(4a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}$$

output
```
1/8*(4*a^2-b^2)*arctanh(sin(d*x+c))/d+5/12*a*b*sec(d*x+c)^3/d+1/8*(4*a^2-b
^2)*sec(d*x+c)*tan(d*x+c)/d+1/4*b*sec(d*x+c)^3*(a+b*tan(d*x+c))/d
```

3.525.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{a^2\text{arctanh}(\sin(c+dx))}{2d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{b^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2
*a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*
Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.525.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3991, 27, 3042, 3086, 15, 4159, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sec^3(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \sec^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sec^3(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3\left(a^2+b^2\tan(c+dx)^2\right)dx+2ab\int \sec(c+dx)^3\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$\int \sec(c+dx)^3\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\int \sec^2(c+dx)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \sec(c+dx)^3\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{(1-\sin^2(c+dx))^3}d\sin(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}(4a^2-b^2)\int \frac{1}{(1-\sin^2(c+dx))^2}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{4(1-\sin^2(c+dx))^2}}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}(4a^2-b^2)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{b^2\sin(c+dx)}{4(1-\sin^2(c+dx))^2}}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}(4a^2-b^2)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{b^2\sin(c+dx)}{4(1-\sin^2(c+dx))^2}}{d}+\frac{2ab\sec^3(c+dx)}{3d}$$

input

```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output

```
(2*a*b*Sec[c + d*x]^3)/(3*d) + ((b^2*Sin[c + d*x])/(4*(1 - Sin[c + d*x]^2)
^2) + ((4*a^2 - b^2)*(ArcTanh[Sin[c + d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c
 + d*x]^2))))/4)/d
```

3.525.3.1 Defintions of rubi rules used

rule 15

```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.525. $\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$

3.525.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^{3}}+b^{2}\left(\frac{\sin^{3}(dx+c)}{4\cos(dx+c)^{4}}+\frac{\sin^{3}(dx+c)}{8\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
default	$\frac{a^{2}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^{3}}+b^{2}\left(\frac{\sin^{3}(dx+c)}{4\cos(dx+c)^{4}}+\frac{\sin^{3}(dx+c)}{8\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\left(12a^{2}\mathrm{e}^{6i(dx+c)}-3b^{2}\mathrm{e}^{6i(dx+c)}+12a^{2}\mathrm{e}^{4i(dx+c)}+21b^{2}\mathrm{e}^{4i(dx+c)}+64iab\,\mathrm{e}^{4i(dx+c)}-12a^{2}\mathrm{e}^{2i(dx+c)}-21b^{2}\mathrm{e}^{2i(dx+\right.}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b
/cos(d*x+c)^3+b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+
c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \sec^{3}(c+dx)(a+b\tan(c+dx))^{2}\,dx$$
$$=\frac{3\left(4a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(4a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+32}{48\,d\cos\left(dx+c\right)^{4}}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^
2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2
 - b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.525.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

3.525.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log{(\sin{(dx+c)}+1)} + \log{(\sin{(dx+c)}-1)}\right) - 12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log{(\mathrm{s}}\right.}{48\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*
sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 32*a*b/cos(d*x + c)^3)/d
```

3.525.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(91) = 182.

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.52

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{3\left(4\,a^2-b^2\right)\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)} - 3\left(4\,a^2-b^2\right)\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)} + \frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3
*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1
/2*d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1
/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 -
16*a*b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/
2*d*x + 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.525.9 Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.18

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(\frac{7\,b^2}{4}-a^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(\frac{7\,b^2}{4}-\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\right.}$$

$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(a^2-\frac{b^2}{4}\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
((4*a*b)/3 + tan(c/2 + (d*x)/2)*(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2
+ b^2/4) - tan(c/2 + (d*x)/2)^3*(a^2 - (7*b^2)/4) - tan(c/2 + (d*x)/2)^5*(
a^2 - (7*b^2)/4) - (4*a*b*tan(c/2 + (d*x)/2)^2)/3 + 4*a*b*tan(c/2 + (d*x)/
2)^4 - 4*a*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2
+ (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atan
h(tan(c/2 + (d*x)/2))*(a^2 - b^2/4))/d
```

3.526 $\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$

3.526.1 Optimal result 3631
3.526.2 Mathematica [A] (verified) 3631
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3.526.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{3ab\sec(c+dx)}{2d} + \frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}$$

output
```
1/2*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+3/2*a*b*sec(d*x+c)/d+1/2*b*sec(d*x+c
)*(a+b*tan(d*x+c))/d
```

3.526.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{b^2\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{2ab\sec(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b
*Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.526.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3991, 27, 3042, 3086, 24, 4159, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)(a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \ 3991$$

$$\int \sec(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \ 27$$

$$\int \sec(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)\left(a^2+b^2\tan(c+dx)^2\right)dx+2ab\int \sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \ 3086$$

$$\int \sec(c+dx)\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\int 1d\sec(c+dx)}{d}$$

$$\Big\downarrow \ 24$$

$$\int \sec(c+dx)\left(a^2+b^2\tan(c+dx)^2\right)dx+\frac{2ab\sec(c+dx)}{d}$$

$$\Big\downarrow \ 4159$$

$$\frac{\int \frac{a^2-\left(a^2-b^2\right)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}$$

$$\Big\downarrow \ 298$$

$$\frac{\frac{1}{2}\left(2a^2-b^2\right)\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}}{d}+\frac{2ab\sec(c+dx)}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{2}\left(2a^2-b^2\right)\operatorname{arctanh}(\sin(c+dx))+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}}{d}+\frac{2ab\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*a*b*Sec[c + d*x])/d + (((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/2 + (b^2*S
in[c + d*x])/(2*(1 - Sin[c + d*x]^2)))/d
```

3.526.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.526.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{b^{2}\left(\frac{\sin^{3}\left(dx+c\right)}{2\cos\left(dx+c\right)^{2}}+\frac{\sin\left(dx+c\right)}{2}-\frac{\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2}\right)+\frac{2ab}{\cos\left(dx+c\right)}+a^{2}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$
default	$\frac{b^{2}\left(\frac{\sin^{3}\left(dx+c\right)}{2\cos\left(dx+c\right)^{2}}+\frac{\sin\left(dx+c\right)}{2}-\frac{\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2}\right)+\frac{2ab}{\cos\left(dx+c\right)}+a^{2}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$
risch	$\frac{b\,{\mathrm e}^{i\left(dx+c\right)}\left(-ib\,{\mathrm e}^{2i\left(dx+c\right)}+4a\,{\mathrm e}^{2i\left(dx+c\right)}+ib+4a\right)}{d\left({\mathrm e}^{2i\left(dx+c\right)}+1\right)^{2}}+\frac{a^{2}\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)}{d}-\frac{\ln\left({\mathrm e}^{i\left(dx+c\right)}+i\right)b^{2}}{2d}-\frac{a^{2}\ln\left({\mathrm e}^{i\left(dx+c\right)}-i\right)}{d}+$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+t
an(d*x+c)))+2*a*b/cos(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{(2\,a^2-b^2)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-(2\,a^2-b^2)\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)}+8\,ab\,\mathrm{co}}{4\,d\cos{(dx+c)}^2}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

3.526.6 Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx=\int (a+b\tan{(c+dx)})^2\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*sec(c + d*x), x)
```

3.526.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx=$$
$$-\frac{b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}\right)-4\,a^2\log{(\sec{(dx+c)}+\tan{(dx+c)})}}{4\,d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b/cos
(d*x + c))/d
```

3.526.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(59) = 118$.

Time = 0.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2(b^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-4ab\tan}{(\tan(}}{2d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan
(1/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2
*c)^2 - 1)^2)/d
```

3.526.9 Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4ab}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(2a^2-b^2)}{d}$$

input
```
int((a + b*tan(c + d*x))^2/cos(c + d*x),x)
```

output
```
(4*a*b + b^2*tan(c/2 + (d*x)/2)^3 + b^2*tan(c/2 + (d*x)/2) - 4*a*b*tan(c/2
 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (
atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d
```

3.527 $\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$

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3.527.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{b^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{(a^2-b^2)\sin(c+dx)}{d}$$

output
```
b^2*arctanh(sin(d*x+c))/d-2*a*b*cos(d*x+c)/d+(a^2-b^2)*sin(d*x+c)/d
```

3.527.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{-2ab\cos(c+dx)+b^2\bigl(-\log\bigl(\cos\bigl(\tfrac{1}{2}(c+dx)\bigr)-\sin\bigl(\tfrac{1}{2}(c+dx)\bigr)\bigr)+\log\bigl(\cos\bigl(\tfrac{1}{2}(c+dx)\bigr)+\sin\bigl(\tfrac{1}{2}(c+dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

3.527.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = $\{3042, 3991, 27, 3042, 3118, 4159, 299, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{ 3991}$$

$$\int \cos(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\sin(c+dx)dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \cos(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \sin(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)}dx+2ab\int \sin(c+dx)dx$$

$$\Big\downarrow \text{ 3118}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)}dx-\frac{2ab\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{a^2-(a^2-b^2)\sin^2(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{b^2\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\left(a^2-b^2\right)\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\left(a^2-b^2\right)\sin(c+dx)+b^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x])/d + (b^2*ArcTanh[Sin[c + d*x]] + (a^2 - b^2)*Sin[c +
 d*x])/d
```

3.527.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.527.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ab\cos(dx+c)+a^2\sin(dx+c)}{d}$
default	$\frac{b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ab\cos(dx+c)+a^2\sin(dx+c)}{d}$
risch	$-\frac{e^{i(dx+c)}ab}{d}-\frac{ia^2e^{i(dx+c)}}{2d}+\frac{ie^{i(dx+c)}b^2}{2d}-\frac{e^{-i(dx+c)}ab}{d}+\frac{ie^{-i(dx+c)}a^2}{2d}-\frac{ie^{-i(dx+c)}b^2}{2d}+\frac{\ln(e^{i(dx+c)}+i)b^2}{d}$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*a*b*cos(d*x+c)+a^2*sin(
d*x+c))
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int\cos(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{4\,ab\cos(dx+c)-b^2\log(\sin(dx+c)+1)+b^2\log(-\sin(dx+c)+1)-2\,(a^2-b^2)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x +
c) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

3.527.6 Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx=\int (a+b\tan{(c+dx)})^2\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x), x)
```

3.527.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{b^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}-2\sin{(dx+c)})-4\,ab\cos{(dx+c)}+2\,a^2\sin{(dx+c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
- 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

3.527.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(47) = 94$.

Time = 0.70 (sec) , antiderivative size = 1100, normalized size of antiderivative = 23.40

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/
(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2
*d*x)^2 - b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/
2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) -
4*b^2*tan(1/2*d*x)^2*tan(1/2*c) + b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(
1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - b^2*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*...
```

3.527.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,b^2\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

output
```
(2*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (4*a*b - tan(c/2 + (d*x)/2)*(2*a^2 -
 2*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.528 $\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$

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3.528.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{ab\cos^3(c+dx)}{6d}+\frac{(2a^2+b^2)\sin(c+dx)}{2d}-\frac{(2a^2+b^2)\sin^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}$$

output
```
-1/6*a*b*cos(d*x+c)^3/d+1/2*(2*a^2+b^2)*sin(d*x+c)/d-1/6*(2*a^2+b^2)*sin(d
*x+c)^3/d-1/2*b*cos(d*x+c)^3*(a+b*tan(d*x+c))/d
```

3.528.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3
)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

3.528.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3991, 27, 3042, 3045, 15, 4159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\cos^2(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \cos^3(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \cos^2(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^3}dx+2ab\int \cos(c+dx)^2\sin(c+dx)dx$$

$$\Big\downarrow \text{3045}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^3}dx-\frac{2ab\int \cos^2(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^3}dx-\frac{2ab\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(a^2-\left(a^2-b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin(c+dx)-\frac{1}{3}\left(a^2-b^2\right)\sin^3(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x] - ((a^2 - b^2)*Sin[c + d
*x]^3)/3)/d
```

3.528.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.528.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	52
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	52
risch	$-\frac{ab\cos(dx+c)}{2d}+\frac{3a^2\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^2}{4d}-\frac{ab\cos(3dx+3c)}{6d}+\frac{\sin(3dx+3c)a^2}{12d}-\frac{\sin(3dx+3c)b^2}{12d}$	93

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^2*sin(d*x+c)^3-2/3*a*b*cos(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*si
n(d*x+c))
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=-\frac{2\,ab\cos\left(dx+c\right)^3-\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+2\,a^2+b^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*si
n(d*x + c))/d
```

3.528.6 Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

3.528.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\,ab\cos(dx+c)^3 - b^2\sin(dx+c)^3 + \left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)a^2}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(
d*x + c))*a^2)/d
```

3.528.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11162 vs. 2(82) = 164.

Time = 16.59 (sec) , antiderivative size = 11162, normalized size of antiderivative = 124.02

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/48*(3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1
/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 ...
```

3.528.9 Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{2\left(\frac{\sin(c+d\,x)\,a^2\cos(c+d\,x)^2}{2}+\sin\left(c+d\,x\right)\,a^2-a\,b\cos\left(c+d\,x\right)^3-\frac{\sin(c+d\,x)\,b^2\cos(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)\,b^2}{2}\right)}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

output
```
(2*(a^2*sin(c + d*x) + (b^2*sin(c + d*x))/2 + (a^2*cos(c + d*x)^2*sin(c +
d*x))/2 - (b^2*cos(c + d*x)^2*sin(c + d*x))/2 - a*b*cos(c + d*x)^3))/(3*d)
```

3.529 $\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$

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3.529.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx=-\frac{3ab\cos^5(c+dx)}{20d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d}$$

output
```
-3/20*a*b*cos(d*x+c)^5/d+1/4*(4*a^2+b^2)*sin(d*x+c)/d-1/6*(4*a^2+b^2)*sin(
d*x+c)^3/d+1/20*(4*a^2+b^2)*sin(d*x+c)^5/d-1/4*b*cos(d*x+c)^5*(a+b*tan(d*x
+c))/d
```

3.529.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{-6ab\cos^5(c+dx)+15a^2\sin(c+dx)+5(-2a^2+b^2)\sin^3(c+dx)+3(a^2-b^2)\sin^5(c+dx)}{15d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-6*a*b*Cos[c + d*x]^5 + 15*a^2*Sin[c + d*x] + 5*(-2*a^2 + b^2)*Sin[c + d*
x]^3 + 3*(a^2 - b^2)*Sin[c + d*x]^5)/(15*d)
```

3.529.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3991, 27, 3042, 3045, 15, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\cos^4(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$\int \cos^5(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \cos^4(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^5}dx+2ab\int \cos(c+dx)^4\sin(c+dx)dx$$

$$\Big\downarrow \text{3045}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^5}dx-\frac{2ab\int \cos^4(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^5}dx-\frac{2ab\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)\left(a^2-\left(a^2-b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left((a-b)(a+b)\sin^4(c+dx)-\left(2a^2-b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}(a^2-b^2)\sin^5(c+dx)-\frac{1}{3}(2a^2-b^2)\sin^3(c+dx)+a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x] - ((2*a^2 - b^2)*Sin[c +
 d*x]^3)/3 + ((a^2 - b^2)*Sin[c + d*x]^5)/5)/d
```

3.529.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.529.4 Maple [A] (verified)

Time = 21.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
risch	$-\frac{ab\cos(dx+c)}{4d}+\frac{5a^2\sin(dx+c)}{8d}+\frac{\sin(dx+c)b^2}{8d}-\frac{ab\cos(5dx+5c)}{40d}+\frac{\sin(5dx+5c)a^2}{80d}-\frac{\sin(5dx+5c)b^2}{80d}-ab\,c$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2
/5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)
```

3.529.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\left(a^2-b^2\right)\cos\left(dx+c\right)^4+\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^2+8\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2
)*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

3.529.6 Sympy [F]

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\cos^5\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**5, x)
```

3.529.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^2+\left(3\,\sin\left(dx+c\right)^5-5\,\sin}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

3.529.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28204 vs. $2(104) = 208$.

Time = 34.08 (sec) , antiderivative size = 28204, normalized size of antiderivative = 247.40

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/960*(45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 60*pi*a*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^1
0*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^...
```

3.529.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{2\left(\frac{3\sin(c+d\,x)\,a^2\cos(c+d\,x)^4}{2} + 2\sin\left(c+d\,x\right)\,a^2\cos\left(c+d\,x\right)^2 + 4\sin\left(c+d\,x\right)\,a^2 - 3\,a\,b\cos\left(c+d\,x\right)^5 - \frac{3\sin(}{}\right.}{15\,d}$$

input
```
int(cos(c + d*x)^5*(a + b*tan(c + d*x))^2,x)
```

output
```
(2*(4*a^2*sin(c + d*x) + b^2*sin(c + d*x) + 2*a^2*cos(c + d*x)^2*sin(c + d
*x) + (3*a^2*cos(c + d*x)^4*sin(c + d*x))/2 + (b^2*cos(c + d*x)^2*sin(c +
d*x))/2 - (3*b^2*cos(c + d*x)^4*sin(c + d*x))/2 - 3*a*b*cos(c + d*x)^5))/(
15*d)
```

3.529. $\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$

3.530 $\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$

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3.530.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\begin{aligned}\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx &= -\frac{5ab\cos^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d}\\ &\quad-\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d}\\ &\quad-\frac{(6a^2+b^2)\sin^7(c+dx)}{42d}\\ &\quad-\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d}\end{aligned}$$

output
```
-5/42*a*b*cos(d*x+c)^7/d+1/6*(6*a^2+b^2)*sin(d*x+c)/d-1/6*(6*a^2+b^2)*sin(
d*x+c)^3/d+1/10*(6*a^2+b^2)*sin(d*x+c)^5/d-1/42*(6*a^2+b^2)*sin(d*x+c)^7/d
-1/6*b*cos(d*x+c)^7*(a+b*tan(d*x+c))/d
```

3.530.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

$$\begin{aligned}&\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx\\ &=\frac{-30ab\cos^7(c+dx)+105a^2\sin(c+dx)-35(3a^2-b^2)\sin^3(c+dx)+21(3a^2-2b^2)\sin^5(c+dx)-15(a^2}{105d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-30*a*b*Cos[c + d*x]^7 + 105*a^2*Sin[c + d*x] - 35*(3*a^2 - b^2)*Sin[c +
d*x]^3 + 21*(3*a^2 - 2*b^2)*Sin[c + d*x]^5 - 15*(a^2 - b^2)*Sin[c + d*x]^7
)/(105*d)
```

3.530.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 27, 3042, 3045, 15, 4159, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+b\tan(c+dx))^2}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \ 3991$$

$$\int \cos^7(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+\int 2ab\cos^6(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \ 27$$

$$\int \cos^7(c+dx)\left(a^2+b^2\tan^2(c+dx)\right)dx+2ab\int \cos^6(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^7}dx+2ab\int \cos(c+dx)^6\sin(c+dx)dx$$

$$\Big\downarrow \ 3045$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^7}dx-\frac{2ab\int \cos^6(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 15$$

$$\int \frac{a^2+b^2\tan(c+dx)^2}{\sec(c+dx)^7}dx-\frac{2ab\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \left(1-\sin^2(c+dx)\right)^2 \left(a^2-\left(a^2-b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(-((a-b)(a+b)\sin^6(c+dx))+\left(3a^2-2b^2\right)\sin^4(c+dx)-\left(3a^2-b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{7}\left(a^2-b^2\right)\sin^7(c+dx)+\frac{1}{5}\left(3a^2-2b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(3a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}$$

input
```
Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^7)/(7*d) + (a^2*Sin[c + d*x] - ((3*a^2 - b^2)*Sin[c +
 d*x]^3)/3 + ((3*a^2 - 2*b^2)*Sin[c + d*x]^5)/5 - ((a^2 - b^2)*Sin[c + d*x
]^7)/7)/d
```

3.530.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.530.4 Maple [A] (verified)

Time = 68.94 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos$
risch	$-\frac{5ab\cos(dx+c)}{32d}+\frac{35a^2\sin(dx+c)}{64d}+\frac{5\sin(dx+c)b^2}{64d}-\frac{ab\cos(7dx+7c)}{224d}+\frac{\sin(7dx+7c)a^2}{448d}-\frac{\sin(7dx+7c)b^2}{448d}-$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(
d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx =$$
$$-\frac{30\,ab\cos\left(dx+c\right)^7-\left(15\left(a^2-b^2\right)\cos\left(dx+c\right)^6+3\left(6\,a^2+b^2\right)\cos\left(dx+c\right)^4+4\left(6\,a^2+b^2\right)\cos\left(dx+c\right)}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2
+ b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*s
in(d*x + c))/d
```

3.530.6 Sympy [F]

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\cos^7\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**7, x)
```

3.530.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{30\,ab\cos{(dx+c)}^7 + 3\,(5\,\sin{(dx+c)}^7 - 21\,\sin{(dx+c)}^5 + 35\,\sin{(dx+c)}^3 - 35\,\sin{(dx+c)})\,a^2 - (1}{105\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 +
35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x
 + c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

3.530.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52002 vs. 2(126) = 252.

Time = 61.66 (sec) , antiderivative size = 52002, normalized size of antiderivative = 376.83

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output

```
-1/26880*(945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 8400*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta...
```

3.530.9 Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{16\,a^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,b^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^2\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^2\,{\cos\left(c+d\,x\right)}^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{4\,b^2\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{b^2\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^2\,{\cos\left(c+d\,x\right)}^6\,\sin\left(c+d\,x\right)}{7\,d}-\frac{2\,a\,b\,{\cos\left(c+d\,x\right)}^7}{7\,d}$$

input
```
int(cos(c + d*x)^7*(a + b*tan(c + d*x))^2,x)
```

output
```
(16*a^2*sin(c + d*x))/(35*d) + (8*b^2*sin(c + d*x))/(105*d) + (8*a^2*cos(c
 + d*x)^2*sin(c + d*x))/(35*d) + (6*a^2*cos(c + d*x)^4*sin(c + d*x))/(35*d
) + (a^2*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (4*b^2*cos(c + d*x)^2*sin(c
+ d*x))/(105*d) + (b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (b^2*cos(c +
d*x)^6*sin(c + d*x))/(7*d) - (2*a*b*cos(c + d*x)^7)/(7*d)
```

3.530. $\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$

3.531 $\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.531.1 Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a^2b\sec^8(c+dx)}{8d}+\frac{a^3\tan(c+dx)}{d} + \frac{a(a^2+b^2)\tan^3(c+dx)}{d}+\frac{b^3\tan^4(c+dx)}{4d} + \frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d}+\frac{b^3\tan^6(c+dx)}{2d} + \frac{a(a^2+9b^2)\tan^7(c+dx)}{7d}+\frac{3b^3\tan^8(c+dx)}{8d} + \frac{ab^2\tan^9(c+dx)}{3d}+\frac{b^3\tan^{10}(c+dx)}{10d}$$

output

```
3/8*a^2*b*sec(d*x+c)^8/d+a^3*tan(d*x+c)/d+a*(a^2+b^2)*tan(d*x+c)^3/d+1/4*b
^3*tan(d*x+c)^4/d+3/5*a*(a^2+3*b^2)*tan(d*x+c)^5/d+1/2*b^3*tan(d*x+c)^6/d+
1/7*a*(a^2+9*b^2)*tan(d*x+c)^7/d+3/8*b^3*tan(d*x+c)^8/d+1/3*a*b^2*tan(d*x+
c)^9/d+1/10*b^3*tan(d*x+c)^10/d
```

3.531.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\frac{1}{4}(a^2+b^2)^3(a+b\tan(c+dx))^4-\frac{6}{5}a(a^2+b^2)^2(a+b\tan(c+dx))^5+\frac{1}{2}(a^2+b^2)(5a^2+b^2)(a+b\tan(c+}{}$$

input
```
Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[
c + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4
*a*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan
[c + d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10
/10)/(b^7*d)
```

3.531.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 475, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^8(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^3}{b^6}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^3d(b\tan(c+dx))}{b^7d}$$

$$\Big\downarrow \text{475}$$

$$\int \left(\tan^9(c+dx)b^9+3\tan^7(c+dx)b^9+3\tan^5(c+dx)b^9+\tan^3(c+dx)b^9+3a\tan^8(c+dx)b^8+a\left(a^2+9b^2\right)\tan^6$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$a^3b^7\tan(c+dx)+\tfrac{3}{8}a^2\left(b^2\tan^2(c+dx)+b^2\right)^4+\tfrac{1}{7}ab^7\left(a^2+9b^2\right)\tan^7(c+dx)+\tfrac{3}{5}ab^7\left(a^2+3b^2\right)\tan^5(c+dx)+ab$$

input
```
Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

output
```
(a^3*b^7*Tan[c + d*x] + a*b^7*(a^2 + b^2)*Tan[c + d*x]^3 + (b^10*Tan[c + d
*x]^4)/4 + (3*a*b^7*(a^2 + 3*b^2)*Tan[c + d*x]^5)/5 + (b^10*Tan[c + d*x]^6
)/2 + (a*b^7*(a^2 + 9*b^2)*Tan[c + d*x]^7)/7 + (3*b^10*Tan[c + d*x]^8)/8 +
 (a*b^9*Tan[c + d*x]^9)/3 + (b^10*Tan[c + d*x]^10)/10 + (3*a^2*(b^2 + b^2*
Tan[c + d*x]^2)^4)/8)/(b^7*d)
```

3.531.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 475
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp
[d*n*c^(n - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Int[ExpandIntegran
d[((c + d*x)^n - d*n*c^(n - 1)*x)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c,
d}, x] && IGtQ[p, 0] && IGtQ[n, 0] && LeQ[n, p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.531.4 Maple [A] (verified)

Time = 191.80 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3a\,b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c\right.}{21\cos(dx+c)}\right.}{d}$
default	$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3a\,b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c\right.}{21\cos(dx+c)}\right.}{d}$
risch	$-\frac{32\left(-30ia^3\mathrm{e}^{2i(dx+c)}-360ia^3\mathrm{e}^{6i(dx+c)}-315a^2b\,\mathrm{e}^{12i(dx+c)}+105b^3\mathrm{e}^{12i(dx+c)}-3ia^3+ia\,b^2-630a^2b\,\mathrm{e}^{10i(dx+c)}-126b^3\mathrm{e}^{1}\right.}{}$

input
```
int(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*ta
n(d*x+c)+3/8*a^2*b/cos(d*x+c)^8+3*a*b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/2
1*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x
+c)^3/cos(d*x+c)^3)+b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4
/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^
4))
```

3.531.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{84\,b^3+105\,(3\,a^2b-b^3)\cos{(dx+c)}^2+8\,\left(16\,(3\,a^3-ab^2)\cos{(dx+c)}^9+8\,(3\,a^3-ab^2)\cos{(dx+c)}^7+6\,(3\right.}{840\,d\cos{(dx+c)}^1}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)
*cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos
(d*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^10)
```

3.531.6 Sympy [F]

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sec^8{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**8, x)
```

3.531.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{84\,b^3\tan{(dx+c)}^{10} + 280\,ab^2\tan{(dx+c)}^9 + 315\,(a^2b+b^3)\tan{(dx+c)}^8 + 120\,(a^3+9\,ab^2)\tan{(dx+c)}^7}{}$$

input
```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*(a^2*b + b^
3)*tan(d*x + c)^8 + 120*(a^3 + 9*a*b^2)*tan(d*x + c)^7 + 420*(3*a^2*b + b^
3)*tan(d*x + c)^6 + 504*(a^3 + 3*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(d*
x + c)^2 + 210*(9*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 840
*(a^3 + a*b^2)*tan(d*x + c)^3)/d
```

3.531.8 Giac [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{84\,b^3\tan{(dx+c)}^{10} + 280\,ab^2\tan{(dx+c)}^9 + 315\,a^2b\tan{(dx+c)}^8 + 315\,b^3\tan{(dx+c)}^8 + 120\,a^3\tan{(dx}{}$$

input

```
integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d
*x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*t
an(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a
^3*tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4
+ 210*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)
^3 + 1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

3.531.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.90

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan(c+d\,x)^5\left(\frac{3\,a^3}{5}+\frac{9\,a\,b^2}{5}\right)+\tan(c+d\,x)^7\left(\frac{a^3}{7}+\frac{9\,a\,b^2}{7}\right)+\tan(c+d\,x)^6\left(\frac{3\,a^2\,b}{2}+\frac{b^3}{2}\right)+\tan(c+d\,x)^4\Bigg(}{}$$

input

```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^8,x)
```

output

```
(tan(c + d*x)^5*((9*a*b^2)/5 + (3*a^3)/5) + tan(c + d*x)^7*((9*a*b^2)/7 +
a^3/7) + tan(c + d*x)^6*((3*a^2*b)/2 + b^3/2) + tan(c + d*x)^4*((9*a^2*b)/
4 + b^3/4) + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^10)/10 + (3*a^2*b*tan(c
+ d*x)^2)/2 + (a*b^2*tan(c + d*x)^9)/3 + a*tan(c + d*x)^3*(a^2 + b^2) + (3
*b*tan(c + d*x)^8*(a^2 + b^2))/8)/d
```

3.532 $\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.532.8 Giac [A] (verification not implemented) 3674
3.532.9 Mupad [B] (verification not implemented) 3675

3.532.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d} - \frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d} + \frac{(3a^2+b^2)(a+b\tan(c+dx))^6}{3b^5d} - \frac{4a(a+b\tan(c+dx))^7}{7b^5d} + \frac{(a+b\tan(c+dx))^8}{8b^5d}$$

output
```
1/4*(a^2+b^2)^2*(a+b*tan(d*x+c))^4/b^5/d-4/5*a*(a^2+b^2)*(a+b*tan(d*x+c))^
5/b^5/d+1/3*(3*a^2+b^2)*(a+b*tan(d*x+c))^6/b^5/d-4/7*a*(a+b*tan(d*x+c))^7/
b^5/d+1/8*(a+b*tan(d*x+c))^8/b^5/d
```

3.532.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\frac{1}{4}(a^2+b^2)^2(a+b\tan(c+dx))^4 - \frac{4}{5}a(a^2+b^2)(a+b\tan(c+dx))^5 + \frac{1}{3}(3a^2+b^2)(a+b\tan(c+dx))^6 -}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[
c + d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

3.532.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^6(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \Big((a+b\tan(c+dx))^7-4a(a+b\tan(c+dx))^6+2\left(3a^2+b^2\right)(a+b\tan(c+dx))^5-4a\left(a^2+b^2\right)(a+b\tan(c+}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\left(3a^2+b^2\right)(a+b\tan(c+dx))^6-\frac{4}{5}a\left(a^2+b^2\right)(a+b\tan(c+dx))^5+\frac{1}{4}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^4+\frac{1}{8}(a+b}{b^5d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[
c + d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

3.532.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.532.4 Maple [A] (verified)

Time = 69.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3ab^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8(\sin^3(dx+c}{105\cos(dx+c}}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3ab^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8(\sin^3(dx+c}{105\cos(dx+c}}{d}$
risch	$-\frac{16(-56ia^3e^{2i(dx+c)}+24iab^2e^{2i(dx+c)}-210a^2b\,e^{10i(dx+c)}+70b^3e^{10i(dx+c)}-322ia^3e^{6i(dx+c)}-7ia^3-420a^2b\,e^{8i(dx+c)}}{}$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/2*a^2*b/
cos(d*x+c)^6+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(
d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)
^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4))
```

3.532.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{105\,b^3+140\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2+8\left(8\left(7\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^7+4\left(7\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5+\right.}{840\,d\cos\left(dx+c\right)^8}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^
2)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x
+ c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8
)
```

3.532.6 Sympy [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx=\int\left(a+b\tan\left(c+dx\right)\right)^3\sec^6\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**6, x)
```

3.532.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{105\,b^3\tan{(dx+c)}^8 + 360\,ab^2\tan{(dx+c)}^7 + 140\,(3\,a^2b + 2\,b^3)\tan{(dx+c)}^6 + 168\,(a^3 + 6\,ab^2)\tan{(dx+}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 140*(3*a^2*b +
2*b^3)*tan(d*x + c)^6 + 168*(a^3 + 6*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*ta
n(d*x + c)^2 + 210*(6*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) +
 280*(2*a^3 + 3*a*b^2)*tan(d*x + c)^3)/d
```

3.532.8 Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{105\,b^3\tan{(dx+c)}^8 + 360\,ab^2\tan{(dx+c)}^7 + 420\,a^2b\tan{(dx+c)}^6 + 280\,b^3\tan{(dx+c)}^6 + 168\,a^3\tan{(dx}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d
*x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*t
an(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a
^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 +
 840*a^3*tan(d*x + c))/d
```

3.532.9 Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan(c+d\,x)^3\,\Bigg(\frac{2\,a^3}{3}+a\,b^2\Bigg)+\tan(c+d\,x)^5\,\Bigg(\frac{a^3}{5}+\frac{6\,a\,b^2}{5}\Bigg)+\tan(c+d\,x)^6\,\Bigg(\frac{a^2\,b}{2}+\frac{b^3}{3}\Bigg)+\tan(c+d\,x)^4\,\Bigg(\frac{3\,a^2\,b}{2}}{d}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^6,x)
```

output
```
(tan(c + d*x)^3*(a*b^2 + (2*a^3)/3) + tan(c + d*x)^5*((6*a*b^2)/5 + a^3/5)
 + tan(c + d*x)^6*((a^2*b)/2 + b^3/3) + tan(c + d*x)^4*((3*a^2*b)/2 + b^3/
4) + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^8)/8 + (3*a^2*b*tan(c + d*x)^2)/
2 + (3*a*b^2*tan(c + d*x)^7)/7)/d
```

3.533 $\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.533.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a^2+b^2)\,(a+b\tan(c+dx))^4}{4b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}$$

output
```
1/4*(a^2+b^2)*(a+b*tan(d*x+c))^4/b^3/d-2/5*a*(a+b*tan(d*x+c))^5/b^3/d+1/6*
(a+b*tan(d*x+c))^6/b^3/d
```

3.533.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a+b\tan(c+dx))^4\,(a^2+15b^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx))}{60b^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c
+ d*x]^2))/(60*b^3*d)
```

3.533.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \ 3987$$

$$\frac{\int \frac{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}{b^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \ 476$$

$$\frac{\int \left((a+b\tan(c+dx))^5-2a(a+b\tan(c+dx))^4+\left(a^2+b^2\right)(a+b\tan(c+dx))^3\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{4}\left(a^2+b^2\right)(a+b\tan(c+dx))^4+\frac{1}{6}(a+b\tan(c+dx))^6-\frac{2}{5}a(a+b\tan(c+dx))^5}{b^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)*(a + b*Tan[c + d*x])^4)/4 - (2*a*(a + b*Tan[c + d*x])^5)/5 +
 (a + b*Tan[c + d*x])^6/6)/(b^3*d)
```

3.533.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.533.4 Maple [A] (verified)

Time = 16.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3a\,b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+}{12\cos(dx-}}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3a\,b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+}{12\cos(dx-}}{d}$
risch	$-\frac{4(-15ia^3e^{8i(dx+c)}+45ia\,b^2e^{8i(dx+c)}-45a^2b\,e^{8i(dx+c)}+15b^3e^{8i(dx+c)}-50ia^3e^{6i(dx+c)}+30ia\,b^2e^{6i(dx+c)}-90a^2b\,e^{6i(}{15d(e^{2i}}$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3/4*a^2*b/cos(d*x+c)^4+3*a*b^
2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/6*
sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))
```

3.533.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{10\,b^3+15\,(3\,a^2b-b^3)\cos{(dx+c)}^2+4\left(2\,(5\,a^3-3\,ab^2)\cos{(dx+c)}^5+9\,ab^2\cos{(dx+c)}+(5\,a^3-3\,ab^2)\right.}{60\,d\cos{(dx+c)}^6}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*
cos(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*
sin(d*x + c))/(d*cos(d*x + c)^6)
```

3.533.6 Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\int (a+b\tan{(c+dx)})^3\sec^4{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**4, x)
```

3.533.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+90\,a^2b\tan\left(dx+c\right)^2+15\left(3\,a^2b+b^3\right)\tan\left(dx+c\right)^4+60\,a^3\tan}{60\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 90*a^2*b*tan(d*x +
 c)^2 + 15*(3*a^2*b + b^3)*tan(d*x + c)^4 + 60*a^3*tan(d*x + c) + 20*(a^3
+ 3*a*b^2)*tan(d*x + c)^3)/d
```

3.533.8 Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+45\,a^2b\tan\left(dx+c\right)^4+15\,b^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)}{60\,d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
 c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x +
 c)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

3.533.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan(c+d\,x)^3\left(\frac{a^3}{3}+a\,b^2\right)+\tan(c+d\,x)^4\left(\frac{3\,a^2\,b}{4}+\frac{b^3}{4}\right)+a^3\tan(c+d\,x)+\frac{b^3\tan(c+d\,x)^6}{6}+\frac{3\,a^2\,b\tan(c+d\,x)^2}{2}+}{d}$$

input

```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^4,x)
```

output

```
(tan(c + d*x)^3*(a*b^2 + a^3/3) + tan(c + d*x)^4*((3*a^2*b)/4 + b^3/4) + a
^3*tan(c + d*x) + (b^3*tan(c + d*x)^6)/6 + (3*a^2*b*tan(c + d*x)^2)/2 + (3
*a*b^2*tan(c + d*x)^5)/5)/d
```

3.533. $\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$

3.534 $\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.534.9 Mupad [B] (verification not implemented) 3686

3.534.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a+b\tan(c+dx))^4}{4bd}$$

output
```
1/4*(a+b*tan(d*x+c))^4/b/d
```

3.534.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. $2(22)=44$.

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx \\ =\frac{\tan(c+dx)\left(4a^3+6a^2b\tan(c+dx)+4ab^2\tan^2(c+dx)+b^3\tan^3(c+dx)\right)}{4d}$$

input
```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Tan[c + d*x]*(4*a^3 + 6*a^2*b*Tan[c + d*x] + 4*a*b^2*Tan[c + d*x]^2 + b^3
*Tan[c + d*x]^3))/(4*d)
```

3.534.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int (a+b\tan(c+dx))^3d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+b\tan(c+dx))^4}{4bd}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(a + b*Tan[c + d*x])^4/(4*b*d)
```

3.534.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.534.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(20) = 40$.

Time = 4.50 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

method	result
derivativedivides	$\frac{\frac{b^{3} \left(\sin^{4}\left(d x+c\right)\right)}{4 \cos\left(d x+c\right)^{4}}+\frac{a \,b^{2} \left(\sin^{3}\left(d x+c\right)\right)}{\cos\left(d x+c\right)^{3}}+\frac{3 a^{2} b}{2 \cos\left(d x+c\right)^{2}}+a^{3} \tan\left(d x+c\right)}{d}$
default	$\frac{\frac{b^{3} \left(\sin^{4}\left(d x+c\right)\right)}{4 \cos\left(d x+c\right)^{4}}+\frac{a \,b^{2} \left(\sin^{3}\left(d x+c\right)\right)}{\cos\left(d x+c\right)^{3}}+\frac{3 a^{2} b}{2 \cos\left(d x+c\right)^{2}}+a^{3} \tan\left(d x+c\right)}{d}$
risch	$-\frac{2 (-i a^{3} {\mathrm e}^{6 i \left(d x+c\right)}+3 i a \,b^{2} {\mathrm e}^{6 i \left(d x+c\right)}-3 a^{2} b \,{\mathrm e}^{6 i \left(d x+c\right)}+b^{3} {\mathrm e}^{6 i \left(d x+c\right)}-3 i a^{3} {\mathrm e}^{4 i \left(d x+c\right)}+3 i a \,b^{2} {\mathrm e}^{4 i \left(d x+c\right)}-6 a^{2} b \,{\mathrm e}^{4 i \left(d x+c\right)}-3 i a}{d \left({\mathrm e}^{2 i \left(d x+c\right)}+1\right)^{4}}$

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4+a*b^2*sin(d*x+c)^3/cos(d*x+c)^3+3/2
*a^2*b/cos(d*x+c)^2+a^3*tan(d*x+c))
```

3.534.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(20) = 40$.

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.55

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{b^{3}+2\,(3\,a^{2}b-b^{3})\cos\left(dx+c\right)^{2}+4\,\left(ab^{2}\cos\left(dx+c\right)+(a^{3}-ab^{2})\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^{4}}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.534.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

3.534.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(b\tan(dx+c)+a)^4}{4\,bd}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(b*tan(d*x + c) + a)^4/(b*d)
```

3.534.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(20) = 40.

Time = 0.74 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\begin{aligned}&\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx\\&=\frac{b^3\tan(dx+c)^4+4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+4\,a^3\tan(dx+c)}{4\,d}\end{aligned}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2
+ 4*a^3*tan(d*x + c))/d
```

3.534.9 Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan(c+d\,x)+\frac{3\,a^2\,b\tan(c+d\,x)^2}{2}+a\,b^2\tan(c+d\,x)^3+\frac{b^3\tan(c+d\,x)^4}{4}}{d}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(a^3*tan(c + d*x) + (b^3*tan(c + d*x)^4)/4 + (3*a^2*b*tan(c + d*x)^2)/2 +
a*b^2*tan(c + d*x)^3)/d
```

3.535 $\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.535.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{1}{2}a\left(a^2+3b^2\right)x-\frac{b^3\log(\cos(c+dx))}{d}-\frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}$$

output
```
1/2*a*(a^2+3*b^2)*x-b^3*ln(cos(d*x+c))/d-1/2*a*b^2*tan(d*x+c)/d-1/2*cos(d*
x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))^2/d
```

3.535.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. $2(86) = 172$.

Time = 0.84 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.66

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{5a^4b^2+2a^2b^4-b^6+\left(-3a^4b^2-2a^2b^4+b^6\right)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2b^6\log}{}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d
*x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] -
 b*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3
*(-b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt
[-b^2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*
b^6*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*T
an[c + d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3
*(-b^2)^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*
b^4)*Sin[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

3.535.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3987, 27, 495, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int \frac{b^4(a+b\tan(c+dx))^3}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b^3\int \frac{(a+b\tan(c+dx))^3}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 495}$$

$$\frac{b^3\left(\frac{\int \frac{(a+b\tan(c+dx))\left(a^2-b\tan(c+dx)a+2b^2\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))^2\left(b^2-ab\tan(c+dx)\right)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}\right)}{d}$$

$$\Big\downarrow \text{ 657}$$

$$\frac{b^3\left(\frac{\int \left(\frac{a^3+3b^2a+2b^3\tan(c+dx)}{\tan^2(c+dx)b^2+b^2}-a\right)d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))^2(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{b^3\left(\frac{\frac{a(a^2+3b^2)\arctan(\tan(c+dx))}{b}-ab\tan(c+dx)+b^2\log\left(b^2\tan^2(c+dx)+b^2\right)}{2b^2}-\frac{(a+b\tan(c+dx))^2(b^2-ab\tan(c+dx))}{2b^2(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^3*(((a*(a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/b + b^2*Log[b^2 + b^2*Tan[c
+ d*x]^2] - a*b*Tan[c + d*x])/(2*b^2) - ((a + b*Tan[c + d*x])^2*(b^2 - a*b
*Tan[c + d*x]))/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2))))/d
```

3.535.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))    Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.535.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2b\left(\cos^2(dx+c)\right)}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2b\left(\cos^2(dx+c)\right)}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
risch	$ix\,b^3+\frac{a^3x}{2}+\frac{3xa\,b^2}{2}-\frac{3\,\mathrm{e}^{2i(dx+c)}b\,a^2}{8d}+\frac{\mathrm{e}^{2i(dx+c)}b^3}{8d}-\frac{i\mathrm{e}^{2i(dx+c)}a^3}{8d}+\frac{3i\mathrm{e}^{2i(dx+c)}a\,b^2}{8d}-\frac{3\,\mathrm{e}^{-2i(dx+c)}b\,a^2}{8d}$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a*b^2*(-1/2*sin(d*x+c)*cos(d
*x+c)+1/2*d*x+1/2*c)-3/2*a^2*b*cos(d*x+c)^2+a^3*(1/2*sin(d*x+c)*cos(d*x+c)
+1/2*d*x+1/2*c))
```

3.535.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{2\,b^3\log\left(-\cos\left(dx+c\right)\right)-(a^3+3\,ab^2)dx+(3\,a^2b-b^3)\cos\left(dx+c\right)^2-(a^3-3\,ab^2)\cos\left(dx+c\right)\sin\left(dx\right.}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos
(d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

3.535.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**2, x)
```

3.535.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\log\left(\tan(dx+c)^2+1\right) + (a^3+3\,ab^2)(dx+c) - \frac{3\,a^2b-b^3-(a^3-3\,ab^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (3*a^2*b -
b^3 - (a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

3.535.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(81) = 162.

Time = 0.89 (sec) , antiderivative size = 561, normalized size of antiderivative = 6.52

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{2\,a^3dx\tan(dx)^2\tan(c)^2 + 6\,ab^2dx\tan(dx)^2\tan(c)^2 - 2\,b^3\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\,\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx}{}$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(2*a^3*d*x*tan(d*x)^2*tan(c)^2 + 6*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 2*b
^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 2*a^3*d*x*tan(d*x)^2
 + 6*a*b^2*d*x*tan(d*x)^2 + 2*a^3*d*x*tan(c)^2 + 6*a*b^2*d*x*tan(c)^2 - 3*
a^2*b*tan(d*x)^2*tan(c)^2 + b^3*tan(d*x)^2*tan(c)^2 - 2*b^3*log(4*(tan(d*x
)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
tan(c)^2 + 1))*tan(d*x)^2 - 2*a^3*tan(d*x)^2*tan(c) + 6*a*b^2*tan(d*x)^2*t
an(c) - 2*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 - 2*a^3*tan(d*x)*tan(c
)^2 + 6*a*b^2*tan(d*x)*tan(c)^2 + 2*a^3*d*x + 6*a*b^2*d*x + 3*a^2*b*tan(d*
x)^2 - b^3*tan(d*x)^2 + 12*a^2*b*tan(d*x)*tan(c) - 4*b^3*tan(d*x)*tan(c) +
 3*a^2*b*tan(c)^2 - b^3*tan(c)^2 - 2*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*ta
n(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) + 2*
a^3*tan(d*x) - 6*a*b^2*tan(d*x) + 2*a^3*tan(c) - 6*a*b^2*tan(c) - 3*a^2*b
+ b^3)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.535.9 Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.64

$$\begin{aligned}\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{b^3\,\ln\left(\frac{1}{\cos(c+d\,x)^2}\right)}{2\,d}+\frac{b^3\cos\left(c+d\,x\right)^2}{2\,d}\\ &\quad+\frac{a^3\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{2\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^2}{2\,d}\\ &\quad+\frac{3\,a\,b^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{2\,d}\\ &\quad+\frac{a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}\\ &\quad-\frac{3\,a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}\end{aligned}$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

output
```
(b^3*log(1/cos(c + d*x)^2))/(2*d) + (b^3*cos(c + d*x)^2)/(2*d) + (a^3*atan
(sin(c + d*x)/cos(c + d*x)))/(2*d) - (3*a^2*b*cos(c + d*x)^2)/(2*d) + (3*a
*b^2*atan(sin(c + d*x)/cos(c + d*x)))/(2*d) + (a^3*cos(c + d*x)*sin(c + d*
x))/(2*d) - (3*a*b^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.535. $\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$

3.536 $\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.536.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\begin{aligned}\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{3}{8}a\left(a^2+b^2\right)x-\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d}\\&\quad+\frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d}\end{aligned}$$

output
```
3/8*a*(a^2+b^2)*x-3/8*a*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1
/4*cos(d*x+c)^3*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

3.536.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 488 vs. 2(84) = 168.

Time = 1.22 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.81

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{39a^6b^2+41a^4b^4+21a^2b^6+3b^8-4b^2\left(a^2+b^2\right)^2\left(3a^2+b^2\right)\cos(2(c+dx))+b^2\left(-3a^2+b^2\right)\left(a^2+b^2\right)^2\cos(4}{}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
(39*a^6*b^2 + 41*a^4*b^4 + 21*a^2*b^6 + 3*b^8 - 4*b^2*(a^2 + b^2)^2*(3*a^2
 + b^2)*Cos[2*(c + d*x)] + b^2*(-3*a^2 + b^2)*(a^2 + b^2)^2*Cos[4*(c + d*x
)] - 6*a^7*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 18*a^5*(-b^2)^(3/
2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 6*a*b^4*(-b^2)^(3/2)*Log[Sqrt[-b^2]
- b*Tan[c + d*x]] - 18*a^3*(-b^2)^(5/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] +
 6*a^7*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 18*a^3*b^4*Sqrt[-b^2]
*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 6*a*b^6*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*
Tan[c + d*x]] - 18*a^5*(-b^2)^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 8*a
^7*b*Sin[2*(c + d*x)] + 16*a^5*b^3*Sin[2*(c + d*x)] + 8*a^3*b^5*Sin[2*(c +
 d*x)] + a^7*b*Sin[4*(c + d*x)] - a^5*b^3*Sin[4*(c + d*x)] - 5*a^3*b^5*Sin
[4*(c + d*x)] - 3*a*b^7*Sin[4*(c + d*x)])/(32*b*(a^2 + b^2)^2*d)
```

3.536.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3987, 27, 490, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^6(a+b\tan(c+dx))^3}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\int \frac{(a+b\tan(c+dx))^3}{(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{490}$$

$$\frac{b^5\left(\frac{3a\int \frac{(a+b\tan(c+dx))^2}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{4b(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow 487$$

$$\frac{b^5\left(\frac{3a\left(\frac{\left(a^2+b^2\right)\int \frac{1}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}\right)}{4b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{4b(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$$\Big\downarrow 216$$

$$\frac{b^5\left(\frac{3a\left(\frac{\left(a^2+b^2\right)\arctan(\tan(c+dx))}{2b^3}-\frac{(a+b\tan(c+dx))\left(b^2-ab\tan(c+dx)\right)}{2b^2\left(b^2\tan^2(c+dx)+b^2\right)}\right)}{4b^2}+\frac{\tan(c+dx)(a+b\tan(c+dx))^3}{4b(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^5*((Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*(b^2 + b^2*Tan[c + d*x]^2
)^2) + (3*a*(((a^2 + b^2)*ArcTan[Tan[c + d*x]])/(2*b^3) - ((a + b*Tan[c +
d*x])*(b^2 - a*b*Tan[c + d*x]))/(2*b^2*(b^2 + b^2*Tan[c + d*x]^2))))/(4*b^
2)))/d
```

3.536.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 490
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] - Simp[c*(n/(2*a*(
p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.536.4 Maple [A] (verified)

Time = 20.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4}+3a\,b^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b\left(\cos^4(dx+c)\right)}{4}+a^3\left(\left(\cos^3(dx+c)+3\right.\right.}{d}$
default	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4}+3a\,b^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b\left(\cos^4(dx+c)\right)}{4}+a^3\left(\left(\cos^3(dx+c)+3\right.\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3xa\,b^2}{8}-\frac{3b\cos(4dx+4c)a^2}{32d}+\frac{b^3\cos(4dx+4c)}{32d}+\frac{a^3\sin(4dx+4c)}{32d}-\frac{3a\sin(4dx+4c)b^2}{32d}-\frac{3b\cos(2dx+2}{8d}$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*b^3*sin(d*x+c)^4+3*a*b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*
x+c)*cos(d*x+c)+1/8*d*x+1/8*c)-3/4*a^2*b*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)
^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

3.536.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{4\,b^3\cos\left(dx+c\right)^2+2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^4-3\left(a^3+ab^2\right)dx-\left(2\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+3\left(a^3\right.\right.}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a
*b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x +
c))*sin(d*x + c))/d
```

3.536.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\cos^4\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**4, x)
```

3.536.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\left(a^3+ab^2\right)(dx+c)-\frac{4\,b^3\tan(dx+c)^2-3\,\left(a^3+ab^2\right)\tan(dx+c)^3+6\,a^2b+2\,b^3-\left(5\,a^3-3\,ab^2\right)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(3*(a^3 + a*b^2)*(d*x + c) - (4*b^3*tan(d*x + c)^2 - 3*(a^3 + a*b^2)*t
an(d*x + c)^3 + 6*a^2*b + 2*b^3 - (5*a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x
 + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

3.536.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2496 vs. $2(79) = 158$.

Time = 12.25 (sec) , antiderivative size = 2496, normalized size of antiderivative = 29.71

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/64*(9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^3
*d*x*tan(d*x)^4*tan(c)^4 + 24*a*b^2*d*x*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*s
gn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan
(d*x)^4*tan(c)^4 + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan
(c)^2 + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 18*
a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^
4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4
*tan(c)^4 + 48*a^3*d*x*tan(d*x)^4*tan(c)^2 + 48*a*b^2*d*x*tan(d*x)^4*tan(c
)^2 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d
*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 48*a^3*d*x*tan(d*x)^2*tan(c)^4 + 48*
a*b^2*d*x*tan(d*x)^2*tan(c)^4 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 30*a^2*b*t
an(d*x)^4*tan(c)^4 - 6*b^3*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*sgn(2*tan(d*x)
^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*
x) - 2*tan(c))*tan(d*x)^4 + 36*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn
(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d
*x)^2*tan(c)^2 + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -...
```

3.536.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\,a^3\,x}{8} - \frac{6\,a^2\,b-\tan(c+d\,x)^3\,\left(3\,a^3+3\,a\,b^2\right)+2\,b^3+\tan(c+d\,x)\,\left(3\,a\,b^2-5\,a^3\right)+4\,b^3\tan(c+d\,x)^2}{d\,\left(8\tan\left(c+d\,x\right)^4+16\tan\left(c+d\,x\right)^2+8\right)} + \frac{3\,a\,b^2\,x}{8}$$

input

```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^3,x)
```

output

```
(3*a^3*x)/8 - (6*a^2*b - tan(c + d*x)^3*(3*a*b^2 + 3*a^3) + 2*b^3 + tan(c
+ d*x)*(3*a*b^2 - 5*a^3) + 4*b^3*tan(c + d*x)^2)/(d*(16*tan(c + d*x)^2 + 8
*tan(c + d*x)^4 + 8)) + (3*a*b^2*x)/8
```

3.536. $\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$

3.537 $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.537.1 Optimal result

Integrand size = 21, antiderivative size = 159

$$\begin{aligned}\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{3a(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{16d} \\ &\quad+\frac{3a(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} \\ &\quad+\frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d} \\ &\quad+\frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} \\ &\quad+\frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d}\end{aligned}$$

output

```
3/16*a*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+3/16*a*(2*a^2-b^2)*sec(d*x+c)*tan
(d*x+c)/d+1/8*a*(2*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1/7*b*sec(d*x+c)^5*(
a+b*tan(d*x+c))^2/d+1/70*b*sec(d*x+c)^5*(32*a^2-4*b^2+15*a*b*tan(d*x+c))/d
```

3.537.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 637 vs. $2(159) = 318$.

Time = 3.32 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.01

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\sec^7(c+dx)\left(10752a^2b+1536b^3+3584(3a^2b-b^3)\cos(2(c+dx))-4410a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c +
d*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a
*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*C
os[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7
*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2
)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 105*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 4340*a^3*Sin[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4
*(c + d*x)] - 1400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210
*a*b^2*Sin[6*(c + d*x)]))/(35840*d)
```

3.537.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3991, 3042, 4159, 27, 298, 215, 215, 219, 4861, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^5(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sec^5(c+dx)\tan(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^5\left(a^3+3b^2\tan(c+dx)^2a\right)dx+\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{\left(1-\sin^2(c+dx)\right)^4}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\int \frac{a^2-\left(a^2-3b^2\right)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^4}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\int \frac{1}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)^3}\right)}{d}$$

$$\Big\downarrow \text{215}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)+\frac{\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)^3}\right)}{d}$$

$$\Big\downarrow \text{215}$$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)+\frac{b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)^3}\right)}{d}$$

$$\Big\downarrow \text{219}$$

3.537. $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

$$\int \sec(c+dx)^5\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}$$

$$\Big\downarrow \text{4861}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{\int b\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^8(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{b\int \left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^8(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{b\int \left(b^2\sec^8(c+dx)+\left(3a^2-b^2\right)\sec^6(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-b^2\right)\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}\right)+\frac{\sin(c+dx)}{4(1-\sin^2(c+dx))^2}\right)+\frac{b^2\sin(c+dx)}{2(1-\sin^2(c+dx))^3}\right)}{d}-\frac{b(-\frac{1}{5}\left(3a^2-b^2\right)\sec^5(c+dx)-\frac{1}{7}b^2\sec^7(c+dx))}{d}$$

input
```
Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b*(-1/5*((3*a^2 - b^2)*Sec[c + d*x]^5) - (b^2*Sec[c + d*x]^7)/7))/d) +
(a*((b^2*Sin[c + d*x])/(2*(1 - Sin[c + d*x]^2)^3) + ((2*a^2 - b^2)*(Sin[c
+ d*x]/(4*(1 - Sin[c + d*x]^2)^2) + (3*(ArcTanh[Sin[c + d*x]]/2 + Sin[c +
d*x]/(2*(1 - Sin[c + d*x]^2))))/4))/2))/d
```

3.537. $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

3.537.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

3.537. $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

3.537.4 Maple [A] (verified)

Time = 31.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.56

method	result
derivativedivides	$a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3a\,b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3}{8\cos(}\right.$
default	$a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3a\,b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3}{8\cos(}\right.$
risch	$-\frac{e^{i(dx+c)}\left(-1400ia^3e^{2i(dx+c)}+700ia\,b^2e^{2i(dx+c)}+1400ia^3e^{10i(dx+c)}-105ia\,b^2e^{12i(dx+c)}+210ia^3e^{12i(dx+c)}-210ia^3-\right.}{}$

input
```
int(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))+3/5*a^2*b/cos(d*x+c)^5+3*a*b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^
6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*
x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/
35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c
)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

3.537.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{105\,(2\,a^3-ab^2)\cos{(dx+c)}^7\log{(\sin{(dx+c)}+1)}-105\,(2\,a^3-ab^2)\cos{(dx+c)}^7\log{(-\sin{(dx+c)}+1}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*
a^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*
b - b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*c
os(d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
 c)^7)
```

3.537.6 Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sec^5{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**5, x)
```

3.537.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.31

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log{(\sin{(dx+c)}+1)}+3\,\log{(\sin{(dx+c)}-1)}\right)-70\,a^3\Bigg(}{}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x
 + c)^2 - 5)*b^3/cos(d*x + c)^7)/d
```

3.537.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(146) = 292$.

Time = 0.84 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.92

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,(350\,a^3\tan(\frac{1}{2}}{}}{}$$

input
```
integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/
2*c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2
*c)^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)
^11 + 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^1
0 + 630*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5
040*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*
a^2*b*tan(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*t
an(1/2*d*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan
(1/2*d*x + 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x
 + 1/2*c)^3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x +
1/2*c)^2 - 224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) -
 105*a*b^2*tan(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c
)^2 - 1)^7)/d
```

3.537.9 Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.66

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{8\,d}$$

$$-\ \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{6\,a^2\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\mathrm{ta}$$

input

```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^5,x)
```

output

```
(3*a*atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/(8*d) - (tan(c/2 + (d*x)/2)*
((3*a*b^2)/8 + (5*a^3)/4) + (6*a^2*b)/5 + tan(c/2 + (d*x)/2)^3*((11*a*b^2)
/2 - 3*a^3) - tan(c/2 + (d*x)/2)^11*((11*a*b^2)/2 - 3*a^3) - tan(c/2 + (d*
x)/2)^13*((3*a*b^2)/8 + (5*a^3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 +
(9*a^3)/4) - tan(c/2 + (d*x)/2)^9*((31*a*b^2)/8 + (9*a^3)/4) - tan(c/2 + (
d*x)/2)^10*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 - (4*b^
3)/5) + tan(c/2 + (d*x)/2)^8*(18*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^6*(24
*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^4*((66*a^2*b)/5 + (8*b^3)/5) - (4*b^3
)/35 + 6*a^2*b*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(
c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*
tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 -
1))
```

3.538 $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.538.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\begin{aligned}\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{a(4a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{8d}\\ &\quad+\frac{a(4a^2-3b^2)\sec(c+dx)\tan(c+dx)}{8d}\\ &\quad+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d}\\ &\quad+\frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+21ab\tan(c+dx)\right)}{60d}\end{aligned}$$

output
```
1/8*a*(4*a^2-3*b^2)*arctanh(sin(d*x+c))/d+1/8*a*(4*a^2-3*b^2)*sec(d*x+c)*t
an(d*x+c)/d+1/5*b*sec(d*x+c)^3*(a+b*tan(d*x+c))^2/d+1/60*b*sec(d*x+c)^3*(4
8*a^2-8*b^2+21*a*b*tan(d*x+c))/d
```

3.538.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 464 vs. 2(126) = 252.

Time = 2.46 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.68

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{\sec^5(c+dx)\left(960a^2b+64b^3+320(3a^2b-b^3)\cos(2(c+dx))-300a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input

```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output

```
(Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
 - 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225
*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*
Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5
*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^
2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 240*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Si
n[4*(c + d*x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

3.538.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3991, 3042, 4159, 27, 298, 215, 219, 4861, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec^3(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sec^3(c+dx)\tan(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3\left(a^3+3b^2\tan(c+dx)^2a\right)dx+\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\int \frac{a^2-\left(a^2-3b^2\right)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^3}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\int \frac{1}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}$$

$$\Big\downarrow \text{215}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\int \sec(c+dx)^3\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}$$

$$\Big\downarrow \text{4861}$$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{\int b\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^6(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{b\int \left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^6(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

3.538. $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{b\int\left(b^2\sec^6(c+dx)+\left(3a^2-b^2\right)\sec^4(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\left(\frac{1}{4}\left(4a^2-3b^2\right)\left(\frac{1}{2}\text{arctanh}(\sin(c+dx))+\frac{\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)+\frac{3b^2\sin(c+dx)}{4\left(1-\sin^2(c+dx)\right)^2}\right)}{d}-\frac{b\left(-\frac{1}{3}\left(3a^2-b^2\right)\sec^3(c+dx)-\frac{1}{5}b^2\sec^5(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b*(-1/3*((3*a^2 - b^2)*Sec[c + d*x]^3) - (b^2*Sec[c + d*x]^5)/5))/d) +
(a*((3*b^2*Sin[c + d*x])/(4*(1 - Sin[c + d*x]^2)^2) + ((4*a^2 - 3*b^2)*(Ar
cTanh[Sin[c + d*x]]/2 + Sin[c + d*x]/(2*(1 - Sin[c + d*x]^2))))/4))/d
```

3.538.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

3.538. $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$

3.538.4 Maple [A] (verified)

Time = 9.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2b}{\cos(dx+c)^3}+3a\,b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx}{}}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2b}{\cos(dx+c)^3}+3a\,b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx}{}}{d}$
risch	$-\frac{e^{i(dx+c)}(60ia^3e^{8i(dx+c)}-45ia\,b^2e^{8i(dx+c)}+120ia^3e^{6i(dx+c)}+270ia\,b^2e^{6i(dx+c)}-480a^2b\,e^{6i(dx+c)}+160b^3e^{6i(dx+c)}-}{60d(e^{2}}$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*b/c
os(d*x+c)^3+3*a*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*
x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/5*sin(d*x+c)^4
/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-
1/15*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

3.538.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.17

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(-\sin(dx+c)+}{240\,d\cos}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a
^3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b
 - b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(
d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.538.6 Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**3, x)
```

3.538.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\right.}{240\,d}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin
(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*
x + c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/co
s(d*x + c)^5)/d
```

3.538.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(115) = 230.

Time = 0.75 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.64

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(60\,a^3\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1
/2*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)
^8 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 7
20*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2
*b*tan(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/
2*d*x + 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*
x + 1/2*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c)
 - 45*a*b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*
c)^2 - 1)^5)/d
```

3.538.9 Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.33

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(a^3+\frac{3ab^2}{4}\right)-2a^2b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{9ab^2}{2}-2a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{9ab^2}{2}-2a^3\right)+\tan\big(\frac{c}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\big(\frac{c}{2}+\right.}$$

$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3ab^2}{4}-a^3\right)}{d}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^9*((3*a*b^2)/4 + a^3) - 2*a^2*b - tan(c/2 + (d*x)/2)^3
*((9*a*b^2)/2 - 2*a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 - 2*a^3) + tan(
c/2 + (d*x)/2)^2*(4*a^2*b - (4*b^3)/3) - tan(c/2 + (d*x)/2)^4*(8*a^2*b + (
4*b^3)/3) + tan(c/2 + (d*x)/2)^6*(12*a^2*b - 4*b^3) + (4*b^3)/15 - tan(c/2
 + (d*x)/2)*((3*a*b^2)/4 + a^3) - 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(
c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (atanh(tan(c/2 + (d*x)
/2))*((3*a*b^2)/4 - a^3))/d
```

3.539 $\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.539.1 Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a(2a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{b\sec(c+dx)\left(4(4a^2-b^2)+5ab\tan(c+dx)\right)}{6d}$$

output
```
1/2*a*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)*(a+b*tan(d*x+c)
)^2/d+1/6*b*sec(d*x+c)*(16*a^2-4*b^2+5*a*b*tan(d*x+c))/d
```

3.539.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 293 vs. $2(91) = 182$.

Time = 2.44 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{36a^2b-10b^3-6a(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2
- b^2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*
x]^3*Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
2 + b^3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

3.539.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {3042, 3991, 3042, 4159, 27, 298, 219, 4861, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3991}$$

$$\int \sec(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sec(c+dx)\tan(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)\left(a^3+3b^2\tan(c+dx)^2a\right)dx+\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+\frac{a\int \frac{a^2-\left(a^2-3b^2\right)\sin^2(c+dx)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+a\left(\frac{1}{2}\left(2a^2-3b^2\right)\int\frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\int \sec(c+dx)\tan(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx+a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}$$

$$\Big\downarrow \ 4861$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{\int b\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{b\int\left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)\sec^4(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 244$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{b\int\left(b^2\sec^4(c+dx)+\left(3a^2-b^2\right)\sec^2(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{a\left(\frac{1}{2}\left(2a^2-3b^2\right)\text{arctanh}(\sin(c+dx))+\frac{3b^2\sin(c+dx)}{2\left(1-\sin^2(c+dx)\right)}\right)}{d}-\frac{b\left(-\left(3a^2-b^2\right)\sec(c+dx)-\frac{1}{3}b^2\sec^3(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b*(-((3*a^2 - b^2)*Sec[c + d*x]) - (b^2*Sec[c + d*x]^3)/3))/d) + (a*(((
2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/2 + (3*b^2*Sin[c + d*x])/(2*(1 - Sin
[c + d*x]^2))))/d
```

3.539. $\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$

3.539.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4861
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

3.539.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{b^{3}\left(\frac{\sin^{4}(dx+c)}{3\cos(dx+c)^{3}}-\frac{\sin^{4}(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}\right)+3a\,b^{2}\left(\frac{\sin^{3}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$
default	$\frac{b^{3}\left(\frac{\sin^{4}(dx+c)}{3\cos(dx+c)^{3}}-\frac{\sin^{4}(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}\right)+3a\,b^{2}\left(\frac{\sin^{3}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$
risch	$-\frac{b\,\mathrm{e}^{i(dx+c)}\left(9iab\,\mathrm{e}^{4i(dx+c)}-18a^{2}\mathrm{e}^{4i(dx+c)}+6b^{2}\mathrm{e}^{4i(dx+c)}-36a^{2}\mathrm{e}^{2i(dx+c)}+4b^{2}\mathrm{e}^{2i(dx+c)}-9iab-18a^{2}+6b^{2}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}-\frac{a^{3}\ln(\mathrm{e}^{i}}{}$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2
+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(
d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b/cos(d*x+c)+a^3*ln(sec(d*x+c)
+tan(d*x+c)))
```

3.539.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \sec(c+dx)(a+b\tan(c+dx))^{3}\,dx$$
$$=\frac{3\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3
- 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*s
in(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3
)
```

3.539.6 Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx=\int(a+b\tan(c+dx))^3\sec(c+dx)\,dx$$

input

```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

output

```
Integral((a + b*tan(c + d*x))**3*sec(c + d*x), x)
```

3.539.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx=$$

$$-\frac{9\,ab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-12\,a^3\log(\sec(dx+c)+\tan(dx+}{12\,d}$$

input

```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
-1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) - 12*a^3*log(sec(d*x + c) + tan(d*x + c)) - 36*
a^2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

3.539.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(82) = 164$.

Time = 0.71 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{3\left(2a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(2a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(9ab^2\tan\left(\frac{1}{2}dx+\right.\right.}{}}{6d}$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3
*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*
c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
 12*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b +
 4*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.539.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.76

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(3\,a\,b^2-2\,a^3\right)}{d}$$

$$-\frac{6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(12\,a^2\,b-4\,b^3\right)-\frac{4\,b^3}{3}+3\,a\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3\,a\,b^2\tan\left(\frac{c}{2}+\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int((a + b*tan(c + d*x))^3/cos(c + d*x),x)
```

output
```
- (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 - 2*a^3))/d - (6*a^2*b - tan(c/2 + (
d*x)/2)^2*(12*a^2*b - 4*b^3) - (4*b^3)/3 + 3*a*b^2*tan(c/2 + (d*x)/2) + 6*
a^2*b*tan(c/2 + (d*x)/2)^4 - 3*a*b^2*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 +
 (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.540 $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.540.1 Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3ab^2\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} - \frac{b\sec(c+dx)\left(2(a^2-b^2)+ab\tan(c+dx)\right)}{d}$$

output
```
3*a*b^2*arctanh(sin(d*x+c))/d-cos(d*x+c)*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))
^2/d-b*sec(d*x+c)*(2*a^2-2*b^2+a*b*tan(d*x+c))/d
```

3.540.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.56

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\,(-3a^2b+3b^3+(-3a^2b+b^3)\cos(2(c+dx))-6ab^2\cos(c+dx)\,(\log(\cos(\tfrac{1}{2}(c+dx))-\sin(\tfrac{1}{2}}{2d}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*
b^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d
*x)]))/(2*d)
```

3.540.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {3042, 3991, 3042, 4147, 27, 244, 2009, 4159, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\int \sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{4147}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{\int b\cos^2(c+dx)\left(3a^2-b^2+b^2\sec^2(c+dx)\right)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{b\int \cos^2(c+dx)\left(3a^2-b^2+b^2\sec^2(c+dx)\right)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{b\int \left(b^2+\left(3a^2-b^2\right)\cos^2(c+dx)\right)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)}dx+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{a\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\int \frac{a^2-\left(a^2-3b^2\right)\sin^2(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{a\left(3b^2\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)+\left(a^2-3b^2\right)\sin(c+dx)\right)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{a\left(\left(a^2-3b^2\right)\sin(c+dx)+3b^2\text{arctanh}(\sin(c+dx))\right)}{d}+\frac{b\left(b^2\sec(c+dx)-\left(3a^2-b^2\right)\cos(c+dx)\right)}{d}$$

input

```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

output

```
(b*(-((3*a^2 - b^2)*Cos[c + d*x]) + b^2*Sec[c + d*x]))/d + (a*(3*b^2*ArcTa
nh[Sin[c + d*x]] + (a^2 - 3*b^2)*Sin[c + d*x]))/d
```

3.540.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

3.540. $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

3.540. $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$

3.540.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{b^{3}\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+(2+\sin^{2}(dx+c))\cos(dx+c)\right)+3a\,b^{2}(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^{2}b\cos(dx+c)+a^{3}\sin}{d}$
default	$\frac{b^{3}\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+(2+\sin^{2}(dx+c))\cos(dx+c)\right)+3a\,b^{2}(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^{2}b\cos(dx+c)+a^{3}\sin}{d}$
risch	$-\frac{3\,\mathrm{e}^{i(dx+c)}b\,a^{2}}{2d}+\frac{\mathrm{e}^{i(dx+c)}b^{3}}{2d}-\frac{i\mathrm{e}^{i(dx+c)}a^{3}}{2d}+\frac{3i\mathrm{e}^{i(dx+c)}a\,b^{2}}{2d}-\frac{3\,\mathrm{e}^{-i(dx+c)}b\,a^{2}}{2d}+\frac{\mathrm{e}^{-i(dx+c)}b^{3}}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}a}{2d}$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(-s
in(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-3*a^2*b*cos(d*x+c)+a^3*sin(d*x+c))
```

3.540.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int\cos(c+dx)(a+b\tan(c+dx))^{3}\,dx$$

$$=\frac{3\,ab^{2}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^{2}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^{3}-2\left(3\,a^{2}b-b^{3}\right)}{2\,d\cos\left(dx+c\right)}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log
(-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 -
3*a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

3.540.6 Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx=\int (a+b\tan{(c+dx)})^3\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x), x)
```

3.540.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos{(dx+c)}\right)+3\,ab^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}-2\,\sin{(dx+c)})-6}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3
*sin(d*x + c))/d
```

3.540.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4309 vs. 2(83) = 166.

Time = 2.62 (sec) , antiderivative size = 4309, normalized size of antiderivative = 51.30

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-1/4*(3*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*pi
*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 6*a*b^2*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x
)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(
1/2*c)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 -
4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^3
+ 12*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 8*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4
 + 12*pi*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*ar...
```

3.540.9 Mupad [B] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.38

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(6\,a\,b^2-2\,a^3\right)-6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-2\,a^3\right)+4\,b^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

input

```
int(cos(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

output

```
(6*a*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 2
*a^3) - 6*a^2*b - tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^3) + 4*b^3 + 6*a^2*b*t
an(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.541 $\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.541.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\begin{aligned}\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx &= -\frac{2(a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{3d}\\ &\quad -\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}\end{aligned}$$

output

```
-2/3*(a^2+b^2)*cos(d*x+c)*(b-a*tan(d*x+c))/d-1/3*cos(d*x+c)^3*(b-a*tan(d*x
+c))*(a+b*tan(d*x+c))^2/d
```

3.541.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\begin{aligned}&\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx\\ &= \frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin(c}{12d}\end{aligned}$$

input

```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output

```
(-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(
5*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

3.541.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3991, 3042, 4159, 2009, 4857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^3(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \cos^2(c+dx)\sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)^3}dx+\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^2}dx+\frac{\int \left(a^3-a\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^2}dx+\frac{a^3\sin(c+dx)-\frac{1}{3}a\left(a^2-3b^2\right)\sin^3(c+dx)}{d}$$

$$\Big\downarrow \text{4857}$$

$$\frac{a^3\sin(c+dx)-\frac{1}{3}a\left(a^2-3b^2\right)\sin^3(c+dx)}{d}-\frac{\int \left(\left(1-\cos^2(c+dx)\right)b^3+3a^2\cos^2(c+dx)b\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sin(c+dx)-\frac{1}{3}a\left(a^2-3b^2\right)\sin^3(c+dx)}{d}-\frac{a^2b\cos^3(c+dx)-\frac{1}{3}b^3\cos^3(c+dx)+b^3\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

output
```
-((b^3*Cos[c + d*x] + a^2*b*Cos[c + d*x]^3 - (b^3*Cos[c + d*x]^3)/3)/d) +
(a^3*Sin[c + d*x] - (a*(a^2 - 3*b^2)*Sin[c + d*x]^3)/3)/d
```

3.541.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.541. $\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$

3.541.4 Maple [A] (verified)

Time = 10.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{b^{3}\left(2+\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)}{3}+a\,b^{2}\left(\sin^{3}\left(dx+c\right)\right)-a^{2}b\left(\cos^{3}\left(dx+c\right)\right)+\frac{a^{3}\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3}}{d}$
default	$\frac{-\frac{b^{3}\left(2+\sin^{2}\left(dx+c\right)\right)\cos\left(dx+c\right)}{3}+a\,b^{2}\left(\sin^{3}\left(dx+c\right)\right)-a^{2}b\left(\cos^{3}\left(dx+c\right)\right)+\frac{a^{3}\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)}{3}}{d}$
risch	$-\frac{3b\cos\left(dx+c\right)a^{2}}{4d}-\frac{3b^{3}\cos\left(dx+c\right)}{4d}+\frac{3a^{3}\sin\left(dx+c\right)}{4d}+\frac{3a\sin\left(dx+c\right)b^{2}}{4d}-\frac{b\cos\left(3dx+3c\right)a^{2}}{4d}+\frac{b^{3}\cos\left(3dx+3c\right)}{12d}+$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-a^2*b*cos(d*x
+c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

3.541.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \cos^{3}(c+dx)(a+b\tan(c+dx))^{3}\,dx = -\frac{3\,b^{3}\cos\left(dx+c\right)+\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{3}-\left(2\,a^{3}+3\,ab^{2}+\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b
^2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.541.6 Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**3, x)
```

3.541.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{3a^2b\cos(dx+c)^3 - 3ab^2\sin(dx+c)^3 + (\sin(dx+c)^3 - 3\sin(dx+c))a^3 - (\cos(dx+c)^3 - 3\cos(d}{3d}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/3*(3*a^2*b*cos(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 -
3*sin(d*x + c))*a^3 - (cos(d*x + c)^3 - 3*cos(d*x + c))*b^3)/d
```

3.541.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24430 vs. 2(68) = 136.

Time = 142.57 (sec) , antiderivative size = 24430, normalized size of antiderivative = 349.00

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/768*(72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105*pi*b^3*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2
*c)^6 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 72*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 105*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6
+ 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*...
```

3.541.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\frac{\sin(c+d\,x)\,a^3\cos(c+d\,x)^2}{3} + \frac{2\,\sin(c+d\,x)\,a^3}{3} - a^2\,b\cos\left(c+d\,x\right)^3 - \sin\left(c+d\,x\right)\,a\,b^2\cos\left(c+d\,x\right)^2 + \sin\left(c+d\,x\right)\,a\,b}{d}$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

output
```
((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*
b*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x
) - a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

3.542 $\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.542.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\begin{aligned}\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx\\ &= -\frac{2(4a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{15d}\\ &\quad-\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d}\\ &\quad+\frac{\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{5d}\end{aligned}$$

output

```
-2/15*(4*a^2+b^2)*cos(d*x+c)*(b-a*tan(d*x+c))/d-1/15*cos(d*x+c)^3*(b-4*a*t
an(d*x+c))*(a+b*tan(d*x+c))^2/d+1/5*cos(d*x+c)^4*sin(d*x+c)*(a+b*tan(d*x+c
))^3/d
```

3.542.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{-9a^2b\cos^5(c+dx)+15a^3\sin(c+dx)-5a(2a^2-3b^2)\sin^3(c+dx)+3a(a^2-3b^2)\sin^5(c+dx)+b^3\cos(}{15d}$$

input
```
Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
(-9*a^2*b*Cos[c + d*x]^5 + 15*a^3*Sin[c + d*x] - 5*a*(2*a^2 - 3*b^2)*Sin[c
 + d*x]^3 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]^5 + b^3*Cos[c + d*x]*(-2 + 2/Sq
rt[Cos[c + d*x]^2] - Sin[c + d*x]^2 + 3*Sin[c + d*x]^4))/(15*d)
```

3.542.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 3042, 4159, 27, 290, 2009, 4857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^5}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^5(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \cos^4(c+dx)\sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)^5}dx+\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{4159}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{\int a\left(1-\sin^2(c+dx)\right)\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{a\int \left(1-\sin^2(c+dx)\right)\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{a\int \left(\left(a^2-3b^2\right)\sin^4(c+dx)-\left(2a^2-3b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^4}dx+\frac{a\left(\frac{1}{5}\left(a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(2a^2-3b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4857}$$

$$\frac{a\left(\frac{1}{5}\left(a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(2a^2-3b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\int \left(3a^2b\cos^4(c+dx)+b^3\left(1-\cos^2(c+dx)\right)\cos^2(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{1}{5}\left(a^2-3b^2\right)\sin^5(c+dx)-\frac{1}{3}\left(2a^2-3b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\frac{3}{5}a^2b\cos^5(c+dx)-\frac{1}{5}b^3\cos^5(c+dx)+\frac{1}{3}b^3\cos^3(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

output
```
-(((b^3*Cos[c + d*x]^3)/3 + (3*a^2*b*Cos[c + d*x]^5)/5 - (b^3*Cos[c + d*x]
^5)/5)/d) + (a*(a^2*Sin[c + d*x] - ((2*a^2 - 3*b^2)*Sin[c + d*x]^3)/3 + ((
a^2 - 3*b^2)*Sin[c + d*x]^5)/5))/d
```

3.542.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.542.4 Maple [A] (verified)

Time = 40.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{b^{3}\left(-\frac{\left(\cos^{3}(dx+c)\right)\left(\sin^{2}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)+3a\,b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)-3a^{2}b}{d}$
default	$\frac{b^{3}\left(-\frac{\left(\cos^{3}(dx+c)\right)\left(\sin^{2}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)+3a\,b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{5}+\frac{\left(2+\cos^{2}(dx+c)\right)\sin(dx+c)}{15}\right)-3a^{2}b}{d}$
risch	$-\frac{3b\cos(dx+c)a^{2}}{8d}-\frac{b^{3}\cos(dx+c)}{8d}+\frac{5a^{3}\sin(dx+c)}{8d}+\frac{3a\sin(dx+c)b^{2}}{8d}-\frac{3b\cos(5dx+5c)a^{2}}{80d}+\frac{b^{3}\cos(5dx+5c)}{80d}+$

input
```
int(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/5*
sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*a^2*b*cos(d*
x+c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

3.542.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{5\,b^3\cos\left(dx+c\right)^3+3\,(3\,a^2b-b^3)\cos\left(dx+c\right)^5-(3\,(a^3-3\,ab^2)\cos\left(dx+c\right)^4+8\,a^3+6\,ab^2+(4\,a^3+3}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
 3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c
)^2)*sin(d*x + c))/d
```

3.542.6 Sympy [F]

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\cos^5{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**5, x)
```

3.542.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx =$$

$$-\frac{9\,a^2b\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^3+3\left(3\,\sin\left(dx+c\right)^5-5}{15\,d}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15
*sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*co
s(d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

3.542.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56572 vs. 2(102) = 204.

Time = 274.60 (sec) , antiderivative size = 56572, normalized size of antiderivative = 538.78

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/15360*(1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 105*pi*b^3*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^10*tan(1/2*c)^10 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 180
0*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1
/2*c)^10 + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^10*tan(1/2*c)^10 + 1800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - ...
```

3.542.9 Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.40

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{2\left(\frac{3\sin(c+d\,x)\,a^3\cos(c+d\,x)^4}{2} + 2\sin\left(c+d\,x\right)\,a^3\cos\left(c+d\,x\right)^2 + 4\sin\left(c+d\,x\right)\,a^3 - \frac{9\,a^2\,b\cos(c+d\,x)^5}{2} - 9\sin(c+d\,x\right.}{15\,d}$$

input
```
int(cos(c + d*x)^5*(a + b*tan(c + d*x))^3,x)
```

output
```
(2*(4*a^3*sin(c + d*x) - (5*b^3*cos(c + d*x)^3)/2 + (3*b^3*cos(c + d*x)^5)
/2 - (9*a^2*b*cos(c + d*x)^5)/2 + 2*a^3*cos(c + d*x)^2*sin(c + d*x) + (3*a
^3*cos(c + d*x)^4*sin(c + d*x))/2 + 3*a*b^2*sin(c + d*x) + (3*a*b^2*cos(c
+ d*x)^2*sin(c + d*x))/2 - (9*a*b^2*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d
)
```

3.543 $\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$

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3.543.1 Optimal result

Integrand size = 21, antiderivative size = 142

$$\begin{aligned}\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{8a(2a^2+b^2)\sin(c+dx)}{35d} - \frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} \\ &\quad + \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} \\ &\quad - \frac{2\cos^3(c+dx)\left(b(6a^2+b^2)-a(4a^2-b^2)\tan(c+dx)\right)}{35d}\end{aligned}$$

output
```
8/35*a*(2*a^2+b^2)*sin(d*x+c)/d-3/35*cos(d*x+c)^5*(b-2*a*tan(d*x+c))*(a+b*
tan(d*x+c))^2/d+1/7*cos(d*x+c)^6*sin(d*x+c)*(a+b*tan(d*x+c))^3/d-2/35*cos(
d*x+c)^3*(b*(6*a^2+b^2)-a*(4*a^2-b^2)*tan(d*x+c))/d
```

3.543.2 Mathematica [A] (verified)

Time = 3.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\begin{aligned}&\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx \\ &= \frac{-30a^2b\cos^7(c+dx)+b^3\cos^5(c+dx)(-9+5\cos(2(c+dx)))+4b^3\sqrt{\cos^2(c+dx)}\sec(c+dx)+2a\sin(c}{70d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

output
```
(-30*a^2*b*Cos[c + d*x]^7 + b^3*Cos[c + d*x]^5*(-9 + 5*Cos[2*(c + d*x)]) +
 4*b^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x] + 2*a*Sin[c + d*x]*(35*a^2 - 35*(
a^2 - b^2)*Sin[c + d*x]^2 + 21*(a^2 - 2*b^2)*Sin[c + d*x]^4 - 5*(a^2 - 3*b
^2)*Sin[c + d*x]^6))/(70*d)
```

3.543.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3991, 3042, 4159, 27, 290, 2009, 4857, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3}{\sec(c+dx)^7}dx$$

$$\Big\downarrow \text{3991}$$

$$\int \cos^7(c+dx)\left(a^3+3b^2\tan^2(c+dx)a\right)dx+\int \cos^6(c+dx)\sin(c+dx)\left(\tan^2(c+dx)b^3+3a^2b\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^3+3b^2\tan(c+dx)^2a}{\sec(c+dx)^7}dx+\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx$$

$$\Big\downarrow \text{4159}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{\int a\left(1-\sin^2(c+dx)\right)^2\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{a\int \left(1-\sin^2(c+dx)\right)^2\left(a^2-\left(a^2-3b^2\right)\sin^2(c+dx)\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{a\int \left(-\left(\left(a^2-3b^2\right)\sin^6(c+dx)\right)+3\left(a^2-2b^2\right)\sin^4(c+dx)-3\left(a^2-b^2\right)\sin^2(c+dx)+a^2\right)d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{\sin(c+dx)\left(\tan(c+dx)^2b^3+3a^2b\right)}{\sec(c+dx)^6}dx+\frac{a\left(-\frac{1}{7}\left(a^2-3b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(a^2-2b^2\right)\sin^5(c+dx)-\left(a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4857}$$

$$\frac{a\left(-\frac{1}{7}\left(a^2-3b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(a^2-2b^2\right)\sin^5(c+dx)-\left(a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\int \left(3a^2b\cos^6(c+dx)+b^3\left(1-\cos^2(c+dx)\right)\cos^4(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(-\frac{1}{7}\left(a^2-3b^2\right)\sin^7(c+dx)+\frac{3}{5}\left(a^2-2b^2\right)\sin^5(c+dx)-\left(a^2-b^2\right)\sin^3(c+dx)+a^2\sin(c+dx)\right)}{d}-\frac{\frac{3}{7}a^2b\cos^7(c+dx)-\frac{1}{7}b^3\cos^7(c+dx)+\frac{1}{5}b^3\cos^5(c+dx)}{d}$$

input

```
Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

output

```
-(((b^3*Cos[c + d*x]^5)/5 + (3*a^2*b*Cos[c + d*x]^7)/7 - (b^3*Cos[c + d*x]
^7)/7)/d) + (a*(a^2*Sin[c + d*x] - (a^2 - b^2)*Sin[c + d*x]^3 + (3*(a^2 -
2*b^2)*Sin[c + d*x]^5)/5 - ((a^2 - 3*b^2)*Sin[c + d*x]^7)/7))/d
```

3.543.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 290

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3991

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Module[{k}, Int[Sec[e + f*x]^m*Sum[Binomial[n, 2*k]*a^(n -
 2*k)*b^(2*k)*Tan[e + f*x]^(2*k), {k, 0, n/2}], x] + Int[Sec[e + f*x]^m*Tan
[e + f*x]*Sum[Binomial[n, 2*k + 1]*a^(n - 2*k - 1)*b^(2*k + 1)*Tan[e + f*x]
^(2*k), {k, 0, (n - 1)/2}], x]] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2,
 0] && IntegerQ[(m - 1)/2] && IGtQ[n, 0]
```

rule 4159

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

rule 4857

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.543.4 Maple [A] (verified)

Time = 119.54 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{b^{3}\left(-\frac{\left(\cos^{5}(dx+c)\right)\left(\sin^{2}(dx+c)\right)}{7}-\frac{2\left(\cos^{5}(dx+c)\right)}{35}\right)+3a\,b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\mathrm{si}}{35}\right.}{d}$
default	$\frac{b^{3}\left(-\frac{\left(\cos^{5}(dx+c)\right)\left(\sin^{2}(dx+c)\right)}{7}-\frac{2\left(\cos^{5}(dx+c)\right)}{35}\right)+3a\,b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\mathrm{si}}{35}\right.}{d}$
risch	$-\frac{15b\cos(dx+c)a^{2}}{64d}-\frac{3b^{3}\cos(dx+c)}{64d}+\frac{35a^{3}\sin(dx+c)}{64d}+\frac{15a\sin(dx+c)b^{2}}{64d}-\frac{3b\cos(7dx+7c)a^{2}}{448d}+\frac{b^{3}\cos(7dx+7}{448d}$

input
```
int(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3*(-1/7*cos(d*x+c)^5*sin(d*x+c)^2-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/7*
sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c
))-3/7*a^2*b*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*
cos(d*x+c)^2)*sin(d*x+c))
```

3.543.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{7\,b^3\cos\left(dx+c\right)^5+5\,(3\,a^2b-b^3)\cos\left(dx+c\right)^7-\left(5\,(a^3-3\,ab^2)\cos\left(dx+c\right)^6+3\,(2\,a^3+ab^2)\cos\left(dx+\right.\right.}{35\,d}$$

input
```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
 3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a
*b^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.543.6 Sympy [F]

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\cos^7\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**7, x)
```

3.543.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-(15}{35\,d}$$

input

```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
-1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
b^3)/d
```

3.543.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101962 vs. 2(136) = 272.

Time = 84.49 (sec) , antiderivative size = 101962, normalized size of antiderivative = 718.04

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

input

```
integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
1/17920*(945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 210*pi*b^3*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^1
4*tan(1/2*c)^14 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 2205*
pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2
*c)^14 + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^14*tan(1/2*c)^14 + 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*...
```

3.543. $\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$

3.543.9 Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{16\,a^3\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^3\cos\left(c+d\,x\right)^5}{5\,d}\\ &\quad+\frac{b^3\cos\left(c+d\,x\right)^7}{7\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^7}{7\,d}\\ &\quad+\frac{8\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}\\ &\quad+\frac{6\,a^3\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}\\ &\quad+\frac{a^3\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}\\ &\quad+\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)}{35\,d}\\ &\quad+\frac{4\,a\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}\\ &\quad+\frac{3\,a\,b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}\\ &\quad-\frac{3\,a\,b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}\end{aligned}$$

input
```
int(cos(c + d*x)^7*(a + b*tan(c + d*x))^3,x)
```

output
```
(16*a^3*sin(c + d*x))/(35*d) - (b^3*cos(c + d*x)^5)/(5*d) + (b^3*cos(c + d
*x)^7)/(7*d) - (3*a^2*b*cos(c + d*x)^7)/(7*d) + (8*a^3*cos(c + d*x)^2*sin(
c + d*x))/(35*d) + (6*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a^3*cos(c
 + d*x)^6*sin(c + d*x))/(7*d) + (8*a*b^2*sin(c + d*x))/(35*d) + (4*a*b^2*c
os(c + d*x)^2*sin(c + d*x))/(35*d) + (3*a*b^2*cos(c + d*x)^4*sin(c + d*x))
/(35*d) - (3*a*b^2*cos(c + d*x)^6*sin(c + d*x))/(7*d)
```

3.544 $\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$

3.544.1 Optimal result 3753
3.544.2 Mathematica [A] (verified) 3753
3.544.3 Rubi [A] (verified) 3754
3.544.4 Maple [A] (verified) 3755
3.544.5 Fricas [A] (verification not implemented) 3756
3.544.6 Sympy [F] 3756
3.544.7 Maxima [A] (verification not implemented) 3757
3.544.8 Giac [A] (verification not implemented) 3757
3.544.9 Mupad [B] (verification not implemented) 3758

3.544.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d} - \frac{a(a^2+2b^2)\tan(c+dx)}{b^4d} + \frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{\tan^4(c+dx)}{4bd}$$

output
```
(a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^5/d-a*(a^2+2*b^2)*tan(d*x+c)/b^4/d+1/2*(a
^2+2*b^2)*tan(d*x+c)^2/b^3/d-1/3*a*tan(d*x+c)^3/b^2/d+1/4*tan(d*x+c)^4/b/d
```

3.544.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{12(a^2+b^2)^2\log(a+b\tan(c+dx)) + 3b^4\sec^4(c+dx) - 12ab(a^2+2b^2)\tan(c+dx) + 6b^2(a^2+b^2)\tan^2(c}{12b^5d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

output
```
(12*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 3*b^4*Sec[c + d*x]^4 - 12*a*b*
(a^2 + 2*b^2)*Tan[c + d*x] + 6*b^2*(a^2 + b^2)*Tan[c + d*x]^2 - 4*a*b^3*Ta
n[c + d*x]^3)/(12*b^5*d)
```

3.544.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4(a+b\tan(c+dx))}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(b^3\tan^3(c+dx)-ab^2\tan^2(c+dx)+b\left(a^2+2b^2\right)\tan(c+dx)-a\left(a^2+2b^2\right)+\frac{\left(a^2+b^2\right)^2}{a+b\tan(c+dx)}\right)d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2\left(a^2+2b^2\right)\tan^2(c+dx)-ab\left(a^2+2b^2\right)\tan(c+dx)+\left(a^2+b^2\right)^2\log(a+b\tan(c+dx))-\frac{1}{3}ab^3\tan^3(c+dx)+}{b^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] - a*b*(a^2 + 2*b^2)*Tan[c + d*x] +
(b^2*(a^2 + 2*b^2)*Tan[c + d*x]^2)/2 - (a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan
[c + d*x]^4)/4)/(b^5*d)
```

3.544. $\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$

3.544.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.544.4 Maple [A] (verified)

Time = 32.68 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{\frac{-\frac{\left(\tan^{4}\left(dx+c\right)\right)b^{3}}{4}+\frac{a\left(\tan^{3}\left(dx+c\right)\right)b^{2}}{3}-\frac{\left(a^{2}+2b^{2}\right)\left(\tan^{2}\left(dx+c\right)\right)b}{2}+a\left(a^{2}+2b^{2}\right)\tan\left(dx+c\right)}{b^{4}}+\frac{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\ln\left(a+b\tan\left(dx+c\right)\right)}{b^{5}}}{d}$
default	$-\frac{\frac{-\frac{\left(\tan^{4}\left(dx+c\right)\right)b^{3}}{4}+\frac{a\left(\tan^{3}\left(dx+c\right)\right)b^{2}}{3}-\frac{\left(a^{2}+2b^{2}\right)\left(\tan^{2}\left(dx+c\right)\right)b}{2}+a\left(a^{2}+2b^{2}\right)\tan\left(dx+c\right)}{b^{4}}+\frac{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\ln\left(a+b\tan\left(dx+c\right)\right)}{b^{5}}}{d}$
risch	$\frac{-2ia^{3}\mathrm{e}^{6i(dx+c)}-2ia\,b^{2}\mathrm{e}^{6i(dx+c)}+2a^{2}b\,\mathrm{e}^{6i(dx+c)}+2b^{3}\mathrm{e}^{6i(dx+c)}-6ia^{3}\mathrm{e}^{4i(dx+c)}-10ia\,b^{2}\mathrm{e}^{4i(dx+c)}+4a^{2}b\,\mathrm{e}^{4i(dx+c)}+8b^{3}\mathrm{e}}{b^{4}d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/4*tan(d*x+c)^4*b^3+1/3*a*tan(d*x+c)^3*b^2-1/2*(a^2+2*b^2)*
tan(d*x+c)^2*b+a*(a^2+2*b^2)*tan(d*x+c))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*ta
n(d*x+c)))
```

3.544.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{6\left(a^4+2a^2b^2+b^4\right)\cos\left(dx+c\right)^4\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(a^4\right.}{}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(
d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*c
os(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)
^2 - 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x +
 c))/(b^5*d*cos(d*x + c)^4)
```

3.544.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^6\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

3.544.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,(a^2b+2\,b^3)\tan(dx+c)^2-12\,(a^3+2\,ab^2)\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(b\tan(dx+c)+a)}{b^5}}{12\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(a^2*b + 2*b^3)*t
an(d*x + c)^2 - 12*(a^3 + 2*a*b^2)*tan(d*x + c))/b^4 + 12*(a^4 + 2*a^2*b^2
 + b^4)*log(b*tan(d*x + c) + a)/b^5)/d
```

3.544.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(|b\tan(}{b^5}}{12\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c
)^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))
/b^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d
```

3.544.9 Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)^4}{4\,b\,d}+\frac{\tan(c+d\,x)^2\,\left(\frac{1}{b}+\frac{a^2}{2\,b^3}\right)}{d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5\,d} - \frac{a\tan(c+d\,x)^3}{3\,b^2\,d}-\frac{a\tan(c+d\,x)\,\left(\frac{2}{b}+\frac{a^2}{b^3}\right)}{b\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))),x)
```

output
```
tan(c + d*x)^4/(4*b*d) + (tan(c + d*x)^2*(1/b + a^2/(2*b^3)))/d + (log(a +
 b*tan(c + d*x))*(a^4 + b^4 + 2*a^2*b^2))/(b^5*d) - (a*tan(c + d*x)^3)/(3*
b^2*d) - (a*tan(c + d*x)*(2/b + a^2/b^3))/(b*d)
```

3.545 $\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.545.3 Rubi [A] (verified) 3760
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3.545.9 Mupad [B] (verification not implemented) 3763

3.545.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}$$

output
```
(a^2+b^2)*ln(a+b*tan(d*x+c))/b^3/d-a*tan(d*x+c)/b^2/d+1/2*tan(d*x+c)^2/b/d
```

3.545.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx \\ =\frac{(a^2+b^2)\log(a+b\tan(c+dx))-ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x
]^2)/2)/(b^3*d)
```

3.545.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{b^2(a+b\tan(c+dx))}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-a+b\tan(c+dx)+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a^2+b^2)\log(a+b\tan(c+dx))-ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x
]^2)/2)/(b^3*d)
```

3.545. $\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}dx$

3.545.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.545.4 Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{b^{2}}+\frac{\left(a^{2}+b^{2}\right)\ln(a+b\tan(dx+c))}{b^{3}}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^{2}(dx+c)\right)}{2}+a\tan(dx+c)}{b^{2}}+\frac{\left(a^{2}+b^{2}\right)\ln(a+b\tan(dx+c))}{b^{3}}}{d}$
risch	$\frac{-2ia\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}-2ia}{b^{2}d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)a^{2}}{b^{3}d}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{bd}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)a^{2}}{b^{3}d}+\ln\Big(\mathrm{e}^{2i(d}$

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+(a^2+b^2)/b^3*ln(a+b*tan(d*
x+c)))
```

3.545.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(57) = 114.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.98

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)\cos(dx+c)^2\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-(a^2+b^2)\cos(dx}{2\,b^3d\cos(dx+c)^2}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
 - b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c
)^2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

3.545.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

3.545.7 Maxima [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(b*tan(d
*x + c) + a)/b^3)/d
```

3.545.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*t
an(d*x + c) + a))/b^3)/d
```

3.545.9 Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)^2}{2\,b\,d}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^2+b^2\right)}{b^3\,d}-\frac{a\tan(c+d\,x)}{b^2\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

output
```
tan(c + d*x)^2/(2*b*d) + (log(a + b*tan(c + d*x))*(a^2 + b^2))/(b^3*d) - (
a*tan(c + d*x))/(b^2*d)
```

3.546 $\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.546.7 Maxima [A] (verification not implemented) . . . 3767
3.546.8 Giac [A] (verification not implemented) . . . 3767
3.546.9 Mupad [B] (verification not implemented) . . . 3768

3.546.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{\log(a+b\tan(c+dx))}{bd}$$

output
```
ln(a+b*tan(d*x+c))/b/d
```

3.546.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{\log(a+b\tan(c+dx))}{bd}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
Log[a + b*Tan[c + d*x]]/(b*d)
```

3.546.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^2}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int \frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
Log[a + b*Tan[c + d*x]]/(b*d)
```

3.546.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.546.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
risch	$-\frac{\ln(\mathrm{e}^{2i(dx+c)}+1)}{bd}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{bd}$	58

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*tan(d*x+c))/b/d
```

3.546.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^
2) - log(cos(d*x + c)^2))/(b*d)
```

3.546.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^2{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

3.546.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log{(b\tan{(dx+c)}+a)}}{bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*tan(d*x + c) + a)/(b*d)
```

3.546.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log{(|b\tan{(dx+c)}+a|)}}{bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*tan(d*x + c) + a))/(b*d)
```

3.546.9 Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{\ln\left(a+b\tan(c+d\,x)\right)}{b\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))),x)
```

output
```
log(a + b*tan(c + d*x))/(b*d)
```

3.547 $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.547.1 Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(a^2+3b^2)\,x}{2\left(a^2+b^2\right)^2}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d}$$

output
```
1/2*a*(a^2+3*b^2)*x/(a^2+b^2)^2+b^3*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2
)^2/d+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

3.547.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2a^3c+6ab^2c+4ib^3c+2a^3dx+6ab^2dx+4ib^3dx-4ib^3\arctan(\tan(c+dx))+b(a^2+b^2)\cos(2(c+dx))}{4\left(a^2+b^2\right)^2 d}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
(2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d
*x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b
^3*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2
*Sin[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

3.547.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.70, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{1}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}-\frac{\int -\frac{a^2+b\tan(c+dx)a+2b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{a^2+b\tan(c+dx)a+2b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^3\left(\frac{\int \left(\frac{2b^2}{\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{a^3+3b^2a-2b^3\tan(c+dx)}{\left(a^2+b^2\right)\left(\tan^2(c+dx)b^2+b^2\right)}\right)d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{b^3\left(\frac{\frac{a\left(a^2+3b^2\right)\arctan(\tan(c+dx))}{b\left(a^2+b^2\right)}-\frac{b^2\log\left(b^2\tan^2(c+dx)+b^2\right)}{a^2+b^2}+\frac{2b^2\log(a+b\tan(c+dx))}{a^2+b^2}}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

output
```
(b^3*(((a*(a^2 + 3*b^2)*ArcTan[Tan[c + d*x]])/(b*(a^2 + b^2)) + (2*b^2*Log
[a + b*Tan[c + d*x]])/(a^2 + b^2) - (b^2*Log[b^2 + b^2*Tan[c + d*x]^2])/(a
^2 + b^2))/(2*b^2*(a^2 + b^2)) + (b^2 + a*b*Tan[c + d*x])/(2*b^2*(a^2 + b^
2)*(b^2 + b^2*Tan[c + d*x]^2))))/d
```

3.547.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

3.547. $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.547.4 Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2b}{2}+\frac{b^3}{2}}{1+\tan^2(dx+c)}-\frac{b^3\ln\left(1+\tan^2(dx+c)\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2b}{2}+\frac{b^3}{2}}{1+\tan^2(dx+c)}-\frac{b^3\ln\left(1+\tan^2(dx+c)\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
risch	$\frac{2ixb}{4iab-2a^2+2b^2}-\frac{xa}{4iab-2a^2+2b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-ib+a)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{2ib^3c}{d(a^4+2a^2b^2+b^4)}+$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^2*(((1/2*a^3+1/2*a*b^2)*tan(d*x+c)+1/2*a^2*b+1/2*b^3)/(1+
tan(d*x+c)^2)-1/2*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^3+3*a*b^2)*arctan(tan(d*x+
c)))+b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

3.547.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{b^3\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3+3\,ab^2\right)dx+\left(a^2b+b^3\right)\cos\left(dx+\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2
)*cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

3.547.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

3.547.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\,b^3\log\left(b\tan\left(dx+c\right)+a\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}+\frac{a\tan\left(dx+c\right)+b}{\left(a^2+b^2\right)\tan\left(dx+c\right)^2+a^2+b^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*b^3*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(tan(d
*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a^4 +
2*a^2*b^2 + b^4) + (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2
+ b^2))/d
```

3.547.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(89) = 178$.

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.96

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,b^4\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5} - \frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^3+3\,ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+a^2b+2\,b^3}{(a^4+2\,a^2b^2+b^4)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*lo
g(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/
(a^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*t
an(d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)
))/d
```

3.547.9 Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\cos\left(c+d\,x\right)^2\left(\frac{b}{2\,(a^2+b^2)}+\frac{a\tan(c+d\,x)}{2\,(a^2+b^2)}\right)}{d}\\ &\quad - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(2\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}\\ &\quad - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,2\mathrm{i}\right)}{4\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} + \frac{b^3\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)^2}\end{aligned}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

output
```
(cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2))))/d
- (log(tan(c + d*x) + 1i)*(a*1i + 2*b))/(4*d*(a*b*2i - a^2 + b^2)) - (log(
tan(c + d*x) - 1i)*(a + b*2i))/(4*d*(2*a*b - a^2*1i + b^2*1i)) + (b^3*log(
a + b*tan(c + d*x)))/(d*(a^2 + b^2)^2)
```

3.547. $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$

3.548 $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.548.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{a(3a^4+10a^2b^2+15b^4)\,x}{8\,(a^2+b^2)^3}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}\\ &\quad+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d}\\ &\quad+\frac{\cos^2(c+dx)\,(4b^3+a(3a^2+7b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d}\end{aligned}$$

output
```
1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*x/(a^2+b^2)^3+b^5*ln(a*cos(d*x+c)+b*sin(d*
x+c))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d+1/8*cos(
d*x+c)^2*(4*b^3+a*(3*a^2+7*b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

3.548.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.48

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= -\left(\left(8b^6+\sqrt{-b^2}(3a^5+10a^3b^2+15ab^4)\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)\right)+16b^6\log(a+b\tan(c+dx))-(8b$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

output
```
(-((8*b^6 + Sqrt[-b^2]*(3*a^5 + 10*a^3*b^2 + 15*a*b^4))*Log[Sqrt[-b^2] - b
*Tan[c + d*x]]) + 16*b^6*Log[a + b*Tan[c + d*x]] - (8*b^6 - Sqrt[-b^2]*(3*
a^5 + 10*a^3*b^2 + 15*a*b^4))*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 4*b*(a^2
+ b^2)^2*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + 2*(a^2 + b^2)*Cos[c + d*x]^
2*(4*b^4 + a*b*(3*a^2 + 7*b^2)*Tan[c + d*x]))/(16*b*(a^2 + b^2)^3*d)
```

3.548.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.61, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3987, 27, 496, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^4(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^6}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^5\int \frac{1}{(a+b\tan(c+dx))(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}-\frac{\int -\frac{3a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^5\left(\frac{\int \frac{3a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$\downarrow$ 686

$$\frac{b^5\left(\frac{\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int -\frac{3a^4+7b^2a^2+b\left(3a^2+7b^2\right)\tan(c+dx)a+8b^4}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$\downarrow$ 25

$$\frac{b^5\left(\frac{\frac{\int \frac{3a^4+7b^2a^2+b\left(3a^2+7b^2\right)\tan(c+dx)a+8b^4}{(a+b\tan(c+dx))\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}+\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}\right)}{d}$$

$\downarrow$ 657

$$\frac{b^5\left(\frac{\frac{\int \left(\frac{8b^4}{\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{3a^5+10b^2a^3+15b^4a-8b^5\tan(c+dx)}{\left(a^2+b^2\right)\left(\tan^2(c+dx)b^2+b^2\right)}\right)d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}+\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)}\right.}{d}$$

$\downarrow$ 2009

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}+\frac{\frac{ab\left(3a^2+7b^2\right)\tan(c+dx)+4b^4}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}+\frac{-\frac{4b^4\log\left(b^2\tan^2(c+dx)+b^2\right)}{a^2+b^2}+\frac{8b^4\log(a+b\tan(c+dx))}{a^2+b^2}+\frac{a\left(3a^4+10a^2b^2+15b^4\right.}{b\left(a^2\right.}}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

3.548. $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$

output
```
(b^5*((b^2 + a*b*Tan[c + d*x])/(4*b^2*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^
2)^2) + (((a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Tan[c + d*x]])/(b*(a^2 +
 b^2)) + (8*b^4*Log[a + b*Tan[c + d*x]])/(a^2 + b^2) - (4*b^4*Log[b^2 + b^
2*Tan[c + d*x]^2])/(a^2 + b^2))/(2*b^2*(a^2 + b^2)) + (4*b^4 + a*b*(3*a^2
+ 7*b^2)*Tan[c + d*x])/(2*b^2*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^2)))/(4*
b^2*(a^2 + b^2))))/d
```

3.548.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.548. $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.548.4 Maple [A] (verified)

Time = 8.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\dfrac{\dfrac{\dfrac{\left(\frac{3}{8}a^{5}+\frac{5}{4}a^{3}b^{2}+\frac{7}{8}a\,b^{4}\right)\left(\tan^{3}(dx+c)\right)+\left(\frac{1}{2}a^{2}b^{3}+\frac{1}{2}b^{5}\right)\left(\tan^{2}(dx+c)\right)+\left(\frac{7}{4}a^{3}b^{2}+\frac{9}{8}a\,b^{4}+\frac{5}{8}a^{5}\right)\tan(dx+c)+\frac{a^{4}b}{4}+a^{2}b^{3}+\frac{3b^{5}}{4}}{\left(1+\tan^{2}(dx+c)\right)^{2}}-b^{5}\ln(1}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
default	$\dfrac{\dfrac{\dfrac{\left(\frac{3}{8}a^{5}+\frac{5}{4}a^{3}b^{2}+\frac{7}{8}a\,b^{4}\right)\left(\tan^{3}(dx+c)\right)+\left(\frac{1}{2}a^{2}b^{3}+\frac{1}{2}b^{5}\right)\left(\tan^{2}(dx+c)\right)+\left(\frac{7}{4}a^{3}b^{2}+\frac{9}{8}a\,b^{4}+\frac{5}{8}a^{5}\right)\tan(dx+c)+\frac{a^{4}b}{4}+a^{2}b^{3}+\frac{3b^{5}}{4}}{\left(1+\tan^{2}(dx+c)\right)^{2}}-b^{5}\ln(1}{\left(a^{2}+b^{2}\right)^{3}}}{d}$
risch	$\frac{9xab}{8ia^{3}-24ia\,b^{2}+24a^{2}b-8b^{3}}+\frac{3ix\,a^{2}}{8ia^{3}-24ia\,b^{2}+24a^{2}b-8b^{3}}-\frac{8ix\,b^{2}}{8ia^{3}-24ia\,b^{2}+24a^{2}b-8b^{3}}-\frac{3\,\mathrm{e}^{2i(dx+c)}b}{16(-2iab+a^{2}-b^{2})d}-8($

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(((3/8*a^5+5/4*a^3*b^2+7/8*a*b^4)*tan(d*x+c)^3+(1/2*a^2
*b^3+1/2*b^5)*tan(d*x+c)^2+(7/4*a^3*b^2+9/8*a*b^4+5/8*a^5)*tan(d*x+c)+1/4*
a^4*b+a^2*b^3+3/4*b^5)/(1+tan(d*x+c)^2)^2-1/2*b^5*ln(1+tan(d*x+c)^2)+1/8*(
3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tan(d*x+c)))+b^5/(a^2+b^2)^3*ln(a+b*tan(
d*x+c)))
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.37

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{4\,b^5\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^
2 + 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2
 + a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*si
n(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

3.548.6 Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^4\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

3.548.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.78

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{8\,b^5\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{4\,b^3\tan(dx+c)^2+\left(3\,a^3+7\,ab^2\right)\tan(dx+c)^3+2\,a^2b+}{\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+2\,(}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(8*b^5*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4
*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5
+ 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (
4*b^3*tan(d*x + c)^2 + (3*a^3 + 7*a*b^2)*tan(d*x + c)^3 + 2*a^2*b + 6*b^3
+ (5*a^3 + 9*a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4
+ a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

3.548.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(146) = 292$.

Time = 0.40 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.12

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{8\,b^6\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^5+10\,a^3b^2+15\,ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{6\,b^5\tan(dx+c)^4+3\,a^5\tan(dx+c)^3+10\,a^3b^2\tan(dx}{}}{8\,d}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b
^7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x +
c)^3 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x
+ c)^2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x +
c) + 2*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(t
an(d*x + c)^2 + 1)^2))/d
```

3.548.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.09

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx
&= \frac{\frac{a^2\,b+3\,b^3}{4\,(a^4+2\,a^2\,b^2+b^4)}+\frac{b^3\tan(c+d\,x)^2}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)^3\,(3\,a^3+7\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,(5\,a^3+9\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)} \\
&\quad - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-a^2\,3\mathrm{i}+9\,a\,b+b^2\,8\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}+\frac{b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,(a^2+b^2)^3} \\
&\quad - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,9\mathrm{i}+8\,b^2\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}
\end{aligned}
$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

output
```
((a^2*b + 3*b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*tan(c + d*x)^2)/(2*(a^
4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(7*a*b^2 + 3*a^3))/(8*(a^4 + b^4 +
 2*a^2*b^2)) + (tan(c + d*x)*(9*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)
))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) - (log(tan(c + d*x) - 1i)*(
9*a*b - a^2*3i + b^2*8i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (b^
5*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) + 1i)*(a*
b*9i - 3*a^2 + 8*b^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

3.549 $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.549.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = &-\frac{a(2a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^4d}\\&-\frac{(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d}\\&+\frac{(a^2+b^2)\sec(c+dx)}{b^3d}+\frac{\sec^3(c+dx)}{3bd}-\frac{a\sec(c+dx)\tan(c+dx)}{2b^2d}\end{aligned}$$

output
```
-1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/b^4/d-(a^2+b^2)^(3/2)*arctanh(cos
(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/b^4/d+(a^2+b^2)*sec(d*x+c)/b^3/d
+1/3*sec(d*x+c)^3/b/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d
```

3.549.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 321 vs. 2(140) = 280.

Time = 2.59 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.29

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{48(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sec^3(c+dx)\left(12a^2b+20b^3+12b(a^2+b^2)\cos(2(c+dx))+6a\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

output
```
(48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
 Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6
*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*C
os[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*
b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

3.549.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3989, 3042, 3967, 3042, 3989, 3042, 3967, 3042, 3988, 219, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3989}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec^3(c+dx)(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec(c+dx)^3(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \sec^3(c+dx)dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{ 3989}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec(c+dx)(a-b\tan(c+dx))dx}{b^2}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec(c+dx)(a-b\tan(c+dx))dx}{b^2}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{ 3967}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{ 3988}$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{ 219}$$

3.549. $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}dx$

$$\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(a^2+b^2)\left(-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\frac{a\text{arctanh}(\sin(c+dx))}{d}-\frac{b\sec(c+dx)}{d}}{b^2}\right)}{b^2}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)-\frac{b\sec^3(c+dx)}{3d}}{b^2}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-((Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]
))/Sqrt[a^2 + b^2]])/(b^2*d)) - ((a*ArcTanh[Sin[c + d*x]])/d - (b*Sec[c +
d*x])/d)/b^2))/b^2 - (-1/3*(b*Sec[c + d*x]^3)/d + a*(ArcTanh[Sin[c + d*x]]
/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/b^2
```

3.549. $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$

3.549.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3989
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[-(b^2)^(-1)   Int[Sec[e + f*x]^(m - 2)*(a - b*Tan[e + f*x])
, x], x] + Simp[(a^2 + b^2)/b^2   Int[Sec[e + f*x]^(m - 2)/(a + b*Tan[e + f
*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(m - 1)
/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.549. $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$

3.549.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(130) = 260$.

Time = 16.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.92

method	result
derivativedivides	$-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
default	$-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
risch	$\frac{\mathrm{e}^{i(dx+c)}\left(3iab\,\mathrm{e}^{4i(dx+c)}+6a^{2}\mathrm{e}^{4i(dx+c)}+6b^{2}\mathrm{e}^{4i(dx+c)}+12a^{2}\mathrm{e}^{2i(dx+c)}+20b^{2}\mathrm{e}^{2i(dx+c)}-3iab+6a^{2}+6b^{2}\right)}{3d\,b^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}+\frac{a^{3}\ln\left(\mathrm{e}^{i(dx+c}\right.}{d\,b^{4}}$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^2/(tan(1/2*d*x+1/2*c)-1)^
2-1/2*(2*a^2+a*b+3*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2*a*(2*a^2+3*b^2)/b^4
*ln(tan(1/2*d*x+1/2*c)-1)+1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a+b)/b^2/(t
an(1/2*d*x+1/2*c)+1)^2-1/2*(-2*a^2+a*b-3*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1
/2*a*(2*a^2+3*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*(-a^4-2*a^2*b^2-b^4)
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
)
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.85

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{6\left(a^2+b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right) -}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos
(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x
+ c) + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*
a*b^2*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)
/(b^4*d*cos(d*x + c)^3)
```

3.549.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^5{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)), x)
```

3.549.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(132) = 264$.

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.58

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\left(6a^2+8b^2-\frac{3ab\sin(dx+c)}{\cos(dx+c)+1}+\frac{3ab\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{12\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\left(a^2+2b^2\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)}{b^3-\frac{3b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\left(2a^3+3ab^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+3\,(2\,a}{6d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*(6*a^2 + 8*b^2 - 3*a*b*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*b*sin(
d*x + c)^5/(cos(d*x + c) + 1)^5 - 12*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 6*(a^2 + 2*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(b^3 - 3
*b^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*b^3*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 3*(2*a^3 + 3*a*b^2
)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^3 + 3*a*b^2)*log(s
in(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*log((b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c
)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

3.549.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(132) = 264$.

Time = 0.41 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.99

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{\frac{3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{6\,(a^4+2\,a^2b^2+b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2}}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2}}\right.}{\sqrt{a^2+b^2}b^4}}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^
3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 +
 b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2
*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan
(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x +
 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/
2*c)^2 - 1)^3*b^3))/d
```

3.549.9 Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 724, normalized size of antiderivative = 5.17

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{b^3\left(\cos\left(c+d\,x\right)+\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{\cos(3\,c+3\,d\,x)}{3}+\frac{5}{6}\right)-b^2\left(\frac{a\,\sin(2\,c+2\,d\,x)}{4}+\frac{3\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(3\,c+3\,d\,x)}{4}+\frac{9}{}\right.}{}$$

input
```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))),x)
```

output
```
(b^3*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + 5/6) - b^2*
((a*sin(2*c + 2*d*x))/4 + (3*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
)*cos(3*c + 3*d*x))/4 + (9*a*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/4) + b*((3*a^2*cos(c + d*x))/4 + a^2/2 + (a^2*cos(2*c + 2*d*
x))/2 + (a^2*cos(3*c + 3*d*x))/4) + (atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2)
 + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c
/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^
2)^3)^(1/2))/2 - (3*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/2 - (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3
*d*x))/2 + (3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*c
os(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)
/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/2)/(b^4*d*((3*
cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

3.550 $\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.550.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{a\text{arctanh}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d} + \frac{\sec(c+dx)}{bd}$$

output
```
-a*arctanh(sin(d*x+c))/b^2/d+sec(d*x+c)/b/d-arctanh(cos(d*x+c)*(b-a*tan(d*
x+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^2/d
```

3.550.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
(2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

3.550.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = $\{3042, 3989, 3042, 3967, 3042, 3988, 219, 4257\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3989}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec(c+dx)(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{\int \sec(c+dx)(a-b\tan(c+dx))dx}{b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx-\frac{b\sec(c+dx)}{d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}$$

$$\Big\downarrow \text{3988}$$

$$-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{b\sec(c+dx)}{d}}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{\frac{a\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{b\sec(c+dx)}{d}}{b^2}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
-((Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 +
b^2]])/(b^2*d)) - ((a*ArcTanh[Sin[c + d*x]])/d - (b*Sec[c + d*x])/d)/b^2
```

3.550.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3989
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[-(b^2)^(-1)   Int[Sec[e + f*x]^(m - 2)*(a - b*Tan[e + f*x])
, x], x] + Simp[(a^2 + b^2)/b^2   Int[Sec[e + f*x]^(m - 2)/(a + b*Tan[e + f
*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && IGtQ[(m - 1)
/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.550.4 Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{b^2}}{d}$
default	$\frac{-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{b^2}}{d}$
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d\,b^2}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,b^2}+\frac{\sqrt{a^2+b^2}\,\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{d\,b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(\mathrm{e}^{i(dx\right.}{d\,b}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^2*(-a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)
-2*b)/(a^2+b^2)^(1/2))-1/b/(tan(1/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2
*c)-1)+1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1))
```

3.550.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(77) = 154.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.42

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)-\sqrt{a^2+b^2}\cos(dx+c)\log}{2\,b^2d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
 c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(
d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

3.550.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx=\int \frac{\sec^3{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

3.550.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(77) = 154$.

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.06

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^2}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^2 + sqrt(a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x
 + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqr
t(a^2 + b^2)))/b^2 - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

3.550.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*s
qrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b
^2 + 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

3.550.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.92

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{a^2+b^2}}{64\,a^2\,b+\frac{64\,a^4}{b}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^2+\frac{64\,a^4}{b^2}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{}{64\,a^4+12}\right.}{b^2\,d}$$

$$-\frac{2\,a\,\mathrm{atanh}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2+\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4+64\,a^2\,b^2}\right)}{b^2\,d}-\frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

output
```
(2*atanh((64*a^2*(a^2 + b^2)^(1/2))/(64*a^2*b + (64*a^4)/b + 128*a^3*tan(c
/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*
(a^2 + b^2)^(1/2))/(64*a^2 + (64*a^4)/b^2 + (128*a^3*tan(c/2 + (d*x)/2))/b
 + 128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1
/2))/(64*a^4 + 64*a^2*b^2 + 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3*b*tan(c
/2 + (d*x)/2)))*(a^2 + b^2)^(1/2))/(b^2*d) - (2*a*atanh((64*a^2*tan(c/2 +
(d*x)/2))/(64*a^2 + (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 +
64*a^2*b^2)))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.551 $\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.551.9 Mupad [B] (verification not implemented) 3803

3.551.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

output
```
-arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

3.551.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
(2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

3.551.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx$$
$$\Big\downarrow \text{3988}$$
$$-\frac{\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{d}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

input
```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output
```
-(ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]]/(Sqrt[a^2 +
 b^2]*d))
```

3.551.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

3.551.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

3.551.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(44) = 88.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.85

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2
*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 +
b^2)*d)
```

3.551.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x)), x)
```

3.551.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(
d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.551.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1
/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.551.9 Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

input
```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x))),x)
```

output
```
-(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1
/2))
```

3.552 $\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.552.1 Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{b^2\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}+\frac{b\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}$$

output

```
-b^2*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d+b*cos(d*x+c)/(a^2+b^2)/d+a*sin(d*x+c)/(a^2+b^2)/d
```

3.552.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2b^2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}(b\cos(c+dx)+a\sin(c+dx))}{\left(a^2+b^2\right)^{3/2}d}$$

input

```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output

```
(2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

3.552.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3990, 3042, 3967, 3042, 3117, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3990}$$

$$\frac{\int \cos(c+dx)(a-b\tan(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sec(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int \cos(c+dx)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{3988}$$

$$\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

input

```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

output

```
-((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
 + b^2)^(3/2)*d)) + ((b*Cos[c + d*x])/d + (a*Sin[c + d*x])/d)/(a^2 + b^2)
```

3.552.3.1 Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 3990

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[1/(a^2 + b^2)   Int[Sec[e + f*x]^m*(a - b*Tan[e + f*x]), x]
, x] + Simp[b^2/(a^2 + b^2)   Int[Sec[e + f*x]^(m + 2)/(a + b*Tan[e + f*x])
, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[(m - 1)/2,
0]
```

3.552.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result	siz
derivativedivides	$\frac{\frac{2b^{2}\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^{2}+b^{2}\right)\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90
default	$\frac{\frac{2b^{2}\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^{2}+b^{2}\right)\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90
risch	$-\frac{i\mathrm{e}^{i(dx+c)}}{2(-ib+a)d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2(ib+a)d}+\frac{b^{2}\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^{3}+ia\,b^{2}-a^{2}b-b^{3}}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}-\frac{b^{2}\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^{3}+ia\,b^{2}-a^{2}b-b^{3}}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}d}$	17

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b
^2)^(1/2))-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2))
```

3.552.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.08

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\sqrt{a^{2}+b^{2}}b^{2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}+b^{2}}\right)+2\left(a^{2}b+b^{3}\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((
a^4 + 2*a^2*b^2 + b^4)*d)
```

3.552.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx=\int \frac{\cos\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x)), x)
```

3.552.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{\frac{b^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a
*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2
*(b + a*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2))/d
```

3.552.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{\frac{b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*
tan(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

3.552.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}$$

input
```
int(cos(c + d*x)/(a + b*tan(c + d*x)),x)
```

output
```
((2*b)/(a^2 + b^2) + (2*a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(tan(c/2 + (
d*x)/2)^2 + 1)) - (2*b^2*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 +
b^2))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2))
```

3.553 $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$

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3.553.2 Mathematica [A] (verified) 3810
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3.553.1 Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{b^4\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d}+\frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{ab^2\sin(c+dx)}{\left(a^2+b^2\right)^2d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}-\frac{a\sin^3(c+dx)}{3\left(a^2+b^2\right)d}$$

output
```
-b^4*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/
d+b^3*cos(d*x+c)/(a^2+b^2)^2/d+1/3*b*cos(d*x+c)^3/(a^2+b^2)/d+a*b^2*sin(d*
x+c)/(a^2+b^2)^2/d+a*sin(d*x+c)/(a^2+b^2)/d-1/3*a*sin(d*x+c)^3/(a^2+b^2)/d
```

3.553.2 Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{24b^4\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}(3b(a^2+5b^2)\cos(c+dx)+b(a^2+b^2)\cos(3(c+dx))+2a(5a^2}{12\left(a^2+b^2\right)^{5/2}d}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

output
```
(24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^
2]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*
(5*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 +
b^2)^(5/2)*d)
```

3.553.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3990, 3042, 3967, 3042, 3113, 2009, 3990, 3042, 3967, 3042, 3117, 3988, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3990}$$

$$\frac{\int \cos^3(c+dx)(a-b\tan(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sec(c+dx)^3}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int \cos^3(c+dx)dx+\frac{b\cos^3(c+dx)}{3d}}{a^2+b^2}+\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{b\cos^3(c+dx)}{3d}}{a^2+b^2}+\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3113}$$

$$\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\int (1-\sin^2(c+dx))d(-\sin(c+dx))}{d}}{a^2+b^2}+\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{3990}$$

$$\frac{b^2\left(\frac{\int \cos(c+dx)(a-b\tan(c+dx))dx}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sec(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{3967}$$

$$\frac{b^2\left(\frac{a\int \cos(c+dx)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx+\frac{b\cos(c+dx)}{d}}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{b^2\left(\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{3988}$$

$$\frac{b^2\left(\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\int \frac{1}{a^2+b^2-\cos^2(c+dx)(b-a\tan(c+dx))^2}d(\cos(c+dx)(b-a\tan(c+dx)))}{d(a^2+b^2)}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b^2\left(\frac{\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)}{d}}{a^2+b^2}-\frac{b^2\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}\right)}{a^2+b^2}+\frac{\frac{b\cos^3(c+dx)}{3d}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d}}{a^2+b^2}$$

input

```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

3.553. $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$

output
```
(b^2*(-((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])
/((a^2 + b^2)^(3/2)*d)) + ((b*Cos[c + d*x])/d + (a*Sin[c + d*x])/d)/(a^2 +
 b^2)))/(a^2 + b^2) + ((b*Cos[c + d*x]^3)/(3*d) - (a*(-Sin[c + d*x] + Sin[
c + d*x]^3/3))/d)/(a^2 + b^2)
```

3.553.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3988
```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Simp[-f^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f
*x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

3.553. $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$

rule 3990
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_
Symbol] :> Simp[1/(a^2 + b^2)   Int[Sec[e + f*x]^m*(a - b*Tan[e + f*x]), x]
, x] + Simp[b^2/(a^2 + b^2)   Int[Sec[e + f*x]^(m + 2)/(a + b*Tan[e + f*x])
, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[(m - 1)/2,
0]
```

3.553.4 Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}}{d}$
default	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}}{d}$
risch	$-\frac{5\,\mathrm{e}^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3i\mathrm{e}^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{5\,\mathrm{e}^{-i(dx+c)}b}{8(ib+a)^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a}{8(ib+a)^2d}+\frac{b^4\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4-}{\left(a^2+b^2\right.}\right.}{(a^2+b^2)^{\frac{5}{2}}d}$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*
x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1
/2*d*x+1/2*c)^5+(-a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*t
an(1/2*d*x+1/2*c)^3-2*b^3*tan(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+
1/2*c)-1/3*a^2*b-4/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

3.553.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.59

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{3\sqrt{a^2+b^2}b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^4b+2\right.}{6\left(a^6\right.}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*
sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5
)*cos(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4
)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

3.553.6 Sympy [F]

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^3{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

3.553.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(159) = 318$.

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.30

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{3\,b^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^2b+4\,b^3+\frac{6\,b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^3+4\,ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^2b+2\,b^3\right)\sin(dx+c}{(\cos(dx+c)+1)^4}\right.}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^4+2\,a^2b^2\right.}{(\cos(}}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(3*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b
^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^2*b + 4*b^3 + 6*b^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(a
^3 + 4*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^2*b + 2*b^3)*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6))/d
```

3.553.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{(a^4}}{3\,d}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2
*d*x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2
*c)^4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*
b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*
d*x + 1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*
c)^2 + 1)^3))/d
```

3.553.9 Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.07

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{\frac{\frac{2a^2b}{3}+\frac{8b^3}{3}}{a^4+2a^2b^2+b^4}+\frac{4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+4ab^2)}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{4a^3}{3}+\frac{16ab^2}{3}\right)}{a^4+2a^2b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^3+2ab^2)}{a^4+2a^2b^2+b^4}+}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

$$-\frac{2b^4\,\mathrm{atanh}\left(\frac{a^4b+b^5+2a^2b^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

input
```
int(cos(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

output
```
(((2*a^2*b)/3 + (8*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x
)/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(4*a*b^2 + 2*a^3))
/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((16*a*b^2)/3 + (4*a^3)/3
))/(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(2*a*b^2 + a^3))/(a^4 +
 b^4 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 + 2*b^2))/(a^4 + b^4 +
2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2
+ (d*x)/2)^6 + 1)) - (2*b^4*atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (
d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.554 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.554.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = &-\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d}\\ &+\frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d}-\frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d}\\ &+\frac{(a^2+b^2)\tan^3(c+dx)}{b^4d}-\frac{a\tan^4(c+dx)}{2b^3d}\\ &+\frac{\tan^5(c+dx)}{5b^2d}-\frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-6*a*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^7/d+(5*a^4+9*a^2*b^2+3*b^4)*tan(d*x+
c)/b^6/d-a*(2*a^2+3*b^2)*tan(d*x+c)^2/b^5/d+(a^2+b^2)*tan(d*x+c)^3/b^4/d-1
/2*a*tan(d*x+c)^4/b^3/d+1/5*tan(d*x+c)^5/b^2/d-(a^2+b^2)^3/b^7/d/(a+b*tan(
d*x+c))
```

3.554.2 Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.29

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2b^6\sec^6(c+dx)+b^4\sec^4(c+dx)\left(a^2+4b^2-3ab\tan(c+dx)\right)-2\Big(8(a^2+b^2)^3+30a^2(a^2+b^2)^2\log(a+}{}$$

input
```
Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

output
```
(2*b^6*Sec[c + d*x]^6 + b^4*Sec[c + d*x]^4*(a^2 + 4*b^2 - 3*a*b*Tan[c + d*
x]) - 2*(8*(a^2 + b^2)^3 + 30*a^2*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] +
2*a*b*(-11*a^4 - 18*a^2*b^2 - 4*b^4 + 15*(a^2 + b^2)^2*Log[a + b*Tan[c + d
*x]])*Tan[c + d*x] - b^2*(15*a^4 + 29*a^2*b^2 + 8*b^4)*Tan[c + d*x]^2 + a*
b^3*(5*a^2 + 7*b^2)*Tan[c + d*x]^3 - 2*a^2*b^4*Tan[c + d*x]^4))/(10*b^7*d*
(a + b*Tan[c + d*x]))
```

3.554.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sec(c+dx)^8}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \ 3987$$
$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^3}{b^6(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^3}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{b^7d}$$

$$\Big\downarrow 476$$

$$\frac{\int \left(5\left(\frac{3\left(3a^2+b^2\right)b^2}{5a^4}+1\right)a^4-2b^3\tan^3(c+dx)a-2b\left(2a^2+3b^2\right)\tan(c+dx)a-\frac{6\left(a^2+b^2\right)^2a}{a+b\tan(c+dx)}+b^4\tan^4(c+dx)+3b}{b^7d}$$

$$\Big\downarrow 2009$$

$$\frac{-ab^2\left(2a^2+3b^2\right)\tan^2(c+dx)-\frac{\left(a^2+b^2\right)^3}{a+b\tan(c+dx)}-6a\left(a^2+b^2\right)^2\log(a+b\tan(c+dx))+b^3\left(a^2+b^2\right)\tan^3(c+dx)+}{b^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-6*a*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + b*(5*a^4 + 9*a^2*b^2 + 3*b^4
)*Tan[c + d*x] - a*b^2*(2*a^2 + 3*b^2)*Tan[c + d*x]^2 + b^3*(a^2 + b^2)*Ta
n[c + d*x]^3 - (a*b^4*Tan[c + d*x]^4)/2 + (b^5*Tan[c + d*x]^5)/5 - (a^2 +
b^2)^3/(a + b*Tan[c + d*x]))/(b^7*d)
```

3.554.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.554.4 Maple [A] (verified)

Time = 265.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{\frac{\left(\tan^5(dx+c)\right)b^4}{5}-\frac{a\,b^3\left(\tan^4(dx+c)\right)}{2}+a^2b^2\left(\tan^3(dx+c)\right)+b^4\left(\tan^3(dx+c)\right)-2a^3b\left(\tan^2(dx+c)\right)-3a\,b^3\left(\tan^2(dx+c)\right)+5a^4\tan(dx+}{b^6}}{d}$
default	$\frac{\frac{\frac{\left(\tan^5(dx+c)\right)b^4}{5}-\frac{a\,b^3\left(\tan^4(dx+c)\right)}{2}+a^2b^2\left(\tan^3(dx+c)\right)+b^4\left(\tan^3(dx+c)\right)-2a^3b\left(\tan^2(dx+c)\right)-3a\,b^3\left(\tan^2(dx+c)\right)+5a^4\tan(dx+}{b^6}}{d}$
risch	$-\frac{4i(25a^2b^3+40b^5e^{4i(dx+c)}+32b^5e^{2i(dx+c)}-33ia\,b^4e^{2i(dx+c)}-30ia^3b^2e^{10i(dx+c)}-135ia^3b^2e^{8i(dx+c)}-15ia\,b^4e^{10i(dx+}}{}$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^6*(1/5*tan(d*x+c)^5*b^4-1/2*a*b^3*tan(d*x+c)^4+a^2*b^2*tan(d*x+c)
^3+b^4*tan(d*x+c)^3-2*a^3*b*tan(d*x+c)^2-3*a*b^3*tan(d*x+c)^2+5*a^4*tan(d*
x+c)+9*a^2*b^2*tan(d*x+c)+3*b^4*tan(d*x+c))-6*a/b^7*(a^4+2*a^2*b^2+b^4)*ln
(a+b*tan(d*x+c))-1/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a+b*tan(d*x+c)))
```

3.554.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(174) = 348.

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.17

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{4\,(15\,a^4b^2+25\,a^2b^4+8\,b^6)\cos{(dx+c)}^6-2\,b^6-2\,(15\,a^4b^2+25\,a^2b^4+8\,b^6)\cos{(dx+c)}^4-(5\,a^2b^4+4}{}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/10*(4*(15*a^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - 2*b^6 - 2*(15*
a^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^4 - (5*a^2*b^4 + 4*b^6)*cos(d*x
 + c)^2 + 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^6 + (a^5*b + 2*a^3*
b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*
cos(d*x + c)^6 + (a^5*b + 2*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*
log(cos(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*(15*a^5*b + 25*a^3*b^3 + 8
*a*b^5)*cos(d*x + c)^5 + 2*(5*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^3)*sin(d*x +
 c))/(a*b^7*d*cos(d*x + c)^6 + b^8*d*cos(d*x + c)^5*sin(d*x + c))
```

3.554.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^8\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**2, x)
```

3.554.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{\frac{10\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}{b^8\tan(dx+c)+ab^7} - \frac{2\,b^4\tan(dx+c)^5-5\,ab^3\tan(dx+c)^4+10\,(a^2b^2+b^4)\tan(dx+c)^3-10\,(2\,a^3b+3\,ab^3)\tan(dx+c)^2+10\,(5\,a^4+9\,a^{2}}{b^6}}{10\,d}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/10*(10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/(b^8*tan(d*x + c) + a*b^7) -
 (2*b^4*tan(d*x + c)^5 - 5*a*b^3*tan(d*x + c)^4 + 10*(a^2*b^2 + b^4)*tan(d
*x + c)^3 - 10*(2*a^3*b + 3*a*b^3)*tan(d*x + c)^2 + 10*(5*a^4 + 9*a^2*b^2
+ 3*b^4)*tan(d*x + c))/b^6 + 60*(a^5 + 2*a^3*b^2 + a*b^4)*log(b*tan(d*x +
c) + a)/b^7)/d
```

3.554.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.42

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{\frac{60\,(a^5+2\,a^3b^2+ab^4)\log(|b\tan(dx+c)+a|)}{b^7}-\frac{10\,(6\,a^5b\tan(dx+c)+12\,a^3b^3\tan(dx+c)+6\,ab^5\tan(dx+c)+5\,a^6+9\,a^4b^2+3\,a^2b^4-b^6)}{(b\tan(dx+c)+a)b^7}-\frac{2\,b^8\,\mathrm{t}}{}}{}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/10*(60*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 10*
(6*a^5*b*tan(d*x + c) + 12*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 5
*a^6 + 9*a^4*b^2 + 3*a^2*b^4 - b^6)/((b*tan(d*x + c) + a)*b^7) - (2*b^8*ta
n(d*x + c)^5 - 5*a*b^7*tan(d*x + c)^4 + 10*a^2*b^6*tan(d*x + c)^3 + 10*b^8
*tan(d*x + c)^3 - 20*a^3*b^5*tan(d*x + c)^2 - 30*a*b^7*tan(d*x + c)^2 + 50
*a^4*b^4*tan(d*x + c) + 90*a^2*b^6*tan(d*x + c) + 30*b^8*tan(d*x + c))/b^1
0)/d
```

3.554.9 Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{\tan(c+d\,x)^2\left(\frac{a^3}{b^5}-\frac{a\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{d} \\ &\quad+\frac{\tan(c+d\,x)^5}{5\,b^2\,d}+\frac{\tan(c+d\,x)^3\left(\frac{1}{b^2}+\frac{a^2}{b^4}\right)}{d} \\ &\quad-\frac{\tan(c+d\,x)\left(\frac{a^2\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{3}{b^2}+\frac{2\,a\left(\frac{2\,a^3}{b^5}-\frac{2\,a\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d} \\ &\quad-\frac{a\tan(c+d\,x)^4}{2\,b^3\,d} \\ &\quad-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{b^7\,d} \\ &\quad-\frac{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^7+a\,b^6\right)}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^2),x)
```

output
```
(tan(c + d*x)^2*(a^3/b^5 - (a*(3/b^2 + (3*a^2)/b^4))/b))/d + tan(c + d*x)^
5/(5*b^2*d) + (tan(c + d*x)^3*(1/b^2 + a^2/b^4))/d - (tan(c + d*x)*((a^2*(
3/b^2 + (3*a^2)/b^4))/b^2 - 3/b^2 + (2*a*((2*a^3)/b^5 - (2*a*(3/b^2 + (3*a
^2)/b^4))/b))/b))/d - (a*tan(c + d*x)^4)/(2*b^3*d) - (log(a + b*tan(c + d*
x))*(6*a*b^4 + 6*a^5 + 12*a^3*b^2))/(b^7*d) - (a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)/(b*d*(a*b^6 + b^7*tan(c + d*x)))
```

3.554. $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.555 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.555.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} + \frac{(3a^2+2b^2)\tan(c+dx)}{b^4d} - \frac{a\tan^2(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d} - \frac{(a^2+b^2)^2}{b^5d(a+b\tan(c+dx))}$$

output
```
-4*a*(a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d+(3*a^2+2*b^2)*tan(d*x+c)/b^4/d-a*t
an(d*x+c)^2/b^3/d+1/3*tan(d*x+c)^3/b^2/d-(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c)
)
```

3.555.2 Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4b(2a^2+b^2)\tan(c+dx) - 2ab^2\tan^2(c+dx) + \frac{b^4\sec^4(c+dx)-4(a^2+b^2)\left(a^2+b^2+3a^2\log(a+b\tan(c+dx))+3ab\log(a+b\tan(c}{a+b\tan(c+dx)}}{3b^5d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

output
```
(4*b*(2*a^2 + b^2)*Tan[c + d*x] - 2*a*b^2*Tan[c + d*x]^2 + (b^4*Sec[c + d*
x]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a^2*Log[a + b*Tan[c + d*x]] + 3*a*b*Lo
g[a + b*Tan[c + d*x]]*Tan[c + d*x]))/(a + b*Tan[c + d*x]))/(3*b^5*d)
```

3.555.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(3\left(\frac{2b^2}{3a^2}+1\right)a^2-2b\tan(c+dx)a-\frac{4\left(a^2+b^2\right)a}{a+b\tan(c+dx)}+b^2\tan^2(c+dx)+\frac{\left(a^2+b^2\right)^2}{(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(3a^2+2b^2\right)\tan(c+dx)-\frac{\left(a^2+b^2\right)^2}{a+b\tan(c+dx)}-4a\left(a^2+b^2\right)\log(a+b\tan(c+dx))-ab^2\tan^2(c+dx)+\frac{1}{3}b^3\tan^3(c+d}{b^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

3.555. $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}dx$

output
```
(-4*a*(a^2 + b^2)*Log[a + b*Tan[c + d*x]] + b*(3*a^2 + 2*b^2)*Tan[c + d*x]
 - a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c + d*x]^3)/3 - (a^2 + b^2)^2/(a + b*Ta
n[c + d*x]))/(b^5*d)
```

3.555.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.555.4 Maple [A] (verified)

Time = 58.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$-\frac{8i(3a^2b\,\mathrm{e}^{4i(dx+c)}+6a^2b\,\mathrm{e}^{2i(dx+c)}-3ia^3\mathrm{e}^{6i(dx+c)}-9ia^3\mathrm{e}^{4i(dx+c)}-9ia^3\mathrm{e}^{2i(dx+c)}+2b^3+3a^2b-2iab^2-3ia^3+4b^3\mathrm{e}^{2i(dx}{3\left(\mathrm{e}^{2i(dx+c)}+1\right)^3\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)b^4d}$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^4*(1/3*b^2*tan(d*x+c)^3-a*b*tan(d*x+c)^2+3*a^2*tan(d*x+c)+2*b^2*t
an(d*x+c))-1/b^5*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))-4*a/b^5*(a^2+b^2)*ln
(a+b*tan(d*x+c)))
```

3.555.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(114) = 228$.

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.42

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{4\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^4-b^4-2\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^2+6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4+\left(a^3b\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*c
os(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*
x + c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)
*cos(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c)
- 2*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c
)^4 + b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

3.555.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^6\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

3.555.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,(a^4+2\,a^2b^2+b^4)}{b^6\tan(dx+c)+ab^5} - \frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+3\,(3\,a^2+2\,b^2)\tan(dx+c)}{b^4} + \frac{12\,(a^3+ab^2)\log(b\tan(dx+c)+a)}{b^5}}{3\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x
+ c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(
a^3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

3.555.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.28

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{12\,(a^3+ab^2)\log(|b\tan(dx+c)+a|)}{b^5} - \frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6} - \frac{3\,(4\,a^3b\tan(dx+c)+4\,ab^3\tan}{(b\tan(dx+}}{3\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c
)^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c)
)/b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2
 - b^4)/((b*tan(d*x + c) + a)*b^5))/d
```

3.555.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)^3}{3\,b^2\,d} + \frac{\tan(c+d\,x)\,\left(\frac{2}{b^2}+\frac{3\,a^2}{b^4}\right)}{d} - \frac{a\tan(c+d\,x)^2}{b^3\,d} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(4\,a^3+4\,a\,b^2\right)}{b^5\,d} - \frac{a^4+2\,a^2\,b^2+b^4}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^5+a\,b^4\right)}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

output
```
tan(c + d*x)^3/(3*b^2*d) + (tan(c + d*x)*(2/b^2 + (3*a^2)/b^4))/d - (a*tan
(c + d*x)^2)/(b^3*d) - (log(a + b*tan(c + d*x))*(4*a*b^2 + 4*a^3))/(b^5*d)
 - (a^4 + b^4 + 2*a^2*b^2)/(b*d*(a*b^4 + b^5*tan(c + d*x)))
```

3.556 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.556.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{2a\log(a+b\tan(c+dx))}{b^3d} + \frac{\tan(c+dx)}{b^2d} - \frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}$$

output

```
-2*a*ln(a+b*tan(d*x+c))/b^3/d+tan(d*x+c)/b^2/d+(-a^2-b^2)/b^3/d/(a+b*tan(d
*x+c))
```

3.556.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-2a\log(a+b\tan(c+dx)) + b\tan(c+dx) - \frac{a^2+b^2}{a+b\tan(c+dx)}}{b^3d}$$

input

```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output

```
(-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c
+ d*x]))/(b^3*d)
```

3.556.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \textbf{3987}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{b^2(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \textbf{476}$$

$$\frac{\int \left(-\frac{2a}{a+b\tan(c+dx)}+\frac{a^2+b^2}{(a+b\tan(c+dx))^2}+1\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{-\frac{a^2+b^2}{a+b\tan(c+dx)}-2a\log(a+b\tan(c+dx))+b\tan(c+dx)}{b^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c
+ d*x]))/(b^3*d)
```

3.556.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.556.4 Maple [A] (verified)

Time = 15.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}}{d}$	57
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}}{d}$	57
risch	$-\frac{4i\left(-ia\,\mathrm{e}^{2i(dx+c)}+b-ia\right)}{\left(\mathrm{e}^{2i(dx+c)}+1\right)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)b^2d}+\frac{2a\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{b^3d}-\frac{2a\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}$	136

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*tan(d*x+c)-2/b^3*a*ln(a+b*tan(d*x+c))-1/b^3*(a^2+b^2)/(a+b*tan(
d*x+c)))
```

3.556.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(61) = 122$.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.92

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,b^2\cos\left(dx+c\right)^2 - 2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right) - b^2 + \left(a^2\cos\left(dx+c\right)^2 + ab\cos\left(dx+c\right)\sin\left(dx+c\right)\right)}{ab^3d\cos}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(
d*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x
 + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(
d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*
d*cos(d*x + c)*sin(d*x + c))
```

3.556.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

3.556.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3} + \frac{2\,a\log(b\tan(dx+c)+a)}{b^3} - \frac{\tan(dx+c)}{b^2}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
 - tan(d*x + c)/b^2)/d
```

3.556.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,a\log(|b\tan(dx+c)+a|)}{b^3} - \frac{\tan(dx+c)}{b^2} - \frac{2\,ab\tan(dx+c)+a^2-b^2}{(b\tan(dx+c)+a)b^3}}{d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
 + c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

3.556.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)}{b^2\,d} - \frac{a^2+b^2}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)} - \frac{2\,a\,\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

output
```
tan(c + d*x)/(b^2*d) - (a^2 + b^2)/(b*d*(a*b^2 + b^3*tan(c + d*x))) - (2*a
*log(a + b*tan(c + d*x)))/(b^3*d)
```

3.557 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.557.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx=-\frac{1}{bd(a+b\tan(c+dx))}$$

output
```
-1/b/d/(a+b*tan(d*x+c))
```

3.557.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx=\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.557.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)^2}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \ 3987$$

$$\frac{\int \frac{1}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{bd(a+b\tan(c+dx))}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
-(1/(b*d*(a + b*Tan[c + d*x])))
```

3.557.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.557.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
default	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}$	47

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/d/(a+b*tan(d*x+c))
```

3.557.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx=-\frac{b\cos{(dx+c)}-a\sin{(dx+c)}}{(a^3+ab^2)d\cos{(dx+c)}+(a^2b+b^3)d\sin{(dx+c)}}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b
+ b^3)*d*sin(d*x + c))
```

3.557.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^2\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

3.557.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{(b\tan\left(dx+c\right)+a)bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/((b*tan(d*x + c) + a)*b*d)
```

3.557.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{(b\tan\left(dx+c\right)+a)bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/((b*tan(d*x + c) + a)*b*d)
```

3.557.9 Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{b\,d\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
```

output
```
-1/(b*d*(a + b*tan(c + d*x)))
```

3.558 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.558.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^4+6a^2b^2-3b^4)\,x}{2\,(a^2+b^2)^3} + \frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} + \frac{b(a^2-3b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
1/2*(a^4+6*a^2*b^2-3*b^4)*x/(a^2+b^2)^3+4*a*b^3*ln(a*cos(d*x+c)+b*sin(d*x+
c))/(a^2+b^2)^3/d+1/2*b*(a^2-3*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))+1/2*cos
(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.558.2 Mathematica [A] (verified)

Time = 4.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.00

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{-\frac{ab\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-2\sqrt{-b^2}\log(a+b\tan(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)}{\sqrt{-b^2}(a^2+b^2)} + \frac{\cos^2(c+dx)(b+a\tan(c+}{a+b\tan(c+dx)}}{2\,(a^2}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-((a*b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*Sqrt[-b^2]
*Log[a + b*Tan[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x
]]))/(Sqrt[-b^2]*(a^2 + b^2))) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(a
+ b*Tan[c + d*x]) + (b*(a^2 - 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[
Sqrt[-b^2] - b*Tan[c + d*x]] - 4*a*Log[a + b*Tan[c + d*x]] + (2*a + (a^2 -
 b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 + b^2))/(a +
b*Tan[c + d*x])))/(2*(a^2 + b^2)^2))/(2*(a^2 + b^2)*d)
```

3.558.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^2(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{1}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))}-\frac{\int -\frac{a^2+2b\tan(c+dx)a+3b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{a^2+2b\tan(c+dx)a+3b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow 657$$

$$\frac{b^3\left(\frac{\int \left(\frac{8ab^2}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{a^4+6b^2a^2-8b^3\tan(c+dx)a-3b^4}{\left(a^2+b^2\right)^2\left(\tan^2(c+dx)b^2+b^2\right)}+\frac{3b^2-a^2}{\left(a^2+b^2\right)(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(}\right.}{d}$$

$$\Big\downarrow 2009$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))}+\frac{\frac{a^2-3b^2}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{4ab^2\log\left(b^2\tan^2(c+dx)+b^2\right)}{\left(a^2+b^2\right)^2}+\frac{8ab^2\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2}+\frac{\left(a^4+6a^2b\right.}{}}{2b^2(a^2+b^2)}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^3*((b^2 + a*b*Tan[c + d*x])/(2*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])*(b^
2 + b^2*Tan[c + d*x]^2)) + (((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Tan[c + d*x]
])/(b*(a^2 + b^2)^2) + (8*a*b^2*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - (
4*a*b^2*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^2 + (a^2 - 3*b^2)/((a^2
 + b^2)*(a + b*Tan[c + d*x])))/(2*b^2*(a^2 + b^2))))/d
```

3.558.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.558.4 Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{4b^3a\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan^2(dx+c)}-2a\,b^3\ln\left(1+\tan^2(dx+c)\right)+\left(a^4+6a^2b^2\right.}{\left(a^2+b^2\right)^3}}{d}$
default	$\frac{-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{4b^3a\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan^2(dx+c)}-2a\,b^3\ln\left(1+\tan^2(dx+c)\right)+\left(a^4+6a^2b^2\right.}{\left(a^2+b^2\right)^3}}{d}$
risch	$\frac{3ixb}{6ib\,a^2-2ib^3-2a^3+6a\,b^2}-\frac{xa}{6ib\,a^2-2ib^3-2a^3+6a\,b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-2iab+a^2-b^2)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(2iab+a^2-b^2)d}-\frac{8ia\,b^3x}{a^6+3a^4b^2+3a^2b}$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+
c))+1/(a^2+b^2)^3*(((1/2*a^4-1/2*b^4)*tan(d*x+c)+a^3*b+a*b^3)/(1+tan(d*x+c
)^2)-2*a*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tan(d*x+c
))))
```

3.558.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.84

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^4b+2\,a^2b^3+b^5)\cos{(dx+c)}^3-(a^2b^3+3\,b^5-(a^5+6\,a^3b^2-3\,ab^4)dx)\cos{(dx+c)}+4\,(a^2b^3\cos{(dx+}}{2\,((a^7+3}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 +
6*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*s
in(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2
*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(
d*x + c))
```

3.558.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.558.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.86

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{8\,ab^3\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,a^2b-2\,b^3+(a^2b-3\,b^3)\tan(}{a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)^3+(a^5+}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^
4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*
a^2*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c
))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (
a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*
x + c)))/d
```

3.558.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{8\,ab^4\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{a^2b\tan(dx+c)^2-3\,b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab}{(a^4+2\,a^2b^2+b^4)\Big(b\tan(dx+c)^3+a\tan(dx+c)^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^
2*tan(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)
^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

3.558.9 Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{a^2\,b-b^3}{{\left(a^2+b^2\right)}^2}+\frac{{\tan(c+d\,x)}^2\,\left(a^2\,b-3\,b^3\right)}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\,\tan(c+d\,x)}{2\,\left(a^2+b^2\right)}}{d\,\left({b\,\tan\left(c+d\,x\right)}^3+a\,{\tan\left(c+d\,x\right)}^2+b\,\tan\left(c+d\,x\right)+a\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-3\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a-b\,3\mathrm{i}\right)}{4\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$
$$+\frac{4\,a\,b^3\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^3}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

output
```
((a^2*b - b^3)/(a^2 + b^2)^2 + (tan(c + d*x)^2*(a^2*b - 3*b^3))/(2*(a^4 +
b^4 + 2*a^2*b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2)))/(d*(a + b*tan(c + d*
x) + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) - 1i)*(a*1i
 - 3*b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log(tan(c + d*x) + 1
i)*(a - b*3i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (4*a*b^3*log(a
+ b*tan(c + d*x)))/(d*(a^2 + b^2)^3)
```

3.559 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.559.1 Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = {} & \frac{3(a^6+5a^4b^2+15a^2b^4-5b^6)\,x}{8\,(a^2+b^2)^4} \\ & + \frac{6ab^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} \\ & + \frac{3b(a^2-b^2)\,(a^2+5b^2)}{8\,(a^2+b^2)^3\,d(a+b\tan(c+dx))} \\ & + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d(a+b\tan(c+dx))} \\ & - \frac{\cos^2(c+dx)\,(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output

```
3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*x/(a^2+b^2)^4+6*a*b^5*ln(a*cos(d*x+c)
+b*sin(d*x+c))/(a^2+b^2)^4/d+3/8*b*(a^2-b^2)*(a^2+5*b^2)/(a^2+b^2)^3/d/(a+
b*tan(d*x+c))+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c
))-1/8*cos(d*x+c)^2*(b*(a^2-5*b^2)-3*a*(a^2+3*b^2)*tan(d*x+c))/(a^2+b^2)^2
/d/(a+b*tan(d*x+c))
```

3.559.2 Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.77

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4b\cos^4(c+dx)(b+a\tan(c+dx))+\frac{2b\cos^2(c+dx)\left(-a^2b+5b^3+3a\left(a^2+3b^2\right)\tan(c+dx)\right)}{a^2+b^2}-\frac{\sqrt{-b^2}\Big(6a\left(a^2+b^2\right)\left(a^2+3b^2\right)\Big(\Big(a-\sqrt{-}}{}}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + (2*b*Cos[c + d*x]^2*(-(a^2*b) +
 5*b^3 + 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(a^2 + b^2) - (Sqrt[-b^2]*(6*a*(
a^2 + b^2)*(a^2 + 3*b^2)*((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]
] + 2*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]] - (a + Sqrt[-b^2])*Log[Sqrt[-b^2]
 + b*Tan[c + d*x]])*(a + b*Tan[c + d*x]) + 3*(a^4 + 4*a^2*b^2 - 5*b^4)*(2*
Sqrt[-b^2]*(a^2 + b^2) + (-a^2 + b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*
Tan[c + d*x]]*(a + b*Tan[c + d*x]) - 4*a*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]
]*(a + b*Tan[c + d*x]) + (a^2 - b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*T
an[c + d*x]]*(a + b*Tan[c + d*x]))))/(a^2 + b^2)^3)/(16*b*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x]))
```

3.559.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3987, 27, 496, 25, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sec(c+dx)^4(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{ 3987}$$

$$\frac{\int \frac{b^6}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \; 27$$

$$\frac{b^5\int \frac{1}{(a+b\tan(c+dx))^2(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 496$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}-\frac{\int -\frac{3a^2+4b\tan(c+dx)a+5b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{b^5\left(\frac{\int \frac{3a^2+4b\tan(c+dx)a+5b^2}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}\right)}{d}$$

$$\Big\downarrow \; 686$$

$$\frac{b^5\Bigg(\frac{-\frac{\int -\frac{3\left(a^4+2b^2a^2+2b\left(a^2+3b^2\right)\tan(c+dx)a+5b^4\right)}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{b^2\left(a^2-5b^2\right)-3ab\left(a^2+3b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{b^5\Bigg(\frac{\frac{3\int \frac{a^4+2b^2a^2+2b\left(a^2+3b^2\right)\tan(c+dx)a+5b^4}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{b^2\left(a^2-5b^2\right)-3ab\left(a^2+3b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(}}{d}$$

$$\Big\downarrow \; 657$$

$$\frac{b^5\Bigg(\frac{\frac{3\int \left(\frac{16ab^4}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{a^6+5b^2a^4+15b^4a^2-16b^5\tan(c+dx)a-5b^6}{\left(a^2+b^2\right)^2\left(\tan^2(c+dx)b^2+b^2\right)}+\frac{-a^4-4b^2a^2+5b^4}{\left(a^2+b^2\right)(a+b\tan(c+dx))^2}\right)d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{b^2\left(a^2-5b^2\right)-3ab\left(a^2+3}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+}}{4b^2(a^2+b^2)}}{d}$$

3.559. $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

$$\Big\downarrow \text{2009}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))}+\frac{\frac{3\left(-\frac{8ab^4\log\left(b^2\tan^2(c+dx)+b^2\right)}{\left(a^2+b^2\right)^2}+\frac{16ab^4\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2}+\frac{a^4+4a^2b^2-5b^4}{\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{(a^6}{}\right.}{2b^2\left(a^2+b^2\right)}}{4b^2(a}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b^5*((b^2 + a*b*Tan[c + d*x])/(4*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])*(b^
2 + b^2*Tan[c + d*x]^2)^2) + (-1/2*(b^2*(a^2 - 5*b^2) - 3*a*b*(a^2 + 3*b^2
)*Tan[c + d*x])/(b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])*(b^2 + b^2*Tan[c + d
*x]^2)) + (3*(((a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*ArcTan[Tan[c + d*x]]
)/(b*(a^2 + b^2)^2) + (16*a*b^4*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - (
8*a*b^4*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2 + b^2)^2 + (a^4 + 4*a^2*b^2 -
5*b^4)/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a
^2 + b^2))))/d
```

3.559.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.559.4 Maple [A] (verified)

Time = 16.90 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{\left(\frac{3}{8}a^{6}+\frac{15}{8}a^{4}b^{2}+\frac{5}{8}a^{2}b^{4}-\frac{7}{8}b^{6}\right)\left(\tan^{3}(dx+c)\right)+\left(2a^{3}b^{3}+2a\,b^{5}\right)\left(\tan^{2}(dx+c)\right)+\left(\frac{17}{8}a^{4}b^{2}+\frac{3}{8}a^{2}b^{4}-\frac{9}{8}b^{6}+\frac{5}{8}a^{6}\right)\tan(dx+c)+\frac{a^{5}b}{2}+3a^{3}}{\left(1+\tan^{2}(dx+c)\right)^{2}}}{\left(a^{2}+b^{2}\right)^{4}}$
default	$\frac{\frac{\left(\frac{3}{8}a^{6}+\frac{15}{8}a^{4}b^{2}+\frac{5}{8}a^{2}b^{4}-\frac{7}{8}b^{6}\right)\left(\tan^{3}(dx+c)\right)+\left(2a^{3}b^{3}+2a\,b^{5}\right)\left(\tan^{2}(dx+c)\right)+\left(\frac{17}{8}a^{4}b^{2}+\frac{3}{8}a^{2}b^{4}-\frac{9}{8}b^{6}+\frac{5}{8}a^{6}\right)\tan(dx+c)+\frac{a^{5}b}{2}+3a^{3}}{\left(1+\tan^{2}(dx+c)\right)^{2}}}{\left(a^{2}+b^{2}\right)^{4}}$
risch	$\frac{12ixab}{32ia^{3}b-32ia\,b^{3}-8a^{4}+48a^{2}b^{2}-8b^{4}}-\frac{3x\,a^{2}}{32ia^{3}b-32ia\,b^{3}-8a^{4}+48a^{2}b^{2}-8b^{4}}+\frac{15x\,b^{2}}{32ia^{3}b-32ia\,b^{3}-8a^{4}+48a^{2}b^{2}-8b^{4}}-$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^4*(((3/8*a^6+15/8*a^4*b^2+5/8*a^2*b^4-7/8*b^6)*tan(d*x+c)
^3+(2*a^3*b^3+2*a*b^5)*tan(d*x+c)^2+(17/8*a^4*b^2+3/8*a^2*b^4-9/8*b^6+5/8*
a^6)*tan(d*x+c)+1/2*a^5*b+3*a^3*b^3+5/2*a*b^5)/(1+tan(d*x+c)^2)^2-3*a*b^5*
ln(1+tan(d*x+c)^2)+3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*arctan(tan(d*x+c))
)-b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))+6*b^5/(a^2+b^2)^4*a*ln(a+b*tan(d*x+c)))
```

3.559.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.80

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^5-2\left(a^6b-3\,a^4b^3-9\,a^2b^5-5\,b^7\right)\cos\left(dx+c\right)^3+\left(3\,a^6b+8\,a^4b}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 2*(a^6*b -
3*a^4*b^3 - 9*a^2*b^5 - 5*b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 - 9*a
^2*b^5 - 30*b^7 + 6*(a^7 + 5*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6)*d*x)*cos(d*x
+ c) + 48*(a^2*b^5*cos(d*x + c) + a*b^6*sin(d*x + c))*log(2*a*b*cos(d*x +
c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (3*a^5*b^2 + 22*a^3*
b^4 + 3*a*b^6 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^4 - 6
*(a^6*b + 5*a^4*b^3 + 15*a^2*b^5 - 5*b^7)*d*x - 6*(a^7 + 5*a^5*b^2 + 7*a^3
*b^4 + 3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^
4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4
*a^2*b^7 + b^9)*d*sin(d*x + c))
```

3.559.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.559.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(228) = 456$.

Time = 0.48 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.14

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{48\,ab^5\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{24\,ab^5\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{4\,a^4}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6+(a^6b+3}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*(48*a*b^5*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^
4 + 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x + c)/
(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (4*a^4*b + 20*a^2*b^3 -
8*b^5 + 3*(a^4*b + 4*a^2*b^3 - 5*b^5)*tan(d*x + c)^4 + 3*(a^5 + 4*a^3*b^2
+ 3*a*b^4)*tan(d*x + c)^3 + (5*a^4*b + 28*a^2*b^3 - 25*b^5)*tan(d*x + c)^2
 + (5*a^5 + 16*a^3*b^2 + 11*a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*t
an(d*x + c)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

3.559.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(228) = 456$.

Time = 0.47 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.97

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{48\,ab^6\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} - \frac{24\,ab^5\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{8\,(6\,ab^6\tan(dx+c)+7\,a^2}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(}}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(48*a*b^6*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5
+ 4*a^2*b^7 + b^9) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*
x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(6*a*b^6*tan(d*
x + c) + 7*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
*(b*tan(d*x + c) + a)) + (36*a*b^5*tan(d*x + c)^4 + 3*a^6*tan(d*x + c)^3 +
 15*a^4*b^2*tan(d*x + c)^3 + 5*a^2*b^4*tan(d*x + c)^3 - 7*b^6*tan(d*x + c)
^3 + 16*a^3*b^3*tan(d*x + c)^2 + 88*a*b^5*tan(d*x + c)^2 + 5*a^6*tan(d*x +
 c) + 17*a^4*b^2*tan(d*x + c) + 3*a^2*b^4*tan(d*x + c) - 9*b^6*tan(d*x + c
) + 4*a^5*b + 24*a^3*b^3 + 56*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2
*b^6 + b^8)*(tan(d*x + c)^2 + 1)^2))/d
```

3.559.9 Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.97

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{3\tan(c+dx)^4\,(a^4\,b+4\,a^2\,b^3-5\,b^5)}{8\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{a^4\,b+5\,a^2\,b^3-2\,b^5}{2\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+dx)\,(5\,a^3+11\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{3\tan(c+dx)^3\,(a^3+3\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+dx)^2\,(}{8\,(a^2+b^2}}{d\,\left(b\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^4+2\,b\tan\left(c+d\,x\right)^3+2\,a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-a^2+a\,b\,4\mathrm{i}+5\,b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a^2+a\,b\,4\mathrm{i}-5\,b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}+\frac{6\,a\,b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^4}$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

output
```
((3*tan(c + d*x)^4*(a^4*b - 5*b^5 + 4*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)) + (a^4*b - 2*b^5 + 5*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*
a^2*b^2)) + (tan(c + d*x)*(11*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2))
+ (3*tan(c + d*x)^3*(3*a*b^2 + a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c
+ d*x)^2*(5*a^4*b - 25*b^5 + 28*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^
2*b^2)))/(d*(a + b*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 +
2*b*tan(c + d*x)^3 + b*tan(c + d*x)^5)) + (3*log(tan(c + d*x) + 1i)*(a*b*4
i - a^2 + 5*b^2))/(16*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)
) + (3*log(tan(c + d*x) - 1i)*(a*b*4i + a^2 - 5*b^2))/(16*d*(4*a*b^3 - 4*a
^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)) + (6*a*b^5*log(a + b*tan(c + d*x)))/
(d*(a^2 + b^2)^4)
```

3.560 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.560.1 Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{5(8a^4+12a^2b^2+3b^4)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} \\ &\quad + \frac{5a(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^6d\sqrt{\sec^2(c+dx)}} \\ &\quad - \frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} \\ &\quad - \frac{5\sec(c+dx)\left(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx)\right)}{8b^5d}\end{aligned}$$

output
```
5/8*(8*a^4+12*a^2*b^2+3*b^4)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x
+c)^2)^(1/2)+5*a*(a^2+b^2)^(3/2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/
(sec(d*x+c)^2)^(1/2))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1/2)-5/12*sec(d*x+c
)^3*(4*a-3*b*tan(d*x+c))/b^3/d-sec(d*x+c)^5/b/d/(a+b*tan(d*x+c))-5/8*sec(d
*x+c)*(8*a*(a^2+b^2)-b*(4*a^2+3*b^2)*tan(d*x+c))/b^5/d
```

3.560.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.76 (sec) , antiderivative size = 1152, normalized size of antiderivative = 4.90

$$\begin{aligned}
\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= -\frac{(a-ib)^2(a+ib)^2\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{b^5d(a+b\tan(c+dx))^2} \\
&- \frac{a(12a^2+13b^2)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^5d(a+b\tan(c+dx))^2} \\
&+ \frac{10ia(a+ib)(ia+b)\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}\left(-b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\cos\left(\frac{1}{2}(c+dx)\right)+b^2\cos\left(\frac{1}{2}(c+dx)\right)}\right)\sec^2(c+dx)(a\cos(c+dx)+}{b^6d(a+b\tan(c+dx))^2} \\
&- \frac{5(8a^4+12a^2b^2+3b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx}{8b^6d(a+b\tan(c+dx))^2} \\
&+ \frac{5(8a^4+12a^2b^2+3b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx}{8b^6d(a+b\tan(c+dx))^2} \\
&+ \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{16b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a+b\tan(c+dx))^2} \\
&+ \frac{(36a^2-8ab+21b^2)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{48b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2} \\
&- \frac{a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+b\tan(c+dx))^2} \\
&- \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{16b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a+b\tan(c+dx))^2} \\
&+ \frac{a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+b\tan(c+dx))^2} \\
&+ \frac{(-36a^2-8ab-21b^2)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{48b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2} \\
&+ \frac{\sec^2(c+dx)\left(-12a^3\sin\left(\frac{1}{2}(c+dx)\right)-13ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^5d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2} \\
&+ \frac{\sec^2(c+dx)\left(12a^3\sin\left(\frac{1}{2}(c+dx)\right)+13ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}
\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

output
```
-(((a - I*b)^2*(a + I*b)^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]
))/(b^5*d*(a + b*Tan[c + d*x])^2)) - (a*(12*a^2 + 13*b^2)*Sec[c + d*x]^2*(
a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(a + b*Tan[c + d*x])^2) + ((1
0*I)*a*(a + I*b)*(I*a + b)*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a^2 + b^2]*(-(b*C
os[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c + d*x)/2] + b^2*Cos[(c
 + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^6*d*(a
 + b*Tan[c + d*x])^2) - (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(8*b^6*d*(a + b*Tan[c + d*x])^2) + (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(8*b^6*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^2)/(16*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^4*(a + b*Tan[c + d*x])^2) + ((36*a^2 - 8*a*b + 21*b^2)*Sec[c + d*x]^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2)/(48*b^4*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2*(a + b*Tan[c + d*x])^2) - (a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^3*(a + b*Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(16*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a + b*Tan
[c + d*x])^2) + (a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(3*b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + b*T...
```

3.560.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3992, 492, 591, 25, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^7}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{5\int \frac{b\tan(c+dx)\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{591}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\int -\frac{\left(a-\left(\frac{4a^2}{b^2}+3\right)b\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))-\frac{1}{12}\left(\tan^2(c+dx)+1\right)^{3/2}(4a-3b\tan(c+dx))\right)}{b^2}-\frac{(\tan^2(c+dx)}{a+b\tan(c}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(-\frac{1}{4}\int \frac{\left(a-\left(\frac{4a^2}{b^2}+3\right)b\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))-\frac{1}{12}\left(\tan^2(c+dx)+1\right)^{3/2}(4a-3b\tan(c+dx))\right)}{b^2}-\frac{(\tan^2(c+dx)}{a+b\tan(c}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{682}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{1}{2}b^2\int \frac{ab^2\left(4a^2+5b^2\right)-b\left(8a^4+12b^2a^2+3b^4\right)\tan(c+dx)}{b^6(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{2b^2}\right)\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{\int \frac{ab^2\left(4a^2+5b^2\right)-b\left(8a^4+12b^2a^2+3b^4\right)\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{2b^2}\right)-\frac{1}{12}\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{719}$$

3.560. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{8a\left(a^2+b^2\right)^2\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\left(8a^4+12a^2b^2+3b^4\right)\int\frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(c+}{}\right.\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \color{blue}{222}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{8a\left(a^2+b^2\right)^2\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-b\left(8a^4+12a^2b^2+3b^4\right)\text{arcsinh}(\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\right.}{}\right.\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \color{blue}{488}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{-8a\left(a^2+b^2\right)^2\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}-b\left(8a^4+12a^2b^2+3b^4\right)\text{arcsinh}(\tan(c+dx))}{2b^4}-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2+\right.\right.}{}\right.\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{1}{4}\left(-\frac{\sqrt{\tan^2(c+dx)+1}\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{2b^2}-\frac{-8ab\left(a^2+b^2\right)^{3/2}\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)-b\left(8a^4+12a^2b^2+3b^4\right)\text{arcs}}{2b^4}\right.\right.}{b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input

```
Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

3.560. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

output

```
(Sec[c + d*x]*(-((1 + Tan[c + d*x]^2)^(5/2)/(a + b*Tan[c + d*x])) + (5*(-1
/12*((4*a - 3*b*Tan[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2)) + (-1/2*(-(b*(8*
a^4 + 12*a^2*b^2 + 3*b^4)*ArcSinh[Tan[c + d*x]]) - 8*a*b*(a^2 + b^2)^(3/2)
*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/b^4 - ((8*a*(a^2 + b^2) - b*
(4*a^2 + 3*b^2)*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])/(2*b^2))/4))/b^2))
/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.560.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

3.560. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.560. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.560.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(219) = 438$.

Time = 152.14 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.93

method	result
derivativedivides	$\frac{-\frac{1}{4b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{4a-3b}{6b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{12a^{2}-8ab+11b^{2}}{8b^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{\left(40a^{4}+60a^{2}b^{2}+15b^{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^{6}}-\frac{32a^{3}-1}{8b^{5}(\mathrm{t}}}{}$
default	$\frac{-\frac{1}{4b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{4a-3b}{6b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{12a^{2}-8ab+11b^{2}}{8b^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{\left(40a^{4}+60a^{2}b^{2}+15b^{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^{6}}-\frac{32a^{3}-1}{8b^{5}(\mathrm{t}}}{}$
risch	$-\frac{120ia^{3}b\,e^{3i(dx+c)}+60ia^{3}b\,e^{i(dx+c)}+180a^{2}b^{2}e^{9i(dx+c)}+640a^{2}b^{2}e^{7i(dx+c)}+920a^{2}b^{2}e^{5i(dx+c)}+640a^{2}b^{2}e^{3i(dx+c)}+180}{}$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/b^2/(tan(1/2*d*x+1/2*c)+1)^4-1/6*(4*a-3*b)/b^3/(tan(1/2*d*x+1/2*
c)+1)^3-1/8*(12*a^2-8*a*b+11*b^2)/b^4/(tan(1/2*d*x+1/2*c)+1)^2+1/8/b^6*(40
*a^4+60*a^2*b^2+15*b^4)*ln(tan(1/2*d*x+1/2*c)+1)-1/8*(32*a^3-12*a^2*b+40*a
*b^2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)+1)+1/4/b^2/(tan(1/2*d*x+1/2*c)-1)^4-1/
6*(-4*a-3*b)/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/8*(-12*a^2-8*a*b-11*b^2)/b^4/(
tan(1/2*d*x+1/2*c)-1)^2+1/8/b^6*(-40*a^4-60*a^2*b^2-15*b^4)*ln(tan(1/2*d*x
+1/2*c)-1)-1/8*(-32*a^3-12*a^2*b-40*a*b^2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)-1
)+2/b^6*(((a^4+2*a^2*b^2+b^4)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^4+2*a^2*b^2+b^
4))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-5*a*(a^4+2*a^2*b^2+b
^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))))
```

3.560.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(220) = 440$.

Time = 0.42 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.01

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{12\,b^5 - 30\,(8\,a^4b + 12\,a^2b^3 + 3\,b^5)\cos{(dx+c)}^4 + 10\,(4\,a^2b^3 + 3\,b^5)\cos{(dx+c)}^2 + 120\,((a^4 + a^2b^2)\cos{(d}{}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(12*b^5 - 30*(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4 + 10*(4*a^
2*b^3 + 3*b^5)*cos(d*x + c)^2 + 120*((a^4 + a^2*b^2)*cos(d*x + c)^5 + (a^3
*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^
2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^5 + (8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x
+ c))*log(sin(d*x + c) + 1) - 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos(d*x +
 c)^5 + (8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*log(-s
in(d*x + c) + 1) - 10*(2*a*b^4*cos(d*x + c) + 3*(4*a^3*b^2 + 5*a*b^4)*cos(
d*x + c)^3)*sin(d*x + c))/(a*b^6*d*cos(d*x + c)^5 + b^7*d*cos(d*x + c)^4*s
in(d*x + c))
```

3.560.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^7{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**2, x)
```

3.560.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(220) = 440$.

Time = 0.32 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.52

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/24*(2*(120*a^5 + 160*a^3*b^2 + 24*a*b^4 + (180*a^4*b + 245*a^2*b^3 + 24
*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*(48*a^5 + 68*a^3*b^2 + 15*a*b^4
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(300*a^4*b + 385*a^2*b^3 + 48*b^
5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(72*a^5 + 100*a^3*b^2 + 15*a*b
^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 48*(15*a^4*b + 20*a^2*b^3 + 3*b^
5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 30*(16*a^5 + 20*a^3*b^2 + 3*a*b^4
)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(60*a^4*b + 85*a^2*b^3 + 16*b^5)
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 30*(4*a^5 + 4*a^3*b^2 - a*b^4)*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*(20*a^4*b + 25*a^2*b^3 + 8*b^5)*sin(d*
x + c)^9/(cos(d*x + c) + 1)^9)/(a^2*b^5 + 2*a*b^6*sin(d*x + c)/(cos(d*x +
c) + 1) - 5*a^2*b^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a*b^6*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 10*a^2*b^5*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 12*a*b^6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10*a^2*b^5*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 - 8*a*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5
*a^2*b^5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 2*a*b^6*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 - a^2*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*(a
^4 + 2*a^2*b^2 + b^4)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(
a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sq
rt(a^2 + b^2)*b^6) - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/b^6 + 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x +...
```

3.560.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(220) = 440$.

Time = 0.53 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.26

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{15\,(8\,a^4+12\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6} - \frac{15\,(8\,a^4+12\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6} + \frac{120\,(a^5+2\,a^3b^2+ab^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\right.\right.}{\left|2\,a\tan\left(\frac{1}{2}\right.\right.}\right.}{\sqrt{a^2+b^2}b^6}}{}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b
^6 - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
6 + 120*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b -
 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*b^6) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) + 2*a^2*b^3*tan(
1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 + 2*a^3*b^2 + a*b^4)/((a
*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)*a*b^5) + 2*(36*a^2
*b*tan(1/2*d*x + 1/2*c)^7 + 27*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2
*d*x + 1/2*c)^6 + 144*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 36*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 - 288*a^3*tan(1/2*d*x + 1/2*c)^4
 - 336*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*
b^3*tan(1/2*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 + 304*a*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 36*a^2*b*tan(1/2*d*x + 1/2*c) + 27*b^3*tan(1/2*d*x
+ 1/2*c) - 96*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^5))/d
```

3.560.9 Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 2654, normalized size of antiderivative = 11.29

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^2),x)
```

output
```
-((9*a*b^5)/64 + (15*a^5*b)/8 + (b^6*sin(c + d*x))/8 + (115*a^3*b^3)/48 +
(3*b^6*sin(3*c + 3*d*x))/16 + (b^6*sin(5*c + 5*d*x))/16 + (a^6*cos(c + d*x
)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*25i)/4 + (5*a*b^5*cos(2
*c + 2*d*x))/8 + (5*a^5*b*cos(2*c + 2*d*x))/2 + (5*a*b^5*cos(3*c + 3*d*x))
/16 + (25*a^5*b*cos(3*c + 3*d*x))/16 + (15*a*b^5*cos(4*c + 4*d*x))/64 + (5
*a^5*b*cos(4*c + 4*d*x))/8 + (a*b^5*cos(5*c + 5*d*x))/16 + (5*a^5*b*cos(5*
c + 5*d*x))/16 + (25*a^3*b^3*cos(c + d*x))/6 + (5*a^2*b^4*sin(c + d*x))/6
+ (5*a^4*b^2*sin(c + d*x))/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 +
 (d*x)/2))*cos(3*c + 3*d*x)*25i)/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*cos(5*c + 5*d*x)*5i)/8 + (10*a^3*b^3*cos(2*c + 2*d*x))/3
+ (25*a^3*b^3*cos(3*c + 3*d*x))/12 + (15*a^3*b^3*cos(4*c + 4*d*x))/16 + (5
*a^3*b^3*cos(5*c + 5*d*x))/12 + (95*a^2*b^4*sin(2*c + 2*d*x))/96 + (5*a^4*
b^2*sin(2*c + 2*d*x))/8 + (5*a^2*b^4*sin(3*c + 3*d*x))/4 + (15*a^4*b^2*sin
(3*c + 3*d*x))/16 + (25*a^2*b^4*sin(4*c + 4*d*x))/64 + (5*a^4*b^2*sin(4*c
+ 4*d*x))/16 + (5*a^2*b^4*sin(5*c + 5*d*x))/12 + (5*a^4*b^2*sin(5*c + 5*d*
x))/16 + (5*a*b^5*cos(c + d*x))/8 + (25*a^5*b*cos(c + d*x))/8 + (a*b^5*ata
n((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*45i)/64 + (
a^5*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*15
i)/8 + (a*b^5*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5
*d*x)*15i)/64 + (a^5*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))...
```

3.560. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.561 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.561.1 Optimal result

Integrand size = 21, antiderivative size = 176

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{3(2a^2+b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} \\ &\quad+\frac{3a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} \\ &\quad-\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d}-\frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))}\end{aligned}$$

output
```
3/2*(2*a^2+b^2)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^4/d/(sec(d*x+c)^2)^(1/2)+
3*a*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*sec(d*x
+c)*(a^2+b^2)^(1/2)/b^4/d/(sec(d*x+c)^2)^(1/2)-3/2*sec(d*x+c)*(2*a-b*tan(d
*x+c))/b^3/d-sec(d*x+c)^3/b/d/(a+b*tan(d*x+c))
```

3.561.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.58 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.03

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= -\frac{(a-ib)(a+ib)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{b^3d(a+b\tan(c+dx))^2}\\
&\quad-\frac{2a\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d(a+b\tan(c+dx))^2}\\
&\quad-\frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}\left(-b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\cos\left(\frac{1}{2}(c+dx)\right)+b^2\cos\left(\frac{1}{2}(c+dx)\right)}\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^4d(a+b\tan(c+dx))^2}\\
&\quad-\frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2}\\
&\quad+\frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2}\\
&\quad+\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2}\\
&\quad-\frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}\\
&\quad-\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2}\\
&\quad+\frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

3.561. $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

output
```
-(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(
b^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcT
anh[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Co
s[(c + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(
a + b*Tan[c + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a +
 b*Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(4*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2)
+ (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
)/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

3.561.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3992, 492, 591, 25, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{3\int\frac{b\tan(c+dx)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 591}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\int-\frac{a-\left(\frac{2a^2}{b^2}+1\right)b\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c+dx))\right)}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(-\frac{1}{2}\int\frac{a-\left(\frac{2a^2}{b^2}+1\right)b\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c+dx))\right)}{b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(\left(\frac{2a^2}{b^2}+1\right)\int\frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-2a\left(\frac{a^2}{b^2}+1\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))\right)-\frac{1}{2}\sqrt{\tan^2(c+dx}\right.\right.}{b^2}}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(b\left(\frac{2a^2}{b^2}+1\right)\text{arcsinh}(\tan(c+dx))-2a\left(\frac{a^2}{b^2}+1\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))\right)-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-\right.\right.}{b^2}}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(2a\left(\frac{a^2}{b^2}+1\right)\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}+b\left(\frac{2a^2}{b^2}+1\right)\text{arcsinh}(\tan(c+dx))\right)-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c+\right.\right.}{b^2}}{bd\sqrt{\sec^2(c+dx)}}$$

3.561. $\int\frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{1}{2}\left(b\left(\frac{2a^2}{b^2}+1\right)\operatorname{arcsinh}(\tan(c+dx))+\frac{2ab\left(\frac{a^2}{b^2}+1\right)\operatorname{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)-\frac{1}{2}\sqrt{\tan^2(c+dx)+1}(2a-b\tan(c+dx))\right)}{b^2}-\frac{(t}{}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-((1 + Tan[c + d*x]^2)^(3/2)/(a + b*Tan[c + d*x])) + (3*(((
1 + (2*a^2)/b^2)*b*ArcSinh[Tan[c + d*x]] + (2*a*(1 + a^2/b^2)*b*ArcTanh[(b
^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2])/2 - ((2*a - b*Tan[c +
d*x])*Sqrt[1 + Tan[c + d*x]^2])/2))/b^2))/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.561.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.561. $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.561.4 Maple [A] (verified)

Time = 32.08 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\frac{1}{2b^2(\tan(\frac{dx}{2}+\frac{c}{2})-1)^2}-\frac{-4a-b}{2b^3(\tan(\frac{dx}{2}+\frac{c}{2})-1)}+\frac{(-6a^2-3b^2)\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)}{2b^4}-\frac{1}{2b^2(\tan(\frac{dx}{2}+\frac{c}{2})+1)^2}-\frac{4a-b}{2b^3(\tan(\frac{dx}{2}+\frac{c}{2})+1}}{d}$
default	$\frac{\frac{1}{2b^2(\tan(\frac{dx}{2}+\frac{c}{2})-1)^2}-\frac{-4a-b}{2b^3(\tan(\frac{dx}{2}+\frac{c}{2})-1)}+\frac{(-6a^2-3b^2)\ln(\tan(\frac{dx}{2}+\frac{c}{2})-1)}{2b^4}-\frac{1}{2b^2(\tan(\frac{dx}{2}+\frac{c}{2})+1)^2}-\frac{4a-b}{2b^3(\tan(\frac{dx}{2}+\frac{c}{2})+1}}{d}$
risch	$-\frac{-3iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}+3b^2e^{i(dx+c}}{(e^{2i(dx+c)}+1)^2(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a)\,d\,b^3}$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)
-1)+1/2/b^4*(-6*a^2-3*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b^2/(tan(1/2*d*x+1
/2*c)+1)^2-1/2*(4*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)+1/2/b^4*(6*a^2+3*b^2)*ln
(tan(1/2*d*x+1/2*c)+1)+2/b^4*(((a^2+b^2)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^2+b
^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-3*a*(a^2+b^2)^(1/2)
*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

3.561.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(163) = 326.

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.02

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{6\,ab^2\cos(dx+c)\sin(dx+c)-2\,b^3+6\,(2\,a^2b+b^3)\cos(dx+c)^2-6\,(a^2\cos(dx+c)^3+ab\cos(dx+c}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*
x + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a
^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*
a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2
*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c)
)*log(sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b
^3)*cos(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x
+ c)^3 + b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

3.561.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^5{\left(c+dx\right)}}{\left(a+b\tan{\left(c+dx\right)}\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**2, x)
```

3.561.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(163) = 326.

Time = 0.33 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.68

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\left(6\,a^{3}+2\,ab^{2}+\frac{6\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\left(9\,a^{2}b+2\,b^{3}\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\left(2\,a^{3}+ab^{2}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{4\left(3\,a^{2}b+b^{3}\right)\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\left(3\,a^{2}b+2\,b^{3}\right)\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{2}b^{3}+\frac{2\,ab^{4}\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\,a^{2}b^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{4\,ab^{4}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,a^{2}b^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{2\,ab^{4}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{a^{2}b^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{6\,\sqrt{a^{2}}}{}}{2\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(6*a^3 + 2*a*b^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (9*
a^2*b + 2*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^3 + a*b^2)*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^2*b + b^3)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + (3*a^2*b + 2*b^3)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2*b
^3 + 2*a*b^4*sin(d*x + c)/(cos(d*x + c) + 1) - 3*a^2*b^3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 4*a*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*b
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*b^4*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - a^2*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 6*sqrt(a^2 + b
^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a
*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/b^4 - 3*(2*a^2 + b^2)
*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^2 + b^2)*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/b^4)/d
```

3.561.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.59

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4} - \frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4} + \frac{6\,(a^3+ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4} + 2\,(b}{2\,d}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c
) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1
/2*c)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(
1/2*d*x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d
*x + 1/2*c) - a)*a*b^3))/d
```

3.561.9 Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.32

$$
\begin{aligned}
&\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx \\
&= \frac{\operatorname{atanh}\left(\frac{648\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a\,b^2+648\,a^3+\frac{432\,a^5}{b^2}}+\frac{432\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{432\,a^5+648\,a^3\,b^2+216\,a\,b^4}+\frac{216\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a+\frac{648\,a^3}{b^2}+\frac{432\,a^5}{b^4}}\right)\,\left(6\,a^2+3\,b^2\right)}{b^4\,d} \\
&\quad-\frac{\frac{2\,\left(3\,a^2+b^2\right)}{b^3}+\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^3}-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+b^2\right)}{b^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a^2+2\,b^2\right)}{a\,b^2}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^2+b^2\right)}{a\,b^2}+\frac{\tan}{}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.} \\
&\quad-\frac{6\,a\operatorname{atanh}\left(\frac{432\,a^3\,\sqrt{a^2+b^2}}{432\,a^3\,b+\frac{432\,a^5}{b}+864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+864\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{864\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}{432\,a^3+\frac{432\,a^5}{b^2}+864\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}\right.}{b^4\,d}
\end{aligned}
$$

input

```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^2),x)
```

output

```
(atanh((648*a^3*tan(c/2 + (d*x)/2))/(216*a*b^2 + 648*a^3 + (432*a^5)/b^2)
+ (432*a^5*tan(c/2 + (d*x)/2))/(216*a*b^4 + 432*a^5 + 648*a^3*b^2) + (216*
a*tan(c/2 + (d*x)/2))/(216*a + (648*a^3)/b^2 + (432*a^5)/b^4))*(6*a^2 + 3*
b^2))/(b^4*d) - ((2*(3*a^2 + b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3
- (6*tan(c/2 + (d*x)/2)^2*(2*a^2 + b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2
+ 2*b^2))/(a*b^2) - (4*tan(c/2 + (d*x)/2)^3*(3*a^2 + b^2))/(a*b^2) + (tan(
c/2 + (d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2)
- 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/
2)^6 - 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (6*a*atanh(
(432*a^3*(a^2 + b^2)^(1/2))/(432*a^3*b + (432*a^5)/b + 864*a^4*tan(c/2 + (
d*x)/2) + 864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(a
^2 + b^2)^(1/2))/(432*a^3 + (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) +
 (864*a^4*tan(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(a^2 + b^2)
^(1/2))/(432*a^5 + 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) + 864*a^2*b^
3*tan(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2))/(b^4*d)
```

3.562 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.562.1 Optimal result 3878
3.562.2 Mathematica [A] (verified) 3878
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3.562.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d} + \frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^2/d+a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^
(1/2))/b^2/d/(a^2+b^2)^(1/2)-sec(d*x+c)/b/d/(a+b*tan(d*x+c))
```

3.562.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2a\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin(\frac{1}{2}(c}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
-(((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2
] + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

3.562.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3992, 492, 605, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{\tan^2(c+dx)+1}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{b\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{605}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-a\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sec(c+dx)\left(\frac{b\text{arcsinh}(\tan(c+dx))-a\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(\frac{a\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}+b\text{arcsinh}(\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 219$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+b\text{arcsinh}(\tan(c+dx))}{b^2}-\frac{\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*((b*ArcSinh[Tan[c + d*x]] + (a*b*ArcTanh[(b^2*Tan[c + d*x])/
Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2])/b^2 - Sqrt[1 + Tan[c + d*x]^2]/(a + b*T
an[c + d*x])))/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.562.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

3.562. $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.562.4 Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\dfrac{\dfrac{\dfrac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\dfrac{2a\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\dfrac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\dfrac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}}{d}$
default	$\dfrac{\dfrac{\dfrac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\dfrac{2a\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\dfrac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\dfrac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}}{d}$
risch	$-\dfrac{2\,\mathrm{e}^{i(dx+c)}}{db\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}+\dfrac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\dfrac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\dfrac{\ln(\mathrm{e}^{i(dx+c)}-i}{d\,b^2}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*((b^2/a*tan(1/2*d*x+1/2*c)+b)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1
/2*d*x+1/2*c)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b
)/(a^2+b^2)^(1/2)))-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/b^2*ln(tan(1/2*d*x+1/
2*c)+1))
```

3.562.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(87) = 174$.

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b}\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b
^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*co
s(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b
^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a
^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) +
 1))/((a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

3.562.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**2, x)
```

3.562.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\left(a+\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2b+\frac{2\,ab^2\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^2b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}-\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}}{d}$$

input

```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output

```
-(2*(a + b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2*b + 2*a*b^2*sin(d*x + c)/
(cos(d*x + c) + 1) - a^2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - a*log((b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c
)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/b^2 + log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/b^2)/d
```

3.562.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.82

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

input

```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output

```
(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(
abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
2 + 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/
2*d*x + 1/2*c) - a)*a*b))/d
```

3.562.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.21

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{b^2\sin(c+d\,x)-\frac{2\left(a^2\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\sqrt{a^2+b^2}+a^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}\right)\mathrm{co}\right.}{\sqrt{a^2+b^2}}}{a\,b^2\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^2),x)
```

output
```
-(b^2*sin(c + d*x) - (2*(a^3*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
 + (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b
^2)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*cos(c + d*x)*1i + a
^2*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(
1/2)))/(a^2 + b^2)^(1/2) + (2*b*((a*(a^2 + b^2)^(1/2))/2 + (a*cos(c + d*x)
*(a^2 + b^2)^(1/2))/2 - a^2*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
+ (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^
2)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*sin(c + d*x)*1i - a*
sin(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2
)))/(a^2 + b^2)^(1/2))/(a*b^2*d*(a*cos(c + d*x) + b*sin(c + d*x)))
```

3.563 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.563.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
-a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-
b*sec(d*x+c)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.563.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2a\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b\sec(c+dx)}{(a^2+b^2)(a+b\tan(c+dx))}}{d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2
) - (b*Sec[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])))/d
```

3.563.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3992, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{491}$$

$$\frac{\sec(c+dx)\left(\frac{a\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(-\frac{a\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(-\frac{ab\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{(a^2+b^2)(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input

```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(-((a*b*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 +
b^2)^(3/2)) - (b^2*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan[c + d
*x]))))/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.563.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.563. $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.563.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$-\frac{2ib\,\mathrm{e}^{i(dx+c)}}{(-ia+b)d(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+i}{(a}\right.}{(a^2+b^2)^{\frac{3}{2}}}$

input

```
int(sec(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(-2*(-b^2/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tan(1/2*d*x+1/2
*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan
(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.563.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(78) = 156.

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.62

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2}\right.}{2\left((a^5+2\,a^3b^2+ab^4)d\cos\left(dx+c\right)+(a^4b+2\,a^2b^3+b^5)d\sin\left(dx+c\right)\right)}$$

input

```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output

```
-1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b
^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2
*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a
^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

3.563.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

3.563.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(78) = 156$.

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.22

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(ab+\frac{b^2\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+a^2b^2+\frac{2\left(a^3b+ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4+a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(
a*b + b^2*sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + a^2*b^2 + 2*(a^3*b + a*b
^3)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^4 + a^2*b^2)*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2))/d
```

3.563.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{(a^3+ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*
tan(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)))/d
```

3.563.9 Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.66

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,(a^2+b^2)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)} + \frac{a\,\mathrm{atan}\left(\frac{a^2\,b\,1\mathrm{i}+b^3\,1\mathrm{i}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)\,2\mathrm{i}}{d\,(a^2+b^2)^{3/2}}$$

input
```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^2),x)
```

output
```
(a*atan((a^2*b*1i + b^3*1i - a*tan(c/2 + (d*x)/2)*(a^2 + b^2)*1i)/(a^2 + b
^2)^(3/2))*2i)/(d*(a^2 + b^2)^(3/2)) - ((2*b)/(a^2 + b^2) + (2*b^2*tan(c/2
 + (d*x)/2))/(a*(a^2 + b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 +
 (d*x)/2)^2))
```

3.564 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.564.2 Mathematica [A] (verified) 3891
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3.564.1 Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{3ab^2\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{5/2}\,d} + \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
-3*a*b^2*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*co
s(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(5/2)/d+b*(a^2-2*b^2)*sec(d*x+c)/(
a^2+b^2)^2/d/(a+b*tan(d*x+c))+cos(d*x+c)*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b
*tan(d*x+c))
```

3.564.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\sec(c+dx)\Big(12ab^2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))+(a^2+b^2)\big(3b(a^2 -}{2(a^2+b^2)^3\,d(a+b\tan(c+dx))}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(12*a*b^2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(3*b*(a^2
 - b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*Sin[2*(c + d*x)])
))/(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

3.564.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3992, 496, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{b^2\int -\frac{2b^2+a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{2b^2+a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2b^2+a\tan(c+dx)b}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 679$$

$$\frac{\sec(c+dx)\left(\frac{\frac{3ab^2\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{b^2\left(a^2-2b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 488$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(a^2-2b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{3ab^2\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 219$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(a^2-2b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{3ab^3\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])*
Sqrt[1 + Tan[c + d*x]^2]) + ((-3*a*b^3*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2
 + b^2]])/(a^2 + b^2)^(3/2) + (b^2*(a^2 - 2*b^2)*Sqrt[1 + Tan[c + d*x]^2])
/((a^2 + b^2)*(a + b*Tan[c + d*x])))/(a^2 + b^2)))/(b*d*Sqrt[Sec[c + d*x]^
2])
```

3.564. $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.564.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.564. $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.564.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\ \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}}{2(-2iab+a^2-b^2)d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2(2iab+a^2-b^2)d}-\frac{2ib^3\mathrm{e}^{i(dx+c)}}{(-ia+b)^2d(ia+b)^2\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{3b^2a\ln\left(\mathrm{e}^{i(dx+}\right.}{}$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*d*x+1/2*c)-2*a*b)/(1+tan(1
/2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2
*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*
(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

3.564.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.92

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{2\,a^4b-2\,a^2b^3-4\,b^5+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2+2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)d\,\mathrm{c}\right.}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c
)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*c
os(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)
*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^
2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (
a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*
d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

3.564.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cos{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

3.564.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(151) = 302$.

Time = 0.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.22

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,ab^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(2\,a^3b-ab^3-\frac{3\,ab^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^4+3\,a^2b^2-b^4\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4-a^2b^2+b^4\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+2\,a^4b^2+a^2b^4+\frac{2\left(a^5b+2\,a^3b^3+ab^5\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^5b+2\,a^3b^3+ab^5\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\left(a^6+2\,a^4b^2+a^2b^4\right)}{(\cos(dx+c)+}}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(3*a*b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3*b - a*b^3 - 3*a*b^3*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + (a^4 + 3*a^2*b^2 - b^4)*sin(d*x + c)/(cos(d*x + c) + 1
) - (a^4 - a^2*b^2 + b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + 2*a^
4*b^2 + a^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)/(cos(d*x + c)
 + 1) + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- (a^6 + 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

3.564.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}{(a^5+2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,d\right.\right.}}{d}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(
2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x
 + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2
- a^4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*
d*x + 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x
+ 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/
d
```

3.564.9 Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{4a^2b-2b^3}{a^4+2a^2b^2+b^4} - \frac{6b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4+2a^2b^2+b^4} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+3a^2b^2-b^4)}{a(a^4+2a^2b^2+b^4)} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^4-2a^2b^2+2b^4)}{a(a^4+2a^2b^2+b^4)}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$
$$- \frac{6ab^2\,\mathrm{atanh}\left(\frac{a^4b+b^5+2a^2b^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

input

```
int(cos(c + d*x)/(a + b*tan(c + d*x))^2,x)
```

output

```
((4*a^2*b - 2*b^3)/(a^4 + b^4 + 2*a^2*b^2) - (6*b^3*tan(c/2 + (d*x)/2)^2)/
(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(a^4 - b^4 + 3*a^2*b^2))/(
a*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 2*a^2*
b^2))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (6*a*b^2*atanh((a^4*b + b^
5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^
(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.565 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

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3.565.1 Optimal result

Integrand size = 21, antiderivative size = 241

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = &-\frac{5ab^4\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{7/2}\,d}\\ &+\frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}\\ &+\frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))}\\ &-\frac{\cos(c+dx)\,(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx))}{3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-5*a*b^4*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*co
s(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(7/2)/d+1/3*b*(2*a^4+9*a^2*b^2-8*b
^4)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))+1/3*cos(d*x+c)^3*(b+a*tan(d*
x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(a^2-4*b^2)-a*(2*a^2+
7*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

3.565.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\sec(c+dx)\Bigl(240ab^4\sqrt{a^2+b^2}\,\mathrm{arctanh}\Bigl(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\Bigr)(a\cos(c+dx)+b\sin(c+dx))+(a^2+b^2)\Bigl(15a^4b}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*(240*a*b^4*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(15*a^4*
b + 90*a^2*b^3 - 45*b^5 + 20*b^3*(a^2 + b^2)*Cos[2*(c + d*x)] + b*(a^2 + b
^2)^2*Cos[4*(c + d*x)] + 10*a^5*Sin[2*(c + d*x)] + 40*a^3*b^2*Sin[2*(c + d
*x)] + 30*a*b^4*Sin[2*(c + d*x)] + a^5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*
(c + d*x)] + a*b^4*Sin[4*(c + d*x)])))/(24*(a^2 + b^2)^4*d*(a + b*Tan[c +
d*x]))
```

3.565.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3992, 496, 25, 27, 686, 25, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(c+dx)^3(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3992}$$
$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^2(\tan^2(c+dx)+1)^{5/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$
$$\Big\downarrow \text{496}$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}-\frac{b^2\int -\frac{2\left(\frac{a^2}{b^2}+2\right)b^2+3a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{2\left(a^2+2b^2\right)+3ab\tan(c+dx)}{b^2(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2\left(a^2+2b^2\right)+3ab\tan(c+dx)}{(a+b\tan(c+dx))^2\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 686$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{b^4\int -\frac{2\left(4-\frac{a^2}{b^2}\right)b^4+a\left(2a^2+7b^2\right)\tan(c+dx)b}{b^4(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+dx)}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^4\int -\frac{2b^2\left(a^2-4b^2\right)-ab\left(2a^2+7b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}}{3(a^2+b^2)}+\frac{ab\tan(c+dx)}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

3.565. $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{b^4\int\frac{2b^2\left(a^2-4b^2\right)-ab\left(2a^2+7b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{\int\frac{2b^2\left(a^2-4b^2\right)-ab\left(2a^2+7b^2\right)\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 679$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{-\frac{15ab^4\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{b^2\left(2a^4+9a^2b^2-8b^4\right)\sqrt{\tan^2(c+dx)+}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 488$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{\frac{15ab^4\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{b^2\left(2a^4+9a^2b^2-8b^4\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}}{3(a^2+b^2)}+\frac{}{3}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 219$$

3.565. $\int\frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))}+\frac{\frac{ab\left(2a^2+7b^2\right)\tan(c+dx)+b^4\left(4-\frac{a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))}-\frac{\frac{15ab^5\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b^2(2}{}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/(3*(a^2 + b^2)*(a + b*Tan[c + d*x]
)*(1 + Tan[c + d*x]^2)^(3/2)) + (((4 - a^2/b^2)*b^4 + a*b*(2*a^2 + 7*b^2)*
Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])
- ((15*a*b^5*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2
) - (b^2*(2*a^4 + 9*a^2*b^2 - 8*b^4)*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2
)*(a + b*Tan[c + d*x])))/(a^2 + b^2))/(3*(a^2 + b^2))))/(b*d*Sqrt[Sec[c +
d*x]^2])
```

3.565.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.565. $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.565.4 Maple [A] (verified)

Time = 8.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^3b-6a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^4-6a^2b^2+\frac{8}{3}b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}{\left(a^2+b^2\right)\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$
default	$-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^3b-6a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^4-6a^2b^2+\frac{8}{3}b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}{\left(a^2+b^2\right)\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$
risch	$-\frac{i\,\mathrm{e}^{3i(dx+c)}}{24(-2iab+a^2-b^2)d}-\frac{7\,\mathrm{e}^{i(dx+c)}b}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}-\frac{3i\,\mathrm{e}^{i(dx+c)}a}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}-\frac{7\,\mathrm{e}^{-i(dx+c)}b}{8(ib+a)^3d}+\frac{3i\,\mathrm{e}^{-i(dx+}}{8(ib+a)^3}$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+
1/2*c)^5+(-2*a^3*b-6*a*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^4-6*a^2*b^2+8/3*b
^4)*tan(1/2*d*x+1/2*c)^3-8*a*b^3*tan(1/2*d*x+1/2*c)^2+(-a^4-3*a^2*b^2+2*b^
4)*tan(1/2*d*x+1/2*c)-2/3*a^3*b-14/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3-2*b
^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d
*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2
*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{4\,a^6b+22\,a^4b^3+2\,a^2b^5-16\,b^7+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos{(dx+c)}^4-2\,(a^6b-2\,a^4b^3-7\,a^2b^5-}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(4*a^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 2*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(a^6*b - 2*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*
cos(d*x + c)^2 + 15*(a^2*b^4*cos(d*x + c) + a*b^5*sin(d*x + c))*sqrt(a^2 +
 b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b
*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*((a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 11*a^5*b^2 + 16
*a^3*b^4 + 7*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*
b^4 + 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9)*d*sin(d*x + c))
```

3.565.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.565.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. 2(229) = 458.

Time = 0.46 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.20

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{\frac{15\,ab^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^5b+14\,a^3b^3-3\,ab^5-\frac{15\,ab^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\left(3\,a^6+13\,a^4b^2+22\,a^2b^4-3\,b^6\right)\sin(dx+c)}{\cos(dx+c)+1}+\right.}{a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6+\frac{2\left(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6\right)\sin(dx+}{(\cos(dx+c)+1)^2}}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(15*a*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2
))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(2*a^5*b + 14*a^3*b^3 - 3*a*
b^5 - 15*a*b^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (3*a^6 + 13*a^4*b^2 +
 22*a^2*b^4 - 3*b^6)*sin(d*x + c)/(cos(d*x + c) + 1) + (4*a^5*b + 28*a^3*b
^3 - 21*a*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^6 - 9*a^4*b^2 - 46
*a^2*b^4 + 9*b^6)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*(2*a^5*b + 6*a^3
*b^3 - 5*a*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^6 + 3*a^4*b^2 + 3
8*a^2*b^4 - 9*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^6 + 3*a^4*b^
2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^8 + 3*a^6*b^2
 + 3*a^4*b^4 + a^2*b^6 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x
 + c)/(cos(d*x + c) + 1) + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^
4 + a^2*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 2*(a^7*b + 3*a^5*b^3 +
3*a^3*b^5 + a*b^7)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - (a^8 + 3*a^6*b^2
+ 3*a^4*b^4 + a^2*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8))/d
```

3.565.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.82

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{15\,ab^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{6\,(b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab^5)}{(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)} - \frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,d\right.\right.}{}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(15*a*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))
/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 6*(b^6*tan(1/2*d*x + 1/2*c) + a*b^
5)/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*
tan(1/2*d*x + 1/2*c) - a)) - 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*t
an(1/2*d*x + 1/2*c)^5 - 6*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b*tan(1/2*d*x
 + 1/2*c)^4 + 18*a*b^3*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)
^3 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*b^4*tan(1/2*d*x + 1/2*c)^3 + 24
*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) + 9*a^2*b^2*tan
(1/2*d*x + 1/2*c) - 6*b^4*tan(1/2*d*x + 1/2*c) + 2*a^3*b + 14*a*b^3)/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

3.565.9 Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.80

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{2\,(2\,a^4\,b+14\,a^2\,b^3-3\,b^5)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}-\frac{10\,b^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{10\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(2\,a^4+6\,a^2\,b^2-5\,b^4)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(4\,a^4\,b+28\,a^2\,b^3-21\,b^5)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\,a\,\tan\left(\frac{c}{2}+\right.\right.}$$

$$-\frac{10\,a\,b^4\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}{2\,{(a^2+b^2)}^{7/2}}\right)}{d\,{(a^2+b^2)}^{7/2}}$$

input
```
int(cos(c + d*x)^3/(a + b*tan(c + d*x))^2,x)
```

output

```
((2*(2*a^4*b - 3*b^5 + 14*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)
) - (10*b^5*tan(c/2 + (d*x)/2)^6)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (1
0*b*tan(c/2 + (d*x)/2)^4*(2*a^4 - 5*b^4 + 6*a^2*b^2))/(3*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4*b - 21*b^5 + 28*a^2*b
^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^7*(a^
6 + b^6 - 2*a^2*b^4 + 3*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
+ (2*tan(c/2 + (d*x)/2)*(3*a^6 - 3*b^6 + 22*a^2*b^4 + 13*a^4*b^2))/(3*a*(a
^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(a^6 - 9*b^6
+ 38*a^2*b^4 + 3*a^4*b^2))/(3*a*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*
tan(c/2 + (d*x)/2)^3*(a^6 + 9*b^6 - 46*a^2*b^4 - 9*a^4*b^2))/(3*a*(a^2 + b
^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2
 + (d*x)/2)^2 - 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8 + 6*b*ta
n(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7))
 - (10*a*b^4*atanh((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2
+ (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d
*(a^2 + b^2)^(7/2))
```

3.565. $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

3.566 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.566.1 Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{3(a^2+b^2)\,(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d}\\ &\quad-\frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d}+\frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d}\\ &\quad-\frac{a\tan^3(c+dx)}{b^4d}+\frac{\tan^4(c+dx)}{4b^3d}\\ &\quad-\frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2}+\frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))}\end{aligned}$$

output
```
3*(a^2+b^2)*(5*a^2+b^2)*ln(a+b*tan(d*x+c))/b^7/d-a*(10*a^2+9*b^2)*tan(d*x+
c)/b^6/d+3/2*(2*a^2+b^2)*tan(d*x+c)^2/b^5/d-a*tan(d*x+c)^3/b^4/d+1/4*tan(d
*x+c)^4/b^3/d-1/2*(a^2+b^2)^3/b^7/d/(a+b*tan(d*x+c))^2+6*a*(a^2+b^2)^2/b^7
/d/(a+b*tan(d*x+c))
```

3.566.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.47

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2(a^2+b^2)\left(19a^4+16a^2b^2-3b^4+6a^2(5a^2+b^2)\log(a+b\tan(c+dx))\right)+b^6\sec^6(c+dx)+4ab(4a^4+17}{}$$

input
```
Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

output
```
(2*(a^2 + b^2)*(19*a^4 + 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 + b^2)*Log[a +
b*Tan[c + d*x]]) + b^6*Sec[c + d*x]^6 + 4*a*b*(4*a^4 + 17*a^2*b^2 + 11*b^4
 + 6*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 4*b
^2*(-13*a^4 - 10*a^2*b^2 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d
*x]])*Tan[c + d*x]^2 - 20*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 4*a^2*b^4*Tan
[c + d*x]^4 + b^4*Sec[c + d*x]^4*(a^2 + 3*b^2 - 2*a*b*Tan[c + d*x]))/(4*b^
7*d*(a + b*Tan[c + d*x])^2)
```

3.566.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sec(c+dx)^8}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \; 3987$$
$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^3}{b^6(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^3}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{b^7d}$$

$$\Big\downarrow 476$$

$$\frac{\int \left(-10\left(\frac{9b^2}{10a^2}+1\right)a^3-3b^2\tan^2(c+dx)a-\frac{6\left(a^2+b^2\right)^2a}{(a+b\tan(c+dx))^2}+b^3\tan^3(c+dx)+3b\left(2a^2+b^2\right)\tan(c+dx)+\frac{3\left(5a^4-\right.}{a+b\left.\right.}\right.}{b^7d}$$

$$\Big\downarrow 2009$$

$$\frac{\frac{3}{2}b^2\left(2a^2+b^2\right)\tan^2(c+dx)-ab\left(10a^2+9b^2\right)\tan(c+dx)+\frac{6a\left(a^2+b^2\right)^2}{a+b\tan(c+dx)}-\frac{\left(a^2+b^2\right)^3}{2(a+b\tan(c+dx))^2}+3\left(a^2+b^2\right)\left(5a^2+b^2\right.}{b^7d}$$

input
```
Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

output
```
(3*(a^2 + b^2)*(5*a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*(10*a^2 + 9*b^2
)*Tan[c + d*x] + (3*b^2*(2*a^2 + b^2)*Tan[c + d*x]^2)/2 - a*b^3*Tan[c + d*
x]^3 + (b^4*Tan[c + d*x]^4)/4 - (a^2 + b^2)^3/(2*(a + b*Tan[c + d*x])^2) +
 (6*a*(a^2 + b^2)^2)/(a + b*Tan[c + d*x]))/(b^7*d)
```

3.566.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.566. $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.566.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.05

$$\frac{\frac{\frac{\left(\tan^{4}(dx+c)\right)b^{3}}{4}-a\left(\tan^{3}(dx+c)\right)b^{2}+3a^{2}b\left(\tan^{2}(dx+c)\right)+\frac{3b^{3}\left(\tan^{2}(dx+c)\right)}{2}-10a^{3}\tan(dx+c)-9a\,b^{2}\tan(dx+c)}{b^{6}}+\frac{6a\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}{b^{7}(a+b\tan(dx+c))}+\frac{(15a^{4}+18}{}}{d}$$

input
```
int(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x)
```

output
```
1/d*(1/b^6*(1/4*tan(d*x+c)^4*b^3-a*tan(d*x+c)^3*b^2+3*a^2*b*tan(d*x+c)^2+3
/2*b^3*tan(d*x+c)^2-10*a^3*tan(d*x+c)-9*a*b^2*tan(d*x+c))+6*a/b^7*(a^4+2*a
^2*b^2+b^4)/(a+b*tan(d*x+c))+(15*a^4+18*a^2*b^2+3*b^4)/b^7*ln(a+b*tan(d*x+
c))-1/2/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a+b*tan(d*x+c))^2)
```

3.566.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(179) = 358.

Time = 0.31 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.57

$$\int \frac{\sec^{8}(c+dx)}{(a+b\tan(c+dx))^{3}}\,dx$$
$$=\frac{8\left(15\,a^{4}b^{2}+13\,a^{2}b^{4}\right)\cos\left(dx+c\right)^{6}+b^{6}-2\left(45\,a^{4}b^{2}+44\,a^{2}b^{4}+3\,b^{6}\right)\cos\left(dx+c\right)^{4}+\left(5\,a^{2}b^{4}+3\,b^{6}\right)\cos\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44
*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*
((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b
^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*co
s(d*x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x +
c)^2 + b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5
*a^5*b + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a
^2*b^4 + b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c)
+ 2*(15*a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5
)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b
^9*d*cos(d*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

3.566.6 Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**3, x)
```

3.566.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(11\,a^6+21\,a^4b^2+9\,a^2b^4-b^6+12\,(a^5b+2\,a^3b^3+ab^5)\tan(dx+c))}{b^9\tan(dx+c)^2+2\,ab^8\tan(dx+c)+a^2b^7}+\frac{b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,(2\,a^2b+b^3)\tan(dx+c)^2-4\,(10\,a^3+9\,ab}{b^6}}{4\,d}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(2*(11*a^6 + 21*a^4*b^2 + 9*a^2*b^4 - b^6 + 12*(a^5*b + 2*a^3*b^3 + a*
b^5)*tan(d*x + c))/(b^9*tan(d*x + c)^2 + 2*a*b^8*tan(d*x + c) + a^2*b^7) +
 (b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(2*a^2*b + b^3)*tan(d*x
+ c)^2 - 4*(10*a^3 + 9*a*b^2)*tan(d*x + c))/b^6 + 12*(5*a^4 + 6*a^2*b^2 +
b^4)*log(b*tan(d*x + c) + a)/b^7)/d
```

3.566.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.31

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{12\left(5a^4+6a^2b^2+b^4\right)\log(|b\tan(dx+c)+a|)}{b^7} - \frac{2\left(45a^4b^2\tan(dx+c)^2+54a^2b^4\tan(dx+c)^2+9b^6\tan(dx+c)^2+78a^5b\tan(dx+c)+84a^3b^3\tan(dx\right.}{(b\tan(dx+c)+a)^2b^7}}{}$$

input
```
integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45
*a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2
 + 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c)
+ 34*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x +
 c)^4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x
 + c)^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

3.566.9 Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{\frac{11\,a^6+21\,a^4\,b^2+9\,a^2\,b^4-b^6}{2\,b}+\tan(c+d\,x)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{d\,\left(a^2\,b^6+2\,a\,b^7\,\tan\left(c+d\,x\right)+b^8\,{\tan\left(c+d\,x\right)}^2\right)} \\ &\quad+\frac{{\tan(c+d\,x)}^2\,\left(\frac{3}{2\,b^3}+\frac{3\,a^2}{b^5}\right)}{d}+\frac{{\tan(c+d\,x)}^4}{4\,b^3\,d} \\ &\quad+\frac{\tan(c+d\,x)\,\left(\frac{8\,a^3}{b^6}-\frac{3\,a\,\left(\frac{3}{b^3}+\frac{6\,a^2}{b^5}\right)}{b}\right)}{d}-\frac{a\,{\tan(c+d\,x)}^3}{b^4\,d} \\ &\quad+\frac{\ln\left(a+b\,\tan(c+d\,x)\right)\,\left(15\,a^4+18\,a^2\,b^2+3\,b^4\right)}{b^7\,d}\end{aligned}$$

input
```
int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^3),x)
```

output
```
((11*a^6 - b^6 + 9*a^2*b^4 + 21*a^4*b^2)/(2*b) + tan(c + d*x)*(6*a*b^4 + 6
*a^5 + 12*a^3*b^2))/(d*(a^2*b^6 + b^8*tan(c + d*x)^2 + 2*a*b^7*tan(c + d*x
))) + (tan(c + d*x)^2*(3/(2*b^3) + (3*a^2)/b^5))/d + tan(c + d*x)^4/(4*b^3
*d) + (tan(c + d*x)*((8*a^3)/b^6 - (3*a*(3/b^3 + (6*a^2)/b^5))/b))/d - (a*
tan(c + d*x)^3)/(b^4*d) + (log(a + b*tan(c + d*x))*(15*a^4 + 3*b^4 + 18*a^
2*b^2))/(b^7*d)
```

3.566. $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.567 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.567.1 Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{2(3a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} - \frac{3a\tan(c+dx)}{b^4d} + \frac{\tan^2(c+dx)}{2b^3d} - \frac{(a^2+b^2)^2}{2b^5d(a+b\tan(c+dx))^2} + \frac{4a(a^2+b^2)}{b^5d(a+b\tan(c+dx))}$$

output
```
2*(3*a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d-3*a*tan(d*x+c)/b^4/d+1/2*tan(d*x+c)
^2/b^3/d-1/2*(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c))^2+4*a*(a^2+b^2)/b^5/d/(a+b
*tan(d*x+c))
```

3.567.2 Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{b^4\sec^4(c+dx)}{2(a+b\tan(c+dx))^2} - 2a\left(-2a\log(a+b\tan(c+dx)) + b\tan(c+dx) - \frac{a^2+b^2}{a+b\tan(c+dx)}\right) + 2(a^2+b^2)\left(\log(a+b\,t\right.}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
((b^4*Sec[c + d*x]^4)/(2*(a + b*Tan[c + d*x])^2) - 2*a*(-2*a*Log[a + b*Tan
[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c + d*x])) + 2*(a^2 +
 b^2)*(Log[a + b*Tan[c + d*x]] + (3*a^2 - b^2 + 4*a*b*Tan[c + d*x])/(2*(a
+ b*Tan[c + d*x])^2)))/(b^5*d)
```

3.567.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^6}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(\tan^2(c+dx)b^2+b^2\right)^2}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\frac{\left(a^2+b^2\right)^2}{(a+b\tan(c+dx))^3}-\frac{4a\left(a^2+b^2\right)}{(a+b\tan(c+dx))^2}-3a+b\tan(c+dx)+\frac{2\left(3a^2+b^2\right)}{a+b\tan(c+dx)}\right)d(b\tan(c+dx))}{b^5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(a^2+b^2\right)^2}{2(a+b\tan(c+dx))^2}+\frac{4a\left(a^2+b^2\right)}{a+b\tan(c+dx)}+2\left(3a^2+b^2\right)\log(a+b\tan(c+dx))-3ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)}{b^5d}$$

input
```
Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

output
```
(2*(3*a^2 + b^2)*Log[a + b*Tan[c + d*x]] - 3*a*b*Tan[c + d*x] + (b^2*Tan[c
 + d*x]^2)/2 - (a^2 + b^2)^2/(2*(a + b*Tan[c + d*x])^2) + (4*a*(a^2 + b^2)
)/(a + b*Tan[c + d*x]))/(b^5*d)
```

3.567.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.567.4 Maple [A] (verified)

Time = 174.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^{2}(dx+c)\right)}{2}+3a\tan(dx+c)}{b^{4}}-\frac{a^{4}+2a^{2}b^{2}+b^{4}}{2b^{5}(a+b\tan(dx+c))^{2}}+\frac{\left(6a^{2}+2b^{2}\right)\ln(a+b\tan(dx+c))}{b^{5}}+\frac{4a\left(a^{2}+b^{2}\right)}{b^{5}(a+b\tan(dx+c))}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^{2}(dx+c)\right)}{2}+3a\tan(dx+c)}{b^{4}}-\frac{a^{4}+2a^{2}b^{2}+b^{4}}{2b^{5}(a+b\tan(dx+c))^{2}}+\frac{\left(6a^{2}+2b^{2}\right)\ln(a+b\tan(dx+c))}{b^{5}}+\frac{4a\left(a^{2}+b^{2}\right)}{b^{5}(a+b\tan(dx+c))}}{d}$
risch	$\frac{-36a^{2}b\,e^{2i(dx+c)}+36ia^{3}e^{2i(dx+c)}+4ia\,b^{2}e^{6i(dx+c)}+12ia\,b^{2}e^{4i(dx+c)}+12a^{2}b\,e^{6i(dx+c)}+12ia^{3}e^{6i(dx+c)}+36ia^{3}e^{4i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^{2}\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^{2}b}$

input
```
int(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/2*b*tan(d*x+c)^2+3*a*tan(d*x+c))-1/2/b^5*(a^4+2*a^2*b^2+b^
4)/(a+b*tan(d*x+c))^2+(6*a^2+2*b^2)/b^5*ln(a+b*tan(d*x+c))+4*a/b^5*(a^2+b^
2)/(a+b*tan(d*x+c)))
```

3.567.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(117) = 234$.

Time = 0.30 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.93

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{24\,a^2b^2\cos{(dx+c)}^4 + b^4 - 2\,(9\,a^2b^2 + b^4)\cos{(dx+c)}^2 + 2\,((3\,a^4 - 2\,a^2b^2 - b^4)\cos{(dx+c)}^4 + 2\,(3\,a^3b +}{}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2
+ 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*
x + c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^
2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x +
 c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos
(d*x + c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos
(d*x + c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*
x + c)^4)
```

3.567.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^6{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{7\,a^4+6\,a^2b^2-b^4+8\,(a^3b+ab^3)\tan(dx+c)}{b^7\tan(dx+c)^2+2\,ab^6\tan(dx+c)+a^2b^5}+\frac{b\tan(dx+c)^2-6\,a\tan(dx+c)}{b^4}+\frac{4\,(3\,a^2+b^2)\log(b\tan(dx+c)+a)}{b^5}}{2\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((7*a^4 + 6*a^2*b^2 - b^4 + 8*(a^3*b + a*b^3)*tan(d*x + c))/(b^7*tan(d
*x + c)^2 + 2*a*b^6*tan(d*x + c) + a^2*b^5) + (b*tan(d*x + c)^2 - 6*a*tan(
d*x + c))/b^4 + 4*(3*a^2 + b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{4\,(3\,a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^5}+\frac{b^3\tan(dx+c)^2-6\,ab^2\tan(dx+c)}{b^6}-\frac{18\,a^2b^2\tan(dx+c)^2+6\,b^4\tan(dx+c)^2+28\,a^3b\tan(dx+c)+4\,ab^3\tan(d}{(b\tan(dx+c)+a)^2b^5}}{2\,d}$$

input
```
integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^
2 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x
 + c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b
*tan(d*x + c) + a)^2*b^5))/d
```

3.567.9 Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{7\,a^4+6\,a^2\,b^2-b^4}{2\,b}+\tan(c+d\,x)\,\left(4\,a^3+4\,a\,b^2\right)}{d\,\left(a^2\,b^4+2\,a\,b^5\,\tan\left(c+d\,x\right)+b^6\,\tan\left(c+d\,x\right)^2\right)} + \frac{\tan(c+d\,x)^2}{2\,b^3\,d} - \frac{3\,a\tan(c+d\,x)}{b^4\,d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^2+2\,b^2\right)}{b^5\,d}$$

input
```
int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

output
```
((7*a^4 - b^4 + 6*a^2*b^2)/(2*b) + tan(c + d*x)*(4*a*b^2 + 4*a^3))/(d*(a^2
*b^4 + b^6*tan(c + d*x)^2 + 2*a*b^5*tan(c + d*x))) + tan(c + d*x)^2/(2*b^3
*d) - (3*a*tan(c + d*x))/(b^4*d) + (log(a + b*tan(c + d*x))*(6*a^2 + 2*b^2
))/(b^5*d)
```

3.568 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.568.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\log(a+b\tan(c+dx))}{b^3d} - \frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2} + \frac{2a}{b^3d(a+b\tan(c+dx))}$$

output
```
ln(a+b*tan(d*x+c))/b^3/d+1/2*(-a^2-b^2)/b^3/d/(a+b*tan(d*x+c))^2+2*a/b^3/d
/(a+b*tan(d*x+c))
```

3.568.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\log(a+b\tan(c+dx)) - \frac{a^2+b^2}{2(a+b\tan(c+dx))^2} + \frac{2a}{a+b\tan(c+dx)}}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/
(a + b*Tan[c + d*x]))/(b^3*d)
```

3.568.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^4}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{b^2(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\tan^2(c+dx)b^2+b^2}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-\frac{2a}{(a+b\tan(c+dx))^2}+\frac{1}{a+b\tan(c+dx)}+\frac{a^2+b^2}{(a+b\tan(c+dx))^3}\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2+b^2}{2(a+b\tan(c+dx))^2}+\frac{2a}{a+b\tan(c+dx)}+\log(a+b\tan(c+dx))}{b^3d}$$

input
```
Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/
(a + b*Tan[c + d*x]))/(b^3*d)
```

3.568. $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}dx$

3.568.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.568.4 Maple [A] (verified)

Time = 37.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}}{d}$	63
default	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}}{d}$	63
risch	$\frac{-2a^2\mathrm{e}^{2i(dx+c)}+2b^2\mathrm{e}^{2i(dx+c)}+4iab\,\mathrm{e}^{2i(dx+c)}-2a^2-2iab}{b^2(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)^2d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{b^3d}$	160

input
```
int(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*ln(a+b*tan(d*x+c))-1/2*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^2+2/b^3*a
/(a+b*tan(d*x+c)))
```

3.568.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 284, normalized size of antiderivative = 4.12

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{4\,a^2b^2\cos\left(dx+c\right)^2 - 3\,a^2b^2 - b^4 - 2\left(a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^2b^2+b^4+\left(a^4-b^4\right)\cos\left(dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*
x + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3
*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*
x + c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2
))/((a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*
sin(d*x + c) + (a^2*b^5 + b^7)*d)
```

3.568.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{4\,ab\tan(dx+c)+3\,a^2-b^2}{b^5\tan(dx+c)^2+2\,ab^4\tan(dx+c)+a^2b^3}+\frac{2\,\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((4*a*b*tan(d*x + c) + 3*a^2 - b^2)/(b^5*tan(d*x + c)^2 + 2*a*b^4*tan(
d*x + c) + a^2*b^3) + 2*log(b*tan(d*x + c) + a)/b^3)/d
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2\,\log(|b\tan(dx+c)+a|)}{b^3}-\frac{3\,b\tan(dx+c)^2+2\,a\tan(dx+c)+b}{(b\tan(dx+c)+a)^2b^2}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*
x + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

3.568.9 Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{3\,a^2-b^2}{2\,b^3}+\frac{2\,a\tan(c+d\,x)}{b^2}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)} + \frac{\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d}$$

input
```
int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

output
```
((3*a^2 - b^2)/(2*b^3) + (2*a*tan(c + d*x))/b^2)/(d*(a^2 + b^2*tan(c + d*x
)^2 + 2*a*b*tan(c + d*x))) + log(a + b*tan(c + d*x))/(b^3*d)
```

3.569 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.569.9 Mupad [B] (verification not implemented) 3932

3.569.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx=-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

output
```
-1/2/b/d/(a+b*tan(d*x+c))^2
```

3.569.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx=-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*1/(b*d*(a + b*Tan[c + d*x])^2)
```

3.569.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{1}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*1/(b*d*(a + b*Tan[c + d*x])^2)
```

3.569.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.569.4 Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
default	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2 d(ia+b)^2}$	77

input

```
int(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2/b/d/(a+b*tan(d*x+c))^2
```

3.569.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{4a^2b\cos(dx+c)^2-a^2b+b^3-2(a^3-ab^2)\cos(dx+c)\sin(dx+c)}{2((a^6+a^4b^2-a^2b^4-b^6)d\cos(dx+c)^2+2(a^5b+2a^3b^3+ab^5)d\cos(dx+c)\sin(dx+c)+(a^4b^2+2}$$

input

```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output

```
-1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*
sin(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b
 + 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 +
 b^6)*d)
```

3.569.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2\,(b\tan(dx+c)+a)^2bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2/((b*tan(d*x + c) + a)^2*b*d)
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2\,(b\tan(dx+c)+a)^2bd}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2/((b*tan(d*x + c) + a)^2*b*d)
```

3.569.9 Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{d\,\left(2\,a^2\,b+4\,a\,b^2\,\tan\left(c+d\,x\right)+2\,b^3\,\tan\left(c+d\,x\right)^2\right)}$$

input

```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
```

output

```
-1/(d*(2*a^2*b + 2*b^3*tan(c + d*x)^2 + 4*a*b^2*tan(c + d*x)))
```

3.570 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.570.7 Maxima [B] (verification not implemented) . . . 3938
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3.570.9 Mupad [B] (verification not implemented) . . . 3940

3.570.1 Optimal result

Integrand size = 21, antiderivative size = 202

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{a(a^4+10a^2b^2-15b^4)\,x}{2\left(a^2+b^2\right)^4} + \frac{2b^3(5a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{b(a^2-2b^2)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right) d(a+b\tan(c+dx))^2} + \frac{ab(a^2-11b^2)}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))}$$

output
```
1/2*a*(a^4+10*a^2*b^2-15*b^4)*x/(a^2+b^2)^4+2*b^3*(5*a^2-b^2)*ln(a*cos(d*x
+c)+b*sin(d*x+c))/(a^2+b^2)^4/d+1/2*b*(a^2-2*b^2)/(a^2+b^2)^2/d/(a+b*tan(d
*x+c))^2+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+
1/2*a*b*(a^2-11*b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

3.570.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. $2(202) = 404$.

Time = 6.37 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{b^3\left(\frac{\cos^2(c+dx)\left(b^2+ab\tan(c+dx)\right)}{2b^4(a^2+b^2)(a+b\tan(c+dx))^2} - \frac{(2a^2-4b^2)\left(-\frac{\left(3a^2-b^2-\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3}+\frac{\left(3a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3}-\frac{\left(3a^2-b^2+\frac{a^3}{\sqrt{}}\right.}{}\right.}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^3*((Cos[c + d*x]^2*(b^2 + a*b*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*
Tan[c + d*x])^2) - ((2*a^2 - 4*b^2)*(-1/2*((3*a^2 - b^2 - (a^3 - 3*a*b^2)/
Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)^3 + ((3*a^2 - b^
2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - ((3*a^2 - b^2 + (a^3 - 3*a*b^2
)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^3) - 1/(2*(
a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^2 + b^2)^2*(a + b*Tan[c + d
*x]))) - 3*a*(-1/2*((2*a - (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[
c + d*x]])/(a^2 + b^2)^2 + (2*a*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - (
(2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 +
 b^2)^2) - 1/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2))))/d
```

3.570.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sec(c+dx)^2(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{b^4}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))^2}-\frac{\int -\frac{a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{a^2+3b\tan(c+dx)a+4b^2}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^3\left(\frac{\int \left(-\frac{a\left(a^2-11b^2\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{4\left(5a^2b^2-b^4\right)}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{a\left(a^4+10b^2a^2-15b^4\right)-4b^3\left(5a^2-b^2\right)\tan(c+dx)}{\left(a^2+b^2\right)^3\left(\tan^2(c+dx)b^2+b^2\right)}-\frac{2\left(a^2-2b^2\right)}{\left(a^2+b^2\right)(a+b\tan(c+dx))^3}\right)d(b\tan(c}{2b^2(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\left(\frac{ab\tan(c+dx)+b^2}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)(a+b\tan(c+dx))^2}+\frac{\frac{a\left(a^2-11b^2\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{a^2-2b^2}{\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2b^2\left(5a^2-b^2\right)\log\left(b^2\tan^2(c+dx)+b^2\right.}{\left(a^2+b^2\right)^3}}{2b^2(a^2+b^2)}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

3.570. $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}dx$

output
```
(b^3*((b^2 + a*b*Tan[c + d*x])/(2*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2*(
b^2 + b^2*Tan[c + d*x]^2)) + ((a*(a^4 + 10*a^2*b^2 - 15*b^4)*ArcTan[Tan[c
+ d*x]])/(b*(a^2 + b^2)^3) + (4*b^2*(5*a^2 - b^2)*Log[a + b*Tan[c + d*x]])
/(a^2 + b^2)^3 - (2*b^2*(5*a^2 - b^2)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a^2
+ b^2)^3 + (a^2 - 2*b^2)/((a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (a*(a^2 -
11*b^2))/((a^2 + b^2)^2*(a + b*Tan[c + d*x])))/(2*b^2*(a^2 + b^2))))/d
```

3.570.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.570.4 Maple [A] (verified)

Time = 10.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}a\,b^4\right)\tan(dx+c)+\frac{3a^4b}{2}+a^2b^3-\frac{b^5}{2}}{1+\tan^2(dx+c)}+\frac{\left(-20a^2b^3+4b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^5+10a^3b^2-15a\,b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^4}}{d}$
default	$\frac{\frac{\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}a\,b^4\right)\tan(dx+c)+\frac{3a^4b}{2}+a^2b^3-\frac{b^5}{2}}{1+\tan^2(dx+c)}+\frac{\left(-20a^2b^3+4b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^5+10a^3b^2-15a\,b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^4}}{d}$
risch	$\frac{4ixb}{8ia^3b-8ia\,b^3-2a^4+12a^2b^2-2b^4}-\frac{xa}{8ia^3b-8ia\,b^3-2a^4+12a^2b^2-2b^4}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}+\frac{i\mathrm{e}^{-2i}}{8(3ib\,a^2-ib^3}$

input
```
int(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^4*(((1/2*a^5-a^3*b^2-3/2*a*b^4)*tan(d*x+c)+3/2*a^4*b+a^2*
b^3-1/2*b^5)/(1+tan(d*x+c)^2)+1/4*(-20*a^2*b^3+4*b^5)*ln(1+tan(d*x+c)^2)+1
/2*(a^5+10*a^3*b^2-15*a*b^4)*arctan(tan(d*x+c)))-1/2*b^3/(a^2+b^2)^2/(a+b*
tan(d*x+c))^2-4*b^3/(a^2+b^2)^3*a/(a+b*tan(d*x+c))+2*b^3*(5*a^2-b^2)/(a^2+
b^2)^4*ln(a+b*tan(d*x+c)))
```

3.570.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(194) = 388.

Time = 0.31 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.49

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$
$$-\frac{3\,a^4b^3-16\,a^2b^5+b^7-2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos{(dx+c)}^4-2\,(a^5b^2+10\,a^3b^4-15\,ab^6)dx-(a}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(3*a^4*b^3 - 16*a^2*b^5 + b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7)*cos(d*x + c)^4 - 2*(a^5*b^2 + 10*a^3*b^4 - 15*a*b^6)*d*x - (a^6*b - a^4
*b^3 - 45*a^2*b^5 - 3*b^7 + 2*(a^7 + 9*a^5*b^2 - 25*a^3*b^4 + 15*a*b^6)*d*
x)*cos(d*x + c)^2 - 4*(5*a^2*b^5 - b^7 + (5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos
(d*x + c)^2 + 2*(5*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 - 3*a^3*b^4 + 6*
a*b^6 - (a^6*b + 10*a^4*b^3 - 15*a^2*b^5)*d*x)*cos(d*x + c))*sin(d*x + c))
/((a^10 + 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x
+ c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x +
 c)*sin(d*x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

3.570.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.570.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(194) = 388$.

Time = 0.31 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(a^5+10a^3b^2-15ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{4(5a^2b^3-b^5)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{2(5a^2b^3-b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{}{a^8+3a^6b^2+3a^4b^4+a^2b^6+(a^6b}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 4*(5*a^2*b^3 - b^5)*log(b*tan(d*x + c) + a)/(a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x +
 c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (3*a^4*b - 10
*a^2*b^3 - b^5 + (a^3*b^2 - 11*a*b^4)*tan(d*x + c)^3 + 2*(a^4*b - 6*a^2*b^
3 - b^5)*tan(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 10*a*b^4)*tan(d*x + c))/(a^8
+ 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8
)*tan(d*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)^
3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^
7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

3.570.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(194) = 388$.

Time = 0.64 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.17

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(a^5+10\,a^3b^2-15\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{2\,(5\,a^2b^3-b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{4\,(5\,a^2b^4-b^6)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} + \frac{10\,a^2b^3\tan(dx+c)^2-2\,b^5\tan(}{}}{}$$

input
```
integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*
a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 4*(5*a^2*b^4 - b^6)*log(abs(b*tan
(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (10*a^
2*b^3*tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 + a^5*tan(d*x + c) - 2*a^3*b^2
*tan(d*x + c) - 3*a*b^4*tan(d*x + c) + 3*a^4*b + 12*a^2*b^3 - 3*b^5)/((a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d*x + c)^2 + 1)) - (30*a^
2*b^5*tan(d*x + c)^2 - 6*b^7*tan(d*x + c)^2 + 68*a^3*b^4*tan(d*x + c) - 4*
a*b^6*tan(d*x + c) + 39*a^4*b^3 + 4*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

3.570.9 Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.07

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{10\,b^3}{{\left(a^2+b^2\right)}^3}-\frac{12\,b^5}{{\left(a^2+b^2\right)}^4}\right)}{d}$$
$$-\frac{\frac{-3\,a^4\,b+10\,a^2\,b^3+b^5}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{{\tan(c+d\,x)}^3\,\left(11\,a\,b^4-a^3\,b^2\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^2\,\left(-a^4\,b+6\,a^2\,b^3+b^5\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{a\,\tan(c+d\,x)\,\left(a^4+3\,a^2\,b^2-10\,b^4\right)}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(a^2+b^2\right)+a^2+b^2\,{\tan\left(c+d\,x\right)}^4+2\,a\,b\,\tan\left(c+d\,x\right)+2\,a\,b\,{\tan\left(c+d\,x\right)}^3\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(b+\frac{a\,1{}\mathrm{i}}{4}\right)}{d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,4{}\mathrm{i}\right)}{4\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(a + b*tan(c + d*x))*((10*b^3)/(a^2 + b^2)^3 - (12*b^5)/(a^2 + b^2)^4)
)/d - ((b^5 - 3*a^4*b + 10*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)
) + (tan(c + d*x)^3*(11*a*b^4 - a^3*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (tan(c + d*x)^2*(b^5 - a^4*b + 6*a^2*b^3))/((a^2 + b^2)*(a^4 + b
^4 + 2*a^2*b^2)) - (a*tan(c + d*x)*(a^4 - 10*b^4 + 3*a^2*b^2))/(2*(a^2 + b
^2)*(a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*t
an(c + d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(tan(c +
 d*x) + 1i)*((a*1i)/4 + b))/(d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^
2)) + (log(tan(c + d*x) - 1i)*(a + b*4i))/(4*d*(4*a*b^3 - 4*a^3*b + a^4*1i
 + b^4*1i - a^2*b^2*6i))
```

3.571 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.571.1 Optimal result

Integrand size = 21, antiderivative size = 295

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{3a(a^6+7a^4b^2+35a^2b^4-35b^6)\,x}{8\,(a^2+b^2)^5}\\ &+\frac{3b^5(7a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d}\\ &+\frac{3b(a^4+5a^2b^2-4b^4)}{8\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^2}\\ &+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}\\ &+\frac{3ab(a^4+6a^2b^2-27b^4)}{8\,(a^2+b^2)^4\,d(a+b\tan(c+dx))}\\ &-\frac{\cos^2(c+dx)\,(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}\end{aligned}$$

output
```
3/8*a*(a^6+7*a^4*b^2+35*a^2*b^4-35*b^6)*x/(a^2+b^2)^5+3*b^5*(7*a^2-b^2)*ln
(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d+3/8*b*(a^4+5*a^2*b^2-4*b^4)/(a^2
+b^2)^3/d/(a+b*tan(d*x+c))^2+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d
/(a+b*tan(d*x+c))^2+3/8*a*b*(a^4+6*a^2*b^2-27*b^4)/(a^2+b^2)^4/d/(a+b*tan(
d*x+c))-1/8*cos(d*x+c)^2*(2*b*(a^2-3*b^2)-a*(3*a^2+11*b^2)*tan(d*x+c))/(a^
2+b^2)^2/d/(a+b*tan(d*x+c))^2
```

3.571.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 596 vs. 2(295) = 590.

Time = 6.29 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.02

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{b^5\left(\frac{\cos^4(c+dx)\left(b^2+ab\tan(c+dx)\right)}{4b^6(a^2+b^2)(a+b\tan(c+dx))^2} - \frac{\frac{\cos^2(c+dx)\left(5a^2b^2-3b^2\left(a^2+2b^2\right)+b\left(-5ab^2-3a\left(a^2+2b^2\right)\right)\tan(c+dx)\right)}{2b^4\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\left(-3a^2\left(3a^2+11b^2\right)+3\left(a^4+a^2b^2+8b^4\right)\right)}{}}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^5*((Cos[c + d*x]^4*(b^2 + a*b*Tan[c + d*x]))/(4*b^6*(a^2 + b^2)*(a + b*
Tan[c + d*x])^2) - ((Cos[c + d*x]^2*(5*a^2*b^2 - 3*b^2*(a^2 + 2*b^2) + b*(
-5*a*b^2 - 3*a*(a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*Tan
[c + d*x])^2) - ((-3*a^2*(3*a^2 + 11*b^2) + 3*(a^4 + a^2*b^2 + 8*b^4))*(-1
/2*((3*a^2 - b^2 - (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c +
d*x]])/(a^2 + b^2)^3 + ((3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)
^3 - ((3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c
+ d*x]])/(2*(a^2 + b^2)^3) - 1/(2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2
*a)/((a^2 + b^2)^2*(a + b*Tan[c + d*x]))) + 3*a*(3*a^2 + 11*b^2)*(-1/2*((2
*a - (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)
^2 + (2*a*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (a^2 - b^2)/Sqr
t[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2) - 1/((a^2 + b
^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

3.571.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3987, 27, 496, 25, 686, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sec(c+dx)^4(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \ 3987$$

$$\frac{\int \frac{b^6}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \ 27$$

$$\frac{b^5\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)b^2+b^2)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \ 496$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))^2}-\frac{\int -\frac{3\left(a^2+2b^2\right)+5ab\tan(c+dx)}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{b^5\left(\frac{\int \frac{3\left(a^2+2b^2\right)+5ab\tan(c+dx)}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))^2}\right)}{d}$$

$$\Big\downarrow \ 686$$

$$\frac{b^5\Bigg(\frac{-\frac{\int -\frac{3\left(a^4+b^2a^2+b\left(3a^2+11b^2\right)\tan(c+dx)a+8b^4\right)}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{2b^2\left(a^2-3b^2\right)-ab\left(3a^2+11b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))^2}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+d}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)-}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{b^5\Bigg(\frac{\frac{3\int \frac{a^4+b^2a^2+b\left(3a^2+11b^2\right)\tan(c+dx)a+8b^4}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}-\frac{2b^2\left(a^2-3b^2\right)-ab\left(3a^2+11b^2\right)\tan(c+dx)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)(a+b\tan(c+dx))^2}}{4b^2(a^2+b^2)}+\frac{ab\tan(c+dx)+b}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2}}{d}$$

3.571. $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}dx$

$$\Big\downarrow \text{657}$$

$$\frac{b^5\left(\frac{\frac{3\int\left(-\frac{a\left(a^4+6b^2a^2-27b^4\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{8\left(7a^2b^4-b^6\right)}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{a\left(a^6+7b^2a^4+35b^4a^2-35b^6\right)-8b^5\left(7a^2-b^2\right)\tan(c+dx)}{\left(a^2+b^2\right)^3\left(\tan^2(c+dx)b^2+b^2\right)}-\frac{2\left(a^4+5b^2a^2-4b^4\right)}{\left(a^2+b^2\right)(a+b\tan(c+dx))^3}\right)d(}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^5\left(\frac{ab\tan(c+dx)+b^2}{4b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)^2(a+b\tan(c+dx))^2}+\frac{3\left(-\frac{4b^4\left(7a^2-b^2\right)\log\left(b^2\tan^2(c+dx)+b^2\right)}{\left(a^2+b^2\right)^3}+\frac{8b^4\left(7a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3}+\frac{a\left(a^4+6a^2\right.}{\left(a^2+b^2\right)^2(a}\right.}{2b^2\left(a\right.}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b^5*((b^2 + a*b*Tan[c + d*x])/(4*b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2*(
b^2 + b^2*Tan[c + d*x]^2)^2) + (-1/2*(2*b^2*(a^2 - 3*b^2) - a*b*(3*a^2 + 1
1*b^2)*Tan[c + d*x])/(b^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2*(b^2 + b^2*Ta
n[c + d*x]^2)) + (3*((a*(a^6 + 7*a^4*b^2 + 35*a^2*b^4 - 35*b^6)*ArcTan[Tan
[c + d*x]])/(b*(a^2 + b^2)^3) + (8*b^4*(7*a^2 - b^2)*Log[a + b*Tan[c + d*x
]])/(a^2 + b^2)^3 - (4*b^4*(7*a^2 - b^2)*Log[b^2 + b^2*Tan[c + d*x]^2])/(a
^2 + b^2)^3 + (a^4 + 5*a^2*b^2 - 4*b^4)/((a^2 + b^2)*(a + b*Tan[c + d*x])^
2) + (a*(a^4 + 6*a^2*b^2 - 27*b^4))/((a^2 + b^2)^2*(a + b*Tan[c + d*x]))))
/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

3.571.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.571. $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.571.4 Maple [A] (verified)

Time = 43.90 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{b^5}{2\left(a^2+b^2\right)^3(a+b\tan(dx+c))^2}-\frac{6b^5a}{\left(a^2+b^2\right)^4(a+b\tan(dx+c))}+\frac{3b^5\left(7a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}a\,b}{}}{}$
default	$\frac{-\frac{b^5}{2\left(a^2+b^2\right)^3(a+b\tan(dx+c))^2}-\frac{6b^5a}{\left(a^2+b^2\right)^4(a+b\tan(dx+c))}+\frac{3b^5\left(7a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}a\,b}{}}{}$
risch	$\frac{15xab}{8ia^5-80ia^3b^2+40ia\,b^4+40a^4b-80a^2b^3+8b^5}-\frac{24ix\,b^2}{8ia^5-80ia^3b^2+40ia\,b^4+40a^4b-80a^2b^3+8b^5}-\frac{ie^{2i(dx+c}}{8(-4ia^3b+4ia\,b^3+a^4}$

input
```
int(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))^2-6*b^5/(a^2+b^2)^4*a/(a+b*tan(
d*x+c))+3*b^5*(7*a^2-b^2)/(a^2+b^2)^5*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((
3/8*a^7+21/8*a^5*b^2-15/8*a^3*b^4-33/8*a*b^6)*tan(d*x+c)^3+(5*a^4*b^3+4*a^
2*b^5-b^7)*tan(d*x+c)^2+(19/8*a^5*b^2-39/8*a*b^6+5/8*a^7-25/8*a^3*b^4)*tan
(d*x+c)+3/4*a^6*b+25/4*a^4*b^3+17/4*b^5*a^2-5/4*b^7)/(1+tan(d*x+c)^2)^2+3/
16*(-56*a^2*b^5+8*b^7)*ln(1+tan(d*x+c)^2)+3/8*(a^7+7*a^5*b^2+35*a^3*b^4-35
*a*b^6)*arctan(tan(d*x+c))))
```

3.571.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(284) = 568$.

Time = 0.33 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.27

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$
$$-\frac{9\,a^6b^3+95\,a^4b^5-141\,a^2b^7-3\,b^9-8\left(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9\right)\cos\left(dx+c\right)^6+8\left(a^8b-6\,a^4}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/32*(9*a^6*b^3 + 95*a^4*b^5 - 141*a^2*b^7 - 3*b^9 - 8*(a^8*b + 4*a^6*b^3
 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 8*(a^8*b - 6*a^4*b^5 - 8*
a^2*b^7 - 3*b^9)*cos(d*x + c)^4 - 12*(a^7*b^2 + 7*a^5*b^4 + 35*a^3*b^6 - 3
5*a*b^8)*d*x - (15*a^8*b + 82*a^6*b^3 + 68*a^4*b^5 - 498*a^2*b^7 - 51*b^9
+ 12*(a^9 + 6*a^7*b^2 + 28*a^5*b^4 - 70*a^3*b^6 + 35*a*b^8)*d*x)*cos(d*x +
 c)^2 - 48*(7*a^2*b^7 - b^9 + (7*a^4*b^5 - 8*a^2*b^7 + b^9)*cos(d*x + c)^2
 + 2*(7*a^3*b^6 - a*b^8)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)
*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*(4*(a^9 + 4*a^7*b^2
+ 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^5 + 2*(3*a^9 + 20*a^7*b^2 +
42*a^5*b^4 + 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^3 - (3*a^7*b^2 + 53*a^5*b
^4 - 15*a^3*b^6 + 159*a*b^8 - 12*(a^8*b + 7*a^6*b^3 + 35*a^4*b^5 - 35*a^2*
b^7)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*
a^4*b^8 - 4*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10
*a^7*b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) +
(a^10*b^2 + 5*a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

3.571.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.571.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(284) = 568$.

Time $= 0.53$ (sec) , antiderivative size $= 738$, normalized size of antiderivative $= 2.50$

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,(a^7+7\,a^5b^2+35\,a^3b^4-35\,ab^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(7\,a^2b^5-b^7)\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(7\,a^2b^5-b^7)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} +}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b
^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(7*a^2*b^5 - b^7)*lo
g(b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*
b^8 + b^10) - 12*(7*a^2*b^5 - b^7)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b
^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + (6*a^6*b + 44*a^4*b^3 -
 62*a^2*b^5 - 4*b^7 + 3*(a^5*b^2 + 6*a^3*b^4 - 27*a*b^6)*tan(d*x + c)^5 +
6*(a^6*b + 6*a^4*b^3 - 13*a^2*b^5 - 2*b^7)*tan(d*x + c)^4 + (3*a^7 + 23*a^
5*b^2 + 61*a^3*b^4 - 151*a*b^6)*tan(d*x + c)^3 + 2*(5*a^6*b + 37*a^4*b^3 -
 73*a^2*b^5 - 9*b^7)*tan(d*x + c)^2 + (5*a^7 + 26*a^5*b^2 + 49*a^3*b^4 - 6
8*a*b^6)*tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8
 + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2
*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^1
0 + 6*a^8*b^2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)
^4 + 4*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3
+ (2*a^10 + 9*a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*
x + c)^2 + 2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x +
 c)))/d
```

3.571.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(284) = 568$.

Time = 0.61 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.99

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{3\,(a^7+7\,a^5b^2+35\,a^3b^4-35\,ab^6)\,(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(7\,a^2b^5-b^7)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(7\,a^2b^6-b^8)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}} + 3}{}$$

input
```
integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b
^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(7*a^2*b^5 - b^7)*lo
g(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*
b^8 + b^10) + 24*(7*a^2*b^6 - b^8)*log(abs(b*tan(d*x + c) + a))/(a^10*b +
5*a^8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) + (3*a^5*b^2*tan(d
*x + c)^5 + 18*a^3*b^4*tan(d*x + c)^5 - 81*a*b^6*tan(d*x + c)^5 + 6*a^6*b*
tan(d*x + c)^4 + 36*a^4*b^3*tan(d*x + c)^4 - 78*a^2*b^5*tan(d*x + c)^4 - 1
2*b^7*tan(d*x + c)^4 + 3*a^7*tan(d*x + c)^3 + 23*a^5*b^2*tan(d*x + c)^3 +
61*a^3*b^4*tan(d*x + c)^3 - 151*a*b^6*tan(d*x + c)^3 + 10*a^6*b*tan(d*x +
c)^2 + 74*a^4*b^3*tan(d*x + c)^2 - 146*a^2*b^5*tan(d*x + c)^2 - 18*b^7*tan
(d*x + c)^2 + 5*a^7*tan(d*x + c) + 26*a^5*b^2*tan(d*x + c) + 49*a^3*b^4*ta
n(d*x + c) - 68*a*b^6*tan(d*x + c) + 6*a^6*b + 44*a^4*b^3 - 62*a^2*b^5 - 4
*b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 +
 a*tan(d*x + c)^2 + b*tan(d*x + c) + a)^2))/d
```

3.571.9 Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.42

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{3\,a^6\,b+22\,a^4\,b^3-31\,a^2\,b^5-2\,b^7}{4\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{\tan(c+d\,x)\,\left(5\,a^7+26\,a^5\,b^2+49\,a^3\,b^4-68\,a\,b^6\right)}{8\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{3\tan(c+d\,x)^5\,\left(a^5\,b^2+6\,a^3\,b^4-27\,a\,b^6\right)}{8\,\left(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8\right)}+\frac{\tan(c+d\,x)^3}{8\,(a^8+}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(2\,a^2+b^2\right)+{\tan\left(c+d\,x\right)}^4\,\left(a^2+2\,b^2\right)+a^2+b^2\,\tan\left(c+d\right.}$$

$$+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{21\,b^5}{{\left(a^2+b^2\right)}^4}-\frac{24\,b^7}{{\left(a^2+b^2\right)}^5}\right)}{d}$$

$$+\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-a^2\,1\mathrm{i}+5\,a\,b+b^2\,8\mathrm{i}\right)}{16\,d\,\left(a^5+a^4\,b\,5\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10\mathrm{i}+5\,a\,b^4+b^5\,1\mathrm{i}\right)}$$

$$+\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}+5\,a\,b-b^2\,8\mathrm{i}\right)}{16\,d\,\left(a^5-a^4\,b\,5\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10\mathrm{i}+5\,a\,b^4-b^5\,1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

output

```
((3*a^6*b - 2*b^7 - 31*a^2*b^5 + 22*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6
*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*a^7 - 68*a*b^6 + 49*a^3*b^4 + 26
*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c
+ d*x)^5*(6*a^3*b^4 - 27*a*b^6 + a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a
^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(3*a^7 - 151*a*b^6 + 61*a^3*b^4 + 2
3*a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c
 + d*x)^4*(a^6*b - 2*b^7 - 13*a^2*b^5 + 6*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*
b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^2*(5*a^6*b - 9*b^7 - 73*a^2*
b^5 + 37*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)))/(d
*(tan(c + d*x)^2*(2*a^2 + b^2) + tan(c + d*x)^4*(a^2 + 2*b^2) + a^2 + b^2*
tan(c + d*x)^6 + 2*a*b*tan(c + d*x) + 4*a*b*tan(c + d*x)^3 + 2*a*b*tan(c +
 d*x)^5)) + (log(a + b*tan(c + d*x))*((21*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a
^2 + b^2)^5))/d + (3*log(tan(c + d*x) - 1i)*(5*a*b - a^2*1i + b^2*8i))/(16
*d*(5*a*b^4 + a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (3*lo
g(tan(c + d*x) + 1i)*(5*a*b + a^2*1i - b^2*8i))/(16*d*(5*a*b^4 - a^4*b*5i
+ a^5 - b^5*1i + a^2*b^3*10i - 10*a^3*b^2))
```

3.571. $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.572 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.572.1 Optimal result 3951
3.572.2 Mathematica [C] (verified) 3952
3.572.3 Rubi [A] (warning: unable to verify) 3952
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3.572.9 Mupad [B] (verification not implemented) 3961

3.572.1 Optimal result

Integrand size = 21, antiderivative size = 239

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = &-\frac{5a(4a^2+3b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} \\ &-\frac{5\sqrt{a^2+b^2}(4a^2+b^2)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} \\ &-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} \\ &+\frac{5\sec(c+dx)\left(4a^2+b^2-2ab\tan(c+dx)\right)}{2b^5d}\end{aligned}$$

output
```
-5/2*a*(4*a^2+3*b^2)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(
1/2)-5/2*(4*a^2+b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^
2)^(1/2))*sec(d*x+c)*(a^2+b^2)^(1/2)/b^6/d/(sec(d*x+c)^2)^(1/2)-1/2*sec(d*
x+c)^5/b/d/(a+b*tan(d*x+c))^2+5/6*sec(d*x+c)^3*(4*a+b*tan(d*x+c))/b^3/d/(a
+b*tan(d*x+c))+5/2*sec(d*x+c)*(4*a^2+b^2-2*a*b*tan(d*x+c))/b^5/d
```

3.572.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.61 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.88

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^2(a^2+b^2)^2\sin(c+dx)}{a}+\frac{6(a-ib)(a+ib)b(8a^2-b^2)(a\cos(c+dx)+b\sin(c+dx))}{a}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Si
n[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b
*Sin[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^2 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/S
qrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^2 - 30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/
2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^3 + (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]
*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3 + (b^2*(9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b
^6*d*(a + b*Tan[c + d*x])^3)
```

3.572.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3992, 492, 590, 25, 27, 682, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)^7}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \ 3992$$

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 492$$

$$\frac{\sec(c+dx)\left(\frac{5\int \frac{b\tan(c+dx)\left(\tan^2(c+dx)+1\right)^{3/2}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 590$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\tan(c+dx))}{3(a+b\tan(c+dx))}-\int -\frac{\left(b^2-4ab\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{b^2(a+b\tan(c+dx))}d(b\tan(c+dx))\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\int \frac{\left(b^2-4ab\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{b^2(a+b\tan(c+dx))}d(b\tan(c+dx))+\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\tan(c+dx))}{3(a+b\tan(c+dx))}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\int \frac{\left(b^2-4ab\tan(c+dx)\right)\sqrt{\tan^2(c+dx)+1}}{a+b\tan(c+dx)}d(b\tan(c+dx))}{b^2}+\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\tan(c+dx))}{3(a+b\tan(c+dx))}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{5/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

3.572. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}dx$

$$\Big\downarrow 682$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{1}{2}b^2\int\frac{2\left(\left(\frac{2a^2}{b^2}+1\right)b^4-ab\left(4a^2+3b^2\right)\tan(c+dx)\right)}{b^4(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)-2ab\tan(c+dx)\right)}{b^2}+\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{3(a+b\tan(}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{\int\frac{b^2\left(2a^2+b^2\right)-ab\left(4a^2+3b^2\right)\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)-2ab\tan(c+dx)\right)}{b^2}+\frac{\left(\tan^2(c+dx)+1\right)^{3/2}(4a+b\tan}{3(a+b\tan(c+dx))}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 719$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{\left(a^2+b^2\right)\left(4a^2+b^2\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-a\left(4a^2+3b^2\right)\int\frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(\right.}{b^2}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 222$$

3.572. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{\left(a^2+b^2\right)\left(4a^2+b^2\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-ab\left(4a^2+3b^2\right)\text{arcsinh}(\tan(c+dx))}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+\right.\right.}{b^2}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{-\left(a^2+b^2\right)\left(4a^2+b^2\right)\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}-ab\left(4a^2+3b^2\right)\text{arcsinh}(\tan(c+dx))}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)-2\right.}{b^2}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(\frac{5\left(\frac{\frac{-ab\left(4a^2+3b^2\right)\text{arcsinh}(\tan(c+dx))-b\sqrt{a^2+b^2}\left(4a^2+b^2\right)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2}+\sqrt{\tan^2(c+dx)+1}\left(b^2\left(\frac{4a^2}{b^2}+1\right)-2ab\tan(c+dx)\right)}{b^2}\right.}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(1 + Tan[c + d*x]^2)^(5/2)/(a + b*Tan[c + d*x])^2 + (5
*(((4*a + b*Tan[c + d*x])*(1 + Tan[c + d*x]^2)^(3/2))/(3*(a + b*Tan[c + d*
x])) + ((-(a*b*(4*a^2 + 3*b^2)*ArcSinh[Tan[c + d*x]]) - b*Sqrt[a^2 + b^2]*
(4*a^2 + b^2)*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/b^2 + ((1 + (4*
a^2)/b^2)*b^2 - 2*a*b*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])/b^2))/(2*b^2
)))/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.572. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.572.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

3.572. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.572.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(219) = 438$.

Time = 299.23 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.86

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{b^2\left(7a^4+5a^2b^2-2b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\left(8a^6-9a^4b^2-15a^2b^4+2b^6\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2\left(25a^4+23a^2b^2-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^6}$
default	$-\frac{2\left(\frac{\frac{b^2\left(7a^4+5a^2b^2-2b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\left(8a^6-9a^4b^2-15a^2b^4+2b^6\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2\left(25a^4+23a^2b^2-2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^6}$
risch	$15a^2b^2\mathrm{e}^{9i(dx+c)}+140a^2b^2\mathrm{e}^{7i(dx+c)}+250a^2b^2\mathrm{e}^{5i(dx+c)}+140a^2b^2\mathrm{e}^{3i(dx+c)}+15a^2b^2\mathrm{e}^{i(dx+c)}+240a^4\mathrm{e}^{3i(dx+c)}+60a^4\mathrm{e}^{i(dx}$

input
```
int(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^6*((1/2*b^2*(7*a^4+5*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)^3+1/2*b
*(8*a^6-9*a^4*b^2-15*a^2*b^4+2*b^6)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(25*a
^4+23*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)-4*a^4*b-7/2*a^2*b^3+1/2*b^5)/(ta
n(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(4*a^4+5*a^2*b^2+b^4)
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
)+1/3/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-3*a+b)/b^4/(tan(1/2*d*x+1/2*c)+1)
^2-1/2*(-12*a^2+3*a*b-5*b^2)/b^5/(tan(1/2*d*x+1/2*c)+1)-5/2*a*(4*a^2+3*b^2
)/b^6*ln(tan(1/2*d*x+1/2*c)+1)-1/3/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(3*a+b
)/b^4/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(12*a^2+3*a*b+5*b^2)/b^5/(tan(1/2*d*x+1
/2*c)-1)+5/2*a*(4*a^2+3*b^2)/b^6*ln(tan(1/2*d*x+1/2*c)-1))
```

3.572.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(223) = 446$.

Time = 0.35 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.36

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{4\,b^5+30\left(4\,a^4b+a^2b^3-b^5\right)\cos\left(dx+c\right)^4+20\left(2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2+15\left(\left(4\,a^4-3\,a^2b^2-b^4\right)\cos\left(d\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3
+ b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(
4*a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x +
 c)^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2)) - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a
^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3
)*log(sin(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 +
 2*(4*a^4*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*
a^3*b^2 + 2*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4
*sin(d*x + c) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

3.572.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^7\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**3, x)
```

3.572.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(223) = 446.

Time = 0.61 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.77

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6*(2*(60*a^6 + 35*a^4*b^2 - 3*a^2*b^4 + (210*a^5*b + 125*a^3*b^3 - 6*a*b
^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^6 - 10*a^4*b^2 - 55*a^2*b^4
 + 3*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(330*a^5*b + 205*a^3*b^3
 - 12*a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^6 - 95*a^4*b^2
 - 120*a^2*b^4 + 9*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*(60*a^5*b
 + 35*a^3*b^3 - 3*a*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*(40*a^6 -
 30*a^4*b^2 - 35*a^2*b^4 + 3*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*
(50*a^5*b + 25*a^3*b^3 - 4*a*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*
(20*a^6 - 15*a^4*b^2 - 15*a^2*b^4 + 2*b^6)*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 + 3*(10*a^5*b + 5*a^3*b^3 - 2*a*b^5)*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9)/(a^4*b^5 + 4*a^3*b^6*sin(d*x + c)/(cos(d*x + c) + 1) - 16*a^3*b^6*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 24*a^3*b^6*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 16*a^3*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 4*a^3*b^6*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - a^4*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10 - (5*a^4*b^5 - 4*a^2*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(5*
a^4*b^5 - 6*a^2*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(5*a^4*b^5 -
6*a^2*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^4*b^5 - 4*a^2*b^7)*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8) - 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/b^6 + 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^6 - 15*(4*a^4 + 5*a^2*b^2 + b^4)*log((b - a*sin(d...
```

3.572.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(223) = 446$.

Time = 0.65 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.13

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6}-\frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6}+\frac{15\,(4\,a^4+5\,a^2b^2+b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-\right.}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+\right.}\right.}{\sqrt{a^2+b^2}\,b^6}}{}$$

input
```
integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*
a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*
b^2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6
) + 2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b
^2*tan(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2
*d*x + 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3
*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d
*x + 1/2*c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x
+ 1/2*c)^2 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c)
- 23*a^3*b^3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 -
 7*a^4*b^2 + a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c
) - a)^2*a^2*b^5))/d
```

3.572.9 Mupad [B] (verification not implemented)

Time = 7.83 (sec) , antiderivative size = 1203, normalized size of antiderivative = 5.03

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^3),x)
```

output

```
((60*a^4 - 3*b^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/2)*(210*a^4 - 6*
b^4 + 125*a^2*b^2))/(3*a*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^6 + 2*b^6 - 15
*a^2*b^4 - 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*(40*a^6 + 3*b^
6 - 35*a^2*b^4 - 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^2*(120*a^6
 + 3*b^6 - 55*a^2*b^4 - 10*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/2 + (d*x)/2)^4
*(180*a^6 + 9*b^6 - 120*a^2*b^4 - 95*a^4*b^2))/(3*a^2*b^5) + (tan(c/2 + (d
*x)/2)^9*(10*a^4 - 2*b^4 + 5*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(
50*a^4 - 4*b^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(60*a^4 -
3*b^4 + 35*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(330*a^4 - 12*b^4 +
 205*a^2*b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^8*(5*a^2 - 4*b^2) - tan(c
/2 + (d*x)/2)^2*(5*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 - 12*b^2) -
 tan(c/2 + (d*x)/2)^6*(10*a^2 - 12*b^2) - a^2*tan(c/2 + (d*x)/2)^10 + a^2
- 16*a*b*tan(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 - 16*a*b*tan(c
/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)))
- (atanh((3000*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 + (7000*a^4)/b^2 + (4000*
a^6)/b^4) + (7000*a^4*tan(c/2 + (d*x)/2))/(7000*a^4 + 3000*a^2*b^2 + (4000
*a^6)/b^2) + (4000*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 + 7000
*a^4*b^2))*(15*a*b^2 + 20*a^3))/(b^6*d) + (5*atanh((1000*a^2*(a^2 + b^2)^(
1/2))/(1000*a^2*b + (5000*a^4)/b + (4000*a^6)/b^3 + 10000*a^3*tan(c/2 + (d
*x)/2) + 2000*a*b^2*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/...
```

3.572. $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.573 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.573.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = &-\frac{3a\operatorname{arctanh}(\sin(c+dx))}{b^4d}\\ &-\frac{3(2a^2+b^2)\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4\sqrt{a^2+b^2}d}\\ &-\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-3*a*arctanh(sin(d*x+c))/b^4/d-3/2*(2*a^2+b^2)*arctanh((b*cos(d*x+c)-a*sin
(d*x+c))/(a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)-1/2*sec(d*x+c)^3/b/d/(a+b*
tan(d*x+c))^2+3/2*sec(d*x+c)*(2*a+b*tan(d*x+c))/b^3/d/(a+b*tan(d*x+c))
```

3.573.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 396 vs. 2(148) = 296.

Time = 3.68 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.68

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2(a^2+b^2)\sin(c+dx)}{a}+\frac{(2a-b)b(2a+b)(a\cos(c+dx)+b\sin(c+dx))}{a}+2b(a\,\mathrm{co}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c
+ d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a +
2*b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/S
qrt[a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c +
d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])))/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

3.573.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3992, 492, 590, 25, 27, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^5}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{\sec(c+dx)\left(\frac{3\int \frac{b\tan(c+dx)\sqrt{\tan^2(c+dx)+1}}{(a+b\tan(c+dx))^2}d(b\tan(c+dx))}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{590}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}-\int -\frac{b^2-2ab\tan(c+dx)}{b^2(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\int \frac{b^2-2ab\tan(c+dx)}{b^2(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\int \frac{b^2-2ab\tan(c+dx)}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\left(2a^2+b^2\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-2a\int \frac{1}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 222$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{\left(2a^2+b^2\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))-2ab\,\text{arcsinh}(\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{(\tan^2}{2(a+}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 488$$

3.573. $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{-\left(2a^2+b^2\right)\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}-2ab\,\text{arcsinh}(\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{(\tan^2(c+}{2(a+b\tan}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(\frac{3\left(\frac{-\frac{b\left(2a^2+b^2\right)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-2ab\,\text{arcsinh}(\tan(c+dx))}{b^2}+\frac{\sqrt{\tan^2(c+dx)+1}(2a+b\tan(c+dx))}{a+b\tan(c+dx)}\right)}{2b^2}-\frac{\left(\tan^2(c+dx)+1\right)^{3/2}}{2(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(1 + Tan[c + d*x]^2)^(3/2)/(a + b*Tan[c + d*x])^2 + (3
*((-2*a*b*ArcSinh[Tan[c + d*x]] - (b*(2*a^2 + b^2)*ArcTanh[(b^2*Tan[c + d*
x])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2])/b^2 + ((2*a + b*Tan[c + d*x])*Sqrt[
1 + Tan[c + d*x]^2])/(a + b*Tan[c + d*x])))/(2*b^2)))/(b*d*Sqrt[Sec[c + d*
x]^2])
```

3.573.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

---

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.573. $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.573.4 Maple [A] (verified)

Time = 94.52 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{1}{b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^{4}}-\frac{2\left(\frac{\frac{b^{2}\left(3a^{2}-2b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\left(4a^{4}-9a^{2}b^{2}+2b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}}-\frac{b^{2}\left(13a^{2}\right.}{}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}\right.}{b^{4}}}{d}$
default	$\frac{\frac{1}{b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^{4}}-\frac{2\left(\frac{\frac{b^{2}\left(3a^{2}-2b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b\left(4a^{4}-9a^{2}b^{2}+2b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^{2}}-\frac{b^{2}\left(13a^{2}\right.}{}}{\left(\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}\right.}{b^{4}}}{d}$
risch	$\frac{-9iab\,\mathrm{e}^{5i(dx+c)}+6a^{2}\mathrm{e}^{5i(dx+c)}-3b^{2}\mathrm{e}^{5i(dx+c)}+12a^{2}\mathrm{e}^{3i(dx+c)}+2b^{2}\mathrm{e}^{3i(dx+c)}+9iab\,\mathrm{e}^{i(dx+c)}+6a^{2}\mathrm{e}^{i(dx+c)}-3b^{2}\mathrm{e}^{i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)^{2}b^{3}d}-$

input
```
int(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*(
(1/2*b^2*(3*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-9*a^2*b^2+2*b^4
)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(13*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)-2*a
^2*b+1/2*b^3)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(2*a
^2+b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2)))-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*x+1/2*c)-1))
```

3.573.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(138) = 276$.

Time = 0.34 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.47

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{4\,a^2b^3 + 4\,b^5 + 6\,(2\,a^4b + a^2b^3 - b^5)\cos{(dx+c)}^2 + 18\,(a^3b^2 + ab^4)\cos{(dx+c)}\sin{(dx+c)} + 3\,((2\,a^4 - a}{}$$

input
```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(
a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*co
s(d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^
2 + b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*
cos(d*x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*
b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin
(d*x + c) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos
(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x +
c) + 1))/((a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x
 + c)^2*sin(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

3.573.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^5{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**3, x)
```

3.573.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(138) = 276$.

Time = 0.34 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.50

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{2\left(6a^4-a^2b^2+\frac{(21a^3b-2ab^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(6a^4-9a^2b^2+b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(6a^3b-ab^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(6a^4-9a^2b^2+2b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(3a^3b-2ab^3)\,\mathrm{si}}{(\cos(dx+c)}\right.}{a^4b^3+\frac{4a^3b^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{8a^3b^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4a^3b^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^4b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{2d}$$

input

```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output

```
1/2*(2*(6*a^4 - a^2*b^2 + (21*a^3*b - 2*a*b^3)*sin(d*x + c)/(cos(d*x + c)
+ 1) - 2*(6*a^4 - 9*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4
*(6*a^3*b - a*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (6*a^4 - 9*a^2*b^
2 + 2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (3*a^3*b - 2*a*b^3)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4*b^3 + 4*a^3*b^4*sin(d*x + c)/(cos(d*x
 + c) + 1) - 8*a^3*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*a^3*b^4*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^4*b^3*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 - (3*a^4*b^3 - 4*a^2*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^
4*b^3 - 4*a^2*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 6*a*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/b^4 + 6*a*log(sin(d*x + c)/(cos(d*x + c) + 1)
 - 1)/b^4 - 3*(2*a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + s
qrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^4))/d
```

3.573.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(138) = 276$.

Time = 0.61 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.12

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{6a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{6a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}+\frac{3(2a^2+b^2)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\right.}}{}$$

input

```
integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2
*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3
*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan
(1/2*d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x
 + 1/2*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c)
 - 4*a^4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)
- a)^2*a^2*b^3))/d
```

3.573.9 Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 1311, normalized size of antiderivative = 8.86

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^3),x)
```

output
```
((6*a^2 - b^2)/b^3 - (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 - 9*a^2*b^2))/(a
^2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/(a*b^2) + (tan(c/2 + (d*x)
/2)^4*(6*a^4 + 2*b^4 - 9*a^2*b^2))/(a^2*b^3) - (4*tan(c/2 + (d*x)/2)^3*(6*
a^2 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3*a^2 - 2*b^2))/(a*b^2))/(d*(
tan(c/2 + (d*x)/2)^4*(3*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(3*a^2 - 4*b^2
) - a^2*tan(c/2 + (d*x)/2)^6 + a^2 - 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*ta
n(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (6*a*atanh(tan(c/2 + (d*
x)/2)))/(b^4*d) + (atan((((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 +
 (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a
^2 + b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8
))/b^9 - 48*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(
c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b
^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)) + ((2*a^2 + b^2)*(a^2 + b^2)^(1/2)
*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^
3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/
2)*(12*a*b^10 + 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*
a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 +
a^2*b^4))))/(2*(b^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)))/((16*(54*a^4 + 2
7*a^2*b^2))/b^8 - (16*tan(c/2 + (d*x)/2)*(216*a^5 + 108*a^3*b^2))/b^9 - (3
*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)...
```

3.573. $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.574 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.574.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

output
```
-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d-1/2*sec(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2
```

3.574.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\left(a^2+b^2\right)(-b\cos(c+dx)+a\sin(c+dx))+2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output
```
((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcT
anh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
)
```

3.574.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3992, 486, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)^3}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3992}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{\tan^2(c+dx)+1}}{(a+b\tan(c+dx))^3}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 486}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{\sqrt{\tan^2(c+dx)+1}(b^2-ab\tan(c+dx))}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{\sec(c+dx)\left(-\frac{\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{2(a^2+b^2)}-\frac{\sqrt{\tan^2(c+dx)+1}(b^2-ab\tan(c+dx))}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sec(c+dx)\left(-\frac{b\,\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{\sqrt{\tan^2(c+dx)+1}\,(b^2-ab\tan(c+dx))}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(b*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 +
 b^2)^(3/2) - ((b^2 - a*b*Tan[c + d*x])*Sqrt[1 + Tan[c + d*x]^2])/(2*(a^2
+ b^2)*(a + b*Tan[c + d*x])^2)))/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.574.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 486
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*(a*d - b*c*x)*((a + b*x^2)^p/((n + 1)*(b*c^2 + a*d^2))),
x] - Simp[2*a*b*(p/((n + 1)*(b*c^2 + a*d^2)))   Int[(c + d*x)^(n + 2)*(a +
b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0] &&
GtQ[p, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.574.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(87) = 174$.

Time = 17.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^2+b^2\right)}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+b^2\right)a}+\frac{b}{2a^2+2b^2}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^2+b^2\right)}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+b^2\right)a}+\frac{b}{2a^2+2b^2}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b^2}}\right.}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{\mathrm{e}^{i(dx+c)}\left(ia\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{2i(dx+c)}-ia+b\right)}{\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)^2(-ia+b)d(ia+b)}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{2(a^2+b^2)^{\frac{3}{2}}d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+ia\,b^2-}{\left(a^2+b^2\right.}\right.}{2(a^2+b^2)^{\frac{3}{2}}d}$

input
```
int(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a^2+2*b^2)/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^
2)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*
d*x+1/2*c)+1/2*b/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)
-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2)))
```

3.574.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.09

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos}{2\,ab\cos(dx+c)\,\mathrm{si}}\right.}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+a\right.\right.}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*
sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c
)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
- 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4
*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*c
os(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

3.574.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^3{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**3, x)
```

3.574.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(88) = 176$.

Time = 0.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.43

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\left(a^2b-\frac{\left(a^3-2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^2b-2\,b^3\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(a^3+2\,ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+a^4b^2+\frac{4\left(a^5b+a^3b^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(a^6-a^4b^2-2\,a^2b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(a^5b+a^3b^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\left(a^6+a^4b^2\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b
- 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c
)/(cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 (a^6 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x
 + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

3.574.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time = 0.61 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\,\mathrm{ta}\right.}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^
3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*
d*x + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) -
 2*a*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1
/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.574.9 Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.74

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-2\,b^2)}{a\,(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2+2\,b^2)}{a\,(a^2+b^2)}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^2-2\,b^2)}{a^2\,(a^2+b^2)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+\frac{\operatorname{atanh}\left(\frac{\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

input
```
int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan
(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2
*(a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2
+ (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*ta
n(c/2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a
^2 + b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

3.575 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

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3.575.1 Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{(2a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{5/2}d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}$$

output
```
-1/2*(2*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(
1/2))*sec(d*x+c)/(a^2+b^2)^(5/2)/d/(sec(d*x+c)^2)^(1/2)-1/2*b*sec(d*x+c)/(
a^2+b^2)/d/(a+b*tan(d*x+c))^2-3/2*a*b*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*
x+c))
```

3.575.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2(2a^2-b^2)\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\sec(c+dx)(4a^2+b^2+3ab\tan(c+dx))}{(a^2+b^2)^2(a+b\tan(c+dx))^2}}{2d}$$

input
```
Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
 + b^2)^(5/2) - (b*Sec[c + d*x]*(4*a^2 + b^2 + 3*a*b*Tan[c + d*x]))/((a^2
+ b^2)^2*(a + b*Tan[c + d*x])^2))/(2*d)
```

3.575.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3992, 498, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{498}$$

$$\frac{\sec(c+dx)\left(-\frac{\int -\frac{2a-b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2a-b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{679}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{\left(2a^2-b^2\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{3ab^2\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(\frac{-\frac{\left(2a^2-b^2\right)\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{3ab^2\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(\frac{-\frac{b\left(2a^2-b^2\right)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{3ab^2\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{2(a^2+b^2)}-\frac{b^2\sqrt{\tan^2(c+dx)+1}}{2(a^2+b^2)(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-1/2*(b^2*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan
[c + d*x])^2) + (-((b*(2*a^2 - b^2)*ArcTanh[(b^2*Tan[c + d*x])/Sqrt[a^2 +
b^2]])/(a^2 + b^2)^(3/2)) - (3*a*b^2*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2
)*(a + b*Tan[c + d*x])))/(2*(a^2 + b^2))))/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.575.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.575. $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.575.4 Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.81

method	result
derivativedivides	$-\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
default	$-\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
risch	$-\frac{b\,\mathrm{e}^{i(dx+c)}\left(-3iab\,\mathrm{e}^{2i(dx+c)}+4a^2\mathrm{e}^{2i(dx+c)}+b^2\mathrm{e}^{2i(dx+c)}+3iab+4a^2+b^2\right)}{(ib+a)^2\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)^2d(-ib+a)^2}+\frac{\ln\Bigg(\mathrm{e}^{i(dx+c)}+\frac{ia^5+2ia^3b^2+iab^4-a^4b-2a^2}{(a^2+b^2)^{\frac{5}{2}}}}{(a^2+b^2)^{\frac{5}{2}}d}$

input
```
int(sec(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3
-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^
2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4
*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2
*c)-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.575.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(143) = 286$.

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.27

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{(2\,a^2b^2-b^4+(2\,a^4-3\,a^2b^2+b^4)\cos(dx+c)^2+2\,(2\,a^3b-ab^3)\cos(dx+c)\sin(dx+c))\sqrt{a^2+b^2}\log}{4\,((a^8+2\,a^6b^2-2\,a^2b^6-b^8)d\cos(dx+c)^2+2\,(}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a
^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^
2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*co
s(d*x + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b
^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*c
os(d*x + c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

3.575.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

3.575.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(143) = 286$.

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.66

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(2a^2-b^2)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(4a^4b+a^2b^3+\frac{(11a^3b^2+2ab^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(4a^4b-7a^2b^3-2b^5)\sin(dx+c)}{(\cos(dx+c)+1)^2}\right.}{a^8+2a^6b^2+a^4b^4+\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^8-3a^4b^4-2a^2b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^7b+2a}{(c}}}{2d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*((2*a^2 - b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2
+ b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 +
 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(4*a^4*b + a^2*b^3 + (11*a^3*b^2 +
2*a*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - (4*a^4*b - 7*a^2*b^3 - 2*b^5)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - (5*a^3*b^2 + 2*a*b^4)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 +
a^3*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4))/d
```

3.575.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(143) = 286$.

Time = 0.54 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.89

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(2a^2-b^2)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\big(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a^2b^3\tan(}{(a^6+2a^4b^2+a^2b^4)\big(a\tan(}}{2d}$$

input
```
integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 +
b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a
^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a
*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*t
an(1/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*
d*x + 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2
*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) -
 a)^2))/d
```

3.575.9 Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.86

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\ln\bigl(\left(a^2+b^2\right)^{5/2} - a^4 b - b^5 - 2\,a^2 b^3 + a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 2\,a^3 b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\bigr)\,\bigl(a^2 -}{d\left(a^2+b^2\right)^{5/2}}$$

$$- \frac{\ln\bigl(\left(a^2+b^2\right)^{5/2} + a^4 b + b^5 + 2\,a^2 b^3 - a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 2\,a^3 b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\bigr)\,\bigl(2\,a^2}{2\,d\left(a^2+b^2\right)^{5/2}}$$

$$- \frac{\frac{4\,a^2 b+b^3}{a^4+2\,a^2 b^2+b^4} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(a^2-2\,b^2\right)\left(4\,a^2 b+b^3\right)}{a^2\left(a^4+2\,a^2 b^2+b^4\right)} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(11\,a^2 b+2\,b^3\right)}{a\left(a^4+2\,a^2 b^2+b^4\right)} - \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(5\,a^2 b+2\,b^3\right)}{a\left(a^4+2\,a^2 b^2+b^4\right)}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2-4\,b^2\right) + a^2 - 4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

input

```
int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^3),x)
```

output

```
(log((a^2 + b^2)^(5/2) - a^4*b - b^5 - 2*a^2*b^3 + a^5*tan(c/2 + (d*x)/2)
+ a*b^4*tan(c/2 + (d*x)/2) + 2*a^3*b^2*tan(c/2 + (d*x)/2))*(a^2 - b^2/2))/
(d*(a^2 + b^2)^(5/2)) - (log((a^2 + b^2)^(5/2) + a^4*b + b^5 + 2*a^2*b^3 -
 a^5*tan(c/2 + (d*x)/2) - a*b^4*tan(c/2 + (d*x)/2) - 2*a^3*b^2*tan(c/2 + (
d*x)/2))*(2*a^2 - b^2))/(2*d*(a^2 + b^2)^(5/2)) - ((4*a^2*b + b^3)/(a^4 +
b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2)*(4*a^2*b + b^3))/(a
^2*(a^4 + b^4 + 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)*(11*a^2*b + 2*b^3))/(a
*(a^4 + b^4 + 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3))/(a*
(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2
)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (
d*x)/2)))
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.576.1 Optimal result 3988
3.576.2 Mathematica [A] (verified) 3989
3.576.3 Rubi [A] (warning: unable to verify) 3989
3.576.4 Maple [A] (verified) 3993
3.576.5 Fricas [B] (verification not implemented) 3994
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3.576.8 Giac [A] (verification not implemented) 3995
3.576.9 Mupad [B] (verification not implemented) 3996

3.576.1 Optimal result

Integrand size = 19, antiderivative size = 221

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{3b^2(4a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\,(a^2+b^2)^{7/2}\,d}\\ &\quad+\frac{b(2a^2-3b^2)\sec(c+dx)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}\\ &\quad+\frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{ab(2a^2-13b^2)\sec(c+dx)}{2\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-3/2*b^2*(4*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^
2)^(1/2))*cos(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(7/2)/d+1/2*b*(2*a^2-3
*b^2)*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+cos(d*x+c)*(b+a*tan(d*x+
c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2*a*b*(2*a^2-13*b^2)*sec(d*x+c)/(a^2+
b^2)^3/d/(a+b*tan(d*x+c))
```

3.576.2 Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{-\frac{12b^2(-4a^2+b^2)\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\sec^2(c+dx)\Big(b(11a^4-22a^2b^2-3b^4)\cos(c+dx)+b(a^2+b^2)^2\cos(3(c+dx))+2a\Big(a^4+4a^2b^2}{(a^2+b^2)^3(a+b\tan(c+dx))^2}}{4d}$$

input
```
Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
((-12*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]
])/(a^2 + b^2)^(7/2) + (Sec[c + d*x]^2*(b*(11*a^4 - 22*a^2*b^2 - 3*b^4)*Co
s[c + d*x] + b*(a^2 + b^2)^2*Cos[3*(c + d*x)] + 2*a*(a^4 + 4*a^2*b^2 - 12*
b^4 + (a^2 + b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x]))/((a^2 + b^2)^3*(a + b
*Tan[c + d*x])^2))/(4*d)
```

3.576.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 3992, 496, 25, 27, 688, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sec(c+dx)(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \; 3992$$
$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)+1)^{3/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$
$$\Big\downarrow \; 496$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{b^2\int -\frac{3b^2+2a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 25$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{3b^2+2a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{3b^2+2a\tan(c+dx)b}{(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 688$$

$$\frac{\sec(c+dx)\left(\frac{\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}-\frac{b^2\int -\frac{10a-\left(3-\frac{2a^2}{b^2}\right)b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\int \frac{10a-\left(3-\frac{2a^2}{b^2}\right)b\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{ab\tan(c+dx)+b^2}{(a^2+b^2)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 679$$

3.576. $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(\frac{3\left(4a^2-b^2\right)\int \frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{a\left(2a^2-13b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}\right)}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{}{(a^2-}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 488$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(\frac{a\left(2a^2-13b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{3\left(4a^2-b^2\right)\int \frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}\right)}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{}{(a^2+b^2)\sqrt{}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 219$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^2\left(\frac{a\left(2a^2-13b^2\right)\sqrt{\tan^2(c+dx)+1}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{3b\left(4a^2-b^2\right)\text{arctanh}\left(\frac{b^2\tan(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}\right)}{2\left(a^2+b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\tan^2(c+dx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{ab\tan(c+}{(a^2+b^2)\sqrt{\tan^2(c+dx)-}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])^
2*Sqrt[1 + Tan[c + d*x]^2]) + (((2*a^2 - 3*b^2)*Sqrt[1 + Tan[c + d*x]^2])/
(2*(1 + a^2/b^2)*(a + b*Tan[c + d*x])^2) + (b^2*((-3*b*(4*a^2 - b^2)*ArcTa
nh[(b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (a*(2*a^2 - 13
*b^2)*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*(a
^2 + b^2)))/(a^2 + b^2)))/(b*d*Sqrt[Sec[c + d*x]^2])
```

3.576. $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.576.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.576. $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.576.4 Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{2\left(\left(-a^{3}+3 a\, b^{2}\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)-3 a^{2} b+b^{3}\right)}{\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}-\frac{2 b^{2}\left(\frac{-\frac{b^{2}\left(9 a^{2}+2 b^{2}\right)\left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2 a}-\frac{b\left(8 a^{4}-15 a^{2} b^{2}-2 b^{4}\right)\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2 a^{2}}+b^{2}}{\left(\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right) a-2 b \tan\left(\frac{d x}{2}+\frac{c}{2}\right)-a\right)^{2}}\right.}{\left(a^{2}\right.}}{d}$
default	$\frac{-\frac{2\left(\left(-a^{3}+3 a\, b^{2}\right) \tan\left(\frac{d x}{2}+\frac{c}{2}\right)-3 a^{2} b+b^{3}\right)}{\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}-\frac{2 b^{2}\left(\frac{-\frac{b^{2}\left(9 a^{2}+2 b^{2}\right)\left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2 a}-\frac{b\left(8 a^{4}-15 a^{2} b^{2}-2 b^{4}\right)\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{2 a^{2}}+b^{2}}{\left(\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right) a-2 b \tan\left(\frac{d x}{2}+\frac{c}{2}\right)-a\right)^{2}}\right.}{\left(a^{2}\right.}}{d}$
risch	$-\frac{i \mathrm{e}^{i(d x+c)}}{2(-3 i b\, a^{2}+i b^{3}+a^{3}-3 a\, b^{2}) d}+\frac{i \mathrm{e}^{-i(d x+c)}}{2(3 i b\, a^{2}-i b^{3}+a^{3}-3 a\, b^{2}) d}+\frac{b^{3} \mathrm{e}^{i(d x+c)}\left(-7 i a b\, \mathrm{e}^{2 i(d x+c)}+8 a^{2} \mathrm{e}^{2 i(d x+c)}+b^{2} \mathrm{e}^{2 i(d x+\right.}{(-i a+b)^{3}\left(b\, \mathrm{e}^{2 i(d x+c)}+i a\, \mathrm{e}^{2 i(d x+c)}-b+i a\right)^{2}}$

input
```
int(cos(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*b^2)*tan(1/2*d*x+1/2*c)-3
*a^2*b+b^3)/(1+tan(1/2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2
*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*
x+1/2*c)^2+1/2*b^2*(23*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(t
an(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

3.576.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(209) = 418$.

Time = 0.31 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.17

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{4\left(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}\right)\cos\left(dx+c\right)^{3}-3\left(4\,a^{2}b^{4}-b^{6}+\left(4\,a^{4}b^{2}-5\,a^{2}b^{4}+b^{6}\right)\cos\left(dx+c\right)^{2}+2\left(4\,a^{3}\right.\right.}{4\left(\left(a^{10}+\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
 - b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b
^5)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin
(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b
^7)*cos(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b
^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 +
 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4
*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a
^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

3.576.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{3}}\,dx$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

3.576.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(209) = 418$.

Time = 0.32 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.98

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{3\,(4\,a^2b^2-b^4)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(6\,a^6b-10\,a^4b^3-a^2b^5+\frac{(2\,a^7+18\,a^5b^2-31\,a^3b^4-2\,ab^6)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(2\,a^6b-2\,a^4b^3+\right.}{(\cos(}\right.}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6+\frac{4\,(a^9b+3\,a^7b^3+3\,a^5b^5+a^3b^7)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^{10}-a^8b^2-9\,a^6b^4-11\,a^{\,}\right.}{(\cos(dx+}}}{}$$

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(3*(4*a^2*b^2 - b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^6*b - 10*a^4
*b^3 - a^2*b^5 + (2*a^7 + 18*a^5*b^2 - 31*a^3*b^4 - 2*a*b^6)*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*(2*a^6*b - 2*a^4*b^3 + 12*a^2*b^5 + b^7)*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 2*(2*a^7 + 2*a^5*b^2 + 15*a^3*b^4)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 - (2*a^6*b - 30*a^4*b^3 + 15*a^2*b^5 + 2*b^7)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + (2*a^7 - 6*a^5*b^2 + 9*a^3*b^4 + 2*a*b^
6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^
4*b^6 + 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*sin(d*x + c)/(cos(d*x
+ c) + 1) - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4
*a^2*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(a^9*b + 3*a^7*b^3 + 3*a
^5*b^5 + a^3*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (a^10 + 3*a^8*b^2
+ 3*a^6*b^4 + a^4*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

3.576.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.81

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{3\,(4\,a^2b^2-b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{4\,(a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-3\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+3\,a^2b-b^3)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+1\right)}-\frac{2\left(9\,a^3b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

---

input
```
integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/
2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x +
 1/2*c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c
)^2 - 15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2
3*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3
- a^2*b^5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c
)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.576.9 Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.76

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{-6\,a^4\,b+10\,a^2\,b^3+b^5}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5+2\,a^3\,b^2+15\,a\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^6\,b-30\,a^4\,b^3+15\,a^2\,b^5+2\,b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^6\right.}{a\,\left(a^6+3\,a\right.}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

$$-\frac{\mathrm{atan}\left(\frac{-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^7+a^6\,b\,1\mathrm{i}-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5\,b^2+a^4\,b^3\,3\mathrm{i}-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^4+a^2\,b^5\,3\mathrm{i}-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^6+b^7\,1\mathrm{i}}{\left(a^2+b^2\right)^{7/2}}\right)\,\left(3\,b^4-\right.}{d\,\left(a^2+b^2\right)^{7/2}}$$

input
```
int(cos(c + d*x)/(a + b*tan(c + d*x))^3,x)
```

output

```
- ((b^5 - 6*a^4*b + 10*a^2*b^3)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*t
an(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 + 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b + 2*b^7 + 15*a^2*b^5 - 30*a
^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)*(
2*a^6 - 2*b^6 - 31*a^2*b^4 + 18*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) - (tan(c/2 + (d*x)/2)^5*(2*a^6 + 2*b^6 + 9*a^2*b^4 - 6*a^4*b^2))/(
a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*a^6*b
+ b^7 + 12*a^2*b^5 - 2*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
)/(d*(a^2*tan(c/2 + (d*x)/2)^6 + a^2 - tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2)
- tan(c/2 + (d*x)/2)^4*(a^2 - 4*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*
tan(c/2 + (d*x)/2))) - (atan((a^6*b*1i + b^7*1i + a^2*b^5*3i + a^4*b^3*3i
- a^7*tan(c/2 + (d*x)/2)*1i - a*b^6*tan(c/2 + (d*x)/2)*1i - a^3*b^4*tan(c/
2 + (d*x)/2)*3i - a^5*b^2*tan(c/2 + (d*x)/2)*3i)/(a^2 + b^2)^(7/2))*(3*b^4
 - 12*a^2*b^2)*1i)/(d*(a^2 + b^2)^(7/2))
```

3.576. $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.577 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.577.1 Optimal result 3998
3.577.2 Mathematica [A] (verified) 3999
3.577.3 Rubi [A] (warning: unable to verify) 3999
3.577.4 Maple [A] (verified) 4005
3.577.5 Fricas [B] (verification not implemented) 4005
3.577.6 Sympy [F(-1)] 4006
3.577.7 Maxima [B] (verification not implemented) 4006
3.577.8 Giac [B] (verification not implemented) 4007
3.577.9 Mupad [B] (verification not implemented) 4008

3.577.1 Optimal result

Integrand size = 21, antiderivative size = 310

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{5b^4(6a^2-b^2)\,\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{9/2}d} \\ &\quad + \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} \\ &\quad + \frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} \\ &\quad - \frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}\end{aligned}$$

output
```
-5/2*b^4*(6*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^
2)^(1/2))*cos(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(9/2)/d+1/6*b*(4*a^4+2
4*a^2*b^2-15*b^4)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2+1/3*cos(d*x+
c)^3*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/6*a*b*(4*a^4+28*a^2
*b^2-81*b^4)*sec(d*x+c)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(
2*a^2-5*b^2)-a*(2*a^2+9*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2
```

3.577.2 Mathematica [A] (verified)

Time = 3.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{9b(a^4+14a^2b^2-3b^4)(a\cos(c+dx)+b\sin(c+dx))^2}{(a^2+b^2)^4}+\frac{6b^6\tan(c+dx)}{a(a^2+b^2)^3}+\frac{9a(a^4+6}{}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*((9*b*(a^4 + 14*a^2*b^2
- 3*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2 + b^2)^4 + (6*b^6*Tan[c
 + d*x])/(a*(a^2 + b^2)^3) + (9*a*(a^4 + 6*a^2*b^2 - 11*b^4)*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 + b^2)^4 - (6*b^5*(12*a^2 + b^2)
*(a + b*Tan[c + d*x]))/(a*(a^2 + b^2)^4) - (60*b^4*(-6*a^2 + b^2)*ArcTanh[
(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]*(a + b*Tan[c + d*x
])^2)/(a^2 + b^2)^(9/2) - (b*(-3*a^2 + b^2)*Cos[c + d*x]*Cos[3*(c + d*x)]*
(a + b*Tan[c + d*x])^2)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[
3*(c + d*x)]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2)^3))/(12*d*(a + b*Tan[c +
d*x])^3)
```

3.577.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3992, 496, 25, 27, 686, 25, 25, 27, 688, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(c+dx)^3(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{1}{(a+b\tan(c+dx))^3(\tan^2(c+dx)+1)^{5/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 496$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}-\frac{b^2\int -\frac{\left(\frac{2a^2}{b^2}+5\right)b^2+4a\tan(c+dx)b}{b^2(a+b\tan(c+dx))^3\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 25$$

$$\frac{\sec(c+dx)\left(\frac{b^2\int \frac{2a^2+4b\tan(c+dx)a+5b^2}{b^2(a+b\tan(c+dx))^3\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sec(c+dx)\left(\frac{\int \frac{2a^2+4b\tan(c+dx)a+5b^2}{(a+b\tan(c+dx))^3\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{3(a^2+b^2)}+\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 686$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{b^4\int -\frac{3\left(5-\frac{2a^2}{b^2}\right)b^4+2a\left(2a^2+9b^2\right)\tan(c+dx)b}{b^4(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+}{3(a^2+b^2)(\tan^2(c+dx)+1}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{b^4\int -\frac{3b^2\left(2a^2-5b^2\right)-2ab\left(2a^2+9b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}+\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}}{3(a^2+b^2)}+\frac{ab\tan(c+}{3(a^2+b^2)(\tan^2(c+dx)+}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

3.577. $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

$$\Big\downarrow 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{b^4\int\frac{3b^2\left(2a^2-5b^2\right)-2ab\left(2a^2+9b^2\right)\tan(c+dx)}{b^4(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+d}{3(a^2+b^2)(\tan^2(c+dx)+1)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\int\frac{3b^2\left(2a^2-5b^2\right)-2ab\left(2a^2+9b^2\right)\tan(c+dx)}{(a+b\tan(c+dx))^3\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{ab\tan(c+dx)}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/}}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 688$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{-\frac{b^2\int-\frac{2ab^2\left(2a^2-33b^2\right)-b\left(4a^4+24b^2a^2-15b^4\right)\tan(c+dx)}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}-\frac{b^2\left(4a^4+24a^2b^2-\right.}{2\left(a^2+b^2\right)(}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 25$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{b^2\int\frac{2ab^2\left(2a^2-33b^2\right)-b\left(4a^4+24b^2a^2-15b^4\right)\tan(c+dx)}{b^2(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}-\frac{b^2\left(4a^4+24a^2b^2-15b\right.}{2\left(a^2+b^2\right)(a+b}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow 27$$

3.577. $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{\int\frac{2ab^2\left(2a^2-33b^2\right)-b\left(4a^4+24b^2a^2-15b^4\right)\tan(c+dx)}{(a+b\tan(c+dx))^2\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{2\left(a^2+b^2\right)}-\frac{b^2\left(4a^4+24a^2b^2-15b^4\right)}{2\left(a^2+b^2\right)(a+b\,\mathrm{ta}}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{679}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{-\frac{15b^4\left(6a^2-b^2\right)\int\frac{1}{(a+b\tan(c+dx))\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{a^2+b^2}-\frac{ab^2\left(4a^4+28a^2b^2-81b^4\right)}{\left(a^2+b^2\right)(a+b\tan}}{2\left(a^2+b^2\right)}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sec(c+dx)\left(\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{\frac{15b^4\left(6a^2-b^2\right)\int\frac{1}{\frac{a^2}{b^2}-b^2\tan^2(c+dx)+1}d\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\tan^2(c+dx)+1}}}{a^2+b^2}-\frac{ab^2\left(4a^4+28a^2b^2-81b^4\right)\sqrt{\tan^2(c-}}{\left(a^2+b^2\right)(a+b\tan(c+dx))}}{2\left(a^2+b^2\right)}}{a^2+b^2}}{3(a^2+b^2)}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(c+dx)\left(\frac{ab\tan(c+dx)+b^2}{3(a^2+b^2)(\tan^2(c+dx)+1)^{3/2}(a+b\tan(c+dx))^2}+\frac{\frac{ab\left(2a^2+9b^2\right)\tan(c+dx)+b^4\left(5-\frac{2a^2}{b^2}\right)}{\left(a^2+b^2\right)\sqrt{\tan^2(c+dx)+1}(a+b\tan(c+dx))^2}-\frac{\frac{\frac{15b^5\left(6a^2-b^2\right)\text{arctanh}\left(\frac{b^2\tan(c+}{\sqrt{a^2+b}}\right.}{\left(a^2+b^2\right)^{3/2}}}{}}{3}\right.}{bd\sqrt{\sec^2(c+dx)}}$$

3.577. $\int\frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

input

```
Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

output

```
(Sec[c + d*x]*((b^2 + a*b*Tan[c + d*x])/(3*(a^2 + b^2)*(a + b*Tan[c + d*x]
)^2*(1 + Tan[c + d*x]^2)^(3/2)) + (((5 - (2*a^2)/b^2)*b^4 + a*b*(2*a^2 + 9
*b^2)*Tan[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])^2*Sqrt[1 + Tan[c + d
*x]^2]) - (-1/2*(b^2*(4*a^4 + 24*a^2*b^2 - 15*b^4)*Sqrt[1 + Tan[c + d*x]^2
])/((a^2 + b^2)*(a + b*Tan[c + d*x])^2) + ((15*b^5*(6*a^2 - b^2)*ArcTanh[(
b^2*Tan[c + d*x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (a*b^2*(4*a^4 + 28
*a^2*b^2 - 81*b^4)*Sqrt[1 + Tan[c + d*x]^2])/((a^2 + b^2)*(a + b*Tan[c + d
*x])))/(2*(a^2 + b^2)))/(a^2 + b^2))/(3*(a^2 + b^2))))/(b*d*Sqrt[Sec[c + d
*x]^2])
```

3.577.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.577. $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.577.4 Maple [A] (verified)

Time = 23.38 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.47

method	result
derivativedivides	$-\frac{2b^4\left(\frac{-\frac{b^2(13a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(12a^4-23a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(35a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+6a^2b+\frac{b^3}{2}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\right.}{(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}$
default	$-\frac{2b^4\left(\frac{-\frac{b^2(13a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(12a^4-23a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(35a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+6a^2b+\frac{b^3}{2}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\right.}{(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}$
risch	$-\frac{ie^{3i(dx+c)}}{24(-3iba^2+ib^3+a^3-3ab^2)d}-\frac{9\,e^{i(dx+c)}b}{8(-4ia^3b+4iab^3+a^4-6a^2b^2+b^4)d}-\frac{3ie^{i(dx+c)}a}{8(-4ia^3b+4iab^3+a^4-6a^2b^2+b^4)d}-$

input
```
int(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^4/(a^2+b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-1/2*b^2*(13*a^2+2*b
^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(12*a^4-23*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x
+1/2*c)^2+1/2*b^2*(35*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+6*a^2*b+1/2*b^3)/(ta
n(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(6*a^2-b^2)/(a^2+b^2)
^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^6+3
*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)*((-a^5-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/
2*c)^5+(-3*a^4*b-12*a^2*b^3+3*b^5)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^5-32/3*a^3
*b^2+14*a*b^4)*tan(1/2*d*x+1/2*c)^3+(-20*a^2*b^3+4*b^5)*tan(1/2*d*x+1/2*c)
^2+(-a^5-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/2*c)-a^4*b-32/3*a^2*b^3+7/3*b^5)
/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

3.577.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(294) = 588.

Time = 0.33 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$=\frac{4\,(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\cos{(dx+c)}^5-4\,(2\,a^8b+a^6b^3-9\,a^4b^5-13\,a^2b^7-5\,b^9)\cos{(dx+c}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 -
 4*(2*a^8*b + a^6*b^3 - 9*a^4*b^5 - 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^3 - 1
5*(6*a^2*b^6 - b^8 + (6*a^4*b^4 - 7*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(6*a
^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(
a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(8*a^8*b + 64*a^6*b^3 - 16*a
^4*b^5 - 87*a^2*b^7 - 15*b^9)*cos(d*x + c) + 2*(4*a^7*b^2 + 32*a^5*b^4 - 5
3*a^3*b^6 - 81*a*b^8 + 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)
*cos(d*x + c)^4 + 2*(2*a^9 + 15*a^7*b^2 + 33*a^5*b^4 + 29*a^3*b^6 + 9*a*b^
8)*cos(d*x + c)^2)*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b
^8 - 4*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*
b^5 + 10*a^5*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10
*b^2 + 5*a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

3.577.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

3.577.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(294) = 588$.

Time = 0.36 (sec) , antiderivative size = 1229, normalized size of antiderivative = 3.96

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/6*(15*(6*a^2*b^4 - b^6)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sq
rt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/
((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 2*(6*a
^8*b + 64*a^6*b^3 - 50*a^4*b^5 - 3*a^2*b^7 + (6*a^9 + 48*a^7*b^2 + 202*a^5
*b^4 - 161*a^3*b^6 - 6*a*b^8)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(6*a^8*b
 + 56*a^6*b^3 - 14*a^4*b^5 - 67*a^2*b^7 - 3*b^9)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(2*a^9 - 4*a^7*b^2 - 86*a^5*b^4 + 133*a^3*b^6 + 3*a*b^8)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(8*a^8*b + 28*a^6*b^3 + 188*a^4*b^5
- 156*a^2*b^7 - 9*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*(2*a^9 + 4*
a^7*b^2 + 62*a^5*b^4 - 255*a^3*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
2*(14*a^8*b + 56*a^6*b^3 - 246*a^4*b^5 + 141*a^2*b^7 + 9*b^9)*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 4*(2*a^9 + 8*a^7*b^2 + 42*a^5*b^4 + 33*a^3*b^6 -
 3*a*b^8)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*(2*a^8*b + 8*a^6*b^3 - 7
8*a^4*b^5 + 23*a^2*b^7 + 2*b^9)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*(2
*a^9 + 8*a^7*b^2 - 18*a^5*b^4 + 13*a^3*b^6 + 2*a*b^8)*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9)/(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*a^6*b^6 + a^4*b^8 + 4*
(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)/(cos(d
*x + c) + 1) + (a^12 + 8*a^10*b^2 + 22*a^8*b^4 + 28*a^6*b^6 + 17*a^4*b^8 +
 4*a^2*b^10)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*(a^11*b + 4*a^9*b^3 +
 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 -...
```

3.577.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(294) = 588$.

Time = 0.63 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.06

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{15\,(6\,a^2b^4-b^6)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)\sqrt{a^2+b^2}} - \frac{6\,\bigl(13\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-23}{(a^{10}+4\,a^8b^2+6\,a^6b^4+4\,a^4b}}{}$$

input
```
integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-1/6*(15*(6*a^2*b^4 - b^6)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt
(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 6*(13*a^3*
b^6*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*t
an(1/2*d*x + 1/2*c)^2 - 23*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*
d*x + 1/2*c)^2 - 35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1
/2*c) - 12*a^4*b^5 - a^2*b^7)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 +
 a^2*b^8)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2) - 4
*(3*a^5*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*
b^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*t
an(1/2*d*x + 1/2*c)^4 - 9*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x +
 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 42*a*b^4*tan(1/2*d*x + 1/2
*c)^3 + 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2
+ 3*a^5*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*
tan(1/2*d*x + 1/2*c) + 3*a^4*b + 32*a^2*b^3 - 7*b^5)/((a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

3.577.9 Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.64

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input

```
int(cos(c + d*x)^3/(a + b*tan(c + d*x))^3,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^5*(2*a^7 - 255*a*b^6 + 62*a^3*b^4 + 4*a^5*b^2))/(3*
(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (6*a^6*b - 3*b^7 - 50*a
^2*b^5 + 64*a^4*b^3)/(3*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) +
 (2*tan(c/2 + (d*x)/2)^2*(6*a^6*b - 3*b^7 - 64*a^2*b^5 + 50*a^4*b^3))/(3*a
^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^9*(2*a^8 + 2
*b^8 + 13*a^2*b^6 - 18*a^4*b^4 + 8*a^6*b^2))/(a*(a^8 + b^8 + 4*a^2*b^6 + 6
*a^4*b^4 + 4*a^6*b^2)) - (4*tan(c/2 + (d*x)/2)^7*(2*a^6 - 3*b^6 + 36*a^2*b
^4 + 6*a^4*b^2))/(3*a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d
*x)/2)^8*(2*a^8*b + 2*b^9 + 23*a^2*b^7 - 78*a^4*b^5 + 8*a^6*b^3))/(a^2*(a^
8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (2*tan(c/2 + (d*x)/2)^4*(8
*a^8*b - 9*b^9 - 156*a^2*b^7 + 188*a^4*b^5 + 28*a^6*b^3))/(3*a^2*(a^2 + b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(14*a^8*
b + 9*b^9 + 141*a^2*b^7 - 246*a^4*b^5 + 56*a^6*b^3))/(3*a^2*(a^2 + b^2)*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(6*a^8 - 6*b^8 -
161*a^2*b^6 + 202*a^4*b^4 + 48*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^3*(2*a^8 + 3*b^8 + 133*a^2*b^
6 - 86*a^4*b^4 - 4*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*(2*a^2 - 12
*b^2) - tan(c/2 + (d*x)/2)^4*(2*a^2 - 12*b^2) + a^2 + tan(c/2 + (d*x)/2)^2
*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 + 4*b^2) + 8*a*b*tan(c/2 + (...
```

3.577. $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

3.578 $\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$

3.578.1 Optimal result 4010
3.578.2 Mathematica [A] (verified) 4010
3.578.3 Rubi [A] (verified) 4011
3.578.4 Maple [C] (verified) 4013
3.578.5 Fricas [C] (verification not implemented) 4014
3.578.6 Sympy [F(-1)] 4015
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3.578.9 Mupad [F(-1)] 4016

3.578.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = -\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(d\sec(e+fx))^{7/2}}{7f}+\frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f}$$

output
```
2/7*b*(d*sec(f*x+e))^(7/2)/f+2/5*a*d*(d*sec(f*x+e))^(5/2)*sin(f*x+e)/f-6/5
*a*d^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+6/5*a*d^3*sin(f
*x+e)*(d*sec(f*x+e))^(1/2)/f
```

3.578.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{7/2}\left(40b-168a\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+70a\sin(2(e+fx))+2\right.}{140f}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(7/2)*(40*b - 168*a*Cos[e + f*x]^(7/2)*EllipticE[(e + f*
x)/2, 2] + 70*a*Sin[2*(e + f*x)] + 21*a*Sin[4*(e + f*x)]))/(140*f)
```

3.578.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{7/2}dx+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}d^2\int (d\sec(e+fx))^{3/2}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}d^2\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int\frac{1}{\sqrt{d\sec(e+fx)}}dx\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int\frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int\sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{3}{5}d^2\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2d\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*(d*Sec[e + f*x])^(7/2))/(7*f) + a*((2*d*(d*Sec[e + f*x])^(5/2)*Sin[e
+ f*x])/(5*f) + (3*d^2*((-2*d^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e +
 f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/f))
/5)
```

3.578. $\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$

3.578.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.578.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.10 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.60

method	result
default	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^3\left(3iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^2(fx+e))-3iF(i(\csc(fx+e)-\cot(fx+e)\right.}{}$
parts	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^3\left(3iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^2(fx+e))-3iF(i(\csc(fx+e)-\cot(fx+e)\right.}{}$

input
```
int((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a/f*(d*sec(f*x+e))^(1/2)*d^3/(cos(f*x+e)+1)*(3*I*EllipticE(I*(csc(f*x
+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*cos(f*x+e)^2-3*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2+6*I*cos(f*x+e)*E
llipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),
I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*(1/(cos(
f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)-3*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x
+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-3*sin(f*x+e)-tan(f*x+e)-sec(f*
x+e)*tan(f*x+e))+2/7*b*(d*sec(f*x+e))^(7/2)/f
```

3.578.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{-21i\sqrt{2}ad^{\frac{7}{2}}\cos(fx+e)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+}{}$$

input
```
integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/35*(-21*I*sqrt(2)*a*d^(7/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*a*d^(7
/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) - I*sin(f*x + e))) + 2*(5*b*d^3 + 7*(3*a*d^3*cos(f*x + e)^3 + a*d^
3*cos(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

3.578.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)*(a+b*tan(f*x+e)),x)
```

output
```
Timed out
```

3.578.7 Maxima [F]

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{7}{2}}\,(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

3.578.8 Giac [F]

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{7}{2}}\,(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

3.578.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx=\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{7/2}\,\left(a+b\tan(e+f\,x)\right)\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)), x)
```

3.579 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$

3.579.1 Optimal result . . . 4017
3.579.2 Mathematica [A] (verified) . . . 4017
3.579.3 Rubi [A] (verified) . . . 4018
3.579.4 Maple [C] (verified) . . . 4020
3.579.5 Fricas [C] (verification not implemented) . . . 4020
3.579.6 Sympy [F] . . . 4021
3.579.7 Maxima [F] . . . 4021
3.579.8 Giac [F] . . . 4021
3.579.9 Mupad [F(-1)] . . . 4022

3.579.1 Optimal result

Integrand size = 23, antiderivative size = 92

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx=\frac{2ad^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2b(d\sec(e+fx))^{5/2}}{5f}+\frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

output
```
2/5*b*(d*sec(f*x+e))^(5/2)/f+2/3*a*d*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/f+2/3
*a*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f
*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/f
```

3.579.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.63

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx=\frac{(d\sec(e+fx))^{5/2}\left(6b+10a\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+5a\sin(2(e+fx))\right)}{15f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(5/2)*(6*b + 10*a*Cos[e + f*x]^(5/2)*EllipticF[(e + f*x)
/2, 2] + 5*a*Sin[2*(e + f*x)]))/(15*f)
```

3.579.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{5/2}dx+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{1}{3}d^2\int \sqrt{d\sec(e+fx)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}d^2\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{2d^2\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*(d*Sec[e + f*x])^(5/2))/(5*f) + a*((2*d^2*Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*d*(d*Sec[e + f*x])^(3/2
)*Sin[e + f*x])/(3*f))
```

3.579.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.579.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 24.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^2\left(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-c\right.}{3f}$
parts	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^2\left(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-c\right.}{3f}$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a/f*(d*sec(f*x+e))^(1/2)*d^2*(I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I
)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+I*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(
1/(cos(f*x+e)+1))^(1/2)-tan(f*x+e))+2/5*b*(d*sec(f*x+e))^(5/2)/f
```

3.579.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{-5i\sqrt{2}ad^{\frac{5}{2}}\cos\left(fx+e\right)^2\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right))+5i}{}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/15*(-5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierst
rassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(5*a*d^2*cos(f*x +
e)*sin(f*x + e) + 3*b*d^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

3.579.6 Sympy [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int (d\sec(e+fx))^{\frac{5}{2}}(a+b\tan(e+fx))\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x)), x)
```

3.579.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

3.579.8 Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

3.579.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/2}\,\left(a+b\tan(e+f\,x)\right)\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)), x)
```

3.580 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$

3.580.1 Optimal result 4023
3.580.2 Mathematica [A] (verified) 4023
3.580.3 Rubi [A] (verified) 4024
3.580.4 Maple [C] (verified) 4026
3.580.5 Fricas [C] (verification not implemented) 4026
3.580.6 Sympy [F] 4027
3.580.7 Maxima [F] 4027
3.580.8 Giac [F] 4027
3.580.9 Mupad [F(-1)] 4028

3.580.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = -\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f}$$

output

```
2/3*b*(d*sec(f*x+e))^(3/2)/f-2*a*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec
(f*x+e))^(1/2)+2*a*d*sin(f*x+e)*(d*sec(f*x+e))^(1/2)/f
```

3.580.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{3/2}\left(2b-6a\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+3a\sin(2(e+fx))\right)}{3f}$$

input

```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

output

```
((d*Sec[e + f*x])^(3/2)*(2*b - 6*a*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/
2, 2] + 3*a*Sin[2*(e + f*x)]))/(3*f)
```

3.580.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{3/2}dx+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{4255}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\sec(e+fx)}}dx\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3119}$$

$$a\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*(d*Sec[e + f*x])^(3/2))/(3*f) + a*((-2*d^2*EllipticE[(e + f*x)/2, 2])
/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*d*Sqrt[d*Sec[e + f*x]]*S
in[e + f*x])/f)
```

3.580.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.580.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.59 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.68

method	result
default	$-\frac{2a\left(iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^2(fx+e))-iF(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\right.}{}$
parts	$-\frac{2a\left(iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^2(fx+e))-iF(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\right.}{}$

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2*a/f*(I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticF(I*(csc(f*x+e)-c
ot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
os(f*x+e)^2+2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(f*
x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(f*
x+e))*(d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)+2/3*b*(d*sec(f*x+e))^(3/2)/f
```

3.580.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \frac{-3i\,\sqrt{2}ad^{\frac{3}{2}}\cos{(fx+e)}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(-3*I*sqrt(2)*a*d^(3/2)*cos(f*x + e)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*a*d^(3/2)*c
os(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e)
 - I*sin(f*x + e))) + 2*(3*a*d*cos(f*x + e)*sin(f*x + e) + b*d)*sqrt(d/cos
(f*x + e)))/(f*cos(f*x + e))
```

3.580.6 Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx=\int (d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})\,\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x)), x)
```

3.580.7 Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx=\int (d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

3.580.8 Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx=\int (d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

3.580.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(a+b\tan(e+f\,x)\right)\,dx$$

input

```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)),x)
```

output

```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)), x)
```

3.581 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$

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3.581.1 Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2b\sqrt{d\sec(e+fx)}}{f}+\frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}$$

output
```
2*b*(d*sec(f*x+e))^(1/2)/f+2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))
^(1/2)/f
```

3.581.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2\left(b+a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{d\sec(e+fx)}}{f}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

output
```
(2*(b + a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[d*Sec[e + f*x
]])/f
```

3.581.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \sqrt{d\sec(e+fx)}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{4258}$$

$$a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

output
```
(2*b*Sqrt[d*Sec[e + f*x]])/f + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)
/2, 2]*Sqrt[d*Sec[e + f*x]])/f
```

3.581.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.581.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

method	result
parts	$-\frac{2 i a (\cos(fx+e)+1) F(i(\csc(fx+e)-\cot(fx+e)),i) \sqrt{d\sec(fx+e)}\, \sqrt{\frac{1}{\cos(fx+e)+1}}\, \sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}}{f}+\frac{2b\sqrt{d\sec(fx+e)}}{f}$
default	$-\frac{2\left(i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, \sqrt{\frac{1}{\cos(fx+e)+1}}\, F(i(\csc(fx+e)-\cot(fx+e)),i) a\cos(fx+e)+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, \sqrt{\frac{1}{\cos(fx+e)+1}}\, F(i(\csc(fx+e)-\cot\right.}{f}$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2*I*a/f*(cos(f*x+e)+1)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(d*sec(f*x+
e))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*b*(
d*sec(f*x+e))^(1/2)/f
```

3.581.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$$
$$= \frac{-i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\sin{(fx+e)})+i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,}{f}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e)) + 2*b*sqrt(d/cos(f*x + e)))/f
```

3.581.6 Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec{(e+fx)}}(a+b\tan{(e+fx)})\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e)),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x)), x)
```

3.581.7 Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

3.581.8 Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx=\int \sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

3.581.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx=\frac{2\left(b+a\sqrt{\cos\left(e+f\,x\right)}\,\mathrm{F}\left(\frac{e}{2}+\frac{f\,x}{2}\middle|2\right)\right)\,\sqrt{\frac{d}{\cos(e+f\,x)}}}{f}$$

input
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x)),x)
```

output
```
(2*(b + a*cos(e + f*x)^(1/2)*ellipticF(e/2 + (f*x)/2, 2))*(d/cos(e + f*x))
^(1/2))/f
```

3.582 $\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$

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3.582.2 Mathematica [A] (verified) 4034
3.582.3 Rubi [A] (verified) 4035
3.582.4 Maple [C] (verified) 4036
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3.582.8 Giac [F] 4038
3.582.9 Mupad [F(-1)] 4038

3.582.1 Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx=-\frac{2b}{f\sqrt{d\sec(e+fx)}}+\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

output
```
-2*b/f/(d*sec(f*x+e))^(1/2)+2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f*x+
e))^(1/2)
```

3.582.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx=\frac{-2b\sqrt{\cos(e+fx)}+2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-2*b*Sqrt[Cos[e + f*x]] + 2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e +
f*x]]*Sqrt[d*Sec[e + f*x]])
```

3.582.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{\sqrt{d\sec(e+fx)}}dx-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

input

```
Int[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

output

```
(-2*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[
Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

3.582.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.582.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.44 (sec) , antiderivative size = 306, normalized size of antiderivative = 5.28

method	result
risch	$-\frac{i(-ib+a)\sqrt{2}}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}}-\frac{ia\left(-\frac{2\left(e^{2i(fx+e)}d+d\right)}{d\sqrt{e^{i(fx+e)}\left(e^{2i(fx+e)}d+d\right)}}+\frac{i\sqrt{-i\left(e^{i(fx+e)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(fx+e)}-i\right)}\sqrt{ie^{i(fx+e)}}\,(-2iE(\sqrt{-i\left(e^{i(fx+e)}+i\right)}}{\sqrt{d\,e^{3i(fx+e)}+d\,e^{i(fx+e)}}}\right.}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)}$
parts	$\frac{2a(i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e}}}{}$
default	Expression too large to display

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(a-I*b)/f*2^(1/2)/(d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2+1))^(1/2)-I*a/f*(
-2*(exp(I*(f*x+e))^2*d+d)/d/(exp(I*(f*x+e))*(exp(I*(f*x+e))^2*d+d))^(1/2)+
I*(-I*(exp(I*(f*x+e))+I))^(1/2)*2^(1/2)*(I*(exp(I*(f*x+e))-I))^(1/2)*(I*ex
p(I*(f*x+e)))^(1/2)/(d*exp(I*(f*x+e))^3+d*exp(I*(f*x+e)))^(1/2)*(-2*I*Elli
pticE((-I*(exp(I*(f*x+e))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(f
*x+e))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/(d*exp(I*(f*x+e))/(exp(I*(f*x+e))^
2+1))^(1/2)*(d*exp(I*(f*x+e))*(exp(I*(f*x+e))^2+1))^(1/2)/(exp(I*(f*x+e))^
2+1)
```

3.582.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \frac{i\sqrt{2}a\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))-i\sqrt{2}a\sqrt{d}\text{wei}}{d}$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e))) - I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*b*sqrt(d/co
s(f*x + e))*cos(f*x + e))/(d*f)
```

3.582.6 Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan{(e+fx)}}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

3.582.7 Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

3.582.8 Giac [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

3.582.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{a+b\tan(e+f\,x)}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2), x)
```

3.583 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$

3.583.1 Optimal result 4039
3.583.2 Mathematica [A] (verified) 4039
3.583.3 Rubi [A] (verified) 4040
3.583.4 Maple [C] (verified) 4042
3.583.5 Fricas [C] (verification not implemented) 4042
3.583.6 Sympy [F] 4043
3.583.7 Maxima [F] 4043
3.583.8 Giac [F] 4043
3.583.9 Mupad [F(-1)] 4044

3.583.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f} + \frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}$$

output
```
-2/3*b/f/(d*sec(f*x+e))^(3/2)+2/3*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(1/2)+2/
3*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+
1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^2/f
```

3.583.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{d\sec(e+fx)}\left(b+b\cos(2(e+fx))-2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-a\sin(2(e+fx))\right)}{3d^2f}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

output
```
-1/3*(Sqrt[d*Sec[e + f*x]]*(b + b*Cos[2*(e + f*x)] - 2*a*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2] - a*Sin[2*(e + f*x)]))/(d^2*f)
```

3.583.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3967}$$

$$a\int \frac{1}{(d\sec(e+fx))^{3/2}}dx-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$a\left(\frac{\int \sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$a\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$a\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$a\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

input

```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

output

```
(-2*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + a*((2*Sqrt[Cos[e + f*x]]*EllipticF[(
e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f*x])/(3*d*f*S
qrt[d*Sec[e + f*x]]))
```

3.583.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.583.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.85 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

method	result
default	$\frac{\frac{2iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,a}{3}+\frac{2i\sec(fx+e)F(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,a}{3}+}{df\sqrt{d\sec(fx+e)}}$
parts	$-\frac{2a\left(i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{}\right.}{3f\sqrt{d\sec(fx+e)}\,d}$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d/f/(d*sec(f*x+e))^(1/2)*(I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*a+I*sec(f*x+e)*Ell
ipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(
cos(f*x+e)+1))^(1/2)*a+sin(f*x+e)*a-b*cos(f*x+e))
```

3.583.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \frac{-i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}a\sqrt{d}}{}$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(
f*x + e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) -
I*sin(f*x + e)) - 2*(b*cos(f*x + e)^2 - a*cos(f*x + e)*sin(f*x + e))*sqrt(
d/cos(f*x + e)))/(d^2*f)
```

3.583.6 Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{a+b\tan{(e+fx)}}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(3/2), x)
```

3.583.7 Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

3.583.8 Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

3.583.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2), x)
```

3.584 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx$

3.584.1 Optimal result 4045
3.584.2 Mathematica [A] (verified) 4045
3.584.3 Rubi [A] (verified) 4046
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3.584.5 Fricas [C] (verification not implemented) 4048
3.584.6 Sympy [F] 4049
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3.584.8 Giac [F] 4049
3.584.9 Mupad [F(-1)] 4050

3.584.1 Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2b}{5f(d\sec(e+fx))^{5/2}} + \frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}$$

output
```
-2/5*b/f/(d*sec(f*x+e))^(5/2)+2/5*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(3/2)+6/
5*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+
1/2*e),2^(1/2))/d^2/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)
```

3.584.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\sqrt{d\sec(e+fx)}\Big(3a\sqrt{\cos(e+fx)}E\big(\frac{1}{2}(e+fx)\big|2\big)+\cos^2(e+fx)(-b\cos(e+f}{5d^3f}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*Sqrt[d*Sec[e + f*x]]*(3*a*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
+ Cos[e + f*x]^2*(-(b*Cos[e + f*x]) + a*Sin[e + f*x])))/(5*d^3*f)
```

3.584.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \ 3967$$

$$a\int \frac{1}{(d\sec(e+fx))^{5/2}}dx-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4256$$

$$a\left(\frac{3\int \frac{1}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$a\left(\frac{3\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4258$$

$$a\left(\frac{3\int \sqrt{\cos(e+fx)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$a\left(\frac{3\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3119$$

$$a\left(\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + a*((6*EllipticE[(e + f*x)/2, 2])/(5*
d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*Sin[e + f*x])/(5*d*f*(
d*Sec[e + f*x])^(3/2)))
```

3.584.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.584.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.15 (sec) , antiderivative size = 438, normalized size of antiderivative = 4.66

method	result
default	$2a\left(3i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-3i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx}}$
parts	$2a\left(3i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-3i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx}}$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*a/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(3*I*cos(f*x+e)*EllipticE(
I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)-3*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+
e)),I)-6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f
*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+3*I*sec(f*x+e)*EllipticE(I*(csc(f*x+e)-
cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
3*I*sec(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e)*
cos(f*x+e)+3*sin(f*x+e))-2/5*b/f/(d*sec(f*x+e))^(5/2)
```

3.584.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \frac{3i\sqrt{2}a\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin}{}$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(b*
cos(f*x + e)^3 - a*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3
*f)
```

3.584.6 Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{a+b\tan{(e+fx)}}{(d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

3.584.7 Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

3.584.8 Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

3.584.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input

```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2),x)
```

output

```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2), x)
```

3.585 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$

3.585.1 Optimal result 4051
3.585.2 Mathematica [A] (verified) 4051
3.585.3 Rubi [A] (verified) 4052
3.585.4 Maple [C] (verified) 4054
3.585.5 Fricas [C] (verification not implemented) 4055
3.585.6 Sympy [F] 4055
3.585.7 Maxima [F] 4056
3.585.8 Giac [F] 4056
3.585.9 Mupad [F(-1)] 4056

3.585.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = {} & -\frac{2b}{7f(d\sec(e+fx))^{7/2}} \\ & + \frac{10a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} \\ & + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}\end{aligned}$$

output
```
-2/7*b/f/(d*sec(f*x+e))^(7/2)+2/7*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(5/2)+10
/21*a*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(1/2)+10/21*a*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)
^(1/2)*(d*sec(f*x+e))^(1/2)/d^4/f
```

3.585.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{d\sec(e+fx)}\left(-9b-12b\cos(2(e+fx))-3b\cos(4(e+fx))+40a\sqrt{\cos(e+}\right.}{84d^4f}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

output

```
(Sqrt[d*Sec[e + f*x]]*(-9*b - 12*b*Cos[2*(e + f*x)] - 3*b*Cos[4*(e + f*x)]
 + 40*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 26*a*Sin[2*(e + f*x
)] + 3*a*Sin[4*(e + f*x)]))/(84*d^4*f)
```

3.585.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(d\sec(e+fx))^{7/2}}dx-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\int \frac{1}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$a\left(\frac{5\left(\frac{\int \sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$a\left(\frac{5\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(-2*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + a*((2*Sin[e + f*x])/(7*d*f*(d*Sec[e
+ f*x])^(5/2)) + (5*((2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[
d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f*x])/(3*d*f*Sqrt[d*Sec[e + f*x]])
))/(7*d^2))
```

3.585. $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$

3.585.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.585.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.45

method	result
default	$-\frac{2a\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\right.}{21f\sqrt{d\sec(fx+e)}\,d^3}$
parts	$-\frac{2a\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\right.}{21f\sqrt{d\sec(fx+e)}\,d^3}$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*a/f/(d*sec(f*x+e))^(1/2)/d^3*(5*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+5*I*sec(f*
x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e))
,I)*(1/(cos(f*x+e)+1))^(1/2)-3*sin(f*x+e)*cos(f*x+e)^2-5*sin(f*x+e))-2/7*b
/f/(d*sec(f*x+e))^(7/2)
```

3.585.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \frac{-5i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5i\sqrt{2}a\sqrt{}}{}$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*s
in(f*x + e)) + 5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x +
e) - I*sin(f*x + e)) - 2*(3*b*cos(f*x + e)^4 - (3*a*cos(f*x + e)^3 + 5*a*c
os(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

3.585.6 Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{a+b\tan{(e+fx)}}{(d\sec{(e+fx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(7/2),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(7/2), x)
```

3.585.7 Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

3.585.8 Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx=\int \frac{a+b\tan(e+f\,x)}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{7/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2), x)
```

3.586 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$

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3.586.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\frac{2(7a^2-2b^2)\,d^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21f}+\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2(7a^2-2b^2)\,d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f}+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

output
```
18/35*a*b*(d*sec(f*x+e))^(5/2)/f+2/21*(7*a^2-2*b^2)*d*(d*sec(f*x+e))^(3/2)
*sin(f*x+e)/f+2/21*(7*a^2-2*b^2)*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f
*x+e))^(1/2)/f+2/7*b*(d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))/f
```

3.586.2 Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\frac{2d^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\bigl(5(7a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\bigl(\frac{1}{2}(e+f}{105f(a\cos(e+fx)+b\sin(e}$$

input

```
Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

output

```
(2*d^2*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2*(5*(7*a^2 - 2*b^2)*Cos[
e + f*x]^(5/2)*EllipticF[(e + f*x)/2, 2] + (5*(7*a^2 - 2*b^2)*Sin[2*(e + f
*x)])/2 + 3*b*(14*a + 5*b*Tan[e + f*x])))/(105*f*(a*Cos[e + f*x] + b*Sin[e
 + f*x])^2)
```

3.586.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{2}{7}\int \frac{1}{2}(d\sec(e+fx))^{5/2}\left(7a^2+9b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int (d\sec(e+fx))^{5/2}\left(7a^2+9b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int (d\sec(e+fx))^{5/2}\left(7a^2+9b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\int (d\sec(e+fx))^{5/2}dx+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 4255$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\int\sqrt{d\sec(e+fx)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\int\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{18ab(d\sec(e+fx))^{5/2}}{5f}\right)$$
$$\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)+\right.$$
$$\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{1}{3}d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)\right.$$
$$\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

$$\Big\downarrow \; 3120$$

$$\frac{1}{7}\left(\left(7a^2-2b^2\right)\left(\frac{2d^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2d\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}\right)\right.$$
$$\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

3.586. $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$

output
```
((18*a*b*(d*Sec[e + f*x])^(5/2))/(5*f) + (7*a^2 - 2*b^2)*((2*d^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*d*(d*
Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)))/7 + (2*b*(d*Sec[e + f*x])^(5/2)*
(a + b*Tan[e + f*x]))/(7*f)
```

3.586.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.586.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 146.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.22

method	result
default	$-\frac{2d^2\sqrt{d\sec(fx+e)}\left(35i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,a^2-10i\cos(fx+e)F(i(\csc(fx+e)-\right.}{}$
parts	$-\frac{2a^2\sqrt{d\sec(fx+e)}\,d^2\left(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\csc(fx+e)-\right.}{3f}$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105*d^2/f*(d*sec(f*x+e))^(1/2)*(35*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)
-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)
*a^2-10*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*b^2+35*I*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e))
,I)*a^2-10*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*b^2-35*tan(f*x+e)*a^2+10*tan(f*x+e)*b
^2-42*sec(f*x+e)^2*a*b-15*tan(f*x+e)*sec(f*x+e)^2*b^2)
```

3.586.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\frac{-5i\sqrt{2}(7a^2-2b^2)d^{\frac{5}{2}}\cos(fx+e)^3\ \mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(f}{}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/105*(-5*I*sqrt(2)*(7*a^2 - 2*b^2)*d^(5/2)*cos(f*x + e)^3*weierstrassPInv
erse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*(7*a^2 - 2*b^2)*d
^(5/2)*cos(f*x + e)^3*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
+ e)) + 2*(42*a*b*d^2*cos(f*x + e) + 5*((7*a^2 - 2*b^2)*d^2*cos(f*x + e)^2
 + 3*b^2*d^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

3.586.6 Sympy [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(e+fx)})^{\frac{5}{2}}\,(a+b\tan{(e+fx)})^2\;dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2, x)
```

3.586.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

3.586.8 Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

3.586.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/2}\,{\left(a+b\tan\left(e+f\,x\right)\right)}^2 \,dx$$

input
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2, x)
```

3.587 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$

3.587.1 Optimal result 4064
3.587.2 Mathematica [A] (verified) 4064
3.587.3 Rubi [A] (verified) 4065
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3.587.8 Giac [F] 4070
3.587.9 Mupad [F(-1)] 4070

3.587.1 Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = -\frac{2(5a^2-2b^2)\,d^2E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

output
```
14/15*a*b*(d*sec(f*x+e))^(3/2)/f-2/5*(5*a^2-2*b^2)*d^2*(cos(1/2*f*x+1/2*e)
^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f
*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+2/5*(5*a^2-2*b^2)*d*sin(f*x+e)*(d*sec(f*x
+e))^(1/2)/f+2/5*b*(d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))/f
```

3.587.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = -\frac{2d^2(a+b\tan(e+fx))^2\Bigg(3(5a^2-2b^2)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|\,2\right)+\left(-\frac{15a^2}{2}+3b^2\right)\sin(2(e+fx))-b(}{15f\sqrt{d\sec(e+fx)}(a\cos(e+fx)+b\sin(e+fx))^2}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

output
```
(-2*d^2*(a + b*Tan[e + f*x])^2*(3*(5*a^2 - 2*b^2)*Cos[e + f*x]^(3/2)*Ellip
ticE[(e + f*x)/2, 2] + ((-15*a^2)/2 + 3*b^2)*Sin[2*(e + f*x)] - b*(10*a +
3*b*Tan[e + f*x])))/(15*f*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e +
 f*x])^2)
```

3.587.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{2}{5}\int \frac{1}{2}(d\sec(e+fx))^{3/2}\left(5a^2+7b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int (d\sec(e+fx))^{3/2}\left(5a^2+7b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int (d\sec(e+fx))^{3/2}\left(5a^2+7b\tan(e+fx)a-2b^2\right)dx+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\int (d\sec(e+fx))^{3/2}dx+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \ 4255$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\sec(e+fx)}}dx\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \ 4258$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{d^2\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

$$\Big\downarrow \ 3119$$

$$\frac{1}{5}\left(\left(5a^2-2b^2\right)\left(\frac{2d\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\right)+\frac{14ab(d\sec(e+fx))^{3/2}}{3f}\right)+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

3.587. $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

output
```
((14*a*b*(d*Sec[e + f*x])^(3/2))/(3*f) + (5*a^2 - 2*b^2)*((-2*d^2*Elliptic
E[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*d*Sqrt
[d*Sec[e + f*x]]*Sin[e + f*x])/f))/5 + (2*b*(d*Sec[e + f*x])^(3/2)*(a + b*
Tan[e + f*x]))/(5*f)
```

3.587.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.587.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.00 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.83

method	result	size
parts	Expression too large to display	833
default	Expression too large to display	844

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/f*(I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticF(I*(csc(f*x+e)
-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*cos(f*x+e)^2+2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(
f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(
f*x+e))*(d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)-2/5*b^2/f*(d*sec(f*x+e))^(1/
2)*d/(cos(f*x+e)+1)*(2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-2*I*Elliptic
E(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*cos(f*x+e)^2+4*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*
x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4*I*co
s(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-2*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-
cot(f*x+e)),I)+2*sin(f*x+e)-tan(f*x+e)-sec(f*x+e)*tan(f*x+e))+4/3*a*b*(...
```

3.587.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \frac{-3i\sqrt{2}(5\,a^2-2\,b^2)d^{\frac{3}{2}}\cos{(fx+e)}^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/15*(-3*I*sqrt(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sq
rt(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(10*a*b*d*cos(f*x
 + e) + 3*((5*a^2 - 2*b^2)*d*cos(f*x + e)^2 + b^2*d)*sin(f*x + e))*sqrt(d/
cos(f*x + e)))/(f*cos(f*x + e)^2)
```

3.587.6 Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \int \left(d\sec{\left(e+fx\right)}\right)^{\frac{3}{2}}\left(a+b\tan{\left(e+fx\right)}\right)^2\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2, x)
```

3.587.7 Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

3.587.8 Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{3}{2}}(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^{3/2}(a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2, x)
```

3.588 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

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3.588.1 Optimal result

Integrand size = 25, antiderivative size = 103

$$\begin{aligned}\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx &= \frac{10ab\sqrt{d\sec(e+fx)}}{3f} \\ &\quad+\frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} \\ &\quad+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}\end{aligned}$$

output
```
10/3*a*b*(d*sec(f*x+e))^(1/2)/f+2/3*(3*a^2-2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(
1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(
1/2)*(d*sec(f*x+e))^(1/2)/f+2/3*b*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))/f
```

3.588.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$

$$= \frac{2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\left((3a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+b\cos(e+fx)(6a\cos(e\right.}{3f}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

output
```
(2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*((3*a^2 - 2*b^2)*Cos[e + f*x]^(5/2)
*EllipticF[(e + f*x)/2, 2] + b*Cos[e + f*x]*(6*a*Cos[e + f*x] + b*Sin[e +
f*x]))/(3*f)
```

3.588.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow 3042$$

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow 3993$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{d\sec(e+fx)}\left(3a^2+5b\tan(e+fx)a-2b^2\right)dx+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\int \sqrt{d\sec(e+fx)}\left(3a^2+5b\tan(e+fx)a-2b^2\right)dx+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\int \sqrt{d\sec(e+fx)}\left(3a^2+5b\tan(e+fx)a-2b^2\right)dx+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow 3967$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\int \sqrt{d\sec(e+fx)}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{3}\left(\frac{2\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{10ab\sqrt{d\sec(e+fx)}}{f}\right)+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

output
```
((10*a*b*Sqrt[d*Sec[e + f*x]])/f + (2*(3*a^2 - 2*b^2)*Sqrt[Cos[e + f*x]]*E
llipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/f)/3 + (2*b*Sqrt[d*Sec[e +
f*x]]*(a + b*Tan[e + f*x]))/(3*f)
```

3.588.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.588. $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.588.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.54 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.32

method	result
parts	$-\frac{2ia^2(\cos(fx+e)+1)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{d\sec(fx+e)}\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}}{f}+\frac{2b^2\sqrt{d\sec(fx+e)}\left(2i\cos(fx-\right.}{}$
default	$-\frac{2\sqrt{d\sec(fx+e)}\left(3i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}a^2-2i\cos(fx+e)F(i(\csc(fx+e)-\cot(\right.}{}$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*a^2/f*(cos(f*x+e)+1)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(d*sec(f*
x+e))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2/3
*b^2/f*(d*sec(f*x+e))^(1/2)*(2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+2*I*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/
(cos(f*x+e)+1))^(1/2)+tan(f*x+e))+4*a*b*(d*sec(f*x+e))^(1/2)/f
```

3.588.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$

$$= \frac{\sqrt{2}(-3i\,a^2+2i\,b^2)\sqrt{d}\cos(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2}(3i\,a^2-}{}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(d)*cos(f*x + e)*weierstrassPInverse
(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(
d)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))
+ 2*(6*a*b*cos(f*x + e) + b^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f
*x + e))
```

3.588.6 Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2, x)
```

3.588.7 Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

3.588.8 Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

3.588.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int \sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,{\left(a+b\tan(e+f\,x)\right)}^2\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2, x)
```

3.589 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$

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3.589.7 Maxima [F] 4081
3.589.8 Giac [F] 4082
3.589.9 Mupad [F(-1)] 4082

3.589.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{6ab}{f\sqrt{d\sec(e+fx)}} + \frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

output
```
-6*a*b/f/(d*sec(f*x+e))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)
/(d*sec(f*x+e))^(1/2)+2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/2)
```

3.589.2 Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\frac{2(a^2-2b^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} + 2b(-2a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

output
```
((2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 2*b*(-2*
a + b*Tan[e + f*x]))/(f*Sqrt[d*Sec[e + f*x]])
```

3.589.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3993}$$

$$2\int \frac{a^2+3b\tan(e+fx)a-2b^2}{2\sqrt{d\sec(e+fx)}}dx+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{a^2+3b\tan(e+fx)a-2b^2}{\sqrt{d\sec(e+fx)}}dx+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2+3b\tan(e+fx)a-2b^2}{\sqrt{d\sec(e+fx)}}dx+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3967}$$

$$(a^2-2b^2)\int \frac{1}{\sqrt{d\sec(e+fx)}}dx-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(a^2-2b^2)\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{(a^2-2b^2)\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-2b^2\right)\int \sqrt{\sin \left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-6*a*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2
, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*b*(a + b*Tan[e + f*
x]))/(f*Sqrt[d*Sec[e + f*x]])
```

3.589.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

3.589. $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.589.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 16.29 (sec) , antiderivative size = 805, normalized size of antiderivative = 8.47

method	result
parts	$2a^2\left(i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+}}\right.$
default	Expression too large to display

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)*(I*EllipticE(I*(csc(f*x+e)-cot
(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos
(f*x+e)-I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-2
*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I
)*(1/(cos(f*x+e)+1))^(1/2)+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*sec(f*x+
e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I
)*(1/(cos(f*x+e)+1))^(1/2)+sin(f*x+e))-2*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e
))^(1/2)*(2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(f*x+
e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*cos(f*x+e)+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-4*I*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+2*I*
sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(csc(f*x+e)-cot(f*x+e)),I)-2*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+
sin(f*x+e)-tan(f*x+e))-4*a*b/f/(d*sec(f*x+e))^(1/2)
```

3.589.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$$

$$= \frac{\sqrt{2}(i\,a^2 - 2i\,b^2)\sqrt{d}\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos{(fx+e)} + i\,\sin{(fx+e)})) + \sqrt{}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*(I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-I*a^2 + 2*I*b^2)*sq
rt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e))) - 2*(2*a*b*cos(f*x + e) - b^2*sin(f*x + e))*sqrt(d/cos(f*x +
 e)))/(d*f)
```

3.589.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/sqrt(d*sec(e + f*x)), x)
```

3.589.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

3.589.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(b\tan\left(fx+e\right)+a)^2}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

3.589.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(a+b\tan(e+f\,x))^2}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2), x)
```

3.590 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$

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3.590.9 Mupad [F(-1)] 4089

3.590.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx &= \frac{2ab}{3f(d\sec(e+fx))^{3/2}}\\ &+ \frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}\\ &+ \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}\end{aligned}$$

output
```
2/3*a*b/f/(d*sec(f*x+e))^(3/2)+2/3*(a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(f*x+e
))^(1/2)+2/3*(a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*E
llipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)
/d^2/f-2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(3/2)
```

3.590.2 Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(-2ab-2ab\cos(2(e+fx))+2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{Elliptic}\right.}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(Sec[e + f*x]^2*(-2*a*b - 2*a*b*Cos[2*(e + f*x)] + 2*(a^2 + 2*b^2)*Sqrt[Co
s[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^2*Sin[2*(e + f*x)] - b^2*Sin[2*(
e + f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

3.590.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3993}$$

$$-2\int -\frac{a^2-b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{3/2}}dx-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{a^2-b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{3/2}}dx-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a^2-b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{3/2}}dx-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3967}$$

$$\left(a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{3/2}}dx+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\left(a^2+2b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4256}$$

$$(a^2+2b^2)\left(\frac{\int \sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$(a^2+2b^2)\left(\frac{\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$(a^2+2b^2)\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$(a^2+2b^2)\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$(a^2+2b^2)\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

input

```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

output

```
(2*a*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (a^2 + 2*b^2)*((2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f
*x])/(3*d*f*Sqrt[d*Sec[e + f*x]])) - (2*b*(a + b*Tan[e + f*x]))/(f*(d*Sec[
e + f*x])^(3/2))
```

3.590. $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$

3.590.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.590. $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$

3.590.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.54 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.12

method	result
default	$\frac{2iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,a^2}{3}+\frac{4iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,b^2}{3}+$ $2i\sec(f$
parts	$-\frac{2a^2\left(i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\csc(fx+e)-\cot(fx+e)),i)\right.}{3f\sqrt{d\sec(fx+e)}\,d}$

input

```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/3/d/f/(d*sec(f*x+e))^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*a^2+2*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-c
sc(f*x+e)),I)*b^2+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*a^2+2*I*sec(f*x+e)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(co
t(f*x+e)-csc(f*x+e)),I)*b^2-2*cos(f*x+e)*a*b+a^2*sin(f*x+e)-sin(f*x+e)*b^2
)
```

3.590.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{2}(-i\,a^2-2i\,b^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})}{}$$

input

```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output

```
1/3*(sqrt(2)*(-I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x
 + e) + I*sin(f*x + e)) + sqrt(2)*(I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassPIn
verse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(2*a*b*cos(f*x + e)^2 - (a
^2 - b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^2*f)
```

3.590.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(3/2), x)
```

3.590.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

3.590.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

3.590.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2), x)
```

3.591 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$

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3.591.8 Giac [F] 4096
3.591.9 Mupad [F(-1)] 4096

3.591.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

output
```
-2/15*a*b/f/(d*sec(f*x+e))^(5/2)+2/15*(3*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(
f*x+e))^(3/2)+2/5*(3*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/d^2/f/cos(f*x+e)^(1/2)/(d*sec(
f*x+e))^(1/2)-2/3*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(5/2)
```

3.591.2 Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \frac{(6a^2+4b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)+2\cos^{\frac{3}{2}}(e+fx)\,(-2ab\cos(e+fx)+(a^2-b^2)\,\mathrm{s}}{5f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{5/2}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

output
```
((6*a^2 + 4*b^2)*EllipticE[(e + f*x)/2, 2] + 2*Cos[e + f*x]^(3/2)*(-2*a*b*
Cos[e + f*x] + (a^2 - b^2)*Sin[e + f*x]))/(5*f*Cos[e + f*x]^(5/2)*(d*Sec[e
 + f*x])^(5/2))
```

3.591.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3993}$$

$$-\frac{2}{3}\int -\frac{3a^2+b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{5/2}}dx-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{5/2}}dx-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{3a^2+b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{5/2}}dx-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{5/2}}dx-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{1}{3}\left(\left(3a^2+2b^2\right)\left(\frac{3\int \frac{1}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left((3a^2+2b^2)\left(\frac{3\int \frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}\left((3a^2+2b^2)\left(\frac{3\int \sqrt{\cos(e+fx)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left((3a^2+2b^2)\left(\frac{3\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{3}\left((3a^2+2b^2)\left(\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)-\frac{2ab}{5f(d\sec(e+fx))^{5/2}}\right)-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

output
```
((-2*a*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + (3*a^2 + 2*b^2)*((6*EllipticE[(e
+ f*x)/2, 2])/(5*d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*Sin[e
 + f*x])/(5*d*f*(d*Sec[e + f*x])^(3/2))))/3 - (2*b*(a + b*Tan[e + f*x]))/(
3*f*(d*Sec[e + f*x])^(5/2))
```

3.591. $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$

3.591.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.591. $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$

3.591.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.41 (sec) , antiderivative size = 863, normalized size of antiderivative = 5.95

method	result	size
parts	Expression too large to display	863
default	Expression too large to display	890

input

```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/5*a^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(3*I*cos(f*x+e)*Elliptic
E(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)-3*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*
x+e)),I)-6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot
(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+3*I*sec(f*x+e)*EllipticE(I*(csc(f*x+e
)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-3*I*sec(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e
)*cos(f*x+e)+3*sin(f*x+e))-2/5*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d
^2*(-2*I*cos(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*EllipticF(I*(csc(f*x+e)-co
t(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*co
s(f*x+e)-4*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+4*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e
)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e
)*cos(f*x+e)^2-2*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+2*I*sec(f*x+e)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/...
```

3.591.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}(3i\,a^2+2i\,b^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(fx}}{}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(sqrt(2)*(3*I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-3*I*a^2 - 2*I
*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x +
e) - I*sin(f*x + e))) - 2*(2*a*b*cos(f*x + e)^3 - (a^2 - b^2)*cos(f*x + e)
^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3*f)
```

3.591.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/2), x)
```

3.591.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{(d\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

3.591.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2), x)
```

3.592 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$

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3.592.1 Optimal result

Integrand size = 25, antiderivative size = 184

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx &= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}}\\ &+\frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f}\\ &+\frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}}+\frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}\end{aligned}$$

output
```
-6/35*a*b/f/(d*sec(f*x+e))^(7/2)+2/35*(5*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(
f*x+e))^(5/2)+2/21*(5*a^2+2*b^2)*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(1/2)+2/2
1*(5*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^4/f-2/
5*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(7/2)
```

3.592.2 Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \frac{-18ab\cos(e+fx)-6ab\cos(3(e+fx))+\frac{4(5a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{\sqrt{\cos(e+fx)}}+23a^2\,\mathrm{si}}{42d^3f\sqrt{d\sec(e}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(-18*a*b*Cos[e + f*x] - 6*a*b*Cos[3*(e + f*x)] + (4*(5*a^2 + 2*b^2)*Ellipt
icF[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 23*a^2*Sin[e + f*x] + 5*b^2*Sin[
e + f*x] + 3*a^2*Sin[3*(e + f*x)] - 3*b^2*Sin[3*(e + f*x)])/(42*d^3*f*Sqrt
[d*Sec[e + f*x]])
```

3.592.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3993}$$

$$-\frac{2}{5}\int -\frac{5a^2+3b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{7/2}}dx-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{5a^2+3b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{7/2}}dx-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{5a^2+3b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{7/2}}dx-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{5}\left(\left(5a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{7/2}}dx-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left((5a^2+2b^2)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 4256$$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\int \frac{1}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 4256$$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{\int \sqrt{d\sec(e+fx)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{\int \sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{6ab}{7f(d\sec(e+fx))^{7/2}}\right)-\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-\frac{}{7f(d\sec(} \quad \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

3.592. $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-7f(d\right.$$

$$\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{1}{5}\left((5a^2+2b^2)\left(\frac{5\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\right)}{7d^2}+\frac{2\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}\right)-7f(\right.$$

$$\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

output
```
((-6*a*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + (5*a^2 + 2*b^2)*((2*Sin[e + f*x])
/(7*d*f*(d*Sec[e + f*x])^(5/2)) + (5*((2*Sqrt[Cos[e + f*x]]*EllipticF[(e +
 f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*Sin[e + f*x])/(3*d*f*Sqrt
[d*Sec[e + f*x]])))/(7*d^2))/5 - (2*b*(a + b*Tan[e + f*x]))/(5*f*(d*Sec[e
 + f*x])^(7/2))
```

3.592.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.592. $\int\frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.592.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.61 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.83

method	result
default	$\frac{10iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,a^2}{21}+\frac{4iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,b^2}{21}-$
parts	$-\frac{2a^2\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\right.}{21f\sqrt{d\sec(fx+e)}\,d^3}$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d^3/f/(d*sec(f*x+e))^(1/2)*(5*I*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x
+e)-csc(f*x+e)),I)*b^2-6*a*b*cos(f*x+e)^3+3*a^2*cos(f*x+e)^2*sin(f*x+e)-3*
b^2*cos(f*x+e)^2*sin(f*x+e)+5*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I
*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*b^2+5*a^2*sin(f*x+e)+2*sin(f*x+e)*b^2)
```

3.592.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}(-5i\,a^2-2i\,b^2)\sqrt{d}\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)}{}$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(sqrt(2)*(-5*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^2 + 2*I*b^2)*sqrt(d)*weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(6*a*b*cos(f*x + e)^4
 - (3*(a^2 - b^2)*cos(f*x + e)^3 + (5*a^2 + 2*b^2)*cos(f*x + e))*sin(f*x +
 e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

3.592.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(7/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(7/2), x)
```

3.592.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

3.592.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2), x)
```

3.593 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx$

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3.593.1 Optimal result

Integrand size = 25, antiderivative size = 184

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx &= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} \\ &+ \frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} \\ &+ \frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}\end{aligned}$$

output
```
-10/63*a*b/f/(d*sec(f*x+e))^(9/2)+2/63*(7*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec
(f*x+e))^(7/2)+2/45*(7*a^2+2*b^2)*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(3/2)+2/
15*(7*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))/d^4/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)-2
/7*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(9/2)
```

3.593.2 Mathematica [A] (verified)

Time = 6.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\frac{48(7a^2+2b^2)E\left(\frac{1}{2}(e+fx)|2\right)}{\sqrt{\cos(e+fx)}} + 4\cos(e+fx)\left(-30ab\cos(e+fx) - 10ab\cos(3(e+f\right.}{360d^4f\sqrt{d\sec(e+f}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((48*(7*a^2 + 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 4*Cos
[e + f*x]*(-30*a*b*Cos[e + f*x] - 10*a*b*Cos[3*(e + f*x)] + 2*(19*a^2 - b^
2 + 5*(a^2 - b^2)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(360*d^4*f*Sqrt[d*Sec[e
 + f*x]])
```

3.593.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3993}$$

$$-\frac{2}{7}\int -\frac{7a^2+5b\tan(e+fx)a+2b^2}{2(d\sec(e+fx))^{9/2}}dx-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{7a^2+5b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{9/2}}dx-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{7a^2+5b\tan(e+fx)a+2b^2}{(d\sec(e+fx))^{9/2}}dx-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int \frac{1}{(d\sec(e+fx))^{9/2}}dx-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{9/2}}dx-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 4256$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\int\frac{1}{(d\sec(e+fx))^{5/2}}dx}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\int\frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 4256$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{3\int\frac{1}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)-\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{3\int\frac{1}{\sqrt{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5d^2}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))^{9/2}}\right)$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 4258$$

$$\frac{1}{7}\left(\left(7a^2+2b^2\right)\left(\frac{7\left(\frac{3\int\sqrt{\cos(e+fx)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx))}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

3.593. $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left((7a^2+2b^2)\left(\frac{7\left(\frac{3\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx)}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{1}{7}\left((7a^2+2b^2)\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(e+fx)|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}\right)}{9d^2}+\frac{2\sin(e+fx)}{9df(d\sec(e+fx))^{7/2}}\right)-\frac{10ab}{9f(d\sec(e+fx)}\right.$$

$$\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((-10*a*b)/(9*f*(d*Sec[e + f*x])^(9/2)) + (7*a^2 + 2*b^2)*((2*Sin[e + f*x]
)/(9*d*f*(d*Sec[e + f*x])^(7/2)) + (7*((6*EllipticE[(e + f*x)/2, 2])/(5*d^
2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*Sin[e + f*x])/(5*d*f*(d*
Sec[e + f*x])^(3/2))))/(9*d^2))/7 - (2*b*(a + b*Tan[e + f*x]))/(7*f*(d*Se
c[e + f*x])^(9/2))
```

3.593.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.593.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 33.43 (sec) , antiderivative size = 931, normalized size of antiderivative = 5.06

method	result	size
parts	Expression too large to display	931
default	Expression too large to display	968

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

---

output

```
2/45*a^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(21*I*cos(f*x+e)*Ellipt
icE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-21*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^4*s
in(f*x+e)+42*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-42*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+5*cos(
f*x+e)^3*sin(f*x+e)+21*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-21*I*sec(f*x+e
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)
*(1/(cos(f*x+e)+1))^(1/2)+7*sin(f*x+e)*cos(f*x+e)^2+7*sin(f*x+e)*cos(f*x+e
)+21*sin(f*x+e))+2/45*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(6*I*c
os(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-c
ot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5
*cos(f*x+e)^4*sin(f*x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-12*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*
x+e)),I)-5*cos(f*x+e)^3*sin(f*x+e)+6*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),...
```

3.593.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx =$$

$$-\frac{3\sqrt{2}(-7i\,a^2-2i\,b^2)\sqrt{d}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))}{}$$

input

```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output

```
-1/45*(3*sqrt(2)*(-7*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a^2
 + 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) - I*sin(f*x + e))) + 2*(10*a*b*cos(f*x + e)^5 - (5*(a^2 - b^2)*co
s(f*x + e)^4 + (7*a^2 + 2*b^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*
x + e)))/(d^5*f)
```

3.593.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.593.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan\left(fx+e\right)+a)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

3.593.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan\left(fx+e\right)+a)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2), x)
```

3.594 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$

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3.594.1 Optimal result

Integrand size = 25, antiderivative size = 198

$$\begin{aligned}\int (d\sec(e+fx))^{5/2}(a&\\ +b\tan(e+fx))^3\,dx &= \frac{2a(7a^2-6b^2)\,d^2\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{21f\sqrt[4]{\sec^2(e+fx)}}\\ &+\frac{2a(7a^2-6b^2)\,d^2\sqrt{d\sec(e+fx)}\tan(e+fx)}{21f}\\ &+\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f}\\ &+\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(14(11a^2-2b^2)+65ab\tan(e+fx))}{315f}\end{aligned}$$

output
```
2/21*a*(7*a^2-6*b^2)*d^2*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arc
tan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x
+e))^(1/2)/f/(sec(f*x+e)^2)^(1/4)+2/21*a*(7*a^2-6*b^2)*d^2*(d*sec(f*x+e))^
(1/2)*tan(f*x+e)/f+2/9*b*d^2*sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*
x+e))^2/f+2/315*b*d^2*sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)*(154*a^2-28*b^2+65
*a*b*tan(f*x+e))/f
```

3.594.2 Mathematica [A] (verified)

Time = 4.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.79

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{2d(d\sec(e+fx))^{3/2}\Big(63b(-3a^2+b^2)\cos^2(e+fx)-15a(7a^2-6b^2)\cos^{\frac{9}{2}}(e+fx)\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right.}{315f(a\cos(e+fx)+}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

output
```
(-2*d*(d*Sec[e + f*x])^(3/2)*(63*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 - 15*a*(7
*a^2 - 6*b^2)*Cos[e + f*x]^(9/2)*EllipticF[(e + f*x)/2, 2] - 15*a*(7*a^2 -
 6*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - (5*b^2*(14*b + 27*a*Sin[2*(e + f*x)]
))/2)*(a + b*Tan[e + f*x])^3)/(315*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

3.594.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3994, 497, 27, 25, 676, 211, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int (a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{497}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2}{9}b^2\int -\frac{(a+b\tan(e+fx))\left(\left(4-\frac{9a^2}{b^2}\right)b^2-13ab\tan(e+fx)\right)\sqrt[4]{\tan^2(e+fx)+1}}{2b^2}d(b\tan(e+fx))+\frac{2}{9}b^2(\tan}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2}{9}b^2\left(\tan^2(e+fx)+1\right)^{5/4}(a+b\tan(e+fx))^2-\frac{1}{9}\int -\Big((a+b\tan(e+fx))\left(9a^2+13b\tan(e\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 25$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\int (a+b\tan(e+fx))\left(9a^2+13b\tan(e+fx)a-4b^2\right)\sqrt[4]{\tan^2(e+fx)+1}d(b\tan(e+fx))+\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 676$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\left(\frac{9}{7}a\left(7a^2-6b^2\right)\int \sqrt[4]{\tan^2(e+fx)+1}d(b\tan(e+fx))+\frac{4}{5}b^2\left(11a^2-2b^2\right)\left(\tan^2(e+fx)+1\right.\right.\right.}{bf\sqrt[4]{\sec^2(e+f}}$$

$$\Big\downarrow 211$$

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\left(\frac{9}{7}a\left(7a^2-6b^2\right)\left(\frac{1}{3}\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+\frac{2}{3}b\sqrt[4]{\tan^2(e+fx)+1}\tan(e+fx)\right)\right.\right.$$

$$\Big\downarrow 229$$

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{1}{9}\left(\frac{9}{7}a\left(7a^2-6b^2\right)\left(\frac{2}{3}b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)+\frac{2}{3}b\tan(e+fx)\sqrt[4]{\tan^2(e+fx)+\right.\right.\right.$$

input

```
Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

3.594. $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*((2*b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]
^2)^(5/4))/9 + ((4*b^2*(11*a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(5/4))/5 + (2
6*a*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(5/4))/7 + (9*a*(7*a^2 - 6*b^2)*
((2*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2])/3 + (2*b*Tan[e + f*x]*(1 + Tan
[e + f*x]^2)^(1/4))/3))/7)/9))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

3.594.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.594.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 619.10 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2d^2\sqrt{d\sec(fx+e)}\,(105i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)a^3-90i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}}{}$
parts	$-\frac{2a^3\sqrt{d\sec(fx+e)}\,d^2\,(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-}{3f}$

input
```
int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/315*d^2/f*(d*sec(f*x+e))^(1/2)*(105*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I
)*a^3-90*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a*b^2+105*I*(1/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*a^3-90*I*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+
e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-105*tan(f*x+e)*a^3+90*tan(f
*x+e)*a*b^2-189*sec(f*x+e)^2*a^2*b-135*tan(f*x+e)*sec(f*x+e)^2*a*b^2+63*b^
3*sec(f*x+e)^2-35*sec(f*x+e)^4*b^3)
```

3.594.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{-15i\sqrt{2}(7\,a^3-6\,ab^2)d^{\frac{5}{2}}\cos\left(fx+e\right)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin}{}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/315*(-15*I*sqrt(2)*(7*a^3 - 6*a*b^2)*d^(5/2)*cos(f*x + e)^4*weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*I*sqrt(2)*(7*a^3 - 6*a*
b^2)*d^(5/2)*cos(f*x + e)^4*weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e)) + 2*(35*b^3*d^2 + 63*(3*a^2*b - b^3)*d^2*cos(f*x + e)^2 + 15*(
9*a*b^2*d^2*cos(f*x + e) + (7*a^3 - 6*a*b^2)*d^2*cos(f*x + e)^3)*sin(f*x +
 e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^4)
```

3.594.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**3,x)
```

output
```
Timed out
```

3.594.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \int (d\sec\left(fx+e\right))^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

3.594.8 Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx=\int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))^3\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3, x)
```

3.595 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$

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3.595.1 Optimal result

Integrand size = 25, antiderivative size = 176

$$\begin{aligned}\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = &-\frac{2a(5a^2-6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)(d\sec(e+fx))^{3/2}}{5f\sec^2(e+fx)^{3/4}} \\ &+\frac{2a(5a^2-6b^2)\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{5f} \\ &+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} \\ &+\frac{2b(d\sec(e+fx))^{3/2}\left(10(9a^2-2b^2)+33ab\tan(e+fx)\right)}{105f}\end{aligned}$$

output

```
-2/5*a*(5*a^2-6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(
tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))
^(3/2)/f/(sec(f*x+e)^2)^(3/4)+2/5*a*(5*a^2-6*b^2)*cos(f*x+e)*(d*sec(f*x+e)
)^(3/2)*sin(f*x+e)/f+2/7*b*(d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2/f+2/105
*b*(d*sec(f*x+e))^(3/2)*(90*a^2-20*b^2+33*a*b*tan(f*x+e))/f
```

3.595.2 Mathematica [A] (verified)

Time = 3.86 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{d\sqrt{d\sec(e+fx)}\Big(70b(-3a^2+b^2)\cos^2(e+fx)+42a(5a^2-6b^2)\cos^{\frac{7}{2}}(e+fx)E\big(\frac{1}{2}(e+fx)\big|\,2\big)-42a(5a^2}{105f(a\cos(e+fx)+b\sin(}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

output
```
-1/105*(d*Sqrt[d*Sec[e + f*x]]*(70*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 + 42*a*
(5*a^2 - 6*b^2)*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x)/2, 2] - 42*a*(5*a^2
 - 6*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - 3*b^2*(10*b + 21*a*Sin[2*(e + f*x)
]))*(a + b*Tan[e + f*x])^3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

3.595.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3994, 497, 27, 25, 676, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{(d\sec(e+fx))^{3/2}\int \frac{(a+b\tan(e+fx))^3}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{497}$$

$$\frac{(d\sec(e+fx))^{3/2}\Bigg(\frac{2}{7}b^2\int -\frac{(a+b\tan(e+fx))\left(\left(4-\frac{7a^2}{b^2}\right)b^2-11ab\tan(e+fx)\right)}{2b^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{2}{7}b^2\left(\tan^2(e+fx)+1\right)^{3/4}(}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 27$$

$$\frac{(d\sec(e+fx))^{3/2}\Bigg(\frac{2}{7}b^2\left(\tan^2(e+fx)+1\right)^{3/4}(a+b\tan(e+fx))^2-\frac{1}{7}\int -\frac{(a+b\tan(e+fx))(7a^2+11b\tan(e+fx)a-4b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}d(}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 25$$

$$\frac{(d\sec(e+fx))^{3/2}\Bigg(\frac{1}{7}\int \frac{(a+b\tan(e+fx))(7a^2+11b\tan(e+fx)a-4b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{2}{7}b^2\left(\tan^2(e+fx)+1\right)^{3/4}(a+b\,\mathrm{ta}}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 676$$

$$\frac{(d\sec(e+fx))^{3/2}\Bigg(\frac{1}{7}\Bigg(\frac{7}{5}a(5a^2-6b^2)\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{4}{3}b^2(9a^2-2b^2)\left(\tan^2(e+fx)+1\right.}{bf\sec^2(e+fx)}$$

$$\Big\downarrow \ 225$$

$$\frac{(d\sec(e+fx))^{3/2}\Bigg(\frac{1}{7}\Bigg(\frac{7}{5}a(5a^2-6b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))\right)+\frac{4}{3}b^2(9a^2-}{bf\,\mathrm{s}}$$

$$\Big\downarrow \ 212$$

$$\frac{(d\sec(e+fx))^{3/2}\Bigg(\frac{1}{7}\Bigg(\frac{7}{5}a(5a^2-6b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)+\frac{4}{3}b^2(9a^2-2b}{bf\,\mathrm{sec}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

3.595. $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$

output

```
((d*Sec[e + f*x])^(3/2)*((2*b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2
)^(3/4))/7 + ((4*b^2*(9*a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(3/4))/3 + (22*a
*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(3/4))/5 + (7*a*(5*a^2 - 6*b^2)*(-2
*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e +
f*x]^2)^(1/4)))/5)/7))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

3.595.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 497

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.595.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 22.07 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.95

method	result	size
parts	Expression too large to display	872
default	Expression too large to display	899

input
```
int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*a^3/f*(I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticF(I*(csc(f*x+e)
-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*cos(f*x+e)^2+2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(
f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(
f*x+e))*(d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)+2*b^3/f/d^2*(1/7*(d*sec(f*x+
e))^(7/2)-1/3*d^2*(d*sec(f*x+e))^(3/2))+2*a^2*b/f*(d*sec(f*x+e))^(3/2)+6/5
*a*b^2/f*(d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)*(2*I*EllipticE(I*(csc(f*x+e
)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*cos(f*x+e)^2-2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2+4*I*cos(f*x+e)*Ell
ipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)-4*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-c
ot(f*x+e)),I)-2*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*...
```

3.595.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.14

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \frac{-21i\sqrt{2}(5a^3-6ab^2)d^{\frac{3}{2}}\cos(fx+e)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output

```
1/105*(-21*I*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21
*I*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(15*b^3*d
+ 35*(3*a^2*b - b^3)*d*cos(f*x + e)^2 + 21*(3*a*b^2*d*cos(f*x + e) + (5*a^
3 - 6*a*b^2)*d*cos(f*x + e)^3)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(
f*x + e)^3)
```

3.595.6 Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx=\int \left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)^{3}\,dx$$

input

```
integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**3,x)
```

output

```
Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3, x)
```

3.595.7 Maxima [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx=\text{Timed out}$$

input

```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output

```
Timed out
```

3.595.8 Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx=\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)^{3}\,dx$$

input

```
integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output

```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3, x)
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx=\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(a+b\tan(e+f\,x)\right)^3\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3, x)
```

3.596 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$

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3.596.1 Optimal result

Integrand size = 25, antiderivative size = 129

$$\begin{aligned}\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx &= \frac{2a(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}}\\ &\quad+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f}\\ &\quad+\frac{2b\sqrt{d\sec(e+fx)}(2(7a^2-2b^2)+3ab\tan(e+fx))}{5f}\end{aligned}$$

output

```
2*a*(a^2-2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f
*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2
)/f/(sec(f*x+e)^2)^(1/4)+2/5*b*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2/f+2
/5*b*(d*sec(f*x+e))^(1/2)*(14*a^2-4*b^2+3*a*b*tan(f*x+e))/f
```

3.596.2 Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{2\sqrt{d\sec(e+fx)}\left(5b(-3a^2+b^2)\cos^3(e+fx)-5a(a^2-2b^2)\cos^{\frac{7}{2}}(e+fx)\,\mathrm{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-\right.}{5f(a\cos(e+fx)+b\sin(e+fx))^3}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]]*(5*b*(-3*a^2 + b^2)*Cos[e + f*x]^3 - 5*a*(a^2 - 2
*b^2)*Cos[e + f*x]^(7/2)*EllipticF[(e + f*x)/2, 2] - (b^2*Cos[e + f*x]*(2*
b + 5*a*Sin[2*(e + f*x)]))/2)*(a + b*Tan[e + f*x])^3)/(5*f*(a*Cos[e + f*x]
 + b*Sin[e + f*x])^3)
```

3.596.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3994, 497, 27, 25, 676, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sqrt{d\sec(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 497$$

$$\frac{\sqrt{d\sec(e+fx)}\Big(\frac{2}{5}b^2\int -\frac{(a+b\tan(e+fx))\left(\left(4-\frac{5a^2}{b^2}\right)b^2-9ab\tan(e+fx)\right)}{2b^2(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+\frac{2}{5}b^2\sqrt[4]{\tan^2(e+fx)+1}(a+b\,\mathrm{ta}}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d\sec(e+fx)}\Big(\frac{2}{5}b^2\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2-\frac{1}{5}\int -\frac{(a+b\tan(e+fx))(5a^2+9b\tan(e+fx)a-4b^2)}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d\sec(e+fx)}\Big(\frac{1}{5}\int \frac{(a+b\tan(e+fx))(5a^2+9b\tan(e+fx)a-4b^2)}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+\frac{2}{5}b^2\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 676$$

$$\frac{\sqrt{d\sec(e+fx)}\Big(\frac{1}{5}\Big(5a(a^2-2b^2)\int \frac{1}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+4b^2(7a^2-2b^2)\sqrt[4]{\tan^2(e+fx)+1}+6ab^3}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sqrt{d\sec(e+fx)}\Big(\frac{1}{5}\Big(10ab(a^2-2b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)+4b^2(7a^2-2b^2)\sqrt[4]{\tan^2(e+fx)+1}+6}{bf\sqrt[4]{\sec^2(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*((2*b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^
(1/4))/5 + (10*a*b*(a^2 - 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 4*
b^2*(7*a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(1/4) + 6*a*b^3*Tan[e + f*x]*(1 +
 Tan[e + f*x]^2)^(1/4))/5))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

3.596. $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$

3.596.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.596.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.31

method	result
default	$\frac{2\sqrt{d\sec(fx+e)}\left(-5i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)a^3+10i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\right.}{}$
parts	$-\frac{2ia^3(\cos(fx+e)+1)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{d\sec(fx+e)}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}}{f}-\frac{b^3\sqrt{d\sec(fx+e)}\left(20\cos(fx\right.}{}$

input
```
int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
2/5/f*(d*sec(f*x+e))^(1/2)*(-5*I*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+10
*I*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a*b^2-5*I*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^
3+10*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(csc(f*x+e)-cot(f*x+e)),I)*a*b^2+5*tan(f*x+e)*a*b^2+15*a^2*b-5*b^3+b^3
*sec(f*x+e)^2)
```

3.596.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.27

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^3-2i\,ab^2)\sqrt{d}\cos(fx+e)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+5\sqrt{2}(-}{}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/5*(5*sqrt(2)*(I*a^3 - 2*I*a*b^2)*sqrt(d)*cos(f*x + e)^2*weierstrassPInv
erse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-I*a^3 + 2*I*a*b^2
)*sqrt(d)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f
*x + e)) - 2*(5*a*b^2*cos(f*x + e)*sin(f*x + e) + b^3 + 5*(3*a^2*b - b^3)*
cos(f*x + e)^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

3.596.6 Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx=\int \sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^3\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3, x)
```

3.596.7 Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx=\int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

3.596.8 Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx=\int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^3\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

3.596.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx=\int \sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,{\left(a+b\tan\left(e+f\,x\right)\right)}^3\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3, x)
```

3.597 $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$

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3.597.1 Optimal result

Integrand size = 25, antiderivative size = 178

$$
\begin{aligned}
\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = {} & \frac{2a(a^2-6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{f\sqrt{d\sec(e+fx)}} \\
& - \frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} \\
& - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} \\
& - \frac{2b\sec^2(e+fx)\left(2(3a^2-2b^2)+3ab\tan(e+fx)\right)}{3f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

output
```
2*a*(a^2-6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f
*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4
)/f/(d*sec(f*x+e))^(1/2)-2*a*(a^2-6*b^2)*tan(f*x+e)/f/(d*sec(f*x+e))^(1/2)
-2*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/f/(d*sec(f*x+e))^(1/2)-2/3*b*sec(f*
x+e)^2*(6*a^2-4*b^2+3*a*b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/2)
```

3.597.2 Mathematica [A] (verified)

Time = 4.90 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= \frac{d\Big(6a(a^2-6b^2)\cos^{\frac{3}{2}}(e+fx)E\big(\frac{1}{2}(e+fx)\big|2\big)+b(-9a^2+5b^2+(-9a^2+3b^2)\cos(2(e+fx))+9ab\sin(2(}{3f(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(d*(6*a*(a^2 - 6*b^2)*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2, 2] + b*(-9
*a^2 + 5*b^2 + (-9*a^2 + 3*b^2)*Cos[2*(e + f*x)] + 9*a*b*Sin[2*(e + f*x)])
)*(a + b*Tan[e + f*x])^3)/(3*f*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*
Sin[e + f*x])^3)
```

3.597.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3994, 495, 27, 25, 676, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 495}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(2b^2\int \frac{(a+b\tan(e+fx))\left(\left(4-\frac{a^2}{b^2}\right)b^2-5ab\tan(e+fx)\right)}{2b^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2\left(b^2-ab\tan(e+fx)\right)}{\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\int -\frac{(a+b\tan(e+fx))\left(a^2+5b\tan(e+fx)a-4b^2\right)}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2\left(b^2-ab\tan(e+fx)\right)}{\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-\int \frac{(a+b\tan(e+fx))\left(a^2+5b\tan(e+fx)a-4b^2\right)}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{2\left(b^2-ab\tan(e+fx)\right)(a+b\tan(e+fx))^2}{\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{676}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-a\left(a^2-6b^2\right)\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{4}{3}b^2\left(3a^2-2b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}-\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-a\left(a^2-6b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))\right)-\frac{4}{3}b^2\left(3a^2-2b^2\right)\left(\right.\right.}{bf\sqrt{d\sec(e+f}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(-a\left(a^2-6b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-\frac{4}{3}b^2\left(3a^2-2b^2\right)\left(\tan^2\right.\right.}{bf\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

3.597. $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$

output
```
((Sec[e + f*x]^2)^(1/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(1 + Tan[e + f*x]^2)^(1/4) - (4*b^2*(3*a^2 - 2*b^2)*(1 + Tan[e + f*x]^
2)^(3/4))/3 - 2*a*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(3/4) - a*(a^2 - 6
*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 +
 Tan[e + f*x]^2)^(1/4))))/(b*f*Sqrt[d*Sec[e + f*x]])
```

3.597.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

3.597. $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.597.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.30 (sec) , antiderivative size = 1114, normalized size of antiderivative = 6.26

method	result	size
parts	Expression too large to display	1114
default	Expression too large to display	2626

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^3/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)*(I*EllipticE(I*(csc(f*x+e)-cot
(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos
(f*x+e)-I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-2
*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I
)*(1/(cos(f*x+e)+1))^(1/2)+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*sec(f*x+
e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I
)*(1/(cos(f*x+e)+1))^(1/2)+sin(f*x+e))-1/6*b^3/f/(cos(f*x+e)+1)/(d*sec(f*x
+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(-12*cos(f*x+e)*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)+3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+
1))-3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-12*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-4*sec(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4
*sec(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-6*a^2*b/(d*sec(f*x+e))
^(1/2)/f-6*a*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)*(2*I*EllipticE(I*(c
sc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos...
```

3.597.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx =$$

$$-\frac{3\sqrt{2}(-i\,a^3+6i\,ab^2)\sqrt{d}\cos{(fx+e)}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*sqrt(2)*(-I*a^3 + 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt
(2)*(I*a^3 - 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(9*a*b^2*cos(f*x
+ e)*sin(f*x + e) + b^3 - 3*(3*a^2*b - b^3)*cos(f*x + e)^2)*sqrt(d/cos(f*x
 + e)))/(d*f*cos(f*x + e))
```

3.597.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/sqrt(d*sec(e + f*x)), x)
```

3.597.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

3.597.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

3.597.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx=\int \frac{(a+b\tan(e+f\,x))^3}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input

```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2),x)
```

output

```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2), x)
```

3.598 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx$

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3.598.1 Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx=\frac{2a(a^2+6b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3f(d\sec(e+fx))^{3/2}}-\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}}-\frac{2b\sec^2(e+fx)\left(2(a^2-2b^2)+ab\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}}$$

output
```
2/3*a*(a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan
(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(3
/4)/f/(d*sec(f*x+e))^(3/2)-2/3*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/f/(d*se
c(f*x+e))^(3/2)-2/3*b*sec(f*x+e)^2*(2*a^2-4*b^2+a*b*tan(f*x+e))/f/(d*sec(f
*x+e))^(3/2)
```

3.598.2 Mathematica [A] (verified)

Time = 3.80 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx=\frac{\sec^2(e+fx)\left(-3a^2b+7b^3+(-3a^2b+b^3)\cos(2(e+fx))+2a(a^2+6b^2)\sqrt{\mathrm{co}}\right.}{3f(d\sec(e}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(Sec[e + f*x]^2*(-3*a^2*b + 7*b^3 + (-3*a^2*b + b^3)*Cos[2*(e + f*x)] + 2*
a*(a^2 + 6*b^2)*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^3*Sin[2*(
e + f*x)] - 3*a*b^2*Sin[2*(e + f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

3.598.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3994, 495, 27, 676, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{(a+b\tan(e+fx))^3}{\left(\tan^2(e+fx)+1\right)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 495}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2}{3}b^2\int \frac{(a+b\tan(e+fx))\left(\left(\frac{a^2}{b^2}+4\right)b^2-3ab\tan(e+fx)\right)}{2b^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2\left(b^2-ab\tan(e+fx)\right)}{3\left(\tan^2(e+fx)+1\right)^{3/4}}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{3}\int \frac{(a+b\tan(e+fx))\left(a^2-3b\tan(e+fx)a+4b^2\right)}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2\left(b^2-ab\tan(e+fx)\right)}{3\left(\tan^2(e+fx)+1\right)^{3/4}}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 676}$$

$$\sec^2(e+fx)^{3/4}\Bigg(\frac{1}{3}\Bigg(a\left(a^2+6b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-4b^2\left(a^2-2b^2\right)\sqrt[4]{\tan^2(e+fx)+1}-2ab^3\ \text{ta}$$

$$bf(d\sec(e+fx))^{3/2}$$

$$\Big\downarrow 229$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{3}\left(2ab(a^2+6b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-4b^2(a^2-2b^2)\sqrt[4]{\tan^2(e+fx)+1}-2ab\right.\right.}{bf(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(3*(1 + Tan[e + f*x]^2)^(3/4)) + (2*a*b*(a^2 + 6*b^2)*EllipticF[ArcTan
[Tan[e + f*x]]/2, 2] - 4*b^2*(a^2 - 2*b^2)*(1 + Tan[e + f*x]^2)^(1/4) - 2*
a*b^3*Tan[e + f*x]*(1 + Tan[e + f*x]^2)^(1/4))/3))/(b*f*(d*Sec[e + f*x])^(
3/2))
```

3.598.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.598.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.96 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.21

method	result
default	$-\frac{2\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^{3}+6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+}}\right.}{}$
parts	Expression too large to display

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d/f/(d*sec(f*x+e))^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+6*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-
cot(f*x+e)),I)*a*b^2+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+6*I*sec(f*x+
e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(csc(f*x+e)-cot(f*x+e)),I)*a*b^2+3*cos(f*x+e)*a^2*b-cos(f*x+e)*b^3-a^3*sin
(f*x+e)+3*a*b^2*sin(f*x+e)-3*sec(f*x+e)*b^3)
```

3.598.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,a^3-6i\,ab^2)\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e)) + sqrt(2)*(I*a^3 + 6*I*a*b^2)*sqrt(d)*weierstras
sPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(3*b^3 - (3*a^2*b - b^
3)*cos(f*x + e)^2 + (a^3 - 3*a*b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(
f*x + e)))/(d^2*f)
```

3.598.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(3/2), x)
```

3.598.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

3.598.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{(b\tan\left(fx+e\right)+a)^3}{(d\sec\left(fx+e\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

3.598.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2), x)
```

3.599 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$

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3.599.1 Optimal result

Integrand size = 25, antiderivative size = 204

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx &= \frac{6a(a^2+2b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}}\\ &- \frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}}\\ &- \frac{2(2b(a^2+2b^2)-a(3a^2+5b^2)\tan(e+fx))}{5d^2f\sqrt{d\sec(e+fx)}}\end{aligned}$$

output
```
6/5*a*(a^2+2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan
(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1
/4)/d^2/f/(d*sec(f*x+e))^(1/2)-6/5*a*(a^2+2*b^2)*tan(f*x+e)/d^2/f/(d*sec(f
*x+e))^(1/2)-2/5*cos(f*x+e)^2*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/d^2/f/(d
*sec(f*x+e))^(1/2)-2/5*(2*b*(a^2+2*b^2)-a*(3*a^2+5*b^2)*tan(f*x+e))/d^2/f/
(d*sec(f*x+e))^(1/2)
```

3.599.2 Mathematica [A] (verified)

Time = 3.94 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{d\sec(e+fx)}\left(-b(9a^2+17b^2)\cos(e+fx)-3a^2b\cos(3(e+fx))+b^3\cos(3(\right.}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-(b*(9*a^2 + 17*b^2)*Cos[e + f*x]) - 3*a^2*b*Cos[3*
(e + f*x)] + b^3*Cos[3*(e + f*x)] + 12*a*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*
EllipticE[(e + f*x)/2, 2] + a^3*Sin[e + f*x] - 3*a*b^2*Sin[e + f*x] + a^3*
Sin[3*(e + f*x)] - 3*a*b^2*Sin[3*(e + f*x)]))/(10*d^3*f)
```

3.599.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 3994, 495, 27, 675, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{495}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2}{5}b^2\int \frac{(a+b\tan(e+fx))\left(\left(\frac{3a^2}{b^2}+4\right)b^2-ab\tan(e+fx)\right)}{2b^2(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{5(\tan^2(e+fx)+1)^{5/4}}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\int \frac{(a+b\tan(e+fx))(3a^2-b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{5(\tan^2(e+fx)+1)^{5/4}}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{675}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\left(-3a(a^2+2b^2)\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\frac{4b^2(a^2+2b^2)}{\sqrt[4]{\tan^2(e+fx)+1}}+\frac{2ab(3a^2+5b^2)\,\text{ta}}{\sqrt[4]{\tan^2(e+f}}\right.\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\left(-3a(a^2+2b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))\right)-\frac{4b^2(a^2+2b}{\sqrt[4]{\tan^2(e+f}}\right.\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{5}\left(-3a(a^2+2b^2)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-\frac{4b^2(a^2+2b^2)}{\sqrt[4]{\tan^2(e+fx)}}\right.\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(5*(1 + Tan[e + f*x]^2)^(5/4)) + ((-4*b^2*(a^2 + 2*b^2))/(1 + Tan[e +
f*x]^2)^(1/4) + (2*a*b*(3*a^2 + 5*b^2)*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^
(1/4) - 3*a*(a^2 + 2*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*
b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/5))/(b*d^2*f*Sqrt[d*Sec[e + f
*x]])
```

3.599. $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$

3.599.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.599. $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$

3.599.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.00 (sec) , antiderivative size = 1289, normalized size of antiderivative = 6.32

method	result	size
parts	Expression too large to display	1289
default	Expression too large to display	1602

input

```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/5*a^3/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(3*I*cos(f*x+e)*Elliptic
E(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)-3*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*
x+e)),I)-6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot
(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+3*I*sec(f*x+e)*EllipticE(I*(csc(f*x+e
)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-3*I*sec(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e
)*cos(f*x+e)+3*sin(f*x+e))-1/10*b^3/f/(d*sec(f*x+e))^(1/2)/d^2*(5*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+
1))-5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
+1)/(cos(f*x+e)+1))-4*cos(f*x+e)^2+5*sec(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-5*sec(f*x+e)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/...
```

3.599.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}(-i\,a^3-2i\,ab^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(3*sqrt(2)*(-I*a^3 - 2*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(I*a^3 +
2*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) - I*sin(f*x + e))) + 2*(5*b^3*cos(f*x + e) + (3*a^2*b - b^3)*cos(f
*x + e)^3 - (a^3 - 3*a*b^2)*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x +
e)))/(d^3*f)
```

3.599.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan\left(e+fx\right))^3}{(d\sec\left(e+fx\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(5/2), x)
```

3.599.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

3.599.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

3.599.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+fx))^3}{\left(\frac{d}{\cos(e+fx)}\right)^{5/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2), x)
```

3.600 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx$

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3.600.9 Mupad [F(-1)] 4160

3.600.1 Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx=\frac{2a(5a^2+6b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}}-\frac{2(2b(3a^2+2b^2)-a(5a^2+3b^2)\tan(e+fx))}{21d^2f(d\sec(e+fx))^{3/2}}$$

output
```
2/21*a*(5*a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(
tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)
^(3/4)/d^2/f/(d*sec(f*x+e))^(3/2)-2/7*cos(f*x+e)^2*(b-a*tan(f*x+e))*(a+b*t
an(f*x+e))^2/d^2/f/(d*sec(f*x+e))^(3/2)-2/21*(2*b*(3*a^2+2*b^2)-a*(5*a^2+3
*b^2)*tan(f*x+e))/d^2/f/(d*sec(f*x+e))^(3/2)
```

3.600.2 Mathematica [A] (verified)

Time = 5.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx=\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\Big(4(5a^3+6ab^2)\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\sqrt{\cos}}{}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]*(4*(5*a^3 + 6*a*b^2)*EllipticF[(e
 + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(-(b*(27*a^2 + 19*b^2)*Cos[e + f*x]) +
(-9*a^2*b + 3*b^3)*Cos[3*(e + f*x)] + 2*a*(13*a^2 + 3*b^2 + 3*(a^2 - 3*b^2
)*Cos[2*(e + f*x)])*Sin[e + f*x])))/(42*d^4*f)
```

3.600.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3994, 495, 27, 675, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{(a+b\tan(e+fx))^3}{\left(\tan^2(e+fx)+1\right)^{11/4}}d(b\tan(e+fx))}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 495}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2}{7}b^2\int \frac{(a+b\tan(e+fx))\left(\left(\frac{5a^2}{b^2}+4\right)b^2+a\tan(e+fx)b\right)}{2b^2\left(\tan^2(e+fx)+1\right)^{7/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2\left(b^2-ab\tan(e+fx)\right)}{7\left(\tan^2(e+fx)+1\right)^{7/4}}\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{7}\int \frac{(a+b\tan(e+fx))\left(5a^2+b\tan(e+fx)a+4b^2\right)}{\left(\tan^2(e+fx)+1\right)^{7/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2\left(b^2-ab\tan(e+fx)\right)}{7\left(\tan^2(e+fx)+1\right)^{7/4}}\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 675}$$

$$\frac{\sec^2(e+fx)^{3/4}\Bigg(\frac{1}{7}\Bigg(\frac{1}{3}a\left(5a^2+6b^2\right)\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\frac{4b^2\left(3a^2+2b^2\right)}{3\left(\tan^2(e+fx)+1\right)^{3/4}}+\frac{2ab\left(5a^2+3b^2\right)\tan(e+fx)}{3\left(\tan^2(e+fx)+1\right)^{3/4}}}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 229$$

$$\frac{\sec^2(e+fx)^{3/4}\Bigg(\frac{1}{7}\Bigg(\frac{2}{3}ab\left(5a^2+6b^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\frac{4b^2\left(3a^2+2b^2\right)}{3\left(\tan^2(e+fx)+1\right)^{3/4}}+\frac{2ab\left(5a^2+3b^2\right)\tan(e+}{3\left(\tan^2(e+fx)+1\right)^{3/4}}}{bd^2f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(7*(1 + Tan[e + f*x]^2)^(7/4)) + ((2*a*b*(5*a^2 + 6*b^2)*EllipticF[Arc
Tan[Tan[e + f*x]]/2, 2])/3 - (4*b^2*(3*a^2 + 2*b^2))/(3*(1 + Tan[e + f*x]^
2)^(3/4)) + (2*a*b*(5*a^2 + 3*b^2)*Tan[e + f*x])/(3*(1 + Tan[e + f*x]^2)^(
3/4)))/7))/(b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

3.600.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.600.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.65 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.15

method	result
default	$\frac{10i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^3}{21}+\frac{4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a\,b^2}{7}-\frac{6(\mathrm{co}$
parts	$-\frac{2a^3\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\right.}{21f\sqrt{d\sec(fx+e)}\,d^3}$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/d^3/f/(d*sec(f*x+e))^(1/2)*(5*I*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^3+6*I*(1/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*a*b^2-9*cos(f*x+e)^3*a^2*b+3*cos(f*x+e)^3*b^3+3*cos(f*x
+e)^2*sin(f*x+e)*a^3-9*cos(f*x+e)^2*sin(f*x+e)*a*b^2+5*I*sec(f*x+e)*(1/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*a^3+6*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-7*cos(f
*x+e)*b^3+5*a^3*sin(f*x+e)+6*a*b^2*sin(f*x+e))
```

3.600.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}(-5i\,a^3-6i\,ab^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e}}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(sqrt(2)*(-5*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, co
s(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^3 + 6*I*a*b^2)*sqrt(d)*weier
strassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(7*b^3*cos(f*x +
e)^2 + 3*(3*a^2*b - b^3)*cos(f*x + e)^4 - (3*(a^3 - 3*a*b^2)*cos(f*x + e)^
3 + (5*a^3 + 6*a*b^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d
^4*f)
```

3.600.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{(d\sec{(e+fx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(7/2),x)
```

output
```
Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(7/2), x)
```

3.600.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

3.600.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

3.600.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2), x)
```

3.601 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx$

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3.601.1 Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2a(7a^2+6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)\left(2b(5a^2+2b^2)-a(7a^2+b^2)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}}$$

output
```
2/15*a*(7*a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(
tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)
^(1/4)/d^4/f/(d*sec(f*x+e))^(1/2)-2/9*cos(f*x+e)^4*(b-a*tan(f*x+e))*(a+b*t
an(f*x+e))^2/d^4/f/(d*sec(f*x+e))^(1/2)-2/45*cos(f*x+e)^2*(2*b*(5*a^2+2*b^
2)-a*(7*a^2+b^2)*tan(f*x+e))/d^4/f/(d*sec(f*x+e))^(1/2)
```

3.601.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 372 vs. 2(176) = 352.

Time = 9.41 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.11

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\sec^{\frac{3}{2}}(e+fx)\left(\frac{2(56a^3+48ab^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}\sqrt{\sec(e+fx)}} - \frac{2(15a^2b+7b^3)\sin^2(e+fx)}{\sqrt{1-\cos^2(e+fx)}\sqrt{\sec(e+fx)}\sqrt{\cos^2(e+fx)(-1+s}}\right.}{120f(d\sec(e+fx))^{9/2}(a\cos(e+fx)+b\sin(e+f}$$

$$+\frac{\sec^2(e+fx)\left(-\frac{1}{90}b(15a^2+4b^2)\cos(e+fx)-\frac{1}{360}b(75a^2+11b^2)\cos(3(e+fx))-\frac{1}{72}b(3a^2-b^2)\cos(5(e+\right.}{f(d\sec(e+fx)}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(Sec[e + f*x]^(3/2)*((2*(56*a^3 + 48*a*b^2)*EllipticE[(e + f*x)/2, 2])/(Sq
rt[Cos[e + f*x]]*Sqrt[Sec[e + f*x]]) - (2*(15*a^2*b + 7*b^3)*Sin[e + f*x]^
2)/(Sqrt[1 - Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[e + f*x]^2*(-1 +
Sec[e + f*x]^2)]))*(a + b*Tan[e + f*x])^3)/(120*f*(d*Sec[e + f*x])^(9/2)*(
a*Cos[e + f*x] + b*Sin[e + f*x])^3) + (Sec[e + f*x]^2*(-1/90*(b*(15*a^2 +
4*b^2)*Cos[e + f*x]) - (b*(75*a^2 + 11*b^2)*Cos[3*(e + f*x)])/360 - (b*(3*
a^2 - b^2)*Cos[5*(e + f*x)])/72 + (a*(19*a^2 - 3*b^2)*Sin[e + f*x])/180 +
(a*(43*a^2 - 21*b^2)*Sin[3*(e + f*x)])/360 + (a*(a^2 - 3*b^2)*Sin[5*(e + f
*x)])/72)*(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(9/2)*(a*Cos[e + f*x
] + b*Sin[e + f*x])^3)
```

3.601.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3994, 495, 27, 675, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{(a+b\tan(e+fx))^3}{\left(\tan^2(e+fx)+1\right)^{13/4}}d(b\tan(e+fx))}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{495}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2}{9}b^2\int \frac{(a+b\tan(e+fx))\left(\left(\frac{7a^2}{b^2}+4\right)b^2+3a\tan(e+fx)b\right)}{2b^2\left(\tan^2(e+fx)+1\right)^{9/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2\left(b^2-ab\tan(e+fx)\right)}{9\left(\tan^2(e+fx)+1\right)^{9/4}}\right)}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{9}\int \frac{(a+b\tan(e+fx))(7a^2+3b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx)) - \frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{9(\tan^2(e+fx)+1)^{9/4}}\right)}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{675}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{9}\left(\frac{3}{5}a(7a^2+6b^2)\int \frac{1}{(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx)) - \frac{4b^2(5a^2+2b^2)}{5(\tan^2(e+fx)+1)^{5/4}} + \frac{2ab(7a^2+b^2)\tan(e+fx)}{5(\tan^2(e+fx)+1)^{5/4}}\right)\right.}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{1}{9}\left(\frac{6}{5}ab(7a^2+6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right) - \frac{4b^2(5a^2+2b^2)}{5(\tan^2(e+fx)+1)^{5/4}} + \frac{2ab(7a^2+b^2)\tan(e+fx)}{5(\tan^2(e+fx)+1)^{5/4}}\right) - 2(\right.}{bd^4f\sqrt{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((Sec[e + f*x]^2)^(1/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(9*(1 + Tan[e + f*x]^2)^(9/4)) + ((6*a*b*(7*a^2 + 6*b^2)*EllipticE[Arc
Tan[Tan[e + f*x]]/2, 2])/5 - (4*b^2*(5*a^2 + 2*b^2))/(5*(1 + Tan[e + f*x]^
2)^(5/4)) + (2*a*b*(7*a^2 + b^2)*Tan[e + f*x])/(5*(1 + Tan[e + f*x]^2)^(5/
4)))/9))/(b*d^4*f*Sqrt[d*Sec[e + f*x]])
```

3.601.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

3.601. $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx$

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.601.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 32.98 (sec) , antiderivative size = 976, normalized size of antiderivative = 5.55

method	result	size
parts	Expression too large to display	976
default	Expression too large to display	1035

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*a^3/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(21*I*cos(f*x+e)*Ellipt
icE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-21*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^4*s
in(f*x+e)+42*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-42*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+5*cos(
f*x+e)^3*sin(f*x+e)+21*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-21*I*sec(f*x+e
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)
*(1/(cos(f*x+e)+1))^(1/2)+7*sin(f*x+e)*cos(f*x+e)^2+7*sin(f*x+e)*cos(f*x+e
)+21*sin(f*x+e))+2/45*b^3/f/(d*sec(f*x+e))^(1/2)/d^4*(5*cos(f*x+e)^4-9*cos
(f*x+e)^2)+2/15*a*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(6*I*cos(f
*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f
*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5*cos
(f*x+e)^4*sin(f*x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-12*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)
),I)-5*cos(f*x+e)^3*sin(f*x+e)+6*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*...
```

3.601.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx =$$

$$-\frac{3\sqrt{2}(-7i\,a^3-6i\,ab^2)\sqrt{d}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-1/45*(3*sqrt(2)*(-7*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a
^3 + 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(f*x + e) - I*sin(f*x + e))) + 2*(9*b^3*cos(f*x + e)^3 + 5*(3*a^2*b - b
^3)*cos(f*x + e)^5 - (5*(a^3 - 3*a*b^2)*cos(f*x + e)^4 + (7*a^3 + 6*a*b^2)
*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^5*f)
```

3.601.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.601.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

3.601.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

3.601.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{tan}\left(e+f\,x\right)\right)}^3}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{9/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2), x)
```

3.602 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$

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3.602.1 Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{10a(3a^2+2b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\left(2b(7a^2+2b^2)-a(9a^2-b^2)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}$$

output

```
10/77*a*(3*a^2+2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan
(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2
)^(3/4)/d^4/f/(d*sec(f*x+e))^(3/2)+10/77*a*(3*a^2+2*b^2)*tan(f*x+e)/d^4/f/
(d*sec(f*x+e))^(3/2)-2/11*cos(f*x+e)^4*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2
/d^4/f/(d*sec(f*x+e))^(3/2)-2/77*cos(f*x+e)^2*(2*b*(7*a^2+2*b^2)-a*(9*a^2-
b^2)*tan(f*x+e))/d^4/f/(d*sec(f*x+e))^(3/2)
```

3.602.2 Mathematica [A] (verified)

Time = 10.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{10a(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)(a+b\tan(e+fx))^3}{77f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{11/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

$$+\frac{\sec^3(e+fx)\left(-\frac{1}{616}b(105a^2+31b^2)-\frac{b(315a^2+71b^2)\cos(2(e+fx))}{1232}-\frac{1}{616}b(63a^2+b^2)\cos(4(e+fx))-\frac{1}{176}b(3a^2\right.}{f(d\sec(e+}$$

input
```
Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

output
```
(10*a*(3*a^2 + 2*b^2)*EllipticF[(e + f*x)/2, 2]*(a + b*Tan[e + f*x])^3)/(7
7*f*Cos[e + f*x]^(5/2)*(d*Sec[e + f*x])^(11/2)*(a*Cos[e + f*x] + b*Sin[e +
 f*x])^3) + (Sec[e + f*x]^3*(-1/616*(b*(105*a^2 + 31*b^2)) - (b*(315*a^2 +
 71*b^2)*Cos[2*(e + f*x)])/1232 - (b*(63*a^2 + b^2)*Cos[4*(e + f*x)])/616
- (b*(3*a^2 - b^2)*Cos[6*(e + f*x)])/176 + (a*(347*a^2 + 103*b^2)*Sin[2*(e
 + f*x)])/1232 + (a*(16*a^2 - 15*b^2)*Sin[4*(e + f*x)])/308 + (a*(a^2 - 3*
b^2)*Sin[6*(e + f*x)])/176)*(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(1
1/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

3.602.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3994, 495, 27, 675, 215, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{(a+b\tan(e+fx))^3}{(\tan^2(e+fx)+1)^{15/4}}d(b\tan(e+fx))}{bd^4f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 495$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2}{11}b^2\int\frac{(a+b\tan(e+fx))\left(\left(\frac{9a^2}{b^2}+4\right)b^2+5a\tan(e+fx)b\right)}{2b^2(\tan^2(e+fx)+1)^{11/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{11(\tan^2(e+fx)+1)^{11/4}}\right)}{bd^4f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\int\frac{(a+b\tan(e+fx))(9a^2+5b\tan(e+fx)a+4b^2)}{(\tan^2(e+fx)+1)^{11/4}}d(b\tan(e+fx))-\frac{2(a+b\tan(e+fx))^2(b^2-ab\tan(e+fx))}{11(\tan^2(e+fx)+1)^{11/4}}\right)}{bd^4f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 675$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\left(\frac{15}{7}a(3a^2+2b^2)\int\frac{1}{(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))-\frac{4b^2(7a^2+2b^2)}{7(\tan^2(e+fx)+1)^{7/4}}+\frac{2ab(9a^2-b^2)\tan(e+fx}{7(\tan^2(e+fx)+1)^{7/4}}\right.\right.}{bd^4f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 215$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\left(\frac{15}{7}a(3a^2+2b^2)\left(\frac{1}{3}\int\frac{1}{(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+\frac{2b\tan(e+fx)}{3(\tan^2(e+fx)+1)^{3/4}}\right)-\frac{4b^2(7a^2+2b^2)}{7(\tan^2(e+fx)+1}\right.\right.}{bd^4f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 229$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{1}{11}\left(\frac{15}{7}a(3a^2+2b^2)\left(\frac{2}{3}b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)+\frac{2b\tan(e+fx)}{3(\tan^2(e+fx)+1)^{3/4}}\right)-\frac{4b^2(7a^2+2}{7(\tan^2(e+fx)}\right.\right.}{bd^4f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((-2*(a + b*Tan[e + f*x])^2*(b^2 - a*b*Tan[e + f*x
]))/(11*(1 + Tan[e + f*x]^2)^(11/4)) + ((-4*b^2*(7*a^2 + 2*b^2))/(7*(1 + T
an[e + f*x]^2)^(7/4)) + (2*a*b*(9*a^2 - b^2)*Tan[e + f*x])/(7*(1 + Tan[e +
 f*x]^2)^(7/4)) + (15*a*(3*a^2 + 2*b^2)*((2*b*EllipticF[ArcTan[Tan[e + f*x
]]/2, 2])/3 + (2*b*Tan[e + f*x])/(3*(1 + Tan[e + f*x]^2)^(3/4))))/7)/11))/
(b*d^4*f*(d*Sec[e + f*x])^(3/2))
```

3.602. $\int\frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$

3.602.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 675
```
Int[((d_) + (e_.)*(x_))*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[a*(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + (-
Simp[(c*d*f - a*e*g)*x*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] - Simp[(a*
e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1))   Int[(a + c*x^2)^(p + 1), x], x]) /
; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1] &&  !(IntegerQ[p] && NiceSqrtQ
[(-a)*c])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.602. $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$

3.602.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 39.40 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.87

method	result
default	$\frac{-\frac{6a^2b\left(\cos^5(fx+e)\right)}{11}+\frac{2b^3\left(\cos^5(fx+e)\right)}{11}+\frac{2a^3\left(\cos^4(fx+e)\right)\sin(fx+e)}{11}-\frac{6a\,b^2\left(\cos^4(fx+e)\right)\sin(fx+e)}{11}+\frac{30i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc}{77}}{}$
parts	$-\frac{2a^3\left(-7(\cos^4(fx+e))\sin(fx+e)+15i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+15i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\right.}{77f\sqrt{d\sec(fx+e)}\,d^5}$

input
```
int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/77/d^5/f/(d*sec(f*x+e))^(1/2)*(-21*a^2*b*cos(f*x+e)^5+7*b^3*cos(f*x+e)^5
+7*a^3*cos(f*x+e)^4*sin(f*x+e)-21*a*b^2*cos(f*x+e)^4*sin(f*x+e)+15*I*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x
+e)-csc(f*x+e)),I)*a^3+10*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*a*b^2-11*cos(f*x+e)^3*
b^3+9*cos(f*x+e)^2*sin(f*x+e)*a^3+6*cos(f*x+e)^2*sin(f*x+e)*a*b^2+15*I*sec
(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(cot(f*x+e)-csc(f*x+e)),I)*a^3+10*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I
)*a*b^2+15*a^3*sin(f*x+e)+10*a*b^2*sin(f*x+e))
```

3.602.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx =$$

$$-\frac{5\sqrt{2}(3i\,a^3+2i\,ab^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+5\sqrt{2}(-3i\,a^3-2i\,ab^2)\sqrt{}}{}$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="fricas")
```

output
```
-1/77*(5*sqrt(2)*(3*I*a^3 + 2*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0,
cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-3*I*a^3 - 2*I*a*b^2)*sqrt(d)*
weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(11*b^3*cos(
f*x + e)^4 + 7*(3*a^2*b - b^3)*cos(f*x + e)^6 - (7*(a^3 - 3*a*b^2)*cos(f*x
 + e)^5 + 3*(3*a^3 + 2*a*b^2)*cos(f*x + e)^3 + 5*(3*a^3 + 2*a*b^2)*cos(f*x
 + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^6*f)
```

3.602.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(11/2),x)
```

output
```
Timed out
```

3.602.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

3.602.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

3.602.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx=\int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{11/2}}\,dx$$

input
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2),x)
```

output
```
int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2), x)
```

3.603 $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

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3.603.2 Mathematica [B] (warning: unable to verify) 4176
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3.603.8 Giac [F] 4185
3.603.9 Mupad [F(-1)] 4185

3.603.1 Optimal result

Integrand size = 25, antiderivative size = 456

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx=\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}$$

$$+\frac{\left(a^2+b^2\right)^{3/4}d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$

$$-\frac{\left(a^2+b^2\right)^{3/4}d^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$

$$+\frac{2ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$

$$-\frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$

$$-\frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{b^3f\sec^2(e+fx)^{3/4}}$$

$$+\frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+}}{b^3f\sec^2(e+fx)^{3/4}}$$

output

```
2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f+(a^2+b^2)^(3/4)*d^2*arctan((sec(f*x+e)^2)
^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)
^2)^(3/4)-(a^2+b^2)^(3/4)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^
2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)^2)^(3/4)+2*a*d^2*(cos
(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(si
n(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec(f*x+e)^
2)^(3/4)-2*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f-a*d^2*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+
e))^(3/2)*(a^2+b^2)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)
+a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*
sec(f*x+e))^(3/2)*(a^2+b^2)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^
2)^(3/4)
```

3.603.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 11117 vs. 2(456) = 912.

Time = 90.48 (sec) , antiderivative size = 11117, normalized size of antiderivative = 24.38

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```

output

```
Result too large to show
```

3.603.3 Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.71, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3994, 493, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\int \frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 493$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\int \frac{b^2-ab\tan(e+fx)}{b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\frac{2}{3}\left(\tan^2(e+fx)+1\right)^{3/4}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{\int \frac{b^2-ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{b^2}+\frac{2}{3}\left(\tan^2(e+fx)+1\right)^{3/4}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 719$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{\left(a^2+b^2\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{b^2}+\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 225$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{\left(a^2+b^2\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{\left(\tan^2(e+fx)+1\right.}\right.}{b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \; 212$$

3.603. $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}dx$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\right.\right.}{b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 504$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(\right.}\right.}{b^2}\right.}{bf\sec^2(e+fx)^{3/}}$$

$$\Big\downarrow 310$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left((a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\frac{}{\sqrt[4]{\mathrm{t}}}\right.\right.}{bf\,\mathrm{s}}$$

$$\Big\downarrow 353$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left((a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\frac{}{\sqrt[4]{}}\right.\right.}{bf}$$

$$\Big\downarrow 73$$

3.603. $\int\frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int\right.}{b}\right.}{bf\sec}$$

$$\Big\downarrow \color{blue}{827}$$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\right.\right.}{}\right.$$

$$\Big\downarrow \color{blue}{218}$$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\right.\right.}{}\right.$$

$$\Big\downarrow \color{blue}{221}$$

3.603. $\int\frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

$$d^2(d\sec(e+fx))^{3/2}\Bigg(\frac{(a^2+b^2)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\Bigg(a}{}$$

$$bf\,\mathrm{s}$$

$$\Big\downarrow \text{993}$$

$$d^2(d\sec(e+fx))^{3/2}\Bigg(\frac{(a^2+b^2)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b}{b}}{}$$

$$\Big\downarrow \text{1537}$$

$$d^2(d\sec(e+fx))^{3/2}\Bigg(\frac{(a^2+b^2)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e}{}}{}$$

$$\Big\downarrow \text{412}$$

3.603. $\int\frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\mathrm{EllipticPi}\left(-\frac{}{\sqrt{a}}\right.}{}\right.}{b}\right.}{}\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(d^2*(d*Sec[e + f*x])^(3/2)*((2*(1 + Tan[e + f*x]^2)^(3/4))/3 + ((a^2 + b^
2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)
*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2
*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/S
qrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2]
+ (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1]
)/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b) - a*(-2*b*EllipticE[ArcTa
n[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/b^
2))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

3.603.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

3.603. $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.603. $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

3.603.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31612 vs. 2(421) = 842.

Time = 33.06 (sec) , antiderivative size = 31613, normalized size of antiderivative = 69.33

method	result	size
default	Expression too large to display	31613

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.603.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.603.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e)),x)
```

output
```
Timed out
```

3.603.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{7}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

3.603.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{7}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

3.603.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx=\int \frac{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{7/2}}{a+b\,\tan{(e+f\,x)}}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)), x)
```

3.604 $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

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3.604.6 Sympy [F] 4198
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3.604.8 Giac [F] 4198
3.604.9 Mupad [F(-1)] 4199

3.604.1 Optimal result

Integrand size = 25, antiderivative size = 396

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx &= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} \\
&- \frac{\sqrt[4]{a^2+b^2}\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}} \\
&- \frac{\sqrt[4]{a^2+b^2}\,d^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}} \\
&- \frac{2ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} \\
&+ \frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} \\
&+ \frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}
\end{aligned}
$$

output
```
2*d^2*(d*sec(f*x+e))^(1/2)/b/f-(a^2+b^2)^(1/4)*d^2*arctan((sec(f*x+e)^2)^(
1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2
)^(1/4)-(a^2+b^2)^(1/4)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)
^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2)^(1/4)-2*a*d^2*(cos(1
/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(
1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(f*x+e)^2)
^(1/4)+a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2)
,I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)+
a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*s
ec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)
```

3.604.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 27.35 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.73

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\left(-\sqrt{b}\sqrt[4]{a^2+b^2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\tan(e\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(d^2*Cot[e + f*x]*Sqrt[d*Sec[e + f*x]]*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*ArcTan
[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Tan[e + f*x]) - Sqrt[
b]*(a^2 + b^2)^(1/4)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^
(1/4)]*Tan[e + f*x] + 2*b*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x] - a*Hypergeo
metric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2 + a*EllipticPi[-(
b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]
^2] + a*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
Sqrt[-Tan[e + f*x]^2]))/(b^2*f*(Sec[e + f*x]^2)^(1/4))
```

3.604.3 Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.76, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3994, 493, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int \frac{\sqrt[4]{\tan^2(e+fx)+1}}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{493}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\int \frac{b^2-ab\tan(e+fx)}{b^2(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))+2\sqrt[4]{\tan^2(e+fx)+1}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{b^2-ab\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{b^2}+2\sqrt[4]{\tan^2(e+fx)+1}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{719}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{b^2}+2\sqrt[4]{\tan^2(e+}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{229}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{b^2}+2\sqrt[4]{\tan^2(e+}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{312}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{}{\left(\tan^2(e+f\right.}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{118}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{}{\sqrt{1-b^4\tan^4(e+fx)}\left(-\right.}}{b}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

3.604. $\int\frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{}{\left(\tan^2(e+\right.}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 353$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{}{\left(\frac{\tan(e}{}\right.}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx}}$$

$$\Big\downarrow \; 73$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{}{-\tan}\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 756$$

3.604. $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left((a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int\frac{}{\sqrt{a}}\right.\right.\right.}{}$$

$$\Big\downarrow 218$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int\frac{}{\sqrt{a}}\right.\right.}{b^2}\right.}{bf\sqrt[4]{\sec}}$$

$$\Big\downarrow 221$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan}{2}\right.\right.}{b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx}}$$

$$\Big\downarrow 925$$

3.604. $\int\frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int}{}\right.}{b}\right.}{}\right.}{}$$

$$\Big\downarrow 1537$$

3.604. $\int\frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\left(a^2+b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+}}}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \text{412}$$

3.604. $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{(a^2+b^2)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\operatorname{EllipticPi}\left(\right.}{}\right.}{b}\right.}{}\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(2*(1 + Tan[e + f*x]^2)^(1/4) + (-2*a*b*Elliptic
F[ArcTan[Tan[e + f*x]]/2, 2] + (a^2 + b^2)*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e
+ f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2
)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*C
ot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e
+ f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], Ar
cSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]
^2])/b))/b^2))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

3.604.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.604. $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.604. $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.604.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7346 vs. 2(369) = 738.

Time = 30.43 (sec) , antiderivative size = 7347, normalized size of antiderivative = 18.55

method	result	size
default	Expression too large to display	7347

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.604.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*d^2*sec(f*x + e)^2/(b*tan(f*x + e) + a), x)
```

3.604.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(e+fx)})^{\frac{5}{2}}}{a+b\tan{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x)), x)
```

3.604.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{5}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

3.604.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{5}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

3.604.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx=\int \frac{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}}{a+b\,\tan\left(e+f\,x\right)}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)), x)
```

3.605 $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

3.605.1 Optimal result 4200
3.605.2 Mathematica [C] (warning: unable to verify) 4201
3.605.3 Rubi [A] (warning: unable to verify) 4201
3.605.4 Maple [B] (warning: unable to verify) 4206
3.605.5 Fricas [F(-2)] 4207
3.605.6 Sympy [F] 4207
3.605.7 Maxima [F] 4207
3.605.8 Giac [F] 4208
3.605.9 Mupad [F(-1)] 4208

3.605.1 Optimal result

Integrand size = 25, antiderivative size = 334

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx &= \frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}}\\
&- \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}}\\
&- \frac{a\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}\\
&+ \frac{a\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}\end{aligned}$$

output
```
arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/
(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-arctanh((sec(f*x+e)^2)^(1/4
)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(1/4)/f/(sec(f*x
+e)^2)^(3/4)/b^(1/2)-a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+
b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/f/(sec(f*x+e)^2
)^(3/4)/(a^2+b^2)^(1/2)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^
2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/f/(sec(f*x+e)
^2)^(3/4)/(a^2+b^2)^(1/2)
```

3.605.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 3.43 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.83

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx =$$

$$-\frac{12d^2\,\text{AppellF1}\Big(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{a-ib}{a+b\tan(e+fx)},}{bf\sqrt{d\sec(e+fx)}\,\Big((a+ib)\,\text{AppellF1}\Big(\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{5}{2},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\Big)+(a-ib)\,\text{AppellF1}\Big(\frac{3}{2},\frac{5}{4},\frac{1}{4},\frac{5}{2}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

output
```
(-12*d^2*AppellF1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*Sqrt[d*Sec[e + f*x]
]*((a + I*b)*AppellF1[3/2, 1/4, 5/4, 5/2, (a - I*b)/(a + b*Tan[e + f*x]),
(a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[3/2, 5/4, 1/4, 5/2, (
a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 6*AppellF
1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan
[e + f*x])]*(a + b*Tan[e + f*x])))
```

3.605.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.74, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3994, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{(d\sec(e+fx))^{3/2}\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 504$$

$$\frac{(d\sec(e+fx))^{3/2}\left(a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 310$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int \frac{}{\sqrt[4]{\tan^2(e}}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 353$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int \frac{}{\sqrt[4]{\tan(}}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 73$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int \frac{\sqrt[4]{\text{ta}}}{-\tan^4}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 827$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int \frac{}{\sqrt{a^2}}\right.\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow 218$$

3.605. $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2}}}{}\right.\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 221$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\text{arcta}}{2}\right.\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 993$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b\int\frac{}{\left(\tan^2(\right.}\right.}{b}\right.}{bf\sec^2(e+}$$

$$\Big\downarrow \ 1537$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e+fx)}\right.}{b}\right.}{}$$

$$\Big\downarrow \ 412$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\text{arcs}\right.}{}\right.}{b}\right.}{bf\sec^2(e+fx)^{3/4}}$$

3.605. $\int\frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 +
 b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/
(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/
2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)],
-1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e
+ f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b))/(b*
f*(Sec[e + f*x]^2)^(3/4))
```

3.605.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.605. $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.605. $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

3.605.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3633 vs. $2(282) = 564$.

Time = 9.43 (sec) , antiderivative size = 3634, normalized size of antiderivative = 10.88

method	result	size
default	Expression too large to display	3634

input

```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
-1/2*I*d/f*(cos(f*x+e)+1)*(-I*(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(
a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*ln(2)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*a^2-I*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2
*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x+e)+
b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^
3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b^3-I*(b*((a^2
+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/
2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^
(1/2)+b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b
^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^4*b-I*(a^2+b^2)^(1/2)*(b*((a^2+b^2)^(1/2)*
a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e
)*(a^2+b^2)^(1/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(
cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^
2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*b^4+I*(a^2+b^2)^(3/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2
)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1
/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(cos(f*x+e)+...
```

3.605.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.605.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{a+b\tan{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x)), x)
```

3.605.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

3.605.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

3.605.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{a+b\tan{(e+f\,x)}}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)), x)
```

3.606 $\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

3.606.1 Optimal result 4209
3.606.2 Mathematica [A] (verified) 4210
3.606.3 Rubi [A] (warning: unable to verify) 4210
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3.606.1 Optimal result

Integrand size = 25, antiderivative size = 324

$$\begin{aligned}\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx &= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} \\ &\quad - \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} \\ &\quad + \frac{a\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} \\ &\quad + \frac{a\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}\end{aligned}$$

output
```
-arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e
))^(1/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)-arctanh((sec(f*x+e)^2)^(1/
4)*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(3/4)/f
/(sec(f*x+e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2
+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec
(f*x+e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^
(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec(f*x+e
)^2)^(1/4)
```

3.606.2 Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{d\sec(e+fx)}\left(-\sqrt{b}\sqrt[4]{a^2+b^2}\left(\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\right)+a\cot(e}{}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e
 + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(
1/4))/(a^2 + b^2)^(1/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]
), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f
*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt
[-Tan[e + f*x]^2]))/((a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4))
```

3.606.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.77, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules used = {3042, 3994, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}dx$$
$$\Big\downarrow \text{ 3994}$$
$$\frac{\sqrt{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$
$$\Big\downarrow \text{ 504}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(a\int\frac{1}{(\tan^2(e+fx)+1)^{3/4}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{3/4}(a^2-b^2\tan^2(e+fx))}d(b\tan}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 312$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{b}{(\tan^2(e+fx)+}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 118$$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\int\frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{3/4}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{}{\sqrt{1-b^4\tan^4(e+fx}}}{b}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{b}{(\tan^2(e+fx)+1}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 353$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{}{\left(\frac{\tan(e+fx)}{b}+\right.}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 73$$

3.606. $\int\frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{}{-\tan^4(e+}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int\frac{}{\sqrt{a^2+b^2-}}\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int\frac{}{\sqrt{a^2+b^2-}}\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\frac{b^3}{\sqrt[4]{}}\right.}{2\sqrt{b}(a}\right.\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{925}$$

3.606. $\int\frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\frac{}{\left(\frac{\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right.}}{}\right.}{b}\right.}{bf\sqrt[4]{\sec^2(e+fx}}$$

$\Big\downarrow$ 1537

$$\sqrt{d\sec(e+fx)}\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}+1}+1}}d\sqrt[4]{}}{2\left(a^2+b^2\right)}\right.}{}\right.$$

$\Big\downarrow$ 412

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\text{a}\right.}{}\right.}{b}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

3.606. $\int\frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

input

```
Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

output

```
(Sqrt[d*Sec[e + f*x]]*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(
1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2
+ b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^
2*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1]
)/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x
]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(b*f*(Sec[e
+ f*x]^2)^(1/4))
```

3.606.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 312

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

3.606. $\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.606. $\int \frac{\sqrt{d \sec(e+fx)}}{a+b \tan(e+fx)}\,dx$

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.606.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3037 vs. 2(276) = 552.

Time = 8.02 (sec) , antiderivative size = 3038, normalized size of antiderivative = 9.38

method	result	size
default	Expression too large to display	3038

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(cos(f*x+e)+1)*(4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1/(b+(a^2+b^2)^(1/
2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/
a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a
^4)^(1/2)*(a^2+b^2)^(1/2)-4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1/(-b+(a^2+b^2)
^(1/2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b
^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^
3)/a^4)^(1/2)*(a^2+b^2)^(1/2)-4*I*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/
2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(b*((a^2+b^2)^(1/
2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^5-4*I*(-b*((a^2+b
^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc
(f*x+e)),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a
^4)^(1/2)*a^3*b^2-(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2
*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2)*(b*((a^2+b^2)^
(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^4*b+(-b*((a^2+
b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1...
```

3.606.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.606.6 Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{a+b\tan{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x)), x)
```

3.606.7 Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

3.606.8 Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{\sqrt{d\sec(fx+e)}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

3.606.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{a+b\tan(e+f\,x)}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)), x)
```

3.607 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

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3.607.1 Optimal result

Integrand size = 25, antiderivative size = 451

$$
\begin{aligned}
\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx &= \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^{5/4}f\sqrt{d\sec(e+fx)}} \\
&- \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^{5/4}f\sqrt{d\sec(e+fx)}} \\
&+ \frac{2aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{\left(a^2+b^2\right)f\sqrt{d\sec(e+fx)}} \\
&- \frac{ab\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{\left(a^2+b^2\right)^{3/2}f\sqrt{d\sec(e+fx)}} \\
&+ \frac{ab\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{\left(a^2+b^2\right)^{3/2}f\sqrt{d\sec(e+fx)}} \\
&+ \frac{2(b+a\tan(e+fx))}{\left(a^2+b^2\right)f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

output
```
b^(3/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2
)^(1/4)/(a^2+b^2)^(5/4)/f/(d*sec(f*x+e))^(1/2)-b^(3/2)*arctanh((sec(f*x+e)
^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(5/4)/f/
(d*sec(f*x+e))^(1/2)+2*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arc
tan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e
)^2)^(1/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)-a*b*cot(f*x+e)*EllipticPi((sec
(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2
)^(1/2)/(a^2+b^2)^(3/2)/f/(d*sec(f*x+e))^(1/2)+a*b*cot(f*x+e)*EllipticPi((
sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)
^2)^(1/2)/(a^2+b^2)^(3/2)/f/(d*sec(f*x+e))^(1/2)-2*a*tan(f*x+e)/(a^2+b^2)/
f/(d*sec(f*x+e))^(1/2)+2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)
```

3.607.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 4.48 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx =$$

$$-\frac{28d\,\mathrm{AppellF1}\left(\frac{5}{2},\frac{5}{4},\frac{5}{4},\frac{7}{2},\frac{a-ib}{a+b\tan(e+fx)},\right.}{5bf(d\sec(e+fx))^{3/2}\left(5(a+ib)\,\mathrm{AppellF1}\left(\frac{7}{2},\frac{5}{4},\frac{9}{4},\frac{9}{2},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)\,\mathrm{AppellF1}\left(\right.\right.}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

output
```
(-28*d*AppellF1[5/2, 5/4, 5/4, 7/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(5*b*f*(d*Sec
[e + f*x])^(3/2)*(5*(a + I*b)*AppellF1[7/2, 5/4, 9/4, 9/2, (a - I*b)/(a +
b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[7/
2, 9/4, 5/4, 9/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])] + 14*AppellF1[5/2, 5/4, 5/4, 7/2, (a - I*b)/(a + b*Tan[e + f*x]),
(a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

3.607.3 Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.76, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3994, 496, 27, 25, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{2b^2\int -\frac{\left(1-\frac{a^2}{b^2}\right)b^2-ab\tan(e+fx)}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int -\frac{a^2+b\tan(e+fx)a-b^2}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{\int \frac{a^2+b\tan(e+fx)a-b^2}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-b^2\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx}}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))\right)-b^2\int\frac{}{(a+b\tan(e}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\int\frac{}{(a+b\tan(e+fx))}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(a\int\frac{}{\sqrt[4]{\tan^2(e}}\right.}{}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{310}$$

3.607. $\int\frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{}\right.}{bf\sqrt{d\,s}}$$

$$\Big\downarrow 353$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{}\right.}{bf\sqrt{d}}$$

$$\Big\downarrow 73$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{}\right.}{bf\sqrt{d\sec}}$$

$$\Big\downarrow 827$$

3.607. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\Bigg(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}}{}}{}$$

$$\Big\downarrow \text{218}$$

$$\sqrt[4]{\sec^2(e+fx)}\Bigg(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}}{}}{}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\Bigg(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}}{}}{}}{bf\sqrt{d\,\mathrm{s}}}$$

$$\Big\downarrow \text{993}$$

3.607. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{}\right.$$

$$\Big\downarrow \; 1537$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{}\right.$$

$$\Big\downarrow \; 412$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)\right)-b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{}\right.$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

3.607. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/((a^2 + b^2)*(1 + Ta
n[e + f*x]^2)^(1/4)) - (-(b^2*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/
(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e +
f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x
]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1
/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 +
Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b
)) + a*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1
+ Tan[e + f*x]^2)^(1/4)))/(a^2 + b^2)))/(b*f*Sqrt[d*Sec[e + f*x]])
```

3.607.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.607. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.607.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6967 vs. $2(418) = 836$.

Time = 9.99 (sec) , antiderivative size = 6968, normalized size of antiderivative = 15.45

method	result	size
default	Expression too large to display	6968

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.607.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.607.6 Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))), x)
```

3.607.7 Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

3.607.8 Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

3.607.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,(a+b\tan(e+f\,x))}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))), x)
```

3.608 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

3.608.1 Optimal result 4231
3.608.2 Mathematica [C] (verified) 4232
3.608.3 Rubi [A] (warning: unable to verify) 4233
3.608.4 Maple [B] (warning: unable to verify) 4242
3.608.5 Fricas [F(-1)] 4242
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3.608.8 Giac [F] 4243
3.608.9 Mupad [F(-1)] 4244

3.608.1 Optimal result

Integrand size = 25, antiderivative size = 422

$$\begin{aligned}
&\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \\
&-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} \\
&-\frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} \\
&+\frac{2a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} \\
&+\frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} \\
&+\frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} \\
&+\frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}}
\end{aligned}$$

output

```
-b^(5/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^
2)^(3/4)/(a^2+b^2)^(7/4)/f/(d*sec(f*x+e))^(3/2)-b^(5/2)*arctanh((sec(f*x+e
)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(7/4)/f
/(d*sec(f*x+e))^(3/2)+2/3*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*
arctan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*
x+e)^2)^(3/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*EllipticPi
((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x
+e)^2)^(1/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*EllipticP
i((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x
+e)^2)^(1/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+2/3*(b+a*tan(f*x+e))/(a^2+
b^2)/f/(d*sec(f*x+e))^(3/2)
```

3.608.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.17 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \frac{a^2b\sec^2(e+fx)+b^3\sec^2(e+fx)+a^2b\cos(2(e+fx))\sec^2(e}{}$$

input

```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

output

```
(a^2*b*Sec[e + f*x]^2 + b^3*Sec[e + f*x]^2 + a^2*b*Cos[2*(e + f*x)]*Sec[e
+ f*x]^2 + b^3*Cos[2*(e + f*x)]*Sec[e + f*x]^2 - 3*b^(5/2)*(a^2 + b^2)^(1/
4)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x
]^2)^(3/4) - 3*b^(5/2)*(a^2 + b^2)^(1/4)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)
^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4) + 2*a^3*Tan[e + f*x] + 2
*a*b^2*Tan[e + f*x] + a*(a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[
e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Tan[e + f*x] + 3*a*b^2*Cot[e + f*x]*Ell
ipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e +
 f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2] + 3*a*b^2*Cot[e + f*x]*EllipticPi[b/S
qrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)
*Sqrt[-Tan[e + f*x]^2])/(3*(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2))
```

3.608.3 Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.77, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3994, 496, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow 3994$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 496$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}}-\frac{2b^2\int -\frac{\left(\frac{a^2}{b^2}+3\right)b^2+a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\int \frac{a^2+b\tan(e+fx)a+3b^2}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2\left(ab\tan(e+fx)+b^2\right)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 719$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b}{3(a^2+b^2)(\tan^2(e+fx)-}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 229$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{3(a^2+b^2)}+\frac{2\left(ab\tan(e+fx)+b\right.}{3(a^2+b^2)(\tan^2(e+fx)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 504$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))\right)}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 312$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int \frac{b\tan}{\left(\tan^2(e+fx)+1\right)}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 118$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int -\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4\right.}}{b}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 25$$

3.608. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{b\,\text{ta}}{\left(\tan^2(e+fx)+1\right)^{3}}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{}{\left(\frac{\tan(e+fx)}{b}+1\right.}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{1}{-\tan^4(e+fx)}\right.}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{756}$$

3.608. $\displaystyle\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int}{\sqrt{a^2+b^2-b^3}}\right.\right.}{}\right.$$

$$\Big\downarrow 218$$

$$\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int}{\sqrt{a^2+b^2-b^3}}\right.\right.}{3(a^2+b^2)}\right.$$

$$bf(d\sec(e$$

$$\Big\downarrow 221$$

$$\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\frac{b^{3/2}\,\mathrm{t}}{\sqrt[4]{a}}\right.}{2\sqrt{b}\left(a^2+\right.}\right.\right.}{3(a^2+b^2)}\right.$$

$$bf(d\sec(e+fx))^{3/}$$

$$\Big\downarrow 925$$

3.608. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

$$\sec^2(e+fx)^{3/4}\frac{\left(3b^2\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\frac{}{\left(\frac{\tan^2(e+fx}{\sqrt{a^2+b^2}}\right.}}{}\right.}{b}\right.\right.}{}$$

$$\Big\downarrow \text{1537}$$

3.608. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{3b^2\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}+1}+1}}d\sqrt[4]{}}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \; 412$$

3.608. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

$$\sec^2(e+fx)^{3/4}\frac{3b^2\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\right.}{}\right.}{b}\right.}{}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(3*(a^2 + b^2)*(1 +
Tan[e + f*x]^2)^(3/4)) + (2*a*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 3*b
^2*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a
^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sq
rt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sq
rt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^
2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(
2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(3*(a^2 + b^2))))/(b*f*(d*Sec[e
 + f*x])^(3/2))
```

3.608. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

3.608.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

3.608. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.608.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6051 vs. $2(391) = 782$.

Time = 10.47 (sec) , antiderivative size = 6052, normalized size of antiderivative = 14.34

method	result	size
default	Expression too large to display	6052

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.608.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.608.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))), x)
```

3.608.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

3.608.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

3.608.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,\left(a+b\tan\left(e+f\,x\right)\right)}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))), x)
```

3.609 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

3.609.1 Optimal result 4245
3.609.2 Mathematica [C] (warning: unable to verify) 4246
3.609.3 Rubi [A] (warning: unable to verify) 4247
3.609.4 Maple [B] (warning: unable to verify) 4257
3.609.5 Fricas [F(-1)] 4257
3.609.6 Sympy [F] 4258
3.609.7 Maxima [F(-2)] 4258
3.609.8 Giac [F] 4258
3.609.9 Mupad [F(-1)] 4259

3.609.1 Optimal result

Integrand size = 25, antiderivative size = 568

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}\,d^2f\sqrt{d\sec(e+fx)}}$$

$$-\frac{b^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}\,d^2f\sqrt{d\sec(e+fx)}}$$

$$+\frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}}$$

$$-\frac{2a(3a^2+8b^2)\tan(e+fx)}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}}$$

$$-\frac{ab^3\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{5/2}\,d^2f\sqrt{d\sec(e+fx)}}$$

$$+\frac{ab^3\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{5/2}\,d^2f\sqrt{d\sec(e+fx)}}$$

$$+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\,(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}}+\frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}}$$

output

```
b^(7/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2
)^(1/4)/(a^2+b^2)^(9/4)/d^2/f/(d*sec(f*x+e))^(1/2)-b^(7/2)*arctanh((sec(f*
x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(9/4
)/d^2/f/(d*sec(f*x+e))^(1/2)+2/5*a*(3*a^2+8*b^2)*(cos(1/2*arctan(tan(f*x+e
)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+
e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)-
a*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(se
c(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/d^2/f/(d*sec(f*x+e
))^(1/2)+a*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2
),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/d^2/f/(d*s
ec(f*x+e))^(1/2)-2/5*a*(3*a^2+8*b^2)*tan(f*x+e)/(a^2+b^2)^2/d^2/f/(d*sec(f
*x+e))^(1/2)+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+
e))^(1/2)+2/5*(5*b^3+a*(3*a^2+8*b^2)*tan(f*x+e))/(a^2+b^2)^2/d^2/f/(d*sec(
f*x+e))^(1/2)
```

3.609.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 77.21 (sec) , antiderivative size = 2596, normalized size of antiderivative = 4.57

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Result too large to show}$$

input

```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

output
```
(3*a^2*Cos[2*(e + f*x)]*Sec[e + f*x]^(9/2)*(((1/4 + I/4)*(a^4 - b^4)*(2*Ar
cTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])/(a^2 + b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])/(a^2 + b^2)^(1/4)] - Log[Sqrt[a
^2 + b^2] - (1 + I)*Sqrt[b]*(a^2 + b^2)^(1/4)*Sqrt[Sec[e + f*x]] + I*b*Sec
[e + f*x]] + Log[Sqrt[a^2 + b^2] + (1 + I)*Sqrt[b]*(a^2 + b^2)^(1/4)*Sqrt[
Sec[e + f*x]] + I*b*Sec[e + f*x]]))/(Sqrt[b]*(a^2 + b^2)^(9/4)) + (4*(b +
a*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]))/((a^2 + b^2)*Sqrt[Sec[e + f*x]])
 + (2*a*(3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, Sec[e + f*x]^2, (b^2*Sec[
e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(5/2))/(3*(
a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]) - (4*a*b^2*AppellF1[7/4, 1/2, 1, 11
/4, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x
]^2]*Sec[e + f*x]^(9/2))/(7*(a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]))*(a +
b*Sec[e + f*x]*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)])*Sin[e + f*x])/(
10*(a - I*b)*(a + I*b)*f*Sqrt[1 - Cos[e + f*x]^2]*(d*Sec[e + f*x])^(5/2)*(
2 - Sec[e + f*x]^2)*(a + b*Tan[e + f*x])) + (11*b^2*Cos[2*(e + f*x)]*Sec[e
 + f*x]^(9/2)*(((1/4 + I/4)*(a^4 - b^4)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqr
t[Sec[e + f*x]])/(a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[S
ec[e + f*x]])/(a^2 + b^2)^(1/4)] - Log[Sqrt[a^2 + b^2] - (1 + I)*Sqrt[b]*(
a^2 + b^2)^(1/4)*Sqrt[Sec[e + f*x]] + I*b*Sec[e + f*x]] + Log[Sqrt[a^2 + b
^2] + (1 + I)*Sqrt[b]*(a^2 + b^2)^(1/4)*Sqrt[Sec[e + f*x]] + I*b*Sec[e ...
```

3.609.3 Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.75, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3994, 496, 27, 686, 27, 25, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 496$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}}-\frac{2b^2\int-\frac{\left(\frac{3a^2}{b^2}+5\right)b^2+3a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int\frac{3a^2+3b\tan(e+fx)a+5b^2}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}+\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 686$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{2b^4\int-\frac{\left(-\frac{3a^4}{b^4}-\frac{8a^2}{b^2}+5\right)b^4-ab\left(3a^2+8b^2\right)\tan(e+fx)}{2b^4(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{2(ab\tan(e+fx}{5(a^2+b^2)(\tan^2(e+}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{\int-\frac{3a^4+8b^2a^2+b\left(3a^2+8b^2\right)\tan(e+fx)a-5b^4}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}}{5(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 25$$

3.609. $\displaystyle\int\frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{\int\frac{3a^4+8b^2a^2+b\left(3a^2+8b^2\right)\tan(e+fx)a-5b^4}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 719$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-5b^4\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e}}}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 225$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))\right)-5b^4\int\frac{}{(a+}}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 212$$

3.609. $\int\frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\int\frac{}{(a+b\tan(e+}}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \color{blue}{504}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(a\int\frac{}{\sqrt[4]{\tan^2}}\right.}{}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d}}$$

$$\Big\downarrow \color{blue}{310}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.}{}$$

$$\Big\downarrow \color{blue}{353}$$

3.609. $\displaystyle\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

$$\Big\downarrow 73$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

$$\Big\downarrow 827$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

$$\Big\downarrow 218$$

3.609. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \; 221$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \; 993$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

3.609. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

$$\Big\downarrow \text{1537}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{412}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+8b^2\right)\tan(e+fx)+5b^4\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}-\frac{a\left(3a^2+8b^2\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\right)-5b^4\left(\frac{2a\sqrt{-\tan^2(e}}{}\right.}{}}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

3.609. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

output

```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(5*(a^2 + b^2)*(1 +
Tan[e + f*x]^2)^(5/4)) + ((2*(5*b^4 + a*b*(3*a^2 + 8*b^2)*Tan[e + f*x]))/(
(a^2 + b^2)*(1 + Tan[e + f*x]^2)^(1/4)) - (-5*b^4*(-2*b^2*(-1/2*ArcTan[(b^
(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTa
nh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))
) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1
+ Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2
+ b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt
[-Tan[e + f*x]^2])/b) + a*(3*a^2 + 8*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f
*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/(a^2 + b^2))/
(5*(a^2 + b^2))))/(b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

3.609.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

3.609. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.609. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.609.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12840 vs. $2(525) = 1050$.

Time = 14.01 (sec) , antiderivative size = 12841, normalized size of antiderivative = 22.61

method	result	size
default	Expression too large to display	12841

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.609.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.609.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{5}{2}}\,(a+b\tan{(e+fx)})}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))), x)
```

3.609.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.609.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)), x)
```

3.609.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,\left(a+b\tan\left(e+f\,x\right)\right)}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))), x)
```

3.610 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

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3.610.1 Optimal result

Integrand size = 25, antiderivative size = 480

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx &= -\frac{3ad^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
&+ \frac{3ad^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
&- \frac{3d^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} \\
&+ \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} \\
&+ \frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b^3\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
&- \frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b^3\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}} \\
&- \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}
\end{aligned}$$

output
```
-3/2*a*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x
+e))^(3/2)/b^(5/2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)+3/2*a*d^2*arctan
h((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/
2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)-3*d^2*(cos(1/2*arctan(tan(f*x+e)
))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e
))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec(f*x+e)^2)^(3/4)+3*d^2*cos(f*x
+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f+3/2*a^2*d^2*cot(f*x+e)*EllipticP
i((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*
x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(1/2)-3/2*a^2*d^2*cot(f
*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^
(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(1/2)-d^2
*(d*sec(f*x+e))^(3/2)/b/f/(a+b*tan(f*x+e))
```

3.610.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.03 (sec) , antiderivative size = 1129, normalized size of antiderivative = 2.35

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

output

```
(Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(
(3*Cos[e + f*x])/(a*b) + (3*Sin[e + f*x])/b^2 - 1/(b*(a*Cos[e + f*x] + b*S
in[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) + (3*(d*Sec[e + f*x])^(7/2)*(a*
Cos[e + f*x] + b*Sin[e + f*x])^2*(-((a*EllipticE[ArcSin[Tan[(e + f*x)/2]],
 -1]*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan[(e + f*x)/2]^2]) + (2*a*El
lipticF[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1
 - Tan[(e + f*x)/2]^2] + (-2*Sqrt[2]*a*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[
Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]
*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + Sqrt[2]*a^2*Sq
rt[a^2 + b^2]*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b +
Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e
 + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*
x)/2]))] + a^2*(a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b
+ Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 +
Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*
Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] - a^3*EllipticPi[((1 + I)*(a -
I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*
(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (
2*I)*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] - I*a^2*b*EllipticPi[((1 +
 I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sq...
```

3.610.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.68, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3994, 492, 605, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\int \frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 492

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\int\frac{b\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{a+b\tan(e+fx)}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 605

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))\right)}{2b^2}-\frac{(t}{}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 225

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))+\frac{2b\tan}{\sqrt[4]{\tan^2(e}}\right.}{2b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 212

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)+\frac{2b\tan(e+fx}{\sqrt[4]{\tan^2(e+f}}\right.}{2b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$\downarrow$ 504

3.610. $\displaystyle\int\frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{d^2(d\sec(e+fx))^{3/2}\Bigg(\frac{3\Bigg(-a\Bigg(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\,(a^2-b^2\tan^2(e+}}{2b^2}}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \textcolor{blue}{310}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\Bigg(\frac{3\Bigg(-a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\frac{}{\sqrt[4]{\tan}}}{2}}{bf\sec^2(e}$$

$$\Big\downarrow \textcolor{blue}{353}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\Bigg(\frac{3\Bigg(-a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\frac{}{\sqrt[4]{\frac{t}{-}}}}{}}{bf\sec^2(}$$

$$\Big\downarrow \textcolor{blue}{73}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\Bigg(\frac{3\Bigg(-a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int\frac{\sqrt[4]{}}{-}}{2b^2}}{bf\sec^2(e+}$$

$$\Big\downarrow \textcolor{blue}{827}$$

3.610. $\int\frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{218}$$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int\right.\right.\right.}{}\right.$$

$$b$$

$$\Big\downarrow \text{221}$$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\text{ar}}{2}\right.\right.\right.}{}\right.$$

$$bf\sec^2(e$$

$$\Big\downarrow \text{993}$$

3.610. $\int\frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

$$d^2(d\sec(e+fx))^{3/2} \qquad 3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b\int\right.}{b}\right.\right.$$

$$\Big\downarrow \text{ 1537}$$

$$d^2(d\sec(e+fx))^{3/2} \qquad 3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e+\right.}{b}\right.\right.$$

$$\Big\downarrow \text{ 412}$$

$$d^2(d\sec(e+fx))^{3/2} \qquad 3\left(-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+\right.}}{b}\right.\right.$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

3.610. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

output

```
(d^2*(d*Sec[e + f*x])^(3/2)*(-((1 + Tan[e + f*x]^2)^(3/4)/(a + b*Tan[e + f
*x])) + (3*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])
/(1 + Tan[e + f*x]^2)^(1/4) - a*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x]
)/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e
+ f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f
*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^
(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1
+ Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])
/b)))/(2*b^2)))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

3.610.3.1 Defintions of rubi rules used

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

3.610. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.610.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33210 vs. 2(439) = 878.

Time = 132.90 (sec) , antiderivative size = 33211, normalized size of antiderivative = 69.19

method	result	size
default	Expression too large to display	33211

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.610.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(f*x + e))*d^3*sec(f*x + e)^3/(b^2*tan(f*x + e)^2 + 2*a
*b*tan(f*x + e) + a^2), x)
```

3.610.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Timed out
```

3.610.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.610.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^2, x)
```

3.610.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2, x)
```

3.611 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

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3.611.1 Optimal result

Integrand size = 25, antiderivative size = 440

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx &= \frac{ad^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&+ \frac{ad^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&+ \frac{d^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} \\
&- \frac{a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} \\
&- \frac{a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} \\
&- \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}
\end{aligned}$$

output
```
1/2*a*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+
e))^(1/2)/b^(3/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+1/2*a*d^2*arctanh
((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2
)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+d^2*(cos(1/2*arctan(tan(f*x+e)))^
2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e)))
,2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)-1/2*a^2*d^2*cot(
f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e)
)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4)-1/2*a^2
*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*se
c(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4)
-d^2*(d*sec(f*x+e))^(1/2)/b/f/(a+b*tan(f*x+e))
```

3.611.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.02 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.86

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{5/2}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{1}{ab}+\frac{\sin(e+fx)}{a(a\cos(e+fx)+b\sin(e+f}\right.}{f(a+b\tan(e+fx))^2}$$

$$+\frac{\cos^2(e+fx)(d\sec(e+fx))^{5/2}\sec^2(e+fx)^{3/4}(a\cos(e+fx)+b\sin(e+fx))^2\left(\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-(1/(a*b)) +
Sin[e + f*x]/(a*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x
])^2) + (Cos[e + f*x]^2*(d*Sec[e + f*x])^(5/2)*(Sec[e + f*x]^2)^(3/4)*(a*C
os[e + f*x] + b*Sin[e + f*x])^2*(Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e +
 f*x]^2]*Tan[e + f*x] + (a*(a*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec
[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + a*EllipticPi[b/Sqrt[a^2 + b^2], Ar
cSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + Sqrt[b]*(a^2 + b^2)^(1/4)
*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sq
rt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])*Sqrt[-Tan[e + f*x]^2]))/
((a^2 + b^2)*Sqrt[-Tan[e + f*x]^2])))/(2*b^2*f*(a + b*Tan[e + f*x])^2)
```

3.611.3 Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.70, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.680, Rules used = {3042, 3994, 492, 605, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int \frac{\sqrt[4]{\tan^2(e+fx)+1}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{b\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{a+b\tan(e+fx)}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{605}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+}}{a+b\tan(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{229}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+}}{a+b\tan(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\,\text{ta}}{\left(\tan^2(e+fx)+1\right)^{3}}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{312}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right.}}{2b}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{118}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\,\text{c}}{}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\text{ta}}{}}}{b}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{353}$$

3.611. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\text{ta}}}{b}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \ 73$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\text{ta}}}{b}\right.}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \ 756$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\text{ta}}}{b}\right.}{}\right.}{b}$$

$$\Big\downarrow \ 218$$

3.611. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\text{ta}}}{b}}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e}}$$

$$\Big\downarrow 221$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\text{ta}}}{b}}{2b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow 925$$

3.611. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{}}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \text{1537}$$

3.611. $\int\frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}}{\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}}}}{}\right.}{}\right.}{}\right.}$$

$$\Big\downarrow 412$$

3.611. $\int\frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+f}{b}}\right.\right.}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(-((1 + Tan[e + f*x]^2)^(1/4)/(a + b*Tan[e + f*x
])) + (2*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] - a*(-2*b^2*(ArcTan[(b^(3/
2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTan
h[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)))
 - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1
 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 +
 b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan
[e + f*x]^2])/b))/(2*b^2)))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

3.611. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

3.611.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.611. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

3.611. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.611.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4811 vs. $2(405) = 810$.

Time = 128.71 (sec) , antiderivative size = 4812, normalized size of antiderivative = 10.94

method	result	size
default	Expression too large to display	4812

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output

```
-1/4*d^2/f*(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(csc(f*x+e)^2*(1-cos(f*x+
e))^2-1))^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(8*I*(b*((a^2+b^2)^(1/2)
*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*
a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(csc(f*x+e)^2*(1-cos(f
*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticPi(I*(c
sc(f*x+e)-cot(f*x+e)),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*a*b*(csc(f*x+e)-cot
(f*x+e))+4*I*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/
a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a
^4)^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*
x+e))^2+1)^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(-b+(a^2+b^2)^(1/
2))^2*a^2,I)*a^2-4*I*csc(f*x+e)^2*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)
*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-c
sc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e))
,-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*a^2*(1-cos(f*x+e))^2-4*I*(b*((a^2+b^2)^(
1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(csc(f*x+e)^2*(1-c
os(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticPi(
I*(csc(f*x+e)-cot(f*x+e)),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*a^2-8*I*(b*((a^2
+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a...
```

3.611.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.611.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Timed out
```

3.611.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

3.611.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

3.611.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2, x)
```

3.612 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

3.612.1 Optimal result 4287
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3.612.9 Mupad [F(-1)] 4298

3.612.1 Optimal result

Integrand size = 25, antiderivative size = 477

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx &= \frac{a\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\,(a^2+b^2)^{5/4}\,f\sec^2(e+fx)^{3/4}} \\
&- \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\,(a^2+b^2)^{5/4}\,f\sec^2(e+fx)^{3/4}} \\
&- \frac{E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f\sec^2(e+fx)^{3/4}} \\
&+ \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)\,f} \\
&- \frac{a^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b\,(a^2+b^2)^{3/2}\,f\sec^2(e+fx)^{3/4}} \\
&+ \frac{a^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b\,(a^2+b^2)^{3/2}\,f\sec^2(e+fx)^{3/4}} \\
&- \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))}
\end{aligned}
$$

output

```
-(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Ellipti
cE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f/(
sec(f*x+e)^2)^(3/4)+cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/(a^2+b^2)/f
+1/2*a*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))
^(3/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-1/2*a*arctanh((sec(f
*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(5/
4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-1/2*a^2*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2
)/b/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+1/2*a^2*cot(f*x+e)*EllipticPi((
sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)
^2)^(1/2)/b/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-b*(d*sec(f*x+e))^(3/2)/
(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.612.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 27.97 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.36

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

output
```
(Sec[e + f*x]*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(
(b*Cos[e + f*x])/(a*(a - I*b)*(a + I*b)) + Sin[e + f*x]/((a - I*b)*(a + I*
b)) - b/((a - I*b)*(a + I*b)*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a +
b*Tan[e + f*x])^2) + (Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(a*Cos[e +
 f*x] + b*Sin[e + f*x])^2*(-((a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sq
rt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan[(e + f*x)/2]^2]) - (-2*Sqrt[2]*a*
b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(
I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + T
an[(e + f*x)/2]))] + Sqrt[2]*a^2*Sqrt[a^2 + b^2]*EllipticPi[((1 + I)*(a -
I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*
(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((1 + I
*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + a^2*(a + I*b + Sqrt[a^2 + b^
2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2
+ b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2
])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]
))] - a^3*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt
[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f
*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)/2])/(I + Tan[(e + f*
x)/2]))] - I*a^2*b*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a +
 b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I ...
```

3.612.3 Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.72, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules used = {3042, 3994, 498, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{(d\sec(e+fx))^{3/2}\int \frac{1}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 498}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(-\frac{\int -\frac{2a+b\tan(e+fx)}{2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\int \frac{2a+b\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2(\tan^2(}{(a^2+b^2)(a}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))+\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx}}}{2(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-2bE\left(\frac{1}{2}\arctan(\tan(e+fx))|2\right)+\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}}{2(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{ 504}$$

3.612. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\right.}{2(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \color{blue}{310}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\frac{}{\sqrt[4]{\tan^2(e+}}\right.}{2(a^2+b^2)}\right.}{bf\sec^2(e+}$$

$$\Big\downarrow \color{blue}{353}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\frac{}{\sqrt[4]{\frac{\tan(e-}{b}}}\right.}{2(a^2+b^2}\right.}{bf\sec^2(e-}$$

$$\Big\downarrow \color{blue}{73}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int\frac{\sqrt[4]{\tan(e}}{-\tan^4(e+}\right.}{2(a^2+b^2)}\right.}{bf\sec^2(e+f}$$

$$\Big\downarrow \color{blue}{827}$$

3.612. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b^2-}}}{}\right.\right.}{}\right.}{}$$

$$\Big\downarrow 218$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\int\frac{}{\sqrt{a^2+b^2-}}}{}\right.\right.}{}\right.}{bf}$$

$$\Big\downarrow 221$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\frac{\text{arctanh}\left(\right.}{2b^{3/2}}\right.\right.}{2(a^2+b^2)}\right.}{bf\sec^2(e+}$$

$$\Big\downarrow 993$$

3.612. $\int\frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

$$(d\sec(e+fx))^{3/2}\Bigg(a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b\int\frac{}{\big(\tan^2(e+}}{b}$$

$$\Big\downarrow \color{blue}{1537}$$

$$(d\sec(e+fx))^{3/2}\Bigg(a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e+fx)+}{b}$$

$$\Big\downarrow \color{blue}{412}$$

$$(d\sec(e+fx))^{3/2}\Bigg(a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(\frac{b\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\mathrm{EllipticPi}\Big(-\frac{b}{\sqrt{a^2+b^2}},\mathrm{arcsi}}{}}{b}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

3.612. $\int\frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

output
```
((d*Sec[e + f*x])^(3/2)*(-((b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(
a + b*Tan[e + f*x]))) + (-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*
Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4) + a*(-2*b^2*(-1/2*ArcTan[(b^(3/2)
*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b
^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (
2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan
[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2
], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan
[e + f*x]^2])/b))/(2*(a^2 + b^2))))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

3.612.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.612. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.612.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16453 vs. $2(436) = 872$.

Time = 8.87 (sec) , antiderivative size = 16454, normalized size of antiderivative = 34.49

method	result	size
default	Expression too large to display	16454

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.612.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.612.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(e+fx))^{\frac{3}{2}}}{(a+b\tan(e+fx))^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**2, x)
```

3.612.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.612.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^2, x)
```

3.612.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2, x)
```

3.613 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

3.613.1 Optimal result 4299
3.613.2 Mathematica [C] (verified) 4300
3.613.3 Rubi [A] (warning: unable to verify) 4301
3.613.4 Maple [B] (warning: unable to verify) 4310
3.613.5 Fricas [F(-1)] 4310
3.613.6 Sympy [F] 4311
3.613.7 Maxima [F] 4311
3.613.8 Giac [F] 4311
3.613.9 Mupad [F(-1)] 4312

3.613.1 Optimal result

Integrand size = 25, antiderivative size = 430

$$\begin{aligned}\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx =& -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{3a^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{3a^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{b\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))}\end{aligned}$$

output
```
-(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Ellipti
cF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)/f/(
sec(f*x+e)^2)^(1/4)-3/2*a*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)-3
/2*a*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(
f*x+e))^(1/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*cot(f*x+e)*El
lipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(
-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*cot(f*x+e)
*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)
*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-b*(d*sec(f*x+e))
^(1/2)/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.613.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.76 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{b}{a(a-ib)(a+ib)}+\frac{b^2\sin(e+fx)}{a(a-ib)(a+ib)(a\cos(e+fx)+b\sin(e+}\right.}{f(a+b\tan(e+fx))^2}$$

$$+\frac{\sqrt{d\sec(e+fx)}\sec^2(e+fx)^{3/4}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\left((a^2+b^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

output
```
(Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(
-(b/(a*(a - I*b)*(a + I*b))) + (b^2*Sin[e + f*x])/(a*(a - I*b)*(a + I*b)*(
a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) + (Sqrt[d*S
ec[e + f*x]]*(Sec[e + f*x]^2)^(3/4)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-
((a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x
]) + 3*a*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1
/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Se
c[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*EllipticP
i[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x
]^2])))/(2*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])^2)
```

3.613.3 Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.75, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3994, 498, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{498}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{\int -\frac{2a-b\tan(e+fx)}{2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{2a-b\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{(a^2+b^2)(a+b\tan(e+fx))}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+}}{(a^2+b^2)(a+b\tan(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-2b\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+}}{(a^2+b^2)(a+b\tan(e+fx)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))\right)-}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{312}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{b\tan(}{\left(\tan^2(e+fx)+1\right)^{3/4}}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{118}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e\right.}}{b}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{25}$$

3.613. $\int\frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{b\tan}{\left(\tan^2(e+fx)+1\right)^{3/}}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \ 353$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{}{\left(\frac{\tan(e+fx)}{b}+1\right)}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \ 73$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{1}{-\tan^4(e+fx)b}\right.}{2(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \ 756$$

3.613. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

$$\sqrt{d\sec(e+fx)}\left(3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int\frac{1}{\sqrt{a^2+b^2-b^3}}\right.\right.\right.$$

$$b$$

$$\Big\downarrow \ 218$$

$$\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\int\frac{1}{\sqrt{a^2+b^2-b^3}}\right.\right.}{2(a^2+b^2)}\right.$$

$$bf\sqrt[4]{\sec^2(e}$$

$$\Big\downarrow \ 221$$

$$\sqrt{d\sec(e+fx)}\left(\frac{3a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\frac{b^{3/2}\text{ta}}{\sqrt[4]{a^2}}\right.}{2\sqrt{b}\left(a^2+b\right.}\right.\right.}{2(a^2+b^2)}\right.$$

$$bf\sqrt[4]{\sec^2(e+fx)}$$

$$\Big\downarrow \ 925$$

3.613. $\int\frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\Bigg(3a\Bigg(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\frac{}{\Big(\frac{\tan^2(e+fx)b}{\sqrt{a^2+b^2}}}}{}}{b}}{}$$

$$\Big\downarrow \text{1537}$$

3.613. $\int\frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}}{\left(\frac{3a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}_{+1}}d\sqrt[4]{}}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}$$

$$\Big\downarrow \; 412$$

3.613. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\text{ar}\right.}{}\right.}{b}\right.}{}\right.}{}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-((b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a
 + b*Tan[e + f*x]))) + (-2*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 3*a*(-2
*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b
^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*
(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2
 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*Elli
pticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2
 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*(a^2 + b^2))))/(b*f*(Sec[e + f*x]^
2)^(1/4))
```

3.613. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

3.613.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.613. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

3.613. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.613.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12205 vs. $2(395) = 790$.

Time = 11.21 (sec) , antiderivative size = 12206, normalized size of antiderivative = 28.39

method	result	size
default	Expression too large to display	12206

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.613.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.613.6 Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**2, x)
```

3.613.7 Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{(b\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

3.613.8 Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{(b\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

3.613.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2, x)
```

3.614 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

3.614.1 Optimal result 4313
3.614.2 Mathematica [C] (warning: unable to verify) 4314
3.614.3 Rubi [A] (warning: unable to verify) 4314
3.614.4 Maple [B] (warning: unable to verify) 4324
3.614.5 Fricas [F(-1)] 4324
3.614.6 Sympy [F] 4324
3.614.7 Maxima [F] 4325
3.614.8 Giac [F] 4325
3.614.9 Mupad [F(-1)] 4325

3.614.1 Optimal result

Integrand size = 25, antiderivative size = 555

$$
\begin{aligned}
&\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx \\
&= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}} - \frac{(2a^2-3b^2)\tan(e+fx)}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{5/2}f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{5/2}f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{b(2a^2-3b^2)\sec^2(e+fx)}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&\quad + \frac{2(b+a\tan(e+fx))}{\left(a^2+b^2\right)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}
\end{aligned}
$$

output

```
5/2*a*b^(3/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*
x+e)^2)^(1/4)/(a^2+b^2)^(9/4)/f/(d*sec(f*x+e))^(1/2)-5/2*a*b^(3/2)*arctanh
((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b
^2)^(9/4)/f/(d*sec(f*x+e))^(1/2)+(2*a^2-3*b^2)*(cos(1/2*arctan(tan(f*x+e))
)^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e)
)),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)-5/2*a^
2*b*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(
f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x+e))^(1/
2)+5/2*a^2*b*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),
I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x
+e))^(1/2)-(2*a^2-3*b^2)*tan(f*x+e)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)+b*(
2*a^2-3*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+
e))+2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

3.614.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.31 (sec) , antiderivative size = 8379, normalized size of antiderivative = 15.10

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

output

```
Result too large to show
```

3.614.3 Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.78, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3994, 496, 27, 25, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 496$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{2b^2\int -\frac{\left(3-\frac{a^2}{b^2}\right)b^2+a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int -\frac{a^2-b\tan(e+fx)a-3b^2}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{\int \frac{a^2-b\tan(e+fx)a-3b^2}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 688$$

3.614. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\int\frac{2a\left(4-\frac{a^2}{b^2}\right)+\left(3-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}-\frac{b^2\left(2}{(}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\int\frac{2a\left(4-\frac{a^2}{b^2}\right)+\left(3-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{b^2\left(2a}{(}}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{719}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(\left(3-\frac{2a^2}{b^2}\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\int\frac{}{(a+b\,\mathrm{ta}}\right.}{2\left(a^2+b^2\right)}}{}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{225}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(\left(3-\frac{2a^2}{b^2}\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\right.\right.}{2}}{}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \textcolor{blue}{212}$$

3.614. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+\left(3\right.\right.}{2\left(a^2\right.}}{}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 504$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+f\right.\right.}{}}{}\right.$$

$$\Big\downarrow \; 310$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+\right.}}{b}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \; 353$$

3.614. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+\right.}}{b}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \; 73$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+\right.}}{b}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \; 827$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+\right.}}{b}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \; 218$$

3.614. $\displaystyle\int \frac{1}{\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+\right.}}{b}\right.\right.}{}}{}\right.}{}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+\right.}}{b}\right.\right.}{}}{}\right.}{}$$

$$\Big\downarrow \text{993}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{}}\right.}{}\right.\right.}{}}{}\right.}{}$$

$$\Big\downarrow \text{1537}$$

3.614. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2}}\right.}{}\right.\right.}{}}{}\right.$$

$$\Big\downarrow \; 412$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{-\frac{b^2\left(5a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{}\right.\right.}{2\sqrt{a^2}}\right.}{}\right.\right.}{}}{}\right.$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

3.614. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/((a^2 + b^2)*(a + b*
Tan[e + f*x])*(1 + Tan[e + f*x]^2)^(1/4)) - (-((b^2*(2*a^2 - 3*b^2)*(1 + T
an[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x]))) - (b^2*(5*a*(-2*
b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 +
 b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2
)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2
 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*El
lipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sq
rt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b) + (3 - (2*a^2)/b^2)*(-2*b*Ellipt
icE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(
1/4))))/(2*(a^2 + b^2)))/(a^2 + b^2)))/(b*f*Sqrt[d*Sec[e + f*x]])
```

3.614.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.614. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.614. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

3.614.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29177 vs. $2(512) = 1024$.

Time = 11.81 (sec) , antiderivative size = 29178, normalized size of antiderivative = 52.57

method	result	size
default	Expression too large to display	29178

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.614.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.614.6 Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2), x)
```

3.614.7 Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

3.614.8 Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

3.614.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,{\left(a+b\,\tan\left(e+f\,x\right)\right)}^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2), x)
```

3.615 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

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3.615.2 Mathematica [C] (verified) 4327
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3.615.9 Mupad [F(-1)] 4340

3.615.1 Optimal result

Integrand size = 25, antiderivative size = 520

$$\begin{aligned}
&\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \\
&-\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}} \\
&-\frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}} \\
&+\frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}} \\
&+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}} \\
&+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}} \\
&+\frac{b(2a^2-5b^2)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} \\
&+\frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}
\end{aligned}$$

output
```
-7/2*a*b^(5/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f
*x+e)^2)^(3/4)/(a^2+b^2)^(11/4)/f/(d*sec(f*x+e))^(3/2)-7/2*a*b^(5/2)*arcta
nh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2
+b^2)^(11/4)/f/(d*sec(f*x+e))^(3/2)+1/3*(2*a^2-5*b^2)*(cos(1/2*arctan(tan(
f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan
(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)
+7/2*a^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2)
,I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(d*sec(f*x+e)
)^(3/2)+7/2*a^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)
^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(d*sec(
f*x+e))^(3/2)+1/3*b*(2*a^2-5*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e)
)^(3/2)/(a+b*tan(f*x+e))+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(
3/2)/(a+b*tan(f*x+e))
```

3.615.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.23 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sec^4(e+fx)(a\cos(e+fx)+b\sin(e+fx))^2\Big(\frac{b(2a^2-3b^2)}{3a(a-ib)^2(a+ib)^2}}{f(d\sec(e+f}$$

$$+\frac{\sec^2(e+fx)\sec^2(e+fx)^{3/4}(a\cos(e+fx)+b\sin(e+fx))^2\Bigg(\big((2a^4-3a^2b^2-5b^4)\,\text{Hypergeometric2F1}\Big(\frac{1}{2}}{}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

output
```
(Sec[e + f*x]^4*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((b*(2*a^2 - 3*b^2))/(
3*a*(a - I*b)^2*(a + I*b)^2) + (2*a*b*Cos[2*(e + f*x)])/(3*(a - I*b)^2*(a
+ I*b)^2) + (b^4*Sin[e + f*x])/(a*(a - I*b)^2*(a + I*b)^2*(a*Cos[e + f*x]
+ b*Sin[e + f*x])) + ((a^2 - b^2)*Sin[2*(e + f*x)])/(3*(a - I*b)^2*(a + I*
b)^2)))/(f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (Sec[e + f*x]^
2*(Sec[e + f*x]^2)^(3/4)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((2*a^4 - 3*a
^2*b^2 - 5*b^4)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e +
f*x] + 21*a*b^2*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x
]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/
(a^2 + b^2)^(1/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), Arc
Sin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*El
lipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[
e + f*x]^2])))/(6*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*
x])^2)
```

3.615.3 Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.79, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3994, 496, 27, 688, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \ \textcolor{blue}{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \ \textcolor{blue}{3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ \textcolor{blue}{496}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))}-\frac{2b^2\int -\frac{\left(\frac{a^2}{b^2}+5\right)b^2+3a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\int \frac{a^2+3b\tan(e+fx)a+5b^2}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 688$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int -\frac{2a\left(\frac{a^2}{b^2}+8\right)-\left(5-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}}{3(a^2+b^2)}+\frac{2(ab\tan(e}{3(a^2+b^2)(\tan^2(e+fx)-}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\int \frac{2a\left(\frac{a^2}{b^2}+8\right)-\left(5-\frac{2a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{3(a^2+b^2)}+\frac{2(ab\tan(e+}{3(a^2+b^2)(\tan^2(e+fx)+1}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 719$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\left(5-\frac{2a^2}{b^2}\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))\right)}{2\left(a^2+b^2\right)}+\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{}}{\left(a^2+b^2\right)(}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 229$$

3.615. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-2b\left(5-\frac{2a^2}{b^2}\right)\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\right)}{2\left(a^2+b^2\right)}+\frac{b^2\left(2a^2-5b^2\right)\sqrt[4]{\text{ta}}}{\left(a^2+b^2\right)(a-}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))\right)-2\right.}{2\left(a^2+b^2\right)}}{3(a^2+b^2)}\right.}{bf(d\sec(e+}$$

$$\Big\downarrow \text{312}$$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{b}{\left(\tan^2(e+fx)+1\right.}\right.\right.}{2\left(a^2+b^2\right)}}{3(a}\right.$$

$$\Big\downarrow \text{118}$$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4\right.}}{b}\right.\right.}{2\left(a^2+b^2\right)}}{3}\right.$$

$$\Big\downarrow \text{25}$$

3.615. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}dx$

$$\sec^2(e+fx)^{3/4}\Bigg(\frac{\frac{b^2\Bigg(21a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{}{\Big(\tan^2(e+fx)+}}{2\left(a^2+b^2\right)}}{3(a}$$

$$\Big\downarrow 353$$

$$\sec^2(e+fx)^{3/4}\Bigg(\frac{\frac{b^2\Bigg(21a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\frac{}{\Big(\frac{\tan(e+fx)}{b}}}{2\left(a^2+b^2\right)}}{3}$$

$$\Big\downarrow 73$$

$$\sec^2(e+fx)^{3/4}\Bigg(\frac{\frac{b^2\Bigg(21a\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int\frac{}{-\tan^4(e+}}{2\left(a^2+b^2\right)}}{3(a}$$

$$\Big\downarrow 756$$

3.615. $\displaystyle\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int}{\sqrt{a^2+b^2}}\right.\right.\right.}{}}{}\right.$$

$$\Big\downarrow \; 218$$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int}{\sqrt{a^2+b^2}}\right.\right.\right.}{2\left(a^2+b^2\right.}}{}\right.$$

$$\Big\downarrow \; 221$$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\arctan\left(\frac{b^3}{4}\right.}{2\sqrt{b}\left(a\right.}\right.\right.\right.}{2\left(a^2+b^2\right)}}{}\right.$$

$$\Big\downarrow \; 925$$

3.615. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\Bigg(\frac{\frac{b^2\Bigg(21a\Bigg(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\frac{}{\left(\frac{\tan^2(e+}{\sqrt{a^2}}\right.}}{}}{b}}{}}{}}{}$$

$$\Big\downarrow 1537$$

3.615. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}}{\left(\frac{b^2\left(21a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+1_{+1}}}}{2\left(a^2+b^2\right)}\right.}{}\right.\right.}{}\right.}$$

$\Big\downarrow$ 412

3.615. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21a\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+}}\right.}{}\right.}{b}\right.\right.}{}}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

output
```
((Sec[e + f*x]^2)^(3/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(3*(a^2 + b^2)*(a +
b*Tan[e + f*x])*(1 + Tan[e + f*x]^2)^(3/4)) + ((b^2*(2*a^2 - 5*b^2)*(1 + T
an[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a + b*Tan[e + f*x])) + (b^2*(-2*(5 - (
2*a^2)/b^2)*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 21*a*(-2*b^2*(ArcTan[
(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) +
ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(
3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 + b^2]), Arc
Sin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqr
t[-Tan[e + f*x]^2])/b)))/(2*(a^2 + b^2)))/(3*(a^2 + b^2))))/(b*f*(d*Sec[e
+ f*x])^(3/2))
```

3.615. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

3.615.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.615. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.615.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14916 vs. $2(477) = 954$.

Time = 13.29 (sec) , antiderivative size = 14917, normalized size of antiderivative = 28.69

method	result	size
default	Expression too large to display	14917

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.615.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.615.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2), x)
```

3.615.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

3.615.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{(d\sec\left(fx+e\right))^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2), x)
```

3.616 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

3.616.1 Optimal result 4341
3.616.2 Mathematica [C] (warning: unable to verify) 4342
3.616.3 Rubi [A] (warning: unable to verify) 4342
3.616.4 Maple [B] (warning: unable to verify) 4356
3.616.5 Fricas [F(-1)] 4356
3.616.6 Sympy [F] 4357
3.616.7 Maxima [F] 4357
3.616.8 Giac [F] 4357
3.616.9 Mupad [F(-1)] 4358

3.616.1 Optimal result

Integrand size = 25, antiderivative size = 700

$$\begin{aligned}
&\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&+ \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&- \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}
\end{aligned}$$

output
```
9/2*a*b^(7/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*
x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/2*a*b^(7/2)*ar
ctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(
a^2+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)+3/5*(2*a^4+10*a^2*b^2-7*b^4)*(c
os(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(
sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^3/d^2/
f/(d*sec(f*x+e))^(1/2)-9/2*a^2*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1
/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+
b^2)^(7/2)/d^2/f/(d*sec(f*x+e))^(1/2)+9/2*a^2*b^3*cot(f*x+e)*EllipticPi((s
ec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^
2)^(1/2)/(a^2+b^2)^(7/2)/d^2/f/(d*sec(f*x+e))^(1/2)-3/5*(2*a^4+10*a^2*b^2-
7*b^4)*tan(f*x+e)/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)+3/5*b*(2*a^4+10*a
^2*b^2-7*b^4)*sec(f*x+e)^2/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan
(f*x+e))+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^
(1/2)/(a+b*tan(f*x+e))-2/5*(b*(2*a^2-7*b^2)-3*a*(a^2+4*b^2)*tan(f*x+e))/(a
^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

3.616.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.27 (sec) , antiderivative size = 9161, normalized size of antiderivative = 13.09

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```

output
```
Result too large to show
```

3.616.3 Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.77, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.840$, Rules used = {3042, 3994, 496, 27, 686, 27, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \ 3994$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \ 496$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))}-\frac{2b^2\int -\frac{\left(\frac{3a^2}{b^2}+7\right)b^2+5a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int \frac{3a^2+5b\tan(e+fx)a+7b^2}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \ 686$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{2b^4\int \frac{3\left(a^4+6b^2a^2-b\left(a^2+4b^2\right)\tan(e+fx)a-7b^4\right)}{2b^4(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{}{5(a^2}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \ 27$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\int\frac{a^4+6b^2a^2-b\left(a^2+4b^2\right)\tan(e+fx)a-7b^4}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\frac{}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow\ 688$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(-\frac{b^2\int-\frac{2a\left(\frac{a^4}{b^2}+5a^2-11b^2\right)b^2+\left(2a^4+10b^2a^2-7b^4\right)\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+f}{a^2+b^2}\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow\ 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\int\frac{2a\left(a^4+5b^2a^2-11b^4\right)+b\left(2a^4+10b^2a^2-7b^4\right)\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{b^2}{}\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow\ 719$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-15ab^4\int -}{2\left(a^2+b^2\right)}\right.}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d}}$$

$$\Big\downarrow \; 225$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{5/4}}d\right.}{2}\right.}{5(a^2+}\right.}{}$$

$$\Big\downarrow \; 212$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{2\left(a^2\right.}\right.}{5(a^2+b^2)}\right.}{bd}$$

$$\downarrow \text{504}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{}\right.}{}\right.}{}$$

$$\downarrow \text{310}$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{}\right.}{}\right.}{}$$

$\Big\downarrow$ 353

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{}\right.}{}\right.}{}$$

$\Big\downarrow$ 73

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fa\right.\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \; 827$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fa\right.\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \; 218$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{}\right.}{}\right.}{}$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\Big\downarrow \text{993}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \text{1537}$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+fx\right.\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \text{412}$$

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))}+\frac{\frac{2\left(3ab\left(a^2+4b^2\right)\tan(e+fx)+b^4\left(7-\frac{2a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))}-\frac{3\left(\frac{\left(2a^4+10a^2b^2-7b^4\right)}{}\right.}{}}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(5*(a^2 + b^2)*(a +
b*Tan[e + f*x])*(1 + Tan[e + f*x]^2)^(5/4)) + ((2*((7 - (2*a^2)/b^2)*b^4 +
 3*a*b*(a^2 + 4*b^2)*Tan[e + f*x]))/((a^2 + b^2)*(a + b*Tan[e + f*x])*(1 +
 Tan[e + f*x]^2)^(1/4)) - (3*(-((b^2*(2*a^4 + 10*a^2*b^2 - 7*b^4)*(1 + Tan
[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x]))) + (-15*a*b^4*(-2*b
^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 +
b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)
*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2
+ b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*Ell
ipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqr
t[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b) + (2*a^4 + 10*a^2*b^2 - 7*b^4)*(-
2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e +
 f*x]^2)^(1/4)))/(2*(a^2 + b^2))))/(a^2 + b^2))/(5*(a^2 + b^2))))/(b*d^2*f
*Sqrt[d*Sec[e + f*x]])
```

3.616.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

3.616. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.616.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37752 vs. $2(645) = 1290$.

Time = 17.40 (sec) , antiderivative size = 37753, normalized size of antiderivative = 53.93

method	result	size
default	Expression too large to display	37753

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.616.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.616.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2), x)
```

3.616.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

3.616.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,(a+b\tan\left(e+f\,x\right))^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2), x)
```

3.617 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

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3.617.1 Optimal result

Integrand size = 25, antiderivative size = 583

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx &= \frac{3(a^2+2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} \\
&- \frac{3(a^2+2b^2)\,d^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} \\
&+ \frac{3ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} \\
&- \frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2\left(a^2+b^2\right)f} \\
&- \frac{3a(a^2+2b^2)\,d^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan}}{8b^3\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}} \\
&+ \frac{3a(a^2+2b^2)\,d^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{8b^3\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}} \\
&- \frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))}
\end{aligned}$$

output

```
3/8*(a^2+2*b^2)*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(
d*sec(f*x+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)-3/8*(a^
2+2*b^2)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(
f*x+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)+3/4*a*d^2*(co
s(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(s
in(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/(a^2+b^2)/f/(
sec(f*x+e)^2)^(3/4)-3/4*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b
^2/(a^2+b^2)/f-3/8*a*(a^2+2*b^2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^
(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3
/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+3/8*a*(a^2+2*b^2)*d^2*cot(f*x+e)*E
llipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(
-tan(f*x+e)^2)^(1/2)/b^3/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-1/2*d^2*(d
*sec(f*x+e))^(3/2)/b/f/(a+b*tan(f*x+e))^2+3/4*a*d^2*(d*sec(f*x+e))^(3/2)/b
/(a^2+b^2)/f/(a+b*tan(f*x+e))
```

3.617.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.71 (sec) , antiderivative size = 14225, normalized size of antiderivative = 24.40

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

output

```
Result too large to show
```

3.617.3 Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.68, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.720$, Rules used = {3042, 3994, 492, 594, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\int \frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{(a+b\tan(e+fx))^3}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{492}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\int \frac{b\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{594}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int -\frac{2b^2-ab\tan(e+fx)}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{4b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3}}{2(a+b\tan(e+fx)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\int \frac{2b^2-ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}\right)}{4b^2}-\frac{\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{719}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+}{4b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{225}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+\right.}\right.}{2\left(a^2+b^2\right)}\right.}{4b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\mathrm{ta}\right.\right.}{2\left(a^2+b^2\right)}\right.}{4b^2}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{504}$$

3.617. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{(a^2+2b^2)\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\,(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\,(a^2-b^2\tan^2}\right.}{2(a^2+b^2)}\right.}{4b^2}\right.$$

$$bf\sec^2(e+$$

$$\Big\downarrow 310$$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{(a^2+2b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\right.}{2(}\right.}{}\right.$$

$$\Big\downarrow 353$$

3.617. $\int\frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\right.}{}\right.}{}\right.$$

$$\downarrow 73$$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\right.}{2\left(a\right.}\right.}{}\right.$$

$$\downarrow 827$$

3.617. $\int\frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{\left(3\frac{(a^2+2b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\right.}{}\right.}{}\right.$$

$\downarrow$ 218

$$d^2(d\sec(e+fx))^{3/2}\left(\frac{\left(3\frac{(a^2+2b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\right.}{}\right.}{}\right.$$

$\downarrow$ 221

3.617. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

$$d^2 (d\sec(e+fx))^{3/2} \Bigg(3 \Bigg(\left(a^2+2b^2\right) \Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b} - 2b^2$$

$$2$$

$$\downarrow 993$$

$$d^2 (d\sec(e+fx))^{3/2} \Bigg(3 \Bigg(\left(a^2+2b^2\right) \Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int \frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}\right.}{b}$$

$$\downarrow 1537$$

3.617. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

$$d^2(d\sec(e+fx))^{3/2}\Bigg(3\Bigg(\left(a^2+2b^2\right)\Bigg(2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2$$

$$\Big\downarrow 412$$

3.617. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{d^2(d\sec(e+fx))^{3/2}\left(\frac{3\left(\frac{\left(a^2+2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\text{EllipticPi}\left(-\right.}{}\right.}{b}\right.}{}\right.}{}\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

3.617. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

output
```
(d^2*(d*Sec[e + f*x])^(3/2)*(-1/2*(1 + Tan[e + f*x]^2)^(3/4)/(a + b*Tan[e
+ f*x])^2 + (3*((a*b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan
[e + f*x])) + ((a^2 + 2*b^2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x])/(
a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e + f
*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e + f*x]
*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)^(1/
4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + T
an[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2])/b)
 - a*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 +
Tan[e + f*x]^2)^(1/4)))/(2*(a^2 + b^2))))/(4*b^2)))/(b*f*(Sec[e + f*x]^2)^
(3/4))
```

3.617.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.617. $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.617.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72184 vs. $2(532) = 1064$.

Time = 1486.38 (sec) , antiderivative size = 72185, normalized size of antiderivative = 123.82

method	result	size
default	Expression too large to display	72185

input
```
int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.617.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.617.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Timed out
```

3.617.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.617.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^3, x)
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3, x)
```

3.618 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

3.618.1 Optimal result . . . 4374

3.618.2 Mathematica [C] (verified) . . . 4375

3.618.3 Rubi [A] (warning: unable to verify) . . . 4376

3.618.4 Maple [B] (warning: unable to verify) . . . 4386

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3.618.6 Sympy [F(-1)] . . . 4387

3.618.7 Maxima [F(-1)] . . . 4387

3.618.8 Giac [F] . . . 4387

3.618.9 Mupad [F(-1)] . . . 4388

3.618.1 Optimal result

Integrand size = 25, antiderivative size = 532

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{(a^2-2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\,(a^2+b^2)^{7/4}\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$+\frac{(a^2-2b^2)\,d^2\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\,(a^2+b^2)^{7/4}\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$+\frac{ad^2\,\mathrm{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4b^2\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$-\frac{a(a^2-2b^2)\,d^2\cot(e+fx)\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e}}{8b^2\,(a^2+b^2)^2\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$-\frac{a(a^2-2b^2)\,d^2\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+}}{8b^2\,(a^2+b^2)^2\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$-\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2}+\frac{ad^2\sqrt{d\sec(e+fx)}}{4b\,(a^2+b^2)\,f(a+b\tan(e+fx))}$$

output
```
1/8*(a^2-2*b^2)*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(
d*sec(f*x+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/8*(a^
2-2*b^2)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(
f*x+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/4*a*d^2*(co
s(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(s
in(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/(a^2+b^2)/f/(
sec(f*x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2
)/b^2/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*cot(f*x+e)*
EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*
(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/2*d^2*(d*se
c(f*x+e))^(1/2)/b/f/(a+b*tan(f*x+e))^2+1/4*a*d^2*(d*sec(f*x+e))^(1/2)/b/(a
^2+b^2)/f/(a+b*tan(f*x+e))
```

3.618.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.26 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.66

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{d(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3\left(-\frac{2b(a^2+b^2)\sec^2(e+fx)(a^2+2b^2}{(a+b\tan(e+fx)}\right.}{}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
(d*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((-2*b*(a^2
+ b^2)*Sec[e + f*x]^2*(a^2 + 2*b^2 - a*b*Tan[e + f*x]))/(a + b*Tan[e + f*x
])^2 + (Sec[e + f*x]^2)^(3/4)*(a*(a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3
/2, -Tan[e + f*x]^2]*Tan[e + f*x] + ((a^2 - 2*b^2)*(a*EllipticPi[-(b/Sqrt[
a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + a*Elliptic
Pi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + S
qrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])*
Sqrt[-Tan[e + f*x]^2]))/Sqrt[-Tan[e + f*x]^2])))/(8*b^2*(a^2 + b^2)^2*f*(a
 + b*Tan[e + f*x])^3)
```

3.618.3 Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.70, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.760, Rules used = {3042, 3994, 492, 594, 27, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\int \frac{\sqrt[4]{\tan^2(e+fx)+1}}{(a+b\tan(e+fx))^3}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{492}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{b\tan(e+fx)}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{4b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{2(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{594}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int -\frac{2b^2+a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}}{4b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{2(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{\int\frac{2b^2+a\tan(e+fx)b}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4b^2}-\frac{\sqrt[4]{\tan^2(e+fx)+1}}{2(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\big\downarrow \; 719$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-\left(a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\sqrt[4]{\tan^2(e+f}}{\left(a^2+b^2\right)(a+b\tan(e-}}{4b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\big\downarrow \; 229$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}+\frac{ab^2\sqrt[4]{\tan^2(e+fx)}}{\left(a^2+b^2\right)(a+b\tan(e+f}}{4b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\big\downarrow \; 504$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b}{\left(\tan^2(e+fx)+1\right)}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\big\downarrow \; 312$$

3.618. $\int\frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}(a^2-b^2\tan^2(e}}{2b}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.}{bf\sqrt[4]{\sec^2(}}$$

$$\Big\downarrow \; 118$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}}{}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.}{bf\sqrt[4]{\sec}}$$

$$\Big\downarrow \; 25$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{}}{b}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.}{bf\sqrt[4]{\sec^2(}}$$

$$\Big\downarrow \; 353$$

3.618. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{}}{b}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.}{bf\sqrt[4]{\sec}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{}}{b}\right.}{2\left(a^2+b^2\right)}}{4b^2}\right.}{bf\sqrt[4]{\sec^2(}}$$

$$\Big\downarrow \text{ 756}$$

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{}}{b}\right.}{}\right.$$

$$\Big\downarrow \text{ 218}$$

3.618. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

$$d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{}}{b}\right.}{2\left(a^2+b^2\right)}}{}\right.$$

$\downarrow$ 221

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{}}{b}\right.}{2\left(a^2+b^2\right)}}{4b}\right.}{bf\sqrt[4]{\mathrm{se}}}$$

$\downarrow$ 925

3.618. $\int\frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\left(2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}}{2\left(a^2+b^2\right)}\right.}{}\right.}{}\right.}{d^2\sqrt{d\sec(e+fx)}}$$

$\downarrow$ 1537

3.618. $\int\frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\left(\frac{\frac{2ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)-\left(a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+}{b}}}}}{}\right.}{}\right.}{}}{}\right.}{}$$

$$\Big\downarrow \color{blue}{412}$$

3.618. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{d^2\sqrt{d\sec(e+fx)}\,\Bigg(\frac{2ab\,\text{EllipticF}\big(\frac{1}{2}\arctan(\tan(e+fx)),2\big)-\big(a^2-2b^2\big)\Bigg(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(-\frac{b^2\,\text{EllipticPi}\Big(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\Big(\sqrt[4]{\frac{\tan(e}{b}}}{2\big(a^2+b^2\big)}}{}}{}}{}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(-1/2*(1 + Tan[e + f*x]^2)^(1/4)/(a + b*Tan[e +
f*x])^2 + ((a*b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a + b*Tan[e +
f*x])) + (2*a*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] - (a^2 - 2*b^2)*(-2*b
^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2
)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*Sqrt[b]*(a
^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*EllipticPi[-(b/Sqrt[a^2 +
 b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a^2 + b^2) - (b^2*Ellipt
icPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(2*(a^2 +
 b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*(a^2 + b^2)))/(4*b^2)))/(b*f*(Sec[e
+ f*x]^2)^(1/4))
```

3.618. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

3.618.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.618. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

3.618. $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.618.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36013 vs. $2(489) = 978$.

Time = 1496.34 (sec) , antiderivative size = 36014, normalized size of antiderivative = 67.70

method	result	size
default	Expression too large to display	36014

input
```
int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.618.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.618.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Timed out
```

3.618.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.618.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^3, x)
```

3.618.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(a+b\tan\left(e+f\,x\right))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3, x)
```

3.619 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

3.619.1 Optimal result 4389
3.619.2 Mathematica [C] (warning: unable to verify) 4390
3.619.3 Rubi [A] (warning: unable to verify) 4390
3.619.4 Maple [B] (warning: unable to verify) 4400
3.619.5 Fricas [F(-1)] 4400
3.619.6 Sympy [F] 4400
3.619.7 Maxima [F(-1)] 4401
3.619.8 Giac [F] 4401
3.619.9 Mupad [F(-1)] 4401

3.619.1 Optimal result

Integrand size = 25, antiderivative size = 566

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{(3a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)^{3/4}}$$

$$-\frac{(3a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)^{3/4}}$$

$$-\frac{5aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)(d\sec(e+fx))^{3/2}}{4\,(a^2+b^2)^2\,f\sec^2(e+fx)^{3/4}}$$

$$+\frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4\,(a^2+b^2)^2\,f}$$

$$-\frac{a(3a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{8b\,(a^2+b^2)^{5/2}\,f\sec^2(e+fx)^{3/4}}$$

$$+\frac{a(3a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+}}{8b\,(a^2+b^2)^{5/2}\,f\sec^2(e+fx)^{3/4}}$$

$$-\frac{b(d\sec(e+fx))^{3/2}}{2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}-\frac{5ab(d\sec(e+fx))^{3/2}}{4\,(a^2+b^2)^2\,f(a+b\tan(e+fx))}$$

output

```
-5/4*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*E
llipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^
2)^2/f/(sec(f*x+e)^2)^(3/4)+5/4*a*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+
e)/(a^2+b^2)^2/f+1/8*(3*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^
2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)/
b^(1/2)-1/8*(3*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(
1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-
1/8*a*(3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2
)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)^(5/2)/f/
(sec(f*x+e)^2)^(3/4)+1/8*a*(3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)
^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/
b/(a^2+b^2)^(5/2)/f/(sec(f*x+e)^2)^(3/4)-1/2*b*(d*sec(f*x+e))^(3/2)/(a^2+b
^2)/f/(a+b*tan(f*x+e))^2-5/4*a*b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^2/f/(a+b*t
an(f*x+e))
```

3.619.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.27 (sec) , antiderivative size = 14364, normalized size of antiderivative = 25.38

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

output

```
Result too large to show
```

3.619.3 Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.73, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3994, 498, 27, 688, 27, 25, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \ 3994$$

$$\frac{(d\sec(e+fx))^{3/2}\int \frac{1}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 498$$

$$\frac{(d\sec(e+fx))^{3/2}\left(-\frac{\int -\frac{4a-b\tan(e+fx)}{2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 27$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\int \frac{4a-b\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4(a^2+b^2)}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 688$$

$$\frac{(d\sec(e+fx))^{3/2}\Bigg(\frac{-\frac{b^2\int \frac{\left(2-\frac{8a^2}{b^2}\right)b^2-5ab\tan(e+fx)}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}-\frac{5ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx)}}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \ 27$$

3.619. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}dx$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{-\frac{\int -\frac{2\left(4a^2-b^2\right)+5ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{5ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\int \frac{2\left(4a^2-b^2\right)+5ab\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{5ab^2\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\left(\tan^2(e+fx)+1\right)^{3/4}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{719}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\int \frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{5ab^2\left(}{\left(a^2+\right.}}{4(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{225}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{5/}}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{212}$$

3.619. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))+5a\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(e+\right.\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$\Big\downarrow \text{504}$$

$$\frac{(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\left(a\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}\left(a^2-b^2\tan^2(e+fx\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sec^2(e+}$$

$$\Big\downarrow \text{310}$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\int\frac{}{\sqrt[4]{\mathrm{ta}}}\right.}{2\left(a^2+b\right.}}{}\right.$$

$$\Big\downarrow \text{353}$$

3.619. $\int\frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

$$(d\sec(e+fx))^{3/2}\left(\frac{\frac{(3a^2-2b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-\frac{1}{2}\int\right.}{2\left(a^2\right.}}{}\right.$$

$$\Big\downarrow 73$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\frac{(3a^2-2b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\int\right.}{2\left(a^2+b^2\right)}}{}\right.$$

$$\Big\downarrow 827$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\frac{(3a^2-2b^2)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\left(\int\right.\right.}{}}{}\right.$$

$$\Big\downarrow 218$$

3.619. $\int\frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

$$(d\sec(e+fx))^{3/2}\Bigg(\frac{(3a^2-2b^2)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\Bigg(\int$$

$$\Big\downarrow 221$$

$$(d\sec(e+fx))^{3/2}\Bigg(\frac{(3a^2-2b^2)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{b^2\tan^2(e+fx)}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\tan^2(e+fx)+1}}{b}-2b^2\Bigg(\mathrm{ar}}{2\big(a^2+}$$

$$\Big\downarrow 993$$

$$(d\sec(e+fx))^{3/2}\Bigg(\frac{(3a^2-2b^2)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\Bigg(\frac{1}{2}b\int \frac{1}{\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\tan^2(e+fx)+1}-\frac{1}{2}b\int}{b}$$

$$\Big\downarrow 1537$$

3.619. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{1}{2}b\int\frac{1}{\sqrt{1-b^2\tan^2(e+fx)}\sqrt{b^2\tan^2(e+fx)+1}\left(\sqrt{a^2+b^2}-b^3\tan^2(e+fx)\right)}d\sqrt[4]{\tan^2(e\right.\right.}{b\quad}\right.}{}\right.$$

$$\Big\downarrow \textcolor{blue}{412}$$

$$(d\sec(e+fx))^{3/2}\left(\frac{\left(3a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(\frac{b\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\tan^2(e+fx)+1}\right),-1\right)}{2\sqrt{a^2+b^2}}-\frac{b\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2}\right.\right.}{}\right.}{b}\right.}{}\right.$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

3.619. $\int\frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

output
```
((d*Sec[e + f*x])^(3/2)*(-1/2*(b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2
)*(a + b*Tan[e + f*x])^2) + ((-5*a*b^2*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 +
 b^2)*(a + b*Tan[e + f*x])) + ((3*a^2 - 2*b^2)*(-2*b^2*(-1/2*ArcTan[(b^(3/
2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[
(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) +
 (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + T
an[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b
^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-T
an[e + f*x]^2])/b) + 5*a*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*b
*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/(2*(a^2 + b^2)))/(4*(a^2 + b^2
))))/(b*f*(Sec[e + f*x]^2)^(3/4))
```

3.619.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.619. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.619. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.619. $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

3.619.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60918 vs. $2(515) = 1030$.

Time = 13.11 (sec) , antiderivative size = 60919, normalized size of antiderivative = 107.63

method	result	size
default	Expression too large to display	60919

input
```
int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.619.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.619.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**3, x)
```

3.619.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.619.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^3, x)
```

3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3, x)
```

3.620 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

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3.620.1 Optimal result

Integrand size = 25, antiderivative size = 515

$$
\begin{aligned}
\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = &-\frac{3\sqrt{b}(5a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&-\frac{3\sqrt{b}(5a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} \\
&-\frac{7a\,\mathrm{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} \\
&+\frac{3a(5a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2}}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}} \\
&+\frac{3a(5a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}} \\
&-\frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f(a+b\tan(e+fx))}
\end{aligned}
$$

output
```
-7/4*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*E
llipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^
2)^2/f/(sec(f*x+e)^2)^(1/4)-3/8*(5*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*
b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(
sec(f*x+e)^2)^(1/4)-3/8*(5*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)
/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(sec(f*x
+e)^2)^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4
),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^
2)^3/f/(sec(f*x+e)^2)^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec
(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)
^(1/2)/(a^2+b^2)^3/f/(sec(f*x+e)^2)^(1/4)-1/2*b*(d*sec(f*x+e))^(1/2)/(a^2+
b^2)/f/(a+b*tan(f*x+e))^2-7/4*a*b*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^2/f/(a+b*
tan(f*x+e))
```

3.620.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.70 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$$

$$= \frac{(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3\left(-\frac{2b\sec^2(e+fx)(9a^2+2b^2+7ab\tan(e+fx))}{(a-ib)^2(a+ib)^2(a+b\tan(e+fx))^2}+\frac{\sec^2(e+fx)^{3/4}\left(-7a(a\right.}{}\right.}{}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((-2*b*Sec[e +
 f*x]^2*(9*a^2 + 2*b^2 + 7*a*b*Tan[e + f*x]))/((a - I*b)^2*(a + I*b)^2*(a
+ b*Tan[e + f*x])^2) + ((Sec[e + f*x]^2)^(3/4)*(-7*a*(a^2 + b^2)*Hypergeom
etric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + 3*(5*a^2 - 2*b^2)*
(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2
 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4
)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x
]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*EllipticPi[b/Sqrt[
a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2])))/(
a^2 + b^2)^3))/(8*d*f*(a + b*Tan[e + f*x])^3)
```

3.620.3 Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.76, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.840, Rules used = {3042, 3994, 498, 27, 688, 27, 25, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{3/4}}d(b\tan(e+fx))}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{498}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(-\frac{\int -\frac{4a-3b\tan(e+fx)}{2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{4a-3b\tan(e+fx)}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{688}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{-\frac{b^2\int \frac{\left(6-\frac{8a^2}{b^2}\right)b^2+7a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}-\frac{7ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{-\frac{\int -\frac{2\left(4a^2-3b^2\right)-7ab\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{\int \frac{2\left(4a^2-3b^2\right)-7ab\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}}{4(a^2+b^2)}-\frac{b^2\sqrt[4]{\tan^2(e+fx)+1}}{2(a^2+b^2)(a+b\tan(e+fx))^2}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-7a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+f}}{\left(a^2+b^2\right)(a+b\tan(e}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 229$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))-14ab\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{2\left(a^2+b^2\right)}-\frac{7ab^2\sqrt[4]{\tan^2(e+fx}}{\left(a^2+b^2\right)(a+b\tan(e+}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$\Big\downarrow \; 504$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2(e+fx)}}$$

3.620. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

$\downarrow$ 312

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{}{\left(\tan^2(e+f\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec^2}}$$

$\downarrow$ 118

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int-\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\right.}}{b}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2}\right.}{bf\sqrt[4]{\mathrm{se}}}$$

$\downarrow$ 25

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{}{\left(\tan^2(e+\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec}}$$

$\downarrow$ 353

3.620. $\int\frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int \frac{}{\left(\frac{\tan(e}{b}\right.}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b}\right.}{bf\sqrt[4]{s}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\int \frac{}{-\tan^4}\right.}{2\left(a^2+b^2\right)}}{4(a^2+b^2)}\right.}{bf\sqrt[4]{\sec}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3\left(5a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int \frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\left(\frac{\int \frac{}{\sqrt{a^2}}}{}\right.\right.}{}\right.}{}$$

$$\Big\downarrow \text{218}$$

3.620. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\Bigg(\frac{3\left(5a^2-2b^2\right)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\Bigg(\int\frac{}{\sqrt{a^2}}}{2\left(a^2+b^2\right)}}{}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d\sec(e+fx)}\Bigg(\frac{\frac{3\left(5a^2-2b^2\right)\Bigg(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\Bigg(\frac{\arctan}{2\sqrt{}}}{2\left(a^2+b^2\right)}}{4(a^2+}}{bf\sqrt[4]{}}$$

$$\Big\downarrow \text{925}$$

3.620. $\int\frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt{d\sec(e+fx)}\left(\frac{3\left(5a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{2\left(a^2+b^2\right)}-\frac{b^2\int\frac{}{\left(\tan\right.}}{}\right.}{b}\right.}{}\right.}{}$$

$$\Big\downarrow \text{1537}$$

3.620. $\int\frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

$$\sqrt{d\sec(e+fx)}\left(\frac{\frac{3\left(5a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+fx)}{b}}+}}}{2\left(a^2+b^2\right)}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow \; 412$$

3.620. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

$$\sqrt{d\sec(e+fx)}\left(\frac{3\left(5a^2-2b^2\right)\left(-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{EllipticPi}\left(\frac{}{\sqrt{}}\right.}{}\right.}{b}\right.}{}\right.$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(-1/2*(b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*
(a + b*Tan[e + f*x])^2) + ((-7*a*b^2*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b
^2)*(a + b*Tan[e + f*x])) + (-14*a*b*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]
+ 3*(5*a^2 - 2*b^2)*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/
4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 +
b^2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*
EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/
(a^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/
b)^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*(a^2 + b^2)
))/(4*(a^2 + b^2))))/(b*f*(Sec[e + f*x]^2)^(1/4))
```

3.620. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

3.620.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.620. $\int \frac{\sqrt{d \sec(e+fx)}}{(a+b \tan(e+fx))^3} \, dx$

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.620. $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.620.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75606 vs. $2(472) = 944$.

Time = 14.53 (sec) , antiderivative size = 75607, normalized size of antiderivative = 146.81

method	result	size
default	Expression too large to display	75607

input
```
int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.620.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.620.6 Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{\sqrt{d\sec\left(e+fx\right)}}{\left(a+b\tan\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**3, x)
```

3.620.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Timed out
```

3.620.8 Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^3, x)
```

3.620.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3, x)
```

3.621 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

3.621.1 Optimal result 4418
3.621.2 Mathematica [C] (warning: unable to verify) 4419
3.621.3 Rubi [A] (warning: unable to verify) 4419
3.621.4 Maple [B] (warning: unable to verify) 4432
3.621.5 Fricas [F(-1)] 4432
3.621.6 Sympy [F] 4432
3.621.7 Maxima [F] 4433
3.621.8 Giac [F] 4433
3.621.9 Mupad [F(-1)] 4433

3.621.1 Optimal result

Integrand size = 25, antiderivative size = 664

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$$

$$= \frac{5b^{3/2}(7a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{13/4}f\sqrt{d\sec(e+fx)}}$$

$$- \frac{5b^{3/2}(7a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{13/4}f\sqrt{d\sec(e+fx)}}$$

$$+ \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^3f\sqrt{d\sec(e+fx)}}$$

$$- \frac{a(8a^2-37b^2)\tan(e+fx)}{4\left(a^2+b^2\right)^3f\sqrt{d\sec(e+fx)}}$$

$$- \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2}}{8\left(a^2+b^2\right)^{7/2}f\sqrt{d\sec(e+fx)}}$$

$$+ \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^{7/2}f\sqrt{d\sec(e+fx)}}$$

$$+ \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

$$+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

$$+ \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

output

```
5/8*b^(3/2)*(7*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)-5/8*b^(3
/2)*(7*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(s
ec(f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)+1/4*a*(8*a^2-37
*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*El
lipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2
)^3/f/(d*sec(f*x+e))^(1/2)-5/8*a*b*(7*a^2-2*b^2)*cot(f*x+e)*EllipticPi((se
c(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^
2)^(1/2)/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)+5/8*a*b*(7*a^2-2*b^2)*cot(
f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)
^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)-1/4*a*
(8*a^2-37*b^2)*tan(f*x+e)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2)+1/2*b*(4*a^2-
5*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+
2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+1/4
*a*b*(8*a^2-37*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2)/(a+b*t
an(f*x+e))
```

3.621.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 39.23 (sec) , antiderivative size = 14652, normalized size of antiderivative = 22.07

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

output

```
Result too large to show
```

3.621.3 Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.77, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {3042, 3994, 496, 27, 25, 688, 27, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{5/4}}d(b\tan(e+fx))}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{2b^2\int -\frac{\left(5-\frac{a^2}{b^2}\right)b^2+3a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx)}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int -\frac{a^2-3b\tan(e+fx)a-5b^2}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{\int \frac{a^2-3b\tan(e+fx)a-5b^2}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{688}$$

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\int\frac{4a\left(8-\frac{a^2}{b^2}\right)-\left(5-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}-b^2}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\int\frac{4a\left(8-\frac{a^2}{b^2}\right)-\left(5-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4\left(a^2+b^2\right)}-b^2(}{a^2+b^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 688$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int\frac{\left(\frac{8a^4}{b^4}-\frac{72a^2}{b^2}+10\right)b^4+a(8}{2b^4(a+b\tan(e+fx))\sqrt[4]{\text{ta}}}}{}\right.}{4\left(a^2+b^2\right)}}{a^2}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 27$$

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{\int\frac{2\left(4a^4-36b^2a^2+5b^4\right)+ab\left(8a\right.}{(a+b\tan(e+fx))\sqrt[4]{\tan^2}}}{2b^2\left(\right.}\right.}{4\left(a^2+b^2\right)}}{a^2-}\right.}{bf\sqrt{d\sec(e+fx)}}$$

$\downarrow$ 719

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\int\frac{1}{\sqrt[4]{\tan^2(e}}}{}\right.}{}\right.}{bf\sqrt{d\sec(e}}$$

$\downarrow$ 225

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e}}\right.}{}\right.}{}\right.}{bf\sqrt{}}$$

$\downarrow$ 212

3.621. $\int\frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}\right.}{bf\sqrt{d}}$$

$\downarrow$ 504

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}\right.}{}$$

$\downarrow$ 310

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow 353$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow 73$$

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{827}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{218}$$

3.621. $\displaystyle\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{221}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow \text{993}$$

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.}{}$$

$\big\downarrow$ 1537

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-\frac{b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.}{}\right.}{}}{}\right.}{}$$

$\big\downarrow$ 412

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

$$a\left(8a^2-37b^2\right)\left(\frac{2b\tan(e}{\sqrt[4]{\tan^2(e-}}\right.$$

$$b^2\left(\frac{a\left(8a^2-37b^2\right)\left(\tan^2(e+fx)+1\right)^{3/4}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\right.$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{(a^2+b^2)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{-}{}\right.$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

output
```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/((a^2 + b^2)*(a + b*
Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(1/4)) - (-1/2*(b^2*(4*a^2 - 5*b^2)*(
1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x])^2) - (b^2*((a
*(8*a^2 - 37*b^2)*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e +
f*x])) - (-5*b^2*(7*a^2 - 2*b^2)*(-2*b^2*(-1/2*ArcTan[(b^(3/2)*Tan[e + f*x
])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + ArcTanh[(b^(3/2)*Tan[e
 + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/4))) + (2*a*Cot[e +
f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]^2)
^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1
 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*Sqrt[-Tan[e + f*x]^2]
)/b) + a*(8*a^2 - 37*b^2)*(-2*b*EllipticE[ArcTan[Tan[e + f*x]]/2, 2] + (2*
b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/4)))/(2*b^2*(a^2 + b^2))))/(4*(a^2
 + b^2)))/(a^2 + b^2)))/(b*f*Sqrt[d*Sec[e + f*x]])
```

3.621.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.621. $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.621. $\int \frac{1}{\sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

3.621.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78531 vs. $2(611) = 1222$.

Time = 15.80 (sec) , antiderivative size = 78532, normalized size of antiderivative = 118.27

method	result	size
default	Expression too large to display	78532

input
```
int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.621.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.621.6 Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3), x)
```

3.621.7 Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

3.621.8 Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3), x)
```

3.622 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

3.622.1 Optimal result 4434
3.622.2 Mathematica [C] (verified) 4435
3.622.3 Rubi [A] (warning: unable to verify) 4436
3.622.4 Maple [B] (warning: unable to verify) 4453
3.622.5 Fricas [F(-1)] 4453
3.622.6 Sympy [F] 4453
3.622.7 Maxima [F] 4454
3.622.8 Giac [F] 4454
3.622.9 Mupad [F(-1)] 4454

3.622.1 Optimal result

Integrand size = 25, antiderivative size = 620

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = -\frac{7b^{5/2}(9a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}}$$

$$-\frac{7b^{5/2}(9a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}}$$

$$+\frac{a(8a^2-69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}}$$

$$+\frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^4f(d\sec(e+fx))^{3/2}}$$

$$+\frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^4f(d\sec(e+fx))^{3/2}}$$

$$+\frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$

$$+\frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$

$$+\frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$

output
```
-7/8*b^(5/2)*(9*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(
1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)-7/8*b^(
5/2)*(9*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(
sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)+1/12*a*(8*a^2-
69*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*
EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(3/4)/(a^2+b
^2)^3/f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-2*b^2)*cot(f*x+e)*EllipticPi
((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x
+e)^2)^(1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-2*b^2)*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^
2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)+1/6*b*(4
*a^2-7*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e
))^2+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)
)^2+1/12*a*b*(8*a^2-69*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3/2
)/(a+b*tan(f*x+e))
```

3.622.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.85 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.66

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \frac{\frac{2(a^2+b^2)\sec^3(e+fx)\Big(b(22a^4-63a^2b^2-8b^4)\cos(e+fx)+2b(a^2+b^2)^2\cos(3(e+fx)}{(a+b\tan(e+fx))^2}}{}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

output
```
((2*(a^2 + b^2)*Sec[e + f*x]^3*(b*(22*a^4 - 63*a^2*b^2 - 8*b^4)*Cos[e + f*
x] + 2*b*(a^2 + b^2)^2*Cos[3*(e + f*x)] + a*(4*a^4 + 16*a^2*b^2 - 65*b^4 +
 4*(a^2 + b^2)^2*Cos[2*(e + f*x)])*Sin[e + f*x]))/(a + b*Tan[e + f*x])^2 +
 (Sec[e + f*x]^2)^(3/4)*(a*(8*a^4 - 61*a^2*b^2 - 69*b^4)*Hypergeometric2F1
[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (21*b^2*(-9*a^2 + 2*b^2)*(
a*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan
[e + f*x] + a*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)]
, -1]*Tan[e + f*x] + Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f
*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4)
)/(a^2 + b^2)^(1/4)])*Sqrt[-Tan[e + f*x]^2]))/Sqrt[-Tan[e + f*x]^2]))/(24*
(a^2 + b^2)^4*f*(d*Sec[e + f*x])^(3/2))
```

3.622.3 Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.80, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.920$, Rules used = {3042, 3994, 496, 27, 688, 27, 688, 27, 25, 719, 229, 504, 312, 118, 25, 353, 73, 756, 218, 221, 925, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{3/4}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{7/4}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))^2}-\frac{2b^2\int -\frac{\left(\frac{a^2}{b^2}+7\right)b^2+5a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\int \frac{a^2+5b\tan(e+fx)a+7b^2}{(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{3(a^2+b^2)}+\frac{2(ab\tan(e+fx)+b^2)}{3(a^2+b^2)(\tan^2(e+fx)+1)^{3/4}(a+b\tan(e+fx))^2}\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{688}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{\left(4a^2-7b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(e+fx))^2}-\frac{b^2\int -\frac{4a\left(\frac{a^2}{b^2}+12\right)-3\left(7-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{2(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2\left(a^2+b^2\right)}}{3(a^2+b^2)}+\frac{2(ab\tan(e}{3(a^2+b^2)(\tan^2(e+fx)+}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\downarrow \text{27}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\int \frac{4a\left(\frac{a^2}{b^2}+12\right)-3\left(7-\frac{4a^2}{b^2}\right)b\tan(e+fx)}{(a+b\tan(e+fx))^2\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{2\left(\frac{a^2}{b^2}+1\right)(a+b\tan(e+fx))^2}}{3(a^2+b^2)}+\frac{2(ab\tan(e+f}{3(a^2+b^2)(\tan^2(e+fx)+1)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\downarrow \text{688}$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(\frac{a\left(8a^2-69b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{b^2\int \frac{\left(-\frac{8a^4}{b^4}-\frac{120a^2}{b^2}+42\right)b^4-ab\left(8a^2-69b^2\right)\tan(e+fx)}{2b^4(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{a^2+b^2}\right)}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7b^2\right.}{2\left(\frac{a^2}{b^2}\right.}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\downarrow \text{27}$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(\frac{a\left(8a^2-69b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}-\frac{\int -\frac{2\left(4a^4+60b^2a^2-21b^4\right)+ab\left(8a^2-69b^2\right)\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2\left(a^2+b^2\right)}\right)}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7b^2\right)\sqrt[4]{}}{2\left(\frac{a^2}{b^2}+\right.}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 25$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(\frac{\int \frac{2\left(4a^4+60b^2a^2-21b^4\right)+ab\left(8a^2-69b^2\right)\tan(e+fx)}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2\left(a^2+b^2\right)}+\frac{a\left(8a^2-69b^2\right)\sqrt[4]{\tan^2(e+fx)+1}}{\left(a^2+b^2\right)(a+b\tan(e+fx))}\right)}{4\left(a^2+b^2\right)}+\frac{\left(4a^2-7b^2\right)\sqrt[4]{}}{2\left(\frac{a^2}{b^2}+1\right.}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow 719$$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+a\left(8a^2-69b^2\right)\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))}{2b^2\left(a^2+b^2\right)}+\frac{a\left(8\right.}{}\right.}{4\left(a^2+b^2\right)}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx)}$$

$$\Big\downarrow 229$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\int\frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{3/4}}d(b\tan(e+fx))+2ab\left(8a^2-69b^2\right)\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{2b^2\left(a^2+b^2\right)}+\frac{a\left(8a^2\right.}{}\right.}{4\left(a^2+b^2\right)}}{3(a^2+b^2)}\right.}{bf(d\sec(e+fx))}$$

$$\Big\downarrow \; 504$$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(a\int\frac{1}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan\right.\right.}{2b^2\left(a^2+b^2\right)}\right.}{4\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 312$$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{-\frac{\tan(e+fx)}{b}}\left(\frac{\tan(e+fx)}{b}+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{2b}-\int\frac{}{\left(\text{ta}\right.}\right.}{2b^2\left(a^2+b^2\right)}\right.}{}\right.$$

$$\Big\downarrow \; 118$$

3.622. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{3/4}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int -\frac{}{\sqrt{1-b^4\tan^4(e+}}}{}\right.}{2b^2\left(a^2+b^2\right)}\right.}{}\right.}{}$$

$$\Big\downarrow \color{blue}{25}$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\int\frac{}{\left(\right.}\right.}{2b^2\left(a^2+b^2\right)}\right.}{}\right.}{}$$

$$\Big\downarrow \; 353$$

3.622. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-\frac{1}{2}\int\right.}{2b^2\left(a^2+b^2\right)}\right.}{}\right.$$

$$\Big\downarrow \; 73$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\right.}{2b^2\left(a^2+b^2\right)}\right.}{}}{}\right.$$

$$\Big\downarrow \textcolor{blue}{756}$$

3.622. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\right.\right.\right.}{}}{}\right.$$

$$\Big\downarrow \text{218}$$

3.622. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{b^2\left(21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\right.\right.}{2}\right.$$

$$\Big\downarrow \textcolor{blue}{221}$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\sec^2(e+fx)^{3/4}\frac{\left(\frac{b^2\left(\frac{21b^2\left(9a^2-2b^2\right)\left(\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\int\frac{1}{\sqrt{1-b^4\tan^4(e+fx)}\left(-b^4\tan^4(e+fx)+\frac{a^2}{b^2}+1\right)}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}}{b}-2b^2\right.}{2b^2\left(a^2+b^2\right)}\right.}{}\right.}{}$$

$$\Big\downarrow \textcolor{blue}{925}$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\sec^2(e+fx)^{3/4}\left(\frac{\frac{b^2\left(21b^2\left(9a^2-2b^2\right)-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-b^4\tan^4(e+fx)}}d\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\quad b^2}{2\left(a^2+b^2\right)}-- \right.}{b}\right.}{}}{}\right.$$

$$\Big\downarrow \text{1537}$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$b^2\int\frac{1}{\left(1-\frac{b^3\tan^2(e+fx)}{\sqrt{a^2+b^2}}\right)\sqrt{1-\sqrt[4]{\frac{\tan(e+fx)}{b}}+1}\sqrt{\sqrt[4]{\frac{\tan(e+f}{b}}}}$$

$$2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{}{2\left(a^2+b^2\right)}\right.$$

$$21b^2\left(9a^2-2b^2\right)-$$

$$b^2$$

$$\sec^2(e+fx)^{3/4}$$

3.622. $\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

$$\Big\downarrow {\color{blue}412}$$

$$\sec^2(e+fx)^{3/4}\Bigg(\frac{b^2\Bigg(21b^2\left(9a^2-2b^2\right)-\frac{2a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\left(-\frac{b^2\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\frac{\tan(e+fx)}{b}+1}\right),-1\right)}{2\left(a^2+b^2\right)}-\frac{b^2\,\text{Ellipt}}{}\right.}{b}}{}$$

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

input

```
Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

output

```
((Sec[e + f*x]^2)^(3/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(3*(a^2 + b^2)*(a +
b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(3/4)) + (((4*a^2 - 7*b^2)*(1 + Tan
[e + f*x]^2)^(1/4))/(2*(1 + a^2/b^2)*(a + b*Tan[e + f*x])^2) + (b^2*((a*(8
*a^2 - 69*b^2)*(1 + Tan[e + f*x]^2)^(1/4))/((a^2 + b^2)*(a + b*Tan[e + f*x
])) + (2*a*b*(8*a^2 - 69*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2] + 21*b^
2*(9*a^2 - 2*b^2)*(-2*b^2*(ArcTan[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)
]/(2*Sqrt[b]*(a^2 + b^2)^(3/4)) + ArcTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^
2)^(1/4)]/(2*Sqrt[b]*(a^2 + b^2)^(3/4))) - (2*a*Cot[e + f*x]*(-1/2*(b^2*El
lipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(1 + Tan[e + f*x]/b)^(1/4)], -1])/(a
^2 + b^2) - (b^2*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(1 + Tan[e + f*x]/b)
^(1/4)], -1])/(2*(a^2 + b^2)))*Sqrt[-Tan[e + f*x]^2])/b))/(2*b^2*(a^2 + b^
2))))/(4*(a^2 + b^2)))/(3*(a^2 + b^2))))/(b*f*(d*Sec[e + f*x])^(3/2))
```

3.622.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 118

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_] :> Simp[-4   Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e/f) +
 d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& GtQ[-f/(d*e - c*f), 0]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 312
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[Sqrt[(-b)*(x^2/a)]/(2*x)   Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*
(c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.622. $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

3.622.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76276 vs. 2(573) = 1146.

Time = 17.74 (sec) , antiderivative size = 76277, normalized size of antiderivative = 123.03

method	result	size
default	Expression too large to display	76277

input
```
int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.622.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.622.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)^{3}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3), x)
```

3.622.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

3.622.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))^3}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3),x)
```

output
```
int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3), x)
```

3.623 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

3.623.1 Optimal result 4456
3.623.2 Mathematica [C] (warning: unable to verify) 4457
3.623.3 Rubi [A] (warning: unable to verify) 4458
3.623.4 Maple [B] (warning: unable to verify) 4476
3.623.5 Fricas [F(-1)] 4476
3.623.6 Sympy [F] 4477
3.623.7 Maxima [F(-2)] 4477
3.623.8 Giac [F] 4477
3.623.9 Mupad [F(-1)] 4478

3.623.1 Optimal result

Integrand size = 25, antiderivative size = 814

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \frac{9b^{7/2}(11a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}$$

$$-\frac{9b^{7/2}(11a^2-2b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}$$

$$+\frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}}$$

$$-\frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}}$$

$$-\frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}}$$

$$+\frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}}$$

$$+\frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

$$+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

$$+\frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

$$-\frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

output

```
9/8*b^(7/2)*(11*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(
1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/8
*b^(7/2)*(11*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/
4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)+3/20*
a*(8*a^4+64*a^2*b^2-139*b^4)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2
*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f
*x+e)^2)^(1/4)/(a^2+b^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)-9/8*a*b^3*(11*a^2-2*
b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec
(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(9/2)/d^2/f/(d*sec(f*x+e)
)^(1/2)+9/8*a*b^3*(11*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4
),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2
)^(9/2)/d^2/f/(d*sec(f*x+e))^(1/2)-3/20*a*(8*a^4+64*a^2*b^2-139*b^4)*tan(f
*x+e)/(a^2+b^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)+3/10*b*(4*a^4+28*a^2*b^2-15*b
^4)*sec(f*x+e)^2/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2
+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)/(a
+b*tan(f*x+e))^2+3/20*a*b*(8*a^4+64*a^2*b^2-139*b^4)*sec(f*x+e)^2/(a^2+b^2
)^4/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))-2/5*(b*(4*a^2-9*b^2)-a*(3*
a^2+16*b^2)*tan(f*x+e))/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*
x+e))^2
```

3.623.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.97 (sec) , antiderivative size = 15481, normalized size of antiderivative = 19.02

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

input

```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

output

```
Result too large to show
```

3.623.3 Rubi [A] (warning: unable to verify)

Time = 1.20 (sec) , antiderivative size = 626, normalized size of antiderivative = 0.77, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.960, Rules used = {3042, 3994, 496, 27, 686, 27, 25, 688, 27, 688, 27, 719, 225, 212, 504, 310, 353, 73, 827, 218, 221, 993, 1537, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^3(\tan^2(e+fx)+1)^{9/4}}d(b\tan(e+fx))}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{496}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))^2}-\frac{2b^2\int -\frac{3\left(\frac{a^2}{b^2}+3\right)b^2+7a\tan(e+fx)b}{2b^2(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\int \frac{3\left(a^2+3b^2\right)+7ab\tan(e+fx)}{(a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{5/4}}d(b\tan(e+fx))}{5(a^2+b^2)}+\frac{2\left(ab\tan(e+fx)+b^2\right)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))^2}\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{686}$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{2b^4\int -\frac{3\left(\left(-\frac{a^4}{b^4}-\frac{12a^2}{b^2}+15\right)b^4+a\left(3a^2+16b^2\right)\tan(e+fx)b\right)}{2b^4(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{3\int -\frac{a^4+12b^2a^2-b\left(3a^2+16b^2\right)\tan(e+fx)a-15b^4}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}+\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}}{5(a^2+b^2)}+\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 25$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\int \frac{a^4+12b^2a^2-b\left(3a^2+16b^2\right)\tan(e+fx)a-15b^4}{(a+b\tan(e+fx))^3\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{a^2+b^2}}{5(a^2+b^2)}+\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow 688$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(-\frac{b^2\int -\frac{4ab^2\left(\frac{a^4}{b^2}+9a^2-31b^2\right)-b\left(4a^4+28b^2a^2-15b^4\right)\tan(e+fx)}{2b^2(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e}{2\left(a^2+b^2\right)}\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\int\frac{4a\left(a^4+9b^2a^2-31b^4\right)-b\left(4a^4+28b^2a^2-15b^4\right)\tan(e+fx)}{(a+b\tan(e+fx))^2\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{4\left(a^2+b^2\right)}-\right.}{a^2+b^2}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \; 688$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{-\frac{b^2\int-\frac{\left(\frac{8a^6}{b^2}+64a^4-304b^2a^2+30b^4\right)b^2+a\left(8a^4+64b^2a^2-139b^4\right)\tan(e+f}{2b^2(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}}{a^2+b^2}}{4\left(a^2+b^2\right.}\right.}{}}{5(a^2+b^2)}\right.}{bd^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{\int\frac{2\left(4a^6+32b^2a^4-152b^4a^2+15b^6\right)+ab\left(8a^4+64b^2a^2-139b^4\right)\tan(e+fx)}{(a+b\tan(e+fx))\sqrt[4]{\tan^2(e+fx)+1}}d(b}{2\left(a^2+b^2\right)}}{4\left(a^2+b^2\right)}\right.}{}}{5(a^2+b^2)}\right.}{bd^2f\sqrt{}}$$

$$\Big\downarrow \; 719$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\int\frac{1}{\sqrt[4]{\tan^2(e+fx)+1}}d(b\tan(e+fx))-15b^4}{2\left(a^2\right.}\right.}{}}{}\right.$$

$$\Big\downarrow 225$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-\int\frac{1}{\left(\tan^2(e+fx)+1\right)}\right.}{}\right.}{}}{}\right.$$

$$\Big\downarrow 212$$

3.623. $\displaystyle\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}\right.}{}\right.$$

$$\big\downarrow \text{504}$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}\right.}{}\right.$$

$$\big\downarrow \text{310}$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}}{}\right.}{}\right.}{}$$

$$\Big\downarrow \; 353$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}\right.}{}\right.}{}$$

$$\Big\downarrow \text{73}$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{}{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}\,(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}}{}\right.}{}\right.}}$$

$$\Big\downarrow \; 827$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.\right.}{}}{}\right.}{}}{}\right.}{}$$

$$\Big\downarrow \ 218$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}}{}\right.}{}\right.}{}$$

$$\Big\downarrow \color{blue}{221}$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow 993$$

$$3.623. \qquad \int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow \color{blue}{1537}$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\frac{\sqrt[4]{\sec^2(e+fx)}\left(\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{a\left(8a^4+64a^2b^2-139b^4\right)\left(\frac{2b\tan(e+fx)}{\sqrt[4]{\tan^2(e+fx)+1}}-2bE\left(\frac{1}{2}\arctan(\tan(\right.\right.}{}}{}\right.}{}\right.}{}$$

$$\Big\downarrow 412$$

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

$$\sqrt[4]{\sec^2(e+fx)}\left(\frac{2(ab\tan(e+fx)+b^2)}{5(a^2+b^2)(\tan^2(e+fx)+1)^{5/4}(a+b\tan(e+fx))^2}+\frac{\frac{2\left(ab\left(3a^2+16b^2\right)\tan(e+fx)+b^4\left(9-\frac{4a^2}{b^2}\right)\right)}{\left(a^2+b^2\right)\sqrt[4]{\tan^2(e+fx)+1}(a+b\tan(e+fx))^2}-\frac{3\left(\frac{\frac{a\left(8a^4+64a^2b^2-1\right.}{}}{}\right.}{}}{}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

output

```
((Sec[e + f*x]^2)^(1/4)*((2*(b^2 + a*b*Tan[e + f*x]))/(5*(a^2 + b^2)*(a +
b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(5/4)) + ((2*((9 - (4*a^2)/b^2)*b^4
 + a*b*(3*a^2 + 16*b^2)*Tan[e + f*x]))/((a^2 + b^2)*(a + b*Tan[e + f*x])^2
*(1 + Tan[e + f*x]^2)^(1/4)) - (3*(-1/2*(b^2*(4*a^4 + 28*a^2*b^2 - 15*b^4)
*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2)*(a + b*Tan[e + f*x])^2) + (-((a*
b^2*(8*a^4 + 64*a^2*b^2 - 139*b^4)*(1 + Tan[e + f*x]^2)^(3/4))/((a^2 + b^2
)*(a + b*Tan[e + f*x]))) + (-15*b^4*(11*a^2 - 2*b^2)*(-2*b^2*(-1/2*ArcTan[
(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(b^(3/2)*(a^2 + b^2)^(1/4)) + Ar
cTanh[(b^(3/2)*Tan[e + f*x])/(a^2 + b^2)^(1/4)]/(2*b^(3/2)*(a^2 + b^2)^(1/
4))) + (2*a*Cot[e + f*x]*(-1/2*(b*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[
(1 + Tan[e + f*x]^2)^(1/4)], -1])/Sqrt[a^2 + b^2] + (b*EllipticPi[b/Sqrt[a
^2 + b^2], ArcSin[(1 + Tan[e + f*x]^2)^(1/4)], -1])/(2*Sqrt[a^2 + b^2]))*S
qrt[-Tan[e + f*x]^2])/b) + a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*(-2*b*Elliptic
E[ArcTan[Tan[e + f*x]]/2, 2] + (2*b*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^(1/
4)))/(2*(a^2 + b^2)))/(4*(a^2 + b^2))))/(a^2 + b^2))/(5*(a^2 + b^2))))/(b*
d^2*f*Sqrt[d*Sec[e + f*x]])
```

3.623.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 310
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[2*(Sqrt[(-b)*(x^2/a)]/x)   Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*
x^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1537
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Simp[Sqrt[-c]   Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqr
t[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] &
& GtQ[a, 0] && LtQ[c, 0]
```

3.623. $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.623.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89814 vs. 2(755) = 1510.

Time = 22.57 (sec) , antiderivative size = 89815, normalized size of antiderivative = 110.34

method	result	size
default	Expression too large to display	89815

input
```
int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.623.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.623.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\int \frac{1}{(d\sec{(e+fx)})^{\frac{5}{2}}\,(a+b\tan{(e+fx)})^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**3), x)
```

3.623.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.623.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)^3}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3), x)
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,(a+b\tan\left(e+f\,x\right))^3}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3), x)
```

3.624 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$

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3.624.9 Mupad [F(-1)] 4483

3.624.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx=\frac{3b(d\sec(e+fx))^{5/3}}{5f}+\frac{3ad\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

output
```
3/5*b*(d*sec(f*x+e))^(5/3)/f+3/2*a*d*hypergeom([-1/3, 1/2],[2/3],cos(f*x+e
)^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/2)
```

3.624.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx=\frac{3d(d\sec(e+fx))^{2/3}\Big(b\sec(e+fx)+a\csc(e+fx)\,\text{Hypergeometric2F1}\big(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(}{5f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*d*(d*Sec[e + f*x])^(2/3)*(b*Sec[e + f*x] + a*Csc[e + f*x]*Hypergeometri
c2F1[1/2, 5/6, 11/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(5*f)
```

3.624.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^{5/3}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\cos(e+fx)}{d}\right)^{5/3}}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}}dx+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3ad\sin(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*a*d*Hypergeometric2F1[-1/3, 1/2, 2
/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(2*f*Sqrt[Sin[e +
 f*x]^2])
```

3.624.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.624.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{5}{3}}(a+b\tan(fx+e))\,dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

3.624.5 Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*d*sec(f*x + e)*tan(f*x + e) + a*d*sec(f*x + e))*(d*sec(f*x + e
))^(2/3), x)
```

3.624.6 Sympy [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx=\int (d\sec(e+fx))^{\frac{5}{3}}(a+b\tan(e+fx))\;dx$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x)), x)
```

3.624.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

3.624.8 Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx=\int (d\sec{(fx+e)})^{\frac{5}{3}}\,(b\tan{(fx+e)}+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

3.624.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^{5/3}\,(a+b\tan(e+f\,x))\,\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)), x)
```

3.625 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$

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3.625.8 Giac [F] 4488
3.625.9 Mupad [F(-1)] 4488

3.625.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sin(e+fx)}{2f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}}$$

output
```
3*b*(d*sec(f*x+e))^(1/3)/f-3/2*a*d*hypergeom([1/3, 1/2],[4/3],cos(f*x+e)^2
)*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)
```

3.625.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3\sqrt[3]{d\sec(e+fx)}\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*(d*Sec[e + f*x])^(1/3)*(b + a*Cot[e + f*x]*Hypergeometric2F1[1/6, 1/2,
7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

3.625.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{d \sec (e+f x)}(a+b \tan (e+f x)) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt[3]{d \sec (e+f x)}(a+b \tan (e+f x))dx$$

$$\Big\downarrow \text{ 3967}$$

$$a \int \sqrt[3]{d \sec (e+f x)}dx+\frac{3 b \sqrt[3]{d \sec (e+f x)}}{f}$$

$$\Big\downarrow \text{ 3042}$$

$$a \int \sqrt[3]{d \csc \left(e+f x+\frac{\pi }{2}\right)}dx+\frac{3 b \sqrt[3]{d \sec (e+f x)}}{f}$$

$$\Big\downarrow \text{ 4259}$$

$$a \sqrt[3]{\frac{\cos (e+f x)}{d}} \sqrt[3]{d \sec (e+f x)} \int \frac{1}{\sqrt[3]{\frac{\cos (e+f x)}{d}}}dx+\frac{3 b \sqrt[3]{d \sec (e+f x)}}{f}$$

$$\Big\downarrow \text{ 3042}$$

$$a \sqrt[3]{\frac{\cos (e+f x)}{d}} \sqrt[3]{d \sec (e+f x)} \int \frac{1}{\sqrt[3]{\frac{\sin \left(e+f x+\frac{\pi }{2}\right)}{d}}}dx+\frac{3 b \sqrt[3]{d \sec (e+f x)}}{f}$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3 b \sqrt[3]{d \sec (e+f x)}}{f}-\frac{3 a d \sin (e+f x) \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos ^2(e+f x)\right)}{2 f \sqrt{\sin ^2(e+f x)} (d \sec (e+f x))^{2/3}}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

output
```
(3*b*(d*Sec[e + f*x])^(1/3))/f - (3*a*d*Hypergeometric2F1[1/3, 1/2, 4/3, C
os[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)*Sqrt[Sin[e + f*x]
^2])
```

3.625.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

3.625.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{1}{3}} (a+b\tan (fx+e))\, dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

3.625.5 Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

3.625.6 Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\;dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x)), x)
```

3.625.7 Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

3.625.8 Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

3.625.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+b\tan(e+f\,x))\;dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)), x)
```

3.626 $\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

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3.626.9 Mupad [F(-1)] 4493

3.626.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)\sin(e+fx)}{4f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}}$$

output

```
-3*b/f/(d*sec(f*x+e))^(1/3)-3/4*a*d*hypergeom([1/2, 2/3],[5/3],cos(f*x+e)^
2)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)
```

3.626.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

input

```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

output

```
(-3*(b + a*Cot[e + f*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3))
```

3.626.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\sqrt[3]{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\cos(e+fx)}{d}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}}dx-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3ad\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

3.626. $\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$

output
```
(-3*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*a*d*Hypergeometric2F1[1/2, 2/3, 5/3
, Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4/3)*Sqrt[Sin[e + f
*x]^2])
```

3.626.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.626.4 Maple [F]

$$\int \frac{a+b\tan(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

3.626.5 Fricas [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*(b*tan(f*x + e) + a)/(d*sec(f*x + e)), x)
```

3.626.6 Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(1/3), x)
```

3.626.7 Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

3.626.8 Giac [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3), x)
```

3.627 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

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3.627.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = & -\frac{3b}{5f(d\sec(e+fx))^{5/3}} \\ & -\frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output
```
-3/5*b/f/(d*sec(f*x+e))^(5/3)-3/8*a*d*hypergeom([1/2, 4/3],[7/3],cos(f*x+e
)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)
```

3.627.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = & \\ & -\frac{3\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{5f(d\sec(e+fx))^{5/3}}\end{aligned}$$

input
```
Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*(b + a*Cot[e + f*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*
Sqrt[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3))
```

3.627.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3967}$$

$$a\int \frac{1}{(d\sec(e+fx))^{5/3}}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$a\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4259}$$

$$a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\cos(e+fx)}{d}\right)^{5/3}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}dx-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*a*d*Hypergeometric2F1[1/2, 4/3, 7
/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8/3)*Sqrt[Sin[e +
 f*x]^2])
```

3.627.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.627.4 Maple [F]

$$\int \frac{a+b\tan(fx+e)}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

3.627.5 Fricas [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)/(d^2*sec(f*x + e)^2),
 x)
```

3.627.6 Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{a+b\tan{(e+fx)}}{(d\sec{(e+fx)})^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

output
```
Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/3), x)
```

3.627.7 Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

3.627.8 Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

3.627.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{a+b\tan\left(e+f\,x\right)}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/3}}\,dx$$

input

```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3),x)
```

output

```
int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3), x)
```

3.628 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$

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3.628.9 Mupad [F(-1)] 4504

3.628.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{33ab(d\sec(e+fx))^{5/3}}{40f} + \frac{3(8a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{16f\sqrt{\sin^2(e+fx)}} + \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

output
```
33/40*a*b*(d*sec(f*x+e))^(5/3)/f+3/16*(8*a^2-3*b^2)*d*hypergeom([-1/3, 1/2
],[2/3],cos(f*x+e)^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/
2)+3/8*b*(d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))/f
```

3.628.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{3(d\sec(e+fx))^{5/3}\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e+fx)\right)\tan(e+fx)+a\right.}{5f\sqrt{-\tan}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
(3*(d*Sec[e + f*x])^(5/3)*(b^2*Hypergeometric2F1[-1/2, 5/6, 11/6, Sec[e +
f*x]^2]*Tan[e + f*x] + a*(-(a*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[e + f*
x]^2]*Tan[e + f*x]) + 2*b*Sqrt[-Tan[e + f*x]^2])))/(5*f*Sqrt[-Tan[e + f*x]
^2])
```

3.628.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{3}{8}\int \frac{1}{3}(d\sec(e+fx))^{5/3}\left(8a^2+11b\tan(e+fx)a-3b^2\right)dx+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int (d\sec(e+fx))^{5/3}\left(8a^2+11b\tan(e+fx)a-3b^2\right)dx+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int (d\sec(e+fx))^{5/3}\left(8a^2+11b\tan(e+fx)a-3b^2\right)dx+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\int (d\sec(e+fx))^{5/3}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{4259}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\cos(e+fx)}{d}\right)^{5/3}}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\left(8a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \frac{1}{\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}}dx+\frac{33ab(d\sec(e+fx))^{5/3}}{5f}\right)+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

$$\Big\downarrow \text{3122}$$

$$\frac{1}{8}\left(\frac{3d\left(8a^2-3b^2\right)\sin(e+fx)(d\sec(e+fx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}+\frac{33ab(d\sec(e+fx}{5f}\right.$$
$$\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

input
```
Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
((33*a*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*(8*a^2 - 3*b^2)*d*Hypergeometr
ic2F1[-1/3, 1/2, 2/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])
/(2*f*Sqrt[Sin[e + f*x]^2]))/8 + (3*b*(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e
+ f*x]))/(8*f)
```

3.628. $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$

3.628.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.628.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{5}{3}} (a+b\tan(fx+e))^2\,dx$$

input
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

3.628.5 Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}\,(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*d*sec(f*x + e)*tan(f*x + e)^2 + 2*a*b*d*sec(f*x + e)*tan(f*x
 + e) + a^2*d*sec(f*x + e))*(d*sec(f*x + e))^(2/3), x)
```

3.628.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e))**2,x)
```

output
```
Timed out
```

3.628.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}\,(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

3.628.8 Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2, x)
```

3.629 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

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3.629.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\begin{aligned}\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx &= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} \\ &\quad - \frac{3(4a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}} \\ &\quad + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}\end{aligned}$$

output
```
21/4*a*b*(d*sec(f*x+e))^(1/3)/f-3/8*(4*a^2-3*b^2)*d*hypergeom([1/3, 1/2],[
4/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)+
3/4*b*(d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))/f
```

3.629.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{3\sqrt[3]{d\sec(e+fx)}\bigg(\frac{b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\tan(e+fx)}{\sqrt{-\tan^2(e+fx)}}+a\Big(2b+a\cot(e+fx)\,\text{Hypergeometric2F1}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
(3*(d*Sec[e + f*x])^(1/3)*((b^2*Hypergeometric2F1[-1/2, 1/6, 7/6, Sec[e +
f*x]^2]*Tan[e + f*x])/Sqrt[-Tan[e + f*x]^2] + a*(2*b + a*Cot[e + f*x]*Hype
rgeometric2F1[1/6, 1/2, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2])))/f
```

3.629.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{3}{4}\int \frac{1}{3}\sqrt[3]{d\sec(e+fx)}\left(4a^2+7b\tan(e+fx)a-3b^2\right)dx+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \sqrt[3]{d\sec(e+fx)}\left(4a^2+7b\tan(e+fx)a-3b^2\right)dx+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sqrt[3]{d\sec(e+fx)}\left(4a^2+7b\tan(e+fx)a-3b^2\right)dx+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\int \sqrt[3]{d\sec(e+fx)}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\int \sqrt[3]{d\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 4259$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \frac{1}{\sqrt[3]{\frac{\cos(e+fx)}{d}}}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\left(\left(4a^2-3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \frac{1}{\sqrt[3]{\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}}}dx+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

$$\Big\downarrow \; 3122$$

$$\frac{1}{4}\left(\frac{21ab\sqrt[3]{d\sec(e+fx)}}{f}-\frac{3d\left(4a^2-3b^2\right)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}\right)+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

output
```
((21*a*b*(d*Sec[e + f*x])^(1/3))/f - (3*(4*a^2 - 3*b^2)*d*Hypergeometric2F
1[1/3, 1/2, 4/3, Cos[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)
*Sqrt[Sin[e + f*x]^2]))/4 + (3*b*(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x
]))/(4*f)
```

3.629. $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

3.629.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.629.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} (a+b\tan(fx+e))^2 \,dx$$

input
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

3.629.5 Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{1}{3}}\left(b\tan{(fx+e)}+a\right)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
(1/3), x)
```

3.629.6 Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int \sqrt[3]{d\sec{(e+fx)}}(a+b\tan{(e+fx)})^2\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2, x)
```

3.629.7 Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{1}{3}}\left(b\tan{(fx+e)}+a\right)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

3.629.8 Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{1}{3}}\,(b\tan{(fx+e)}+a)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^{1/3}\,{\left(a+b\tan(e+f\,x)\right)}^2\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2, x)
```

3.630 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

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3.630.8 Giac [F] . . . 4516
3.630.9 Mupad [F(-1)] . . . 4516

3.630.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx &= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}}\\ &\quad-\frac{3(2a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}}\\ &\quad+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}\end{aligned}$$

output
```
-15/2*a*b/f/(d*sec(f*x+e))^(1/3)-3/8*(2*a^2-3*b^2)*d*hypergeom([1/2, 2/3],
[5/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)
+3/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)
```

3.630.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)+a\left(-a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6}\right.\right.\right.}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*(b^2*Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] +
 a*(-(a*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Tan[e + f*x]) +
2*b*Sqrt[-Tan[e + f*x]^2])))/(f*(d*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^
2])
```

3.630.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \; 3993$$

$$\frac{3}{2}\int \frac{2a^2+5b\tan(e+fx)a-3b^2}{3\sqrt[3]{d\sec(e+fx)}}dx+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{2a^2+5b\tan(e+fx)a-3b^2}{\sqrt[3]{d\sec(e+fx)}}dx+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\int \frac{2a^2+5b\tan(e+fx)a-3b^2}{\sqrt[3]{d\sec(e+fx)}}dx+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 3967$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\int \frac{1}{\sqrt[3]{d\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 4259$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\cos(e+fx)}{d}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 3042$$

$$\frac{1}{2}\left(\left(2a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\int \sqrt[3]{\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}}dx-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 3122$$

$$\frac{1}{2}\left(-\frac{3d\left(2a^2-3b^2\right)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{15ab}{f\sqrt[3]{d\sec(e+fx)}}\right)+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

input

```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

output

```
((-15*a*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*(2*a^2 - 3*b^2)*d*Hypergeometri
c2F1[1/2, 2/3, 5/3, Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4
/3)*Sqrt[Sin[e + f*x]^2]))/2 + (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e +
f*x])^(1/3))
```

3.630. $\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}dx$

3.630.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.630.4 Maple [F]

$$\int \frac{(a+b\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

3.630.5 Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
(2/3)/(d*sec(f*x + e)), x)
```

3.630.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(1/3), x)
```

3.630.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

3.630.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan\left(fx+e\right)+a)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3), x)
```

3.631 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

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3.631.2 Mathematica [A] (verified) 4517
3.631.3 Rubi [A] (verified) 4518
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3.631.7 Maxima [F] 4521
3.631.8 Giac [F] 4522
3.631.9 Mupad [F(-1)] 4522

3.631.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3(2a^2+3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sin(e+fx)}{16f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

output
```
3/10*a*b/f/(d*sec(f*x+e))^(5/3)-3/16*(2*a^2+3*b^2)*d*hypergeom([1/2, 4/3],
[7/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)
-3/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)
```

3.631.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx =$$

$$-\frac{3\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sin(e+fx)+a\left(-a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\text{se}\right.\right.}{5df(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
(-3*(b^2*Hypergeometric2F1[-5/6, -1/2, 1/6, Sec[e + f*x]^2]*Sin[e + f*x] +
 a*(-(a*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*Sin[e + f*x]) +
2*b*Cos[e + f*x]*Sqrt[-Tan[e + f*x]^2])))/(5*d*f*(d*Sec[e + f*x])^(2/3)*Sq
rt[-Tan[e + f*x]^2])
```

3.631.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3993, 27, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3993}$$

$$-\frac{3}{2}\int -\frac{2a^2-b\tan(e+fx)a+3b^2}{3(d\sec(e+fx))^{5/3}}dx-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{2a^2-b\tan(e+fx)a+3b^2}{(d\sec(e+fx))^{5/3}}dx-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{2a^2-b\tan(e+fx)a+3b^2}{(d\sec(e+fx))^{5/3}}dx-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\int \frac{1}{(d\sec(e+fx))^{5/3}}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\int \frac{1}{\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/3}}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{4259}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\cos(e+fx)}{d}\right)^{5/3}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(2a^2+3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{5/3}dx+\frac{3ab}{5f(d\sec(e+fx))^{5/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{1}{2}\left(\frac{3ab}{5f(d\sec(e+fx))^{5/3}}-\frac{3d\left(2a^2+3b^2\right)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}\right)-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

input
```
Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

output
```
((3*a*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*(2*a^2 + 3*b^2)*d*Hypergeometri
c2F1[1/2, 4/3, 7/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8
/3)*Sqrt[Sin[e + f*x]^2]))/2 - (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e +
f*x])^(5/3))
```

3.631.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.631. $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] && !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

3.631.4 Maple [F]

$$\int \frac{(a+b\tan(fx+e))^{2}}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

input
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

output
```
int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

3.631.5 Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
(1/3)/(d^2*sec(f*x + e)^2), x)
```

3.631.6 Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

output
```
Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/3), x)
```

3.631.7 Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

3.631.8 Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(b\tan\left(fx+e\right)+a)^2}{(d\sec\left(fx+e\right))^{\frac{5}{3}}}\,dx$$

input
```
integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

3.631.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx=\int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

input
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3),x)
```

output
```
int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3), x)
```

3.632 $\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$

3.632.1 Optimal result 4523
3.632.2 Mathematica [C] (warning: unable to verify) 4524
3.632.3 Rubi [A] (warning: unable to verify) 4525
3.632.4 Maple [F] 4530
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3.632.8 Giac [F] 4531
3.632.9 Mupad [F(-1)] 4532

3.632.1 Optimal result

Integrand size = 25, antiderivative size = 552

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = & -\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
& +\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
& -\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
& +\frac{\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
& -\frac{\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
& +\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}}
\end{aligned}$$

output
```
-arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(5/3
)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(5/6)+1/4*ln((a^2+b^2)^(1/3)-b^
(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(
d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(5/6)-1/4*ln(
(a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(
f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e
)^2)^(5/6)+1/2*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b
^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)^(1/6)/f/
(sec(f*x+e)^2)^(5/6)+1/2*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/
6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)
^(1/6)/f/(sec(f*x+e)^2)^(5/6)+AppellF1(1/2,1,1/6,3/2,b^2*tan(f*x+e)^2/a^2,
-tan(f*x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(5/6)
```

3.632.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.91 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.50

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx =$$

$$-\frac{24d^2\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{6},\frac{1}{6},\frac{4}{3},\frac{a-ib}{a+b\tan(e+fx)},\right.}{bf\sqrt[3]{d\sec(e+fx)}\left((a+ib)\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{6},\frac{7}{6},\frac{7}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)\,\text{AppellF1}\left(\frac{4}{3},\frac{7}{6},\frac{1}{6},\right.\right.}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

output
```
(-24*d^2*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*(d*Sec[e + f*x])^(1
/3)*((a + I*b)*AppellF1[4/3, 1/6, 7/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x])
, (a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[4/3, 7/6, 1/6, 7/3,
 (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 8*Appel
lF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*(a + b*Tan[e + f*x])))
```

3.632.3 Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.69, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3994, 504, 333, 353, 73, 825, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{1}{(a+b\tan(e+fx))\sqrt[6]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{504}$$

$$\frac{(d\sec(e+fx))^{5/3}\left(a\int \frac{1}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+}\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{333}$$

$$\frac{(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))}d(\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{353}$$

$$\frac{(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\sqrt[6]{\frac{\tan(e+fx)}{b}+1}(a^2-b^2\tan^2(e+fx))}d\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\int\frac{b^4\tan^4(e+fx)}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+fx)}{b}}}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \text{825}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e}{b}}}{3b^{4/3}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e}{b}}}{3b^{4/3}}\right.\right.$$

$$\Big\downarrow \text{221}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\int\frac{\tan(e+fx)b^{4/3}+\sqrt[6]{a^2+b}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^4}}{6b^{4/3}\sqrt[6]{}}\right.\right.$$

$$\Big\downarrow \text{1142}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+}}}{}\right.\right.$$

$$\Big\downarrow \text{25}$$

3.632. $\int\frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}dx$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+}}}{}\right.\right.$$

$$\Big\downarrow \color{blue}{27}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+}}}{}\right.\right.$$

$$\Big\downarrow \color{blue}{1082}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{3\int\frac{1}{\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}-4}d\left(1-\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}\right.}{\sqrt[3]{b}}}{}\right.\right.$$

$$\Big\downarrow \color{blue}{217}$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{-\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+f}}{}\right.\right.$$

$$\Big\downarrow \color{blue}{1103}$$

3.632. $\int\frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\,\mathrm{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(-\frac{\frac{\log\left(\sqrt[3]{a^2+b^2}-b^{4/3}\sqrt[6]{a^2+b^2}\tan(e+f\right.}{2\sqrt[3]{b}}}{6b^4}\right.\right.$$

input
```
Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(5/3)*(-3*b^2*(ArcTanh[(b^(4/3)*Tan[e + f*x])/(a^2 + b^2
)^(1/6)]/(3*b^(5/3)*(a^2 + b^2)^(1/6)) - (-((Sqrt[3]*ArcTan[(1 - (2*b^(4/3
)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3)) + Log[(a^2 + b^2)^(1
/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*
b^(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6)) - ((Sqrt[3]*ArcTan[(1 + (2*b^(4/3)
*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) - Log[(a^2 + b^2)^(1/3
) + b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*b^
(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6))) + (b*AppellF1[1/2, 1, 1/6, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a))/(b*f*(Sec[e + f*
x]^2)^(5/6))
```

3.632.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.632. $\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.632.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{a+b\tan(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

3.632.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.632.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
Timed out
```

3.632.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

3.632.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

3.632.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)), x)
```

3.633 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

3.633.1 Optimal result . . . 4533
3.633.2 Mathematica [C] (warning: unable to verify) . . . 4534
3.633.3 Rubi [A] (warning: unable to verify) . . . 4535
3.633.4 Maple [F] . . . 4540
3.633.5 Fricas [F(-1)] . . . 4540
3.633.6 Sympy [F] . . . 4541
3.633.7 Maxima [F] . . . 4541
3.633.8 Giac [F] . . . 4541
3.633.9 Mupad [F(-1)] . . . 4542

3.633.1 Optimal result

Integrand size = 25, antiderivative size = 552

$$\begin{aligned}\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = {} & \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\ & -\frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\ & -\frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\ & +\frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\ & -\frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\ & +\frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}}\end{aligned}$$

output
```
-b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+
e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(1/6)+1/4*b^(2/3)*ln((a^2+b^2)^
(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^
(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(1/6)-1/4*b^(
2/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/
3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e
)^2)^(1/6)-1/2*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6
)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)^(5/6)/f/
(sec(f*x+e)^2)^(1/6)-1/2*b^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e
)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)
^(5/6)/f/(sec(f*x+e)^2)^(1/6)+AppellF1(1/2,1,5/6,3/2,b^2*tan(f*x+e)^2/a^2,
-tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(1/6)
```

3.633.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.56 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = -\frac{48d^2\,\text{AppellF1}\left(\frac{5}{3},\frac{5}{6},\frac{5}{6},\frac{8}{3},\frac{a}{a+b\,\text{ta}}\right.}{5bf(d\sec(e+fx))^{5/3}\left(5(a+ib)\,\text{AppellF1}\left(\frac{8}{3},\frac{5}{6},\frac{11}{6},\frac{11}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)\,\text{AppellF1}\right.}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

output
```
(-48*d^2*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(5*b*f*(d*Sec[e + f*x])^
(5/3)*(5*(a + I*b)*AppellF1[8/3, 5/6, 11/6, 11/3, (a - I*b)/(a + b*Tan[e +
 f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[8/3, 11/6,
5/6, 11/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]
 + 16*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*
b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

3.633.3 Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.67, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3994, 504, 333, 353, 73, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{5/6}}d(b\tan(e+fx))}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\Bigg(a\int \frac{1}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))}d(b\tan}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\Bigg(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\Bigg(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}(a^2-b^2\tan^2(e+fx))}d\big(b^2\tan}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\int\frac{1}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+fx)}{b}+}\right.}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \text{754}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+f}{b}}}{3(a^2+b^2)^{2/3}}\right.\right.}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+f}{b}}}{3(a^2+b^2)^{2/3}}\right.\right.}{}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\int\frac{2\sqrt[6]{a^2+b^2}-b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{}}}{6(a^2+b^2)^{5/6}}\right.\right.}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan}}{}\right.\right.}{}$$

$$\Big\downarrow \text{25}$$

3.633. $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan}\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+}\right.\right.$$

$$\Big\downarrow \text{1103}$$

3.633. $\int\frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\frac{b\tan(e+fx)\,\mathrm{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-3b^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a^2+\right.}{}}{6(a^2+}\right.\right.}{}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^(1/3)*(-3*b^2*(ArcTanh[(b^(4/3)*Tan[e + f*x])/(a^2 + b^2
)^(1/6)]/(3*b^(1/3)*(a^2 + b^2)^(5/6)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(4/3
)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3)) - Log[(a^2 + b^2)^(1
/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*
b^(1/3)))/(6*(a^2 + b^2)^(5/6)) + ((Sqrt[3]*ArcTan[(1 + (2*b^(4/3)*Tan[e +
 f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) + Log[(a^2 + b^2)^(1/3) + b^(4
/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f*x]^2]/(2*b^(1/3)))/
(6*(a^2 + b^2)^(5/6))) + (b*AppellF1[1/2, 1, 5/6, 3/2, (b^2*Tan[e + f*x]^2
)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a))/(b*f*(Sec[e + f*x]^2)^(1/6))
```

3.633.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

---

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.633. $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.633.4 Maple [F]

$$\int \frac{(d \sec (f x+e))^{\frac{1}{3}}}{a+b \tan (f x+e)}dx$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

3.633.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d \sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.633.6 Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{\sqrt[3]{d\sec{(e+fx)}}}{a+b\tan{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x)), x)
```

3.633.7 Maxima [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{1}{3}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

3.633.8 Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec{(fx+e)})^{\frac{1}{3}}}{b\tan{(fx+e)}+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

3.633.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+b\tan\left(e+f\,x\right)}\,dx$$

input

```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)),x)
```

output

```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)), x)
```

3.634 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

3.634.1 Optimal result 4543
3.634.2 Mathematica [C] (warning: unable to verify) 4544
3.634.3 Rubi [A] (warning: unable to verify) 4545
3.634.4 Maple [F] 4553
3.634.5 Fricas [F(-1)] 4553
3.634.6 Sympy [F] 4553
3.634.7 Maxima [F] 4554
3.634.8 Giac [F] 4554
3.634.9 Mupad [F(-1)] 4554

3.634.1 Optimal result

Integrand size = 25, antiderivative size = 579

$$
\begin{aligned}
&\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx \\
&= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} - \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&\quad + \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&\quad - \frac{b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&\quad + \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&\quad - \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^{7/6}f\sqrt[3]{d\sec(e+fx)}} \\
&\quad + \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}}
\end{aligned}
$$

output
```
3*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1/3)-b^(4/3)*arctanh(b^(1/3)*(sec(f*x+e)^2
)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x
+e))^(1/3)+1/4*b^(4/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x
+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^
(7/6)/f/(d*sec(f*x+e))^(1/3)-1/4*b^(4/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b
^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2
)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/2*b^(4/3)*arctan(-1/3*3^(
1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)
^2)^(1/6)*3^(1/2)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/2*b^(4/3)*arcta
n(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(s
ec(f*x+e)^2)^(1/6)*3^(1/2)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+AppellF1
(1/2,1,7/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)*ta
n(f*x+e)/a/f/(d*sec(f*x+e))^(1/3)
```

3.634.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 52.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx =$$

$$-\frac{60d\,\mathrm{AppellF1}\left(\frac{7}{3},\frac{7}{6},\frac{7}{6},\frac{10}{3},\frac{a-ib}{a+b\tan(e+}\right.}{7bf(d\sec(e+fx))^{4/3}\left(7(a+ib)\,\mathrm{AppellF1}\left(\frac{10}{3},\frac{7}{6},\frac{13}{6},\frac{13}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+7(a-ib)\,\mathrm{AppellF}\right.}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

output
```
(-60*d*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(7*b*f*(d*Se
c[e + f*x])^(4/3)*(7*(a + I*b)*AppellF1[10/3, 7/6, 13/6, 13/3, (a - I*b)/(
a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 7*(a - I*b)*AppellF
1[10/3, 13/6, 7/6, 13/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*
Tan[e + f*x])] + 20*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e +
 f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

3.634.3 Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.73, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3042, 3994, 504, 333, 353, 61, 73, 825, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))\left(\tan^2(e+fx)+1\right)^{7/6}}d(b\tan(e+fx))}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{504}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(a\int \frac{1}{\left(\tan^2(e+fx)+1\right)^{7/6}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{7/6}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{333}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{\left(\tan^2(e+fx)+1\right)^{7/6}\left(a^2-b^2\tan^2(e+fx)\right)}d(b\tan(e\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{7/6}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2\right.\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{61}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{b^2\int\frac{1}{\sqrt[6]{\frac{\tan(e+fx)}{b}+1}\left(a^2-b^2\tan^2(e+fx)\right)}d\left(b^2\tan^2(e+fx)\right)}{a^2+b^2}\right)+\frac{b\tan(e+}{}\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 73$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\int\frac{b^4\tan^4(e+fx)}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{a^2+b^2}\right)+\frac{b\tan(e+fx)\,\mathrm{AppellF}}{}\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow 825$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3b^{4/3}}+\frac{\int-\frac{}{2\left(\tan^2(e+fx\right.}}{}\right.}{}\right.\right.$$

$$\Big\downarrow 27$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3b^{4/3}}-\frac{\int\frac{}{\tan^2(e+fx)b^{8/3}}}{}\right.}{}\right.\right.$$

$$\Big\downarrow 221$$

3.634. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{\int \frac{\tan(e+fx)b^{4/3}+\sqrt[6]{a^2+b^2}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e+}{b}}\right.}{6b^{4/3}\sqrt[6]{a^2+b^2}}\right.\right.\right.$$

$$\Big\downarrow \text{1142}$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int \frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

3.634. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}}{}\right.}{}\right.\right.}{}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(-\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}}{}\right.}{}\right.\right.}{}$$

$$\Big\downarrow \ 1082$$

3.634. $\displaystyle\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{\frac{3\int\frac{1}{\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}-4}d\left(1-\frac{2b^{4/3}\tan(e+fx)}{\sqrt[6]{a^2+b^2}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[6]{a^2+}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2}}}{6b^{4/3}\sqrt[6]{a^2+b}}\right.}{}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}}+1}-\frac{6b^4\left(-\frac{-\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(}{}}}{6b^{4/3}\sqrt[6]{a^2+b^2}}\right.}{}\right.\right.$$

$$\Big\downarrow \text{1103}$$

3.634. $\displaystyle\int\frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}+\frac{1}{2}\left(\frac{6b^2}{(a^2+b^2)\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}-\frac{6b^4\left(-\frac{\log\left(\sqrt[3]{\right.}{}\right.}{}\right.\right.}{}$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

output
```
((Sec[e + f*x]^2)^(1/6)*((b*AppellF1[1/2, 1, 7/6, 3/2, (b^2*Tan[e + f*x]^2
)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a + ((-6*b^4*(ArcTanh[(b^(4/3)*Tan[e
 + f*x])/(a^2 + b^2)^(1/6)]/(3*b^(5/3)*(a^2 + b^2)^(1/6)) - (-((Sqrt[3]*Ar
cTan[(1 - (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3)) +
 Log[(a^2 + b^2)^(1/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*
Tan[e + f*x]^2]/(2*b^(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6)) - ((Sqrt[3]*Arc
Tan[(1 + (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) - L
og[(a^2 + b^2)^(1/3) + b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Ta
n[e + f*x]^2]/(2*b^(1/3)))/(6*b^(4/3)*(a^2 + b^2)^(1/6))))/(a^2 + b^2) + (
6*b^2)/((a^2 + b^2)*(1 + Tan[e + f*x]/b)^(1/6)))/2))/(b*f*(d*Sec[e + f*x])
^(1/3))
```

3.634. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

3.634.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.634. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.634. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

3.634.4 Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+b\tan\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

3.634.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.634.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))), x)
```

3.634.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}\,(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

3.634.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}\,(b\tan(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+b\tan(e+f\,x))}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))), x)
```

3.635 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

3.635.1 Optimal result 4555
3.635.2 Mathematica [B] (warning: unable to verify) 4556
3.635.3 Rubi [A] (warning: unable to verify) 4556
3.635.4 Maple [F] 4564
3.635.5 Fricas [F(-1)] 4564
3.635.6 Sympy [F] 4565
3.635.7 Maxima [F] 4565
3.635.8 Giac [F] 4565
3.635.9 Mupad [F(-1)] 4566

3.635.1 Optimal result

Integrand size = 25, antiderivative size = 581

$$\begin{aligned}
\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx &= \frac{3b}{5\left(a^2+b^2\right)f(d\sec(e+fx))^{5/3}} \\
&+ \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\left(a^2+b^2\right)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&- \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\left(a^2+b^2\right)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&- \frac{b^{8/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{\left(a^2+b^2\right)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&+ \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4\left(a^2+b^2\right)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&- \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4\left(a^2+b^2\right)^{11/6}f(d\sec(e+fx))^{5/3}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}
\end{aligned}$$

output
```
3/5*b/(a^2+b^2)/f/(d*sec(f*x+e))^(5/3)-b^(8/3)*arctanh(b^(1/3)*(sec(f*x+e)
^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(
f*x+e))^(5/3)+1/4*b^(8/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(
f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^
2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/4*b^(8/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a
^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+
e)^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/2*b^(8/3)*arctan(-1/
3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f
*x+e)^2)^(5/6)*3^(1/2)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/2*b^(8/3)
*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/
2))*(sec(f*x+e)^2)^(5/6)*3^(1/2)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)+A
ppellF1(1/2,1,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^
(5/6)*tan(f*x+e)/a/f/(d*sec(f*x+e))^(5/3)
```

3.635.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6862 vs. $2(581) = 1162$.

Time = 111.46 (sec) , antiderivative size = 6862, normalized size of antiderivative = 11.81

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

output
```
Result too large to show
```

3.635.3 Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.71, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3994, 504, 333, 353, 61, 73, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sec^2(e+fx)^{5/6}\int \frac{1}{(a+b\tan(e+fx))(\tan^2(e+fx)+1)^{11/6}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \; 504$$

$$\frac{\sec^2(e+fx)^{5/6}\left(a\int \frac{1}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(e+fx))-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b\,\mathrm{ta}\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \; 333$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int \frac{b\tan(e+fx)}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b\tan(\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \; 353$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{11/6}(a^2-b^2\tan^2(e+fx))}d(b^2\,\mathrm{ta}\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \; 61$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{b^2\int \frac{1}{\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}\left(a^2-b^2\tan^2(e+fx)\right)}d(b^2\tan^2(e+fx))}{a^2+b^2}\right)+\frac{b\tan(e+fx)\,\mathrm{App}}{}\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\int \frac{1}{-\tan^6(e+fx)b^8+b^2+a^2}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{a^2+b^2}\right)+\frac{b\tan(e+fx)\,\text{AppellF1}\Big(}{}\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \; 754$$

3.635. $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3\left(a^2+b^2\right)^{2/3}}+\frac{\int\frac{2}{2\left(\tan^2(e+fx)b^{8/3}\right.}}{}\right.}{}\right.\right.}{}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{8/3}\tan^2(e+fx)}d\sqrt[6]{\frac{\tan(e+fx)}{b}+1}}{3\left(a^2+b^2\right)^{2/3}}+\frac{\int\frac{2\sqrt[6]{}}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{}}}{}\right.}{}\right.\right.}{}$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\int\frac{2\sqrt[6]{a^2+b^2}-b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e+fx)}{b}}}{6\left(a^2+b^2\right)^{5/6}}\right.}{}\right.\right.}{}$$

$$\Big\downarrow \color{blue}{1142}$$

3.635. $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{t}}{}\right.}{}\right.\right.}{}$$

$\downarrow$ 25

$$\frac{\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{t}}{}\right.}{}\right.\right.}{}$$

$\downarrow$ 27

3.635. $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{3}{2}\sqrt[6]{a^2+b^2}\int\frac{1}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\right.}{}\right.\right.$$

$\downarrow$ 1082

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e+f}{b}}}{6\left(a^2+b^2\right)^{5/6}}\right.}{}\right.\right.$$

$\downarrow$ 217

3.635. $\int\frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

$$\sec^2(e+fx)^{5/6}\left(\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a^2+b^2}-2b^{4/3}\tan(e+fx)}{\tan^2(e+fx)b^{8/3}-\sqrt[6]{a^2+b^2}\tan(e+fx)b^{4/3}+\sqrt[3]{a^2+b^2}}d\sqrt[6]{\frac{\tan(e+f}{b}}}{6\left(a^2+b^2\right)^{5/6}}\right.}{}\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}+\frac{1}{2}\left(\frac{6b^2}{5(a^2+b^2)\left(\frac{\tan(e+fx)}{b}+1\right)^{5/6}}-\frac{6b^4\left(\frac{-\frac{\sqrt{3}\arctan}{}}{}\right.}{}\right.\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

3.635. $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

output
```
((Sec[e + f*x]^2)^(5/6)*((b*AppellF1[1/2, 1, 11/6, 3/2, (b^2*Tan[e + f*x]^
2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a + ((-6*b^4*(ArcTanh[(b^(4/3)*Tan[
e + f*x])/(a^2 + b^2)^(1/6)]/(3*b^(1/3)*(a^2 + b^2)^(5/6)) + (-((Sqrt[3]*A
rcTan[(1 - (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3))
- Log[(a^2 + b^2)^(1/3) - b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)
*Tan[e + f*x]^2]/(2*b^(1/3)))/(6*(a^2 + b^2)^(5/6)) + ((Sqrt[3]*ArcTan[(1
+ (2*b^(4/3)*Tan[e + f*x])/(a^2 + b^2)^(1/6))/Sqrt[3]])/b^(1/3) + Log[(a^2
 + b^2)^(1/3) + b^(4/3)*(a^2 + b^2)^(1/6)*Tan[e + f*x] + b^(8/3)*Tan[e + f
*x]^2]/(2*b^(1/3)))/(6*(a^2 + b^2)^(5/6))))/(a^2 + b^2) + (6*b^2)/(5*(a^2
+ b^2)*(1 + Tan[e + f*x]/b)^(5/6)))/2))/(b*f*(d*Sec[e + f*x])^(5/3))
```

3.635.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.635. $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.635.4 Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x +e \right)\right)^{\frac{5}{3}} \left(a +b \tan \left(f x +e \right)\right)}d x$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

3.635.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
Timed out
```

3.635.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(e+fx)})^{\frac{5}{3}}\,(a+b\tan{(e+fx)})}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))), x)
```

3.635.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{5}{3}}\,(b\tan{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

3.635.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{5}{3}}\,(b\tan{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,(a+b\tan\left(e+f\,x\right))}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))), x)
```

3.636 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$

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3.636.1 Optimal result

Integrand size = 25, antiderivative size = 687

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx &= -\frac{a\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&+\frac{a\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&-\frac{a\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&+\frac{a\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&-\frac{a\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&+\frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} \\
&+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}} \\
&-\frac{ab(d\sec(e+fx))^{5/3}}{\left(a^2+b^2\right)f\left(a^2-b^2\tan^2(e+fx)\right)}
\end{aligned}
$$

output
```
-1/3*a*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e)
)^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6)+1/12*a*ln((a^2+b^2)
^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)
^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6
)-1/12*a*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b
^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/
f/(sec(f*x+e)^2)^(5/6)+1/6*a*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2
)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7
/6)/f/(sec(f*x+e)^2)^(5/6)*3^(1/2)+1/6*a*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(s
ec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(
a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6)*3^(1/2)+AppellF1(1/2,2,1/6,3/2,b^2*t
an(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a^2/f/(sec(
f*x+e)^2)^(5/6)+1/3*b^2*AppellF1(3/2,2,1/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f
*x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(5/6)-a*b*
(d*sec(f*x+e))^(5/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

3.636.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 129.13 (sec) , antiderivative size = 8003, normalized size of antiderivative = 11.65

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

output
```
Result too large to show
```

3.636.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \ 3994$$

$$\frac{(d\sec(e+fx))^{5/3}\int \frac{1}{(a+b\tan(e+fx))^2\sqrt[6]{\tan^2(e+fx)+1}}d(b\tan(e+fx))}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \ 505$$

$$\frac{(d\sec(e+fx))^{5/3}\int \left(\frac{a^2}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{\sqrt[6]{\tan^2(e+fx)+1}(a^2-b^2\tan^2(e+fx))^2}+\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$\Big\downarrow \ 2009$$

$$(d\sec(e+fx))^{5/3}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}-\frac{a\sqrt[3]{b}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+1}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}(a^2+b^2)^{7/6}}\right.$$

input
```
Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

3.636. $\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$

output
```
((d*Sec[e + f*x])^(5/3)*(-1/2*(a*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1
+ Tan[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(Sqrt[3]*(a^2 + b^2
)^(7/6)) + (a*b^(1/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x]^2)^(
1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(7/6)) - (a*b^(
1/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)])/(3*(
a^2 + b^2)^(7/6)) + (a*b^(1/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)
^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^(1/3)])/(
12*(a^2 + b^2)^(7/6)) - (a*b^(1/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 +
b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^(1/3)
])/(12*(a^2 + b^2)^(7/6)) + (b*AppellF1[1/2, 2, 1/6, 3/2, (b^2*Tan[e + f*x
]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[3/2, 2, 1/6,
5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^3)/(3*a^4) -
(a*b^2*(1 + Tan[e + f*x]^2)^(5/6))/((a^2 + b^2)*(a^2 - b^2*Tan[e + f*x]^2)
)))/(b*f*(Sec[e + f*x]^2)^(5/6))
```

3.636.3.1 Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.636.4 Maple [F]

$$\int \frac{(d \sec (fx+e))^{\frac{5}{3}}}{(a+b \tan (fx+e))^{2}}dx$$

input
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

3.636.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(d \sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.636.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d \sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Timed out
```

3.636.7 Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.636.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a)^2, x)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2, x)
```

3.637 $\int \frac{\sqrt[3]{d \sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

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3.637.1 Optimal result

Integrand size = 25, antiderivative size = 687

$$
\begin{aligned}
\int \frac{\sqrt[3]{d \sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx &= \frac{5ab^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{5ab^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{5ab^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2 f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{ab\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)\,f\,(a^2-b^2\tan^2(e+fx))}
\end{aligned}
$$

output
```
-5/3*a*b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*se
c(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)+5/12*a*b^(2/3)*ln(
(a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(
f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1
/6)-5/12*a*b^(2/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^
2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/
6)/f/(sec(f*x+e)^2)^(1/6)-5/6*a*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(s
ec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)
^(11/6)/f/(sec(f*x+e)^2)^(1/6)*3^(1/2)-5/6*a*b^(2/3)*arctan(1/3*3^(1/2)+2/
3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/
3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)*3^(1/2)+AppellF1(1/2,2,5/6,3/2,
b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a^2/f/
(sec(f*x+e)^2)^(1/6)+1/3*b^2*AppellF1(3/2,2,5/6,5/2,b^2*tan(f*x+e)^2/a^2,-
tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(1/6)
-a*b*(d*sec(f*x+e))^(1/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

3.637.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7801 vs. $2(687) = 1374$.

Time = 78.55 (sec) , antiderivative size = 7801, normalized size of antiderivative = 11.36

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

output
```
Result too large to show
```

3.637.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \ 3994$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{5/6}}d(b\tan(e+fx))}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \ 505$$

$$\frac{\sqrt[3]{d\sec(e+fx)}\int \Bigg(\frac{a^2}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{(\tan^2(e+fx)+1)^{5/6}(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e+fx)}{(\tan^2(e+fx)+1)^{5/6}(b^2\tan^2}}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$\Big\downarrow \ 2009$$

$$\sqrt[3]{d\sec(e+fx)}\Bigg(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}+\frac{5ab^{5/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+1}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}(a^2+b^2)^{11/6}}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

3.637. $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}dx$

output

```
((d*Sec[e + f*x])^(1/3)*((5*a*b^(5/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1 + T
an[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)
^(11/6)) - (5*a*b^(5/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x]^2)
^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(11/6)) - (5*
a*b^(5/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)])
/(3*(a^2 + b^2)^(11/6)) + (5*a*b^(5/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^
2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^(
1/3)])/(12*(a^2 + b^2)^(11/6)) - (5*a*b^(5/3)*Log[(a^2 + b^2)^(1/3) + b^(1
/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*
x]^2)^(1/3)])/(12*(a^2 + b^2)^(11/6)) + (b*AppellF1[1/2, 2, 5/6, 3/2, (b^2
*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[3
/2, 2, 5/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^3
)/(3*a^4) - (a*b^2*(1 + Tan[e + f*x]^2)^(1/6))/((a^2 + b^2)*(a^2 - b^2*Tan
[e + f*x]^2))))/(b*f*(Sec[e + f*x]^2)^(1/6))
```

3.637.3.1 Defintions of rubi rules used

rule 505

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.637. $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

3.637.4 Maple [F]

$$\int \frac{(d\sec (fx+e))^{\frac{1}{3}}}{(a+b\tan (fx+e))^{2}}dx$$

input
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

3.637.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.637.6 Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{\sqrt[3]{d\sec (e+fx)}}{(a+b\tan (e+fx))^{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x))**2, x)
```

3.637.7 Maxima [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

3.637.8 Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

3.637.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2, x)
```

3.638 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

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3.638.1 Optimal result

Integrand size = 25, antiderivative size = 715

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$$

$$\begin{aligned}
&= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&\quad + \frac{7ab^{4/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&\quad - \frac{7ab^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&\quad + \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&\quad - \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&\quad + \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f\sqrt[3]{d\sec(e+fx)}} \\
&\quad + \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[3]{d\sec(e+fx)}} \\
&\quad - \frac{ab}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}
\end{aligned}$$

output
```
7*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/3)-7/3*a*b^(4/3)*arctanh(b^(1/3)*(se
c(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f
/(d*sec(f*x+e))^(1/3)+7/12*a*b^(4/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^
(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1
/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)-7/12*a*b^(4/3)*ln((a^2+b^2)^(1
/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1
/3))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)+7/6*a*b^
(4/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)
*3^(1/2))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)*3^(
1/2)+7/6*a*b^(4/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^
2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e
))^(1/3)*3^(1/2)+AppellF1(1/2,2,7/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2
)*(sec(f*x+e)^2)^(1/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(1/3)+1/3*b^2*Appel
lF1(3/2,2,7/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)
*tan(f*x+e)^3/a^4/f/(d*sec(f*x+e))^(1/3)-a*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1
/3)/(a^2-b^2*tan(f*x+e)^2)
```

3.638.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 137.80 (sec) , antiderivative size = 18832, normalized size of antiderivative = 26.34

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

output
```
Result too large to show
```

3.638.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 574, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{7/6}}d(b\tan(e+fx))}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{505}$$

$$\frac{\sqrt[6]{\sec^2(e+fx)}\int \Big(\frac{a^2}{(\tan^2(e+fx)+1)^{7/6}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{(\tan^2(e+fx)+1)^{7/6}(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e+fx)}{(\tan^2(e+fx)+1)^{7/6}(b^2\tan^2(}}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$\Big\downarrow \text{2009}$$

$$\sqrt[6]{\sec^2(e+fx)}\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}-\frac{7ab^{7/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+1}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}(a^2+b^2)^{13/6}}-\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

3.638. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}dx$

output

```
((Sec[e + f*x]^2)^(1/6)*((-7*a*b^(7/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1 +
Tan[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2
)^(13/6)) + (7*a*b^(7/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x]^2
)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(13/6)) - (7
*a*b^(7/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]
)/(3*(a^2 + b^2)^(13/6)) + (7*a*b^(7/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a
^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f*x]^2)^
(1/3)])/(12*(a^2 + b^2)^(13/6)) - (7*a*b^(7/3)*Log[(a^2 + b^2)^(1/3) + b^(
1/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f
*x]^2)^(1/3)])/(12*(a^2 + b^2)^(13/6)) + (b*AppellF1[1/2, 2, 7/6, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[
3/2, 2, 7/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^
3)/(3*a^4) + (7*a*b^2)/((a^2 + b^2)^2*(1 + Tan[e + f*x]^2)^(1/6)) - (a*b^2
)/((a^2 + b^2)*(1 + Tan[e + f*x]^2)^(1/6)*(a^2 - b^2*Tan[e + f*x]^2))))/(b
*f*(d*Sec[e + f*x])^(1/3))
```

3.638.3.1 Defintions of rubi rules used

rule 505

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.638. $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

3.638.4 Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x +e \right)\right)^{\frac{1}{3}} \left(a +b \tan \left(f x +e \right)\right)^{2}}dx$$

input
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

3.638.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.638.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\sqrt[3]{d \sec{\left(e + f x \right)}} \left(a + b \tan{\left(e + f x \right)}\right)^{2}}\, dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2), x)
```

3.638.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}\,(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

3.638.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}\,(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2), x)
```

3.639 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$

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3.639.9 Mupad [F(-1)] 4591

3.639.1 Optimal result

Integrand size = 25, antiderivative size = 717

$$\begin{aligned}
&\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \frac{11ab}{5\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{5/3}} \\
&+ \frac{11ab^{8/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{3\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&+ \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&+ \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2 f(d\sec(e+fx))^{5/3}} \\
&+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4 f(d\sec(e+fx))^{5/3}} \\
&- \frac{ab}{\left(a^2+b^2\right) f(d\sec(e+fx))^{5/3}\left(a^2-b^2\tan^2(e+fx)\right)}
\end{aligned}$$

output
```
11/5*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(5/3)-11/3*a*b^(8/3)*arctanh(b^(1/3)
*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/
6)/f/(d*sec(f*x+e))^(5/3)+11/12*a*b^(8/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+
b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^
2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11/12*a*b^(8/3)*ln((a^2+b
^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)
^2)^(1/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11
/6*a*b^(8/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2
)^(1/6)*3^(1/2))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5
/3)*3^(1/2)-11/6*a*b^(8/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(
1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*s
ec(f*x+e))^(5/3)*3^(1/2)+AppellF1(1/2,2,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan
(f*x+e)^2)*(sec(f*x+e)^2)^(5/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(5/3)+1/3*
b^2*AppellF1(3/2,2,11/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e
)^2)^(5/6)*tan(f*x+e)^3/a^4/f/(d*sec(f*x+e))^(5/3)-a*b/(a^2+b^2)/f/(d*sec(
f*x+e))^(5/3)/(a^2-b^2*tan(f*x+e)^2)
```

3.639.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 11783 vs. $2(717) = 1434$.

Time = 81.40 (sec) , antiderivative size = 11783, normalized size of antiderivative = 16.43

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

output
```
Result too large to show
```

3.639.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{5/6}\int \frac{1}{(a+b\tan(e+fx))^2(\tan^2(e+fx)+1)^{11/6}}d(b\tan(e+fx))}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{505}$$

$$\frac{\sec^2(e+fx)^{5/6}\int \Bigg(\frac{a^2}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))^2}-\frac{2b\tan(e+fx)a}{(\tan^2(e+fx)+1)^{11/6}(a^2-b^2\tan^2(e+fx))^2}+\frac{b^2\tan^2(e+f}{(\tan^2(e+fx)+1)^{11/6}(b^2\ \mathrm{ta}}}{bf(d\sec(e+fx))^{5/3}}$$

$$\Big\downarrow \text{2009}$$

$$\sec^2(e+fx)^{5/6}\left(\frac{b\tan(e+fx)\,\mathrm{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}+\frac{11ab^{11/3}\arctan\Bigg(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\tan^2(e+fx)+1}}{\sqrt{3}\sqrt[6]{a^2+b^2}}}{2\sqrt{3}(a^2+b^2)^{17/6}}\right.$$

input
```
Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

3.639. $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$

output
```
((Sec[e + f*x]^2)^(5/6)*((11*a*b^(11/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(1 +
 Tan[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^
2)^(17/6)) - (11*a*b^(11/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(1 + Tan[e + f*x
]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))])/(2*Sqrt[3]*(a^2 + b^2)^(17/6)) -
 (11*a*b^(11/3)*ArcTanh[(b^(1/3)*(1 + Tan[e + f*x]^2)^(1/6))/(a^2 + b^2)^(
1/6)])/(3*(a^2 + b^2)^(17/6)) + (11*a*b^(11/3)*Log[(a^2 + b^2)^(1/3) - b^(
1/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 + Tan[e + f
*x]^2)^(1/3)])/(12*(a^2 + b^2)^(17/6)) - (11*a*b^(11/3)*Log[(a^2 + b^2)^(1
/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(1 + Tan[e + f*x]^2)^(1/6) + b^(2/3)*(1 +
Tan[e + f*x]^2)^(1/3)])/(12*(a^2 + b^2)^(17/6)) + (b*AppellF1[1/2, 2, 11/6
, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3
*AppellF1[3/2, 2, 11/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*a^4) + (11*a*b^2)/(5*(a^2 + b^2)^2*(1 + Tan[e + f*x]^2)^(
5/6)) - (a*b^2)/((a^2 + b^2)*(1 + Tan[e + f*x]^2)^(5/6)*(a^2 - b^2*Tan[e +
 f*x]^2))))/(b*f*(d*Sec[e + f*x])^(5/3))
```

3.639.3.1 Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.639.4 Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(a+b \tan \left(f x+e\right)\right)^{2}}dx$$

input
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

output
```
int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

3.639.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.639.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))**2), x)
```

3.639.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{(d\sec\left(fx+e\right))^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

3.639.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{(d\sec\left(fx+e\right))^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

3.639.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{5/3}\,{\left(a+b\tan\left(e+f\,x\right)\right)}^2}\,dx$$

input
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2),x)
```

output
```
int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2), x)
```

3.640 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$

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3.640.1 Optimal result

Integrand size = 23, antiderivative size = 173

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{a(3b^2-a^2(1+m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m}}{f(1+m)}$$

$$+\frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)}$$

$$-\frac{b(d\sec(e+fx))^m\left(2(1+m)\left(b^2-a^2(3+m)\right)-abm(4+m)\tan(e+fx)\right)}{fm\left(2+3m+m^2\right)}$$

output

```
-a*(3*b^2-a^2*(1+m))*hypergeom([1/2, 1-1/2*m],[3/2],-tan(f*x+e)^2)*(d*sec(
f*x+e))^m*tan(f*x+e)/f/(1+m)/((sec(f*x+e)^2)^(1/2*m))+b*(d*sec(f*x+e))^m*(
a+b*tan(f*x+e))^2/f/(2+m)-b*(d*sec(f*x+e))^m*(2*(1+m)*(b^2-a^2*(3+m))-a*b*
m*(4+m)*tan(f*x+e))/f/m/(m^2+3*m+2)
```

3.640.2 Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$$

$$=\frac{(d\sec(e+fx))^m\left(3ab^2(2+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\tan(e+fx)-a^3(2+m)\right.}{fm(2}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^m*(3*a*b^2*(2 + m)*Hypergeometric2F1[-1/2, m/2, (2 + m)/
2, Sec[e + f*x]^2]*Tan[e + f*x] - a^3*(2 + m)*Hypergeometric2F1[1/2, m/2,
(2 + m)/2, Sec[e + f*x]^2]*Tan[e + f*x] + b*((3*a^2 - b^2)*(2 + m) + b^2*m
*Sec[e + f*x]^2)*Sqrt[-Tan[e + f*x]^2]))/(f*m*(2 + m)*Sqrt[-Tan[e + f*x]^2
])
```

3.640.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3994, 497, 25, 27, 676, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3(d\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^3(d\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int (a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \text{497}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\int -\frac{(a+b\tan(e+fx))\left(b^2\left(2-\frac{a^2(m+2)}{b^2}\right)-ab(m+4)\tan(e+fx)\right)\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{b^2}d(b\tan(e+fx))}{m+2}\right.}{bf}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{b^2\int\frac{(a+b\tan(e+fx))\left(-\left((m+2)a^2\right)-b(m+4)\tan(e+fx)a}{b^2}}{m+2}\right.}{bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{\int(a+b\tan(e+fx))\left(-\left((m+2)a^2\right)-b(m+4)\tan(e+fx}{m+2}\right.}{bf}$$

$$\Big\downarrow \text{676}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{\frac{a(m+2)\left(3b^2-a^2(m+1)\right)\int\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}d(b\tan(e}{m+1}}{}\right.}{bf}$$

$$\Big\downarrow \text{237}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{m/2}(a+b\tan(e+fx))^2}{m+2}-\frac{\frac{ab(m+2)\left(3b^2-a^2(m+1)\right)\tan(e+fx)\,\text{Hypergeometric2F1}\left(\right.}{m+1}}{}\right.}{bf}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Sec[e + f*x])^m*((b^2*(a + b*Tan[e + f*x])^2*(1 + Tan[e + f*x]^2)^(m/2
))/(2 + m) - ((a*b*(2 + m)*(3*b^2 - a^2*(1 + m))*Hypergeometric2F1[1/2, (2
 - m)/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(1 + m) + (2*b^2*(b^2 - a^2*(
3 + m))*(1 + Tan[e + f*x]^2)^(m/2))/m - (a*b^3*(4 + m)*Tan[e + f*x]*(1 + T
an[e + f*x]^2)^(m/2))/(1 + m))/(2 + m)))/(b*f*(Sec[e + f*x]^2)^(m/2))
```

3.640.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.640.4 Maple [F]

$$\int (d\sec (fx+e))^m (a+b\tan (fx+e))^3\,dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

3.640.5 Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int (b\tan{(fx+e)}+a)^3(d\sec{(fx+e)})^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x +
e) + a^3)*(d*sec(f*x + e))^m, x)
```

3.640.6 Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int (d\sec{(e+fx)})^m\,(a+b\tan{(e+fx)})^3\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

3.640.7 Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int (b\tan{(fx+e)}+a)^3(d\sec{(fx+e)})^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

3.640.8 Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int (b\tan\left(fx+e\right)+a)^3(d\sec\left(fx+e\right))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

3.640.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m(a+b\tan(e+f\,x))^3\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

3.641 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$

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3.641.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\begin{aligned}\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx &= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)}\\ &+ \frac{d(b^2-a^2(1+m))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+fx)}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}}\\ &+ \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)}\end{aligned}$$

output
```
a*b*(2+m)*(d*sec(f*x+e))^m/f/m/(1+m)+d*(b^2-a^2*(1+m))*hypergeom([1/2, -1/
2*m+1/2],[3/2-1/2*m],cos(f*x+e)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(-m^
2+1)/(sin(f*x+e)^2)^(1/2)+b*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))/f/(1+m)
```

3.641.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$$

$$= \frac{(d\sec(e+fx))^m\,\Big(b^2\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\tan(e+fx)+a\Big(-a\,\text{Hypergeometr}}{fm\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^m*(b^2*Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[e + f
*x]^2]*Tan[e + f*x] + a*(-(a*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e
+ f*x]^2]*Tan[e + f*x]) + 2*b*Sqrt[-Tan[e + f*x]^2])))/(f*m*Sqrt[-Tan[e +
f*x]^2])
```

3.641.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3993, 25, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(d\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2(d\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3993}$$

$$\frac{\int -(d\sec(e+fx))^m\left(-\left((m+1)a^2\right)-b(m+2)\tan(e+fx)a+b^2\right)dx}{m+1}+\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\int (d\sec(e+fx))^m\left(-\left((m+1)a^2\right)-b(m+2)\tan(e+fx)a+b^2\right)dx}{m+1}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\int (d\sec(e+fx))^m\left(-\left((m+1)a^2\right)-b(m+2)\tan(e+fx)a+b^2\right)dx}{m+1}$$

$$\Big\downarrow \text{3967}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\int(d\sec(e+fx))^mdx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^mdx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{4259}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\left(\frac{\cos(e+fx)}{d}\right)^m(d\sec(e+fx))^m\int\left(\frac{\cos(e+fx)}{d}\right)^{-m}dx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{\left(b^2-a^2(m+1)\right)\left(\frac{\cos(e+fx)}{d}\right)^m(d\sec(e+fx))^m\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-m}dx-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

$$\Big\downarrow \text{3122}$$

$$\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}-\frac{-\frac{d\left(b^2-a^2(m+1)\right)\sin(e+fx)(d\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}-\frac{ab(m+2)(d\sec(e+fx))^m}{fm}}{m+1}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output
```
-((-((a*b*(2 + m)*(d*Sec[e + f*x])^m)/(f*m)) - (d*(b^2 - a^2*(1 + m))*Hype
rgeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 + m)*Sin[e + f*x])/(f*(1 - m)*Sqrt[Sin[e + f*x]^2]))/(1 + m)) + (b*(d*
Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 + m))
```

3.641. $\int(d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$

3.641.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.641.4 Maple [F]

$$\int \left(d \sec \left(f x +e \right)\right)^{m} \left(a +b \tan \left(f x +e \right)\right)^{2}d x$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

3.641.5 Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^
m, x)
```

3.641.6 Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (d\sec(e+fx))^m\,(a+b\tan(e+fx))^2\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

3.641.7 Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

3.641.8 Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan{(fx+e)}+a)^2(d\sec{(fx+e)})^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

3.641.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^m(a+b\tan(e+f\,x))^2\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

3.642 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx$

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3.642.1 Optimal result

Integrand size = 21, antiderivative size = 93

$$\begin{aligned}\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx &= \frac{b(d\sec(e+fx))^m}{fm}\\ &\quad -\frac{ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+fx)}{f(1-m)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

output

```
b*(d*sec(f*x+e))^m/f/m-a*d*hypergeom([1/2, -1/2*m+1/2],[3/2-1/2*m],cos(f*x
+e)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(1-m)/(sin(f*x+e)^2)^(1/2)
```

3.642.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\begin{aligned}\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx\\ = \frac{(d\sec(e+fx))^m\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{fm}\end{aligned}$$

input

```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^m*(b + a*Cot[e + f*x]*Hypergeometric2F1[1/2, m/2, (2 + m
)/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(f*m)
```

3.642.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(d\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))(d\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3967}$$

$$a\int (d\sec(e+fx))^m dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{3042}$$

$$a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^m dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{4259}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^m(d\sec(e+fx))^m\int \left(\frac{\cos(e+fx)}{d}\right)^{-m}dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{\cos(e+fx)}{d}\right)^m(d\sec(e+fx))^m\int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^{-m}dx+\frac{b(d\sec(e+fx))^m}{fm}$$

$$\Big\downarrow \text{3122}$$

$$\frac{b(d\sec(e+fx))^m}{fm}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
(b*(d*Sec[e + f*x])^m)/(f*m) - (a*d*Hypergeometric2F1[1/2, (1 - m)/2, (3 -
 m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(f*(1 - m)*
Sqrt[Sin[e + f*x]^2])
```

3.642.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.642.4 Maple [F]

$$\int (d\sec(fx+e))^m (a+b\tan(fx+e))\,dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

3.642.5 Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx=\int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

3.642.6 Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx=\int (d\sec(e+fx))^m\,(a+b\tan(e+fx))\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

3.642.7 Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx=\int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

3.642.8 Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx=\int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

3.642.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^m(a+b\tan(e+f\,x))\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

3.643 $\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$

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3.643.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)\,fm} + \frac{\text{AppellF1}\left(\frac{1}{2},1,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{af}$$

output
```
-b*hypergeom([1, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f*x+e
))^m/(a^2+b^2)/f/m+AppellF1(1/2,1,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*
x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a/f/((sec(f*x+e)^2)^(1/2*m))
```

3.643.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.16 (sec) , antiderivative size = 1158, normalized size of antiderivative = 8.21

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

$$= \frac{}{f(a+b\tan(e+fx))\Big(am\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)-bm\sec^2(e+}$$

input
```
Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^m*(b - b*(Sec[e + f*x]^2)^(m/2) + a*m*Hypergeometric2F1[
1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (b*AppellF1[-m, -1/2*m,
 -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f
*x])]*(Sec[e + f*x]^2)^(m/2))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x
]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))))/(f*(a + b
*Tan[e + f*x])*(a*m*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*
Sec[e + f*x]^2 - b*m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] + (b*m*AppellF1[-
m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b
*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(((b*(-I + Tan[e + f*
x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*
x]))^(m/2)) + (b*(Sec[e + f*x]^2)^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1
- m, 1 - m/2, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
+ b*Tan[e + f*x])]*Sec[e + f*x]^2)/((1 - m)*(a + b*Tan[e + f*x])^2) - ((a
+ I*b)*b*m^2*AppellF1[1 - m, -1/2*m, 1 - m/2, 2 - m, (a - I*b)/(a + b*Tan[
e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 - m)*(a +
 b*Tan[e + f*x])^2)))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2
)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) - (b*m*AppellF1[-m,
 -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e
 + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b...
```

3.643.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3994, 504, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3994}$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int\frac{\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow 504$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(a\int\frac{\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{a^2-b^2\tan^2(e+fx)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{a^2-b^2\tan^2(e+fx)}d(b\tan}{bf}$$

$$\Big\downarrow 333$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int\frac{b\tan(e+fx)(\tan^2(e+fx)}{a^2-b^2\tan^2(e+f}\right.}{bf}$$

$$\Big\downarrow 353$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int\frac{\left(\frac{\tan(e+fx)}{b}+1\right)^{\frac{m-2}{2}}}{a^2-b^2\tan^2(e+fx)}d\right.}{bf}$$

$$\Big\downarrow 78$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{b^2\left(\frac{\tan(e+fx)}{b}+1\right)^{m/2}\text{Hype}}{}\right.}{bf}$$

input
```
Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Sec[e + f*x])^m*((b*AppellF1[1/2, 1, (2 - m)/2, 3/2, (b^2*Tan[e + f*x]
^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a - (b^2*Hypergeometric2F1[1, m/2,
 (2 + m)/2, (b^2 + b^2*Tan[e + f*x]^2)/(a^2 + b^2)]*(1 + Tan[e + f*x]/b)^(
m/2))/((a^2 + b^2)*m)))/(b*f*(Sec[e + f*x]^2)^(m/2))
```

3.643. $\int\frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$

3.643.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.643. $\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$

3.643.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^m}{a+b\tan(fx+e)}dx$$

input
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

output
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

3.643.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.643.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

3.643.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.643.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.643.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{a+b\tan(e+f\,x)}\,dx$$

input
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)),x)
```

output
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)), x)
```

3.644 $\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

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3.644.1 Optimal result

Integrand size = 23, antiderivative size = 227

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = -\frac{2ab\,\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{\left(a^2+b^2\right)^2 fm} + \frac{\text{AppellF1}\left(\frac{1}{2},2,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{a^2 f} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,1-\frac{m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan^3(e+f}{3a^4 f}$$

output
```
-2*a*b*hypergeom([2, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f
*x+e))^m/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-
tan(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a^2/f/((sec(f*x+e)^2)^(1/2*m))+1
/3*b^2*AppellF1(3/2,2,1-1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*s
ec(f*x+e))^m*tan(f*x+e)^3/a^4/f/((sec(f*x+e)^2)^(1/2*m))
```

3.644.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.79 (sec) , antiderivative size = 2453, normalized size of antiderivative = 10.81

$$\int \frac{(d \sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output

```
((d*Sec[e + f*x])^m*((-2*a*b*(-1 + (Sec[e + f*x]^2)^(m/2)))/m + (a^2 - b^2
)*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (2*
a*b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a
 + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2))/(m*((b*(-I + Tan[e +
 f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e +
 f*x]))^(m/2)) + (b*(a^2 + b^2)*AppellF1[1 - m, -1/2*m, -1/2*m, 2 - m, (a
- I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]
^2)^(m/2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])))
)/(f*(a + b*Tan[e + f*x])^2*((a^2 - b^2)*Hypergeometric2F1[1/2, 1 - m/2, 3
/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2 - 2*a*b*(Sec[e + f*x]^2)^(m/2)*Tan[e +
 f*x] + (2*a*b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e
+ f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*
x])/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e +
 f*x]))/(a + b*Tan[e + f*x]))^(m/2)) - (b^2*(a^2 + b^2)*AppellF1[1 - m, -1
/2*m, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[
e + f*x])]*(Sec[e + f*x]^2)^(1 + m/2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/
(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
(m/2)*(a + b*Tan[e + f*x])^2) + (b*(a^2 + b^2)*m*AppellF1[1 - m, -1/2*m, -
1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + ...
```

3.644.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int \frac{\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \; 505$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\int \left(\frac{a^2\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{\left(a^2-b^2\tan^2(e+fx)\right)^2}-\frac{2ab\tan(e+fx)\left(\tan^2(e+fx)+1\right)^{\frac{m-2}{2}}}{\left(a^2-b^2\tan^2(e+fx)\right)^2}+\frac{b^2\tan^2(e+fx)(\tan^2(e+f}{(b^2\tan^2(e+fx)-a}\right.}{bf}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(\frac{b\tan(e+fx)\,\text{AppellF1}\left(\frac{1}{2},2,\frac{2-m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}-\frac{2ab^2\left(\tan^2(e+fx)+1\right)^{m/2}\,\text{H}}{}\right.}{bf}$$

input
```
Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Sec[e + f*x])^m*((b*AppellF1[1/2, 2, (2 - m)/2, 3/2, (b^2*Tan[e + f*x]
^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*AppellF1[3/2, 2, (2 - m
)/2, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x]^3)/(3*a^
4) - (2*a*b^2*Hypergeometric2F1[2, m/2, (2 + m)/2, (b^2 + b^2*Tan[e + f*x]
^2)/(a^2 + b^2)]*(1 + Tan[e + f*x]^2)^(m/2))/((a^2 + b^2)^2*m)))/(b*f*(Sec
[e + f*x]^2)^(m/2))
```

3.644. $\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

3.644.3.1 Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.644.4 Maple [F]

$$\int \frac{(d\sec(fx+e))^m}{(a+b\tan(fx+e))^2}dx$$

input
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

3.644.5 Fricas [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2
), x)
```

3.644.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec{(e+fx)})^m}{(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

3.644.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec{(fx+e)})^m}{(b\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

3.644.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec{(fx+e)})^m}{(b\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

3.644.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

3.645 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$

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3.645.1 Optimal result

Integrand size = 23, antiderivative size = 181

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$= \frac{b\operatorname{AppellF1}\left(1+n,1-\frac{m}{2},1-\frac{m}{2},2+n,\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)(d\sec(e+fx))^m(a+b\tan(e+fx))^{1+}}{(a^2+b^2)\,f(1+n)}$$

output
```
b*AppellF1(1+n,1-1/2*m,1-1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*
tan(f*x+e))/(a+(-b^2)^(1/2)))*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))^(1+n)/(a^2
+b^2)/f/(1+n)/((1+(a+b*tan(f*x+e))/(-a+(-b^2)^(1/2)))^(1/2*m))/((1+(-a-b*t
an(f*x+e))/(a+(-b^2)^(1/2)))^(1/2*m))
```

3.645.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.00 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.86

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$= \frac{}{f\left(2b\operatorname{AppellF1}\left(1+n,1-\frac{m}{2},1-\frac{m}{2},2+n,\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\sec^2(e+fx)+2n\operatorname{AppellF1}\left(1+\right.\right.}$$

input
```
Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(2*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b)
, (a + b*Tan[e + f*x])/(a + I*b)]*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(1 + n))/(f*(2*b*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f
*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2 + 2*n*Appel
lF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b
*Tan[e + f*x])/(a + I*b)]*(b - a*Tan[e + f*x]) - (b*(-2 + m)*(1 + n)*((a -
 I*b)*AppellF1[2 + n, 1 - m/2, 2 - m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I
*b), (a + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 - m/2,
1 - m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a +
I*b)])*Sec[e + f*x]^2*(a + b*Tan[e + f*x]))/((a - I*b)*(a + I*b)*(2 + n))
+ 2*(m + n)*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Tan[e + f*x]*(a + b*Tan[e + f*x
]) - (m*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a -
 I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*x])
)/(-I + Tan[e + f*x]) - (m*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b
*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(
a + b*Tan[e + f*x]))/(I + Tan[e + f*x])))
```

3.645.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3995, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3995}$$

$$\frac{b(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{-m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{-m/2}\int(a+b\tan(e+fx))^n\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{\frac{m-2}{2}}}{f\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 150$$

$$\frac{b(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{-m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{-m/2}(a+b\tan(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{2-m}{2},\right.}{f(n+1)\left(a^2+b^2\right)}$$

input
```
Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(b*AppellF1[1 + n, (2 - m)/2, (2 - m)/2, 2 + n, (a + b*Tan[e + f*x])/(a -
Sqrt[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*(d*Sec[e + f*x])^m*(a
+ b*Tan[e + f*x])^(1 + n))/((a^2 + b^2)*f*(1 + n)*(1 - (a + b*Tan[e + f*x]
)/(a - Sqrt[-b^2]))^(m/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(m/2
))
```

3.645.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3995
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*(a^2 + b^2)^(IntPart[m/2]
- 1)*((d*Sec[e + f*x])^(2*FracPart[m/2])/(f*b^(2*IntPart[m/2] - 1)*(1 - (a
+ b*Tan[e + f*x])/(a - Rt[-b^2, 2]))^FracPart[m/2]*(1 - (a + b*Tan[e + f*x]
)/(a + Rt[-b^2, 2]))^FracPart[m/2]))   Subst[Int[x^n*(1 - x/(a - Rt[-b^2, 2
]))^(m/2 - 1)*(1 - x/(a + Rt[-b^2, 2]))^(m/2 - 1), x], x, a + b*Tan[e + f*x
]], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 + b^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n]
```

3.645.4 Maple [F]

$$\int (d\sec (fx+e))^{m} (a+b\tan (fx+e))^{n} dx$$

input
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

output
```
int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

3.645.5 Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\sec (fx+e))^{m} (b\tan (fx+e)+a)^{n}\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

3.645.6 Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\sec (e+fx))^{m} (a+b\tan (e+fx))^{n}\,dx$$

input
```
integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
```

output
```
Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

3.645.7 Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\sec{(fx+e)})^m\,(b\tan{(fx+e)}+a)^n\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

3.645.8 Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\sec{(fx+e)})^m\,(b\tan{(fx+e)}+a)^n\,dx$$

input
```
integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

3.645.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^m\,(a+b\tan(e+f\,x))^n\,dx$$

input
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
```

output
```
int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

3.646 $\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.646.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx &= \frac{(a^2+b^2)^2\,(a+b\tan(c+dx))^{1+n}}{b^5d(1+n)}\\ &\quad-\frac{4a(a^2+b^2)\,(a+b\tan(c+dx))^{2+n}}{b^5d(2+n)}\\ &\quad+\frac{2(3a^2+b^2)\,(a+b\tan(c+dx))^{3+n}}{b^5d(3+n)}\\ &\quad-\frac{4a(a+b\tan(c+dx))^{4+n}}{b^5d(4+n)}\\ &\quad+\frac{(a+b\tan(c+dx))^{5+n}}{b^5d(5+n)}\end{aligned}$$

output
```
(a^2+b^2)^2*(a+b*tan(d*x+c))^(1+n)/b^5/d/(1+n)-4*a*(a^2+b^2)*(a+b*tan(d*x+
c))^(2+n)/b^5/d/(2+n)+2*(3*a^2+b^2)*(a+b*tan(d*x+c))^(3+n)/b^5/d/(3+n)-4*a
*(a+b*tan(d*x+c))^(4+n)/b^5/d/(4+n)+(a+b*tan(d*x+c))^(5+n)/b^5/d/(5+n)
```

3.646.2 Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{(a+b\tan(c+dx))^{1+n}\left(b^4\sec^4(c+dx)+4(a^2+b^2)\left(\frac{a^2+b^2}{1+n}-\frac{2a(a+b\tan(c+dx))}{2+n}+\frac{(a+b\tan(c+dx))^2}{3+n}\right)-4a(a+}{b^5d(5+n)}$$

input
```
Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*(b^4*Sec[c + d*x]^4 + 4*(a^2 + b^2)*((a^2 +
b^2)/(1 + n) - (2*a*(a + b*Tan[c + d*x]))/(2 + n) + (a + b*Tan[c + d*x])^2
/(3 + n)) - 4*a*(a + b*Tan[c + d*x])*((a^2 + b^2)/(2 + n) - (2*a*(a + b*Ta
n[c + d*x]))/(3 + n) + (a + b*Tan[c + d*x])^2/(4 + n))))/(b^5*d*(5 + n))
```

3.646.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sec(c+dx)^6(a+b\tan(c+dx))^ndx$$
$$\Big\downarrow \text{3987}$$
$$\frac{\int \frac{(a+b\tan(c+dx))^n\left(\tan^2(c+dx)b^2+b^2\right)^2}{b^4}d(b\tan(c+dx))}{bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int (a+b\tan(c+dx))^n\left(\tan^2(c+dx)b^2+b^2\right)^2d(b\tan(c+dx))}{b^5d}$$
$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\left(a^2+b^2\right)^2 (a+b\tan(c+dx))^n - 4a\left(a^2+b^2\right)(a+b\tan(c+dx))^{n+1} + 2\left(3a^2+b^2\right)(a+b\tan(c+dx))^{n+2} - 4}{b^5 d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^{n+1}}{n+1} - \frac{4a\left(a^2+b^2\right)(a+b\tan(c+dx))^{n+2}}{n+2} + \frac{2\left(3a^2+b^2\right)(a+b\tan(c+dx))^{n+3}}{n+3} - \frac{4a(a+b\tan(c+dx))^{n+4}}{n+4} + \frac{(a+b\tan(c}{n+}}{b^5 d}$$

input
```
Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(1 + n))/(1 + n) - (4*a*(a^2 + b^2)*(
a + b*Tan[c + d*x])^(2 + n))/(2 + n) + (2*(3*a^2 + b^2)*(a + b*Tan[c + d*x
])^(3 + n))/(3 + n) - (4*a*(a + b*Tan[c + d*x])^(4 + n))/(4 + n) + (a + b*
Tan[c + d*x])^(5 + n)/(5 + n))/(b^5*d)
```

3.646.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.646.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(161) = 322$.

Time = 0.12 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.87

$$\frac{(\tan^5(dx+c))\,\mathrm{e}^{n\ln(a+b\tan(dx+c))}}{d(5+n)}+\frac{a(b^4n^4+14b^4n^3+4a^2b^2n^2+71b^4n^2+36a^2b^2n+154b^4n+24a^4+80a^2b}{b^5d(n^5+15n^4+85n^3+225n^2+274n+120)}$$

input
```
int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

output
```
1/d/(5+n)*tan(d*x+c)^5*exp(n*ln(a+b*tan(d*x+c)))+a*(b^4*n^4+14*b^4*n^3+4*a
^2*b^2*n^2+71*b^4*n^2+36*a^2*b^2*n+154*b^4*n+24*a^4+80*a^2*b^2+120*b^4)/b^
5/d/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*exp(n*ln(a+b*tan(d*x+c)))+a*n/b/
d/(n^2+9*n+20)*tan(d*x+c)^4*exp(n*ln(a+b*tan(d*x+c)))-2*(-b^2*n^2+2*a^2*n-
9*b^2*n-20*b^2)/b^2/d/(n^3+12*n^2+47*n+60)*tan(d*x+c)^3*exp(n*ln(a+b*tan(d
*x+c)))-(-b^4*n^4+4*a^2*b^2*n^3-14*b^4*n^3+36*a^2*b^2*n^2-71*b^4*n^2+24*a^
4*n+80*a^2*b^2*n-154*b^4*n-120*b^4)/b^4/(n^5+15*n^4+85*n^3+225*n^2+274*n+1
20)/d*tan(d*x+c)*exp(n*ln(a+b*tan(d*x+c)))+2*(b^2*n^2+9*b^2*n+6*a^2+20*b^2
)*a/b^3/d*n/(n^4+14*n^3+71*n^2+154*n+120)*tan(d*x+c)^2*exp(n*ln(a+b*tan(d*
x+c)))
```

3.646.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.61

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$=\frac{(8(3a^5+10a^3b^2+15ab^4-(a^3b^2-3ab^4)n^2+3(a^3b^2+5ab^4)n)\cos(dx+c)^5+4(2ab^4n^3+3(a^3b^2+3}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
(8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 - (a^3*b^2 - 3*a*b^4)*n^2 + 3*(a^3*b^2 +
 5*a*b^4)*n)*cos(d*x + c)^5 + 4*(2*a*b^4*n^3 + 3*(a^3*b^2 + 3*a*b^4)*n^2 +
 (3*a^3*b^2 + 7*a*b^4)*n)*cos(d*x + c)^3 + (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a
*b^4*n^2 + 6*a*b^4*n)*cos(d*x + c) + (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 +
50*b^5*n + 24*b^5 + 8*(8*b^5 - (3*a^2*b^3 - b^5)*n^2 - 3*(a^4*b + 3*a^2*b^
3 - 2*b^5)*n)*cos(d*x + c)^4 + 4*(8*b^5 - (a^2*b^3 - b^5)*n^3 - (3*a^2*b^3
 - 7*b^5)*n^2 - 2*(a^2*b^3 - 7*b^5)*n)*cos(d*x + c)^2)*sin(d*x + c))*((a*c
os(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^n/((b^5*d*n^5 + 15*b^5*d*n^4 +
 85*b^5*d*n^3 + 225*b^5*d*n^2 + 274*b^5*d*n + 120*b^5*d)*cos(d*x + c)^5)
```

3.646.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**n,x)
```

output
```
Timed out
```

3.646.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.78

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)} + \frac{2\left((n^2+3\,n+2)b^3\tan(dx+c)^3+(n^2+n)ab^2\tan(dx+c)^2-2\,a^2bn\tan(dx+c)+2\,a^3\right)(b\tan(dx+c)+a)^n}{(n^3+6\,n^2+11\,n+6)b^3} + \left.\left((n^4+10\,n^3+\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*tan(d*x
 + c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)
*(b*tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 +
35*n^2 + 50*n + 24)*b^5*tan(d*x + c)^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^
4*tan(d*x + c)^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*tan(d*x + c)^3 + 12*(n^2
+ n)*a^3*b^2*tan(d*x + c)^2 - 24*a^4*b*n*tan(d*x + c) + 24*a^5)*(b*tan(d*x
 + c) + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5))/d
```

3.646.8 Giac [F(-2)]

Exception generated.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,4,0,0,0]%%%}+%%%{2,[0,1,2,2,0,0]%%%}+%%%{-4,[0,1,2,
1,1,0]%%%
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos\left(c+d\,x\right)^6}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^6,x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^6, x)
```

3.647 $\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.647.1 Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(a^2+b^2)\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)}-\frac{2a(a+b\tan(c+dx))^{2+n}}{b^3d(2+n)}+\frac{(a+b\tan(c+dx))^{3+n}}{b^3d(3+n)}$$

output
```
(a^2+b^2)*(a+b*tan(d*x+c))^(1+n)/b^3/d/(1+n)-2*a*(a+b*tan(d*x+c))^(2+n)/b^
3/d/(2+n)+(a+b*tan(d*x+c))^(3+n)/b^3/d/(3+n)
```

3.647.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(a+b\tan(c+dx))^{1+n}\left(\frac{a^2+b^2}{1+n}-\frac{2a(a+b\tan(c+dx))}{2+n}+\frac{(a+b\tan(c+dx))^2}{3+n}\right)}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*((a^2 + b^2)/(1 + n) - (2*a*(a + b*Tan[c + d
*x]))/(2 + n) + (a + b*Tan[c + d*x])^2/(3 + n)))/(b^3*d)
```

3.647.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3987, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^4(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{3987}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^n\left(\tan^2(c+dx)b^2+b^2\right)}{b^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+b\tan(c+dx))^n\left(\tan^2(c+dx)b^2+b^2\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\left(a^2+b^2\right)(a+b\tan(c+dx))^n-2a(a+b\tan(c+dx))^{n+1}+(a+b\tan(c+dx))^{n+2}\right)d(b\tan(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\left(a^2+b^2\right)(a+b\tan(c+dx))^{n+1}}{n+1}-\frac{2a(a+b\tan(c+dx))^{n+2}}{n+2}+\frac{(a+b\tan(c+dx))^{n+3}}{n+3}}{b^3d}$$

input
```
Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(((a^2 + b^2)*(a + b*Tan[c + d*x])^(1 + n))/(1 + n) - (2*a*(a + b*Tan[c +
d*x])^(2 + n))/(2 + n) + (a + b*Tan[c + d*x])^(3 + n)/(3 + n))/(b^3*d)
```

3.647.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.647.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(88) = 176$.

Time = 109.94 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.32

method	result
derivativedivides	$\frac{(\tan^3(dx+c))\mathrm{e}^{n\ln(a+b\tan(dx+c))}}{d(3+n)}+\frac{a(b^2n^2+5b^2n+2a^2+6b^2)\mathrm{e}^{n\ln(a+b\tan(dx+c))}}{b^3d(n^3+6n^2+11n+6)}+\frac{an(\tan^2(dx+c))\mathrm{e}^{n\ln(a+b\tan(dx}}{bd(n^2+5n+6)}$
default	$\frac{(\tan^3(dx+c))\mathrm{e}^{n\ln(a+b\tan(dx+c))}}{d(3+n)}+\frac{a(b^2n^2+5b^2n+2a^2+6b^2)\mathrm{e}^{n\ln(a+b\tan(dx+c))}}{b^3d(n^3+6n^2+11n+6)}+\frac{an(\tan^2(dx+c))\mathrm{e}^{n\ln(a+b\tan(dx}}{bd(n^2+5n+6)}$

input
```
int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/d/(3+n)*tan(d*x+c)^3*exp(n*ln(a+b*tan(d*x+c)))+a*(b^2*n^2+5*b^2*n+2*a^2+
6*b^2)/b^3/d/(n^3+6*n^2+11*n+6)*exp(n*ln(a+b*tan(d*x+c)))+a*n/b/d/(n^2+5*n
+6)*tan(d*x+c)^2*exp(n*ln(a+b*tan(d*x+c)))-(-b^2*n^2+2*a^2*n-5*b^2*n-6*b^2
)/b^2/(n^3+6*n^2+11*n+6)/d*tan(d*x+c)*exp(n*ln(a+b*tan(d*x+c)))
```

3.647.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.00

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{(2\,(2\,ab^2n+a^3+3\,ab^2)\cos{(dx+c)}^3+(ab^2n^2+ab^2n)\cos{(dx+c)}+(b^3n^2+3\,b^3n+2\,b^3+2\,(2\,b^3-(a^2}{(b^3dn^3+6\,b^3dn^2+11\,b^3dn+6\,b^3d)\cos{(dx+c}}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
(2*(2*a*b^2*n + a^3 + 3*a*b^2)*cos(d*x + c)^3 + (a*b^2*n^2 + a*b^2*n)*cos(
d*x + c) + (b^3*n^2 + 3*b^3*n + 2*b^3 + 2*(2*b^3 - (a^2*b - b^3)*n)*cos(d*
x + c)^2)*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^n
/((b^3*d*n^3 + 6*b^3*d*n^2 + 11*b^3*d*n + 6*b^3*d)*cos(d*x + c)^3)
```

3.647.6 Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (a+b\tan{(c+dx)})^n\sec^4{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**4, x)
```

3.647.7 Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)}+\frac{\left((n^2+3\,n+2)b^3\tan(dx+c)^3+(n^2+n)ab^2\tan(dx+c)^2-2\,a^2bn\tan(dx+c)+2\,a^3\right)(b\tan(dx+c)+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}}{d}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*tan(d*x +
 c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)*(
b*tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3))/d
```

3.647.8 Giac [F(-2)]

Exception generated.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0,0,0]%%%}+%%%{1,[0,1,0,2,0,0]%%%}+%%%{-2,[0,1,0,
1,1,0]%%%
```

3.647.9 Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)^4}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^4,x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^4, x)
```

3.648 $\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.648.8 Giac [F(-2)] 4640

3.648.9 Mupad [B] (verification not implemented) 4640

3.648.1 Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

output

```
(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

3.648.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

input

```
Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output

```
(a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

3.648.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^2(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \; 3987$$

$$\frac{\int (a+b\tan(c+dx))^nd(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \; 17$$

$$\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

input
```
Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

3.648.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.648.4 Maple [A] (verified)

Time = 8.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27
default	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27

input
```
int(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

3.648.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a\cos(dx+c)+b\sin(dx+c))\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{(bdn+bd)\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
(a*cos(d*x + c) + b*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d
*x + c))^n/((b*d*n + b*d)*cos(d*x + c))
```

3.648.6 Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**2, x)
```

3.648.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(b\tan(dx+c)+a)^{n+1}}{bd(n+1)}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
(b*tan(d*x + c) + a)^(n + 1)/(b*d*(n + 1))
```

3.648.8 Giac [F(-2)]

Exception generated.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument
Value
```

3.648.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx=\begin{cases}\frac{\ln(a+b\tan(c+d\,x))}{b\,d} & \text{if } n=-1\\ \frac{(a+b\tan(c+d\,x))^{n+1}}{b\,d\,(n+1)} & \text{if } n\neq-1\end{cases}$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^2,x)
```

output
```
piecewise(n == -1, log(a + b*tan(c + d*x))/(b*d), n ~= -1, (a + b*tan(c +
d*x))^(n + 1)/(b*d*(n + 1)))
```

3.649 $\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.649.1 Optimal result

Integrand size = 21, antiderivative size = 272

$$\begin{aligned}\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx =\\ &-\frac{\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{4\left(1+\frac{a^2}{b^2}\right)b\left(a-\sqrt{-b^2}\right)d(1+n)}\\ &+\frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)+an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)d(1+n)}\\ &+\frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d}\end{aligned}$$

output
```
-1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(-a*n+(1+
a^2/b^2-n)*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)/b/d/(1+n)/(a-(
-b^2)^(1/2))+1/4*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/
2)))*(a*n+(1+a^2/b^2-n)*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d/(
1+n)/(a+(-b^2)^(1/2))+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(
1+n)/(a^2+b^2)/d
```

3.649.2 Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{(a+b\tan(c+dx))^{1+n}\left(-\frac{\left(\sqrt{-b^2}(a^2-b^2(-1+n))-ab^2n\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{\left(a-\sqrt{-b^2}\right)(1+n)}+\frac{\left(a^2\sqrt{-b^2}+(-b^2)^{3/2}\right.}{}\right.}{4b\left(a^2+b^2\right)d}$$

input
```
Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*(-(((Sqrt[-b^2]*(a^2 - b^2*(-1 + n)) - a*b^2
*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2
])])/((a - Sqrt[-b^2])*(1 + n))) + ((a^2*Sqrt[-b^2] + (-b^2)^(3/2)*(-1 + n
) + a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a +
Sqrt[-b^2])])/((a + Sqrt[-b^2])*(1 + n)) + 2*b*Cos[c + d*x]^2*(b + a*Tan[c
 + d*x])))/(4*b*(a^2 + b^2)*d)
```

3.649.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3987, 27, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)^2}dx$$

$$\Big\downarrow \; 3987$$

$$\frac{\int \frac{b^4(a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow \; 27$$

$$\frac{b^3\int \frac{(a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{496}$$

$$\frac{b^3\left(\frac{\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}-\frac{\int -\frac{(a+b\tan(c+dx))^n\left(a^2-bn\tan(c+dx)a+b^2(1-n)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{b^3\left(\frac{\int \frac{(a+b\tan(c+dx))^n\left(a^2-bn\tan(c+dx)a+b^2(1-n)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{2b^2(a^2+b^2)(b^2\tan^2(c+dx)+b^2)}\right)}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{b^3\left(\frac{\int \left(\frac{\left(anb^2+\sqrt{-b^2}\left(a^2+b^2(1-n)\right)\right)(a+b\tan(c+dx))^n}{2b^2\left(\sqrt{-b^2}-b\tan(c+dx)\right)}+\frac{\left(\sqrt{-b^2}\left(a^2+b^2(1-n)\right)-ab^2n\right)(a+b\tan(c+dx))^n}{2b^2\left(b\tan(c+dx)+\sqrt{-b^2}\right)}\right)d(b\tan(c+dx))}{2b^2(a^2+b^2)}+\frac{\left(ab\tan(c+dx)+b^2\right)(a+b}{2b^2(a^2+b^2)(b^2\tan^2(}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3\left(\frac{\frac{\left(\sqrt{-b^2}\left(a^2+b^2(1-n)\right)+ab^2n\right)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{2b^2(n+1)\left(a+\sqrt{-b^2}\right)}-\frac{\left(\sqrt{-b^2}\left(a^2+b^2(1-n)\right)-ab^2n\right)(a+b\tan(c+dx))^{n+1}}{2b^2(n+1)\left(\right.}}{2b^2(a^2+b^2)}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b^3*(((a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(2*b^2*(a^2
+ b^2)*(b^2 + b^2*Tan[c + d*x]^2)) + (-1/2*((Sqrt[-b^2]*(a^2 + b^2*(1 - n)
) - a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a -
Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(b^2*(a - Sqrt[-b^2])*(1 + n))
+ ((Sqrt[-b^2]*(a^2 + b^2*(1 - n)) + a*b^2*n)*Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n)
)/(2*b^2*(a + Sqrt[-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2))))/d
```

3.649. $\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$

3.649.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.649.4 Maple [F]

$$\int \left(\cos^2\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

input
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

3.649.5 Fricas [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^n \cos\left(dx+c\right)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

3.649.6 Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^n \cos^2\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*cos(c + d*x)**2, x)
```

3.649.7 Maxima [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan{(dx+c)}+a)^n\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

3.649.8 Giac [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan{(dx+c)}+a)^n\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

3.649.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx=\int \cos{(c+d\,x)}^2\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.650 $\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.650.7 Maxima [F] 4652
3.650.8 Giac [F] 4653
3.650.9 Mupad [F(-1)] 4653

3.650.1 Optimal result

Integrand size = 21, antiderivative size = 434

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{b\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}-\frac{\sqrt{-b^2}\left(3a^4+a^2b^2\left(6-2n-n^2\right)+b^4\left(3-4n+n^2\right)\right)}{b^6}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{16\left(1+\frac{a^2}{b^2}\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$
$$+\frac{b\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}+\frac{\sqrt{-b^2}\left(3a^4+a^2b^2\left(6-2n-n^2\right)+b^4\left(3-4n+n^2\right)\right)}{b^6}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx}{a+\sqrt{-b^2}}\right.}{16\left(1+\frac{a^2}{b^2}\right)^2\left(a+\sqrt{-b^2}\right)d(1+n)}$$
$$+\frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d}$$
$$+\frac{b\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b^2(3-n)+a^2(1+n)+ab\left(5+\frac{3a^2}{b^2}-2n\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d}$$

output
```
1/16*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a*(5+3
*a^2/b^2-2*n)*n/b^2-(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(n^2-4*n+3))*(-b^2)^(1
/2)/b^6)*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/d/(1+n)/(a-(-b^2)^(1/2))+1/1
6*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*(5+3*a^
2/b^2-2*n)*n/b^2+(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(n^2-4*n+3))*(-b^2)^(1/2)
/b^6)*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/d/(1+n)/(a+(-b^2)^(1/2))+1/4*co
s(d*x+c)^4*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d+1/8*b*cos(d
*x+c)^2*(a+b*tan(d*x+c))^(1+n)*(b^2*(3-n)+a^2*(1+n)+a*b*(5+3*a^2/b^2-2*n)*
tan(d*x+c))/(a^2+b^2)^2/d
```

3.650.2 Mathematica [A] (verified)

Time = 4.71 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.83

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{(a+b\tan(c+dx))^{1+n}\left(\frac{\frac{\left(ab^2\left(3a^2+b^2(5-2n)\right)n+\sqrt{-b^2}\left(-3a^4-b^4\left(3-4n+n^2\right)+a^2b^2\left(-6+2n+n^2\right)\right)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c-}{a-\sqrt{-b}}\right.}{a-\sqrt{-b^2}}}{(}\right.}{}$$

input
```
Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*((((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-
b^2]*(-3*a^4 - b^4*(3 - 4*n + n^2) + a^2*b^2*(-6 + 2*n + n^2)))*Hypergeome
tric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])/(a - Sqrt
[-b^2]) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + b^4*(3 -
 4*n + n^2) - a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n
, (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])/(a + Sqrt[-b^2]))/((a^2 + b^2)*(
1 + n)) + 4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) - (2*b*Cos[c + d*x]^2*(b
^3*(-3 + n) - a^2*b*(1 + n) - a*(3*a^2 + b^2*(5 - 2*n))*Tan[c + d*x]))/(a^
2 + b^2)))/(16*b*(a^2 + b^2)*d)
```

3.650.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3987, 27, 496, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int \frac{b^6 (a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{bd}$$

$$\Big\downarrow 27$$

$$\frac{b^5 \int \frac{(a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)b^2+b^2\right)^3}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow 496$$

$$\frac{b^5 \left(\frac{\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{4b^2(a^2+b^2)\left(b^2\tan^2(c+dx)+b^2\right)^2}-\frac{\int -\frac{(a+b\tan(c+dx))^n\left(3a^2+b(2-n)\tan(c+dx)a+b^2(3-n)\right)}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}\right)}{d}$$

$$\Big\downarrow 25$$

$$\frac{b^5 \left(\frac{\int \frac{(a+b\tan(c+dx))^n\left(3a^2+b(2-n)\tan(c+dx)a+b^2(3-n)\right)}{\left(\tan^2(c+dx)b^2+b^2\right)^2}d(b\tan(c+dx))}{4b^2(a^2+b^2)}+\frac{\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{4b^2(a^2+b^2)\left(b^2\tan^2(c+dx)+b^2\right)^2}\right)}{d}$$

$$\Big\downarrow 686$$

$$\frac{b^5 \left(\frac{\frac{(a+b\tan(c+dx))^{n+1}\left(ab\left(3a^2+b^2(5-2n)\right)\tan(c+dx)+b^2\left(a^2(n+1)+b^2(3-n)\right)\right)}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}-\frac{\int -\frac{(a+b\tan(c+dx))^n\left(3a^4+b^2\left(-n^2-2n+6\right)a^2-b\left(3a^2+b^2(5-2n)\right)n\tan(\right.}{\tan^2(c+dx)b^2+b^2}}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow 25$$

$$\frac{b^5 \left(\frac{\frac{\int \frac{(a+b\tan(c+dx))^n\left(3a^4+b^2\left(-n^2-2n+6\right)a^2-b\left(3a^2+b^2(5-2n)\right)n\tan(c+dx)a+b^4\left(n^2-4n+3\right)\right)}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{2b^2\left(a^2+b^2\right)}+\frac{\left(ab\left(3a^2+b^2(5-2n)\right)\tan(c+dx)+b^2\left(a^2(\right.\right.}{2b^2\left(a^2+b^2\right)\left(b^2\,\mathrm{ta}\right.}}{4b^2(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow 657$$

$$b^5 \left(\frac{\frac{\int \left(\frac{\left(a\left(3a^2+b^2(5-2n)\right)nb^2+\sqrt{-b^2}\left(3a^4+b^2\left(-n^2-2n+6\right)a^2+b^4\left(n^2-4n+3\right)\right)\right)(a+b\tan(c+dx))^n}{2b^2\left(\sqrt{-b^2}-b\tan(c+dx)\right)}+\frac{\left(\sqrt{-b^2}\left(3a^4+b^2\left(-n^2-2n+6\right)a^2+b^4\left(n^2-4n+3\right)\right)-ab^2\right.}{2b^2\left(b\tan(c+dx)+\sqrt{-}\right.}\right.}{2b^2\left(a^2+b^2\right)}}{4b^2(a^2+b^2)}\right.$$

3.650. $\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$

$$\Big\downarrow \text{2009}$$

$$b^5\left(\frac{\left(ab\tan(c+dx)+b^2\right)(a+b\tan(c+dx))^{n+1}}{4b^2(a^2+b^2)\left(b^2\tan^2(c+dx)+b^2\right)^2}+\frac{\frac{\left(ab\left(3a^2+b^2(5-2n)\right)\tan(c+dx)+b^2\left(a^2(n+1)+b^2(3-n)\right)\right)(a+b\tan(c+dx))^{n+1}}{2b^2\left(a^2+b^2\right)\left(b^2\tan^2(c+dx)+b^2\right)}+\frac{\left(ab^2n\left(3a^2+b^2(5-2n)\right)\right.}{}}{}\right.$$

input
```
Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

output
```
(b^5*(((a + b*Tan[c + d*x])^(1 + n)*(b^2 + a*b*Tan[c + d*x]))/(4*b^2*(a^2
 + b^2)*(b^2 + b^2*Tan[c + d*x]^2)^2) + (((a + b*Tan[c + d*x])^(1 + n)*(b^2
*(b^2*(3 - n) + a^2*(1 + n)) + a*b*(3*a^2 + b^2*(5 - 2*n))*Tan[c + d*x]))/
(2*b^2*(a^2 + b^2)*(b^2 + b^2*Tan[c + d*x]^2)) + (((a*b^2*(3*a^2 + b^2*(5
 - 2*n))*n - Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n
^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a - Sqrt[-b^2])*(1 + n)) + ((a
*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^
2) + b^4*(3 - 4*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c
 + d*x])/(a + Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(2*b^2*(a + Sqrt[
-b^2])*(1 + n)))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.650.4 Maple [F]

$$\int \left(\cos^{4}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{n}dx$$

input
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

3.650.5 Fricas [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

3.650.6 Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (a+b\tan(c+dx))^n\cos^4(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*cos(c + d*x)**4, x)
```

3.650.7 Maxima [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

3.650.8 Giac [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int \cos(c+d\,x)^4\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

3.651 $\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.651.1 Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{\operatorname{AppellF1}\left(1+n,-\frac{1}{2},-\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\sec(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(1+n)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

output
```
AppellF1(1+n,-1/2,-1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+
c))/(a+(-b^2)^(1/2)))*sec(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)/(1+(-a-b
*tan(d*x+c))/(a-(-b^2)^(1/2)))^(1/2)/(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2))
)^(1/2)
```

3.651.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.54 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.92

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{2(a-ib)(a+ib)(2+n)\operatorname{AppellF1}\Big(1+}{bd(1+n)\left(2\left(a^2+b^2\right)(2+n)\operatorname{AppellF1}\left(1+n,-\frac{1}{2},-\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)-\Big((a-ib)\operatorname{App}\right.}$$

input
```
Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
(2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*T
an[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Sec[c + d*x]*(a +
b*Tan[c + d*x])^(1 + n))/(b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 +
n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])
/(a + I*b)] - ((a - I*b)*AppellF1[2 + n, -1/2, 1/2, 3 + n, (a + b*Tan[c +
d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 +
n, 1/2, -1/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/
(a + I*b)])*(a + b*Tan[c + d*x])))
```

3.651.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(c+dx)^3(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \; 3992$$

$$\frac{\sec(c+dx)\int (a+b\tan(c+dx))^n\sqrt{\tan^2(c+dx)+1}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \; 514$$

$$\frac{\sqrt{\tan^2(c+dx)+1}\sec(c+dx)\int (a+b\tan(c+dx))^n\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}d(a+b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

$$\Big\downarrow \; 150$$

$$\frac{\sqrt{\tan^2(c+dx)+1}\sec(c+dx)(a+b\tan(c+dx))^{n+1}\operatorname{AppellF1}\left(n+1,-\frac{1}{2},-\frac{1}{2},n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)\sqrt{\sec^2(c+dx)}\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

input
```
Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]),
 (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^
(1 + n)*Sqrt[1 + Tan[c + d*x]^2])/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*Sqrt[1
 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[c + d*x])/(a
 + Sqrt[-b^2])])
```

3.651.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.651.4 Maple [F]

$$\int \left(\sec^{3}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{n}dx$$

input
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

3.651.5 Fricas [F]

$$\int \sec^{3}(c+dx)(a+b\tan(c+dx))^{n}\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^{n}\sec\left(dx+c\right)^{3}\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

3.651.6 Sympy [F]

$$\int \sec^{3}(c+dx)(a+b\tan(c+dx))^{n}\,dx=\int \left(a+b\tan\left(c+dx\right)\right)^{n}\sec^{3}\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**3, x)
```

3.651.7 Maxima [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

3.651.8 Giac [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)^3}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^3,x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x)^3, x)
```

3.652 $\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.652.1 Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos(c+dx)(a+b\tan(c+dx))^{1+n}\sqrt{1-\frac{a+b\tan(c+d}{a-\sqrt{-b^2}}}}{bd(1+n)}$$

output
```
AppellF1(1+n,1/2,1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c)
)/(a+(-b^2)^(1/2)))*cos(d*x+c)*(1+(-a-b*tan(d*x+c))/(a-(-b^2)^(1/2)))^(1/2
)*(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(1/2)*(a+b*tan(d*x+c))^(1+n)/b/d/
(1+n)
```

3.652.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.18 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.14

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{2(a^2+b^2)^2(2+n)\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,a\right.}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)\operatorname{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+(\right.}$$

input

```
Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output

```
(2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d
*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^3*(-I + Tan[c
 + d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a +
I*b)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (
a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + ((a - I*b
)*AppellF1[2 + n, 1/2, 3/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*
Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 3/2, 1/2, 3 + n, (a +
 b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c
+ d*x])))
```

3.652.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{(a+b\tan(c+dx))^n}{\sqrt{\tan^2(c+dx)+1}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{514}$$

$$\frac{\sec(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}\int \frac{(a+b\tan(c+dx))^n}{\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}d(a+b\tan(c+dx))}{bd\sqrt{\tan^2(c+dx)+1}\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sec(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}(a+b\tan(c+dx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{1}{2},\frac{1}{2},n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\right.}{bd(n+1)\sqrt{\tan^2(c+dx)+1}\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (
a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[
c + d*x])/(a + Sqrt[-b^2])])/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*Sqrt[1 + Ta
n[c + d*x]^2])
```

3.652.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.652.4 Maple [F]

$$\int \sec(dx+c)\left(a+b\tan(dx+c)\right)^n dx$$

input
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

3.652.5 Fricas [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan(dx+c)+a\right)^n\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

3.652.6 Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(a+b\tan(c+dx)\right)^n\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*sec(c + d*x), x)
```

3.652.7 Maxima [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

3.652.8 Giac [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\sec(dx+c)\,dx$$

input
```
integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)}\,dx$$

input
```
int((a + b*tan(c + d*x))^n/cos(c + d*x),x)
```

output
```
int((a + b*tan(c + d*x))^n/cos(c + d*x), x)
```

3.653 $\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.653.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^3(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c+}{a-\sqrt{-b^2}}}{bd(1+n)}$$

output
```
AppellF1(1+n,3/2,3/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c)
)/(a+(-b^2)^(1/2)))*cos(d*x+c)^3*(a+b*tan(d*x+c))^(1+n)*(1+(-a-b*tan(d*x+c
))/(a-(-b^2)^(1/2)))^(3/2)*(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(3/2)/b/
d/(1+n)
```

3.653.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.12

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{2(a^2+b^2)^2(2+n)\,\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)\,\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+3}$$

input
```
Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
(2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d
*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^5*(-I + Tan[c
 + d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a +
I*b)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (
a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 3*((a - I
*b)*AppellF1[2 + n, 3/2, 5/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a +
b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 5/2, 3/2, 3 + n, (a
 + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[
c + d*x])))
```

3.653.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{(a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)+1\right)^{3/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{514}$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}\int \frac{(a+b\tan(c+dx))^n}{\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}}d(a+b\tan(c+dx}{bd\left(\tan^2(c+dx)+1\right)^{3/2}\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}(a+b\tan(c+dx))^{n+1}\,\mathrm{AppellF1}\left(n+1,\frac{3}{2},\frac{3}{2},n+2,\frac{a+b\,\mathrm{ta}}{a-}\right.}{bd(n+1)\left(\tan^2(c+dx)+1\right)^{3/2}\sqrt{\sec^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

output
```
(AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (
a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(3/2)*(1 - (a + b*Tan[c
+ d*x])/(a + Sqrt[-b^2]))^(3/2))/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*(1 + Ta
n[c + d*x]^2)^(3/2))
```

3.653.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.653.4 Maple [F]

$$\int \cos (dx + c) (a + b\tan (dx + c))^n \, dx$$

input
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

3.653.5 Fricas [F]

$$\int \cos(c + dx)(a + b\tan(c + dx))^n \, dx = \int (b\tan (dx + c) + a)^n \cos (dx + c) \, dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

3.653.6 Sympy [F]

$$\int \cos(c + dx)(a + b\tan(c + dx))^n \, dx = \int (a + b\tan (c + dx))^n \cos (c + dx) \, dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

output
```
Integral((a + b*tan(c + d*x))**n*cos(c + d*x), x)
```

3.653.7 Maxima [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan{(dx+c)}+a)^n\cos{(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

3.653.8 Giac [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan{(dx+c)}+a)^n\cos{(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx=\int \cos{(c+d\,x)}\,{\left(a+b\tan(c+d\,x)\right)}^n\,dx$$

input
```
int(cos(c + d*x)*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.654 $\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$

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3.654.1 Optimal result

Integrand size $= 21$, antiderivative size $= 161$

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^5(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c+}{a-\sqrt{-b^2}}\right.}{bd(1+n)}$$

output
```
AppellF1(1+n,5/2,5/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c)
)/(a+(-b^2)^(1/2)))*cos(d*x+c)^5*(a+b*tan(d*x+c))^(1+n)*(1+(-a-b*tan(d*x+c
))/(a-(-b^2)^(1/2)))^(5/2)*(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(5/2)/b/
d/(1+n)
```

3.654.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time $= 8.43$ (sec) , antiderivative size $= 341$, normalized size of antiderivative $= 2.12$

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{2\left(a^2+b^2\right)^2(2+n)\,\text{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\right.}{(a-ib)(a+ib)bd(1+n)\left(2\left(a^2+b^2\right)(2+n)\,\text{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+5\right.}$$

input
```
Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output
```
(2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d
*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^7*(-I + Tan[c
 + d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a +
I*b)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (
a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 5*((a - I
*b)*AppellF1[2 + n, 5/2, 7/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a +
b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 7/2, 5/2, 3 + n, (a
 + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[
c + d*x])))
```

3.654.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3992, 514, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^n}{\sec(c+dx)^3}dx$$

$$\Big\downarrow \text{3992}$$

$$\frac{\sec(c+dx)\int \frac{(a+b\tan(c+dx))^n}{\left(\tan^2(c+dx)+1\right)^{5/2}}d(b\tan(c+dx))}{bd\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{514}$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}\int \frac{(a+b\tan(c+dx))^n}{\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}}d(a+b\tan(c+dx}{bd\left(\tan^2(c+dx)+1\right)^{5/2}\sqrt{\sec^2(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sec(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}(a+b\tan(c+dx))^{n+1}\,\mathrm{AppellF1}\left(n+1,\frac{5}{2},\frac{5}{2},n+2,\frac{a+b\,\mathrm{ta}}{a-}\right.}{bd(n+1)\left(\tan^2(c+dx)+1\right)^{5/2}\sqrt{\sec^2(c+dx)}}$$

input

```
Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

output

```
(AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (
a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(5/2)*(1 - (a + b*Tan[c
+ d*x])/(a + Sqrt[-b^2]))^(5/2))/(b*d*(1 + n)*Sqrt[Sec[c + d*x]^2]*(1 + Ta
n[c + d*x]^2)^(5/2))
```

3.654.3.1 Defintions of rubi rules used

rule 150

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 514

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(a + b*x^2)^p/(d*(1 - (c + d*x)/(c - d*q))^p*(1 - (
c + d*x)/(c + d*q))^p)   Subst[Int[x^n*Simp[1 - x/(c + d*q), x]^p*Simp[1 -
x/(c - d*q), x]^p, x], x, c + d*x], x]] /; FreeQ[{a, b, c, d, n, p}, x] &&
NeQ[b*c^2 + a*d^2, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3992

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[Sec[e + f*x]/(b*f*Sqrt[Sec[e + f*x]^2])   Subst[Int[(
a + x)^n*(1 + x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b
, e, f, n}, x] && NeQ[a^2 + b^2, 0] && IntegerQ[(m - 1)/2]
```

3.654.4 Maple [F]

$$\int \left(\cos^{3}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{n}dx$$

input
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

output
```
int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

3.654.5 Fricas [F]

$$\int \cos^{3}(c+dx)(a+b\tan(c+dx))^{n}\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^{n}\cos\left(dx+c\right)^{3}\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

3.654.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{3}(c+dx)(a+b\tan(c+dx))^{n}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

output
```
Timed out
```

3.654.7 Maxima [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

3.654.8 Giac [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^3\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.655 $\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

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3.655.1 Optimal result

Integrand size = 26, antiderivative size = 124

$$\begin{aligned}\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx &= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d}\\ &+\frac{10a(e\cos(c+dx))^{7/2}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d}\\ &+\frac{10a(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{21d}\end{aligned}$$

output
```
-2/7*I*a*(e*cos(d*x+c))^(7/2)/d+10/21*a*(e*cos(d*x+c))^(7/2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d
/cos(d*x+c)^(7/2)+2/7*a*(e*cos(d*x+c))^(7/2)*tan(d*x+c)/d+10/21*a*(e*cos(d
*x+c))^(7/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

3.655.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\begin{aligned}&\int (e\cos(c+dx))^{7/2}(a\\ &+ia\tan(c+dx))\,dx=\frac{ae^3\sqrt{e\cos(c+dx)}(\cos(dx)-i\sin(dx))\left(10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\,\mathrm{si}\right.}{}\end{aligned}$$

input
```
Integrate[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*e^3*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*(10*EllipticF[(c + d*x
)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + Sqrt[Cos[c + d*x]]*(-8*I + (2*I)
*Cos[2*(c + d*x)] + 5*Sin[2*(c + d*x)]))*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x
]))/(21*d*Sqrt[Cos[c + d*x]])
```

3.655.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3998, 3042, 3967, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\int \frac{1}{(e\sec(c+dx))^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\int\frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \ 4256$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \ 4258$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2ia}{7d(e\sec(c+d}$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(a\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)-\frac{2i}{7d(e\sec(c}$$

$$\Big\downarrow \ 3120$$

3.655. $\int(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

$$(e\cos(c+dx))^{7/2}(e\sec(c+$$

$$dx))^{7/2}\Bigg(a\Bigg(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\Bigg)-\frac{2}{7d(e\sec(c}$$

input
```
Int[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*((((-2*I)/7)*a)/(d*(e*Sec[c
+ d*x])^(7/2)) + a*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*x])^(5/2)) + (5*(
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d
*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*e^2)))
```

3.655.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.655.4 Maple [A] (verified)

Time = 9.94 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.85

method	result
parts	$-\frac{2a\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,e^4\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{21\sqrt{-e\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\right.}}$
default	$-\frac{2a\,e^4\left(48i\left(\sin^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+48\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-96i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+72i\left(\sin^5\left(\frac{dx}{2}+\right.\right.\right.}{}$

input
```
int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/21*a*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*(48*
cos(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*
cos(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(
-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)
/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d-2/7*I*a*(e*cos(d*x+c))^(7/2)/d
```

3.655.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{\left(-20i\sqrt{2}ae^{\frac{7}{2}}e^{(i\,dx+i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-3i\,ae^3e^{(4i\,dx+4i\,c)}-\right.\right.}{42\,d}$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/42*(-20*I*sqrt(2)*a*e^(7/2)*e^(I*d*x + I*c)*weierstrassPInverse(-4, 0, e
^(I*d*x + I*c)) + sqrt(1/2)*(-3*I*a*e^3*e^(4*I*d*x + 4*I*c) - 16*I*a*e^3*e
^(2*I*d*x + 2*I*c) + 7*I*a*e^3)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*
d*x - 1/2*I*c))*e^(-I*d*x - I*c)/d
```

3.655.6 Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

3.655.7 Maxima [F]

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

3.655.8 Giac [F]

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

3.655.9 Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx=\int (e\ \cos\left(c+d\,x\right))^{7/2}\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)\,dx$$

input
```
int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

3.656 $\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

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3.656.1 Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{6a(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d}$$

output
```
-2/5*I*a*(e*cos(d*x+c))^(5/2)/d+6/5*a*(e*cos(d*x+c))^(5/2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/c
os(d*x+c)^(5/2)+2/5*a*(e*cos(d*x+c))^(5/2)*tan(d*x+c)/d
```

3.656.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.78

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{e^2\sqrt{e\cos(c+dx)}(\cos(dx)-i\sin(dx))\left(\left(9\cos(c-dx-\arctan(\tan(c)))(\csc(c)-i\,\mathrm{sec}\right.\right.}{}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(e^2*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*((9*Cos[c - d*x - ArcTan
[Tan[c]]]*(Csc[c] - I*Sec[c]) + 3*Cos[c + d*x + ArcTan[Tan[c]]]*(Csc[c] -
I*Sec[c]) - Csc[c]*Sqrt[Sec[c]^2]*(Cos[d*x] + I*Sin[d*x])*(6*Cos[c] + 3*Co
s[c + 2*d*x] + 3*Cos[3*c + 2*d*x] + (2*I)*Sin[c] - (4*I)*Sin[c + 2*d*x] -
(2*I)*Sin[3*c + 2*d*x]))*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2] + (6*I)*Hyperge
ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Arc
Tan[Tan[c]]]*(I + Tan[c]))*(a + I*a*Tan[c + d*x]))/(10*d*Sqrt[Sec[c]^2]*Sq
rt[Sin[d*x + ArcTan[Tan[c]]]^2])
```

3.656.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\int \frac{1}{(e\sec(c+dx))^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 4258$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{3\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 3119$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(a\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}\right)$$

input

```
Int[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

output

```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*((((-2*I)/5)*a)/(d*(e*Sec[c
+ d*x])^(5/2)) + a*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*
x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[c + d*x])^(3/2)
)))
```

3.656. $\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

3.656.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.656.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(101) = 202$.

Time = 7.90 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.28

method	result
default	$\frac{2a\,e^3\left(8i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+6i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{5\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}$
parts	$-\frac{2a\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,e^3\left(-8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{5\sqrt{-e\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\left(\cos\right.\right.}}$
risch	$-\frac{i\left(e^{2i(dx+c)}+7\right)\sqrt{2}\,e^2a\sqrt{e\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{10d}-\frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{i}}{}\right.}{}$

input
```
int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(8*I*sin(
1/2*d*x+1/2*c)^7+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*
x+1/2*c)^5-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+6*I*sin(1/2*d*x+1/2*c
)^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-
I*sin(1/2*d*x+1/2*c))/d
```

3.656.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx=\frac{6i\sqrt{2}ae^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{\frac{1}{2}}\left(-i\,ae^2e\right.}{5\,d}$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/5*(6*I*sqrt(2)*a*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, e^(I*d*x + I*c))) + sqrt(1/2)*(-I*a*e^2*e^(2*I*d*x + 2*I*c) + 5*I*a*e^2
)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))/d
```

3.656.6 Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

3.656.7 Maxima [F]

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

3.656.8 Giac [F]

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

3.656.9 Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx=\int (e\,\cos\left(c+d\,x\right))^{5/2}\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)\,dx$$

input
```
int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

3.657 $\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

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3.657.1 Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d}$$

output
```
-2/3*I*a*(e*cos(d*x+c))^(3/2)/d+2/3*a*(e*cos(d*x+c))^(3/2)*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/c
os(d*x+c)^(3/2)+2/3*a*(e*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

3.657.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{2ae\sqrt{\cos(c+dx)}\sqrt{e\cos(c+dx)}\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(i\cos(c)+\sin(c))+\sqrt{\cos(}\right.}{3d}$$

input
```
Integrate[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(2*a*e*Sqrt[Cos[c + d*x]]*Sqrt[e*Cos[c + d*x]]*(EllipticF[(c + d*x)/2, 2]*
(I*Cos[c] + Sin[c]) + Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*(Cos[d*x
] - I*Sin[d*x])*(-I + Tan[c + d*x]))/(3*d)
```

3.657.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{i\tan(c+dx)a+a}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3967}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\int \frac{1}{(e\sec(c+dx))^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 4258}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(a\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)-\frac{2ia}{3d(e\sec(c+dx))^{3/}}\right.$$

input
```
Int[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

output
```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*((((-2*I)/3)*a)/(d*(e*Sec[c
+ d*x])^(3/2)) + a*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e
*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]]))
)
```

3.657.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.657.4 Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

method	result
default	$-\frac{2a\,e^{2}\left(4i\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4i\left(\sin^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$
parts	$-\frac{2a\sqrt{e\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,e^{2}\left(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\right.\right.}\right.}{3\sqrt{-e\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}\sqrt{2}\,ea\sqrt{e\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{3d}+\frac{2\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,F\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)ea\sqrt{e(}}{3d\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}\,\left(\mathrm{e}^{2i(dx+c)}\right.}$

input
```
int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(4*I*sin
(1/2*d*x+1/2*c)^5+4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*I*sin(1/2*d*
x+1/2*c)^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)+I*sin(1/2*d*x+1/2*c))/d
```

3.657.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2\left(i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,aee^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}ae^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\,d}$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e*e^(1/2*I*d*x + 1/2*I
*c) + I*sqrt(2)*a*e^(3/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/d
```

3.657.6 Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

3.657.7 Maxima [F]

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx=\int (e\cos{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

3.657.8 Giac [F]

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx=\int (e\cos{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

3.657.9 Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx=\int (e\,\cos{(c+d\,x)})^{3/2}\,(a+a\tan(c+d\,x)\,\,1\mathrm{i})\,\,dx$$

input
```
int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

3.658 $\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$

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3.658.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx=-\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

output
```
-2*I*a*(e*cos(d*x+c))^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d
*x+c)^(1/2)
```

3.658.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.68 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.20

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$$

$$= \frac{a\cos(c)\sqrt{e\cos(c+dx)}\sin(c)(\cos(dx)-i\sin(dx))\left({}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\right)(-i\csc(c}{}$$

input
```
Integrate[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*Cos[c]*Sqrt[e*Cos[c + d*x]]*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(Hypergeomet
ricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*((-I)*Csc[c] - Se
c[c])*Sec[c]*Sin[d*x + ArcTan[Tan[c]]] + Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]
*(2*Cos[d*x + ArcTan[Tan[c]]]*Csc[c]*(I*Csc[c] + Sec[c]) + Sec[c]*((-2*I)*
Cos[c + d*x]*Csc[c]^2*Sqrt[Sec[c]^2] + (I*Csc[c] + Sec[c])*Sin[d*x + ArcTa
n[Tan[c]]])))*(-I + Tan[c + d*x]))/(d*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2])
```

3.658.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3967, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))\sqrt{e\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))\sqrt{e\cos(c+dx)}dx$$

$$\Big\downarrow \text{3998}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{i\tan(c+dx)a+a}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3967}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(a\int \frac{1}{\sqrt{e\sec(c+dx)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(a\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{a\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow 3042$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{a\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow 3119$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}\right)$$

input
```
Int[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

output
```
Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]*(((-2*I)*a)/(d*Sqrt[e*Sec[c + d*
x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]))
```

3.658.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.658. $\int\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.658.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.80

method	result
default	$\frac{2ae\left(2i\left(\sin^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}-i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$
parts	$\frac{2a\sqrt{e\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,e\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-e\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\,d}-\frac{2ia\sqrt{e\cos(dx+c)}}{d}$
risch	$-\frac{2i\sqrt{2}\,a\sqrt{e\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,(-2iE(}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e}}$

input
```
int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(2*I*sin(1/2*
d*x+1/2*c)^3+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-I*sin(1/2*d*x+1/2*c))/d
```

3.658.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$$
$$=\frac{2i\sqrt{2}a\sqrt{e}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
2*I*sqrt(2)*a*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^
(I*d*x + I*c)))/d
```

3.658.6 Sympy [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = ia\left(\int\left(-i\sqrt{e\cos\left(c+dx\right)}\right)\,dx + \int\sqrt{e\cos\left(c+dx\right)}\tan\left(c+dx\right)dx\right)$$

input
```
integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*sqrt(e*cos(c + d*x)), x) + Integral(sqrt(e*cos(c + d*x))*
tan(c + d*x), x))
```

3.658.7 Maxima [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \int\sqrt{e\cos\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

3.658.8 Giac [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \int\sqrt{e\cos\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)\,dx$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

3.658.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \int \sqrt{e\,\cos\left(c+d\,x\right)}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

input
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i), x)
```

3.659 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

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3.659.9 Mupad [B] (verification not implemented) 4705

3.659.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx=\frac{2ia}{d\sqrt{e\cos(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{e\cos(c+dx)}}$$

output
```
2*I*a/d/(e*cos(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x
+c))^(1/2)
```

3.659.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.38

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx=$$

$$-\frac{\sqrt{2}a\sqrt{e\cos(c+dx)}(-i+\cot(c))\,\Big(\sqrt{2}\sqrt{\csc^2(c)}+i\cos(c+dx)\sqrt{1+\cos(2dx-2\arctan(\cot(c)))}\,\csc(}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

output
```
-((Sqrt[2]*a*Sqrt[e*Cos[c + d*x]]*(-I + Cot[c])*(Sqrt[2]*Sqrt[Csc[c]^2] +
I*Cos[c + d*x]*Sqrt[1 + Cos[2*d*x - 2*ArcTan[Cot[c]]]]*Csc[c]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot
[c]]])*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(-I + Tan[c + d*x]))/(d*e*Sqrt[Csc[c
]^2]))
```

3.659.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3967, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int \sqrt{e\sec(c+dx)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 3042$$

$$\frac{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow 3120$$

$$\frac{\frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

output
```
(((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d)/(Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]])
```

3.659.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.659. $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.659.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result	size
parts	$\frac{2a\sqrt{2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert\sqrt{2}\right)}{d\sqrt{e\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}+\frac{2ia}{d\sqrt{e\cos\left(dx+c\right)}}$	73
default	$-\frac{2\left(\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos\left(dx+c\right)}{2}}-i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\ \sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$	93

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/d/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))+2*I*a/d/(e*cos(d*x+c))^(1/2)
```

3.659.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2\bigg(-2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,ae^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\left(i\sqrt{2}ae^{(2i\,dx+2i\,c)}+i\sqrt{2}a\right)\sqrt{e}\,\mathrm{weierstrassPInverse}\big(-4,0,e^{(i}}{dee^{(2i\,dx+2i\,c)}+de}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*(-2*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e^(1/2*I*d*x + 1/2*I*
c) + (I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(e)*weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c)))/(d*e*e^(2*I*d*x + 2*I*c) + d*e)
```

3.659.6 Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = ia\left(\int\left(-\frac{i}{\sqrt{e\cos{(c+dx)}}}\right)dx+\int\frac{\tan{(c+dx)}}{\sqrt{e\cos{(c+dx)}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

output
```
I*a*(Integral(-I/sqrt(e*cos(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*c
os(c + d*x)), x))
```

3.659.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = \int \frac{i\,a\tan{(dx+c)}+a}{\sqrt{e\cos{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

3.659.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{1,[1]%%%}]%%},[2,1]%%%}+%%%
{%%%{8,[2
```

3.659.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = \frac{2\,a\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{e\,\cos\left(c+d\,x\right)}}+\frac{a\,\cos\left(c+d\,x\right)\,\sqrt{e\,\cos\left(c+d\,x\right)}\,4\mathrm{i}}{d\,e\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(1/2),x)
```

output
```
(2*a*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(d*(e*cos(c + d*x))^(
1/2)) + (a*cos(c + d*x)*(e*cos(c + d*x))^(1/2)*4i)/(d*e*(cos(2*c + 2*d*x)
+ 1))
```

3.660 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

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3.660.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2ia}{3d(e\cos(c+dx))^{3/2}} - \frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

output
```
2/3*I*a/d/(e*cos(d*x+c))^(3/2)-2*a*cos(d*x+c)^(3/2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos(d
*x+c))^(3/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(1/2)
```

3.660.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.02 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.13

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \frac{(\cos(dx)-i\sin(dx))\left(\frac{6\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))(}{\sqrt{\sin^2(dx+\arctan(\tan(c)))}}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

output
```
((Cos[d*x] - I*Sin[d*x])*((6*Cos[c + d*x]*HypergeometricPFQ[{-1/2, -1/4},
{3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*(1 - I*Tan[c
]))/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2] + (Csc[c] - I*Sec[c])*(-3*Cos[c + d*
x]*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]]) + (4*
(I + 3*Cos[d*x]*Cos[c + d*x]*Csc[c])*Tan[c])/Sqrt[Sec[c]^2]))*(a + I*a*Tan
[c + d*x]))/(6*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[Sec[c]^2])
```

3.660.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int (e\sec(c+dx))^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 4258$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow 3119$$

$$\frac{a\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

output
```
((((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + a*((-2*e^2*EllipticE[(c + d*x)/2
, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*
x]]*Sin[c + d*x])/d))/((e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2))
```

3.660.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.660. $\int\frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.660.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(104) = 208$.

Time = 5.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.40

method	result
default	$\frac{2(12(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-6E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}(\sin^2(\frac{dx}{2}+\frac{c}{2}))-6\cos(\frac{dx}{2}+\frac{c}{2})(\mathrm{si}}{3(2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1)\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^2(\frac{dx}{2}}}$
parts	$-\frac{2a(-2\cos(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2}+\frac{c}{2}))e+(\sin^2(\frac{dx}{2}+\frac{c}{2}))e}(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\sqrt{-2(\sin^4}}{e\sqrt{-e(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{e(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-}}$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e*(12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2
+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-I*sin(1/2*d*x+1/2*c))*a/d
```

3.660.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.70

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(3i\,ae^{(4i\,dx+4i\,c)}+i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(i\,\sqrt{2}ae^{(4i\,dx+4i\,c)}+2i\,\sqrt{2}ae^{(2i\,dx+}\right.\right.}{3\left(de^2e^{(4i\,dx+4i\,c)}+2\,de^2e^{(2i\,dx+2i\,c)}\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(2*sqrt(1/2)*(3*I*a*e^(4*I*d*x + 4*I*c) + I*a*e^(2*I*d*x + 2*I*c))*sq
rt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 3*(I*sqrt(2)*a*e^
(4*I*d*x + 4*I*c) + 2*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(
e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d
*e^2*e^(4*I*d*x + 4*I*c) + 2*d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

3.660.6 Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\cos(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

output
```
I*a*(Integral(-I/(e*cos(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*co
s(c + d*x))**(3/2), x))
```

3.660.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{ia\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.660.8 Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{ia\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{a+a\tan(c+d\,x)\,1\mathrm{i}}{{(e\,\cos(c+d\,x))}^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2), x)
```

3.661 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

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3.661.1 Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

output
```
2/5*I*a/d/(e*cos(d*x+c))^(5/2)+2/3*a*cos(d*x+c)^(5/2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos
(d*x+c))^(5/2)+2/3*a*cos(d*x+c)*sin(d*x+c)/d/(e*cos(d*x+c))^(5/2)
```

3.661.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{a\left(6i+10\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))\right)}{15d(e\cos(c+dx))^{5/2}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

output
```
(a*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d
*x)]))/(15*d*(e*Cos[c + d*x])^(5/2))
```

3.661.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3998, 3042, 3967, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int (e\sec(c+dx))^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{a\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 3120$$

$$\frac{a\left(\frac{2e^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

output
```
((((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + a*((2*e^2*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(3*d)))/((e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2
))
```

3.661.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

3.661. $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.661.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(107) = 214$.

Time = 6.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.71

method	result
parts	$-\frac{2a\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2\left(\sin^2\left(\frac{dx}{2}\right.\right.}\right.}{3e^2\sqrt{-e\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\right.}$
default	$-\frac{2\left(20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-20\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{15\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-2*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2))/e^2*(e*(2*cos(1/2*d*x
+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(e*(
2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d+2/5*I*a/d/(e*cos(d*x+c))^(5/2)
```

3.661.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.98

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(5i\,ae^{(5i\,dx+5i\,c)}-12i\,ae^{(3i\,dx+3i\,c)}-5i\,ae^{(i\,dx+i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+5\left(i\sqrt{2}ae^{(6i\,dx+6i}}{15\left(de^{3}e^{(6i\,dx+6i\,c)}+3\,de^{3}e^{(4i\,dx+4i\,c)}+}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(2*sqrt(1/2)*(5*I*a*e^(5*I*d*x + 5*I*c) - 12*I*a*e^(3*I*d*x + 3*I*c)
 - 5*I*a*e^(I*d*x + I*c))*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x -
1/2*I*c) + 5*(I*sqrt(2)*a*e^(6*I*d*x + 6*I*c) + 3*I*sqrt(2)*a*e^(4*I*d*x +
 4*I*c) + 3*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(e)*weierst
rassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^3*e^(6*I*d*x + 6*I*c) + 3*d*e^3
*e^(4*I*d*x + 4*I*c) + 3*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)
```

3.661.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.661.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx=\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.661.8 Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx=\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx=\int \frac{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2), x)
```

3.662 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

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3.662.1 Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx=\frac{2ia}{7d(e\cos(c+dx))^{7/2}}-\frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}}+\frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}}+\frac{6a\cos^{3}(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

output
```
2/7*I*a/d/(e*cos(d*x+c))^(7/2)-6/5*a*cos(d*x+c)^(7/2)*(cos(1/2*d*x+1/2*c)^
2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos
(d*x+c))^(7/2)+2/5*a*cos(d*x+c)*sin(d*x+c)/d/(e*cos(d*x+c))^(7/2)+6/5*a*co
s(d*x+c)^3*sin(d*x+c)/d/(e*cos(d*x+c))^(7/2)
```

3.662.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.52 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.12

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\cos^5(c+dx)\left(\csc(c)\sec(c)\left(\frac{6\cos(c)}{5}-\frac{6}{5}i\sin(c)\right)+\sec^4(c+dx)\left(\frac{2}{7}i\cos(c)+\frac{2\sin(}{7}\right.\right.}{}$$

$$-\frac{3i\cos^{\frac{9}{2}}(c+dx)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^2(c)}}-\right.}{5d(e\cos(c+dx))^{7/2}(\cos(dx)+i\sin(}$$

$$+\frac{3\cos^{\frac{9}{2}}(c+dx)\cot(c)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\tan(c)))\sqrt{1+\tan^2(c)}}\sqrt{1+\tan^{}}}\right.}{5d(e\cos(c+dx))^{7/2}(\cos(dx)+i\,\mathrm{s}}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

output
```
(Cos[c + d*x]^5*(Csc[c]*Sec[c]*((6*Cos[c])/5 - ((6*I)/5)*Sin[c]) + Sec[c +
 d*x]^4*(((2*I)/7)*Cos[c] + (2*Sin[c])/7) + Sec[c]*Sec[c + d*x]^3*((2*Cos[
c])/5 - ((2*I)/5)*Sin[c])*Sin[d*x] + Sec[c]*Sec[c + d*x]*((6*Cos[c])/5 - (
(6*I)/5)*Sin[c])*Sin[d*x] + Sec[c + d*x]^2*((2*Cos[c])/5 - ((2*I)/5)*Sin[c
])*Tan[c])*(a + I*a*Tan[c + d*x]))/(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I
*Sin[d*x])) - (((3*I)/5)*Cos[c + d*x]^(9/2)*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/
(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*S
qrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2
]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])*(a + I*a*Tan[c + d*x
]))/(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*x])) + (3*Cos[c + d*x]^(
9/2)*Cot[c]*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[
c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c
]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]
*Sqrt[1 + Tan[c]^2]])*(a + I*a*Tan[c + d*x]))/(5*d*(e*Cos[c + d*x])^(7/...
```

3.662.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3998, 3042, 3967, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3967}$$

$$\frac{a\int (e\sec(c+dx))^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

input
```
Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

output
```
((((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + a*((2*e*(e*Sec[c + d*x])^(5/2)*S
in[c + d*x])/(5*d) + (3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Co
s[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x]
)/d))/5))/((e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2))
```

3.662. $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

3.662.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.662.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(137) = 274$.

Time = 11.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.96

method	result
parts	$-\frac{2a\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{}\right.}{}$
default	$\frac{2\left(336\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-168E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-504\left(\sin^6\left(\frac{dx}{2}\right.\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2))*(-2*sin(1/2*d*x+1/2*c)
^4*e+sin(1/2*d*x+1/2*c)^2*e)^(1/2)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d+
2/7*I*a/d/(e*cos(d*x+c))^(7/2)
```

3.662.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx =$$
$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(21i\,ae^{(8i\,dx+8i\,c)}+77i\,ae^{(6i\,dx+6i\,c)}+23i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right.}{35\left(de^4e^{(8i\,dx\right.}}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-2/35*(2*sqrt(1/2)*(21*I*a*e^(8*I*d*x + 8*I*c) + 77*I*a*e^(6*I*d*x + 6*I*c
) + 23*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2*I*d*x + 2*I*c))*sqrt(e*e^(2*I*
d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 21*(I*sqrt(2)*a*e^(8*I*d*x +
8*I*c) + 4*I*sqrt(2)*a*e^(6*I*d*x + 6*I*c) + 6*I*sqrt(2)*a*e^(4*I*d*x + 4*
I*c) + 4*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(e)*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^4*e^(8*I*d
*x + 8*I*c) + 4*d*e^4*e^(6*I*d*x + 6*I*c) + 6*d*e^4*e^(4*I*d*x + 4*I*c) +
4*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)
```

3.662.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.662.7 Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.662.8 Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx=\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx=\int \frac{a+a\tan(c+d\,x)\;1\mathrm{i}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2), x)
```

3.663 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

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3.663.1 Optimal result

Integrand size = 28, antiderivative size = 190

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2(e\cos(c+dx))^{7/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{7a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/7*(e*cos(d*x+c))^(7/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(7/2)+2/15*cos(d*x+c
)*(e*cos(d*x+c))^(7/2)*sin(d*x+c)/a^2/d+6/35*(e*cos(d*x+c))^(7/2)*tan(d*x+
c)/a^2/d+2/7*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2*tan(d*x+c)/a^2/d+4/15*I*cos
(d*x+c)^2*(e*cos(d*x+c))^(7/2)/d/(a^2+I*a^2*tan(d*x+c))
```

3.663.2 Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.82

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{e^3\sqrt{e\cos(c+dx)}\Big(-240\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))+i\sin(2(c+}{}$$

input
```
Integrate[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e^3*Sqrt[e*Cos[c + d*x]]*(-240*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)
] + I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-296*I)*Cos[c + d*x] + (68*
I)*Cos[3*(c + d*x)] + (4*I)*Cos[5*(c + d*x)] + 134*Sin[c + d*x] - 117*Sin[
3*(c + d*x)] - 11*Sin[5*(c + d*x)])))/(840*a^2*d*Cos[c + d*x]^(5/2)*(-I +
Tan[c + d*x])^2)
```

3.663.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{1}{(e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{1}{(e\sec(c+dx))^{7/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3981}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\int \frac{1}{(e\sec(c+dx))^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}}dx}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\downarrow 4256$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\int\frac{1}{(e\sec(c+dx))^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\downarrow 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}\right)$$

$$\downarrow 4256$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\int\frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c}\right.$$

$$\downarrow 3042$$

3.663. $\int\frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

$$(e\cos(c+dx))^{7/2}(e\sec(c+$$

$$dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\,\mathrm{se}}\right.$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+$$

$$dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{4}{15d\left(a^2+ia^2\tan(c+\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+$$

$$dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{}{15d\left(a^2+ia^2\tan(\right.}\right.$$

3.663. $\int\frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

$$\Big\downarrow \ 4258$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\frac{}{15}\right.$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\right.$$

$$\Big\downarrow \ 3120$$

3.663. $\int\frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{11e^2\left(\frac{9\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11e^2}+\frac{2\sin(c+dx)}{11de(e\sec(c+dx))^{9/2}}\right)}{15a^2}+\right.$$

input
```
Int[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*((11*e^2*((2*Sin[c + d*x])/(
11*d*e*(e*Sec[c + d*x])^(9/2)) + (9*((2*Sin[c + d*x])/(7*d*e*(e*Sec[c + d*
x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Se
c[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(
7*e^2)))/(11*e^2)))/(15*a^2) + (((4*I)/15)*e^2)/(d*(e*Sec[c + d*x])^(11/2)
*(a^2 + I*a^2*Tan[c + d*x])))
```

3.663.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.663.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(192) = 384$.

Time = 9.44 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.04

method	result
default	$-\frac{2e^4\big(25088i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3584\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3584i\left(\sin^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12544\left(\sin^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+}{}$

input
```
int((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/105/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(250
88*I*sin(1/2*d*x+1/2*c)^11+3584*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16-3
584*I*sin(1/2*d*x+1/2*c)^17-12544*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)
+6272*I*sin(1/2*d*x+1/2*c)^7+19264*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c
)-14*I*sin(1/2*d*x+1/2*c)-16800*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1
568*I*sin(1/2*d*x+1/2*c)^5+9104*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+22
4*I*sin(1/2*d*x+1/2*c)^3-3128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+1433
6*I*sin(1/2*d*x+1/2*c)^15+700*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-1568
0*I*sin(1/2*d*x+1/2*c)^9-90*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+15*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)-25088*I*sin(1/2*d*x+1/2*c)^13)/d
```

3.663.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-480i\,\sqrt{2}e^{\frac{7}{2}}e^{(7i\,dx+7i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-15i\,e^3e\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/1680*(-480*I*sqrt(2)*e^(7/2)*e^(7*I*d*x + 7*I*c)*weierstrassPInverse(-4,
 0, e^(I*d*x + I*c)) + sqrt(1/2)*(-15*I*e^3*e^(10*I*d*x + 10*I*c) - 185*I*
e^3*e^(8*I*d*x + 8*I*c) + 430*I*e^3*e^(6*I*d*x + 6*I*c) + 162*I*e^3*e^(4*I
*d*x + 4*I*c) + 49*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*x
+ 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

3.663.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.663.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.663.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos(c+d\,x))^{7/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.664 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.664.1 Optimal result 4735
3.664.2 Mathematica [C] (warning: unable to verify) 4735
3.664.3 Rubi [A] (verified) 4736
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3.664.9 Mupad [F(-1)] 4742

3.664.1 Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{42(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\,(a^2+ia^2\tan(c+dx))}$$

output
```
42/65*(e*cos(d*x+c))^(5/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(5/2)+2/13*cos(d*x
+c)*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+14/65*(e*cos(d*x+c))^(5/2)*tan(d
*x+c)/a^2/d+4/13*I*cos(d*x+c)^2*(e*cos(d*x+c))^(5/2)/d/(a^2+I*a^2*tan(d*x+
c))
```

3.664.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.00 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.01

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\cos(c+dx))^{5/2}\sec^5(c+dx)(\cos(dx)+i\sin(dx))^2\left(-21\cos(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4}\right.\right.}{}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(Cos[d*x] + I*Sin[d*x])^2*(-21*Cos[
c]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]] - (42*I)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] + (21*I)*(3*Cos
[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x +
 ArcTan[Tan[c]]]^2] + (21*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x +
 ArcTan[Tan[c]]])*Cot[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/2 - (Cos[c + d
*x]*Csc[c]*Sqrt[Sec[c]^2]*(Cos[2*d*x] - I*Sin[2*d*x])*(178*Cos[c + 2*d*x]
+ 158*Cos[3*c + 2*d*x] - 9*Cos[3*c + 4*d*x] + 9*Cos[5*c + 4*d*x] - (88*I)*
Sin[c] + (208*I)*Sin[c + 2*d*x] + (128*I)*Sin[3*c + 2*d*x] - (4*I)*Sin[3*c
 + 4*d*x] + (4*I)*Sin[5*c + 4*d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/8 +
 21*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Si
n[c]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c] - (21*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*Tan[
c])/2))/(65*d*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*(a + I*a*Ta
n[c + d*x])^2)
```

3.664.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \ 3981$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\int \frac{1}{(e\sec(c+dx))^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 4256$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{9e^2\left(\frac{7\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \ 4256$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\Bigg(\frac{9e^2\left(\frac{7\left(\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+da}$$

$$\Big\downarrow \ 3042$$

3.664. $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$

$$dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{3\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+}\right.$$

$$\Big\downarrow \text{4258}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$

$$dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{3\int\sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(}\right.$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$

$$dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(}\right.$$

$$\Big\downarrow \text{3119}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$

$$dx))^{5/2}\left(\frac{9e^2\left(\frac{7\left(\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9e^2}+\frac{2\sin(c+dx)}{9de(e\sec(c+dx))^{7/2}}\right)}{13a^2}+\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec}\right.$$

3.664. $\int\frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

input
```
Int[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*((9*e^2*((2*Sin[c + d*x])/(9
*d*e*(e*Sec[c + d*x])^(7/2)) + (7*((6*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*
Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x])/(5*d*e*(e*Sec[
c + d*x])^(3/2))))/(9*e^2)))/(13*a^2) + (((4*I)/13)*e^2)/(d*(e*Sec[c + d*x
])^(9/2)*(a^2 + I*a^2*Tan[c + d*x])))
```

3.664.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

3.664. $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.664.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(160) = 320$.

Time = 8.42 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.28

method	result
default	$\frac{2e^{3}\Big(840i\big(\sin^{5}\big(\frac{dx}{2}+\frac{c}{2}\big)\big)+1280\big(\sin^{14}\big(\frac{dx}{2}+\frac{c}{2}\big)\big)\cos\big(\frac{dx}{2}+\frac{c}{2}\big)-140i\big(\sin^{3}\big(\frac{dx}{2}+\frac{c}{2}\big)\big)-3840\big(\sin^{12}\big(\frac{dx}{2}+\frac{c}{2}\big)\big)\cos\big(\frac{dx}{2}+\frac{c}{2}\big)-6720i\big(\sin}{}$

input
```
int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/65/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(840*I
*sin(1/2*d*x+1/2*c)^5+1280*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-140*I*
sin(1/2*d*x+1/2*c)^3-3840*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-6720*I*
sin(1/2*d*x+1/2*c)^11+4960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1280*I
*sin(1/2*d*x+1/2*c)^15-3520*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2800*I
*sin(1/2*d*x+1/2*c)^7+1496*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*I*si
n(1/2*d*x+1/2*c)-376*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5600*I*sin(1/
2*d*x+1/2*c)^9+44*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+21*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)+4480*I*sin(1/2*d*x+1/2*c)^13)/d
```

3.664.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\Big(336i\sqrt{2}e^{\frac{5}{2}}e^{(6i\,dx+6i\,c)}\mathrm{weierstrassZeta}\big(-4,0,\mathrm{weierstrassPInverse}\big(-4,0,e^{(i\,dx}}{}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/520*(336*I*sqrt(2)*e^(5/2)*e^(6*I*d*x + 6*I*c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(1/2)*(-13*I*e^2*e^(8*I*d
*x + 8*I*c) + 386*I*e^2*e^(6*I*d*x + 6*I*c) + 88*I*e^2*e^(4*I*d*x + 4*I*c)
 + 30*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)
*e^(-1/2*I*d*x - 1/2*I*c))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

3.664.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.664.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.664.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos\left(c+d\,x\right))^{5/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.665 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

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3.665.1 Optimal result

Integrand size = 28, antiderivative size = 154

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx &= \frac{10(e\cos(c+dx))^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{33a^2d\cos^{\frac{3}{2}}(c+dx)} \\ &+ \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} \\ &+ \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

output

```
10/33*(e*cos(d*x+c))^(3/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(3/2)+2/11*cos(d*x
+c)*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+10/33*(e*cos(d*x+c))^(3/2)*tan(d
*x+c)/a^2/d+4/11*I*cos(d*x+c)^2*(e*cos(d*x+c))^(3/2)/d/(a^2+I*a^2*tan(d*x+
c))
```

3.665.2 Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\cos(c+dx))^{3/2}\Big(-20\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))+i\sin(2(c+}{66a^2d\cos}$$

input
```
Integrate[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((e*Cos[c + d*x])^(3/2)*(-20*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] +
 I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-28*I)*Cos[c + d*x] + (4*I)*Co
s[3*(c + d*x)] + 13*Sin[c + d*x] - 7*Sin[3*(c + d*x)])))/(66*a^2*d*Cos[c +
 d*x]^(7/2)*(-I + Tan[c + d*x])^2)
```

3.665.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3981}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\int \frac{1}{(e\sec(c+dx))^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\int\frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\int\sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/}}\right.$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\int\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx}\right.$$

$$\Big\downarrow \text{4258}$$

3.665. $\int\frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right.}\right.$$

$$\Big\downarrow \; 3042$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4i}{11d\left(a^2+ia^2\tan(c+\right.}\right.$$

$$\Big\downarrow \; 3120$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{7e^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7e^2}+\frac{2\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}\right)}{11a^2}+\frac{4}{11d\left(a^2+ia^2\tan(c+\right.}\right.$$

input
```
Int[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*((7*e^2*((2*Sin[c + d*x])/(7
*d*e*(e*Sec[c + d*x])^(5/2)) + (5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*
Sec[c + d*x]])))/(7*e^2)))/(11*a^2) + (((4*I)/11)*e^2)/(d*(e*Sec[c + d*x])
^(7/2)*(a^2 + I*a^2*Tan[c + d*x])))
```

3.665. $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.665.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.665. $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

3.665.4 Maple [A] (verified)

Time = 7.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.05

method	result
default	$-\frac{2e^2\left(-384i\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+384\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1152i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-960\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-1440}{}$

input
```
int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/33/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-384
*I*sin(1/2*d*x+1/2*c)^13+384*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+1152
*I*sin(1/2*d*x+1/2*c)^11-960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1440
*I*sin(1/2*d*x+1/2*c)^9+1008*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+960*I
*sin(1/2*d*x+1/2*c)^7-552*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-360*I*si
n(1/2*d*x+1/2*c)^5+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+72*I*sin(1/
2*d*x+1/2*c)^3-28*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
c)^2)^(1/2)-6*I*sin(1/2*d*x+1/2*c))/d
```

3.665.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.75

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx=\frac{\left(-40i\sqrt{2}e^{\frac{3}{2}}e^{(5i\,dx+5i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-11i\,ee^{(6i}\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/132*(-40*I*sqrt(2)*e^(3/2)*e^(5*I*d*x + 5*I*c)*weierstrassPInverse(-4, 0
, e^(I*d*x + I*c)) + sqrt(1/2)*(-11*I*e*e^(6*I*d*x + 6*I*c) + 41*I*e*e^(4*
I*d*x + 4*I*c) + 15*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x + 2
*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

3.665.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.665.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.665.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.665.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx=\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.666 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

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3.666.1 Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}} + \frac{2i\sqrt{e\cos(c+dx)}}{9d(a+ia\tan(c+dx))^2} + \frac{2i\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(1/2)+2/9*I*(e*cos(d
*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^2+2/9*I*(e*cos(d*x+c))^(1/2)/d/(a^2+I*a^
2*tan(d*x+c))
```

3.666.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.25 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{e\cos(c+dx)}\sec^3(c+dx)(\cos(dx)+i\sin(dx))^2\left(6\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c)(\mathrm{co}\right.}{}$$

input
```
Integrate[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^2*(6*Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*(Cos[2*
c] + I*Sin[2*c])*Sin[d*x + ArcTan[Tan[c]]] + Sqrt[Sin[d*x + ArcTan[Tan[c]]
]^2]*(Cos[c + d*x]*Csc[c]*Sqrt[Sec[c]^2]*(Cos[2*d*x] - I*Sin[2*d*x])*(7*Co
s[c + 2*d*x] + 5*Cos[3*c + 2*d*x] - (4*I)*(Sin[c] - 2*Sin[c + 2*d*x] - Sin
[3*c + 2*d*x])) - 3*Cos[c + d*x + ArcTan[Tan[c]]]*(I + Cot[c])^2*Tan[c] +
9*Cos[c - d*x - ArcTan[Tan[c]]]*(-2*I - Cot[c] + Tan[c]))))/(18*a^2*d*Sqrt
[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*(-I + Tan[c + d*x])^2)
```

3.666.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3981}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\int \frac{1}{(e\sec(c+dx))^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{5e^2\int\frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \; 4256$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\Bigg(\frac{5e^2\left(\frac{3\int\frac{1}{\sqrt{e\sec(c+dx)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\Bigg(\frac{5e^2\left(\frac{3\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5e^2}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c}$$

$$\Big\downarrow \; 4258$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\Bigg(\frac{5e^2\left(\frac{3\int\sqrt{\cos(c+dx)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\,s}$$

$$\Big\downarrow \; 3042$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\Bigg(\frac{5e^2\left(\frac{3\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e}$$

$$\Big\downarrow \; 3119$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\Bigg(\frac{5e^2\left(\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}\right)}{9a^2}+\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e}$$

input

```
Int[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

3.666. $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

output
```
Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]*((5*e^2*((6*EllipticE[(c + d*x)/
2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*Sin[c + d*x]
)/(5*d*e*(e*Sec[c + d*x])^(3/2))))/(9*a^2) + (((4*I)/9)*e^2)/(d*(e*Sec[c +
 d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x])))
```

3.666.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.666. $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

3.666.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(128) = 256.

Time = 6.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.31

method	result
default	$\frac{2e\left(-64i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+160i\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-160i\left(\sin^{7}\left(\frac{dx}{2}\right.\right.\right.}{}$

input
```
int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/9/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-64*I*si
n(1/2*d*x+1/2*c)^11+64*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+160*I*sin(
1/2*d*x+1/2*c)^9-128*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-160*I*sin(1/2
*d*x+1/2*c)^7+104*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+80*I*sin(1/2*d*x
+1/2*c)^5-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20*I*sin(1/2*d*x+1/2*
c)^3+6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
+2*I*sin(1/2*d*x+1/2*c))/d
```

3.666.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(15i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+12i\,\sqrt{2}\sqrt{e}e^{(4i\,dx+4i\,c)}\text{weierstrassZe}\right.}{18\,a^2d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/18*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(15*I*e^(4*I*d*x + 4*I*c)
+ 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c) + 12*I*sqrt(2)*sqr
t(e)*e^(4*I*d*x + 4*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 e^(I*d*x + I*c))))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

3.666.6 Sympy [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sqrt{e\cos(c+dx)}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(sqrt(e*cos(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x
)/a**2
```

3.666.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.666.8 Giac [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{e\cos(dx+c)}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*cos(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

3.666.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx=\int \frac{\sqrt{e\,\cos\left(c+d\,x\right)}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.667 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

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3.667.1 Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{2i}{7d\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2i}{7d\sqrt{e\cos(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)+2/7*I/d/(e*cos
(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2/7*I/d/(e*cos(d*x+c))^(1/2)/(a^2+I*a^
2*tan(d*x+c))
```

3.667.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(-i\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\Bigl(\sqrt{\cos(c+dx)}\bigl(3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+4i\sin^3\bigl(\frac{1}{2}(c}{7a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{e\cos(c+dx)}(-i+t}$$

input
```
Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sqrt[Cos[c + d*x]]*(3*Cos[(c
+ d*x)/2] + Cos[(3*(c + d*x))/2] + (4*I)*Sin[(c + d*x)/2]^3) + 2*EllipticF
[(c + d*x)/2, 2]*((-I)*Cos[(3*(c + d*x))/2] + Sin[(3*(c + d*x))/2])))/(7*a
^2*d*Cos[c + d*x]^(3/2)*Sqrt[e*Cos[c + d*x]]*(-I + Tan[c + d*x])^2)
```

3.667.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3998, 3042, 3981, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{(i\tan(c+dx)a+a)^2}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{\frac{3e^2\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\int \frac{1}{\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{3e^2\left(\frac{\int \sqrt{e\sec(c+dx)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\left(\frac{\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\left(\frac{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3e^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{3e^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}+\frac{2\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\right)}{7a^2}+\frac{4ie^2}{7d(a^2+ia^2\tan(c+dx))(e\sec(c+dx))^{3/2}}}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((3*e^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*
x]])/(3*d*e^2) + (2*Sin[c + d*x])/(3*d*e*Sqrt[e*Sec[c + d*x]])))/(7*a^2) +
 (((4*I)/7)*e^2)/(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x])))/(S
qrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

3.667. $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

3.667.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.667. $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

3.667.4 Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

method	result
default	$-\frac{2(-32i(\sin^9(\frac{dx}{2}+\frac{c}{2}))+32\cos(\frac{dx}{2}+\frac{c}{2})(\sin^8(\frac{dx}{2}+\frac{c}{2}))+64i(\sin^7(\frac{dx}{2}+\frac{c}{2}))-48(\sin^6(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-48i(\sin^5(\frac{dx}{2}+\frac{c}{2}}{7a}$

input
```
int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/7/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-32*I*sin
(1/2*d*x+1/2*c)^9+32*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+64*I*sin(1/2*
d*x+1/2*c)^7-48*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-48*I*sin(1/2*d*x+1
/2*c)^5+28*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+16*I*sin(1/2*d*x+1/2*c)
^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-2*I
*sin(1/2*d*x+1/2*c))/d
```

3.667.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}(3i\,e^{(2i\,dx+2i\,c)}+i)e^{(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c)}-2i\sqrt{2}\sqrt{e}e^{(3i\,dx+3i\,c)}\mathrm{weierstrassPInverse}(-4,0,e^{(i\,d}}{7\,a^2de}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
1/7*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(3*I*e^(2*I*d*x + 2*I*c) +
I)*e^(-1/2*I*d*x - 1/2*I*c) - 2*I*sqrt(2)*sqrt(e)*e^(3*I*d*x + 3*I*c)*weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-3*I*d*x - 3*I*c)/(a^2*d*e)
```

3.667.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{e\cos{(c+dx)}}\tan^2{(c+dx)}-2i\sqrt{e\cos{(c+dx)}}\tan{(c+dx)}-\sqrt{e\cos{(c+dx)}}}\,dx}{a^2}$$

input
```
integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral(1/(sqrt(e*cos(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*cos(c + d*x
))*tan(c + d*x) - sqrt(e*cos(c + d*x))), x)/a**2
```

3.667.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.667.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cos{(dx+c)}}(i\,a\tan{(dx+c)}+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

3.667.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx=\int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.668 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

3.668.1 Optimal result 4765
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3.668.1 Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^2d(e\cos(c+dx))^{3/2}} + \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
2/5*cos(d*x+c)^(3/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(3/2)+4/5*I*cos(d*x+
c)^2/d/(e*cos(d*x+c))^(3/2)/(a^2+I*a^2*tan(d*x+c))
```

3.668.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.55 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\cos^2(c+dx)\left(1+2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{1}{2}(1\right.\right.}{5a^2d(e\cos(c+}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(2*Cos[c + d*x]^2*(1 + 2^(3/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (1 + I*Ta
n[c + d*x])/2]*(1 - I*Tan[c + d*x])^(1/4)*(1 + I*Tan[c + d*x]) - I*Tan[c +
 d*x]))/(5*a^2*d*(e*Cos[c + d*x])^(3/2)*(-I + Tan[c + d*x]))
```

3.668.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3981, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{3/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{\frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx}{5a^2}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{e^2\int \sqrt{\cos(c+dx)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{e^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2e^2E\left(\frac{1}{2}(c+dx)|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c +
d*x])))/((e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2))
```

3.668.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.668.4 Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.24

method	result
default	$\frac{-\frac{32i\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{32\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+\frac{48i\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{32\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{24i\left(\sin^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\cos\left(\frac{dx}{2}+\right.}{}}{e\,a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}}$

input
```
int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/5/e/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-16*I*si
n(1/2*d*x+1/2*c)^7+16*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*I*sin(1/2
*d*x+1/2*c)^5-16*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*x+
1/2*c)^3+4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)+2*I*sin(1/2*d*x+1/2*c))/d
```

3.668.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-i\,\sqrt{2}\sqrt{e}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weiers}\right.\right.}{5\,a^{2}de^{2}}$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-2/5*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-2*I*e^(2*I*d*x + 2*I*c)
- I)*e^(-1/2*I*d*x - 1/2*I*c) - I*sqrt(2)*sqrt(e)*e^(2*I*d*x + 2*I*c)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-2*I*d
*x - 2*I*c)/(a^2*d*e^2)
```

3.668.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.668.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.668.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{2}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx=\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.669 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

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3.669.1 Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{2\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{5/2}}+\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))}$$

output
```
-2/3*cos(d*x+c)^(5/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(5/2)+4/3*I*cos(d*x
+c)^2/d/(e*cos(d*x+c))^(5/2)/(a^2+I*a^2*tan(d*x+c))
```

3.669.2 Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}(\cos(dx)+i\sin(dx))^2\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),\right.\right.}{3a^2d(e\cos(c+}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*(EllipticF[(c + d*x)/2, 2]
*(Cos[2*c] + I*Sin[2*c]) + 2*Sqrt[Cos[c + d*x]]*((-I)*Cos[c - d*x] + Sin[c
 - d*x])))/(3*a^2*d*(e*Cos[c + d*x])^(5/2)*(-I + Tan[c + d*x])^2)
```

3.669.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3981, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3981}$$

$$\frac{-\frac{e^2\int \sqrt{e\sec(c+dx)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3a^2}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{-\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]]
)/(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c +
 d*x])))/((e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2))
```

3.669.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.669.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\left(-8i\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8i\left(\sin^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}\right.}{3e^{2}a^{2}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$

input
```
int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/e^2/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-8*I*
sin(1/2*d*x+1/2*c)^5+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+8*I*sin(1/2
*d*x+1/2*c)^3-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)-2*I*sin(1/2*d*x+1/2*c))/d
```

3.669.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^{2}}\,dx =$$
$$-\frac{2\left(-i\sqrt{2}\sqrt{e}e^{(i\,dx+i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)-2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{(-i\,dx-i\,c}}{3\,a^{2}de^{3}}$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-2/3*(-I*sqrt(2)*sqrt(e)*e^(I*d*x + I*c)*weierstrassPInverse(-4, 0, e^(I*d
*x + I*c)) - 2*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x -
 1/2*I*c))*e^(-I*d*x - I*c)/(a^2*d*e^3)
```

3.669.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.669.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.669.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx=\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.670 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

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3.670.1 Optimal result

Integrand size = 28, antiderivative size = 122

$$\begin{aligned}\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx &= \frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d(e\cos(c+dx))^{7/2}} \\ &\quad - \frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

output

```
6*cos(d*x+c)^(7/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(7/2)-6*cos(d*x+c)^3*s
in(d*x+c)/a^2/d/(e*cos(d*x+c))^(7/2)+4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/
2)/(a^2+I*a^2*tan(d*x+c))
```

3.670.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.39 (sec) , antiderivative size = 1106, normalized size of antiderivative = 9.07

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}(\cos(dx)+i\sin(dx))^2\Big(-2i\cos(c-dx)\sqrt{\cos(}{d(e\cos(c+dx))^{7/2}(a+ia\tan(}$$

$$+\frac{3\cos(c)\cos^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))^2\Bigg(-\frac{\cos(dx-\arctan(\cot(c)))\cot(c)\,{}_2F_1(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\sin^2(dx-\mathrm{arc}}{\sqrt{1+\cot^2(c)}\sqrt{1-\sin(dx-\arctan(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\arctan(}}}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

$$+\frac{3i\cos^{\frac{3}{2}}(c+dx)\sin(c)(\cos(dx)+i\sin(dx))^2\Bigg(-\frac{\cos(dx-\arctan(\cot(c)))\cot(c)\,{}_2F_1(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\sin^2(dx-\mathrm{ar}}{\sqrt{1+\cot^2(c)}\sqrt{1-\sin(dx-\arctan(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\arctan(}}}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

$$-\frac{3i\cos(c)\cos^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))^2\Bigg(\frac{{}_2F_1(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c))))\sin(dx+\arctan(\mathrm{ta}}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{arctan}}}}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

$$+\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c)(\cos(dx)+i\sin(dx))^2\Bigg(\frac{{}_2F_1(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c))))\sin(dx+\arctan(\tan}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(}}}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*((-2*I)*Cos[c - d*x]*Sqrt[Co
s[c + d*x]] + 2*Sqrt[Cos[c + d*x]]*Sin[c - d*x]))/(d*(e*Cos[c + d*x])^(7/2
)*(a + I*a*Tan[c + d*x])^2) + (3*Cos[c]*Cos[c + d*x]^(3/2)*(Cos[d*x] + I*S
in[d*x])^2*(-((Cos[d*x - ArcTan[Cot[c]]]*Cot[c]*HypergeometricPFQ[{-1/2, -
1/4}, {3/4}, Sin[d*x - ArcTan[Cot[c]]]^2])/(Sqrt[1 + Cot[c]^2]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])) + ((Cos[d*x - ArcTan[Cot
[c]]]*Cot[c])/Sqrt[1 + Cot[c]^2] + (2*Sqrt[1 + Cot[c]^2]*Sin[c]^2*Sin[d*x
- ArcTan[Cot[c]]])/(Cos[c]^2 + Sin[c]^2))/Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]
*Sin[d*x - ArcTan[Cot[c]]])]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c +
d*x])^2) + ((3*I)*Cos[c + d*x]^(3/2)*Sin[c]*(Cos[d*x] + I*Sin[d*x])^2*(-((
Cos[d*x - ArcTan[Cot[c]]]*Cot[c]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Si
n[d*x - ArcTan[Cot[c]]]^2])/(Sqrt[1 + Cot[c]^2]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])) + ((Cos[d*x - ArcTan[Cot[c]]]*Cot[c])/S
qrt[1 + Cot[c]^2] + (2*Sqrt[1 + Cot[c]^2]*Sin[c]^2*Sin[d*x - ArcTan[Cot[c]
]])/(Cos[c]^2 + Sin[c]^2))/Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2) - ((3*
I)*Cos[c]*Cos[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*((HypergeometricPFQ
[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan...
```

3.670.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3998, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{-\frac{3e^2\int(e\sec(c+dx))^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{3e^2\int\left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int\frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{-\frac{3e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\left.\frac{1}{2}(c+dx)\right|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{a^2}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

input

```
Int[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

3.670. $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

output
```
((-3*e^2*((-2*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*
Sec[c + d*x]]) + (2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/a^2 + ((4*I)*
e^2*(e*Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x])))/((e*Cos[c + d*
x])^(7/2)*(e*Sec[c + d*x])^(7/2))
```

3.670.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.670.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11

method	result
default	$-\frac{2(4i(\sin^3(\frac{dx}{2}+\frac{c}{2}))+2\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-3E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}-2i\sin(\frac{dx}{2}+}{e^3a^2\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))e+e}\,d}$

input
```
int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/e^3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*I*sin
(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)-2*I*sin(1/2*d*x+1/2*c))/d
```

3.670.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int\frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=$$

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-3i\,e^{(2i\,dx+2i\,c)}-2i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(-i\,\sqrt{2}e^{(2i\,dx+2i\,c)}-i\,\sqrt{2}\right)\sqrt{e}\mathrm{weierstrassZe}}{a^2de^4e^{(2i\,dx+2i\,c)}+a^2de^4}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-2*(2*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-3*I*e^(2*I*d*x + 2*I*c)
- 2*I)*e^(-1/2*I*d*x - 1/2*I*c) + 3*(-I*sqrt(2)*e^(2*I*d*x + 2*I*c) - I*sq
rt(2))*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x
+ I*c))))/(a^2*d*e^4*e^(2*I*d*x + 2*I*c) + a^2*d*e^4)
```

3.670.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.670.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.670.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.670.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx=\int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}\,{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.671 $\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx$

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3.671.1 Optimal result

Integrand size = 28, antiderivative size = 126

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{10\cos^{\frac{9}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{9/2}} + \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
10/3*cos(d*x+c)^(9/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(9/2)+10/3*cos(d*x+
c)^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(9/2)-4*I*cos(d*x+c)^2/d/(e*cos(d*x+c
))^(9/2)/(a^2+I*a^2*tan(d*x+c))
```

3.671.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{3a^2de^3(e\cos(c+dx))^{3/2}}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(2*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] -
 Sin[c + d*x]))/(3*a^2*d*e^3*(e*Cos[c + d*x])^(3/2))
```

3.671.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3998, 3042, 3981, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{9/2}}dx$$

$$\Big\downarrow \text{ 3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{9/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{\frac{5e^2\int (e\sec(c+dx))^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\int \sqrt{e\sec(c+dx)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\int \sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5e^2\left(\frac{1}{3}e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{5e^2\left(\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2e\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}\right)}{a^2}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((5*e^2*((2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c
+ d*x]])/(3*d) + (2*e*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)))/a^2 - (
(4*I)*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x])))/((e*Cos[
c + d*x])^(9/2)*(e*Sec[c + d*x])^(9/2))
```

3.671.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.671.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.65

method	result
default	$\frac{-\frac{20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-8i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{10\sqrt{2(\mathrm{si}}}{}}{\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,e^4 d}$

input
```
int(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^2*e+e)^(1/2)/e^4*(-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12
*I*sin(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*
d*x+1/2*c)^2)^(1/2)+6*I*sin(1/2*d*x+1/2*c))/d
```

3.671.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{2\,\big(2\,\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\big(5i\,e^{(3i\,dx+3i\,c)}+7i\,e^{(i\,dx+i\,c)}\big)e^{(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c)}+5\,\big(i\,\sqrt{2}e^{(4i\,dx+4i\,c)}+2i\,\sqrt{2}e^{(2i\,dx+2i\,c)}+}{3\,(a^2de^5e^{(4i\,dx+4i\,c)}+2\,a^2de^5e^{(2i\,dx+2i\,c)}+a^2de^5)}$$

input
```
integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

output
```
-2/3*(2*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(5*I*e^(3*I*d*x + 3*I*c)
 + 7*I*e^(I*d*x + I*c))*e^(-1/2*I*d*x - 1/2*I*c) + 5*(I*sqrt(2)*e^(4*I*d*x
 + 4*I*c) + 2*I*sqrt(2)*e^(2*I*d*x + 2*I*c) + I*sqrt(2))*sqrt(e)*weierstra
ssPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^5*e^(4*I*d*x + 4*I*c) + 2*a^2
*d*e^5*e^(2*I*d*x + 2*I*c) + a^2*d*e^5)
```

3.671.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.671.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.671.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(1/((e*cos(d*x + c))^(9/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{9/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.672 $\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx$

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3.672.1 Optimal result

Integrand size = 28, antiderivative size = 164

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}}+\frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}-\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

output
```
-14/5*cos(d*x+c)^(11/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(11/2)+14/15*cos(
d*x+c)^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(11/2)+14/5*cos(d*x+c)^5*sin(d*x+
c)/a^2/d/(e*cos(d*x+c))^(11/2)-4/3*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(11/2)/
(a^2+I*a^2*tan(d*x+c))
```

3.672.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.21 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.52

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\cos^3(c+dx)(\cos(dx)+i\sin(dx))^2\left(7\cos(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4}\right.\right.}{}$$

input

```
Integrate[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output

```
(Cos[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^2*(7*Cos[c]*HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] -
(7*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Cot[c
]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/2 + (Csc[c]*Sqrt[Sec[c]^2]*Sec[c + d*
x]^2*(Cos[2*c] + I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c
 + 3*d*x] + (20*I)*Sin[d*x] - (20*I)*Sin[2*c + d*x])*Sqrt[Sin[d*x + ArcTan
[Tan[c]]]^2])/6 + (7*I)*(2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
+ ArcTan[Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] - (3*Cos[c - d*x - A
rcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[
c]]]^2]) + (7*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[
Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan
[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])*
Tan[c])/2))/(5*d*(e*Cos[c + d*x])^(11/2)*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Arc
Tan[Tan[c]]]^2]*(a + I*a*Tan[c + d*x])^2)
```

3.672.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3998, 3042, 3981, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+ia\tan(c+dx))^2(e\cos(c+dx))^{11/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(e\sec(c+dx))^{11/2}}{(i\tan(c+dx)a+a)^2}dx}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3981}$$

$$\frac{\frac{7e^2\int (e\sec(c+dx))^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\int (e\sec(c+dx))^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\int \left(e\csc\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-e^2\int \frac{1}{\sqrt{e\csc\left(c+dx+\frac{\pi}{2}\right)}}dx\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{e^2\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$\Big\downarrow \text{ 3119}$$

3.672. $\displaystyle\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx$

$$\frac{\frac{7e^2\left(\frac{3}{5}e^2\left(\frac{2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\right)+\frac{2e\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}\right)}{3a^2}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d(a^2+ia^2\tan(c+dx))}}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((7*e^2*((2*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*d) + (3*e^2*((-2*e^2
*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) +
(2*e*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/d))/5))/(3*a^2) - (((4*I)/3)*e^2*(
e*Sec[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x])))/((e*Cos[c + d*x])^(
11/2)*(e*Sec[c + d*x])^(11/2))
```

3.672.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3981
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Simp[d^2*((m - 2)/(b^2*(m + 2*n)))
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[
{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
 && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m +
1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.672.4 Maple [A] (verified)

Time = 7.55 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.96

method	result
default	$\frac{\frac{112\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{56E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{112\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}}{\left(4\left(\sin^{4}\left(\right.\right.\right.}$

input
```
int(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/15/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^5*(168*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)-84*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-168*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+84*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^2+20*I*sin(1/2*d*x+1/2*c)^3+36*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2
*c)^2-21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-10*I*sin(1/2*d*x+1/2*c))/d
```

3.672.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.21

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,(2\,\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}(21i\,e^{(6i\,dx+6i\,c)}+56i\,e^{(4i\,dx+4i\,c)}+47i\,e^{(2i\,dx+2i\,c)})e^{(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c)}+21\,(i\,\sqrt{2}e^{(6i\,dx+6i}}{15\,(a^2de^6e^{(6i\,dx+6i\,c)}+3\,a^2de^6e^{(}}$$

input
```
integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="frica
s")
```

output
```
-2/15*(2*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(21*I*e^(6*I*d*x + 6*I*
c) + 56*I*e^(4*I*d*x + 4*I*c) + 47*I*e^(2*I*d*x + 2*I*c))*e^(-1/2*I*d*x -
1/2*I*c) + 21*(I*sqrt(2)*e^(6*I*d*x + 6*I*c) + 3*I*sqrt(2)*e^(4*I*d*x + 4*
I*c) + 3*I*sqrt(2)*e^(2*I*d*x + 2*I*c) + I*sqrt(2))*sqrt(e)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^2*d*e^6*e^(6*I*d
*x + 6*I*c) + 3*a^2*d*e^6*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^6*e^(2*I*d*x + 2
*I*c) + a^2*d*e^6)
```

3.672.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.672.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.672.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac"
)
```

output
```
integrate(1/((e*cos(d*x + c))^(11/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.672.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{11/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.673 $\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx$

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3.673.2 Mathematica [A] (verified) 4798
3.673.3 Rubi [A] (verified) 4799
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3.673.8 Giac [F] 4804
3.673.9 Mupad [B] (verification not implemented) 4804

3.673.1 Optimal result

Integrand size = 30, antiderivative size = 179

$$\begin{aligned}\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{32ia(e\cos(c+dx))^{7/2}\sec^4(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} \\ &- \frac{16i(e\cos(c+dx))^{7/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}\end{aligned}$$

output
```
12/35*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)+32/
35*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^4/d/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*
(e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/35*I*(e*cos(d*x+c))^(7/
2)*sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.673.2 Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int (e\cos(c +dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e^3\sqrt{e\cos(c+dx)}(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+}{70d}$$

input
```
Integrate[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e^3*Sqrt[e*Cos[c + d*x]]*((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*S
in[c + d*x] + 6*Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d)
```

3.673.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3998, 3042, 3978, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\int\frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\int\frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\left(\frac{2a\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right.$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\left(\frac{2a\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right.$$

$$\Big\downarrow \text{3969}$$

3.673. $\int(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx$

$$(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\left(\frac{6a\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)}{7e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*((((-2*I)/7)*Sqrt[a + I*a*Ta
n[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) + (6*a*(((2*I)/5)/(d*(e*Sec[c + d*
x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)/3)*a*Sqrt[e*Sec[c + d*
x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c +
d*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a)))/(7*e^2))
```

3.673.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.673.4 Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{2 i \sqrt{e \cos \left(d x +c \right)}\, \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}\, e^{3} \left(-6 i \left(\cos^{2}\left(d x +c \right)\right) \sin \left(d x +c \right)+\cos^{3}\left(d x +c \right)-16 i \sin \left(d x +c \right)+8 \cos \left(d x +c \right)\right)}{35 d}$	78

input

```
int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/35*I/d*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*e^3*(-6*I*cos(d*x
+c)^2*sin(d*x+c)+cos(d*x+c)^3-16*I*sin(d*x+c)+8*cos(d*x+c))
```

3.673.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int (e\cos (c+dx))^{7/2}\sqrt {a+ia\tan (c+dx)}\, dx=\frac {\sqrt {2}\sqrt {\frac {1}{2}}\left(-5i\, e^{3}e^{\left(6i\, dx+6i\, c\right)}-35i\, e^{3}e^{\left(4i\, dx+4i\, c\right)}+105i\, e^{3}e^{\left(2i\, dx+2i\, c\right)}+7i\, e^{3}\right)\sqrt{}}{140\, d}$$

input

```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/140*sqrt(2)*sqrt(1/2)*(-5*I*e^3*e^(6*I*d*x + 6*I*c) - 35*I*e^3*e^(4*I*d*
x + 4*I*c) + 105*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*x +
2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/d
```

3.673.6 Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.673.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(7i\,e^3\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,e^3\cos\left(\frac{7}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\right.\right.\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/140*(7*I*e^3*cos(5/2*d*x + 5/2*c) - 5*I*e^3*cos(7/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*e^3*cos(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 105*I*e^3*cos(1/5*arctan2(sin(5/2*d*x + 5
/2*c), cos(5/2*d*x + 5/2*c))) + 7*e^3*sin(5/2*d*x + 5/2*c) + 5*e^3*sin(7/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*e^3*sin(3/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 105*e^3*sin(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)*sqrt(e)/d
```

3.673.8 Giac [F]

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx=\int (e\cos(dx+c))^{\frac{7}{2}}\sqrt{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*cos(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

3.673.9 Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.54

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{e^3\sqrt{e\,\cos(c+d\,x)}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\sin(c+d\,x)+\frac{3\,\sin(3}{3}\right.}{d}$$

input
```
int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e^3*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*((cos(c + d*x)*1i)/2 + sin(c + d*x) + (c
os(3*c + 3*d*x)*1i)/70 + (3*sin(3*c + 3*d*x))/35))/d
```

3.674 $\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx$

3.674.1 Optimal result 4805
3.674.2 Mathematica [A] (verified) 4805
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3.674.7 Maxima [A] (verification not implemented) 4809
3.674.8 Giac [F] 4810
3.674.9 Mupad [B] (verification not implemented) 4810

3.674.1 Optimal result

Integrand size = 30, antiderivative size = 132

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}$$

output
```
8/15*I*a*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-2/5*
I*(e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/15*I*(e*cos(d*x+c))^(
5/2)*sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.674.2 Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.48

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{ie^2\sqrt{e\cos(c+dx)}(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\,\mathrm{ta}}}{15d}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I/15)*e^2*Sqrt[e*Cos[c + d*x]]*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c
+ d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.674.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3998, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$
$$dx))^{5/2}\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$
$$dx))^{5/2}\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3969$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+$$
$$dx))^{5/2}\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*((((-2*I)/5)*Sqrt[a + I*a*Ta
n[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2*I)/3)/(d*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a + I*a*Tan[c + d*x]
])/(a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2))
```

3.674.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.674.4 Maple [A] (verified)

Time = 8.99 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2\left(i\left(\cos^{2}\left(dx+c\right)\right)+4\sin\left(dx+c\right)\cos\left(dx+c\right)-8i\right)\sqrt{e\cos\left(dx+c\right)}\,\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}\,e^{2}}{15d}$	62
risch	$-\frac{i e^{2}\sqrt{2}\,\sqrt{e\cos\left(dx+c\right)}\,\sqrt{\frac{a\,e^{2i\left(dx+c\right)}}{e^{2i\left(dx+c\right)}+1}}\,\left(30-2\cos\left(2dx+2c\right)+8i\sin\left(2dx+2c\right)\right)}{30d}$	74

input
```
int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/d*(I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-8*I)*(e*cos(d*x+c))^(1/2)*(
a*(1+I*tan(d*x+c)))^(1/2)*e^2
```

3.674.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-3i\,e^2e^{(4i\,dx+4i\,c)}-30i\,e^2e^{(2i\,dx+2i\,c)}+5i\,e^2\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{}}{30\,d}$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/30*sqrt(2)*sqrt(1/2)*(-3*I*e^2*e^(4*I*d*x + 4*I*c) - 30*I*e^2*e^(2*I*d*x
 + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/d
```

3.674.6 Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.674.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(5i\,e^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,e^2\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\right.\right.\right.\right.}{}$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/30*(5*I*e^2*cos(3/2*d*x + 3/2*c) - 3*I*e^2*cos(5/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*e^2*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 5*e^2*sin(3/2*d*x + 3/2*c) + 3*e^2*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*e^2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*sqrt(e)/d
```

3.674.8 Giac [F]

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx=\int (e\cos(dx+c))^{\frac{5}{2}}\sqrt{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*cos(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

3.674.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\right.}{15\,d}$$

input
```
int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 4*sin(2*c + 2*d*x
) - 15i))/(15*d)
```

3.675 $\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

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3.675.1 Optimal result

Integrand size = 30, antiderivative size = 85

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{4iae\sqrt{e\cos(c+dx)}\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
4/3*I*a*e*sec(d*x+c)*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-2/3*I
*(e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.675.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2e\sqrt{e\cos(c+dx)}(i\cos(c+dx)+2\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*e*Sqrt[e*Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[a + I*a*T
an[c + d*x]])/(3*d)
```

3.675.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3998, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3969}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*((((4*I)/3)*a*Sqrt[e*Sec[c +
 d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c
 + d*x]])/(d*(e*Sec[c + d*x])^(3/2)))
```

3.675.3.1 Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3998

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && !IntegerQ[m]
```

3.675.4 Maple [A] (verified)

Time = 8.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2(i\cos(dx+c)+2\sin(dx+c))\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e}{3d}$	50
risch	$-\frac{ie\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3d}$	65

input

```
int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c))
)^(1/2)*e
```

3.675.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,d}$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-I*e*e^(2*I*d*x + 2
*I*c) + 3*I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/
d
```

3.675.6 Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.675.7 Maxima [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{\left(-i\,e\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\,e\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+e\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,e\sin}{3\,d}$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/3*(-I*e*cos(3/2*d*x + 3/2*c) + 3*I*e*cos(1/2*d*x + 1/2*c) + e*sin(3/2*d*
x + 3/2*c) + 3*e*sin(1/2*d*x + 1/2*c))*sqrt(a)*sqrt(e)/d
```

3.675.8 Giac [F]

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\int (e\cos(dx+c))^{\frac{3}{2}}\sqrt{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*cos(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

3.675.9 Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2\,e\sqrt{e\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 2\mathrm{i}+2\,\sin\left(c+d\,x\right)-\mathrm{i}\right)\,\sqrt{}}{3\,d}$$

input
```
int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(2*e*(e*(2*cos(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(c + d*x) + cos(c/2 + (d
*x)/2)^2*2i - 1i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c +
 d*x)^2))^(1/2))/(3*d)
```

3.675. $\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

3.676 $\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

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3.676.1 Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

output

```
-2*I*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

3.676.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

input

```
Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.676.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3998, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}dx$$

$$\Big\downarrow \text{3998}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx$$

$$\Big\downarrow \text{3969}$$

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{d}$$

input
```
Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

3.676.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.676.4 Maple [A] (verified)

Time = 7.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2i\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}{d}$	32
risch	$-\frac{2i\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}{d}$	46

input
```
int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)
```

3.676.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=-\frac{2i\,\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/d
```

3.676.6 Sympy [F]

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\cos(c+dx)}\sqrt{ia(\tan(c+dx)-i)}\,dx$$

input
```
integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.676.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{a}\sqrt{e}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{d\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
-2*I*sqrt(a)*sqrt(e)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1)/(d*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 1))
```

3.676.8 Giac [F]

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\cos\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(e*cos(d*x + c))*sqrt(I*a*tan(d*x + c) + a), x)
```

3.676.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.677 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

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3.677.1 Optimal result

Integrand size = 30, antiderivative size = 335

$$\begin{aligned}\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = {} & \frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} \\ & - \frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} \\ & - \frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d\sqrt{e}} \\ & + \frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d\sqrt{e}}\end{aligned}$$

output

```
-1/2*I*ln(a*e^(1/2)-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)
)^(1/2)+cos(d*x+c)*e^(1/2)*(a+I*a*tan(d*x+c)))*a^(1/2)/d*2^(1/2)/e^(1/2)+1
/2*I*ln(a*e^(1/2)+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^
(1/2)+cos(d*x+c)*e^(1/2)*(a+I*a*tan(d*x+c)))*a^(1/2)/d*2^(1/2)/e^(1/2)+I*a
rctan(1-2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1
/2))*2^(1/2)*a^(1/2)/d/e^(1/2)-I*arctan(1+2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+
I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*2^(1/2)*a^(1/2)/d/e^(1/2)
```

3.677.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{ie^{-\frac{3}{2}idx}\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\left(\arctan\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-\operatorname{arctanh}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\cos(c+dx)}}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

output
```
(I*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*(ArcTan[E^((I/2)*d*x)/(
-E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)])*Sqrt
[a + I*a*Tan[c + d*x]])/(d*E^(((3*I)/2)*d*x)*Sqrt[e*Cos[c + d*x]])
```

3.677.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3996, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3996}$$

$$-\frac{4ia\int \frac{e\cos(c+dx)(i\tan(c+dx)a+a)}{a^2e^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{d}$$

$$\Big\downarrow \text{826}$$

$$-\frac{4ia\left(\frac{1}{2}\int \frac{ae+\cos(c+dx)(i\tan(c+dx)a+a)e}{a^2e^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)-\frac{1}{2}\int \frac{ae-e\cos(c+dx)(i\tan(c+dx)a}{a^2e^2+\cos^2(c+dx)(i\tan(c+dx)a-}\right.}{d}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)\right.\right.}{}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e\cos(c+dx)(i\tan(c+dx)a+a)-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-e\cos(c+dx)(i\tan(c+dx)a+a)-1}d\left(\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{i\,\mathrm{ta}}}{\sqrt{a}\sqrt{e}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}\right.\right.}{d}$$

$$\Big\downarrow \text{217}$$

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right)-\frac{1}{2}\int \frac{ae-e\cos(c+dx)(i\tan(c-}{a^2e^2+\cos^2(c+dx)(i\tan(c+}\right.}{d}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}+\frac{\int -\frac{\sqrt{2}}{ae+\cos(c+dx)(i\,\mathrm{t}}}{}\right.\right.}{}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4ia\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{\sqrt{2}(}{ae+\cos(c+dx)(i\,\mathrm{ta}}}{}\right.\right.}{}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4ia\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}}{ae+\cos(c+dx)(i\tan(c+dx)a+a)e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{i\tan(c+dx)a+a}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{\sqrt{}}{ae+\cos(c+dx)(i\,\mathrm{ta}}}{}\right.\right.}{}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{4ia\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(e\cos(c+dx)(a+ia\tan(\right.}{}\right.\right.}{}$$

3.677. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

input

```
Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

output

```
((-4*I)*a*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[
2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]/(Sq
rt[2]*Sqrt[a]*Sqrt[e]))/2 + (Log[a*e - Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e*Cos[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a + I*a*Tan[c + d*x
])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]) - Log[a*e + Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/2))/d
```

3.677.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3996
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-4*(b/f)   Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
 Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f
}, x] && EqQ[a^2 + b^2, 0]
```

3.677.4 Maple [A] (verified)

Time = 9.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.46

method	result
default	$\frac{(-1-i)\sqrt{a(1+i\tan(dx+c))}\left(i\,\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\mathrm{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)\cos(dx+c)}{d(-i\cos(dx+c)+\sin(dx+c)-i)\sqrt{e\cos(dx+c)}\,\sqrt{\frac{1}{\cos(dx+c)+1}}}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1-I)/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+
1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+arctanh(1/2*(-cos(d*x+c)+sin(d
*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)/(-I*cos(d*x+
c)+sin(d*x+c)-I)/(e*cos(d*x+c))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)
```

3.677.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.93

$$\begin{aligned}
\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx
&= -\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\frac{1}{2}i\,de\sqrt{\frac{4i\,a}{d^2e}}\right) \\
&\quad+\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\frac{1}{2}i\,de\sqrt{\frac{4i\,a}{d^2e}}\right) \\
&\quad-\frac{1}{2}\sqrt{-\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right. \\
&\qquad\left.+\frac{1}{2}i\,de\sqrt{-\frac{4i\,a}{d^2e}}\right) \\
&\quad+\frac{1}{2}\sqrt{-\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right. \\
&\qquad\left.-\frac{1}{2}i\,de\sqrt{-\frac{4i\,a}{d^2e}}\right)
\end{aligned}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
+ e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*
e*sqrt(4*I*a/(d^2*e))) + 1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqr
t(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d
*x - 1/2*I*c) - 1/2*I*d*e*sqrt(4*I*a/(d^2*e))) - 1/2*sqrt(-4*I*a/(d^2*e))*
log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*e*sqrt(-4*I*a/(d^2*e))) +
 1/2*sqrt(-4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
 + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) - 1/2*I*d
*e*sqrt(-4*I*a/(d^2*e)))
```

3.677.6 Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\sqrt{e\cos\left(c+dx\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*cos(c + d*x)), x)
```

3.677.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1400 vs. 2(247) = 494.

Time = 0.50 (sec) , antiderivative size = 1400, normalized size of antiderivative = 4.18

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqr
t(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2
*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 1) + I*sqrt(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*...
```

3.677.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx=\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

3.677.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2), x)
```

3.678 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

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3.678.1 Optimal result

Integrand size = 30, antiderivative size = 524

$$\begin{aligned}\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\\ &- \frac{ia^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &+ \frac{ia^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &+ \frac{ia^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &- \frac{ia^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
I*a/d/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*arctan(1
-2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*se
c(d*x+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/
2)+1/2*I*a^(3/2)*arctan(1+2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^
(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)+1/4*I*a^(3/2)*ln(a-2^(1/2)*a^(1/2)*(e*cos(d*x+
c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/e^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*
sec(d*x+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(
1/2)-1/4*I*a^(3/2)*ln(a+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*
x+c))^(1/2)/e^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/e^(3/2)*2^
(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

3.678.2 Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\frac{ie^{-\frac{1}{2}i(c+dx)}\cos^2(c+dx)\left(2\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)+2\arctan\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)\mathrm{co}}{}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

output
```
(I*Cos[c + d*x]^2*(2*Sqrt[2]*Cos[(c + d*x)/2] + 2*ArcTan[1 - Sqrt[2]*E^((I
/2)*(c + d*x))]*Cos[c + d*x] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))]*C
os[c + d*x] + Cos[c + d*x]*Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c +
 d*x))] - Cos[c + d*x]*Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x
))] - (2*I)*Sqrt[2]*Sin[(c + d*x)/2])*(Cos[c + d*x] + I*Sin[c + d*x])*Sqrt
[a + I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(1 + E^((2*I)*(c +
d*x)))*(e*Cos[c + d*x])^(3/2))
```

3.678.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.84, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 3998, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3998$$

$$\frac{\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3979$$

$$\frac{\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3980$$

$$\frac{\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{ae\sec(c+dx)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3976$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\int \frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \ 826$$

3.678. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int \frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx)}{d\sqrt{a+ia\tan(c}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+d}}}{(e\cos(c+dx))^{3/2}(e\,\mathrm{se}}$$

$$\Big\downarrow \text{25}$$

3.678. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec}$$

$$\Big\downarrow 27$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+}$$

$$\Big\downarrow 1103$$

$$\frac{\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx)\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

output
```
((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*a^2*
e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*
Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a -
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*
x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a -
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d
*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]))/((e*Cos[c + d*x])^(3/2)*(e*Sec[c
+ d*x])^(3/2))
```

3.678. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

3.678.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.678. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.678.4 Maple [A] (verified)

Time = 10.44 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.61

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right.\right.}{2}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)-
sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+I*cos(d*x+c)*arctan
h(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2
*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*(1/(cos(d*x+c)+1))^(1/2)+arctan
h(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*c
os(d*x+c)-arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x
+c)+1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2))/(I*cos(d*
x+c)+I-sin(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/e/(e*cos(d*x+c))^(1/2)
```

3.678.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\left(de^2e^{(2i\,dx+2i\,c)}+de^2\right)\sqrt{\;}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="fric
as")
```

output
```
1/2*(4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - (d*e^2*e^(2*I*d*x + 2*I*c) +
d*e^2)*sqrt(I*a/(d^2*e^3))*log(d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/
2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1
/2*I*d*x - 1/2*I*c)) + (d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(I*a/(d^2*e
^3))*log(-d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x
+ 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c))
+ (d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(-I*a/(d^2*e^3))*log(d*e^2*sqrt(
-I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (d*e^2*e^(2*I*d*x
+ 2*I*c) + d*e^2)*sqrt(-I*a/(d^2*e^3))*log(-d*e^2*sqrt(-I*a/(d^2*e^3)) + s
qrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

3.678.6 Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*cos(c + d*x))**(3/2), x)
```

3.678.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1836 vs. $2(396) = 792$.

Time = 0.47 (sec) , antiderivative size = 1836, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

output
```
-8*(2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*ar
ctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(
2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*
x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c) + I
*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*s
in(2*d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(I*sqrt(2)*cos(2*
d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))...
```

3.678.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.678.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2), x)
```

3.679 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

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3.679.1 Optimal result

Integrand size = 30, antiderivative size = 512

$$\begin{aligned}\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{3i\sqrt{a}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\ &- \frac{3i\sqrt{a}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\ &- \frac{3i\sqrt{a}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\ &+ \frac{3i\sqrt{a}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\ &+ \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}\end{aligned}$$

output
```
3/8*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^
(1/2)-3/8*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1
/2)/(e*sec(d*x+c))^(1/2))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5
/2)*2^(1/2)-3/16*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))
^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)/d/(e*co
s(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)+3/16*I*e^(5/2)*ln(a+2^(1/2)*a
^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a
+I*a*tan(d*x+c)))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1
/2)+1/2*I*a/d/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+
c)^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*cos(d*x+c))^(5/2)
```

3.679.2 Mathematica [A] (verified)

Time = 3.71 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\Bigg(\frac{3ie^{-\frac{1}{2}i(2c+5dx)}\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)^2\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\Big(\arctan\Big(\frac{}{\sqrt[4]{}}}{4\sqrt{2}}}{}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((((3*I)/4)*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d
*x)))^2*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcTan[E^((I/2)*d
*x)/(-E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)])
)/(Sqrt[2]*E^((I/2)*(2*c + 5*d*x))) - (3*I)*Cos[c + d*x]^(3/2) + 2*Sqrt[Co
s[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(
4*d*(e*Cos[c + d*x])^(5/2))
```

3.679.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.87, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3998, 3042, 3979, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.679. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3979}$$

$$\frac{\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}a\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3982}$$

$$\frac{\frac{3}{4}a\left(\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}a\left(\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e\left(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2\right)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}\right)+\frac{ia(e\sec(c+dx))^{5/2}}{2d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{826}$$

3.679. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}dx$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+d}}{ad}\right.}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d}{2e}}{2e}\right.}{d}\right.}{(e\cos(c+dx))^{5/2}(e}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a}{a^2}}{}\right.}{d}\right.}{(e\cos(c+dx))^{5/2}(e\sec(c+}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie}{}\right.}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \text{1479}$$

3.679. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.}{(e\cos(}$$

$$\Big\downarrow 25$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.}{(e\cos(c}$$

$$\Big\downarrow 27$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d}{2\sqrt{2}\sqrt{a}e}\right.}{d}\right.}{(e\cos(c+dx}$$

3.679. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{3}{4}a\left(-\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d}\right.}{(e\cos(c+dx))^{5/2}(e\,\mathrm{se}}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

output
```
(((I/2)*a*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*a*((
(-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)))/4)/((e*Cos[c + d*x])^(5/2)*(e*
Sec[c + d*x])^(5/2))
```

3.679.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.679. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.679.4 Maple [A] (verified)

Time = 10.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.70

method	result
default	$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{a(1+i\tan(dx+c))}\left(3i\cos(dx+c)\,\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-3i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+3i\sqrt{\frac{1}{\cos(dx+c)+1}}}{}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/8+1/8*I)/d*(a*(1+I*tan(d*x+c)))^(1/2)/(I*cos(d*x+c)+I-sin(d*x+c))/(1/(c
os(d*x+c)+1))^(1/2)/e^2/(e*cos(d*x+c))^(1/2)*(3*I*cos(d*x+c)*arctanh(1/2*(
cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*I*cos(
d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*
(1/(cos(d*x+c)+1))^(1/2)+2*I*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*arctanh
(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*c
os(d*x+c)+3*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*sin(d*x+c)*(1/(cos(d*x+c
)+1))^(1/2)+2*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+(1/(cos(d*x+c)+1))^(1/
2)+2*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*sec(d*x+c)*(1/(cos(d*x+c)+1))^(
1/2))
```

3.679.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}\right)}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="fric
as")
```

output
```
1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(-3*I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*e^(-1/2*
I*d*x - 1/2*I*c) - (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c
) + d*e^3)*sqrt(9/16*I*a/(d^2*e^5))*log(4/3*I*d*e^3*sqrt(9/16*I*a/(d^2*e^5
)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (d*e^3*e^(4*I*d*x + 4*I*c) + 2*
d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(9/16*I*a/(d^2*e^5))*log(-4/3*I*d*e
^3*sqrt(9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
 + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (d*e^3
*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(-9/16*I*a
/(d^2*e^5))*log(4/3*I*d*e^3*sqrt(-9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*
sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*
I*d*x - 1/2*I*c)) + (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*
c) + d*e^3)*sqrt(-9/16*I*a/(d^2*e^5))*log(-4/3*I*d*e^3*sqrt(-9/16*I*a/(d^2
*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^3*e^(4*I*d*x + 4*I*c) +
 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)
```

3.679.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.679.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2249 vs. $2(384) = 768$.

Time = 0.57 (sec) , antiderivative size = 2249, normalized size of antiderivative = 4.39

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

output
```
-32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*
sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x
+ 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqr
t(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + s
qrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin
(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*cos(4*d*x
 + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(I*sqrt(2...
```

3.679.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.679.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,dx$$

input

```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2),x)
```

output

```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2), x)
```

3.680 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

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3.680.1 Optimal result

Integrand size $=30$, antiderivative size $=719$

$$\begin{aligned}
&\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx=\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\\
&+\frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\\
&-\frac{5ia^{3/2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&+\frac{5ia^{3/2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&+\frac{5ia^{3/2}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&-\frac{5ia^{3/2}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&-\frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}
\end{aligned}$$

output
```
1/3*I*a/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+5/8*I*a*cos(d*x+c)
^2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-5/16*I*a^(3/2)*e^(7/2)*
arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(
1/2))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*
a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/16*I*a^(3/2)*e^(7/2)*arctan
(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*
sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/32*I*a^(3/2)*e^(7/2)*ln(a-2^(1/2)
*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*
(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)
*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/32*I*a^(3/2)*
e^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c
))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/
(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(
1/2)-5/12*I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*cos(d*x+c))^(7/2)
```

3.680.2 Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(-\frac{40}{3}i\cos^{\frac{3}{2}}(c+dx)+\frac{5}{8}ie^{-\frac{7}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^3\sqrt{e^{-i(c+dx)}\left(1\right.}\right.}{}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(((-40*I)/3)*Cos[c + d*x]^(3/2) + (((5*I)/8)*(1 + E^((
2*I)*(c + d*x)))^3*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(2*ArcT
an[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d
*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))] - Log[1 + S
qrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))/E^(((7*I)/2)*(c + d*x)) +
(32*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/3 + 20*Cos[c + d*x
]^(5/2)*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(32*d
*(e*Cos[c + d*x])^(7/2))
```

3.680.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.75, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = $\{3042, 3998, 3042, 3979, 3042, 3982, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int (e\sec(c+dx))^{7/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (e\sec(c+dx))^{7/2}\sqrt{i\tan(c+dx)a+a}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3979}$$

$$\frac{\frac{5}{6}a\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3982}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3979}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/2}}{3d\sqrt{a+ia\tan(c+dx)}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3980}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/}}{3d\sqrt{a+ia\tan(c+d}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}\right)+\frac{ia(e\sec(c+dx))^{7/}}{3d\sqrt{a+ia\tan(c+d}}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/}}{2ad}\right.}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{826}$$

3.680. $\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

$$\frac{\frac{5}{6}a\Bigg(\frac{3e^2\Bigg(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(}{d\sqrt{a+ia}}}{4a}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{5}{6}a\Bigg(\frac{3e^2\Bigg(\frac{2ia^2e^3\sec(c+dx)\Bigg(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-}{}}}{2e}}{2e}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{4a}}{(e\,\mathrm{co}}$$

$$\Big\downarrow \text{1082}$$

3.680. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}\right.}{(e\cos(c+dx)}$$

$$\Big\downarrow 217$$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(}}{\sqrt{e\sec(c+}}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}\right.}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 1479$$

3.680. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\cos(c+dx\right.}}{2\sqrt{2}\sqrt{a}\sqrt{}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ic}}\right.}{}\right.$$

$$\Big\downarrow \; 25$$

3.680. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

$$\frac{5}{6}a\left(\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a}{}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}\right.}{4a}\right.$$

$$\Big\downarrow 27$$

3.680. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

$$\frac{5}{6}a\left(\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\,\mathrm{t}}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+d}}\right.}{4a}}{}\right.$$

$$\Big\downarrow \color{blue}{1103}$$

$$\frac{5}{6}a\left(\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}}{(e\cos(c}\right.$$

3.680. $\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

output
```
(((I/3)*a*(e*Sec[c + d*x])^(7/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (5*a*((
(-1/2*I)*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) + (3
*e^2*((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)
*a^2*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt
[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2
]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqr
t[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c
 + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqr
t[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[
c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a - I
*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/(4*a)))/6)/((e*Cos[c + d*x]
)^(7/2)*(e*Sec[c + d*x])^(7/2))
```

3.680.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

3.680. $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.680.4 Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.64

method	result
default	$\left(\frac{1}{48}-\frac{i}{48}\right)\sqrt{a(1+i\tan(dx+c))}\,\Bigg(10i\tan(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\cos(dx+c)\operatorname{arctanh}\Bigg(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\Bigg)-8i\big(\sec^2(dx+$

input
```
int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(1/48-1/48*I)/d*(a*(1+I*tan(d*x+c)))^(1/2)/(I*cos(d*x+c)+I-sin(d*x+c))/(1/
(cos(d*x+c)+1))^(1/2)/e^3/(e*cos(d*x+c))^(1/2)*(10*I*tan(d*x+c)*(1/(cos(d*
x+c)+1))^(1/2)+15*I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(
d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-8*I*sec(d*x+c)^2*(1/(cos(d*x+c)+1))^(1
/2)-5*I*(1/(cos(d*x+c)+1))^(1/2)+8*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+
1))^(1/2)-15*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)-15*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-15*si
n(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+15*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+2*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-5*(1/(cos(d*x+c)+1))^(1/2)-10*tan
(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-15*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+
2*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-8*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+
c)+1))^(1/2)-8*sec(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2))
```

3.680.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 657, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(5i\,dx+5i\,c)}+42i\,e^{(3i\,dx+3i\,c)}+15i\,e\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="fric
as")
```

output

```
1/12*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(-5*I*e^(5*I*d*x + 5*I*c) + 42*I*e^(3*I*d*x + 3*I*c) + 15*
I*e^(I*d*x + I*c))*e^(-1/2*I*d*x - 1/2*I*c) - 6*(d*e^4*e^(6*I*d*x + 6*I*c)
 + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt
(25/64*I*a/(d^2*e^7))*log(8/5*d*e^4*sqrt(25/64*I*a/(d^2*e^7)) + sqrt(2)*sq
rt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
e^(-1/2*I*d*x - 1/2*I*c)) + 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*
d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(25/64*I*a/(d^2*e^
7))*log(-8/5*d*e^4*sqrt(25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^
(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1
/2*I*c)) + 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*
d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(-25/64*I*a/(d^2*e^7))*log(8/5*d*e^
4*sqrt(-25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c
) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - 6*(d*
e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x
 + 2*I*c) + d*e^4)*sqrt(-25/64*I*a/(d^2*e^7))*log(-8/5*d*e^4*sqrt(-25/64*I
*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^4*e^(6*I*d*x + 6
*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)
```

3.680.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input

```
integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

output

```
Timed out
```

3.680.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2661 vs. 2(549) = 1098.

Time = 0.59 (sec) , antiderivative size = 2661, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxi
ma")
```

output
```
-192*(30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2
)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x +
4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*
sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*
x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + s
qrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
 + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*c
os(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c
) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqr
t(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x +
 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt
(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
30*(-I*sqrt(2)*cos(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(
2)*cos(2*d*x + 2*c) + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + ...
```

3.680.8 Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx=\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.680.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx=\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}} \,d x$$

input
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2),x)
```

output
```
int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2), x)
```

3.681 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.681.1 Optimal result

Integrand size = 30, antiderivative size = 175

$$\begin{aligned}\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}}\\ &\quad+\frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}-\frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad}\\ &\quad-\frac{32i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35ad}\end{aligned}$$

output
```
2/7*I*(e*cos(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/35*I*(e*cos(d*x+c
))^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(e*cos(d*x+c))^(5
/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d-32/35*I*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2
*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.681.2 Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.46

$$\begin{aligned}&\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx =\\ &-\frac{ie^3(35\cos(c+dx)+\cos(3(c+dx))+70i\sin(c+dx)+6i\sin(3(c+dx)))}{70d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

input
```
Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-1/70*I)*e^3*(35*Cos[c + d*x] + Cos[3*(c + d*x)] + (70*I)*Sin[c + d*x] +
 (6*I)*Sin[3*(c + d*x)]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]])
```

3.681.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3998, 3042, 3983, 3042, 3978, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{5/2}}dx}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\,\mathrm{se}}\right.$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\,\mathrm{se}}\right.$$

$$\Big\downarrow \text{3969}$$

3.681. $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

$$(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\left(\frac{6\left(\frac{4a\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)}{5e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}\right)}{7a}+\frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c}\right.$$

input
```
Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)*(((2*I)/7)/(d*(e*Sec[c + d*x
])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (6*((((-2*I)/5)*Sqrt[a + I*a*Tan[c
+ d*x]])/(d*(e*Sec[c + d*x])^(5/2)) + (4*a*(((2*I)/3)/(d*Sqrt[e*Sec[c + d*
x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(
a*d*Sqrt[e*Sec[c + d*x]])))/(5*e^2)))/(7*a))
```

3.681.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.681.4 Maple [A] (verified)

Time = 7.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{2 e^{2} \sqrt{e \cos \left(d x +c \right)}\, \left(i \left(\cos^{2}\left(d x +c \right)\right)-6 \sin \left(d x +c \right) \cos \left(d x +c \right)+8 i -16 \tan \left(d x +c \right)\right)}{35 d \sqrt{a \left(1+i \tan \left(d x +c \right)\right)}}$	70

input
```
int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/d*e^2*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)^
2-6*sin(d*x+c)*cos(d*x+c)+8*I-16*tan(d*x+c))
```

3.681.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-7i\,e^{2}e^{(6i\,dx+6i\,c)} - 105i\,e^{2}e^{(4i\,dx+4i\,c)} + 35i\,e^{2}e^{(2i\,dx+2i\,c)} + 5i\,e^{2}\right)\sqrt{ee^{(2i}}}{140\,ad}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/140*sqrt(2)*sqrt(1/2)*(-7*I*e^2*e^(6*I*d*x + 6*I*c) - 105*I*e^2*e^(4*I*d
*x + 4*I*c) + 35*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x +
2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*d*x - 7/2*I*c)/(a*
d)
```

3.681.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.681.7 Maxima [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\bigl(5i\,e^2\cos\bigl(\frac{7}{2}\,dx+\frac{7}{2}\,c\bigr)-7i\,e^2\cos\bigl(\frac{5}{7}\,\arctan\bigl(\sin\bigl(\frac{7}{2}\,dx+\frac{7}{2}\,c\bigr),\cos\bigl(\frac{7}{2}\,dx+\frac{7}{2}\,c\bigr)}{}$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/140*(5*I*e^2*cos(7/2*d*x + 7/2*c) - 7*I*e^2*cos(5/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*I*e^2*cos(3/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c))) - 105*I*e^2*cos(1/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))) + 5*e^2*sin(7/2*d*x + 7/2*c) + 7*e^2*sin(5/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*e^2*sin(3/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 105*e^2*sin(1/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(sqrt(a)*d)
```

3.681.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*cos(d*x + c))^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.681.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.63

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,28\mathrm{i}+\cos(}{140\,a\,d}$$

input
```
int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*28i + cos(4*c + 4*d*x)
*5i + 42*sin(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) - 105i))/(140*a*d)
```

3.682 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.682.1 Optimal result 4877
3.682.2 Mathematica [A] (verified) 4877
3.682.3 Rubi [A] (verified) 4878
3.682.4 Maple [A] (verified) 4880
3.682.5 Fricas [A] (verification not implemented) 4880
3.682.6 Sympy [F] 4881
3.682.7 Maxima [A] (verification not implemented) 4881
3.682.8 Giac [F] 4882
3.682.9 Mupad [B] (verification not implemented) 4882

3.682.1 Optimal result

Integrand size = 30, antiderivative size = 126

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad}$$

output

```
2/5*I*(e*cos(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*cos(d*x+c
))^(3/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(e*cos(d*x+c))^(3/
2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.682.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ie^2(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output

```
((-1/15*I)*e^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))/(d*Sqrt[
e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.682.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3998, 3042, 3983, 3042, 3978, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3983}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}}{(e\sec(c+dx))^{3/2}}dx}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3978}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\left(\frac{2a\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\left(\frac{2a\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{3e^2}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3969}$$

$$(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\left(\frac{4\left(\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\right)}{5a}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\right)$$

input
```
Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*(((2*I)/5)/(d*(e*Sec[c + d*x
])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (4*((((4*I)/3)*a*Sqrt[e*Sec[c + d*x
]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((2*I)/3)*Sqrt[a + I*a*Tan[c + d
*x]])/(d*(e*Sec[c + d*x])^(3/2))))/(5*a))
```

3.682.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3978
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Simp[a*((m + n)/(m*d^2))   Int[(d*Sec[e + f*x])^(m + 2)*(a +
b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b
^2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

3.682. $\int\frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}dx$

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.682.4 Maple [A] (verified)

Time = 8.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2e\sqrt{e\cos(dx+c)}\,(i\cos(dx+c)-4\sin(dx+c)-8i\sec(dx+c))}{15d\sqrt{a(1+i\tan(dx+c))}}$	59

input
```
int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/d*e*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)-4*
sin(d*x+c)-8*I*sec(d*x+c))
```

3.682.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-5i\,ee^{(4i\,dx+4i\,c)}+30i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+}}}{30\,ad}$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/30*sqrt(2)*sqrt(1/2)*(-5*I*e*e^(4*I*d*x + 4*I*c) + 30*I*e*e^(2*I*d*x + 2
*I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a*d)
```

3.682.6 Sympy [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos{(c+dx)})^{\frac{3}{2}}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*cos(c + d*x))**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.682.7 Maxima [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(3i\,e\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,e\cos\left(\frac{3}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)}{}$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/30*(3*I*e*cos(5/2*d*x + 5/2*c) - 5*I*e*cos(3/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))) + 30*I*e*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c),
 cos(5/2*d*x + 5/2*c))) + 3*e*sin(5/2*d*x + 5/2*c) + 5*e*sin(3/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*e*sin(1/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(e)/(sqrt(a)*d)
```

3.682.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate((e*cos(d*x + c))^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.682.9 Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{e\,\sqrt{e\,\cos(c+d\,x)}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(35\,\sin(c+d\,x)+3\,\sin(3\,c}{30\,a\,d}$$

input
```
int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(e*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*25i + 35*sin(c + d*x) + cos(
3*c + 3*d*x)*3i + 3*sin(3*c + 3*d*x)))/(30*a*d)
```

3.683 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.683.1 Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad}$$

output
```
2/3*I*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(e*cos(d*x+c))
^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

3.683.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{e\cos(c+dx)}(-i+2\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(2*Sqrt[e*Cos[c + d*x]]*(-I + 2*Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d
*x]])
```

3.683.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3998, 3042, 3983, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3983}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}dx}{3a}+\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\right)$$

$$\Big\downarrow \text{3969}$$

$$\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\left(\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\right)$$

input
```
Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

---

output
```
Sqrt[e*Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]*(((2*I)/3)/(d*Sqrt[e*Sec[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(a
*d*Sqrt[e*Sec[c + d*x]]))
```

3.683.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3983
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Simp[Simplify[m + n]/(a*(m + 2*n))   Int[(d*Sec[e +
f*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x
] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*
n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.683.4 Maple [A] (verified)

Time = 7.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2\sqrt{e\cos\left(dx+c\right)}\,\left(i-2\tan\left(dx+c\right)\right)}{3d\sqrt{a\left(1+i\tan\left(dx+c\right)\right)}}$	42
risch	$-\frac{i\sqrt{2}\,\sqrt{e\cos\left(dx+c\right)}\,\left(3\,{\mathrm e}^{2i\left(dx+c\right)}-1\right)}{3\left({\mathrm e}^{2i\left(dx+c\right)}+1\right)\sqrt{\frac{a\,{\mathrm e}^{2i\left(dx+c\right)}}{{\mathrm e}^{2i\left(dx+c\right)}+1}}\,d}$	72

input
```
int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/d*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I-2*tan(d*x+c))
```

3.683.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ad}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fric
as")
```

output
```
1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(-3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2*I*c)/(a*d
)
```

3.683.6 Sympy [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\cos{(c+dx)}}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(e*cos(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.683.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e}\big(i\cos\big(\frac{3}{2}\,dx+\frac{3}{2}\,c\big)-3i\cos\big(\frac{1}{3}\arctan\big(\sin\big(\frac{3}{2}\,dx+\frac{3}{2}\,c\big),\cos\big(\frac{3}{2}\,dx+\frac{3}{2}\,c\big)\big)\big)+\sin\big(\frac{3}{2}\,dx+\frac{3}{2}\,c\big)+3\sin\big(}{3\sqrt{a}d}$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

output
```
1/3*sqrt(e)*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

3.683.8 Giac [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\cos(dx+c)}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac
")
```

output
```
integrate(sqrt(e*cos(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

3.683.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)-3{}\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}{3\,a\,d}$$

input
```
int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
((e*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) - 3i)*((a*
(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2
))/(3*a*d)
```

3.683. $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

3.684 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.684.8 Giac [F] 4893
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3.684.1 Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output

```
2*I/d/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

3.684.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input

```
Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output

```
(2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.684.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3998, 3042, 3969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$\Big\downarrow \text{3969}$$

$$\frac{2i}{d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.684.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3969
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.684.4 Maple [A] (verified)

Time = 10.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2i}{d\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}$	32
risch	$\frac{i\sqrt{2}}{\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	46

input
```
int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
2*I/d/(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

3.684.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ade}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d*e)
```

3.684.6 Sympy [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\cos\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

3.684.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{e}\sqrt{-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
2*I*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt(e)*sqrt
(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 1))
```

3.684.8 Giac [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/(sqrt(e*cos(d*x + c))*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.685 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.685.1 Optimal result

Integrand size = 30, antiderivative size = 495

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$+\frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$+\frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$-\frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
1/2*I*ln(a*e^(1/2)-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))
^(1/2)+cos(d*x+c)*e^(1/2)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/e^(3/2)
*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*ln(a*e^(1
/2)+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)+cos(d*x+
c)*e^(1/2)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/e^(3/2)*2^(1/2)/(a-I*a
*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*arctan(1-2^(1/2)*(e*cos(d*x+
c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)*2^(1/2)*a^(
1/2)/d/e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+I*arctan(
1+2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*s
ec(d*x+c)*2^(1/2)*a^(1/2)/d/e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*
x+c))^(1/2)
```

3.685.2 Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.42

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{\frac{1}{2}i(c+dx)}\Big(2\arctan\Big(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\Big)-2\arctan\Big(1+\sqrt{2}e^{\frac{1}{2}}}{\sqrt{2}de\sqrt{\frac{ae^{2i(}}{1+e^{2i(}}}}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(I*E^((I/2)*(c + d*x))*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcT
an[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x))
+ E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))
]))/(Sqrt[2]*d*e*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*S
qrt[(e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))])
```

3.685.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.75, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3997, 3042, 3996, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3997}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(c+dx)\int \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3996}$$

$$\frac{4ia\sec(c+dx)\int \frac{e\cos(c+dx)(a-ia\tan(c+dx))}{a^2e^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\int \frac{ae+\cos(c+dx)(a-ia\tan(c+dx))e}{a^2e^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2e^2}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)-\frac{1}{2}\int \frac{ae-e\cos(c+dx)(}{a^2e^2+\cos^2(c+dx)(}\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\,\mathrm{tar}\right.\right.\right.}{}$$

$$\Big\downarrow \text{1082}$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\int \frac{1}{-e\cos(c+dx)(a-ia\tan(c+dx))-1}d\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-e\cos(c+dx)(a-ia\tan(c+dx))-1}d\left(\frac{\sqrt{2}\sqrt{e}}{}\right.}{\sqrt{2}\sqrt{a}\sqrt{e}}\right.\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{}}$$

$$\Big\downarrow \text{217}$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right)-\frac{1}{2}\int \frac{ae-e\cos(c}{a^2e^2+\cos^2(c-}\right.}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

3.685. $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\Big\downarrow \; 1479$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}+\frac{\int -\frac{}{ae}}{}\right.\right.}{}$$

$$\Big\downarrow \; 25$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{}{ae+}}{}\right.\right.}{}$$

$$\Big\downarrow \; 27$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{ae+\cos(c+dx)(a-ia\tan(c+dx))e-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\sqrt{e}}d\left(\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{}{ae+}}{}\right.\right.}{}$$

$$\Big\downarrow \; 1103$$

$$\frac{4ia\sec(c+dx)\left(\frac{1}{2}\left(\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}\right)+\frac{1}{2}\left(\frac{\log\left(e\cos(c+d\right.}{}\right.\right.}{de\sqrt{a-ia\tan(}}$$

input
```
Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
((4*I)*a*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt[2
]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]/(Sqr
t[2]*Sqrt[a]*Sqrt[e]))/2 + (Log[a*e - Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e*Cos[c
 + d*x]]*Sqrt[a - I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a - I*a*Tan[c + d*x]
)]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]) - Log[a*e + Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[e*
Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]] + e*Cos[c + d*x]*(a - I*a*Tan[c +
 d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/2)*Sec[c + d*x])/(d*e*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

3.685. $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.685.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.685. $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3996
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-4*(b/f)   Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
 Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f
}, x] && EqQ[a^2 + b^2, 0]
```

rule 3997
```
Int[1/((cos[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)*Sqrt[(a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]]), x_Symbol] :> Simp[1/(d*Cos[e + f*x]*Sqrt[a - b*Tan[e + f*x]]
*Sqrt[a + b*Tan[e + f*x]])   Int[Sqrt[a - b*Tan[e + f*x]]/Sqrt[d*Cos[e + f*
x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.685.4 Maple [A] (verified)

Time = 12.92 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.33

method	result	s
default	$\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i\,\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-\operatorname{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)(\cos(dx+c)+1+i\sin(dx+c))}{d(\cos(dx+c)+1)\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,e\sqrt{e\cos(dx+c)}}$	1

input
```
int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
(-1/2-1/2*I)/d*(I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/
(cos(d*x+c)+1))^(1/2))-arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+
1)/(1/(cos(d*x+c)+1))^(1/2)))*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(
a*(1+I*tan(d*x+c)))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/e/(e*cos(d*x+c))^(1/2)
```

3.685.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.69

$$\begin{aligned}
\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(\frac{1}{2}\,ade^2\sqrt{\frac{4i}{ad^2e^3}}\right. \\
&\quad \left.+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) \\
&\quad +\frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(-\frac{1}{2}\,ade^2\sqrt{\frac{4i}{ad^2e^3}}\right. \\
&\quad \left.+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) \\
&\quad +\frac{1}{2}\sqrt{-\frac{4i}{ad^2e^3}}\log\left(\frac{1}{2}\,ade^2\sqrt{-\frac{4i}{ad^2e^3}}\right. \\
&\quad \left.+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) \\
&\quad -\frac{1}{2}\sqrt{-\frac{4i}{ad^2e^3}}\log\left(-\frac{1}{2}\,ade^2\sqrt{-\frac{4i}{ad^2e^3}}\right. \\
&\quad \left.+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)
\end{aligned}$$

input

```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output

```
-1/2*sqrt(4*I/(a*d^2*e^3))*log(1/2*a*d*e^2*sqrt(4*I/(a*d^2*e^3)) + sqrt(2)
*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*e^(-1/2*I*d*x - 1/2*I*c)) + 1/2*sqrt(4*I/(a*d^2*e^3))*log(-1/2*a*d*e^2*
sqrt(4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + 1/2*sqrt(-4*
I/(a*d^2*e^3))*log(1/2*a*d*e^2*sqrt(-4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*
sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*
I*d*x - 1/2*I*c)) - 1/2*sqrt(-4*I/(a*d^2*e^3))*log(-1/2*a*d*e^2*sqrt(-4*I/
(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c))
```

3.685.6 Sympy [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\cos{(c+dx)})^{\frac{3}{2}}\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/((e*cos(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

3.685.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.44

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\text{Too large to display}$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
-1/4*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/
2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1
, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2
*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2
(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*
sqrt(2)*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1
/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/2*d
*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) +
 1) + I*sqrt(2)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(
2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x
 + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x +
1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^
2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c...
```

3.685.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*cos(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\,\cos(c+d\,x))^{3/2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.686 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

3.686.1 Optimal result 4903
3.686.2 Mathematica [A] (verified) 4904
3.686.3 Rubi [A] (verified) 4904
3.686.4 Maple [B] (warning: unable to verify) 4909
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3.686.1 Optimal result

Integrand size = 30, antiderivative size = 470

$$\begin{aligned}\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&- \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&- \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&+ \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&- \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}}\end{aligned}$$

output
```
1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^
(1/2)-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1
/2)/(e*sec(d*x+c))^(1/2))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1
/2)/a^(1/2)-1/4*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^
(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/(e*cos(d*x+c))
^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1/2)+1/4*I*e^(5/2)*ln(a+2^(1/2)*a^(
1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I
*a*tan(d*x+c)))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1/2
)-I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(d*x+c))^(5/2)
```

3.686.2 Mathematica [A] (verified)

Time = 3.37 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{ic-\frac{idx}{2}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-2e^{\frac{3idx}{2}}+\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\mathrm{arc}\right.}{d\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\sqrt{\cos(c+dx)}}$$

input
```
Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(I*E^(I*c - (I/2)*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-2
*E^(((3*I)/2)*d*x) + (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*ArcTa
n[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)] - (-E^((-2*I)*c))^(3/4)*(1 + E^((2*
I)*(c + d*x)))*ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(d*Sqrt[(1 +
 E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*Sqrt[Cos[c + d*x]]*(e*Cos[c + d*x])
^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

3.686.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.85, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3998, 3042, 3982, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow 3998$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 3982$$

$$\frac{\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 3976$$

$$\frac{-\frac{2ie^4\int \frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e\left(a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2\right)}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 826$$

$$\frac{-\frac{2ie^4\left(\frac{\int \frac{a+\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\,s}}{ad}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow 1476$$

$$\frac{-\frac{2ie^4\left(\frac{\frac{\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan}}{\sqrt{e\,s}}}{2e}}{2e}\right.}{d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))}$$

$$\Big\downarrow 1082$$

3.686. $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c}{a^2+\cos^2(}}{}}{d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(i\tan(c+dx)a+a)}{a^2+\cos^2(c+dx)(i\tan(c+dx)a+a)^2}d\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d}-\frac{ie^2\sqrt{a+}}{}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$\Big\downarrow \textcolor{blue}{1479}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}}{d}}{(e\cos(c+dx))^{5/2}(e}$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}\right)}d\frac{\sqrt{i}}{\sqrt{}}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}}{d}}{(e\cos(c+dx))^{5/2}(e\,\mathrm{se}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{i\tan(c+dx)a+a}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{i\tan(c+dx)a+a}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(i\tan(c+dx)a+a)}{e}}d\frac{\sqrt{i\tan(c+}}{\sqrt{e\sec(}}}{2\sqrt{2}\sqrt{a}e}}{}}{d}}{(e\cos(c+dx))^{5/2}(e\sec(c}$$

3.686. $\displaystyle\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\Big\downarrow 1103$$

$$-\frac{\frac{2ie^4\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{e}}-\log(}{2e}\right.}{d}}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

input
```
Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output
```
(((-2*I)*e^4*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(
Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sq
rt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*
Tan[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e)))/d - (I*e^2*Sqrt[e*Sec[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d))/((e*Cos[c + d*x])^(5/2)*(e*Se
c[c + d*x])^(5/2))
```

3.686.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

3.686. $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 3998

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.686.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(370) = 740.

Time = 12.39 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.63

method	result	size
default	Expression too large to display	765

input

```
int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output

```
-1/4/d*(I*csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc
(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2+I*csc(d*x+c)
^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x
+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*arctanh(1/2*(-cot(d
*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/
2)*(1-cos(d*x+c))^2+csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^
(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-I*
arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c)
)^2+1)^(1/2))*2^(1/2)-I*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(cs
c(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)-4*I*(csc(d*x+c)^2*(1-cos(d*x
+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*arctanh(1/2*(-cot(d*x+c)+c
sc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))-2^(1/2)*arct
anh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+
1)^(1/2))-4*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*(csc(d*x+c)^2*(1-cos(
d*x+c))^2-1)*(-csc(d*x+c)+cot(d*x+c)+I)/(-a*(2*I*(csc(d*x+c)-cot(d*x+c))-c
sc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1/2)/(
csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(5/2)/(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2-
1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(5/2)
```

3.686.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.07

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}-(ade}{}$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output
```
1/2*(-4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - (a*d*e^3*e^(2*I*d*x + 2*I*c)
 + a*d*e^3)*sqrt(I/(a*d^2*e^5))*log(I*a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2
)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sq
rt(I/(a*d^2*e^5))*log(-I*a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*s
qrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I
*d*x - 1/2*I*c)) - (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*
e^5))*log(I*a*d*e^3*sqrt(-I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I
*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I
*c)) + (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*e^5))*log(-I
*a*d*e^3*sqrt(-I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*
c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*
e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)
```

3.686.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.686.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2147 vs. $2(358) = 716$.

Time = 0.49 (sec) , antiderivative size = 2147, normalized size of antiderivative = 4.57

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input

```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output

```
-8*(2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2
(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1,
 -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1
) + 2*(-I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co...
```

3.686.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*cos(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.686.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\,\cos(c+d\,x))^{5/2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.687 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.687.1 Optimal result

Integrand size = 30, antiderivative size = 682

$$
\begin{aligned}
\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3i\sqrt{a}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3i\sqrt{a}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3i\sqrt{a}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3i\sqrt{a}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}}
\end{aligned}
$$

output

```
3/4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-3/8*I*e
^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*
x+c))^(1/2))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2
)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/8*I*e^(7/2)*
arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(
1/2))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/
2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/16*I*e^(7/2)*ln(a-2
^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d
*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec
(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-3
/16*I*e^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/(e*cos(
d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a
*tan(d*x+c))^(1/2)-1/2*I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(
d*x+c))^(7/2)
```

3.687.2 Mathematica [A] (verified)

Time = 3.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.36

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}ie^{\frac{1}{2}i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{5/2}\left(2\,\mathrm{arct}\right.\right.}{}$$

input

```
Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

output

```
(Sqrt[Cos[c + d*x]]*(((3*I)/4)*E^((I/2)*(c + d*x))*((1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x)))^(5/2)*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*
ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*
x)) + E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d
*x))]) + 4*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])))/(16*d*(e
*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

3.687.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 493, normalized size of antiderivative = 0.72, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3998, 3042, 3982, 3042, 3979, 3042, 3980, 3042, 3976, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{1}{\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow 3998$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{i\tan(c+dx)a+a}}dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3982$$

$$\frac{\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{3e^2\int (e\sec(c+dx))^{3/2}\sqrt{i\tan(c+dx)a+a}dx}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3979$$

$$\frac{\frac{3e^2\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{3e^2\left(\frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3980}$$

$$\frac{\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3e^2\left(\frac{ae\sec(c+dx)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{3976}$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\int\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e\left(a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2\right)}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\right)}{4a}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2ad}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\int\frac{a+\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}-\frac{\int\frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))}{d\sqrt{a+ia\tan(c+}}}{4a}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}+\frac{\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}}}{2e}}{2e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}}{(e\cos(c+dx))^{7/2}}$$

$$\Big\downarrow \text{1082}$$

3.687. $\displaystyle\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\frac{\frac{3e^2\Bigg(\frac{2ia^2e^3\sec(c+dx)\Bigg(\frac{\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\int \frac{1}{-\frac{\cos(c+dx)(a-ia\tan(c+dx))}{e}-1}d\left(\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{4a}}{(e\cos(c+dx))^{7/2}(e\sec(c+}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{3e^2\Bigg(\frac{2ia^2e^3\sec(c+dx)\Bigg(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int \frac{a-\cos(c+dx)(a-ia\tan(c+dx))}{a^2+\cos^2(c+dx)(a-ia\tan(c+dx))^2}d\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{2e}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{4a}}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{3e^2\Bigg(\frac{2ia^2e^3\sec(c+dx)\Bigg(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}}{}}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c-}}}{4a}}{(e\,co}$$

$$\Big\downarrow \; 25$$

3.687. $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\sqrt{e}\left(\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan}{e}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+d}}\right.}{4a}}{(e\cos(}$$

$$\Big\downarrow \color{blue}{27}$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{a}}{\sqrt{e}\sqrt{e\sec(c+dx)}}+\frac{\cos(c+dx)(a-ia\tan(c+}{e}}}{2\sqrt{2}\sqrt{a}e}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}}{(e\cos(c+}$$

$$\Big\downarrow \color{blue}{1103}$$

$$\frac{\frac{3e^2\left(\frac{2ia^2e^3\sec(c+dx)\left(\frac{\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{e}}}{2e}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+\right.}{2\sqrt{2}\sqrt{a}\sqrt{e}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\right.}{4a}}{(e\cos(c+dx))^{7/2}(e}$$

input

```
Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

3.687. $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

output
```
(((-1/2*I)*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) +
(3*e^2*((I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*
I)*a^2*e^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[e])) + ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]/(S
qrt[2]*Sqrt[a]*Sqrt[e]))/(2*e) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan
[c + d*x])]/(Sqrt[2]*Sqrt[a]*Sqrt[e]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*S
qrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Ta
n[c + d*x])]/(2*Sqrt[2]*Sqrt[a]*Sqrt[e]))/(2*e))*Sec[c + d*x])/(d*Sqrt[a -
 I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/(4*a))/((e*Cos[c + d*x])^
(7/2)*(e*Sec[c + d*x])^(7/2))
```

3.687.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.687. $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3976
```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[-4*b*(d^2/f)   Subst[Int[x^2/(a^2 + d^2*x^4), x]
, x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d,
e, f}, x] && EqQ[a^2 + b^2, 0]
```

rule 3979
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Simp[a*((m + 2*n - 2)/(m + n - 1))   Int[(d*Se
c[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
 m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ
[2*m, 2*n]
```

rule 3980
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Simp[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]]))   Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
 x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

3.687. $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

rule 3982
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Simp[d^2*((m - 2)/(a*(m + n - 1)))
 Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ
[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !IL
tQ[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.687.4 Maple [A] (verified)

Time = 12.96 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.74

method	result
default	$\frac{\left(-\frac{1}{16}-\frac{i}{16}\right)\left(3i\sin(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+3i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+3i\,\mathrm{arcta}\right.}{}$

input
```
int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
(-1/16-1/16*I)/d/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)/(1/(cos(d*x+c)+
1))^(1/2)/e^3/(e*cos(d*x+c))^(1/2)*(3*I*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)
-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+3*I*cos(d*x+c)*arc
tanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
)+3*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+
1))^(1/2))-2*I*(1/(cos(d*x+c)+1))^(1/2)+4*I*tan(d*x+c)*(1/(cos(d*x+c)+1))^
(1/2)+3*arctanh(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c
)+1))^(1/2))*cos(d*x+c)-3*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)
/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*I*sec(d*x+c)*(1/(cos(d*x+c)+1)
)^(1/2)+4*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*arctanh(1/2*(
cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*(1/(co
s(d*x+c)+1))^(1/2)-4*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*sec(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)-4*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2))
```

3.687.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.89

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(3i\,dx+3i\,c)}+3i\,e\right.}{}$$

input

```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fr
icas")
```

output

```
1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(-I*e^(3*I*d*x + 3*I*c) + 3*I*e^(I*d*x + I*c))*e^(-1/2*I*d*
x - 1/2*I*c) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c
) + a*d*e^4)*sqrt(9/16*I/(a*d^2*e^7))*log(4/3*a*d*e^4*sqrt(9/16*I/(a*d^2*e
^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^4*e^(4*I*d*x + 4*I*c)
+ 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(9/16*I/(a*d^2*e^7))*log(-4
/3*a*d*e^4*sqrt(9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x
+ 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c))
+ (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*
sqrt(-9/16*I/(a*d^2*e^7))*log(4/3*a*d*e^4*sqrt(-9/16*I/(a*d^2*e^7)) + sqrt
(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4
*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(-9/16*I/(a*d^2*e^7))*log(-4/3*a*d*e^4
*sqrt(-9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
+ e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*e^4
*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)
```

3.687.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input

```
integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output

```
Timed out
```

3.687.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2254 vs. $2(522) = 1044$.

Time = 0.87 (sec) , antiderivative size = 2254, normalized size of antiderivative = 3.30

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="ma
xima")
```

output
```
-32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*
sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x
+ 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqr
t(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + s
qrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin
(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(-I*sqrt(2)*cos(4*d*x
 + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt
(2)*sin(2*d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(I*sqrt(2...
```

3.687.8 Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(1/((e*cos(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{(e\cos(c+d\,x))^{7/2}\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

output
```
int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.688 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

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3.688.1 Optimal result

Integrand size = 26, antiderivative size = 105

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{m}{2}+n}(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1}{2}(2+m-2n),1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i}{dm}$$

output
```
-I*2^(-1/2*m+n)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1+1/2*m-n],[1-1/2*m],1
/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m-n)*(a+I*a*tan(d*x+c))^n/d/m
```

3.688.2 Mathematica [A] (verified)

Time = 14.50 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.92

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^{-m+n}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)(e\cos(c+dx))^{}}{}$$

input
```
Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
(I*2^(-m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*
(1 + E^((2*I)*(c + d*x)))^(-m + n)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*
x)))^m*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m + n, -1/2*m + n, 1 - m/2 +
n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(m - 2*n)*Cos[c + d*
x]^m*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

3.688.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(e\cos(c+dx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^n dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^n dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{n-\frac{m}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{n-\frac{m}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+a}{d}$$

$$\Big\downarrow 80$$

$$\frac{a2^{-\frac{m}{2}+n-1}(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}\int\left(\frac{1}{2}i\tan(c+\right.}{d}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{n-\frac{m}{2}}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1}{2}(m-2n+2\right.}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

output
```
((-I)*2^(-1/2*m + n)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (2 + m -
 2*n)/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((m - 2*n)/
2)*(a + I*a*Tan[c + d*x])^n)/(d*m)
```

3.688.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.688.4 Maple [F]

$$\int (e\cos(dx+c))^{m} (a+ia\tan(dx+c))^{n} dx$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

3.688.5 Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos(dx+c))^{m} (i\,a\tan(dx+c)+a)^{n}\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*m*x
+ I*c*m + m*log(a*e) - m*log(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c)
+ 1))), x)
```

3.688.6 Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx=\int (e\cos{(c+dx)})^m\,(ia(\tan{(c+dx)}-i))^n\,dx$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

output
```
Integral((e*cos(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

3.688.7 Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx=\int (e\cos{(dx+c)})^m\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

3.688.8 Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx=\int (e\cos{(dx+c)})^m\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

3.688.9 Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx=\int (e\,\cos(c+d\,x))^m\,(a+a\tan(c+d\,x)\,\,1\mathrm{i})^n\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

3.689 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx$

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3.689.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan}{dm}$$

output
```
-I*2^(2-1/2*m)*a^2*(e*cos(d*x+c))^m*hypergeom([-1/2*m, -1+1/2*m],[1-1/2*m]
,1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

3.689.2 Mathematica [A] (verified)

Time = 9.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-\frac{m}{2},1-\frac{m}{2},-\frac{1}{2}i(i+\tan(c+dx))\right)(1+i\tan}{dm}$$

input
```
Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -1/
2*m, 1 - m/2, (-1/2*I)*(I + Tan[c + d*x])]*(1 + I*Tan[c + d*x])^(m/2))/(d*
m)
```

3.689.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2(e\cos(c+dx))^mdx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{2-\frac{m}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{2-\frac{m}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+a}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^3 2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{1-\frac{m}{2}}(a-ia\tan(c+}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia^2 2^{2-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(\frac{m-2}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -1/
2*m, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

3.689.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.689.4 Maple [F]

$$\int (e\cos (dx+c))^{m} (a+ia\tan (dx+c))^{2} dx$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

3.689.5 Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int (i\,a\tan (dx+c)+a)^2(e\cos (dx+c))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a^2*e^(4*I
*d*x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

3.689.6 Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx=-a^2\left(\int (-(e\cos (c+dx))^m)\,dx+\int (e\cos (c+dx))^m\tan^2 (c+dx)\,dx+\int (-2i(e\cos (c+dx))^m\tan (c+dx))\,dx\right)$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

output
```
-a**2*(Integral(-(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*ta
n(c + d*x)**2, x) + Integral(-2*I*(e*cos(c + d*x))**m*tan(c + d*x), x))
```

3.689.7 Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int (i\,a\tan\left(dx+c\right)+a)^2(e\cos\left(dx+c\right))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

3.689.8 Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int (i\,a\tan\left(dx+c\right)+a)^2(e\cos\left(dx+c\right))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

3.689.9 Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx=\int \left(e\,\cos\left(c+d\,x\right)\right)^m\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

3.690 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx$

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3.690.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = -\frac{i2^{1-\frac{m}{2}}a(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m}}{dm}$$

output
```
-I*2^(1-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([1/2*m, -1/2*m],[1-1/2*m],1/2-
1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

3.690.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = -\frac{a(e\cos(c+dx))^m\left(i(1+m)+m\cot(c+dx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c}\right.}{dm(1+m)}$$

input
```
Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
-((a*(e*Cos[c + d*x])^m*(I*(1 + m) + m*Cot[c + d*x]*Hypergeometric2F1[1/2,
 (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(d*m*(1 + m)
))
```

3.690.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+ia\tan(c+dx))(e\cos(c+dx))^m dx$$

$$\Big\downarrow \ 3998$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)dx$$

$$\Big\downarrow \ 3042$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}(i\tan(c+dx)a+a)dx$$

$$\Big\downarrow \ 3986$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{1-\frac{m}{2}}dx$$

$$\Big\downarrow \ 3042$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{1-\frac{m}{2}}dx$$

$$\Big\downarrow \ 4006$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+a}{d}$$

$$\Big\downarrow \ 80$$

$$\frac{a^2 2^{-m/2}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{-m/2}(a-ia\tan(c}{d}$$

$$\Big\downarrow \ 79$$

$$-\frac{ia2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(1 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, m/2, 1 -
m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

3.690.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.690.4 Maple [F]

$$\int \left(e \cos \left(d x +c \right)\right)^{m} \left(a +i a \tan \left(d x +c \right)\right)d x$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

3.690.5 Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx=\int (i\,a\tan\left(dx+c\right)+a)(e\cos\left(dx+c\right))^{m}\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a*e^(2*I*d
*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

3.690.6 Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx=ia\left(\int\left(-i(e\cos\left(c+dx\right))^{m}\right)\,dx+\int(e\cos\left(c+dx\right))^{m}\tan\left(c+dx\right)dx\right)$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*ta
n(c + d*x), x))
```

3.690.7 Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx=\int (i\,a\tan\left(dx+c\right)+a)(e\cos\left(dx+c\right))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

3.690.8 Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx=\int (i\,a\tan\left(dx+c\right)+a)(e\cos\left(dx+c\right))^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

3.690.9 Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx=\int (e\,\cos\left(c+d\,x\right))^m\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

3.691 $\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

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3.691.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i2^{-1-\frac{m}{2}}(e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{4+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{adm}$$

output
```
-I*2^(-1-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 2+1/2*m],[1-1/2*m],1/2
-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a/d/m
```

3.691.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 433 vs. 2(86) = 172.

Time = 7.52 (sec) , antiderivative size = 433, normalized size of antiderivative = 5.03

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{2^{-m/2}\cos(c+dx)(e\cos(c+dx))^m\left(1-2\cos^2(c+dx)+i\sin(2(c+dx))\right)^{m/2}\left(2^{m/2}(2+m)\,\text{Hypergeom}\right.}{ad(1+m)(2+m)\left(-i\sin(c+dx)\left(\left(1-2\cos^2(c+dx)+i\sin(2(c+dx))\right)^{m/2}\left((\cos(c)+i\sin\right.\right.\right.}$$

input
```
Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output

```
-((Cos[c + d*x]*(e*Cos[c + d*x])^m*(1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*
x)])^(m/2)*(2^(m/2)*(2 + m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, 2*C
os[c + d*x]*(Cos[c + d*x] - I*Sin[c + d*x])]*((Cos[c] - I*Sin[c])*Sin[c]*(
I + Tan[d*x]))^(m/2) - 2*(1 + m)*Hypergeometric2F1[-1 - m/2, m/2, -1/2*m,
(1 + I*Tan[c + d*x])/2]*((Cos[c] + I*Sin[c])*Sin[c]*(-I + Tan[d*x]))^(m/2)
*(1 - I*Tan[c + d*x])^(m/2)))/(2^(m/2)*a*d*(1 + m)*(2 + m)*((-I)*Sin[c + d
*x]*((1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*x)])^(m/2)*((Cos[c] + I*Sin[c]
)*Sin[c]*(-I + Tan[d*x]))^(m/2) - ((Cos[c] - I*Sin[c])*Sin[c]*(I + Tan[d*x
]))^(m/2)) + Cos[c + d*x]*((1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*x)])^(m/
2)*((Cos[c] + I*Sin[c])*Sin[c]*(-I + Tan[d*x]))^(m/2) + ((Cos[c] - I*Sin[c
])*Sin[c]*(I + Tan[d*x]))^(m/2)))*(-I + Tan[c + d*x])))
```

3.691.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-1}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-1}dx$$

$$\Big\downarrow 4006$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+a}{d}$$

$$\Big\downarrow 80$$

$$\frac{2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{-\frac{m}{2}-2}(a-ia\tan(c+}{d}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{-\frac{m}{2}-1}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+4}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

input
```
Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

output
```
((-I)*2^(-1 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (4 + m)/2,
 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a*d*m)
```

3.691.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.691.4 Maple [F]

$$\int \frac{(e\cos(dx+c))^m}{a+ia\tan(dx+c)}dx$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

output
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

3.691.5 Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\cos(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(1/2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(2*I*
d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

3.691.6 Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\cos(c+dx))^m}{\tan(c+dx)-i}\,dx}{a}$$

input
```
integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*Integral((e*cos(c + d*x))**m/(tan(c + d*x) - I), x)/a
```

3.691.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.691.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\cos(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

3.691.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx=\int \frac{{\left(e\,\cos\left(c+d\,x\right)\right)}^m}{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

3.692 $\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

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3.692.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-2-\frac{m}{2}}(e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{6+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{a^2dm}$$

output
```
-I*2^(-2-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3+1/2*m],[1-1/2*m],1/2
-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a^2/d/m
```

3.692.2 Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-m/2}(e\cos(c+dx))^m \,\text{Hypergeometric2F1}\left(-2-\frac{m}{2},\frac{2+m}{2},-1-\frac{m}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(1-i\tan(c}{a^2d(4+m)(-i+\tan(c+dx))^2}$$

input
```
Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-2 - m/2, (2 + m)/2, -1 - m/2,
(1 + I*Tan[c + d*x])/2]*(1 - I*Tan[c + d*x])^(m/2))/(2^(m/2)*a^2*d*(4 + m)
*(-I + Tan[c + d*x])^2)
```

3.692.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{(i\tan(c+dx)a+a)^2}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-2}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{-\frac{m}{2}-2}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+a)}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{2^{-\frac{m}{2}-3}(1+i\tan(c+dx))^{m/2}(a-ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int \left(\frac{1}{2}i\tan(c+dx)+\frac{1}{2}\right)^{-\frac{m}{2}-3}(a-ia\tan(c+}{ad}$$

$$\Big\downarrow \text{79}$$

$$-\frac{i2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+6}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

input
```
Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-I)*2^(-2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (6 + m)/2,
 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a^2*d*m)
```

3.692.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.692.4 Maple [F]

$$\int \frac{(e\cos(dx+c))^m}{(a+ia\tan(dx+c))^2}dx$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

output
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

3.692.5 Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
integral(1/4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(4*I*
d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

3.692.6 Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\cos(c+dx))^m}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-Integral((e*cos(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

3.692.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.692.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

3.692.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos(c+d\,x))^m}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

3.693 $\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$

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3.693.1 Optimal result

Integrand size = 28, antiderivative size = 105

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{dm\sqrt{a+ia\tan(c+dx)}}$$

output
```
-I*2^(-1/2*m+1/2)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1/2+1/2*m],[1-1/2*
m],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2+1/2*m)/d/m/(a+I*a*tan(d*x+c
))^(1/2)
```

3.693.2 Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i2^{-m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}-m}\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1-m}{2},\frac{3-m}{2},-e^{2i(c+dx)}\right)\sqrt{}}{d(-1+m)}$$

input
```
Integrate[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*(1 + E^((2*I)*(c + d*x)))^(1/2 - m)*((e*(1 + E^((2*I)*(c + d*x))))/E^(I
*(c + d*x)))^m*Hypergeometric2F1[1/2 - m, (1 - m)/2, (3 - m)/2, -E^((2*I)*
(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])/(2^m*d*(-1 + m))
```

3.693.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int (e\sec(c+dx))^{-m}\sqrt{i\tan(c+dx)a+a}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1-m}{2}}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1-m}{2}}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+a}{d}$$

$$\Big\downarrow \text{80}$$

$$\frac{a^2 2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan}{d}$$

$$\Big\downarrow \text{79}$$

$$-\frac{ia2^{\frac{1}{2}-\frac{m}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+1}{2},}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(1/2 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (1 + m)
/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2)*(a +
 I*a*Tan[c + d*x])^((-1 - m)/2 + m/2))/(d*m)
```

3.693.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.693. $\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.693.4 Maple [F]

$$\int (e\cos (dx+c))^{m} \sqrt{a+ia\tan (dx+c)}dx$$

input
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

3.693.5 Fricas [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan (dx+c)+a}(e\cos (dx+c))^{m}\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

3.693.6 Sympy [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int (e\cos{(c+dx)})^m\,\sqrt{ia\,(\tan{(c+dx)}-i)}\,dx$$

input
```
integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*cos(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.693.7 Maxima [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan{(dx+c)}+a}(e\cos{(dx+c)})^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

3.693.8 Giac [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan{(dx+c)}+a}(e\cos{(dx+c)})^m\,dx$$

input
```
integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

3.693.9 Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx=\int (e\,\cos\left(c+d\,x\right))^m\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.694 $\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

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3.694.1 Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i2^{-\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{3+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{dm\sqrt{a+ia\tan(c+dx)}}$$

output
```
-I*2^(-1/2-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3/2+1/2*m],[1-1/2*m]
,1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2+1/2*m)/d/m/(a+I*a*tan(d*x+c))
^(1/2)
```

3.694.2 Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i4^{-m}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)\,\mathrm{Hypergeometric}}{d(1+m)\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*(1 + E^((2*I)*(c + d*x)))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m
*((e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Hypergeometric2F1[1, (2
 + m)/2, (1 - m)/2, -E^((2*I)*(c + d*x))])/(4^m*d*(1 + m)*Cos[c + d*x]^m*S
qrt[a + I*a*Tan[c + d*x]])
```

3.694.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3998, 3042, 3986, 3042, 4006, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3998}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$(e\cos(c+dx))^m(e\sec(c+dx))^m\int \frac{(e\sec(c+dx))^{-m}}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3986}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1}{2}(-m-1)}dx$$

$$\Big\downarrow \text{3042}$$

$$(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int (a-ia\tan(c+dx))^{-m/2}(i\tan(c+dx)a+a)^{\frac{1}{2}(-m-1)}dx$$

$$\Big\downarrow \text{4006}$$

$$\frac{a^2(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}(e\cos(c+dx))^m\int(a-ia\tan(c+dx))^{-\frac{m}{2}-1}(i\tan(c+dx)a+a}{d}$$

$$\Big\downarrow 80$$

$$\frac{a2^{-\frac{m}{2}-\frac{3}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\int\left(\frac{1}{2}i\tan(\right.}{d}$$

$$\Big\downarrow 79$$

$$-\frac{i2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(a+ia\tan(c+dx))^{\frac{1}{2}(-m-1)+\frac{m}{2}}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+3}{2},\right.}{dm}$$

input
```
Int[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*2^(-1/2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (3 + m)/
2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2)*(a +
I*a*Tan[c + d*x])^((-1 - m)/2 + m/2))/(d*m)
```

3.694.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3986
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Simp[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2))   Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*
Tan[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
 b^2, 0]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4006
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

3.694.4 Maple [F]

$$\int \frac{(e\cos(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}dx$$

input
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

3.694.5 Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(1/2*sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*
c)/a, x)
```

3.694.6 Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\cos(c+dx))^m}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((e*cos(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

3.694.7 Maxima [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*cos(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

3.694.8 Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

input
```
integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*cos(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(e\,\cos\left(c+d\,x\right))^m}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.695 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx$

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3.695.1 Optimal result

Integrand size = 23, antiderivative size = 183

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{a(3b^2-a^2(1-m))\,(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\,\mathrm{t}}{f(1-m)}$$

$$+\frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)}$$

$$+\frac{b(d\cos(e+fx))^m\,(2(b^2-a^2(3-m))\,(1-m)+ab(4-m)m\tan(e+fx))}{fm\,(2-3m+m^2)}$$

output
```
-a*(3*b^2-a^2*(1-m))*(d*cos(f*x+e))^m*hypergeom([1/2, 1+1/2*m],[3/2],-tan(
f*x+e)^2)*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/f/(1-m)+b*(d*cos(f*x+e))^m*(a+
b*tan(f*x+e))^2/f/(2-m)+b*(d*cos(f*x+e))^m*(2*(b^2-a^2*(3-m))*(1-m)+a*b*(4
-m)*m*tan(f*x+e))/f/(1-m)/(2-m)/m
```

3.695.2 Mathematica [A] (verified)

Time = 6.57 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.16

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx$$

$$=\frac{\cos(e+fx)(d\cos(e+fx))^m\left(-\frac{b^3}{-2+m}+\frac{b(-3a^2+b^2)\cos^2(e+fx)}{m}-\frac{a(a^2-3b^2)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\right.}{(1+m)\sqrt{\sin^2(e+fx)}}\right.}{f(a\cos(e+fx)+b\,\mathrm{si}}$$

input

```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output

```
(Cos[e + f*x]*(d*Cos[e + f*x])^m*(-(b^3/(-2 + m)) + (b*(-3*a^2 + b^2)*Cos[
e + f*x]^2)/m - (a*(a^2 - 3*b^2)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/((1 + m)*Sqrt[Sin[e + f*x
]^2]) - (3*a*b^2*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[e + f*x
]^2]*Sin[2*(e + f*x)])/(2*(-1 + m)*Sqrt[Sin[e + f*x]^2]))*(a + b*Tan[e + f
*x])^3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

3.695.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3998, 3042, 3994, 497, 25, 27, 676, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^3(d\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\tan(e+fx))^3(d\cos(e+fx))^m dx$$

$$\Big\downarrow \text{ 3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \text{ 3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^3dx$$

$$\Big\downarrow \text{ 3994}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\int (a+b\tan(e+fx))^3\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \text{ 497}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b^2\int -\frac{(a+b\tan(e+fx))\left(b^2\left(2-\frac{a^2(2-m)}{b^2}\right)-ab(4-m)\tan(e+fx)\right)\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{b^2}d(b\tan(e+fx))}{2-m}}{bf}$$

$$\Big\downarrow 25$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{b^2\int\frac{(a+b\tan(e+fx))\left(-\left((2-m)a^2\right)-b(4-m)\tan(e+fx)a}{b^2}}{2-m}\right.}{bf}$$

$$\Big\downarrow 27$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{\int(a+b\tan(e+fx))(-((2-m)a^2)-b(4-m)\tan(e+fx}{2-m}\right.}{bf}$$

$$\Big\downarrow 676$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{\frac{a(2-m)\left(3b^2-a^2(1-m)\right)\int\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}d(b\tan}{1-m}}{}\right.}{bf}$$

$$\Big\downarrow 237$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b^2\left(\tan^2(e+fx)+1\right)^{-m/2}(a+b\tan(e+fx))^2}{2-m}-\frac{\frac{ab(2-m)\left(3b^2-a^2(1-m)\right)\tan(e+fx)\,\text{Hypergeometric2F1}(}{1-m}}{}\right.}{bf}$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

output
```
((d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*((b^2*(a + b*Tan[e + f*x])^2)/(
(2 - m)*(1 + Tan[e + f*x]^2)^(m/2)) - ((a*b*(3*b^2 - a^2*(1 - m))*(2 - m)*
Hypergeometric2F1[1/2, (2 + m)/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(1 -
 m) - (2*b^2*(b^2 - a^2*(3 - m)))/(m*(1 + Tan[e + f*x]^2)^(m/2)) - (a*b^3*
(4 - m)*Tan[e + f*x])/((1 - m)*(1 + Tan[e + f*x]^2)^(m/2)))/(2 - m)))/(b*f
)
```

3.695. $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx$

3.695.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.695.4 Maple [F]

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))^3\,dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

3.695.5 Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan(fx+e)+a)^3(d\cos(fx+e))^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x +
e) + a^3)*(d*cos(f*x + e))^m, x)
```

3.695.6 Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (d\cos(e+fx))^m (a+b\tan(e+fx))^3\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

3.695.7 Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int (b\tan(fx+e)+a)^3(d\cos(fx+e))^m\,dx$$

input

```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

output

```
integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

3.695.8 Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int (b\tan(fx+e)+a)^3(d\cos(fx+e))^m\,dx$$

input

```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

output

```
integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

3.695.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx=\int (d\,\cos(e+f\,x))^m\,(a+b\tan(e+f\,x))^3\,dx$$

input

```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
```

output

```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

3.696 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$

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3.696.1 Optimal result

Integrand size = 23, antiderivative size = 155

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m}$$
$$+\frac{(b^2-a^2(1-m))\cos(e+fx)(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)}$$

output
```
-a*b*(2-m)*(d*cos(f*x+e))^m/f/(1-m)/m+(b^2-a^2*(1-m))*cos(f*x+e)*(d*cos(f*
x+e))^m*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/f/
(-m^2+1)/(sin(f*x+e)^2)^(1/2)+b*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))/f/(1-m)
```

3.696.2 Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$$
$$=\frac{(d\cos(e+fx))^m\left(2ab\left(-1+\sec^2(e+fx)^{m/2}\right)+b^2m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e}{}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Cos[e + f*x])^m*(2*a*b*(-1 + (Sec[e + f*x]^2)^(m/2)) + b^2*m*Hypergeom
etric2F1[1/2, m/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*
x] + (a^2 - b^2)*m*Hypergeometric2F1[1/2, (2 + m)/2, 3/2, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]))/(f*m)
```

3.696.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3998, 3042, 3993, 25, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))^2(d\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))^2(d\cos(e+fx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^2dx$$

$$\Big\downarrow \text{3993}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\Bigg(\frac{\int -(d\sec(e+fx))^{-m}\left(-\left((1-m)a^2\right)-b(2-m)\tan(e+fx)a+b^2\right)dx}{1-m}+\frac{b(a+b\tan(e+fx))(d\sec(e+}{f(1-m)}$$

$$\Big\downarrow \text{25}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\Bigg(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{\int (d\sec(e+fx))^{-m}\left(-\left((1-m)a^2\right)-b(2-m)\tan(e+fx)a+\right.}{1-m}$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{\int(d\sec(e+fx))^{-m}\left(-((1-m)a^2)-b(2-m)\tan(e+fx)a+\right.}{1-m}\right.$$

$$\Big\downarrow \text{ 3967}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\int(d\sec(e+fx))^{-m}dx+\frac{ab(2-m)(d\sec(e+fx))^{-}}{fm}}{1-m}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\int\left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}dx+\frac{ab(2-m)(d\sec(e}{fm}}{1-m}\right.$$

$$\Big\downarrow \text{ 4259}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int\left(\frac{\cos(e+fx}{d}\right.}{1-m}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{(b^2-a^2(1-m))\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int\left(\frac{\sin(e+fx}{d}\right.}{1-m}\right.$$

$$\Big\downarrow \text{ 3122}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(\frac{b(a+b\tan(e+fx))(d\sec(e+fx))^{-m}}{f(1-m)}-\frac{\frac{ab(2-m)(d\sec(e+fx))^{-m}}{fm}-\frac{d(b^2-a^2(1-m))\sin(e+fx)(d\sec(e+fx))^{-m-1}\text{H}}{f(m+1)\sqrt{\sin}}}{1-m}\right.$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

output
```
(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m*(-(((a*b*(2 - m))/(f*m*(d*Sec[e + f*
x])^m) - (d*(b^2 - a^2*(1 - m))*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 - m)*Sin[e + f*x])/(f*(1 + m)*Sqrt
[Sin[e + f*x]^2]))/(1 - m)) + (b*(a + b*Tan[e + f*x]))/(f*(1 - m)*(d*Sec[e
 + f*x])^m))
```

3.696. $\int(d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$

3.696.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3993
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Simp[1/(m + 1)   Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 +
a*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^
2 + b^2, 0] &&  !IntegerQ[m]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.696.4 Maple [F]

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))^2\,dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

3.696.5 Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan(fx+e)+a)^2(d\cos(fx+e))^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*cos(f*x + e))^
m, x)
```

3.696.6 Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (d\cos(e+fx))^m (a+b\tan(e+fx))^2\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

3.696.7 Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan{(fx+e)}+a)^2(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

3.696.8 Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan{(fx+e)}+a)^2(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

3.696.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int {\left(d\,\cos\left(e+f\,x\right)\right)}^m\,{\left(a+b\,\tan(e+f\,x)\right)}^2\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

3.697 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx$

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3.697.1 Optimal result

Integrand size = 21, antiderivative size = 90

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = -\frac{b(d\cos(e+fx))^m}{fm} - \frac{a(d\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{df(1+m)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*(d*cos(f*x+e))^m/f/m-a*(d*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],
[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+m)/(sin(f*x+e)^2)^(1/2)
```

3.697.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = -\frac{(d\cos(e+fx))^m\left(b+bm+am\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt{\sin^2(e}\right.}{fm(1+m)}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
-(((d*Cos[e + f*x])^m*(b + b*m + a*m*Cot[e + f*x]*Hypergeometric2F1[1/2, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]))/(f*m*(1 + m)))
```

3.697.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3998, 3042, 3967, 3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(e+fx))(d\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(e+fx))(d\cos(e+fx))^m dx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))dx$$

$$\Big\downarrow \text{3967}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\int (d\sec(e+fx))^{-m}dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\int \left(d\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{-m}dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \text{4259}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int \left(\frac{\cos(e+fx)}{d}\right)^m dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\sec(e+fx))^{-m}\int\left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{d}\right)^m dx-\frac{b(d\sec(e+fx))^{-m}}{fm}\right)$$

$$\Big\downarrow \; 3122$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\left(-\frac{ad\sin(e+fx)(d\sec(e+fx))^{-m-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}}-\frac{b(d\sec(e+fx)}{fm}\right.$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

output
```
(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m*(-(b/(f*m*(d*Sec[e + f*x])^m)) - (a*
d*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(d*Sec[e +
f*x])^(-1 - m)*Sin[e + f*x])/(f*(1 + m)*Sqrt[Sin[e + f*x]^2]))
```

3.697.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3967
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Simp[a   Int[(d
*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m]
|| NeQ[a^2 + b^2, 0])
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

3.697.4 Maple [F]

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))\,dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

3.697.5 Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\cos(fx+e))^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

3.697.6 Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (d\cos(e+fx))^m (a+b\tan(e+fx))\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

3.697.7 Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan{(fx+e)}+a)(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

3.697.8 Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan{(fx+e)}+a)(d\cos{(fx+e)})^m\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

3.697.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (d\,\cos{(e+f\,x)})^m\,(a+b\tan(e+f\,x))\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

3.698 $\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$

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3.698.1 Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \frac{b(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(1,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)\,fm} + \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)}{af}$$

output
```
b*(d*cos(f*x+e))^m*hypergeom([1, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a^2+b
^2))/(a^2+b^2)/f/m+AppellF1(1/2,1,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*
x+e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a/f
```

3.698.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.21 (sec) , antiderivative size = 1132, normalized size of antiderivative = 8.09

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

$$= \frac{}{f(a+b\tan(e+fx))\Bigg(am\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)-bm\sec^2(e+}$$

input
```
Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

output
```
((d*Cos[e + f*x])^m*(b*(-1 + (Sec[e + f*x]^2)^(-1/2*m)) + a*m*Hypergeometr
ic2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (b*AppellF1[m, m/
2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*
x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e +
 f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2)))/(f*(a + b*Ta
n[e + f*x])*(a*m*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec
[e + f*x]^2 - (b*m*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (b*m*AppellF1[m,
 m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*Tan[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2
) - (b*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e
 + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1 +
m, 1 + m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*Sec[e + f*x]^2)/((1 + m)*(a + b*Tan[e + f*x])^2) - ((a + I*b
)*b*m^2*AppellF1[1 + m, m/2, 1 + m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]
), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 + m)*(a + b*Tan[e
 + f*x])^2)))/(Sec[e + f*x]^2)^(m/2) - (b*m*AppellF1[m, m/2, m/2, 1 + m, (
a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + T
an[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*((b*(I + Tan[e + f*x]))/(a
+ b*Tan[e + f*x]))^(m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a...
```

3.698.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3998, 3042, 3994, 504, 333, 353, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int\frac{(d\sec(e+fx))^{-m}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \color{blue}{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int\frac{(d\sec(e+fx))^{-m}}{a+b\tan(e+fx)}dx$$

$$\Big\downarrow \color{blue}{3994}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\int\frac{\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{a+b\tan(e+fx)}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \color{blue}{504}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(a\int\frac{\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{a^2-b^2\tan^2(e+fx)}d(b\tan(e+fx))-\int\frac{b\tan(e+fx)\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{a^2-b^2\tan^2(e+fx)}d(b\,\mathrm{ta}\right.}{bf}$$

$$\Big\downarrow \color{blue}{333}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\int\frac{b\tan(e+fx)\left(\tan^2(e+fx)+\right.}{a^2-b^2\tan^2(e+fx}\right.}{bf}$$

$$\Big\downarrow \color{blue}{353}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}-\frac{1}{2}\int\frac{\left(\frac{\tan(e+fx)}{b}+1\right)^{-\frac{m}{2}-1}}{a^2-b^2\tan^2(e+fx)}d\right.}{bf}$$

$$\Big\downarrow \color{blue}{78}$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\left(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a}+\frac{b^2\left(\frac{\tan(e+fx)}{b}+1\right)^{-m/2}\mathrm{Hype}}{}\right.}{bf}$$

input
```
Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

3.698. $\int\frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$

output

```
((d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*((b*AppellF1[1/2, 1, (2 + m)/2,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a + (b^2*Hy
pergeometric2F1[1, -1/2*m, 1 - m/2, (b^2 + b^2*Tan[e + f*x]^2)/(a^2 + b^2)
])/((a^2 + b^2)*m*(1 + Tan[e + f*x]/b)^(m/2))))/(b*f)
```

3.698.3.1 Defintions of rubi rules used

rule 78

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 353

```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

3.698. $\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.698.4 Maple [F]

$$\int \frac{(d\cos(fx+e))^m}{a+b\tan(fx+e)}dx$$

input
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

output
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

3.698.5 Fricas [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.698.6 Sympy [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

input
```
integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e)),x)
```

output
```
Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

3.698.7 Maxima [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.698.8 Giac [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.698.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\,\cos(e+f\,x))^m}{a+b\tan(e+f\,x)}\,dx$$

input
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)),x)
```

output
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)), x)
```

3.699 $\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

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3.699.1 Optimal result

Integrand size = 23, antiderivative size = 227

$$\begin{aligned}\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx &= \frac{2ab(d\cos(e+fx))^m \operatorname{Hypergeometric2F1}\left(2,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)^2 fm} \\ &\quad + \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)}{a^2f} \\ &\quad + \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{2+m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan^3(e+fx)}{3a^4f}\end{aligned}$$

output

```
2*a*b*(d*cos(f*x+e))^m*hypergeom([2, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a
^2+b^2))/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-
tan(f*x+e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a^2/f+1/3
*b^2*AppellF1(3/2,2,1+1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*cos
(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)^3/a^4/f
```

3.699.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.71 (sec) , antiderivative size = 2502, normalized size of antiderivative = 11.02

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output

```
((d*Cos[e + f*x])^m*((2*a*b*(-1 + (Sec[e + f*x]^2)^(-1/2*m)))/m + a^2*Hype
rgeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - b^2*Hyper
geometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (2*a*b*App
ellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b
*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(
I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(m*(Sec[e + f*x]^2)^(m/2))
 - (b*(a^2 + b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e
+ f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*T
an[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(
(1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x]))))/(f*(a + b*Tan[e + f
*x])^2*(a^2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e +
f*x]^2 - b^2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e +
 f*x]^2 - (2*a*b*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (2*a*b*AppellF1[m,
 m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*Tan[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2
) + (b^2*(a^2 + b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan
[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(1 - m/2)*((b
*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/
(a + b*Tan[e + f*x]))^(m/2))/((1 + m)*(a + b*Tan[e + f*x])^2) + (b*(a^2...
```

3.699.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3998, 3042, 3994, 505, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3998$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int \frac{(d\sec(e+fx))^{-m}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3042$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int \frac{(d\sec(e+fx))^{-m}}{(a+b\tan(e+fx))^2}dx$$

$$\Big\downarrow \; 3994$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\int \frac{(\tan^2(e+fx)+1)^{-\frac{m}{2}-1}}{(a+b\tan(e+fx))^2}d(b\tan(e+fx))}{bf}$$

$$\Big\downarrow \; 505$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\int \Bigg(\frac{a^2\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{\left(a^2-b^2\tan^2(e+fx)\right)^2}-\frac{2ab\tan(e+fx)\left(\tan^2(e+fx)+1\right)^{-\frac{m}{2}-1}}{\left(a^2-b^2\tan^2(e+fx)\right)^2}+\frac{b^2\tan^2(e+fx)(\tan^2(e+}{(b^2\tan^2(e+fx)}}{bf}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\Bigg(\frac{b\tan(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2}+\frac{2ab^2\left(\tan^2(e+fx)+1\right)^{-m/2}\,\mathrm{H}}{}}{bf}$$

input
```
Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

output
```
((d*Cos[e + f*x])^m*(Sec[e + f*x]^2)^(m/2)*((b*AppellF1[1/2, 2, (2 + m)/2,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*Tan[e + f*x])/a^2 + (b^3*
AppellF1[3/2, 2, (2 + m)/2, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2
]*Tan[e + f*x]^3)/(3*a^4) + (2*a*b^2*Hypergeometric2F1[2, -1/2*m, 1 - m/2,
 (b^2 + b^2*Tan[e + f*x]^2)/(a^2 + b^2)])/((a^2 + b^2)^2*m*(1 + Tan[e + f*
x]^2)^(m/2))))/(b*f)
```

3.699. $\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}dx$

3.699.3.1 Defintions of rubi rules used

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3994
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]))   Subst[Int[(a + x)^n*(1 +
x^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.699.4 Maple [F]

$$\int \frac{(d\cos (fx+e))^m}{(a+b\tan (fx+e))^2}dx$$

input
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

output
```
int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

3.699.5 Fricas [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\cos{(fx+e)})^m}{(b\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2
), x)
```

3.699.6 Sympy [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\cos{(e+fx)})^m}{(a+b\tan{(e+fx)})^2}\,dx$$

input
```
integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
```

output
```
Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

3.699.7 Maxima [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\cos{(fx+e)})^m}{(b\tan{(fx+e)}+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

3.699.8 Giac [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\cos(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

3.699.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\,\cos(e+f\,x))^m}{(a+b\tan(e+f\,x))^2}\,dx$$

input
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
```

output
```
int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

3.700 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$

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3.700.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$= \frac{\text{AppellF1}\left(1+n,\frac{2+m}{2},\frac{2+m}{2},2+n,\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos^2(e+fx)(d\cos(e+fx))^m(a+b\tan(e+}{bf(1+n)}$$

output
```
AppellF1(1+n,1+1/2*m,1+1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*ta
n(f*x+e))/(a+(-b^2)^(1/2)))*cos(f*x+e)^2*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))
^(1+n)*(1+(-a-b*tan(f*x+e))/(a-(-b^2)^(1/2)))^(1+1/2*m)*(1+(-a-b*tan(f*x+e
))/(a+(-b^2)^(1/2)))^(1+1/2*m)/b/f/(1+n)
```

3.700.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.41 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.95

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$= \frac{2(a-ib)(a+ib)(2+n)\,\text{AppellF1}\left(1+n,1+\frac{m}{2},1\right.}{bf(1+n)\left(2\left(a^2+b^2\right)(2+n)\,\text{AppellF1}\left(1+n,1+\frac{m}{2},1+\frac{m}{2},2+n,\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)+(2+m\right.}$$

input
```
Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a
 + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Cos[e + f*x]
*(d*Cos[e + f*x])^m*(a*Cos[e + f*x] + b*Sin[e + f*x])*(a + b*Tan[e + f*x])
^n)/(b*f*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1 + m/2, 1 + m/2,
2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)] + (
2 + m)*((a - I*b)*AppellF1[2 + n, 1 + m/2, 2 + m/2, 3 + n, (a + b*Tan[e +
f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 +
n, 2 + m/2, 1 + m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e +
 f*x])/(a + I*b)])*(a + b*Tan[e + f*x])))
```

3.700.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3998, 3042, 3995, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3998}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3042}$$

$$(d\cos(e+fx))^m(d\sec(e+fx))^m\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3995}$$

$$\frac{b(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{m/2}\int (a+b\tan(e+fx))^n\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{-\frac{m}{2}-1}(}{f\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{150}$$

$$\frac{b(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{m/2}(a+b\tan(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{m+2}{2},\right.}{f(n+1)\left(a^2+b^2\right)}$$

input
```
Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

output
```
(b*AppellF1[1 + n, (2 + m)/2, (2 + m)/2, 2 + n, (a + b*Tan[e + f*x])/(a -
Sqrt[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*(d*Cos[e + f*x])^m*(a
+ b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b^2]))^(m/2
)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(m/2))/((a^2 + b^2)*f*(1 + n
))
```

3.700.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3995
```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^(2*IntPart[m/2])*(a^2 + b^2)^(IntPart[m/2]
 - 1)*((d*Sec[e + f*x])^(2*FracPart[m/2])/(f*b^(2*IntPart[m/2] - 1)*(1 - (a
 + b*Tan[e + f*x])/(a - Rt[-b^2, 2]))^FracPart[m/2]*(1 - (a + b*Tan[e + f*x]
)/(a + Rt[-b^2, 2]))^FracPart[m/2]))   Subst[Int[x^n*(1 - x/(a - Rt[-b^2, 2
]))^(m/2 - 1)*(1 - x/(a + Rt[-b^2, 2]))^(m/2 - 1), x], x, a + b*Tan[e + f*x
]], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 + b^2, 0] &&  !Integer
Q[m] &&  !IntegerQ[n]
```

rule 3998
```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Simp[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m   Int[(
a + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] &&  !IntegerQ[m]
```

3.700.4 Maple [F]

$$\int (d\cos (fx+e))^{m} (a+b\tan (fx+e))^{n}dx$$

input
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

output
```
int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

3.700.5 Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\cos (fx+e))^{m} (b\tan (fx+e)+a)^{n}\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

3.700.6 Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\cos (e+fx))^{m} (a+b\tan (e+fx))^{n}\,dx$$

input
```
integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
```

output
```
Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

3.700.7 Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\cos(fx+e))^m(b\tan(fx+e)+a)^n\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

3.700.8 Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\cos(fx+e))^m(b\tan(fx+e)+a)^n\,dx$$

input
```
integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

3.700.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\,\cos(e+f\,x))^m\,(a+b\tan(e+f\,x))^n\,dx$$

input
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
```

output
```
int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 4998

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != −1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
   print ("Enter grade_antiderivative for sagemath")
   print("Enter grade_antiderivative, result=",result)
   print("Enter grade_antiderivative, optimal=",optimal)
   print("type(anti)=",type(result))
   print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation =" "
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```